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† We appreciate discussions by Yasuhiro Nakagami and comments from Keisaku Higashida for the previous version of this paper. All remaining errors are mine.

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1. Introduction

In recent years, population aging has been observed in some developed countries and is predicted to occur in developing countries in the next 50 years. One driving force of this phenomenon is a reduction in the fertility rate and thus a decline in the population. Existing studies have found that population decline may change the properties of equilibrium paths from those identified for a population-increasing economy in neoclassical and semi-endogenous growth models (Ritschl, 1985; Felderer, 1988; Christiaans, 2008, 2011, 2017; Ferrara, 2011; Sasaki, 2014, 2015, 2019a,b; Sasaki and Hoshida, 2017; Jones,2019).[1] While these are interesting and important findings, the rates of population decline (i.e., the absolute value of negative population growth rates) needed to generate richer dynamics seem too large given the empirical data and population prospects available.

This paper investigates whether the range of such theoretically derived rates of population decline may be empirically relevant and, if it is too high, explores how to reduce the critical level of the rate of population decline below which richer dynamics emerge in semi-endogenous and Solow models.

More specifically, Sasaki (2019a) has shown, using a Solow growth model with the CES production function, that with a negative population growth rate ($n < 0$), the long-run growth rate of per capita output is given by the rate of technological progress ($\mu > 0$) if the elasticity of substitution between capital and labor is less than unity ($\sigma < 1$). Then, if the rate of technological progress is zero, the growth rate of per capita output is zero. He also shows that the long-run growth rate of per capita output can be positive even without technological progress if the elasticity of substitution is unity ($\sigma = 1$).

While these results are interesting, they are derived under the condition that the rate of population decline (the absolute value of $n < 0$) is larger than the sum of the rates of capital depreciation ($\delta > 0$) and technological progress ($\mu > 0$), i.e., $n < -(\delta + \mu)$. Empirically, as Jones (2019) points out, the rates of population decline are 1% or smaller[2],

[1] There is extensive literature investigating the effects of a decline in the population growth rate in growth models (Casey and Galor, 2014; Futagami and Nakajima, 2001; Futagami and Hori, 2010; Naito and Zhao, 2009; Prettner and Prslarwetz, 2010; Prettner, 2013; Prettner and Trimborn, 2016; Fukuda, 2017). However, many of them focus on population aging and consider a decline in the positive population growth rate. The literature that explicitly analyzes the effects of negative population growth is relatively small.

[2] Appendix 1 provides detailed estimations from *World Population Prospects 2019*.

whereas depreciation rates are 3% or 5% or more. According to Barro and Sala-i-Martin (2004), the rates of technological progress are 1% or 2% (see TFP growth rate in Table 10.1 on p. 439). Sasaki's (2019a) case, therefore, may seem empirically implausible.[3] Because it is easy to observe that this empirical irrelevance comes from the presence of capital depreciation, one might think that it would be better to eliminate capital depreciation from the growth model, as in the standard population-increasing neoclassical growth models.

However, it seems essential to explicitly incorporate the rate of capital depreciation in the model once we proceed to analyze the properties of growth paths under population decline. Ferrara (2011) found that depreciation plays a fundamental role in the phenomena of convergence in the Solow model with AK technology. Christiaans (2011) showed in a semi-endogenous growth model that the condition for the emergence of richer dynamics under negative population growth crucially depends on the rate of capital depreciation. From a theoretical viewpoint, an intuitive reason for this relationship is straightforward: the rate of capital depreciation affects the direction of movement of the capital-labor ratio $k = K/L$, which is a key variable in determining per capita real income. In a population-*increasing* economy, an increase in population (L) and an increase in the rate of capital depreciation both decrease the capital-labor ratio (K/L). Thus, eliminating capital depreciation does not affect the direction of movement of the capital-labor ratio. In contrast, in a population-*declining* economy, these two forces work in the opposite direction: population decline tends to increase the capital-labor ratio, while an increase in the capital depreciation rate tends to decrease it.[4] The properties of the equilibrium paths will then be determined through interactions between movements in k and the other factors in the model, such as externalities. Therefore, the condition for the emergence of richer dynamics under negative population growth involves the rate of capital depreciation.[5]

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[3] Despite this fact, the qualitative result of Sasaki (2017) could be valid for (the absolute value of) the rates of population decline smaller than $\delta + \mu$ if we introduced child rearing costs. See the last paragraph of section 3.

[4] Jones (2019) makes the same explanation with somewhat different expressions (p.4).

[5] Sasaki (2019b) analyzes a growth model with capital-input externalities by introducing non-renewable resources. He considers both positive and zero rates of capital depreciation and shows that positive per capita output growth is possible not only under increasing returns-to-scale (in capital and labor) production and positive population growth but also under diminishing returns-to-scale production and

As a study that explicitly incorporates capital depreciation, Christiaans (2011) has shown that the growth rate of per capita output ($g_{Y/L}$) exhibits a nonmonotonous dependency on negative population growth rates ($n < 0$) in a semi-endogenous growth model with an aggregate capital externality. In his model, the growth rate of per capita output becomes negative when the population growth rate becomes negative. He derives the critical value of the rate of population decline that separates positive and negative relations between $g_{Y/L}$ and $n < 0$: if $-(\delta/\gamma) < n < 0$ holds, the faster the population declines, the faster the per capita output decreases, and if $n < -(\delta/\gamma)$ holds, the faster the population declines, the slower the per capita output decreases.[6] This result provides another important lesson: a positive externality represented by parameter $\gamma > 1$ reduces the critical rate of population decline below which richer dynamics emerge. However, no studies have investigated whether this critical value is empirically relevant.[7]

This paper first shows that the critical value $-(\delta/\gamma)$ of the rates of population decline derived by Christiaans (2011) may be empirically relevant in the sense that it is sufficiently small to be consistent with available population estimates (*World Population Prospects 2019*). Then, it explores how we can reduce the corresponding rates of population decline in the Solow growth model without such externalities. We find that an introduction of a child rearing cost could make the absolute value of such a rate of population decline smaller. Finally, the economic implications of the presence of a child rearing cost are discussed for the Solow growth model.

2. Semi-Endogenous Growth Model and Empirical Relevance

Let us first summarize the semi-endogenous growth model of Christiaans (2011) and his results. The reason for choosing this model is that it explicitly incorporates capital

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negative population growth.

[6] See Figure 1 in section 2, which is cited from Christiaans (2011) for further details.

[7] Another study investigating negative population growth in a semi-endogenous growth model is Sasaki and Hoshida (2017). They show that within a finite time horizon, the employment share of the R&D sector reaches zero, and thus the rate of technological change falls to zero, and that the growth rate of per capita output asymptotically approaches a positive value. Although these results are interesting, they assume a zero depreciation rate for capital.

depreciation as well as the aggregate capital-input externality, which is a common feature among the advanced countries that have been experiencing population decline. After deriving a lesson from his analysis, I show that his model applies to empirically relevant ranges of rates of population decline by numerical examples.

