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ジョブのローテーションと特化―動学マッチングモデル分析―

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【要旨】

企業がジョブを労働者に割り当てる際にローテーションと特化のどちらが利潤上望ましいのかという問いに答えるため、本研究は労働者とジョブをマッチさせる動学企業モデルを構築する。まず毎期全ての労働者が訓練中か訓練後という世代間重複のない基本モデルを作り、次に毎期訓練中と訓練後の労働者が混在する世代間重複のあるモデルに拡張する。どちらのモデルにおいても、ローテーションと特化の二つの形態のみが企業の利潤最大化から生じることを示す。さらに、どちらが望ましいかは、将来のジョブの存続に関する不確実性や訓練コストの程度に依存し、年功賃金の度合いが大きくなるほどローテーションが望ましくなる可能性が増えることも示す。

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Job rotation or specialization? A dynamic matching model analysis *

Morimitsu Kurino[†] Yoshinori Kurokawa[‡]

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Abstract

Which works better when making a job assignment in firms, rotation or specialization? To answer this question, we develop a dynamic firm model of matching workers with indivisible jobs as in an overlapping generations (OLG) matching model à la Kurino (2014). First, we build a simple benchmark model without OLG in which all workers are either under-training or fully trained in each period, and then we show that either the rotation or the specialization of jobs for a worker emerges from the firms' profit maximization. We extend the benchmark model to the OLG model in which workers under- and post-training coexist in each period. We show that the profit-maximizing allocation is either a rotation or a specialization in this extended model as well. Hence, in both the benchmark and the extended models, the rotation and specialization schemes are the only variations that can be optimal in terms of profits. Moreover, the rotation scheme is better when the training cost is smaller, the uncertainty about job continuation in the future is larger, or the slope of seniority wages is larger.

Journal of Economic Literature Classification Numbers: C78, P51.

Keywords: Job assignment, rotation, specialization, overlapping generations

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1 Introduction

It is often said that, under globalization, the era of the Japanese system of employment is over and the Japanese system is now converging to the U.S. system of employment.[1] The Japanese employment system is supported by flexible job assignments, lifetime employment, and seniority wages. The flexible job assignment is possible because firms rotate their workers through a variety of jobs in the early stages of the workers' careers. When sales of automobiles decreased in the mid-1980s, for example, Nissan moved those workers in idle production lines to other sections, such as sales, and thus maintained employment without firing its workers. This is known as a typical example of Japanese lifetime employment that is supported by flexible job assignment due to job rotation (Ito, 1992). On the other hand, the U.S. employment system is supported by specialized job assignment, non-lifetime employment, and non-seniority wages. In this system, the shutdown of production lines means firing workers there, not moving them to other sections.

So is the era of the Japanese system really over? The answer, it seems, is no. As Iwai (2014) points out, the 2001 data in Jacoby et al. (2005) show that to the question of "What is important to you in your job? Share price or employees' jobs?" most of the U.S. human resource executives answered "share price" while the Japanese ones answered "employees' jobs." This is also implied by the fact that from the Lehman Shock (2008) to the trough of the subsequent downturn, the increase in the unemployment rate was large in the U.S. but small in Japan. As indicated by these observations, the Japanese system has not ended. Yet why do we still observe both types of systems?

We start with an individual firm's behavior of adjusting job assignments, including job rotation and specialization, to maximize its profit. To discuss job rotation or specialization adequately, we have to take into account the fact that jobs are indivisible in most cases, if not all, and a worker can do one job. Specifically, we build a single-firm's matching model with overlapping generations (OLG) and analyze a problem of dynamically assigning different jobs (indivisible goods) to workers for its profit maximization. In the model, one of the characteristics of the job assignment is that in each period a worker can only do one job, not multiple jobs. Thus, the firm faces an assignment problem of rotating workers through a variety of jobs over time, or to keep assigning one specialized job to him. Since the firm incurs a cost of training workers who do a new job, affecting the profit, the assignment problem is not trivial. Job rotation enables workers to do a variety of jobs but needs more training costs, whereas job specialization needs less training costs but does not enable workers to do

[1]For examples of this debate, see Katz and Darbishire (2000); Hansmann and Kraakman (2001); Jacoby et al. (2005); Sako (2005); Kambayashi and Kato (2012).

Table 1: Specialization and rotation when job B disappears

Specialization	$t=1$	$t=2$	$t=3$ (no B)
Worker 1	A	A	A
Worker 2	B	B	$\emptyset$

Rotation	$t=1$	$t=2$	$t=3$ (no B)
Worker 1	A	B	A
Worker 2	B	A	A

Note: The left table shows the specialization scheme for job assignment for each worker where $\emptyset$ means that worker 2 is not assigned any job. The right table shows the rotation scheme.

a variety of jobs.

We first introduce a simple finite-horizon model without OLG in which all workers are either under- or post-training in each period. Then we show that the job-assignment scheme that can result from a firm's profit maximization is either the rotation scheme or the specialization scheme.

The essence of our model is as follows. Suppose that there are three periods and that a firm has two types of jobs, job A and job B, and employs two workers, worker 1 and worker 2 (see Table 1). There is an uncertainty over whether job B might be discontinued in the third period. Under the scheme of job specialization, in the first two periods job A is assigned to worker 1 (A, A) while job B is given to worker 2 (B, B). Under the job rotation scheme, on the other hand, job A and job B are alternately assigned to worker 1 (A, B) while job B and job A are assigned to worker 2 (B, A). Both workers can thus handle job A and job B in the third period. Here, we assume that any job needs training costs when a worker is first assigned to it. Then, if job B is discontinued in the third period, under the scheme of job specialization, the firm fires worker 2 who is specialized in job B, which corresponds to the U.S. non-lifetime employment. Under the scheme of job rotation, however, the firm does not have to fire either worker because both workers can handle job A due to job rotation, which corresponds to the Japanese lifetime employment.

In this benchmark model, we compare the firm's profits under all possible job allocations. Then, we show that the rotation scheme and the specialization scheme are the only variations that can be optimal in terms of profits, and we also show how selecting which scheme is better depends on the uncertainty about job continuation in the future and the training costs. We note that the larger the slope of seniority wages, or the lower the time discount rate, the larger the possibility that the rotation scheme is better. Thus, this simple model provides a unified explanation for the systems of employment in terms of (1) job assignment, (2) lifetime employment, and (3) wages.

Based on the benchmark model, we introduce the OLG structure in which workers under- and post-training coexist in each period. Then, we show that the profit-maximizing allocation is either a rotation or a specialization. We also show that the qualitative results are basically the same as in the benchmark model and are thus robust.

Thus, our paper makes the following contributions to the literature. The first is on the job rotation literature. According to Campion et al. (1994), previous studies have provided mainly two explanations for job rotation: the employee learning theory and the employee motivation theory. These theories emphasize the effects of job rotation on employees' actions. The former argues that job rotation is beneficial because it fosters employee learning and encourages human capital accumulation, while the latter argues that job rotation raises the employees' motivation and reduces their boredom. On the other hand, Ortega (2001) proposes the firm learning theory that emphasizes the effects of job rotation on firm learning. It argues that job rotation enables firms to learn effectively about their employees. Recently, Brünner et al. (2019) introduced a model of an employee learning about his or her talents in a labor market, and showed that competitive labor markets encourage experimentation (corresponding to job rotation) in learning talents whereas monopsonistic labor markets induce job specialization.[2] In light of these studies, our paper now proposes an alternative firm model to Ortega's.

Second, our paper also makes a contribution to matching theory. Kurino (2014) was the first to develop a dynamic matching model. His model, however, does not include pecuniary transactions, and thus it is not appropriate as a firm model. In this line, our paper is the first to introduce a dynamic matching model with pecuniary aspects to the literature.

Third, our paper also makes a contribution to the argument about the Japanese employment system vs. the U.S. employment system. The Japanese employment system is characterized by job rotation, lifetime employment, and seniority wages, while the U.S. employment system is characterized by job specialization, non-lifetime employment, and non-seniority wages. Carmichael and MacLeod (1993) relate job rotation (resp. specialization) to lifetime employment (resp. non-lifetime employment). Their model shows that under the rotation (resp. specialization) scheme, workers will cooperate (resp. will not cooperate) with labor-saving technological change because they will not be fired (resp. will be fired). On the basis of mutual complementarity, Aoki and Okuno (1996) provide a theoretical answer to the essential question of why lifetime employment and seniority wages should come as a package, as has been evident in Japan (Ito, 1992). We go further and develop a firm model of matching workers with indivisible jobs that can provide a unified explanation for the Japanese and U.S. employment systems in terms of all three characteristics: job rotation/specialization, lifetime/non-lifetime employment, and seniority/non-seniority wages.[3]

[2] On a related note, Anderson (2012) constructs a model of a worker's choice to be a specialist or generalist, and shows that it is rational to be a generalist when there are barriers to working on problems in other disciplines but problems are relatively simple.

[3] In the same spirit, Iwai (1999, 2014) provides a unified theory of the Japanese and U.S. *corporation*

The organization of this paper is as follows. In Section 2 we set up our benchmark model, and then compare the firm's profits under all possible job allocations. Section 3 incorporates the OLG structure to the benchmark model. Section 4 concludes.

2 Benchmark: Finite-period Model

We first introduce a benchmark finite-horizon model without OLG in which all workers are either under-training or are post-training in each period.

2.1 The model

There are three periods, $t \in \{1, 2, 3\}$. There is a firm that has two jobs, $J \in \{A, B\}$. The firm hires two workers, $i \in \{1, 2\}$, and assigns each worker at most one job, i.e., job A, job B, or neither.

We make the following assumptions.

Assumption 1. *For a job $J \in \{A, B\}$, if it is the first time assigned to her, each worker*

- *needs to be trained for job J (on-the-job training) in period $t \in \{1, 2\}$;*
- *cannot have any training at period $t = 3$. Thus, if she has had no experience in job J in the past, she cannot be assigned job J in period 3.*

Assumption 2. *A job $J \in \{A, B\}$ disappears with probability p_J. The probability that no job disappears is $1 - p_A - p_B$. We assume that the two jobs are symmetric, that is, $p := p_A = p_B$.*

Note that there is no chance that both jobs will disappear at the same time.[4]

Assumption 3. *The firm pays w^* to a worker under-training, and w to a worker post-training. We assume $w^* < w$.*

We express an allocation by

$$x = \begin{pmatrix} x^1 \\ x^2 \end{pmatrix} = \begin{pmatrix} x^1_{1,} & x^1_2 & x^1_{3,p_A} & x^1_{3,p_B} & x^1_{3,1-p_A-p_B} \\ x^2_1 & x^2_2 & x^2_{3,p_A} & x^2_{3,p_B} & x^2_{3,1-p_A-p_B} \end{pmatrix}.$$

systems. Aoki and Okuno (1996) also provide a unified explanation for the Japanese and U.S. systems; however, they do not involve job rotation vs. specialization.

[4]This implies that the event of one job loss is not independent of that of another job loss, making the job assignment problem well-defined. Otherwise there would be no room for job assignments.

Here, for each $i \in \{1,2\}$ and each $t \in \{1,2\}$, x_t^i denotes the job assigned to worker i in period t. On the other hand, x_{3,p_A}^i (x_{3,p_B}^i) is the job assigned to worker i in the event of job A (job B) disappearing in period 3 with probability p_A (p_B), while $x_{3,1-p_A-p_B}^i$ is the job to worker i in the event of no job disappearing in period 3 with probability $1-p_A-p_B$. To simplify the notation, we denote an allocation by, for example,

- The rotation scheme

$$x^{R1} = \begin{pmatrix} A^* & B^* & B & A & A/B \\ B^* & A^* & B & A & B/A \end{pmatrix} \text{ or } x^{R2} = \begin{pmatrix} B^* & A^* & B & A & A/B \\ A^* & B^* & B & A & B/A \end{pmatrix}$$

where A^* (B^*) means that job A (B) is assigned to the corresponding worker for the first time, and A/B means the assignment of either job A or job B.

