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「人口大国の時代」とマルクス派最適成長論

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【要旨】

アジア NIES が勃興した時代は「人口小国に有利な時代」であったということができるが、その後、アジアの高成長は ASEAN 諸国への伝播を経て、中国、インドなど「人口大国」にももたらされつつある。BRICS とは後で加わった南アフリカを除けば、すべて 1 億人を超える人口大国であり、その時代を象徴している。本稿はその傾向を最初に確認した後、まずはその傾向の背景にある法則を明らかとし、最後にそれが「マルクス派最適成長論」で展開されている成長理論と整合的であることを見る。どの国もが高度成長期を必ず持ち、その後におよそ類似の一人当たり GDP を実現するのであれば、それはそのうちに各国 GDP 比が各国人口比に接近していくことが必然となるからである。

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Summary
While it appears that small-population economies were advantageous for growth when Asia's newly industrialized economies (NIEs) were expanding rapidly, we are now seeing a different trend in which large-population countries like China and India have become the most rapidly growing nations in the world. This is true for the BRIC states (Brazil, Russia, India, and China) as a whole. Brazil and Russia also have large populations of over 100 million, and their geopolitical and economic influence is crucial. This is one of the most important features of the present geopolitical economy. The present paper first demonstrates this trend statistically and then proposes the hidden historical law underlying these phenomena. This anti-Malthusian law can be explained by Marxian Optimal Growth Theory, as developed by our research group. This shows that each country experiences its own rapid growth phase over a certain period and finally realizes a higher per capita GDP similar to that of the present advanced nations. Under this trend, GDP balance among countries will become closer to the population balance among countries. It should be a much more equal world.

Key Words
Large-Population Countries, NIEs, BRIC, Marxian Optimal Growth Theory

Introduction
Asia's high economic growth area has moved from Japan in the 1950s and 1960s to NIEs that can be identified as small-population states. In this sense, this stage could be called "The Age of Small-Population Countries." However, Asia's high economic growth area expanded through the ASEAN countries to China and India, whose population sizes are not small. In other words, if we focus on China, India, or Indonesia, we see that large-population countries have advantages in realizing a higher GDP. From this viewpoint, the newly emerging and crucially geopolitical BRIC states form a representative group of large-population countries..[1] This is one of the most important features of the present geopolitical economy. Therefore, this paper first analyzes these trends, then finds a historical law behind these phenomena, and finally discusses how these trends are consistent with the Marxian Optimal Growth Theory. Phenomena cannot be understood without knowing the hidden laws that give them their meaning.

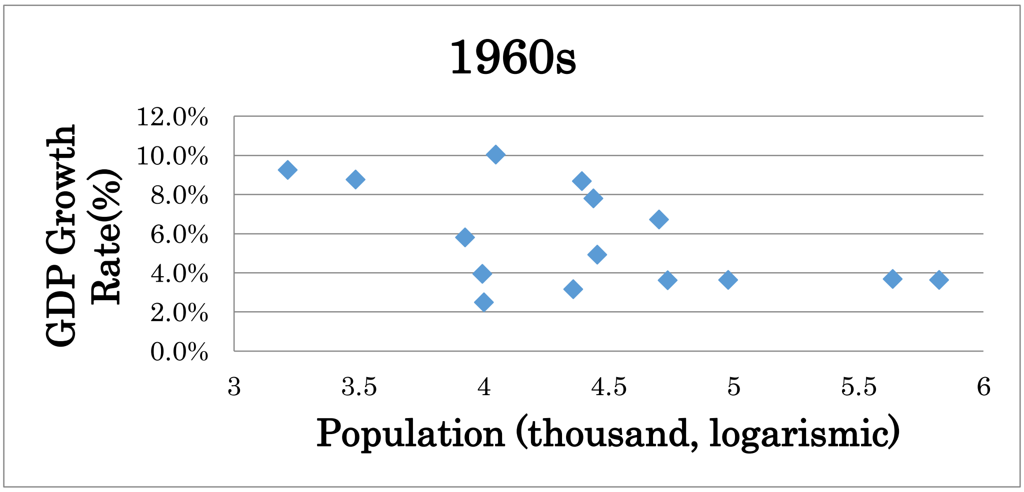
Post-War Asian Growth Started From Small-Population Countries
Kuwait, UAE, and Brunei showed that their small populations were better able to realize higher GDP per capita with their given natural resources. However, a different type of small-population economy has become much more notable in Asia. For example, Hong Kong, Singapore, South Korea, and Taiwan, which were originally called newly industrialized countries (NICs) by the OECD (1979), and were later designated as "NIEs." When the OECD coined this term in 1979, the populations of Hong Kong and Singapore were only 5 million and 2.4 million respectively, while those of South Korea and Taiwan were 18 million and 38 million. However, these limited populations did not create any difficulties for industrial development, because if capital can be accumulated,

[1] This is the point of Desai (2013), Chapter 9.