2.1 Christiaans' (2011) Model and Its Implications

In a one-sector competitive-economy model with the aggregate capital externality, the saving rate s ($0 < s < 1$) is assumed to be exogenous. An individual firm *j* has a production function $Y_j = K_j^{\alpha}(K^{\beta/(1-\alpha)}L_j)^{1-\alpha}$, where K_j and L_j are capital and labor inputs, respectively, and $K = \sum_j K_j$ is the aggregate capital stock, with β representing the positive externality due to learning by investment ($0 < \alpha < 1$, $0 \le \beta < 1$, $\alpha + \beta < 1$). Because all firms are identical and choose the same capital-labor ratio $(K_j / L_j) = (K / L)$, the aggregate production function is $Y = K^{\alpha+\beta}L^{1-\alpha}$, where $Y = \sum_j Y_j$ is the gross domestic product, and $L = \sum_j L_j$ is the total labor force with a constant growth rate $n = \dot{L} / L$ (the dot represents a time derivative). Dividing the aggregate production function by L^{γ}, we obtain the *scale-adjusted* per capita production function $y = k^{\alpha+\beta}$ with $y = Y / L^{\gamma}$ and $k = K / L^{\gamma}$. The term $\gamma = (1-\alpha)/(1-\alpha-\beta)$ represents the strength of the positive externality from knowledge accumulation; $\gamma > 1$ if $\beta > 0$ (semi-endogenous growth model) and $\gamma = 1$ if $\beta = 0$ (Solow model).

Capital is accumulated by $\dot{K} = sY - \delta K$, where $\delta > 0$ is the rate of capital depreciation. Using the accumulation rate $g_K = \dot{K} / K = s(Y / K) - \delta$, we obtain:

$$\dot{k} = sk^{\alpha+\beta} - (\delta + \gamma n)k \tag{1}$$

When $\delta + \gamma n > 0$ holds, there exists a unique steady state ($\dot{k} = 0$) that is globally asymptotically stable for any initial value $k_0 > 0$. Even under $n < 0$, this property holds as long as the absolute value of the rate of population decline is sufficiently small that

$-(\delta/\gamma) < n < 0$ holds. Then, the growth rate of per capita income (not $y = Y/L^{\gamma}$ but *Y/L*) in the steady state is:

$$g_{Y/L} = (\gamma - 1)n \tag{2}$$

In the presence of the externality ($\gamma > 1$), the steady-state growth rate of per capita income is positive if $n > 0$, zero if $n = 0$ and negative if $-(\delta/\gamma) < n < 0$.

However, the main finding of Christiaans (2011) occurs when $\delta + \gamma n < 0$, i.e., $n < -(\delta/\gamma) = -\left(\frac{1-\alpha-\beta}{1-\alpha}\right)\delta$ holds. Then, the scale-adjusted per capita capital k grows indefinitely. More precisely, there exists no steady state with a positive and constant growth rate of per capita income, but an asymptotic steady state exists (i.e., the growth rate of per capita income converges to a constant value as time goes to infinity). Because $Y/L = K^{\alpha+\beta}L^{-\alpha}$, we have

$$g_{Y/L} = (\alpha+\beta)g_K - \alpha g_L = (\alpha+\beta)(sY/K - \delta) - \alpha n = (\alpha+\beta)s\left(\frac{Y/L^{\gamma}}{K/L^{\gamma}} - \delta\right) - \alpha n \tag{3}$$

Therefore, the growth rate of per capita income is:

$$g_{Y/L} = (\alpha+\beta)sk^{\alpha+\beta-1} - (\alpha+\beta)\delta - \alpha n \tag{4}$$

Because $\alpha + \beta < 1$, the growth rate of per capita income in the asymptotic steady state is:

$$g_{Y/L} = -(\alpha+\beta)\delta - \alpha n \tag{5}$$

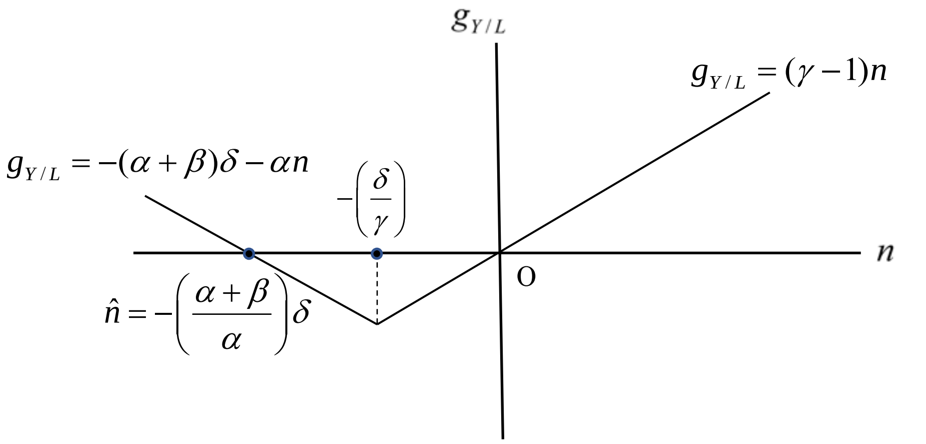
This long-run growth rate is positive if and only if

$$n < -\left(\frac{\alpha+\beta}{\alpha}\right)\delta \tag{6}$$

The threshold value $\hat{n} = -[(\alpha+\beta)/\alpha]\delta$, which separates positive and negative values of $g_{Y/L}$, is lower than the kink at $n = -(\delta/\gamma)$.

Christiaans (2011) has found the nonmonotonous dependency of the long-run per capita income growth rate $g_{Y/L}$ on population growth rates $n < 0$ in Figure 1. An important lesson must then be that the critical value $n = -(\delta/\gamma)$, which separates the positive and negative relations of $g_{Y/L}$ to $n < 0$, is reduced by the positive externality ($\gamma > 1$).

Figure 1. Dependency of Long-Run Per Capita Income Growth Rate on the Population Growth Rate



Source: Figure 2 on p.2671 in Christiaans (2011)

2.2 Empirical Relevance of the Theoretical Rates of Population Decline

We investigate whether the critical values for the rates of population decline derived in Christiaans (2011) can be empirically relevant by calculating the values of $n = -(\delta/\gamma)$ and $\hat{n} = -[(\alpha + \beta)/\alpha]\delta$ with numerical examples that satisfy $\alpha + \beta < 1$: the relative share of capital is approximately $\alpha = 1/3$ and the rate of capital depreciation is $\delta \in (0.05, 0.12)$.

Let us look at the values of $-(\delta/\gamma)$ in Table 1. The absolute values are smaller than 1%.[8] According to the United Nations' *World Population Prospects 2019*, the absolute values of the rates of population decline estimated in most countries are 1% or smaller. For example, the estimated average annual rates of population change (as medium variants) in Japan, Greece, Italy and Germany for 2020-2025 are -0.40, -0.52, -0.20 and -0.06%, while they are -0.53, -0.47, -0.28 and -0.09% for 2025-2030, respectively.[9] From

8 See Appendix 2 for other numerical examples.

9 See Appendix 1 for estimated values of other countries.

this observation, the nonmonotonous dependency derived by Christiaans (2011) seems empirically relevant.[10]

Table 1. Dependency of the Range for the Rates of Population Decline on Capital Depreciation Rates

(a) α=0.33, β=0.65 and thus γ=33.5

Δ	-(δ/γ) (%)	$\hat{n}$ = -(α+β)δ/α (%)
0.12	-0.36	-35.6
0.1	-0.30	-29.7
0.09	-0.27	-26.7
0.08	-0.24	-23.8
0.07	-0.21	-20.8
0.06	-0.18	-17.8
0.05	-0.15	-14.8