- The specialization scheme:

$$x^{S1} = \begin{pmatrix} A^* & A & \emptyset & A & A \\ B^* & B & B & \emptyset & B \end{pmatrix} \text{ or } x^{S2} = \begin{pmatrix} B^* & B & B & \emptyset & B \\ A^* & A & \emptyset & A & A \end{pmatrix}$$

where $\emptyset$ means that no job is assigned.

We define the production function for each period. Let an allocation x be given. We define $\bar{x}_{t,J}^i$ as the contribution of worker i in period t with job $J \in \{A,B\}$ to the production. Any job needs training for its first assignment. We call it the worker i's **training period** for job $J \in \{A,B\}$. Regarding the training period, we make the following two assumptions. First, period $t=3$ cannot be any worker's training period for each job. Second, in the training period, the contribution is reduced. Formally, for each worker $i \in \{1,2\}$, each period $t \in \{1,2,3\}$, and each job $J \in \{A,B\}$,

$$\bar{x}_{t,J}^i = \begin{cases} 0 & \text{if } x_t^i \neq J, \\ \lambda & \text{if } x_t^i = J \text{ and } t \text{ is agent } i\text{'s training period for job } J, \\ 1 & \text{if } x_i^i = J \text{ and } t \text{ is not agent } i\text{'s training period for job } J, \end{cases} \tag{1}$$

where $0 < \lambda < 1$. The parameter λ indicates the training cost in terms of the labor contribution to production. The larger λ is, the smaller the training cost is. Then, the output, $f_t(x)$, produced by the firm is

$$f_t(x) = \begin{cases} (\bar{x}^1_{t,B} + \bar{x}^2_{t,B})^\alpha & \text{if } t = 3 \text{ and job } A \text{ disappears,} \\ (\bar{x}^1_{t,A} + \bar{x}^2_{t,A})^\alpha & \text{if } t = 3 \text{ and job } B \text{ disappears,} \\ \sqrt{(\bar{x}^1_{t,A} + \bar{x}^2_{t,A})(\bar{x}^1_{t,B} + \bar{x}^2_{t,B})} & \text{otherwise,} \end{cases}$$

where $0 < \alpha < 1$. Note that $\bar{x}$ is a function of x as we define it in (1). The production form is different when one or two jobs are present. For example, when two trained workers are assigned jobs, the output is 1 when both jobs are available; the output is 2^α when only one job is available. It is possible that $1 < 2^\alpha$, which might seem odd at first sight. It is, however, reasonable if we interpret this change in production form as the labor-saving technological change (e.g., Carmichael and MacLeod, 1993).

Assumption 4 (No-free-disposal). $0 < 1 - w < 2^\alpha - 2w$.

Consider the case where in period 3 one job disappears and the other surviving job can be done by both workers. This assumption says that the profit $(2^\alpha - 2w)$ from employing both workers is greater than the one $(1 - w)$ from employing one worker, which is also greater than the profit of zero. This is viewed as a no-free-disposal, since the firm does not dispose of any trained worker for a surviving job. Note that this assumption does not impose no-free-disposal of untrained workers for a surviving job.

2.2 Comparison in profits

We denote the discount factor by δ, $0 < \delta < 1$, and also the profit at an allocation x by $\pi(x)$. Then, we can calculate the profit for the rotation x^{R1} and the specialization x^{S1}.

$$\begin{aligned} \pi(x^{R1}) &= [\lambda - 2w^*] + \delta[\lambda - 2w^*] \\ &\quad + \delta^2[p \cdot 2^\alpha + p \cdot 2^\alpha + (1 - 2p) - \{p \cdot 2w + p \cdot 2w + (1 - 2p) \cdot 2w\}], \\ \pi(x^{S1}) &= [\lambda - 2w^*] + \delta[1 - 2w] \\ &\quad + \delta^2[p + p + (1 - 2p) - \{p \cdot w + p \cdot w + (1 - 2p) \cdot 2w\}]. \end{aligned}$$

Thus, we have the following proposition.

Proposition 1. *For any parameters, α, δ, p, w, w^*, and λ, the profit-maximizing allocation is either the rotation or the specialization. In particular, the profit-maximizing allocation is*

1. *the rotation if $p > \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$;*

k	Allocation x^k	Profit $\pi(x)$	$\pi(x^k)$ is do- -minated by
1	$\begin{pmatrix} A^* & A & \emptyset & A & A \\ A^* & A & \emptyset & A & A \end{pmatrix}$	$(0-2w^*)+\delta(0-2w)+\delta^2[0+2^\alpha p$ $+(1-2p)\cdot 0-\{0+p\cdot 2w+(1-2p)\cdot 2w\}]$	$\pi(x^{R1})$
2	$\begin{pmatrix} A^* & A & \emptyset & A & A \\ A^* & B^* & B & A & B \end{pmatrix}$	$(0-2w^*)+\delta[\lambda^{1/2}-w-w^*]+\delta^2[1\cdot p+2^\alpha\cdot p$ $+(1-2p)-\{pw+p\cdot 2w+(1-2p)\cdot 2w\}]$	Not dominated
3	$\begin{pmatrix} A^* & B^* & B & A & A/B \\ A^* & B^* & B & A & B/A \end{pmatrix}$	$(0-2w^*)+\delta(0-2w^*)+\delta^2[2^\alpha p+2^\alpha p$ $+(1-2p)-\{p\cdot 2w+p\cdot 2w+(1-2p)\cdot 2w\}]$	$\pi(x^{R1})$
4	$\begin{pmatrix} A^* & A & \emptyset & A & A \\ B^* & A^* & B & A & B \end{pmatrix}$	$(\lambda-2w^*)+\delta[0-w-w^*]+\delta^2[p+2^\alpha p$ $+(1-2p)-\{pw+p\cdot 2w+(1-2p)\cdot 2w\}]$	$\pi(x^{R1})$
5	$\begin{pmatrix} A^* & B^* & B & A & A \\ B^* & B & B & \emptyset & B \end{pmatrix}$	$(\lambda-2w^*)+\delta(0-w-w^*)+\delta^2[2^\alpha p+p$ $+(1-2p)-\{p\cdot 2w+p\cdot w+(1-2p)\cdot 2w\}]$	$\pi(x^{R1})$
6	$\begin{pmatrix} B^* & A^* & B & A & A/B \\ B^* & A^* & B & A & B/A \end{pmatrix}$	$(0-2w^*)+\delta(0-2w^*)+\delta^2[2^\alpha p+2^\alpha p$ $+(1-2p)-\{p\cdot 2w+p\cdot 2w+(1-2p)\cdot 2w\}]$	$\pi(x^{R1})$
7	$\begin{pmatrix} B^* & A^* & B & A & A \\ B^* & B & B & \emptyset & B \end{pmatrix}$	$(0-2w^*)+\delta(\lambda^{1/2}-w-w^*)+\delta^2[2^\alpha p+p$ $+(1-2p)-\{p\cdot 2w+p\cdot w+(1-2p)\cdot 2w\}]$	Not dominated
8	$\begin{pmatrix} B^* & B & B & \emptyset & B \\ B^* & B & B & \emptyset & B \end{pmatrix}$	$(0-2w^*)+\delta(0-2w)+\delta^2[2^\alpha p+0$ $+(1-2p)\cdot 0-\{p\cdot 2w+0+(1-2p)\cdot 2w\}]$	$\pi(x^{R1})$

Table 2: Profit calculations

2. *the specialization if* $p < \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$;

3. *the rotation and the specialization if* $p = \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$.

The firm diversifies the risk of lower profits due to job loss. The proposition says that there are two allocations for such risk diversification, rotation or specialization.

Proof. We have eight allocations, x^k, $k \in \{1,\ldots,8\}$, in Table 2, to compare with the rotation x^{R1} or the specialization x^{S1}. The profits from each of these eight are calculated in Table 2. Note that by symmetry, each of the other allocations is profit-equivalent with some of the ten allocations—the above eight allocations, the rotation x^{R1}, and the specialization x^{S1}.

Claim 1. $\pi(x^{R1}) \gtreqless \pi(x^{S1}) \Leftrightarrow p \gtreqless \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$.
Since $\pi(x^{R1})-\pi(x^{S1}) = \delta[(\lambda-1)-2(w^*-w)]+\delta^2[2p(2^\alpha-1)-2pw]$,

$$\pi(x^{R1}) \gtreqless \pi(x^{S1}) \Leftrightarrow p \gtreqless \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}, \tag{2}$$

where the fraction is well defined as $w < 2^\alpha - 1$ by Assumption 4. This completes the proof of Claim 1.

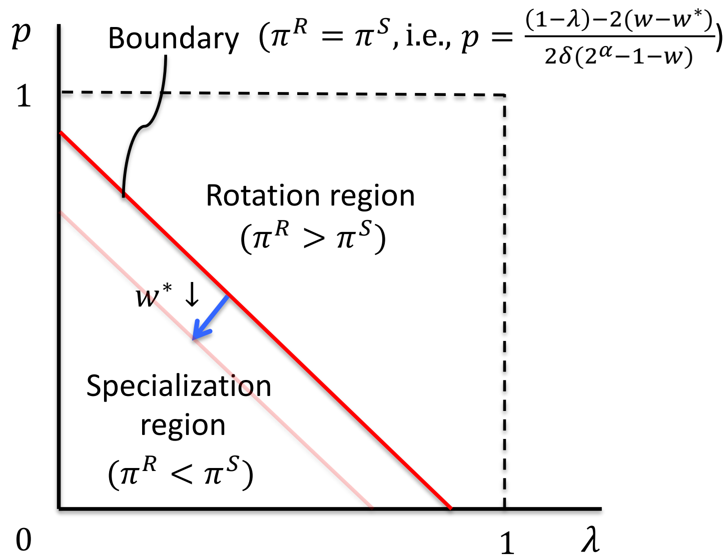

Figure 1: Profit comparison in the graph (λ, p)

Note: The rotation region is when the profit from the rotation is larger, while the specialization region is when the profit from the specialization is larger. The boundary is when the profits from the rotation and the specialization are equal. The above blue arrow indicates that the boundary shifts to the origin as w^* is smaller, that is, $w - w^*$ is larger.

Claim 1 tells us the boundary of p regarding the profits of the rotation and the specialization. Moreover, we can show that when $p > \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$, the rotation is profit maximizing, i.e., for each $k \in \{1, \ldots, 8\}$, $\pi(x^{R1}) > \pi(x^k)$; when $p < \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$, the specialization is profit maximizing, i.e., for each $k \in \{1, \ldots, 8\}$, $\pi(x^{S1}) > \pi(x^k)$; and when $p = \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$, the specialization and the rotation are profit maximizing. The verification is delegated to Appendix A. $\square$

Here, it would be useful to do the comparative statics to see how the parameters affect whether rotation or specialization is better in terms of profit. To this end, we identify the regions of rotation and specialization in a (λ, p)-diagram given the other parameters w, w^*, α, δ (Figure 1). Recall that λ are the training costs and p is the uncertainty about job continuation. By (2), we have

$$p \gtreqless \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}. \tag{3}$$

The rotation region in Figure 1 is when the left-hand side of (3) is larger, i.e., the rotation is better; the specialization region is when the right-hand side is larger, i.e., the specialization is better. The boundary is when both sides are equal, i.e., the profit from the rotation is the same as that from the specialization.

Then, we have the following implications. First, the firm faces the risk of profit decline from the job loss, and thus allocates jobs to diversify the risk. The specialization scheme would be a simple solution to risk diversifications when the uncertainty of job loss (p) is low and the training cost is high (λ is low). However, as the uncertainty is high or the training cost is low (λ is high), the rotation turns to be the best risk diversification.

Second, suppose that w^* (the wage during the training period) declines, that is, $w - w^*$ (the difference between wages under- and post-training) rises. Then, the rotation region expands (see Figure 1). This is consistent with the Japanese practice of the seniority-wage with the rotation. Finally, suppose that δ (the discount factor) or α (the technology parameter of the production function when job A or B disappears) is larger. Then, the p-intercept is smaller and the slope is also smaller. This means that for a small λ, the rotation region expands; for a large λ the specialization region expands.