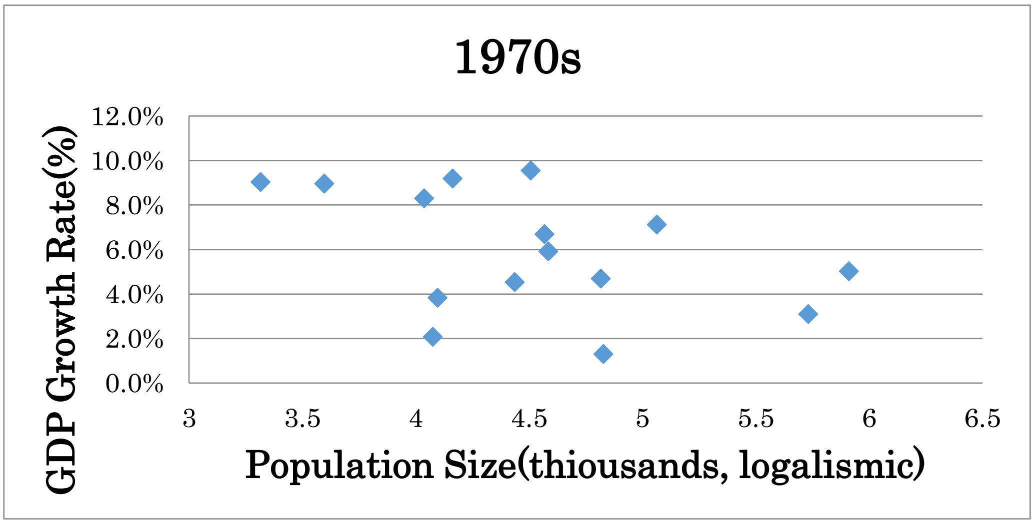
smaller populations are not significant for an industrial economy. In other words, even if a country has a large-population, if it does not have enough capital, its industrial economy cannot be large. Therefore, these small-population countries could undertake industrialization soon after Japan's high growth phase.

This trend can be seen in **Figure 1**, which shows the changing relationship between population size and GDP growth in East and South Asian countries. Here, "East and South Asian countries" includes NIEs, China, major ASEAN countries, and five South Asian countries. Vietnam, Laos, and Cambodia are excluded because the fluctuations created by the Vietnam War were too large. GDP growth rates are calculated from the purchasing power arity base data in Maddison (2006),[2] and the logarithm of the population has been used in order to cover the wide range from several millions to over one billion. Each graph has two plots on the right side and two plots on the left side. The former pair is Singapore and Hong Kong, while the latter pair is China and India.

Figure 1 Changed Relation between Population Size and GDP Growth in East and South Asian Countries



[2] I calculated these 2010 data by multiplying PPP base GDP per capita with each population as taken from the website of the Maddison Project, because Maddison (2006) does not include 2010 data.


1970s
12.0%
10.0%
8.0%
6.0%
4.0%
2.0%
0.0%
3
3.5
4
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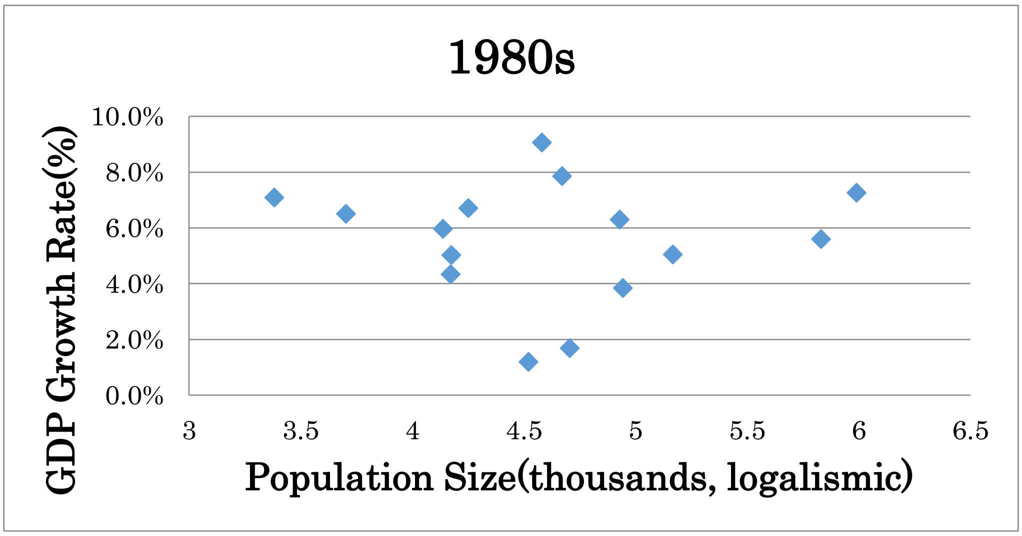
5
5.5
6
6.5
GDP Growth Rate(%)
Population Size(thiousands, logalismic)


1980s
10.0%
8.0%
6.0%
4.0%
2.0%
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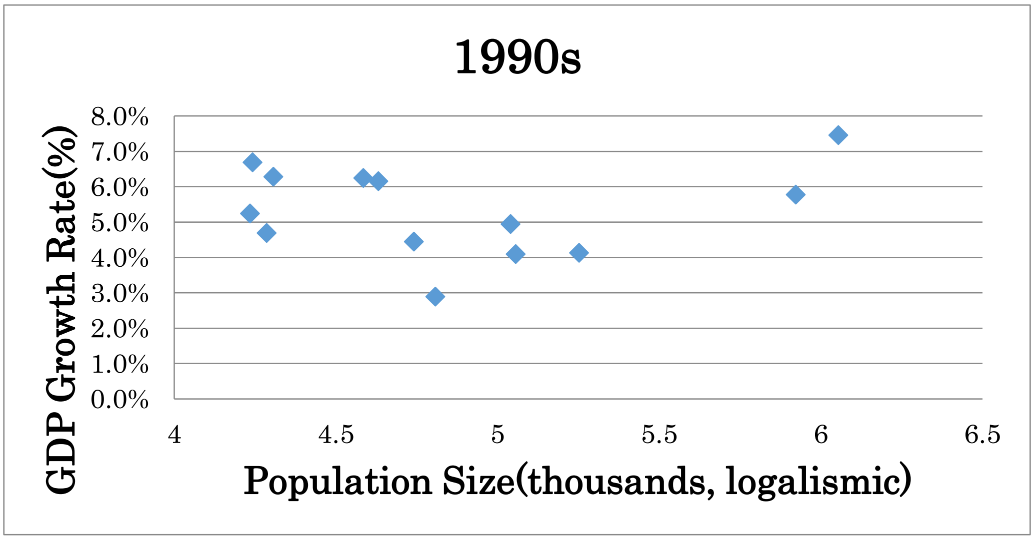
GDP Growth Rate(%)
Population Size(thousands, logalismic)