(b)α=0.3, β=0.69 and thus γ=70

δ	-(δ/γ) (%)	$\hat{n}$ = -(α+β)δ/α (%)
0.12	-0.17	-39.6
0.1	-0.14	-33.0
0.09	-0.13	-29.7
0.08	-0.11	-26.4
0.07	-0.10	-23.1
0.06	-0.09	-19.8
0.05	-0.07	-16.5

Source: Author's calculations

In addition, because the threshold values of $\hat{n}$ in Table 1 are so large in absolute value,

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[10] The absolute values of $g_{Y/L} = -(\gamma - 1)\delta / \gamma$ at $n = -(\delta/\gamma)$ also seem too large from an empirical viewpoint. For example, in case (a) $g_{Y/L} = -4.9\%$ for $\delta = 0.05$ and $g_{Y/L} = -9.7\%$ for $\delta = 0.10$ (I appreciate Yasuhiro Nakagami for his comment on this point). If we exogenously incorporated Harrod-neutral technological progress into Christiaans (2011) model, the absolute values of $g_{Y/L} < 0$ would be smaller.

the long-run per capita income growth rates $g_{Y/L}$ are likely to remain *negative* from an empirical viewpoint. Based on Christiaans (2011), as the absolute value of $n < 0$ increases, the long-run decrease in per capita income will accelerate (the absolute value of $g_{Y/L} < 0$ will become larger) for $-(\delta/\gamma) < n < 0$ while it will slow down (the absolute value of $g_{Y/L} < 0$ will decrease) for $n < -(\delta/\gamma)$.

3. A Solow Growth Model with Child Rearing Cost

We have shown that in the semi-endogenous growth model by Christiaans (2011), positive externalities from knowledge accumulation can reduce the critical rate of population decline, which separates positive and negative relations between the growth rate and the rate of population decline. In this section, we explore how to reduce the corresponding critical value of the rate of population decline in the Solow growth model without such externalities. It is found that an introduction of a child rearing cost could reduce the critical rate of population decline below which a growth path with different properties emerges.

3.1 The Model

Consider the Solow growth model with capital depreciation rate $\delta > 0$. The aggregate production function exhibits constant returns to scale in physical capital $K(t)$ and labor $L(t)$. The exogenous growth rate n of population $L(t)$ can be either positive or negative. Unlike the standard Solow model, we assume that the birth and rearing of each child of the next generation costs an amount η at any point in time *t*. Following Barro and Sala-i-Martin (2004, p.413), the child rearing cost is assumed to be positively related to per capita capital $k(t) = K(t)/L(t)$, i.e., $\eta(k(t))$ with $\eta'(k(t)) > 0$. This is because the cost η tends to rise with parents' wage rates $w(t) = f(k(t)) - k(t)f'(k(t))$, which is increasing in $k(t)$, or other measures of the opportunity costs of parental time. The commodity costs of rearing a child may or may not be increasing in $k(t)$, which could lead to technical complexity in the form of nonlinearity.

Because the aggregate cost of child rearing is $\eta nL(t)$, the aggregate capital stock evolves over time according to $\dot{K}(t) = sY(t) - \delta K(t) - \eta nL(t)$. Thus, the accumulation function of per capita capital is:

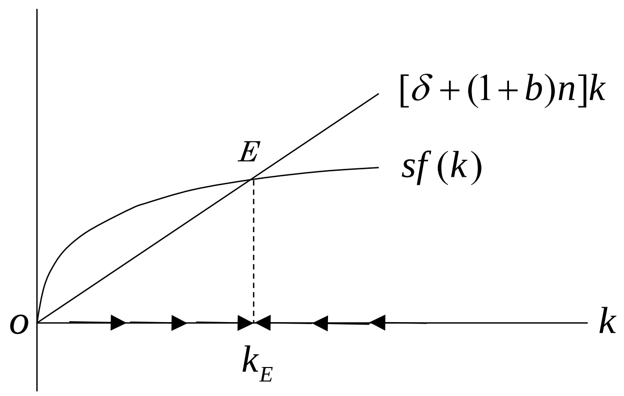
$$\dot{k}(t) = sf(k(t)) - (\delta + n)k(t) - n\eta(k(t)) \qquad (7),$$

where $f(k(t))$ is the per capita production function with $f'(k(t)) > 0$, $f''(k(t)) < 0$, $f(0) = 0$, $f'(0) = \infty$ and $f'(\infty) = 0$. In this section, we show the analysis for a linear child rearing cost $\eta = bk(t)$, where $b > 0$ is a constant. Then, the per capita capital accumulation function is:

$$\dot{k}(t) = sf(k(t)) - [\delta + (1+b)n]k(t) \qquad (8)$$

Even if the population growth rate n is negative, the properties of the equilibrium paths are the same as those of the standard Solow growth model as long as the "effective depreciation rate for *k*", $\delta + (1+b)n > 0$, is positive,[11] that is, a stable steady state k_E uniquely exists, as shown in Figure 2. More generally, when the child rearing cost $\eta(k(t))$ is small enough, the sum of the second and third terms in (7) $\phi(k(t)) = (\delta + n)k(t) + n\eta(k(t))$ will be positive even if $n < 0$ holds. Then, a stable steady state uniquely exists even under negative population growth in the present model.

Figure 2. A Stable Steady State for $\delta + (1+b)n > 0$



[11] We follow Gruescu (2007, p.34) for this expression.

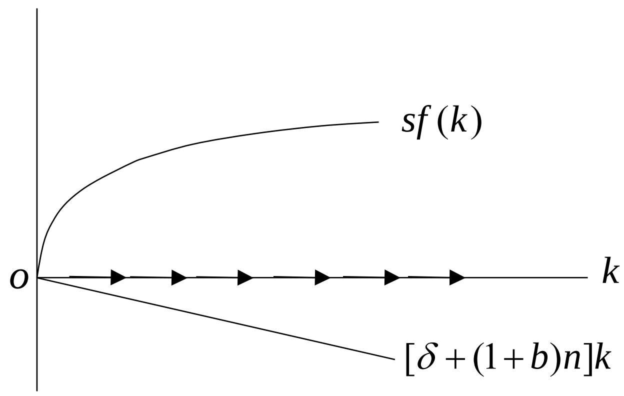
3.2 Rates of Population Decline Inducing the Unbounded Equilibrium Path

The properties of the equilibrium paths will differ substantially when $\delta + (1+b)n \le 0$ holds, as shown in Figure 3. In this case, per capita capital k increases unboundedly because the capital dilution effect, including depreciation, dominates the capital accumulation effect by per capita saving. Although this qualitative result has already been found in Ritschl (1985),[12] the range of rates of population decline for the unbounded equilibrium path turns out to be:

$$n < -\frac{\delta}{1+b} \qquad (9).$$

A higher child rearing cost ($b > 0$) will decrease the absolute value ($\delta/(1+b)$) of the critical rate of population decline below which an equilibrium path is induced with different properties from that of the standard Solow model. Because the child rearing cost must be very high in many developed countries that have already entered the phase of population decline, the critical value $\delta/(1+b)$ is likely to be much lower than $\delta > 0$.