3 Infinite-period Overlapping Generations Model

Section 2 is about a finite-period model. In this section, we develop an infinite-period OLG model.

3.1 The model

In our OLG model, time is discrete, starts at $t = 1$, and lasts forever. Each period t is divided into two subperiods $s \in \{1, 2\}$ (See Table 3). There is one firm who has two workers in each period. In period $t = 1$, there is one worker, a^0, called the **initially old worker** who works only in period 1. In each period $t \geq 1$, one worker, a^t, arrives at the firm and works for two periods, t and $t + 1$. There are two jobs, A and B, initially available, one of which might be lost with some probability as specified below. In each subperiod $s \in \{1, 2\}$ of period t, the firm uses two workers assigned to the available job(s) or nothing to produce an output. Like in our benchmark model, we make the following assumptions.

Assumption 5. *For a job $J \in \{A, B\}$, if it is the first subperiod assigned to her, each worker a^t needs to get trained for job J (on-the-job training) in the subperiod and cannot have any training at period $t + 1$.*

Assumption 6. *Consider the beginning of each period $t \geq 1$.*

1. *When the two jobs remain available, a job $J \in \{A, B\}$ disappears with probability p. The probability that no job will disappear is 1-2p. There is no chance that both jobs will disappear at the same time.*

Table 3: OLG with two generations

	$t=1$		$t=2$		$t=3$		$\dots$
	$s=1$	$s=2$	$s=1$	$s=2$	$s=1$	$s=2$	
a^0	A	A					
a^1	B^*	B	B	B			
a^2			A^*	A	A	A	
a^3					B^*	B	$\dots$
$\vdots$							

2. *When only one of the two jobs, say $J \in \{A, B\}$, remains available, the job J will remain available from the next period on.*

In each period t, we have two kinds of states (ω_t^O, ω_t^A):

- ω_t^O is the set of **assignable jobs** that the firm can assign to the old worker in period t, based on her training in the previous period $t-1$, due to Assumption 5. Note that any ω_t^O contains $\emptyset$ meaning that the firm does not assign any job to the old worker.

- ω_t^A is the set of **available jobs**, ω^A, due to Assumption 6.

For each period $t \geq 1$, a **period-*t* allocation** is the one that assigns jobs to workers in each of its subperiods. We denote it by

$$x_t = \begin{pmatrix} x_{t,1}^{t-1} & x_{t,2}^{t-1} \\ x_{t,1}^{t} & x_{t,2}^{t} \end{pmatrix},$$

where $x_{t,s}^{t-1}$ stands for a job assigned to the old worker in subperiod $s = 1, 2$, while $x_{t,s}^{t}$ for a job assigned to the young worker in subperiod $s = 1, 2$. Thus, the first row is for the old, and the second row for the young; the first column is for the first subperiod, and the second column for the second subperiod.

An **allocation** is a collection of period-t allocations x_t. Like in the previous section, the two types of allocations—a rotation and a specialization—will be important in our analysis. An allocation x is called a **rotation** if for each $t \geq 1$,

$$x_t = \begin{pmatrix} A & B \\ B & A \end{pmatrix} \text{ or } \begin{pmatrix} B & A \\ A & B \end{pmatrix}.$$

On the other hand, it is called a **specialization** if for each $t \geq 1$,

$$x_t = \begin{pmatrix} A & A \\ B & B \end{pmatrix} \text{ or } \begin{pmatrix} B & B \\ A & A \end{pmatrix}.$$

An allocation is history-dependent on the assignability and the availability of jobs which we call states. Due to this history-dependence, the set of allocations is restricted. To make the set non-trivial and as much unrestricted as possible, we make the following assumption.

Assumption 7. *We assume that the initial state is* $\omega_1 = (\omega_1^O, \omega_1^A) = (\{\emptyset, A, B\}, \{A, B\})$ *in period* $t = 1$*, that is, both jobs* A *and* B *can be done by the initially old worker, and are available at* $t = 1$.

Let us now discuss all of the possible states. First, consider the state of available jobs, ω_t^A. Take a sequence of states of available jobs, $\omega^A = (\omega_t^A)_{t=1}^{\infty}$. Then there are three possibilities: (1) $\omega^A = (\{A, B\}, \{A, B\}, \{A, B\}, \ldots)$, i.e., for each period jobs A and B are available. (2) $\omega^A = (\{A, B\}, \ldots, \{A, B\}, \{A\}, \{A\}, \{A\}, \ldots)$, i.e., for some period $t \geq 2$, job B disappears and after this only job A is available forever. (3) $\omega^A = (\{A, B\}, \ldots, \{A, B\}, \{B\}, \{B\}, \{B\}, \ldots)$, i.e., for some period $t \geq 2$, job A disappears and after this only job B is available forever.

Second, consider the state of assignable jobs, ω_t^O. Take a sequence of assignable jobs, $\omega^O = (\omega_t^O)_{t=1}^{\infty}$. For each $t \geq 2$, the state depends on period $t-1$ allocation as

$$\omega_t^O = \{x_{t-1,1}^{t-1}, x_{t-1,2}^{t-1}\} \cup \{\emptyset\}. \tag{4}$$

Note again that any ω_t^O contains $\emptyset$ – the possibility that the firm does not assign any job to the old worker. Specifically, the following covers all possibilities.

$$\omega_t^O = \begin{cases} \{\emptyset, A, B\} & \text{if } (x_{t-1,1}^{t-1}, x_{t-1,2}^{t-1}) = (A, B) \text{ or } (B, A), \\ \{\emptyset, A\} & \text{if } (x_{t-1,1}^{t-1}, x_{t-1,2}^{t-1}) = (A, \emptyset),\ (\emptyset, A),\ \text{or } (A, A), \\ \{\emptyset, B\} & \text{if } (x_{t-1,1}^{t-1}, x_{t-1,2}^{t-1}) = (B, \emptyset),\ (\emptyset, B),\ \text{or } (B, B), \\ \{\emptyset\} & \text{if } (x_{t-1,1}^{t-1}, x_{t-1,2}^{t-1}) = (\emptyset, \emptyset). \end{cases}$$

We will define the production function for each subperiod s of period t. To do so, given an allocation x, we define $\bar{x}_{\tau,s,J}^t$ as the contribution of worker a^t with job $J \in \{A, B\}$ in subperiod $s \in \{1, 2\}$ of period $\tau \in \{t, t+1\}$ to the production. Any job needs training when the job is assigned for the first time to the young worker a^t. We call it the worker a^t's **training subperiod**. Moreover, the training costs to the firm is represented by the following reduced contribution. Formally, for each worker a^t, each period $\tau \in \{t, t+1\}$, each subperiod $s \in \{1, 2\}$, and each job $J \in \{A, B\}$,

$$\bar{x}^t_{\tau,s,J} = \begin{cases} 0 & \text{if } x^t_{\tau,s} \neq J, \\ \lambda & \text{if } \tau = t,\ x^t_{t,s} = J, \text{ and } s \text{ is worker } a^t\text{'s training subperiod for job } J, \\ 1 & \text{if } \tau \in \{t, t+1\},\ x^t_{\tau,s} = J, \\ & \quad \text{and } s \text{ is not worker } a^t\text{'s training subperiod for job } J, \end{cases}$$

where $0 < \lambda < 1$. The first equality ($\bar{x}^t_{\tau,s,J} = 0$) means that a worker's contribution to any unassigned job is 0. The second and third equalities ($\bar{x}^t_{\tau,s,J} = \lambda$ and $\bar{x}^t_{\tau,s,J} = 1$) mean that a worker's contribution to his assigned job for the first time when young is reduced to λ, while the contribution post-training is 1 at the full level.

We turn to the production function of the firm. One constraint that the firm faces is the availability of jobs due to Assumption 6. Like in the finite-period model, the output $f(x; \omega^A_t)$ with available jobs ω^A_t is

$$f(x; \omega^A_t) = \begin{cases} (\bar{x}^{t-1}_{t,s,B} + \bar{x}^t_{t,s,B})^\alpha & \text{if job } A \text{ disappeared and thus } \omega^A_t = \{B\}, \\ (\bar{x}^{t-1}_{t,s,A} + \bar{x}^t_{t,s,A})^\alpha & \text{if job } B \text{ disappeared and thus } \omega^A_t = \{A\}, \\ \sqrt{(\bar{x}^{t-1}_{t,s,A} + \bar{x}^t_{t,s,A})(\bar{x}^{t-1}_{t,s,B} + \bar{x}^t_{t,s,B})} & \text{if no job disappeared and thus } \omega^A_t = \{A, B\}, \end{cases}$$

where $0 < \alpha < 1$. Note that $\bar{x}$ is a function of x. As shown in Table 3, for example, in subperiod 1 in period 1, the old worker is assigned job A, and its contribution is 1. The young worker is assigned job B for the first time, and its contribution is λ. Thus, $f(x; \{A, B\}) = \sqrt{(1+0)(0+\lambda)} = \sqrt{\lambda}$.

The firm maximizes its profit, which is the sum of discounted profits in subperiods of all periods. The profit for subperiod s in period t is

$$\pi(x^{t-1}_{t,s}, x^t_{t,s}; \omega^A_t) = \begin{cases} pf(x^{t-1}_{t,s}, x^t_{t,s}; \omega^A_t) - w - w & \text{if the two workers are assigned not for the first time,} \\ pf(x^{t-1}_{t,s}, x^t_{t,s}; \omega^A_t) - w - w^* & \text{if the two workers are assigned} \\ & \quad \text{and the young is in the training subperiod,} \\ pf(x^{t-1}_{t,s}, x^t_{t,s}; \omega^A_t) - w & \text{if only one worker is assigned} \\ & \quad \text{and she is not in the training subperiod,} \\ pf(x^{t-1}_{t,s}, x^t_{t,s}; \omega^A_t) - w^* & \text{if only one worker is assigned} \\ & \quad \text{and she is in the training subperiod,} \\ 0 & \text{if no worker is assigned.} \end{cases}$$

Therefore, the firm's problem is choosing an allocation $x = (x_t)_{t=1}^{\infty} = (x_{t,s}^{t-1}, x_{t,s}^{t})_{s=1,2;t=1,\ldots,\infty}$ to maximize its profit on an infinite horizon such that the initial state is $\omega_1 = (\omega_1^O, \omega_1^A) = (\{\emptyset, A, B\}, \{A, B\})$; the old worker's assignable jobs in each period t, ω_t^O, are those she experienced in the previous period[5]; the allocation for the old worker in subperiods, $x_{t,1}^{t-1}$ and $x_{t,2}^{t-1}$, is those jobs that are assignable and available to her; the allocation for the young worker in subperiods, $x_{t,1}^{t}$ and $x_{t,2}^{t}$, is just available to her. That is,

$$\begin{aligned}
\max_x \quad & \sum_{t=1}^{\infty} \delta^{t-1}\{\pi(x_{t,1}^{t-1}, x_{t,1}^{t}; \omega_t^A) + \pi(x_{t,2}^{t-1}, x_{t,2}^{t}; \omega_t^A)\} \\
\text{s.t.} \quad & \omega_1 = (\{\emptyset, A, B\}, \{A, B\}), \\
& \omega_t^O = \{x_{t-1,1}^{t-1}, x_{t-1,2}^{t-1}\} \cup \{\emptyset\},\ t \geq 2, \\
& x_{t,1}^{t-1}, x_{t,2}^{t-1} \in (\omega_t^O \cap \omega_t^A) \cup \{\emptyset\}, \\
& x_{t,1}^{t}, x_{t,2}^{t} \in \omega_t^A \cup \{\emptyset\}.
\end{aligned}$$