1990s
8.0%
7.0%
6.0%
5.0%
4.0%
3.0%
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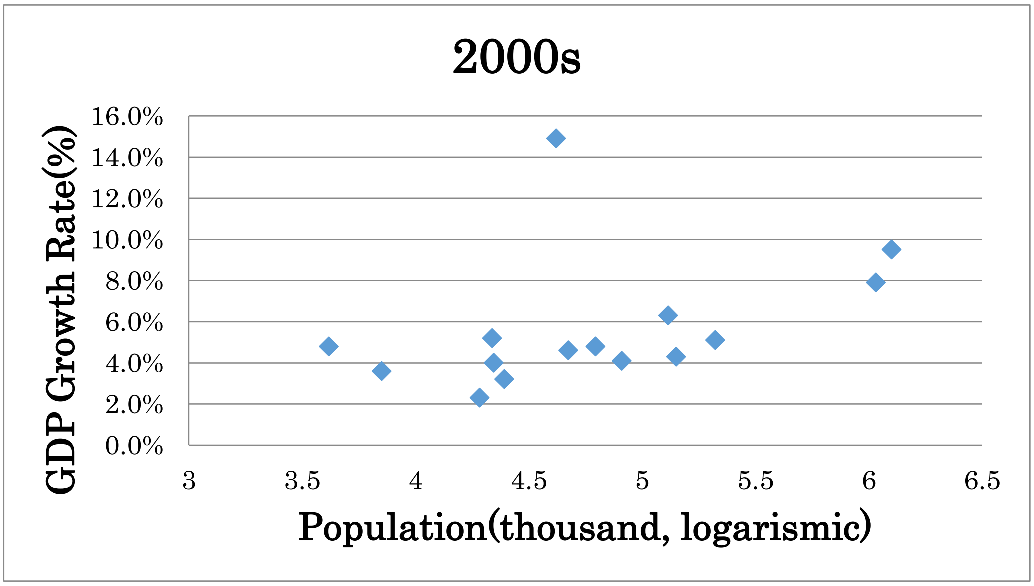
GDP Growth Rate(%)
Population Size(thousands, logalismic)



Data Source: Explained in the text.

Let us then analyze these figures, and take note that high growth in the small-population economies starts in 1960s, which is earlier than the publication of OECD (1979). It appears soon after the Japanese high-growth phase, which starts in the middle of the 1950s. The 1970s also display this characteristic. However, this pattern disappears in 1980s-1990s, and then appears again in the 2000s, but in opposite form. This change can be measured through the correlation coefficients and their *p*-values in parentheses as follows:

1960s	-0.5297 (0.042)
1970s	-0.4470 (0.095)
1980s	-0.0346 (0.902)
1990s	-0.0819 (0.772)
2000s	0.3950 (0.145)

Therefore, we can identify the "Age of Small-Population Economies" only in the 1960s at the 5 percent significance level, and the "Age of Large-Population Countries" in the 2000s at close to the 10 percent significance level. However, if we remove Myanmar from the figure for the 2000s, the correlation becomes significant at the 1 percent level. The outlier in the upper-middle of the graph for the 2000s is Myanmar, which results from specific political factors such as China's special aid and the lifting of economic sanctions. Therefore, there are sufficient reasons to exclude this data point. By removing it, the correlation coefficient and its *p*-value in the 2000s become 0.7719 and 0.001 respectively. This means that Asia has changed to the "Age of Large-Population Countries" in the new century. This contradicts the Malthusian and neo-Malthusian hypotheses.

"The Age of Large-Population Countries"

Therefore, the situation in world politics has changed. The BRIC countries have been rising, followed by Indonesia, Vietnam, Turkey, and Argentina. These new rising countries have been noted by Hirakawa (2014) and named as "Potentially Bigger

Market Economies" (PoBMEs). The point of this designation is that these large-population economies have large potential markets and this condition is very important for growth. It reflects a new world trend in that companies among the industrialized nations are going into these countries to capture these very attractive markets. This is one way to reflect the importance of large-population countries, although my approach is different, as I will discuss here.

Thus, we first identify past trends of world GDP on a PPP basis from the data in the IMF: World Economic Outlook Database (October 2010) and World Bank (2005), and then overview future trends from the projections by Menshikov (2013) based on the data of the World Development Report: Projections in World Bank (2005). The left side figures in **Table 1** are the results of calculations that show the projections of the fifteen countries until 2030 on the PPP base. Take note here that the value for Russia in the row for 1980 reflects data from 1992. This is because the IMF does not have Russian data from before the collapse of the Soviet Union.