Figure 3. An Unbounded Equilibrium Path for $\delta + (1+b)n \le 0$



Let us make a brief comment on the condition $n < -(\delta + \mu)$ in Sasaki (2019a), under which the long-run growth rate of per capita output equals the rate of technological progress ($\mu > 0$) when the elasticity of substitution is less than unity ($\sigma < 1$). If the linear child rearing cost had been introduced in the same way as in this paper, the accumulation function of capital per unit of efficiency labor ($k = K / AL$) would be

[12] See subsection 3.3 for details of Ritschl (1985).

$\dot{k} = sf(k) - \{(1+b)n + \delta + \mu\}$, where A is a technological parameter. The condition $n < -(\delta + \mu)$ would be modified into $(1+b)n + \delta + \mu < 0$, i.e., $n < -(\delta + \mu)/(1+b)$. Then, the qualitative result of Sasaki (2019a) might hold for rates of population decline that seem much lower than $\delta + \mu$.

In addition, it is interesting to show that introducing the child rearing cost $\eta(K/L)$ into the Christiaans (2011) semi-endogenous growth model further reduces the critical rate of population decline. From $\dot{K} = sY - \delta K - \eta(K/L)nL$, the dynamics of the scale-adjusted capital intensity $k \equiv K/L^{\gamma}$ are:

$$\dot{k} = sk^{\alpha+\beta} - [\delta + \{\eta(K/L)/(K/L) + \gamma\}n)]k$$

Then, the critical rate of population decline that separates positive and negative relations between $g_{Y/L}$ and $n < 0$ is given not by $-(\delta/\gamma)$ but by $-\dfrac{\delta}{\gamma + \eta(K/L)/(K/L)}$. Under the linear child rearing cost function, it is $-\{\delta/(\gamma + b)\}$. Their absolute values are smaller than $-(\delta/\gamma)$.

3.3 Previous Studies and Implications of the Present Analysis

We have shown the existence of a stable steady state under negative population growth for $\delta + (1+b)n > 0$. It would be useful to explain how the present model is related to previous studies on Solow models with negative population growth.

The first study that considered negative population growth ($n < 0$) in the Solow model was Ritschl (1985). Using the Solow model without capital depreciation ($\delta = 0$), he found that a steady state does not exist for $n < 0$, but per capita capital k increases unboundedly. He also found that when assuming a negative saving rate ($s \le 0$) in the Solow saving function, there exists an unstable steady state (i.e., the capital-labor ratio either converges to zero or grows unboundedly, depending on the initial conditions). He proceeded to show that a stable steady state exists by introducing a "classical" saving function in which saving is positive when the return on capital is greater than a certain level and falls to zero when a minimum rate of return is reached. Because the "classical" saving function lacks a sound microeconomic foundation, Felderer (1988) introduced simple life-cycle assumptions in a neoclassical framework without capital depreciation and showed that a steady state exists for any sign of the population growth rate.

In the present model, even when $n < 0$, a stable steady state $k_E > 0$ exists as long as $\delta + (1+b)n > 0$ holds. Taking into account that this is the case even if the child rearing cost is absent ($b = 0$), we find that the presence of the capital depreciation rate ($\delta > 0$) enables a stable steady state to exist. Here, again, capital depreciation turns out to play a crucial role in determining the properties of equilibrium dynamics under negative population growth. Ferrara (2011) has found that under negative population growth ($n < 0$), the capital-labor ratio converges to zero when the depreciation rate is sufficiently large (so that $sA < \delta$ holds) using an AK Solow model with capital depreciation.

In the standard Solow growth model with capital depreciation ($\delta > 0$), Gruescu (2007, p.34) had already referred to the possibility that if the workforce declined to a greater extent than capital depreciation ($|n| > \delta$) under negative population growth, it would lead to an increase in the capital-labor ratio *k*. She mentioned, however, that this theoretically possible case is rather unrealistic. Our paper shows that the existence of a child rearing cost makes the emergence of such unbounded growth paths easier ($|n| > \delta/(1+b)$) in a population-declining economy.

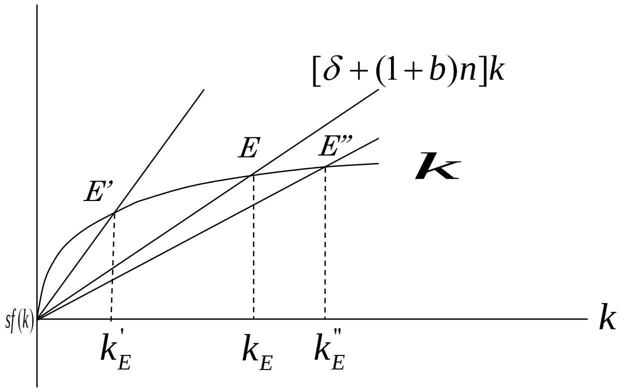
4. Economic Implications of Child Rearing Cost in the Solow Growth Model

A child rearing cost has never been introduced into the Solow growth model. Therefore, we will further explore the economic implications of a child rearing cost based on the Solow growth model, particularly under population decline.

4.1 The Roles of the Child Rearing Cost in Capital Accumulation

The effects of a child rearing cost on the per capita income level in the long run differ between the cases of $n > 0$ and $n < 0$. In Figure 4, the initial steady state is point *E*. In a population-increasing economy ($n > 0$), a higher child rearing cost ($b > 0$) induces a lower level of per capita capital $k_E^{'}$ in the steady state E'. This is because a higher child rearing cost depresses saving and investment, hindering capital accumulation. In contrast, in a population-declining economy ($n < 0$), a higher child rearing cost induces a higher level of per capita capital $k_E^{''}$ in the steady state E''. This is because a higher child rearing cost means that an economy saves more income when the newly born population decreases. In such context, it can afford to save and invest more resources to promote capital accumulation.

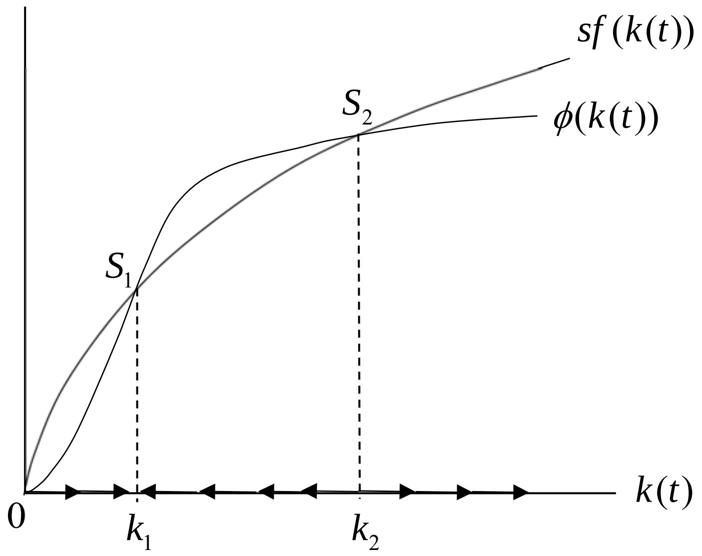
Figure 4. Child Rearing Cost and Capital Accumulation



4.2 Possibility of Multiple Steady States

We do not have any clear evidence showing whether the child rearing cost function $\eta(k(t))$ in the general form will be concave or convex, so it is reasonable to take both possibilities into account. Thus, our Solow growth model has the possibility of multiple equilibria. Figure 5 shows the case when two steady states exist: S_1 is stable and S_2 is unstable. Even under negative population growth ($n < 0$), the model can have steady states with positive values of per capita capital k if the rate of capital depreciation is so large that $\phi(k(t)) = (\delta + n)k(t) + n\eta(k(t))$ is positive.