Note that in any subperiod of period t, assigning no job to the two workers is always feasible. Define the value function $V(\omega_t^O, \omega_t^A)$ as

$$\begin{aligned}
V(\omega_t^O, \omega_t^A) \quad = \quad & \max_{(x_\tau)_{\tau=t}^{\infty}} \sum_{\tau=t}^{\infty} \delta^{\tau-1}\{\pi(x_{\tau,1}^{\tau-1}, x_{\tau,1}^{\tau}; \omega_\tau^A) + \pi(x_{\tau,2}^{\tau-1}, x_{\tau,2}^{\tau}; \omega_\tau^A)\} \\
\text{s.t.} \quad & \omega_\tau^O = \{x_{\tau-1,1}^{\tau-1}, x_{\tau-1,2}^{\tau-1}\} \cup \{\emptyset\},\ \tau \geq t+1, \\
& x_{\tau,1}^{\tau-1}, x_{\tau,2}^{\tau-1} \in (\omega_\tau^O \cap \omega_\tau^A) \cup \{\emptyset\}, \\
& x_{\tau,1}^{\tau}, x_{\tau,2}^{\tau} \in \omega_\tau^A \cup \{\emptyset\}.
\end{aligned}$$

Then, the profit-maximization problem can be stated in the following recursive form.

$$\begin{aligned}
V(\{\emptyset, A, B\}, \{A, B\}) \quad = \quad & \max_{x_1} \pi(x_{1,1}^0, x_{1,1}^1; \{A, B\}) + \pi(x_{1,2}^0, x_{1,2}^1; \{A, B\}) \\
& \quad + \delta\{pV(\omega_2^O, \{A\}) + pV(\omega_2^O, \{B\}) + (1-2p)V(\omega_2^O, \{A, B\})\}, \\
\text{s.t.} \quad & \omega_2^O = \{x_{1,1}^1, x_{1,2}^1\} \cup \{\emptyset\}, \\
& x_{1,1}^0, x_{1,2}^0 \in (\omega_1^O \cap \omega_1^A) \cup \{\emptyset\}, \\
& x_{1,1}^1, x_{1,2}^1 \in \omega_1^A \cup \{\emptyset\}.
\end{aligned}$$

Here, we make the following assumption in the same spirit as the no-free-disposal in Assumption 4 for the benchmark model. When one of the jobs is available, it is profitable

[5] This is the rule that we already explained in (4).

for the firm to employ a worker even if she is young.

Assumption 8. *For each available job $J \in \{A, B\}$, $\pi(\emptyset, \emptyset; \{J\}) < \pi(\emptyset, J^*; \{J\}) < \pi(J, J^*; \{J\})$ and $\pi(\emptyset, \emptyset; \{J\}) < \pi(J, \emptyset; \{J\}) < \pi(J, J^*; \{J\})$.*

3.2 Comparison in profits

Proposition 2. *For any parameters, α, δ, p, w, w^*, and λ, the profit-maximizing allocation is either the rotation or the specialization. In particular, the profit-maximizing allocation is*

1. *the rotation if $p\delta \{V(\{\emptyset, A\}, \{A\}) - V(\{\emptyset\}, \{A\})\} > 1 - w + w^* - \lambda^{\frac{1}{2}}$, i.e., $p > \frac{1-w+w^*-\lambda^{\frac{1}{2}}}{\delta\{(1+\lambda)^\alpha+2^\alpha-\lambda^\alpha-1-2w\}}$,*
2. *the specialization if $p < \frac{1-w+w^*-\lambda^{\frac{1}{2}}}{\delta\{(1+\lambda)^\alpha+2^\alpha-\lambda^\alpha-1-2w\}}$,*
3. *either the rotation or the specialization if $p = \frac{1-w+w^*-\lambda^{\frac{1}{2}}}{\delta\{(1+\lambda)^\alpha+2^\alpha-\lambda^\alpha-1-2w\}}$.*

Proposition 2 argues that either the rotation or the specialization is a profit-maximizing allocation, depending on the parameters. For the proof, we use the following two lemmas whose proofs are delegated to Appendix B.

Lemma 1. *For any parameters, α, δ, p, w, w^*, and λ, we have the following.*

1. *If the profit-maximizing allocation is the rotation, then $p\delta \{V(\{\emptyset, A\}, \{A\}) - V(\{\emptyset\}, \{A\})\} \geq 1 - w + w^* - \lambda^{\frac{1}{2}}$, i.e., $p \geq \frac{1-w+w^*-\lambda^{\frac{1}{2}}}{\delta\{(1+\lambda)^\alpha+2^\alpha-\lambda^\alpha-1-2w\}}$.*
2. *If $p\delta \{V(\{\emptyset, A\}, \{A\}) - V(\{\emptyset\}, \{A\})\} > 1 - w + w^* - \lambda^{\frac{1}{2}}$, then the profit-maximizing allocation is the rotation only.*

For the rotation to be optimal, the first part of Lemma 1 gives the necessary condition, while the second gives an almost sufficient condition.

Lemma 2. *The profit-maximizing allocation is either the rotation or the specialization.*

Now we are ready to prove Proposition 2.

Proof of Proposition 2. If $p\delta \{V(\{\emptyset, A\}, \{A\}) - V(\{\emptyset\}, \{A\})\} > 1 - w + w^* - \lambda^{\frac{1}{2}}$, then Lemma 1-(2) is the desired result. Next, if $p\delta \{V(\{\emptyset, A\}, \{A\}) - V(\{\emptyset\}, \{A\})\} < 1 - w + w^* - \lambda^{\frac{1}{2}}$, then it follows from Lemma 1-(1) that the profit-maximizing allocation is not the rotation. Thus, under Lemma 2, the profit-maximizing one is the specialization. Finally, if $p\delta \{V(\{\emptyset, A\}, \{A\}) - V(\{\emptyset\}, \{A\})\} = 1 - w + w^* - \lambda^{\frac{1}{2}}$, the last part is obvious from Lemma 2. $\square$

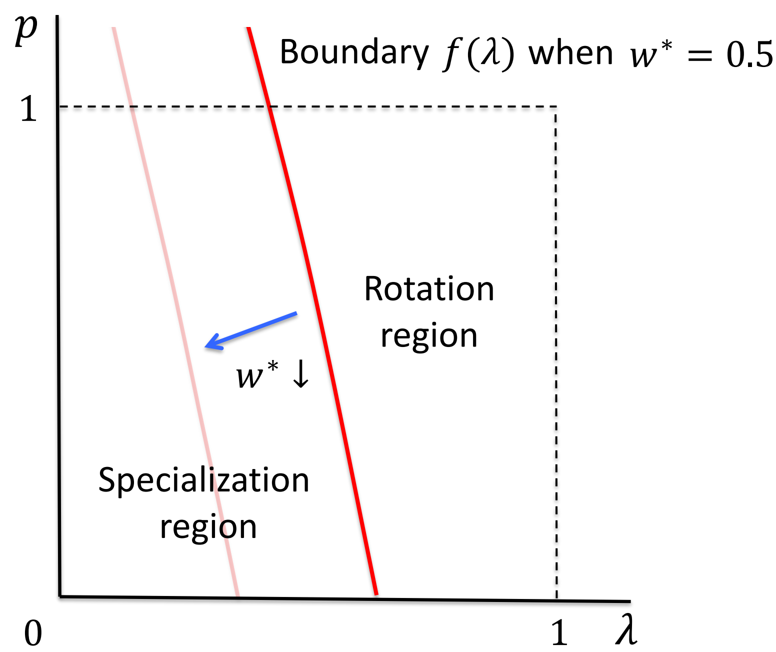

Figure 2: Profit comparison in the graph (λ, p)

Note: The curve shows the function $f(\lambda) = \frac{1-w+w^*-\lambda^{\frac{1}{2}}}{\delta\{(1+\lambda)^\alpha+2^\alpha-\lambda^\alpha-1-2w\}}$ when we change $w^* = 0.3, 0.5$, keeping $w = 0.7$, $\alpha = 0.8$, and $\delta = 0.9$. According to Proposition 2, the rotation region is when the profit of the rotation is larger, while the specialization region is the opposite. The boundary, $f(\lambda)$, is when the profits from the rotation and the specialization are equal.

Here, it would be useful to do the comparative statics to see how parameters affect which—rotation or specialization—is better in terms of profit.

According to Proposition 2, we have

$$p \gtreqless f(\lambda) = \frac{1-w+w^*-\lambda^{\frac{1}{2}}}{\delta\{(1+\lambda)^\alpha+2^\alpha-\lambda^\alpha-1-2w\}}.$$

This can be shown by Figure 2, which describes the profit comparison in the graph of the training costs (λ) and the uncertainty about job continuation (p).

Then, we have the following implications:

1. Suppose that p and λ are large (noting the larger the λ, the smaller the training costs). Then, the rotation is better.

2. As $w-w^*$ (the difference between wages under- and post-training) is larger, the rotation region expands. Under the seniority wage scheme as in Japan, where $w-w^*$ is large, the rotation region is large.

3. Suppose that δ, the discount factor, is larger. Then, the curve of f shifts to the origin.

This means that the rotation region expands.

4. Suppose that α, which is the technology parameter of the production function when job A or B disappears, is larger. Note that function f is decreasing in α, because the numerator of f is constant in α and the denominator of f is increasing in α. To see it for the denominator, its partial derivative with respect to α, $\delta\{(1+\lambda)^\alpha \ln(1+\lambda) + 2^\alpha \ln 2 - \lambda^\alpha \ln \lambda\}$, is positive. Thus, the curve of f shifts to the origin. This means that the rotation region expands.

4 Conclusion

We have developed a firm's decision problem of assigning (indivisible) jobs to workers, with and without OLG. We have shown that only job rotation or specialization can be a profit-maximizing job allocation in both models. Moreover, we have shown that in both models, the rotation is better when the training cost is smaller, the uncertainty about job continuation in the future is larger, or the slope of seniority wages is larger.

Finally, one of the most important implications from our results is that both job rotation and specialization can be an optimal scheme in terms of a firm's profits. Therefore, it is no wonder that we still observe the coexistence of both the Japanese employment system (supported by job rotation) and the U.S. employment system (supported by job specialization).

A Proof of Proposition 1

We complete the proof of Proposition 1 by checking the profit comparison in the following three cases.

Case 1: $p > \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$. We show that for each $k \in \{1,\ldots,8\}$, $\pi(x^{R1}) > \pi(x^k)$.

First, under Claim 1, $\pi(x^{R1}) > \pi(x^{S1})$. Second, from Table 2, for each $k \in \{1,\ldots,8\} \setminus \{2,7\}$, $\pi(x^{R1}) > \pi(x^k)$. It remains to show the inequality for $k \in \{2,7\}$.

$$\begin{aligned}
\pi(x^{R1}) - \pi(x^2) &= \pi(x^{R1}) - \pi(x^7) \\
&= \lambda + \delta\{\lambda - \lambda^{1/2} + w - w^*\} + \delta^2 p(2^\alpha - 1 - w) \\
&> \lambda + \delta\{\lambda - \lambda^{1/2} + w - w^*\} + \frac{\delta}{2}\{(1-\lambda) - 2(w-w^*)\} \\
&= \lambda + \delta\left(\frac{1}{2} - \lambda^{1/2} + \frac{\lambda}{2}\right),
\end{aligned}$$

where the inequality follows from $p > \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$. Let us define the function $f : [0,1] \to \mathbb{R}$

by

$$f(\lambda) = \lambda + \delta\left(\frac{1}{2} - \lambda^{1/2} + \frac{\lambda}{2}\right).$$

Then,

$$\begin{aligned} f(0) &= \frac{\delta}{2} < 1 = f(1), \\ f'(\lambda) &= 1 + \frac{\delta}{2}\left(1 - \frac{1}{\lambda^{1/2}}\right), \\ f''(\lambda) &= \frac{\delta}{4}\lambda^{-3/2} > 0. \end{aligned}$$

Thus, function f is concave and there is a unique λ^* such that $f'(\lambda^*) = 0$, i.e., $\lambda^* = \left(\frac{\delta}{\delta+2}\right)^2 \in (0,1)$. Because our calculation gives us

$$f(\lambda^*) = \frac{2\delta^2 + 4\delta}{2(2+\delta)^2} > 0,$$

we can conclude that for any $\lambda \in [0,1]$, $f(\lambda) > 0$. Thus, $\pi(x^{R1}) > \pi(x^2)$ and $\pi(x^{R1}) > \pi(x^7)$.
Case 2: $p < \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$. We show that for each $k \in \{1,\dots,8\}$, $\pi(x^{S1}) > \pi(x^k)$.