Table 1 Trend of the Ratios of GDP and Populations of the Fifteen Major Countries

	GDP Ratio			Population Ratio		
Year	1980	2003	2030	1980	2003	2030
Brazil	4.8%	2.6%	2.4%	4.5%	5.0%	5.4%
Russia	12.6%*	2.5%	6.9%	5.5%*	4.0%	3.6%
India	3.1%	5.2%	8.7%	25.1%	29.9%	33.3%
China	2.7%	14.0%	29.1%	36.3%	35.7%	34.9%
Canada	2.9%	3.1%	2.9%	0.9%	0.9%	0.8%
France	5.7%	4.0%	3.7%	2.0%	1.7%	1.5%
Germany	8.1%	5.2%	4.2%	2.9%	2.3%	1.7%
Italy	5.4%	2.8%	2.7%	2.1%	1.6%	1.2%
Japan	10.5%	7.7%	7.2%	4.3%	3.5%	2.8%
S. Korea	0.9%	2.1%	2.6%	1.4%	1.3%	1.2%
Mexico	3.6%	1.8%	2.2%	2.5%	2.9%	3.3%
Netherlands	1.6%	1.0%	1.0%	0.5%	0.4%	0.4%
Spain	2.9%	1.7%	1.6%	1.4%	1.1%	0.9%
UK	5.2%	3.5%	3.2%	2.1%	1.7%	1.4%
USA	29.9%	23.0%	21.5%	8.4%	8.0%	7.6%
Total	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Data Source: Explained in the text. *Russian data from 1992 is substituted for the figure in 1980.

The most notable point in this table is the greater size of the Soviet Union before its breakup. Because Russia was only a part of Soviet Union, the figure of 12.6 percent under "1980" means that the Soviet Union had about 20 percent of the total GDP of the fifteen countries at that time. However, in 2003, the Russian ratio had become one fifth of its 1980 value. Therefore, the present increasing trend of Russian GDP represents a recovery.

However, what we need to identify is whether these patterns can be understood as a trend for the "Age of Large-Population Countries." In order to check this point, I also show the trends of the population ratios on the right side of **Table 1**. Here, all of the figures came from the World Bank's datasets such as World Bank (1994). Take note again that the Russian figure for 1992 is substituted for the value under "1980."

Comparing the two sides of **Table 1** enables us to observe big differences among them in 1980, which have however shrunk rapidly as expected. In particular, the Chinese GDP ratio will be almost same as that of its population in 2030. In other words, Chinese GDP per capita in 2030 will catch up with the world standard. Because one more populous country, India, is also in the process of catching up, there will be a "normalization" process of GDP size compared with population size for 2030. This pace can be expressed by calculating the weighted average[3] of the squared "ratio of GDP minus ratio of population" in each year. The results of these calculations are as follows:

1980 0.0576
2003 0.0370
2030 0.0230

The "normalization" process is shown clearly.

However, a much more important question here should be which countries are driving this process and which countries are not contributing well. Therefore, I have created **Table 2,** which indicates the rate of contribution for the total squared ratios of GDP to ratios of population for each country. This table reveals the importance of China and India generally, but also reveals the changing situation across the years. While the low Chinese GDP was critical in 1980, the low Indian GDP will be critical in 2030. India will also grow faster from 2030, but will not catch up sufficiently before then. The ratio of Indian GDP in 2030 will be 8.7 percent, which is almost same as the Chinese rate of around 1990. If this is so, India will be following China with a delay of about 40 years. However, what we should note here is not so much the significant delay, but the fact that India is also catching up with the advanced countries. This is the fact of which Hirakawa (2014) took note.

Table-2 Rate of Contribution for the Total Squared Ratios of GDP to Ratios of Population of Fifteen Countries

	1980	2003	2030
Brazil	0%	0%	0.2%
Russia	0.5%	0.0%	0.2%
India	21.1%	49.3%	87.7%
China	71.1%	45.4%	5.1%
Canada	0.0%	0.0%	0.0%
France	0.0%	0.0%	0.0%
Germany	0.1%	0.1%	0.0%
Italy	0.0%	0.0%	0.0%
Japan	0.3%	0.2%	0.2%
S. Korea	0.0%	0.0%	0.0%

[3] Here, population is used as the weight.

Mexico	0.0%	0.0%	0.0%
Netherlands	0.0%	0.0%	0.0%
Spain	0.0%	0.0%	0.0%
UK	0.0%	0.0%	0.0%
USA	6.7%	4.9%	6.4%
Total	100.0%	100.0%	100%