Figure 5. Possibility of Multiple Steady States



An economic implication of this multiplicity is as follows. If an economy initially lies below k_2, it will converge to the steady state k_1. However, if it initially lies above k_2, an economy will experience the unbounded growth of the capital-labor ratio in the long run. In this process, "capital deepening" (due to the negative "capital dilution effect") takes place without technological progress. It follows that in a competitive market economy, per capita income may increase even if technological progress is absent.[13]

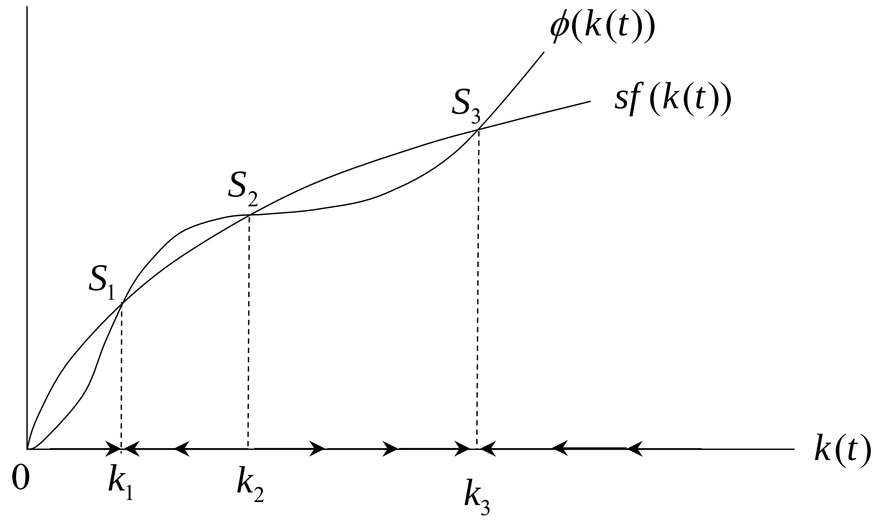
Figure 6 shows the more complicated case when two stable steady states exist (S_1 and S_3). One interpretation could be that high-income countries (such as European countries and Japan) starting from a high level of per capita income (between k_2 and k_3) will reach the high-income steady state k_3, while low-income countries (such as Thailand, Malaysia and Indonesia in Southeast Asia) starting from a lower level of per capita income (below k_2) reach the low-income steady state k_1. History matters in deciding

13 This does not mean, however, that technological progress is less important in a population-declining economy than in a population-increasing economy. If Harrod-neutral technological progress took place at the same time, the growth rate of per capita income would be higher in our Solow growth model with child rearing costs.

the income level that each economy, given the same technologies and preferences, can reach in the long run.

These possibilities have never been considered in traditional studies of the Solow growth model. The child rearing cost might expand our understanding of the importance of initial conditions in deciding the log-run income level in a perfectly competitive economy.

Figure 6. Possibility of Multiple Stable Steady States



Let us finally mention a qualification of our Solow growth model with a child rearing cost. Our model does not explicitly formulate how child rearing activities contribute to the human capital accumulation of children.[14] However, building and analyzing a growth model in which the stock of human capital accumulates through child rearing activities is more complicated and thus beyond the scope of this paper, which attempts to investigate whether the critical rate of population decline is empirically relevant.

5. Concluding Remarks

This paper has investigated whether the theoretically derived critical the rate of population decline, which induces richer dynamics, is empirically relevant and, if it is too

[14] I appreciate Keisaku Higashida (Kwansei Gakuin University) for his comment on this point.

high, explored how we can reduce it in semi-endogenous and Solow models. In the semi-endogenous growth model of Christiaans (2011), positive externalities from knowledge accumulation can make the critical rate of population decline sufficiently small that it is consistent with the United Nations population estimates. In the Solow growth model without such externalities, the introduction of a child rearing cost could reduce the critical rate of population decline, which induces the unbounded growth of per capita income. Finally, an increase in the child rearing cost will promote capital accumulation in a population-declining economy, whereas it hinders capital accumulation in a population-increasing economy. An introduction of a child rearing cost into the Solow growth model may induce multiple steady states.

The main message of this paper is that in realistically relevant ranges of rates of population decline, equilibrium paths may emerge with different properties from those in a population-increasing economy. Changes in the properties of equilibrium paths in growth models under population decline should be not only a theoretical possibility but also a realistically relevant problem in modern economies. Therefore, we should proceed to investigate what kinds of equilibrium paths may emerge in growth models under population decline.

Finally, let us suggest three promising directions for future research, in addition to the existing direction of using the Solow and semi-endogenous R&D growth models. First, it is of fundamental importance to investigate the properties of the equilibrium paths of the Ramsey-type optimal growth model when the population declines exogenously. Second, recalling that complex dynamics such as the indeterminacy of the equilibrium may occur in growth models with externalities, it should be interesting to investigate the kinds of equilibrium paths that can emerge in exogenous growth models with factor-input externalities under negative population growth. One possible step in this direction might be to extend an endogenous growth model with human capital accumulation by exogenously introducing positive and negative population growth.

Third, and most importantly, analyzing endogenous growth models with endogenous fertility will provide deep insights.[15] Recently, Jones (2019) obtained two interesting

[15] Prettner and Prslarwetz (2010) are useful as a survey on interrelations between demographic variables and long-run economic performance in the endogenous growth literature.

findings from his endogenous growth model of this type. First, when the equilibrium fertility rate is negative, the optimal allocation features two stable steady states: the *Expanding Cosmos* outcome of a sustained exponential growth in population, knowledge and living standards, and the *Empty Planet* outcome of stagnant knowledge and living standards combined with a population decline toward zero. Second, if the economy adopts the optimal allocation sufficiently quickly, it converges to the *Expanding Cosmos* outcome while if the economy waits too long to switch, even the optimal allocation converges to the *Empty Planet* outcome. These results may provide an important lesson for policymaking, and they also help us answer the unresolved question of whether population decline is occurring at a steady state or along a transition path.

Negative population growth is just beginning to be analyzed in the field of economic growth theory. This paper is only an early attempt to promote economic research on this theme.