First, under Claim 1, $\pi(x^{S1}) > \pi(x^{R1})$. Second, from Table 2, for each $k \in \{1,\dots,8\} \setminus \{2,7\}$, $\pi(x^{R1}) > \pi(x^k)$, and thus $\pi(x^{S1}) > \pi(x^k)$. It remains to show the inequality for $k \in \{2,7\}$.

$$\begin{aligned} \pi(x^{S1}) - \pi(x^2) &= \pi(x^{S1}) - \pi(x^7) \\ &= \lambda + \delta\{1 - \lambda^{1/2} - (w - w^*)\} - \delta^2 p(2^\alpha - 1 - w) \\ &> \lambda + \delta\{1 - \lambda^{1/2} - (w - w^*)\} - \frac{\delta}{2}\{(1-\lambda) - 2(w - w^*)\} \\ &= f(\lambda), \end{aligned} \tag{5}$$

where the inequality follows from $p < \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$. As shown in Case 1, for each $\lambda \in [0,1]$, $f(\lambda) > 0$. Thus, under inequality (5), $\pi(x^{S1}) > \pi(x^2)$ and $\pi(x^{S1}) > \pi(x^7)$.
Case 3: $p = \frac{(1-\lambda)-2(w-w^*)}{2\delta(2^\alpha-1-w)}$. The last part of the proposition is now straightforward from the arguments in cases 1 and 2. □

B Proof of Proposition 2

To complete the proof of Proposition 2, it remains to show lemmas 1 and 2. We start with some preliminaries.

B.1 Preliminaries

We need to calculate the value function depending on the states ω^O and ω^A. Note that ω^O is the set of assignable jobs at some period, and ω^A is the set of available jobs at some period. For the simplicity of notation, when we write the state (ω^O, ω^A), for example, we denote $(\omega^O, \omega^A) = (\emptyset, A; A, B)$ for $(\{\emptyset, A\}; \{A, B\})$ where we omit the braces for sets. Thus, for value functions, we denote $V(\omega^O, \omega^A) = V(\emptyset, A; A, B)$ for $V(\{\emptyset, A\}, \{A, B\})$. Because of the symmetric roles of jobs A and B, we only need to calculate the value function for the following five cases.

1. $V(\emptyset; A) = V(\emptyset; B) = V(\emptyset, B; A) = V(\emptyset, A; B)$
2. $V(\emptyset; A, B)$
3. $V(\emptyset, A; A) = V(\emptyset, B; B) = V(\emptyset, A, B; B) = V(\emptyset, A, B; A)$
4. $V(\emptyset, A; A, B) = V(\emptyset, B; A, B)$
5. $V(\emptyset, A, B; A, B)$

B.1.1 Calculation of $V(\emptyset; A) = V(\emptyset; B) = V(\emptyset, B; A) = V(\emptyset, A; B)$

We calculate $V(\emptyset; A)$, which is obviously equal to $V(\emptyset; B)$, $V(\emptyset, B; A)$, and $V(\emptyset, A; B)$. We have the following three candidates for $V(\emptyset; A)$, depending on period-t allocations.

V^{11}				V^{12}				V^{13}			
	Alloc. at t		States at $t+1$		Alloc. at t		States at $t+1$		Alloc. at t		States at $t+1$
	$s=1$	$s=2$			$s=1$	$s=2$			$s=1$	$s=2$	
a^{t-1}	$\emptyset$	$\emptyset$	$\omega^A_{t+1} = \{A\}$	a^{t-1}	$\emptyset$	$\emptyset$	$\omega^A_{t+1} = \{A\}$	a^{t-1}	$\emptyset$	$\emptyset$	$\omega^A_{t+1} = \{A\}$
a^t	A^*	A	$\omega^O_{t+1} = \{\emptyset, A\}$	a^t	$\emptyset$ (A^*)	A^* $(\emptyset)$	$\omega^O_{t+1} = \{\emptyset, A\}$	a^t	$\emptyset$	$\emptyset$	$\omega^O_{t+1} = \{\emptyset\}$

Denote

$$
\begin{aligned}
V^{11} &= \pi(\emptyset, A^*; A) + \pi(\emptyset, A; A) + \delta V(\emptyset, A; A), \\
V^{12} &= \pi(\emptyset, \emptyset; A) + \pi(\emptyset, A^*; A) + \delta V(\emptyset, A; A) \\
&\quad \text{or } \pi(\emptyset, A^*; A) + \pi(\emptyset, \emptyset; A) + \delta V(\emptyset, A; A), \\
V^{13} &= \pi(\emptyset, \emptyset; A) + \pi(\emptyset, \emptyset; A) + \delta V(\emptyset; A).
\end{aligned}
$$

Claim 2. $V^{11} > V^{12}$ and $V^{11} > V^{13}$. Thus, $V(\emptyset; A) = V^{11}$.

Proof. First, $V^{11} - V^{12} = \pi(\emptyset, A; A) - \pi(\emptyset, \emptyset; A)$. Thus, since $\pi(\emptyset, A; A) > \pi(\emptyset, \emptyset; A)$ according to Assumption 8, we have $V^{11} > V^{12}$.

We next show $V^{11} > V^{13}$.

$$\begin{aligned} V^{11} - V^{13} &= \{\pi(\emptyset, A^*; A) - \pi(\emptyset, \emptyset; A)\} + \{\pi(\emptyset, A; A) - \pi(\emptyset, \emptyset; A)\} \\ &\quad + \delta\{V(\emptyset, A; A) - V(\emptyset; A)\}. \end{aligned}$$

Under Assumption 8, $\pi(\emptyset, A^*; A) > \pi(\emptyset, \emptyset; A)$ and $\pi(\emptyset, A; A) > \pi(\emptyset, \emptyset; A)$. Moreover, since any allocation under state $(\emptyset; A)$ is possible under state $(\emptyset, A; A)$, we have $V(\emptyset, A; A) \geq V(\emptyset; A)$. Thus, $V^{11} > V^{13}$. □

Thus, we have

$$V(\emptyset; A) = V^{11} = \pi(\emptyset, A^*; A) + \pi(\emptyset, A; A) + \delta V(\emptyset, A; A). \tag{6}$$

Since $V(\emptyset, A; A) = \frac{\pi(A,A^*;A)+\pi(A,A;A)}{1-\delta}$ (this will be shown in Section B.1.3),

$$V(\emptyset; A) = \pi(\emptyset, A^*; A) + \pi(\emptyset, A; A) + \frac{\delta}{1-\delta}\left(\pi(A, A^*; A) + \pi(A, A; A)\right).$$

B.1.2 Calculation of $V(\emptyset; A, B)$

We have the following three candidates for $V(\emptyset; A, B)$, depending on period-t allocations.

V^{21}					
	Alloc. at t		States at $t+1$	States at $t+1$	States at $t+1$
	$s=1$	$s=2$			
a^{t-1}	$\emptyset$	$\emptyset$	$\omega^A_{t+1} = \{A, B\}$	$\omega^A_{t+1} = \{A\}$	$\omega^A_{t+1} = \{B\}$
a^t	B^*	B	$\omega^O_{t+1} = \{\emptyset, B\}$	$\omega^O_{t+1} = \{\emptyset, B\}$	$\omega^O_{t+1} = \{\emptyset, B\}$

$$V^{21} = \pi(\emptyset, B^*; A, B) + \pi(\emptyset, B; A, B) + (1-2p)\delta V(\emptyset, B; A, B) + p\delta V(\emptyset, B; A) + p\delta V(\emptyset, B; B).$$

V^{22}					
	Alloc. at t		States at $t+1$	States at $t+1$	States at $t+1$
	$s=1$	$s=2$			
a^{t-1}	$\emptyset$	$\emptyset$	$\omega^A_{t+1} = \{A, B\}$	$\omega^A_{t+1} = \{A\}$	$\omega^A_{t+1} = \{B\}$
a^t	A^*	A	$\omega^O_{t+1} = \{\emptyset, A\}$	$\omega^O_{t+1} = \{\emptyset, A\}$	$\omega^O_{t+1} = \{\emptyset, A\}$

$V^{22} = \pi(\emptyset, A^*; A, B) + \pi(\emptyset, A; A, B) + (1-2p)\delta V(\emptyset, A; A, B) + p\delta V(\emptyset, A; A) + p\delta V(\emptyset, A; B)$.

V^{23}					
	Alloc. at t		States at $t+1$	States at $t+1$	States at $t+1$
	$s=1$	$s=2$			
a^{t-1}	$\emptyset$	$\emptyset$	$\omega^A_{t+1} = \{A, B\}$	$\omega^A_{t+1} = \{A\}$	$\omega^A_{t+1} = \{B\}$
a^t	A^*	B^*	$\omega^O_{t+1} = \{\emptyset, A, B\}$	$\omega^O_{t+1} = \{\emptyset, A, B\}$	$\omega^O_{t+1} = \{\emptyset, A, B\}$

$$\begin{aligned} V^{23} &= \pi(\emptyset, A^*; A, B) + \pi(\emptyset, B^*; A, B) \\ &\quad +(1-2p)\delta V(\emptyset, A, B; A, B) + p\delta V(\emptyset, A, B; A) + p\delta V(\emptyset, A, B; B) \\ &= \pi(\emptyset, A^*; A, B) + \pi(\emptyset, B^*; A, B) \\ &\quad +(1-2p)\delta V(\emptyset, A, B; A, B) + p\delta V(\emptyset, A; A) + p\delta V(\emptyset, B; B). \end{aligned}$$

Claim 3. $V^{23} > V^{21} = V^{22}$. Thus, $V(\emptyset; A, B) = V^{23}$.

Proof. First, $V^{21} = V^{22}$ by symmetry. Next, we have

$$\begin{aligned} V^{23} - V^{22} &= \pi(\emptyset, B^*; A, B) - \pi(\emptyset, A; A, B) + (1-2p)\delta(V(\emptyset, A, B; A, B) - V(\emptyset, A; A, B)) \\ &\quad +p\delta(V(\emptyset, B; B) - V(\emptyset, A; B)) \\ &= -w^* + w + (1-2p)\delta(V(\emptyset, A, B; A, B) - V(\emptyset, A; A, B)) + p\delta(V(\emptyset, B; B) - V(\emptyset; B)) \\ &> 0. \end{aligned}$$

The last inequality follows from the fact that $w > w^*$, $V(\emptyset, A, B; A, B) \geq V(\emptyset, A; A, B)$, and $V(\emptyset, B; B) \geq V(\emptyset; B)$. □

B.1.3 Calculation of $V(\emptyset, A; A) = V(\emptyset, B; B) = V(\emptyset, A, B; B) = V(\emptyset, A, B; A)$

We calculate $V(\emptyset, A; A)$, which is obviously equal to $V(\emptyset, B; B)$, $V(\emptyset, A, B; B)$, and $V(\emptyset, A, B; A)$. We have the following three candidates for $V(\emptyset, A; A)$, depending on period-t allocations.