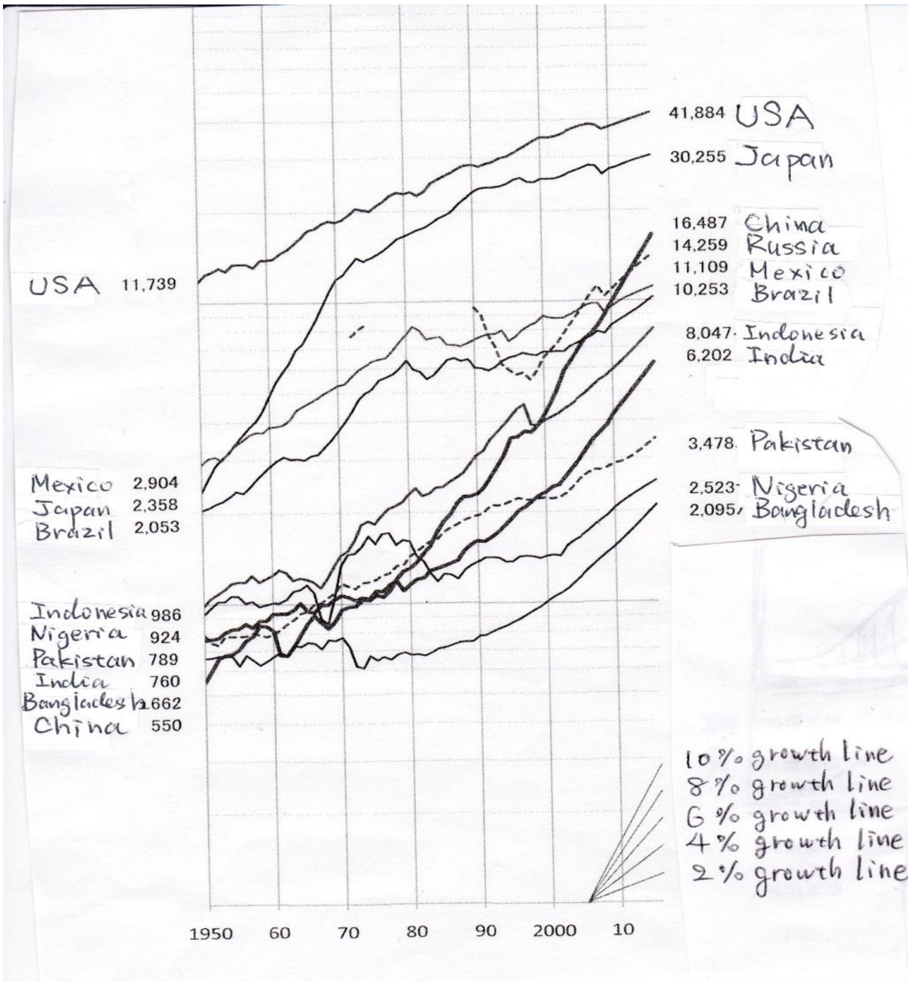
Data Source: **Table 1**

Historically Consistent Explanation of the Trend

In this way, we can identify the change from the "Age of Small-Population Economies" to the "Age of Large-Population Countries." However, it is not enough to understand this trend simply as a shift of economic advantage from small-population economies to large-population countries, because this new trend is actually just reflecting the catching-up of GDP per capita by the large-population countries. In other words, as we saw in **Figure 1**, the point of this change is that high growth has moved from the small-population economies to the large-population countries. Therefore, this is part of the historical principle that all countries follow one historical law and must at some point pass through each stage of development, including a high-growth period. This is the historical law that Karl Marx found and declared[4]. In our case study, this high growth started from the small-population economies, and then the middle- and large-population countries followed one after another. In reality, two of East Asia's four small-population economies are just cities, and in the case of South Korea, one third of its population is concentrated in Seoul, while adding Pusan gives half the population living in megacities. Therefore, we can see that the progress of the East Asian high-growth phase has been from several well-placed big cities to the hinterlands, such as Malaysia or Guangdong Province in China. Of course, this trend has continued beyond such hinterlands to the whole of Asia. Onishi (2007) analyzed the same trend among the Chinese provinces, which starts from megacities to the hinterlands and then spreads further to the whole country. Both within individual countries and crossing over national boundaries, the same developmental law is functioning.

Figure 2　Trends of PPP Base GDP Per Capita of Large-Population Countries

[4] This discourse is contrary to the dependency theory. However, the Marxian understanding was confirmed by Lenin (1917) identifying the law of uneven development of world capitalism, which theorized that the growth rate of backward capitalism is higher than that of more advanced states. See Onishi (2010).



Source: Yao (2012) p.190

To identify this law, **Figure 2** is useful. These data are from Maddison (2006) up to 2008, and as calculated after 2009 by Yao (2012) based on IMF projections. A very interesting point on this figure is that all these growth trends can be classified into "low growth," "middle growth," and "high growth" as shown in **Table 3**. Although some developing countries have never formerly experienced "high growth," if we do not quibble excessively about the differences between "high growth" and "middle growth," basically all these countries experience first "low growth," and then "middle growth" or "high growth," and finally come back to "low growth" at the stage of the advanced countries. This should be a historical law.

Table 3 Changing Stages of Development in Each Country Based on Table 2

Development Stages	Low Growth	Middle Growth	High Growth	Middle Growth	Low Growth
USA					1950-2016
Mexico				1950-1980	1980-2016
Japan			1950-1970	1970-1990	1990-2016

Brazil				1950-2016	
Indonesia		1950-2016			
Nigeria	1950-1970, 1980-2000	1970-1980, 2000-2016			
Pakistan	1950-1960	1960-2016			
India		1950-2000	2000-2016		
Bangladesh	1950-1990	1990-2000	2000-2016		
China		1950-1980	1980-2016		
Russia				2000-2016	

Note: Summarized from **Table 2.** However, "Ten Lost Years" in Russia from 1990-2000 could not be represented here.

Therefore, we can say that all countries can enjoy "high growth" at some point, but it means that "low growth" in advanced countries is inevitable. Furthermore, it implies that GDP per capita will be equalized in the future. It is true that the income gap between the richest, the USA, and the poorest, Bangladesh, had increased before 1990, but since then this gap has started to decrease. This trend is very clear between the USA and China, especially after 1980. It is the catching-up process of the developing countries.

Once this had been understood, it does not seem strange that large-population countries also have their own high-growth periods, and in this way, the catching-up of their GDP per capita will create a more balanced GDP to the ratio of population. The "normalization" process shown in **Table 1** is the result of them catching up. Therefore, the balanced GDP of the large-population countries should be understood as the result of the law of catching-up that is observed in all of these developing countries.

According to development economics, the catching-up process of developing countries is explained by factors such as import substitution industrialization and export-oriented industrialization, which are closely related to the advantages of large-population countries, because large population countries have an advantage for the former type of industrialization, while they do not have any advantage for the latter type of industrialization. As for the newly developing Asian countries, their rapid catching-up processes have been realized by export-oriented industrialization policies, but it is also true that all these countries had passed through the import substitution industrialization stage soon before taking off. This may imply that we need the import substitution industrialization stage first, and this was advantageous for the large-population countries. However, since making the change to export-oriented industrialization, large-population countries have lost their advantage. This may be the reason why export-oriented industrialization policies were adopted first by the small-population countries, and why their rapid growth phases started earlier than those of the large-population countries. Export-oriented industrialization processes in the large-population countries started after the small-population economies lost their international competitive margin against the large-population countries.