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Appendix 1. Excerpt from *World Population Prospects 2019*

(a) Medium Variant

Medium variant	Average annual rate of population change (percentage)															
Region, country or area *	2020-2025	2025-2030	2030-2035	2035-2040	2040-2045	2045-2050	2050-2055	2055-2060	2060-2065	2065-2070	2070-2075	2075-2080	2080-2085	2085-2090	2090-2095	2095-2100
Asia	0.77	0.62	0.49	0.36	0.25	0.14	0.04	-0.05	-0.12	-0.19	-0.25	-0.29	-0.33	-0.35	-0.37	-0.39
Europe	-0.05	-0.12	-0.17	-0.20	-0.23	-0.26	-0.29	-0.33	-0.34	-0.32	-0.28	-0.24	-0.19	-0.16	-0.14	-0.14
Latin America and the Caribbean	0.84	0.70	0.56	0.43	0.32	0.22	0.11	0.02	-0.07	-0.16	-0.24	-0.30	-0.36	-0.40	-0.44	-0.46
Northern America	0.59	0.56	0.53	0.45	0.38	0.34	0.32	0.33	0.34	0.33	0.30	0.27	0.25	0.24	0.24	0.25
Eastern Asia	0.21	0.05	-0.08	-0.20	-0.30	-0.40	-0.49	-0.56	-0.59	-0.60	-0.60	-0.60	-0.59	-0.55	-0.51	-0.50
China	0.26	0.09	-0.05	-0.17	-0.27	-0.38	-0.47	-0.54	-0.57	-0.58	-0.59	-0.59	-0.58	-0.55	-0.51	-0.50
China, Hong Kong SAR	0.68	0.67	0.25	0.05	-0.09	-0.16	-0.17	-0.15	-0.15	-0.17	-0.21	-0.21	-0.15	-0.04	0.08	0.17
Japan	-0.40	-0.53	-0.60	-0.66	-0.69	-0.69	-0.71	-0.76	-0.82	-0.84	-0.80	-0.70	-0.63	-0.57	-0.54	-0.52
Republic of Korea	0.03	-0.07	-0.18	-0.36	-0.53	-0.69	-0.86	-0.99	-1.03	-1.02	-1.00	-0.97	-0.93	-0.88	-0.83	-0.72
South-Eastern Asia	0.91	0.77	0.63	0.49	0.37	0.26	0.16	0.07	-0.01	-0.08	-0.13	-0.18	-0.23	-0.26	-0.30	-0.33
Cambodia	1.26	1.07	0.94	0.84	0.70	0.56	0.41	0.29	0.17	0.05	-0.06	-0.15	-0.21	-0.26	-0.32	-0.38
Indonesia	0.97	0.83	0.69	0.57	0.44	0.32	0.21	0.12	0.05	-0.01	-0.05	-0.10	-0.14	-0.19	-0.24	-0.28
Lao People's Democratic Re	1.33	1.13	0.95	0.79	0.63	0.47	0.31	0.16	0.02	-0.11	-0.24	-0.34	-0.43	-0.51	-0.58	-0.65
Malaysia	1.19	0.99	0.80	0.62	0.50	0.41	0.33	0.24	0.13	0.02	-0.07	-0.14	-0.17	-0.19	-0.19	-0.21
Myanmar	0.76	0.68	0.54	0.38	0.23	0.11	0.02	-0.05	-0.12	-0.18	-0.25	-0.31	-0.35	-0.37	-0.38	-0.38
Philippines	1.28	1.14	1.00	0.84	0.70	0.57	0.44	0.33	0.23	0.14	0.04	-0.05	-0.13	-0.20	-0.26	-0.30
Singapore	0.77	0.60	0.39	0.18	0.01	-0.12	-0.20	-0.25	-0.28	-0.30	-0.30	-0.28	-0.24	-0.19	-0.13	-0.06
Thailand	0.15	0.01	-0.13	-0.26	-0.39	-0.52	-0.63	-0.71	-0.74	-0.75	-0.74	-0.73	-0.73	-0.74	-0.74	-0.71
Viet Nam	0.76	0.60	0.41	0.28	0.20	0.13	0.03	-0.08	-0.18	-0.25	-0.29	-0.31	-0.31	-0.32	-0.32	-0.34
Southern Asia	1.07	0.92	0.77	0.62	0.48	0.36	0.25	0.14	0.03	-0.08	-0.17	-0.24	-0.30	-0.35	-0.39	-0.42
India	0.92	0.80	0.66	0.50	0.35	0.23	0.13	0.03	-0.08	-0.18	-0.27	-0.34	-0.39	-0.42	-0.45	-0.47
Central & South America																
El Salvador	0.48	0.40	0.26	0.18	0.06	-0.05	-0.16	-0.28	-0.41	-0.54	-0.68	-0.83	-0.98	-1.11	-1.22	-1.31
Mexico	0.96	0.81	0.68	0.54	0.41	0.29	0.18	0.08	-0.01	-0.10	-0.17	-0.24	-0.31	-0.38	-0.43	-0.46
Brazil	0.60	0.44	0.30	0.16	0.05	-0.05	-0.15	-0.25	-0.34	-0.43	-0.51	-0.57	-0.61	-0.62	-0.63	-0.62
Chile	0.13	0.22	0.43	0.28	0.14	0.02	-0.08	-0.17	-0.24	-0.29	-0.34	-0.39	-0.41	-0.43	-0.43	-0.41
Eastern Europe	-0.24	-0.35	-0.41	-0.43	-0.41	-0.40	-0.41	-0.44	-0.47	-0.47	-0.43	-0.36	-0.28	-0.23	-0.22	-0.23
Belarus	-0.14	-0.26	-0.35	-0.37	-0.35	-0.34	-0.34	-0.37	-0.40	-0.41	-0.38	-0.32	-0.24	-0.18	-0.17	-0.21
Bulgaria	-0.77	-0.82	-0.88	-0.89	-0.87	-0.86	-0.88	-0.92	-0.94	-0.93	-0.88	-0.80	-0.72	-0.67	-0.68	-0.72
Czechia	0.09	-0.02	-0.10	-0.12	-0.09	-0.06	-0.08	-0.14	-0.20	-0.21	-0.15	-0.06	0.03	0.09	0.11	0.10
Hungary	-0.31	-0.37	-0.45	-0.50	-0.51	-0.49	-0.48	-0.49	-0.52	-0.53	-0.50	-0.44	-0.38	-0.34	-0.30	-0.26
Poland	-0.18	-0.31	-0.42	-0.50	-0.56	-0.60	-0.64	-0.69	-0.75	-0.83	-0.87	-0.86	-0.79	-0.71	-0.64	-0.60
Republic of Moldova	-0.30	-0.45	-0.60	-0.71	-0.78	-0.83	-0.90	-0.98	-1.09	-1.18	-1.21	-1.16	-1.06	-0.95	-0.88	-0.86
Romania	-0.49	-0.50	-0.54	-0.58	-0.61	-0.64	-0.67	-0.71	-0.73	-0.71	-0.66	-0.61	-0.57	-0.55	-0.53	-0.54
Russian Federation	-0.11	-0.25	-0.31	-0.30	-0.25	-0.22	-0.22	-0.25	-0.27	-0.26	-0.20	-0.12	-0.05	-0.02	-0.03	-0.06
Slovakia	-0.04	-0.17	-0.30	-0.40	-0.45	-0.47	-0.49	-0.54	-0.61	-0.67	-0.67	-0.61	-0.52	-0.43	-0.38	-0.37
Ukraine	-0.65	-0.70	-0.73	-0.74	-0.75	-0.77	-0.81	-0.85	-0.87	-0.86	-0.82	-0.74	-0.65	-0.59	-0.57	-0.58
Southern Europe	-0.22	-0.29	-0.32	-0.36	-0.44	-0.54	-0.65	-0.75	-0.79	-0.77	-0.71	-0.62	-0.54	-0.50	-0.49	-0.49
Greece	-0.52	-0.47	-0.42	-0.42	-0.47	-0.56	-0.68	-0.77	-0.81	-0.80	-0.72	-0.62	-0.53	-0.47	-0.46	-0.46
Italy	-0.20	-0.28	-0.31	-0.34	-0.43	-0.57	-0.68	-0.77	-0.80	-0.74	-0.64	-0.56	-0.51	-0.49	-0.48	-0.47
Portugal	-0.27	-0.30	-0.34	-0.39	-0.47	-0.55	-0.62	-0.67	-0.68	-0.65	-0.57	-0.49	-0.42	-0.40	-0.39	-0.36
Spain	-0.08	-0.15	-0.20	-0.24	-0.31	-0.41	-0.55	-0.68	-0.76	-0.77	-0.70	-0.56	-0.42	-0.35	-0.34	-0.36
Western Europe	0.13	0.09	0.05	-0.01	-0.07	-0.12	-0.15	-0.15	-0.13	-0.09	-0.08	-0.07	-0.06	-0.04	-0.02	-0.00
Austria	0.22	0.16	0.08	0	-0.06	-0.12	-0.16	-0.18	-0.16	-0.14	-0.13	-0.13	-0.11	-0.06	0.00	0.05
Belgium	0.29	0.25	0.21	0.16	0.11	0.06	0.01	0.00	0.01	0.03	0.04	0.05	0.06	0.07	0.08	0.08
France	0.24	0.19	0.16	0.10	0.03	-0.03	-0.07	-0.09	-0.08	-0.06	-0.05	-0.04	-0.05	-0.06	-0.06	-0.07
Germany	-0.06	-0.09	-0.12	-0.16	-0.21	-0.26	-0.28	-0.26	-0.22	-0.17	-0.15	-0.14	-0.11	-0.06	-0.02	0.01
Netherlands	0.21	0.15	0.06	-0.05	-0.14	-0.20	-0.23	-0.22	-0.19	-0.15	-0.13	-0.15	-0.17	-0.17	-0.16	-0.14