V^{31}				V^{32}				V^{33}			
	Alloc. at t		States at $t+1$		Alloc. at t		States at $t+1$		Alloc. at t		States at $t+1$
	$s=1$	$s=2$			$s=1$	$s=2$			$s=1$	$s=2$	
a^{t-1}	A	A	$\omega^A_{t+1} = \{A\}$	a^{t-1}	A	A	$\omega^A_{t+1} = \{A\}$	a^{t-1}	A	A	$\omega^A_{t+1} = \{A\}$
a^t	A^*	A	$\omega^O_{t+1} = \{\emptyset, A\}$	a^t	$\emptyset$ (A^*)	A^* ($\emptyset$)	$\omega^O_{t+1} = \{\emptyset, A\}$	a^t	$\emptyset$	$\emptyset$	$\omega^O_{t+1} = \{\emptyset\}$

$$
\begin{aligned}
V^{31} &= \pi(A, A^*; A) + \pi(A, A; A) + \delta V(\emptyset, A; A) \\
V^{32} &= \pi(A, \emptyset; A) + \pi(A, A^*; A) + \delta V(\emptyset, A; A) \\
V^{33} &= \pi(A, \emptyset; A) + \pi(A, \emptyset; A) + \delta V(\emptyset; A).
\end{aligned}
$$

Claim 4. $V^{31} > V^{32}$ and $V^{31} > V^{33}$. Thus, $V(\emptyset, A; A) = V^{31}$.

Proof. First, $V^{31} - V^{32} = \pi(A, A; A) - \pi(A, \emptyset; A)$. Then, since $\pi(A, A; A) > \pi(A, \emptyset; A)$ under Assumption 8, we have $V^{31} > V^{32}$.

We next show $V^{31} > V^{33}$. We have

$$
\begin{aligned}
V^{31} - V^{33} &= \{\pi(A, A; A) - \pi(A, \emptyset; A)\} + \{\pi(A, A^*; A) - \pi(A, \emptyset; A)\} \\
&\quad + \delta\{V(\emptyset, A; A) - V(\emptyset; A)\}.
\end{aligned}
$$

Under Assumption 8, $\pi(A, A; A) > \pi(A, \emptyset; A)$ and $\pi(A, A^*; A) > \pi(A, \emptyset; A)$. Moreover, $V(\emptyset, A; A) \geq V(\emptyset; A)$. Hence, $V^{31} > V^{33}$. □

Thus, we have

$$
\begin{aligned}
V(\emptyset, A; A) &= V^{31} = \pi(A, A^*; A) + \pi(A, A; A) + \delta V(\emptyset, A; A) \\
&= \frac{\pi(A, A^*; A) + \pi(A, A; A)}{1 - \delta}. \qquad (7)
\end{aligned}
$$

B.1.4 Calculation of $V(\emptyset, A; A, B) = V(\emptyset, B; A, B)$

We calculate $V(\emptyset, A; A, B)$, which is obviously equal to $V(\emptyset, B; A, B)$. We have the following three candidates for $V(\emptyset, A; A, B)$, depending on period-t allocations.

V^{41}					
	Alloc. at t		States at $t+1$	States at $t+1$	States at $t+1$
	$s=1$	$s=2$			
a^{t-1}	A	A	$\omega^A_{t+1} = \{A, B\}$	$\omega^A_{t+1} = \{A\}$	$\omega^A_{t+1} = \{B\}$
a^t	B^*	B	$\omega^O_{t+1} = \{\emptyset, B\}$	$\omega^O_{t+1} = \{\emptyset, B\}$	$\omega^O_{t+1} = \{\emptyset, B\}$

$V^{41} = \pi(A, B^*; A, B) + \pi(A, B; A, B) + (1-2p)\delta V(\emptyset, B; A, B) + p\delta V(\emptyset, B; A) + p\delta V(\emptyset, B; B)$. Note that V^{41} is the value of the specialization.

V^{42}					
	Alloc. at t		States at $t+1$	States at $t+1$	States at $t+1$
	$s=1$	$s=2$			
a^{t-1}	A	A	$\omega_{t+1}^A=\{A,B\}$	$\omega_{t+1}^A=\{A\}$	$\omega_{t+1}^A=\{B\}$
a^t	A^*	A	$\omega_{t+1}^O=\{\emptyset,A\}$	$\omega_{t+1}^O=\{\emptyset,A\}$	$\omega_{t+1}^O=\{\emptyset,A\}$

$$V^{42}=\pi(A,A^*;A,B)+\pi(A,A;A,B)+(1-2p)\delta V(\emptyset,A;A,B)+p\delta V(\emptyset,A;A)+p\delta V(\emptyset,A;B).$$

V^{43}					
	Alloc. at t		States at $t+1$	States at $t+1$	States at $t+1$
	$s=1$	$s=2$			
a^{t-1}	A	A	$\omega_{t+1}^A=\{A,B\}$	$\omega_{t+1}^A=\{A\}$	$\omega_{t+1}^A=\{B\}$
a^t	A^*	B^*	$\omega_{t+1}^O=\{\emptyset,A,B\}$	$\omega_{t+1}^O=\{\emptyset,A,B\}$	$\omega_{t+1}^O=\{\emptyset,A,B\}$

$$\begin{aligned}V^{43} &= \pi(A,A^*;A,B)+\pi(A,B^*;A,B)\\ &\quad +(1-2p)\delta V(\emptyset,A,B;A,B)+p\delta V(\emptyset,A,B;A)+p\delta V(\emptyset,A,B;B)\\ &= \pi(A,A^*;A,B)+\pi(A,B^*;A,B)\\ &\quad +(1-2p)\delta V(\emptyset,A,B;A,B)+p\delta V(\emptyset,A;A)+p\delta V(\emptyset,B;B).\end{aligned}$$

Claim 5. $V^{41}>V^{42}$ and $V^{43}>V^{42}$.

Proof. First,

$$\begin{aligned}V^{41}-V^{42} &= \{\pi(A,B^*;A,B)-\pi(A,A^*;A,B)\}+\{\pi(A,B;A,B)-\pi(A,A;A,B)\}\\ &\quad +(1-2p)\delta\{V(\emptyset,B;A,B)-V(\emptyset,A;A,B)\}\\ &\quad +p\delta\{V(\emptyset,B;A)-V(\emptyset,A;B)\}+p\delta\{V(\emptyset,B;B)-V(\emptyset,A;A)\}.\end{aligned}$$

By calculation, $\pi(A,B^*;A,B)>\pi(A,A^*;A,B)$ and $\pi(A,B;A,B)>\pi(A,A;A,B)$. Moreover, by symmetry, $V(\emptyset,B;A,B)=V(\emptyset,A;A,B)$, $V(\emptyset,B;A)=V(\emptyset,A;B)$, and $V(\emptyset,B;B)=V(\emptyset,A;A)$. Thus $V^{41}>V^{42}$.

We next show $V^{43}>V^{42}$.

$$
\begin{aligned}
V^{43}-V^{42} &= \pi(A,B^*;A,B)-\pi(A,A;A,B)+p\delta\left(V(\emptyset,B;B)-V(\emptyset,A;B)\right)\\
&\quad +(1-2p)\delta\left(V(\emptyset,A,B;A,B)-V(\emptyset,A;A,B)\right)\\
&= \pi(A,B^*;A,B)-\pi(A,A;A,B)+p\delta\left(V(\emptyset,B;B)-V(\emptyset;B)\right)\\
&\quad +(1-2p)\delta\left(V(\emptyset,A,B;AB)-V(\emptyset,A;A,B)\right)\\
&> 0.
\end{aligned}
$$

The last inequality follows from the fact that $\pi(A,B^*;A,B)-\pi(A,A;A,B)=(\lambda^{1/2}-w-w^*)+2w=\lambda^{1/2}+w-w^*>0$, $V(\emptyset,B;B)\geq V(\emptyset;B)$, and $V(\emptyset,A,B;A,B)\geq V(\emptyset,A;A,B)$. Thus, $V^{43}>V^{42}$. □

At this stage, we cannot clearly say which is larger, V^{41} or V^{43}.

B.1.5 Calculation of $V(\emptyset,A,B;A,B)$

We have the following four candidates for $V(\emptyset,A,B;A,B)$, depending on period-t allocations.

- Case 1: $(x_{t,1}^{t-1},x_{t,2}^{t-1})=(A,A)$. Then, the old worker is assigned job A though jobs A and B are assignable. Thus, the value candidate V^{51} in this case is equal to the value when only job A is assignable, i.e., $V^{51}=V(\emptyset,A;A,B)$.

- Case 2: $(x_{t,1}^{t-1},x_{t,2}^{t-1})=(B,B)$. Then, similarly, the value candidate V^{52} in this case is $V^{52}=V(\emptyset,B;A,B)$.

- Case 3: $(x_{t,1}^{t-1},x_{t,2}^{t-1})=(A,B)$. Denote the value candidate in this case by V^{53}. Then, we have the following four subcandidates for V^{53}, depending on the young worker's assignment $(x_{t,1}^{t},x_{t,2}^{t})$.

V^{531}					
	Alloc. at t		States at $t+1$	States at $t+1$	States at $t+1$
	$s=1$	$s=2$			
a^{t-1}	A	B	$\omega_{t+1}^A=\{A,B\}$	$\omega_{t+1}^A=\{A\}$	$\omega_{t+1}^A=\{B\}$
a^t	B^*	A^*	$\omega_{t+1}^O=\{\emptyset,A,B\}$	$\omega_{t+1}^O=\{\emptyset,A,B\}$	$\omega_{t+1}^O=\{\emptyset,A,B\}$

$$
\begin{aligned}
V^{531} &= \pi(A,B^*;A,B)+\pi(B,A^*;A,B)\\
&\quad +(1-2p)\delta V(\emptyset,A,B;A,B)+p\delta V(\emptyset,A,B;A)+p\delta V(\emptyset,A,B;B).
\end{aligned}
$$

Note that V^{531} is the value of the rotation.

V^{532}					
	Alloc. at t		States at $t+1$	States at $t+1$	States at $t+1$
	$s=1$	$s=2$			
a^{t-1}	A	B	$\omega^A_{t+1}=\{A,B\}$	$\omega^A_{t+1}=\{A\}$	$\omega^A_{t+1}=\{B\}$
a^t	A^*	B^*	$\omega^O_{t+1}=\{\emptyset,A,B\}$	$\omega^O_{t+1}=\{\emptyset,A,B\}$	$\omega^O_{t+1}=\{\emptyset,A,B\}$

$V^{532}=\pi(A,A^*;A,B)+\pi(B,B^*;A,B)+(1-2p)\delta V(\emptyset,A,B;A,B)+p\delta V(\emptyset,A,B;A)+p\delta V(\emptyset,A,B;B)$.

V^{533}					
	Alloc. at t		States at $t+1$	States at $t+1$	States at $t+1$
	$s=1$	$s=2$			
a^{t-1}	A	B	$\omega^A_{t+1}=\{A,B\}$	$\omega^A_{t+1}=\{A\}$	$\omega^A_{t+1}=\{B\}$
a^t	A^*	A	$\omega^O_{t+1}=\{\emptyset,A\}$	$\omega^O_{t+1}=\{\emptyset,A\}$	$\omega^O_{t+1}=\{\emptyset,A\}$

$V^{533}=\pi(A,A^*;A,B)+\pi(B,A;A,B)+(1-2p)\delta V(\emptyset,A;A,B)+p\delta V(\emptyset,A;A)+p\delta V(\emptyset,A;B)$.

V^{534}					
	Alloc. at t		States at $t+1$	States at $t+1$	States at $t+1$
	$s=1$	$s=2$			
a^{t-1}	A	B	$\omega^A_{t+1}=\{A,B\}$	$\omega^A_{t+1}=\{A\}$	$\omega^A_{t+1}=\{B\}$
a^t	B^*	B	$\omega^O_{t+1}=\{\emptyset,B\}$	$\omega^O_{t+1}=\{\emptyset,B\}$	$\omega^O_{t+1}=\{\emptyset,B\}$

$V^{534}=V^{533}$ by symmetry.

Thus, $V^{53}=\max\{V^{531},V^{532},V^{533},V^{534}\}$.

Claim 6. $V^{531}>V^{532}$.

Proof. This is because $\pi(A,B^*;A,B)>\pi(A,A^*;A,B)$ and $\pi(B,A^*;A,B)>\pi(B,B^*;A,B)$. □

- Case 4: $(x^{t-1}_{t,1},x^{t-1}_{t,2})=(B,A)$. Let V^{54} be the value in this case. Then, by symmetry, $V^{54}=V^{53}$.