Therefore, population size is very important at certain developmental stages, and is not a major factor at other stages of development. That is why I say that population size determines the potential market volume. In fact, if we pay attention to market size as such, we can understand that the real power to absorb FDI into the large-population countries comes from their potential markets. This is the point to which Hirakawa (2014) drew attention.[5]

However, we have to remember the fact that all of these countries experience their own high-growth period at a certain stage. Even if large-population countries cannot adapt easily to export-oriented industrialization, they can follow the formerly developing countries. This is the historical law that we mentioned above. In other words, if a “potentially big market” remains merely potential, it cannot attract FDI from the outside. The point is that only when GDP per capita catches up will its “potential big market” become an actual big market.[6]

Explanation by the Marxian Optimal Growth Model

It needs to be mentioned that the above approach to understanding the historical law of development was clearly explained by the Marxian Optimal Growth Model, which was created and developed by my research group. Onishi (2011, 2012) are the most important papers in this research, explaining that all countries have their own high-growth period at some point even if their starting times are different, but they also decrease their growth rate after experiencing high growth towards a “zero growth” stage. Furthermore, this model can explain the types of advantage held by the large population countries. Therefore, in this paper, I will give a brief introduction to this model.

Given that this model belongs to Marxian economics, “production” is the most important and basic concept, and therefore it is constructed as a growth model that incorporates capital and labor force as the factors of production. Concretely speaking, production functions for the capital goods sector (G) and the consumption goods sector (F) are specified as the form of a Cobb-Douglas function as follows:

$$G(K_1, L_1) = A_1 K_1^{\alpha_1} L_1^{\beta_1}, \quad F(K_2, L_2) = A_2 K_2^{\alpha_2} L_2^{\beta_2} \qquad (1)$$

Here, K represents total capital, L represents total labor force, A_1 and A_2 represent the total factor productivity of each sector, and α and β are the elasticity of production with respect to each sector’s K and L. Furthermore, indices 1 and 2 represent the capital goods and consumption goods sectors respectively. It means that total capital and total labor force are divided into two sectors ($K=K_1+K_2$, $L=L_1+L_2$). In addition, we neglect the difference between total population and total labor force here for simplicity. In this case, we can calculate per capita production of each sector as

[5] See Hirakawa (2014), p.55.

[6] However, I also agree that a large population is also a condition to realize an actual big market. South Korea’s difficulty in changing its growth strategy to domestic demand-oriented growth came from its small population size. It is the same with Japan, which also cannot easily change its growth strategy from export-driven policies to domestic demand-driven policies. Neither of these countries have large populations like China.

$$\frac{G}{L_1} = \left(\frac{K_1}{L_1}\right)^{\alpha_1} A_1 L_1^{\alpha_1+\beta_1-1}, \quad \frac{F}{L_2} = \left(\frac{K_2}{L_2}\right)^{\alpha_2} A_2 L_2^{\alpha_2+\beta_2-1}.$$

This means that per capita production is determined only by per capita capital in each sector except for A and α in the case of constant returns to scale (α+β=1). That is, if we put aside A and α, the problem is not to be seen as a function of the population L, but in terms of the per capita capital K/L. Of course, this is the situation when the returns to scale is constant, and the above equation shows that a larger population (labor force) realizes a larger GDP per capita ($G/L+F/L$) when the returns to scale is increasing (α+β>1).

However, even in the case of constant returns to scale, this model provides a very important implication for the above mentioned historical law that all countries will reach their own stationary per capita capital (K/L) and GDP per capita ($G/L+F/L$) someday, and after that, their growth will stop. This means that economic growth in the advanced countries (where basically stationary K/L and $G/L+F/L$ have been realized) is low or around zero, while economic growth rates in developing countries (whose K/L and $G/L+F/L$ are below the stationary state) are higher than in the advanced countries. This is the catching-up process of the developing countries. It is true that as each country's technology or time preference is different, each K/L and $G/L+F/L$ will differ, but if the differences in technology and time preference are not so large, K/L and $G/L+F/L$ will not ultimately be so different. This assumption appears valid, because as we can see in **Figure 2, the** variance of GDP per capita among the developing countries is larger than that among the advanced countries.

Therefore, we will explain the details of this model, particularly incorporating a production function for the capital goods sector in the form of the Cobb-Douglas function and without assuming constant returns to scale. First, we specify the objective function as the intertemporalized form of the instantaneous utility $U(Y) = \log Y$ under the constraint of the abovementioned two production functions, total capital and total labor force. That is,

$$\max_{K_1,K_2,L_1,L_2 \geq 0} \int_0^\infty e^{-\rho t} U(Y) dt$$

$$s.t. \begin{cases} \dot{K} = G(K_1, L_1) - \delta K \\ Y = F(K_2, L_2) \\ K = K_1 + K_2 \\ L = L_1 + L_2 \end{cases} \qquad (2)$$

Here, ρ is the discount rate of the consumers and δ is the depreciation rate of capital (the same in both sectors), and we assume for the purpose of simplification that the initial value of K is small enough and that the total labor force is constant. In this case, the problem can be solved by setting the following current value of the Hamiltonian:

$$H = U(Y) + \lambda\{G(K_1, L_1) - \delta K\} + R(K - K_1 - K_2) + w(L - L_1 - L_2)$$

Here, λ is the conjugation state variable of K and indicates the price of capital goods measured in utility. R and w are the Lagrange multipliers of K and L respectively and indicate the price of capital and labor cost measured in utility in both sectors. In this case, the first-order conditions for optimizing become as follows:[7]

$$\frac{\partial H}{\partial K_1} = \frac{\partial H}{\partial K_2} = 0 \Leftrightarrow \lambda G_K = U_Y F_K = R \qquad (3)$$

$$\frac{\partial H}{\partial L_1} = \frac{\partial H}{\partial L_2} = 0 \quad \Leftrightarrow \quad \lambda G_L = U_Y F_L = w \qquad (4)$$

$$\frac{\partial H}{\partial K} = \rho\lambda - \dot{\lambda} \Leftrightarrow R - \lambda\delta + \dot{\lambda} = \rho\lambda \qquad (5)$$