(b) Low Variant

Low Variant	Average annual rate of population change (percentage)															
Region, country or area *	**2020-2025**	**2025-2030**	**2030-2035**	**2035-2040**	**2040-2045**	**2045-2050**	**2050-2055**	**2055-2060**	**2060-2065**	**2065-2070**	**2070-2075**	**2075-2080**	**2080-2085**	**2085-2090**	**2090-2095**	**2095-2100**
Asia	0.59	0.35	0.16	0.04	-0.09	-0.24	-0.40	-0.55	-0.69	-0.80	-0.91	-1.01	-1.12	-1.21	-1.30	-1.38
Europe	-0.20	-0.35	-0.45	-0.50	-0.54	-0.60	-0.69	-0.79	-0.86	-0.89	-0.89	-0.88	-0.87	-0.87	-0.89	-0.93
Latin America and the Caribbean	0.65	0.41	0.22	0.09	-0.04	-0.18	-0.33	-0.48	-0.62	-0.75	-0.88	-1.01	-1.14	-1.27	-1.38	-1.48
Northern America	0.42	0.30	0.21	0.14	0.07	0.01	-0.05	-0.09	-0.12	-0.15	-0.19	-0.24	-0.30	-0.34	-0.37	-0.37
Eastern Asia	0.05	-0.18	-0.36	-0.49	-0.61	-0.76	-0.90	-1.04	-1.14	-1.22	-1.28	-1.34	-1.40	-1.44	-1.46	-1.52
China	0.10	-0.14	-0.32	-0.45	-0.58	-0.74	-0.89	-1.03	-1.13	-1.20	-1.27	-1.34	-1.41	-1.45	-1.48	-1.54
China, Hong Kong SAR	0.49	0.41	-0.03	-0.19	-0.33	-0.44	-0.51	-0.57	-0.63	-0.67	-0.72	-0.74	-0.71	-0.63	-0.54	-0.44
China, Taiwan Province of C	-0.05	-0.22	-0.38	-0.54	-0.72	-0.87	-1.00	-1.12	-1.24	-1.34	-1.41	-1.45	-1.42	-1.37	-1.32	-1.25
Japan	-0.53	-0.73	-0.87	-0.94	-0.98	-1.01	-1.06	-1.18	-1.33	-1.43	-1.44	-1.39	-1.35	-1.34	-1.37	-1.43
Republic of Korea	-0.13	-0.32	-0.48	-0.64	-0.81	-0.99	-1.20	-1.41	-1.56	-1.64	-1.68	-1.70	-1.71	-1.74	-1.79	-1.77
South-Eastern Asia	0.73	0.49	0.29	0.16	0.02	-0.14	-0.29	-0.44	-0.58	-0.69	-0.79	-0.89	-1.00	-1.11	-1.21	-1.30
Cambodia	1.06	0.76	0.57	0.46	0.30	0.09	-0.11	-0.29	-0.46	-0.64	-0.82	-0.98	-1.12	-1.25	-1.39	-1.54
Indonesia	0.79	0.55	0.35	0.22	0.08	-0.09	-0.25	-0.40	-0.52	-0.62	-0.71	-0.80	-0.91	-1.02	-1.14	-1.25
Lao People's Democratic Re	1.12	0.82	0.57	0.41	0.23	0.02	-0.20	-0.40	-0.60	-0.79	-0.99	-1.18	-1.36	-1.55	-1.74	-1.93
Malaysia	0.99	0.69	0.45	0.30	0.17	0.07	-0.06	-0.21	-0.37	-0.51	-0.64	-0.75	-0.84	-0.91	-0.96	-1.01
Myanmar	0.57	0.38	0.16	0.01	-0.15	-0.31	-0.47	-0.61	-0.74	-0.86	-0.98	-1.11	-1.23	-1.33	-1.41	-1.46
Philippines	1.09	0.85	0.64	0.49	0.33	0.16	-0.00	-0.16	-0.31	-0.45	-0.59	-0.73	-0.87	-1.00	-1.13	-1.24
Singapore	0.60	0.36	0.13	-0.06	-0.24	-0.40	-0.53	-0.63	-0.70	-0.74	-0.76	-0.77	-0.76	-0.74	-0.70	-0.61
Thailand	-0.01	-0.25	-0.44	-0.57	-0.72	-0.89	-1.06	-1.21	-1.32	-1.38	-1.43	-1.48	-1.57	-1.68	-1.78	-1.84
Viet Nam	0.58	0.33	0.10	-0.02	-0.11	-0.23	-0.38	-0.56	-0.72	-0.85	-0.94	-1.02	-1.10	-1.17	-1.24	-1.32
Southern Asia	0.88	0.63	0.42	0.28	0.13	-0.03	-0.21	-0.38	-0.54	-0.69	-0.83	-0.97	-1.11	-1.23	-1.35	-1.44
India	0.73	0.50	0.30	0.15	-0.00	-0.16	-0.33	-0.50	-0.65	-0.80	-0.94	-1.07	-1.20	-1.32	-1.43	-1.50
Central & South America																
El Salvador	0.26	0.08	-0.13	-0.21	-0.35	-0.51	-0.68	-0.87	-1.06	-1.27	-1.51	-1.79	-2.13	-2.49	-2.87	-3.30
Mexico	0.77	0.51	0.32	0.19	0.04	-0.11	-0.28	-0.43	-0.57	-0.70	-0.83	-0.96	-1.10	-1.25	-1.39	-1.51
Brazil	0.41	0.15	-0.05	-0.17	-0.30	-0.43	-0.58	-0.73	-0.89	-1.04	-1.18	-1.31	-1.42	-1.53	-1.63	-1.71
Chile	-0.05	-0.05	0.11	-0.04	-0.19	-0.33	-0.47	-0.61	-0.74	-0.85	-0.95	-1.04	-1.14	-1.22	-1.28	-1.33
Eastern Europe	-0.39	-0.58	-0.70	-0.74	-0.75	-0.78	-0.85	-0.95	-1.04	-1.11	-1.12	-1.09	-1.05	-1.03	-1.05	-1.12
Belarus	-0.29	-0.49	-0.62	-0.67	-0.69	-0.72	-0.78	-0.87	-0.96	-1.02	-1.05	-1.04	-1.00	-0.97	-0.99	-1.07
Bulgaria	-0.91	-1.04	-1.16	-1.20	-1.21	-1.25	-1.33	-1.45	-1.57	-1.65	-1.68	-1.67	-1.65	-1.68	-1.79	-1.97
Czechia	-0.06	-0.24	-0.37	-0.40	-0.39	-0.39	-0.45	-0.57	-0.69	-0.75	-0.73	-0.65	-0.58	-0.54	-0.54	-0.57
Hungary	-0.46	-0.61	-0.74	-0.80	-0.84	-0.85	-0.89	-0.97	-1.08	-1.16	-1.17	-1.15	-1.14	-1.14	-1.14	-1.15
Poland	-0.33	-0.54	-0.70	-0.79	-0.87	-0.95	-1.05	-1.17	-1.31	-1.46	-1.59	-1.66	-1.67	-1.65	-1.65	-1.68
Republic of Moldova	-0.48	-0.71	-0.90	-1.03	-1.13	-1.24	-1.38	-1.55	-1.76	-1.96	-2.10	-2.17	-2.16	-2.14	-2.16	-2.24
Romania	-0.63	-0.73	-0.83	-0.89	-0.95	-1.01	-1.11	-1.23	-1.33	-1.39	-1.40	-1.41	-1.43	-1.48	-1.56	-1.66
Russian Federation	-0.27	-0.48	-0.60	-0.61	-0.60	-0.61	-0.66	-0.75	-0.83	-0.87	-0.86	-0.82	-0.77	-0.75	-0.79	-0.87
Slovakia	-0.20	-0.40	-0.58	-0.69	-0.77	-0.82	-0.90	-1.02	-1.17	-1.30	-1.37	-1.37	-1.33	-1.29	-1.29	-1.33
Ukraine	-0.80	-0.94	-1.03	-1.06	-1.11	-1.18	-1.28	-1.40	-1.51	-1.59	-1.62	-1.60	-1.57	-1.56	-1.61	-1.70
Southern Europe	-0.36	-0.51	-0.60	-0.65	-0.74	-0.86	-1.02	-1.18	-1.31	-1.37	-1.35	-1.30	-1.26	-1.27	-1.32	-1.39
Greece	-0.66	-0.69	-0.70	-0.72	-0.78	-0.89	-1.04	-1.20	-1.32	-1.38	-1.37	-1.30	-1.24	-1.21	-1.27	-1.34
Italy	-0.33	-0.50	-0.58	-0.62	-0.73	-0.88	-1.04	-1.20	-1.31	-1.31	-1.26	-1.21	-1.19	-1.22	-1.28	-1.34
Portugal	-0.41	-0.52	-0.62	-0.68	-0.76	-0.87	-0.99	-1.11	-1.21	-1.24	-1.21	-1.14	-1.10	-1.12	-1.18	-1.22
Spain	-0.22	-0.36	-0.46	-0.52	-0.60	-0.72	-0.90	-1.09	-1.26	-1.35	-1.33	-1.22	-1.09	-1.05	-1.09	-1.18
Western Europe	-0.02	-0.14	-0.24	-0.29	-0.36	-0.44	-0.52	-0.59	-0.62	-0.62	-0.62	-0.64	-0.67	-0.68	-0.70	-0.72
Austria	0.06	-0.08	-0.21	-0.27	-0.34	-0.43	-0.53	-0.61	-0.64	-0.65	-0.67	-0.70	-0.72	-0.70	-0.66	-0.63
Belgium	0.14	0.01	-0.08	-0.14	-0.19	-0.27	-0.36	-0.43	-0.48	-0.48	-0.49	-0.51	-0.53	-0.55	-0.57	-0.60
France	0.09	-0.04	-0.13	-0.20	-0.28	-0.36	-0.45	-0.53	-0.58	-0.60	-0.61	-0.63	-0.68	-0.73	-0.79	-0.85
Germany	-0.21	-0.32	-0.40	-0.44	-0.50	-0.58	-0.65	-0.70	-0.71	-0.69	-0.69	-0.70	-0.71	-0.69	-0.68	-0.68
Netherlands	0.06	-0.09	-0.24	-0.34	-0.44	-0.53	-0.61	-0.68	-0.70	-0.69	-0.70	-0.74	-0.82	-0.88	-0.93	-0.95