B.2 Proof of Lemma 1: A necessary and almost sufficient condition for the rotation to be optimal

B.2.1 A necessary condition for the rotation to be optimal

Since our initial condition is $(\emptyset, A, B; A, B)$, we focus on Section B.1.5 for $V(\emptyset, A, B; A, B)$. Suppose that the rotation is a profit-maximizing allocation. Then, the value is $V(\emptyset, A, B; A, B) = V^{531}$. In addition to the inequality of Claim 6, we have the following relations.

$$V^{531} \geq V^{533} = V^{534}, \tag{8}$$
$$V^{531} \geq V^{51} = V^{52} = V^{4}, \tag{9}$$

where $V^4 := V(\emptyset, A; A, B) = \max\{V^{41}, V^{42}, V^{43}\}$. Let

$$M := (1-2p)\delta\{V(\emptyset, A, B; A, B) - V(\emptyset, B; A, B)\} + p\delta\{V(\emptyset, A; A) - V(\emptyset; A)\}.$$

Then,

$$V^{43} - V^{41} = M - (1 - w + w^*), \tag{10}$$
$$V^{531} - V^{533} = M - \left(1 - 2\lambda^{\frac{1}{2}} + w^* - w\right). \tag{11}$$

Thus, we have

Claim 7. $V^{43} > V^{41} \Rightarrow V^{531} > V^{533}$.

To know the implications from (8) and (9) using (10) and (11), we explore the value of M. Since $V(\emptyset, A, B; A, B) = V^{531}$ and $V(\emptyset, A; A, B) = V^4$, we have

$$M = (1-2p)\delta(V^{531} - V^4) + p\delta(V(\emptyset, A; A) - V(\emptyset; A)). \tag{12}$$

In the above M, we have the three unknowns: The first unknown is V^{531}. It follows from Section B.1.5 that

$$V^{531} = \pi(A, B^*; A, B) + \pi(B, A^*; A, B) + (1-2p)\delta V^{531} + p\delta V(\emptyset, A; A) + p\delta V(\emptyset, B; B).$$

Thus, solving this equation for V^{531}, we have

$$
\begin{aligned}
V^{531} &= \frac{1}{1-(1-2p)\delta}\Big(\pi(A, B^*; A, B) + \pi(B, A^*; A, B) \\
&\quad + p\delta V(\emptyset, A; A) + p\delta V(\emptyset, B; B)\Big). \qquad (13)
\end{aligned}
$$

The second unknown is

$$
\begin{aligned}
V(\emptyset, A; A) - V(\emptyset; A) &= V(\emptyset, A; A) - \pi(\emptyset, A^*; A) - \pi(\emptyset, A; A) - \delta V(\emptyset, A; A) \quad (\because (6)) \\
&= \pi(A, A^*; A) + \pi(A, A; A) - \pi(\emptyset, A^*; A) - \pi(\emptyset, A; A) \quad (\because (7)) \\
&= (1+\lambda)^\alpha - w - w^* + 2^\alpha - 2w - (\lambda^\alpha - w^*) - (1-w) \\
&= (1+\lambda)^\alpha + 2^\alpha - \lambda^\alpha - 1 - 2w. \qquad (14)
\end{aligned}
$$

The third unknown is V^4 for which we have two cases from (10): $M < 1 + w^* - w$ and $M \geq 1 + w^* - w$.
Case 1: $M < 1 + w^* - w$. Then, $V^{43} - V^{41} = M - (1 - w + w^*) < 0$ and thus, $V^{43} < V^{41}$. Thus, since $V^{41} > V^{42}$ and $V^{43} > V^{42}$ by Claim 5 in Section B.1.4, we have $V^4 = V^{41}$. Now, it follows from Section B.1.4 that

$$
V^4 = V^{41} = \pi(A, B^*; A, B) + \pi(A, B; A, B) + (1-2p)\delta V(\emptyset, B; A, B) + p\delta V(\emptyset, B; A) + p\delta V(\emptyset, B; B).
$$

Since $V(\emptyset, B; A, B) = V^4$ and $V(\emptyset, B; A) = V(\emptyset; A)$, this becomes

$$
\begin{aligned}
& V^4 = \pi(A, B^*; A, B) + \pi(A, B; A, B) + (1-2p)\delta V^4 + p\delta V(\emptyset; A) + p\delta V(\emptyset, B; B) \\
\Rightarrow \quad & V^4 = \frac{1}{1-(1-2p)\delta}\left(\pi(A, B^*; A, B) + \pi(A, B; A, B) + p\delta V(\emptyset; A) + p\delta V(\emptyset, B; B)\right).
\end{aligned}
$$

With this, we have obtained the three unknowns. Hence, $(V^{531} - V^4)$ in M can be calculated as

$$
\begin{aligned}
V^{531} - V^4 &= \frac{1}{1-(1-2p)\delta}\Big(\pi(B, A^*; A, B) - \pi(A, B; A, B) \\
&\quad + p\delta V(\emptyset, A; A) - p\delta V(\emptyset; A)\Big). \qquad (15)
\end{aligned}
$$

Thus, we can calculate the value of M as follows.

$$
\begin{aligned}
M &= \frac{(1-2p)\delta}{1-(1-2p)\delta}\Big\{\pi(B,A^*;A,B)-\pi(A,B;A,B)+p\delta V(\emptyset,A;A)-p\delta V(\emptyset;A)\Big\} \\
&\quad +p\delta\left\{V(\emptyset,A;A)-V(\emptyset;A)\right\} \\
&= \frac{1}{1-(1-2p)\delta}\Big[(1-2p)\delta\left\{\pi(B,A^*;A,B)-\pi(A,B;A,B)\right\}+(1-2p)p\delta^2\Big\{V(\emptyset,A;A) \\
&\quad -V(\emptyset;A))\Big\}+\left\{1-(1-2p)\delta\right\}p\delta(V(\emptyset,A;A)-V(\emptyset;A))\Big] \\
&= \frac{1}{1-(1-2p)\delta}\Big[(1-2p)\delta\Big\{\pi(B,A^*;A,B)-\pi(A,B;A,B)\Big\}+p\delta\Big\{V(\emptyset,A;A)-V(\emptyset;A)\Big\}\Big] \\
&= \frac{1}{1-(1-2p)\delta}\Big[(1-2p)\delta\{\lambda^{\frac{1}{2}}-w-w^*-(1-2w)\}+p\delta\{V(\emptyset,A;A)-V(\emptyset;A)\}\Big] \\
&= \frac{-(1-2p)\delta(1-\lambda^{\frac{1}{2}}+w^*-w)+p\delta(V(\emptyset,A;A)-V(\emptyset;A))}{1-(1-2p)\delta}.
\end{aligned}
$$

Therefore, we substitute this M into (11) to get the following implication from (8).

$$
\begin{aligned}
&\frac{-(1-2p)\delta(1-\lambda^{\frac{1}{2}}+w^*-w)+p\delta(V(\emptyset,A;A)-V(\emptyset;A))}{1-(1-2p)\delta} \geq 1-2\lambda^{\frac{1}{2}}+w^*-w \\
\Rightarrow\ &1-\lambda^{\frac{1}{2}}+w^*-w-\lambda^{\frac{1}{2}}+(1-2p)\delta\lambda^{\frac{1}{2}} \leq p\delta(V(\emptyset,A;A)-V(\emptyset;A)). \qquad (16)
\end{aligned}
$$

On the other hand, we use (15) to get the following implication from (9).

$$
\begin{aligned}
V^{531}-V^{41} &= \frac{1}{1-(1-2p)\delta}\Big\{\pi(B,A^*;A,B)-\pi(A,B;A,B)+p\delta V(\emptyset,A;A)-p\delta V(\emptyset;A)\Big\} \geq 0. \\
&\Rightarrow\ \lambda^{\frac{1}{2}}-w-w^*-(1-2w)+p\delta(V(\emptyset,A;A)-V(\emptyset;A)) \geq 0 \\
&\Rightarrow\ 1-\lambda^{\frac{1}{2}}+w^*-w \leq p\delta(V(\emptyset,A;A)-V(\emptyset;A)). \qquad (17)
\end{aligned}
$$

Therefore, for Case 1, we have the two necessary conditions (16) and (17). In these equations, since $-\lambda^{\frac{1}{2}}+(1-2p)\delta\lambda^{\frac{1}{2}} < 0$, (17) implies (16). Hence, we have (17) as a necessary condition.

Case 2: $1+w^*-w \leq M$. Then, $V^{43}-V^{41} = M-(1-w+w^*) \geq 0$ and thus, $V^{43} \geq V^{41}$. Thus, since $V^{41} > V^{42}$ and $V^{43} > V^{42}$ under Claim 5 in Section B.1.4, we have $V^4 = V^{43}$. Now, it follows from Section B.1.4 that

$$
\begin{aligned}
V^4 &= V^{43} = \pi(A, A^*; A, B) + \pi(A, B^*; A, B) + (1-2p)\delta V^{531} + p\delta V(\emptyset, A; A) + p\delta V(\emptyset, B; B) \\
&= 0 - w - w^* + \lambda^{\frac{1}{2}} - w - w^* + (1-2p)\delta V^{531} + p\delta V(\emptyset, A; A) + p\delta V(\emptyset, B; B).
\end{aligned}
$$

Then, (9) is automatically satisfied as follows.

$$
\begin{aligned}
V^{531} - V^4 &= V^{531} - V^{43} \\
&= (1-(1-2p)\delta)\, V^{531} + 2w + 2w^* - \lambda^{\frac{1}{2}} - p\delta V(\emptyset, A; A) - p\delta V(\emptyset, B; B) \\
&= \pi(A, B^*; A, B) + \pi(B, A^*; A, B) + p\delta V(\emptyset, A; A) + p\delta V(\emptyset, B; B) \\
&\quad + 2w + 2w^* - \lambda^{\frac{1}{2}} - p\delta V(\emptyset, A; A) - p\delta V(\emptyset, B; B) \quad \text{(by (13))} \\
&= \lambda^{\frac{1}{2}} - w - w^* + \lambda^{\frac{1}{2}} - w - w^* + 2w + 2w^* - \lambda^{\frac{1}{2}} \\
&= \lambda^{\frac{1}{2}} > 0.
\end{aligned}
$$

Thus, we substitute this value $(V^{531} - V^4)$ into (12) to get

$$
M = (1-2p)\delta\lambda^{\frac{1}{2}} + p\delta(V(\emptyset, A; A) - V(\emptyset; A)).
$$

We substitute this M into (11) to get the following implication of (8).

$$
\begin{aligned}
& (1-2p)\delta\lambda^{\frac{1}{2}} + p\delta(V(\emptyset, A; A) - V(\emptyset; A) \geq 1 - 2\lambda^{\frac{1}{2}} + w^* - w \\
\Rightarrow\ & 1 - 2\lambda^{\frac{1}{2}} + w^* - w - (1-2p)\delta\lambda^{\frac{1}{2}} \leq p\delta(V(\emptyset, A; A) - V(\emptyset; A)). \qquad (18)
\end{aligned}
$$

Therefore, for Case 2, we have the necessary condition (18).

In sum, we have (17) for Case 1 or (18) for Case 2. Since (17) implies (18), we have (17) as a necessary condition for the rotation to be optimal. Hence, $p \geq \frac{1-w+w^*-\lambda^{\frac{1}{2}}}{\delta\{(1+\lambda)^\alpha + 2^\alpha - \lambda^\alpha - 1 - 2w\}}$.