From the above three conditions, we can introduce capital-labor ratio $\left(\frac{K}{L}\right)^*$, consumption-labor ratio $\left(\frac{Y}{L}\right)^* = \left(\frac{F}{L}\right)^*$, and capital goods production-labor ratio $\left(\frac{G}{L}\right)^*$

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7 These first-order conditions that are measured in utility have following meanings. First, (3) means that

Capital goods price × marginal productivity of capital in the capital goods sector
=consumption goods price × marginal productivity of capital in the consumption goods sector
= rental price of capital goods.

That is, one additional unit of capital goods gives us the same revenue in both the capital goods sector and the consumption goods sector, and this becomes the rental price of capital goods.

Equally, (4) means that

Capital goods price × marginal productivity of labor in the capital goods sector
=consumption goods price × marginal productivity of labor in the consumption goods sector
= rental price of labor force (wage rate).

That is, one additional unit labor input gives us the same revenue in both the capital goods sector and the consumption goods sector, and this becomes the rental cost of the labor force (wage rate).

Condition (5) is the arbitration condition of capital goods. That is, buying one unit of capital goods at λ dollars creates the chance to lend it to entrepreneurs and take R dollars However, its real value depreciates at the rate of δ and fluctuates at the rate $\dot{\lambda}$. This result is shown on the left hand side. The right hand side shows the income flow gained by investing λ dollars at the interest rate ρ. Representative agents are assumed to select the path by balancing these two.

in the stationary state. For this purpose, first, substitution of $\dot{K}=0$ into (2) gives us

$$G^* = \delta K^* . \qquad (6)$$

Furthermore, (5) and (3) gives us

$$\frac{\dot{\lambda}}{\lambda} = (\rho+\delta) - G_K .$$

However, in the stationary state, because $\dot{\lambda}=0$, $G_K{}^* = \rho+\delta$, and because $G_K = \frac{\alpha_1 G}{K_1}$, the above equation becomes

$$\frac{\alpha_1 G^*}{K_1^*} = \rho+\delta . \qquad (7)$$

Then, substitution of (7) into (5) gives $\frac{K^*}{K_1^*} = \frac{\rho+\delta}{\alpha_1\delta}$. Therefore, in the stationary state,

$$K^* : K_1^* : K_2^* = \rho+\delta : \alpha_1\delta : \rho+\delta(1-\alpha_1) . \qquad (8)$$

Furthermore, because (3) and (4) imply $\frac{G_K}{G_L} = \frac{F_K}{F_L}$,

$$\frac{\dfrac{\alpha_1 G}{K_1}}{\dfrac{\beta_1 G}{L_1}} = \frac{\dfrac{\alpha_2 F}{K_2}}{\dfrac{\beta_2 F}{L_2}} .$$

This can be transformed into $\frac{L_1}{L_2} = \frac{\alpha_2\beta_1}{\alpha_1\beta_2}\frac{K_1}{K_2}$, and using (8), we can obtain

$$\frac{L_1^*}{L_2^*} = \frac{\alpha_2\beta_1\delta}{\beta_2\{\rho+\delta(1-\alpha_1)\}} .$$

Therefore, in the stationary state, the ratio of the total labor divided between the two sectors becomes

$$L : L_1^* : L_2^* = \alpha_2\beta_1\delta + \beta_2\{\rho+\delta(1-\alpha_1)\} : \alpha_2\beta_1\delta : \beta_2\{\rho+\delta(1-\alpha_1)\} \qquad (9)$$

Equations (8) and (9) show the ratios of the total capital and total labor between two sectors, and can be transformed into

$$K_1^* = \frac{\alpha_1\delta}{\rho+\delta}K^* , \text{ and } L_1^* = \frac{\alpha_2\beta_1\delta}{\alpha_2\beta_1\delta + \beta_2\{\rho+\delta(1-\alpha_1)\}}L$$

respectively. These two equations are the key relations by which we can introduce capital stock per capita and GDP per capita, because substituting them into $\alpha A_1 (K_1^*)^{\alpha_1 - 1} L_1^{\beta_1} = \rho + \delta$, which is obtained from (7), gives

$$\left(\frac{K}{L}\right)^* = \left[A_1 \left(\frac{\alpha_1 \delta}{\rho + \delta}\right)^{\alpha_1} \left(\frac{\alpha_2 \beta_1}{\alpha_2 \beta_1 \delta + \beta_2 \{\rho + \delta(1-\alpha_1)\}}\right)^{\beta_1} (\delta L)^{\alpha_1 + \beta_1 - 1} \right]^{\frac{1}{1-\alpha_1}} . \qquad (10)$$

Furthermore, substituting (8), (9) and (10) into the production function of the capital goods sector gives us

$$\left(\frac{G}{L}\right)^* = A_1^{\frac{1}{1-\alpha_1}} \left(\frac{\alpha_1 \delta}{\rho + \delta}\right)^{\frac{-\alpha_1^2}{1-\alpha_1}} \left(\frac{\alpha_2 \beta_1 \delta}{\alpha_2 \beta_1 \delta + \beta_2 \{\rho + \delta(1-\alpha_1)\}}\right)^{\frac{\alpha_1 + \beta_1 - \alpha_1 \beta_1}{1-\alpha_1}} \delta^{-\alpha_1} L^{\frac{\alpha_1 + \beta_1 - 1}{1-\alpha_1}} . \qquad (11)$$