Note: The data for “Central & South America” is not available because the data for “Central America” and that for “South America” are provided separately in the *World Population Prospects 2019*.

Appendix 2. Rates of Population Decrease derived from Christiaans (2011)

(c) α=0.33, β＝0.66 and thus γ=67

δ	-(δ/γ) (%)	-(α＋β)δ/α (%)
0.12	-0.18	-36.00
0.1	-0.15	-30.00
0.09	-0.13	-27.00
0.08	-0.12	-24.00
0.07	-0.10	-21.00
0.06	-0.09	-18.00
0.05	-0.07	-15.00

(d) α=0.32, β＝0.67 and thus γ=68

δ	-(δ/γ) (%)	-(α＋β)δ/α (%)
0.12	-0.18	-37.13
0.10	-0.15	-30.94
0.09	-0.13	-27.84
0.08	-0.12	-24.75
0.07	-0.10	-21.66
0.06	-0.09	-18.56
0.05	-0.07	-15.47

(e) α=0.33, β＝0.60 and thus γ=9.57

δ	-(δ/γ) (%)	-(α＋β)δ/α (%)
0.12	-1.25	-33.82
0.10	-1.04	-28.18
0.09	-0.94	-25.36
0.08	-0.84	-22.55
0.07	-0.73	-19.73
0.06	-0.63	-16.91
0.05	-0.52	-14.09

(f) α=0.30, β=0.65 and thus γ=14

δ	$-(\delta/\gamma)$ (%)	$-(\alpha+\beta)\delta/\alpha$ (%)
0.12	-0.86	-38.00
0.10	-0.71	-31.67
0.09	-0.64	-28.50
0.08	-0.57	-25.33
0.07	-0.50	-22.17
0.06	-0.43	-19.00
0.05	-0.36	-15.83

(g) α=0.30, β=0.66 and thus γ=17.5

δ	$-(\delta/\gamma)$ (%)	$-(\alpha+\beta)\delta/\alpha$ (%)
0.12	-0.69	-38.40
0.10	-0.57	-32.00
0.09	-0.51	-28.80
0.08	-0.46	-25.60
0.07	-0.40	-22.40
0.06	-0.34	-19.20
0.05	-0.29	-16.00

(h) α=0.30, β=0.60 and thus γ=7

δ	$-(\delta/\gamma)$ (%)	$-(\alpha+\beta)\delta/\alpha$ (%)
0.12	-1.71	-36.00
0.10	-1.43	-30.00
0.09	-1.29	-27.00
0.08	-1.14	-24.00
0.07	-1.00	-21.00
0.06	-0.86	-18.00
0.05	-0.71	-15.00

Source: Author's Calculation