B.2.2 An almost sufficient condition for the rotation to be optimal

Suppose that

$$
p\delta\left\{V(\emptyset, A; A) - V(\emptyset; A)\right\} > 1 - w + w^* - \lambda^{\frac{1}{2}}. \qquad (19)
$$

The value of the rotation, π^R, is

$$
\begin{aligned}
\pi^R = V^{531} &= \pi(A, B^*; A, B) + \pi(B, A^*; A, B) \\
&\quad + (1-2p)\delta V(\emptyset, A, B; A, B) + p\delta V(\emptyset, A; A) + p\delta V(\emptyset, B; B).
\end{aligned}
$$

We need to show

$$V^{531} \geq V^{532} \tag{20}$$
$$V^{531} \geq V^{533} \tag{21}$$
$$V^{531} \geq V^{51} \tag{22}$$
$$V^{531} \geq V^{52} \tag{23}$$
$$V^{531} \geq V^{54} = V^{53} \tag{24}$$
$$V^{531} \geq V^{41} \tag{25}$$
$$V^{531} \geq V^{42} \tag{26}$$
$$V^{531} \geq V^{43} \tag{27}$$

Here, the last three inequalities follow from the fact that $V^{531} \geq V^{51}$ and $V^{51} = V^{52} = V(\emptyset, A; A, B) \equiv V^4 = \max\{V^{41}, V^{42}, V^{43}\}$.

Claim 8. The three inequalities (21), (25), and (27) are sufficient for all of the above inequalities to hold.

Proof. Suppose that (21), (25), and (27) hold. First of all, Claim 6 implies (20). Note that when (25) is true, by Claim 5 ($V^{41} > V^{42}$), (26) holds. Hence, it follows from (25), (26), and (27) that $V^{531} \geq V^4$. This implies that (22) and (23) are true. On the other hand, since we have (20) and (21), we have $V^{531} = V^{53}$ and thus (24). □

From now on, we will check (21), (25), and (27).

- Check whether $V^{531} \geq V^{533}$, or (21).

$$\begin{aligned}
V^{531} - V^{533} &= (1-2p)\delta\{V(\emptyset, A, B; A, B) - V(\emptyset, B; A, B)\} \\
&\quad + p\delta\{V(\emptyset, A; A) - V(\emptyset; A)\} - (1 - 2\lambda^{\frac{1}{2}} + w^* - w) \\
&> (1-2p)\delta\{V(\emptyset, A, B; A, B) - V(\emptyset, B; A, B)\} \\
&\quad + 1 - w + w^* - \lambda^{\frac{1}{2}} - (1 - 2\lambda^{\frac{1}{2}} + w^* - w) \quad (\because (19)) \\
&= (1-2p)\delta\{V(\emptyset, A, B; A, B) - V(\emptyset, B; A, B)\} + \lambda^{\frac{1}{2}} \\
&\geq \lambda^{\frac{1}{2}} \quad (\because V(\emptyset, A, B; A, B) \geq V(\emptyset, B; A, B)) \\
&> 0.
\end{aligned}$$

- Check whether $V^{531} \geq V^{41}$, or (25).

$$\begin{aligned}
V^{531} - V^{41} &= \pi(B, A^*; A, B) - \pi(A, B; A, B) \\
&\quad +(1-2p)\delta\{V(\emptyset, A, B; A, B) - V(\emptyset, B; A, B)\} + p\delta\{V(\emptyset, A; A) - V(\emptyset; A)\} \\
&\geq \pi(B, A^*; A, B) - \pi(A, B; A, B) \\
&\quad +p\delta\{V(\emptyset, A; A) - V(\emptyset; A)\} \quad (\because V(\emptyset, A, B; A, B) \geq V(\emptyset, B; A, B)) \\
&> \pi(B, A^*; A, B) - \pi(A, B; A, B) + 1 - w + w^* - \lambda^{\frac{1}{2}} \quad (\because (19)) \\
&= \lambda^{\frac{1}{2}} - w - w^* - (1-2w) + 1 - w + w^* - \lambda^{\frac{1}{2}} \\
&= 0.
\end{aligned}$$

- Check whether $V^{531} \geq V^{43}$, or (27).

$$\begin{aligned}
V^{531} - V^{43} &= \pi(B, A^*; A, B) - \pi(A, A^*; A, B) \\
&= \lambda^{\frac{1}{2}} - w - w^* - (0 - w - w^*) \\
&= \lambda^{\frac{1}{2}} \\
&> 0.
\end{aligned}$$

B.3 Proof of Lemma 2

Suppose that the rotation is not optimal. Then, we will check the two cases: $V^{533} > V^{531}$ and $V^{533} \leq V^{531}$.

Case 1: $V^{533} > V^{531}$. Since $V^{531} > V^{532}$ under Claim 6, we have $V^{53} = V^{533}$.

Claim 9. $V^{41} > V^{43}$ and thus $V^4 = V^{41}$.

Proof. Since $V^{533} > V^{531}$ holds, we have

$$\begin{aligned}
V^{533} - V^{531} &= \pi(A, A^*; A, B) + \pi(B, A; A, B) - \pi(A, B^*; A, B) - \pi(B, A^*; A, B) \\
&\quad +(1-2p)\delta\{V(\emptyset, A; A, B) - V(\emptyset, A, B; A, B)\} + p\delta\{V(\emptyset; B) - V(\emptyset, B; B)\} > 0.
\end{aligned}$$

Thus,

$$\begin{aligned}
V^{41} - V^{43} &= \pi(A, B^*; A, B) + \pi(A, B; A, B) - \pi(A, A^*; A, B) - \pi(A, B^*; A, B) \\
&\quad +(1-2p)\{\delta V(\emptyset, B; A, B) - V(\emptyset, A, B; A, B)\} + p\delta V\{(\emptyset; A) - V(\emptyset, A; A)\} \\
&> \pi(A, B^*; A, B) + \pi(A, B; A, B) - \pi(A, A^*; A, B) - \pi(A, B^*; A, B) \\
&\quad -\pi(A, A^*; A, B) - \pi(B, A; A, B) + \pi(A, B^*; A, B) + \pi(B, A^*; A, B) \\
&\quad (\because \text{ by the inequality derived from } V^{533} > V^{531} \text{ in the above}) \\
&= 2\pi(A, B^*; A, B) - 2\pi(A, A^*; A, B) \\
&= 2(\lambda^{\frac{1}{2}} - w - w^*) - 2(0 - w - w^*) \\
&= 2\lambda^{\frac{1}{2}} > 0.
\end{aligned}$$

Thus, $V^{41} > V^{43}$. Moreover, since $V^{41} > V^{42}$ under Claim 5, we have $V^4 = V^{41}$. □

Claim 10. $V^{51} > V^{533} = V^{53}$ and thus $V^5 = V^{51}$.

Proof. We have

$$\begin{aligned}
V^{51} - V^{533} &= V^4 - V^{533} \\
&= V^{41} - V^{533} \quad (\because \text{ Claim 9}) \\
&= \pi(A, B^*; A, B) + \pi(A, B; A, B) - \pi(A, A^*; A, B) - \pi(B, A; A, B) \\
&\quad +(1-2p)\delta\{V(\emptyset, B; A, B) - V(\emptyset, A; A, B)\} \\
&\quad +p\delta\{V(\emptyset, B; A) + V(\emptyset, B; B) - V(\emptyset, A; A) - V(\emptyset, A; B)\} \\
&= \pi(A, B^*; A, B) - \pi(A, A^*; A, B) \\
&= \lambda^{\frac{1}{2}} - w - w^* - (0 - w - w^*) \\
&= \lambda^{\frac{1}{2}} > 0.
\end{aligned}$$

Thus, $V^{51} > V^{533}$. Then, since we know $V^{533} = V^{53}$ and $V^{51} = V^{52}$, we have $V^5 = V^{51}$. □

Note that we have $V^{51} = V^4$, $V^5 = V^{51}$ ($\because$ Claim 10), and $V^4 = V^{41}$ ($\because$ Claim 9). Thus, $V^5 = V^{41}$, which is the specialization value. This means that the specialization is optimal.
Case 2: $V^{533} \leq V^{531}$. Then, since $V^{531} > V^{532}$ under Claim 6, we have $V^{53} = V^{531}$, which is the rotation value.

Claim 11. $V^{51} = V^4 > V^{531}$.

Proof. By definition, $V^{51} = V^4$. Suppose to the contrary that $V^{51} = V^4 \leq V^{531}$. Then, $V^{531} \geq V^4 = V^{51} = V^{52}$, which means that the rotation is optimal. However, this contradicts the initial argument that the rotation is not optimal. $\square$

Claim 12. $V^{41} \geq V^{43}$ and thus $V^4 = V^{41}$.

Proof. Suppose to the contrary that $V^{41} < V^{43}$. Then, since $V^{42} < V^{41}$ ($\because$ Claim 5), we have $V^4 = V^{43}$. Thus, under Claim 11, we have $V^{43} > V^{531}$. However, this inequality contradicts the following.

$$\begin{aligned} V^{43} - V^{531} &= \pi(A, A^*; A, B) + \pi(A, B^*; A, B) - \pi(A, B^*; A, B) - \pi(B, A^*; A, B) \\ &\quad +(1-2p)\delta\{V(\emptyset, A, B; A, B) - V(\emptyset, A, B; A, B)\} \\ &\quad +p\delta\{V(\emptyset, A; A) + V(\emptyset, B; B) - V(\emptyset, A; A) - V(\emptyset, B; B)\} \\ &= \pi(A, A^*; A, B) - \pi(B, A^*; A, B) \\ &= -w - w^* - (\lambda^{\frac{1}{2}} - w - w^*) \\ &= -\lambda^{\frac{1}{2}} < 0 \end{aligned}$$

Hence, we have $V^{41} \geq V^{43}$. Moreover, as $V^{41} > V^{42}$ ($\because$ Claim 5), we have $V^4 = V^{41}$. $\square$

Under claims 11 and 12, we have $V^{51} = V^4 = V^{41} > V^{531} = V^{53}$. This means that the specialization is optimal.

References

Anderson, K. A. (2012): "Specialists and Generalists: Equilibrium Skill Acquisition Decision in Problem-solving Populations," *Journal of Economic Behavior and Organization*, 84, 463–473.

Aoki, M. and M. Okuno (1996): *Comparative Institutional Analysis of Economic Systems*, University of Tokyo Press (in Japanese).

Brünner, T., G. Friebel, R. Holden, and S. Prasad (2019): "Incentives to Discover Talent," Working paper.

Campion, M. A., L. Cheraskin, and M. J. Stevens (1994): "Career-Related Antecedents and the Outcomes of Job Rotation," *Academy of Management Journal*, 37, 1518–1542.

Carmichael, H. L. and W. B. MacLeod (1993): "Multiskilling, Technical Change and the Japanese Firm," *Economic Journal*, 103, 142–160.

Hansmann, H. and R. Kraakman (2001): "The End of History for Corporate Law," *Georgetown Law Journal*, 89, 439–468.

Ito, T. (1992): *The Japanese Economy*, MIT Press.

Iwai, K. (1999): "Persons, Things and Corporations: The Corporate Personality Controversy and Comparative Corporate Governance," *American Journal of Comparative Law*, 47, 583–632.

——— (2014): "Capitalism and Cultures – Universality and Particularity of the Corporate System across Societies," The Clarke Lecture, Cornell Law School.

Jacoby, S. M., E. M. Nason, and K. Saguchi (2005): "The Role of the Senior HR Executive in Japan and the United States: Employment Relations, Corporate Governance, and Values," *Industrial Relations*, 44, 207–241.

Kambayashi, R. and T. Kato (2012): "Trends in Long-term Employment and Job Security in Japan and the United States: the Last Twenty-Five Years," CJEB Working Papers No. 302, Columbia University.

Katz, H. C. and O. Darbishire (2000): *Converging Divergences: Worldwide Changes in Employment Systems*, Cornell University Press.

Kurino, M. (2014): "House Allocation with Overlapping Generations," *American Economic Journal: Microeconomics*, 6, 258–289.

Ortega, J. (2001): "Job Rotation as a Learning Mechanism," *Management Science*, 47, 1361–1370.

Sako, M. (2005): "Does Embeddedness Imply Limits to within-Country Diversity?" *British Journal of Industrial Relations*, 43, 585–592.