On the other hand, (8) and (9) can be transformed into

$$K_2^* = \frac{\rho + \delta(1-\alpha_1)}{\rho + \delta} K^*, \quad L_2^* = \frac{\beta_2 \{\rho + \delta(1-\alpha_1)\}}{\alpha_2 \beta_1 \delta + \beta_2 \{\rho + \delta(1-\alpha_1)\}} L$$

respectively, and substituting these into the production function of the consumption goods sector gives us

$$\left(\frac{Y}{L}\right)^* = \left(\frac{F}{L}\right)^* = \frac{A_2 \beta_2^{\beta_2} \{\rho + \delta(1-\alpha_1)\}^{\alpha_2 + \beta_2}}{[\alpha_2 \beta_1 \delta + \beta_2 \{\rho + \delta(1-\alpha_1)\}]^{\frac{\beta_2 - \alpha_1 \beta_2 + \alpha_2 \beta_1}{1-\alpha_1}}} \left[\frac{A_1 \alpha_1^{\alpha_1} (\alpha_2 \beta_1)^{\beta_1} (\delta L)^{\alpha_1 + \beta_1 - 1}}{\rho + \delta} \right]^{\frac{\alpha_2}{1-\alpha_1}} L^{\alpha_2 + \beta_2 - 1} . \qquad (12)$$

Therefore, we can obtain the capital-labor ratio $\left(\frac{K}{L}\right)^*$, consumption-labor ratio $\left(\frac{Y}{L}\right)^* = \left(\frac{F}{L}\right)^*$ and capital goods production-labor ratio $\left(\frac{G}{L}\right)^*$ in the stationary state, and what we should take note of here is that none of (10), (11), or (12) depends on the population (labor force) size when returns to scale is constant ($\alpha+\beta=1$), but they will depend on it when returns to scale is increasing ($\alpha+\beta>1$). In this sense, GDP per capita ($G/L+F/L$) also increases when population size becomes larger in the latter case. This is the merit of large population countries.

However, it is important to understand that we do not insist that the returns to scale increases in general. Although Krugman (1981) built an international model by assuming increasing return to capital and has many followers for this argument, constantly increasing returns cannot be justified easily, at least at the macro level.[8]

[8] Also at the micro level, increasing returns to scale creates a theoretical difficulty in that each factor of production cannot take its marginal productivity as a portion in decentralized markets. Therefore, in this case, some interventions are necessary, for example from the government. Even in our model, if we assume increasing returns to scale, we also need to assume such interventions.

Therefore, the above result in the case of increasing returns to scale has been demonstrated only as a possibility. Furthermore, we want to assert again that the above model shows the inevitable catching-up process of the developing countries, and it also reveals large-population countries like the BRIC nations to be large-GDP countries with powerful influence in world geopolitics.

APPENDIX Proof of Stability of the Dynamic Optimization Problem in the Case of Increasing Returns to scale

Kanae (2013) showed the stability of the model when the rate of time preference is small enough and returns to scale is constant (in footnote 2 in chapter 2). However, this stability can be shown by Sorger (1989) even when returns to scale is increasing (α+β>1) if return to each factor of production is diminishing (α<1, β<1). Corollary 2(c) in Sorger (1989) is summarized in Yanase (2002) in the following way:[9]

First, we set a dynamic optimization problem with n state variables and state variable vector $x \in R^n$, conjugation state variable vector $p \in R^n$, and maximized Hamiltonian $\overline{H}(x, p)$. In this case, the bounded solution of the system of differential equations

$$\dot{p} = \rho p - \overline{H}x(x, p)$$
$$\dot{x} = \overline{H}p(x, p)$$

asymptotically converges on the stationary state (x^*, p^*) in which $\rho p^* - \overline{H}x(x^*, p^*) = \overline{H}p(x^*, p^*) = 0$, if matrix

$$\begin{bmatrix} \overline{H}_{xx} + \gamma[\overline{H}_{xp} + \overline{H}_{px}] & -\frac{\rho}{2}I + \gamma\overline{H}_{pp} \\ -\frac{\rho}{2}I + \gamma\overline{H}_{pp} & -\overline{H}_{pp} \end{bmatrix}$$

has γ whose sign is negative-definite for any $(x, p) \in R^n \times R^n$.

We use this proposition in the case of $n = 2, \gamma = 0$. Here, the solutions converge asymptotically on the stationary state if

$$A = \begin{bmatrix} \overline{H}_{KK} & -\frac{\rho}{2} \\ -\frac{\rho}{2} & -\overline{H}_{\lambda\lambda} \end{bmatrix}$$

is negative-definite for any K and λ. In our model, because all of U, F and G are

[9] See Yanase (2002), p.147.

convex,[10] $\bar{H}$ becomes convex when plotted against the state variable K and concave against the conjugation state variable λ, while $\overline{H}(x,p)$ has become $\bar{H}_{KK} < 0, \bar{H}_{\lambda\lambda} > 0$. In this case, if ρ>0 is small enough,

$$\Delta = -\bar{H}_{KK}\bar{H}_{\lambda\lambda} - \frac{\rho^2}{4}$$

becomes positive, and therefore, A becomes negative-definite. This means that our model covers all the conditions for stability.

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10 It is because $U_{YY} = -\frac{1}{Y^2} < 0$, $F(K_2, L_2) = \alpha_2(\alpha_2 - 1)AK_2^{\alpha_2 - 2}L_2^{\beta_2} < 0$, $G_{KK} = \alpha_1(\alpha_1 - 1)BK_1^{\alpha_1 - 2}L_1^{\beta_1} < 0$.

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