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Abstract

In the field of commercial capital theory, recently Yano (2006) and Murakami (2014) have tried to incorporate the commercial sector into the reproduction scheme. However, Yano's formulation put the commercial capital outside the reproduction formula, and his explanation is too complex. Moreover, Murakami's formation still has a problem owing to his methodology that uses only the numerical examples. Therefore, this paper incorporates the commercial sector into the reproduction scheme as a perfect equation system based on the equalized profit rate. It is because equalized profit rate is also applied to the commercial sector to determine its weight in the whole economy. As a result of these calculations, this paper also identified that average profit rate is determined by the technological conditions only in the industrial sectors.

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Introduction

Since Marx's reproduction scheme was introduced in Volume 2 of *Capital*, there was no unique commercial sector in his reproduction scheme, even if it discussed circulation process, and hence this scheme is not enough to analyze the real circulation process. However, Yano (1991) and Murakami (2014) argue, Marx planned to add the commercial sector in his 'Tableau" (which later came to fruition as the reproduction scheme) according to his manuscripts of 1861-63. Furthermore, Marx stated, in the first draft of Volume 2 of *Capital*, that he was planning to give his some provisions in the reproduction process of commercial capital at the end of the third volume. In other words, it was an unfinished task that Marx himself acknowledged, and Marxian economists have continued to try to incorporate the commercial sector into the reproduction scheme. In Japan, the most prominent achievement in this field are Yano (2006) and Murakami (2014, 2017).

However, commercial capital is shown outside of the reproduction scheme in Yano (2006), and his formalization is extremely complicated and incomprehensible. Furthermore, Murakami (2014, 2017) also remain a problem that the profit rate is not equalized between the industrial sector and the commercial sector. This shortage comes from his methodological weakness that he could not use any mathematical equations but only numerical examples, and costs such as c,v could be not recalculated to correspond to the price change by the transformation in table-5 and 6 in Murakami (2014).

Therefore, this paper incorporates the commercial sector into Marx's reproduction scheme not using numerical examples but formulating a mathematical equation system. This is decisively inadequate to treat the commercial sector, which only takes place after redistribution of the surplus value produced in industrial capital (productive sector). This is because the commercial sector can exist by being guaranteed the same profit rate as the industrial sector (productive sector) without producing value by itself. In this sense, it is not possible to discuss the commercial sector on the macroeconomic level neglecting the equalization of the profit rates.

In this sense, this paper introduces mathematical variable system instead of numerical examples, and shows a new reproduction scheme which also includes the commercial sector with its equalized profit rate to the industrial sector.

Three Sector Reproduction Scheme with Equalized Profit Rate Including Commercial Sector

Therefore, our first task is to introduce a new value-term reproduction scheme which includes the commercial sector, and it is already tried by above-mentioned Yano (2006) and Murakami (2014) with other sectors, for example, with a sector to produce the materials of money and the materials of means of production. However, in order to simplify this system, we neglect these two sectors and add the commercial sector only into the original reproduction scheme. That is,

$$W_1=c_1+v_1+m_1$$
$$W_2=c_2+v_2+m_2$$
$$W_c=c_c+v_c+m_c$$ [1]

Here, suffixes c indicates the commercial sector which does not produce any value, and this c_c must cover the total amount of W_1+W_2 divided by the number of turn-overs, since the commercial sector must transact all the goods of this society. Here we assume that. Furthermore, what should be explained at first here is that the reproduction scheme in volume 2 of *Capital* is formulated as the two industrial ectors engage its circulation to sell their products by themselves without help by the commercial sector. This can be theoretically assumed. However, this paper does not introduce this assumption, and

[1] This is Obata(2009)'s way to express reproduction scheme.

express explicitly the commercial sector as the third equation which does not produce any value. Therefore, at the level of value, W_c should be zero, while W_1 and W_2 are positive. This fact can be confirmed by examining the intersectoral transactions among three sectors. That is, when we assume simple reproduction,

Intersectoral supply = demand of the means of production $v_1+m_1=c_2+c_c$

Intersectoral supply = demand of the means of consumption $c_2=v_1+m_1+v_c+m_c$

And substituting the second equation for the first equation leads $W_c=c_c+v_c+m_c=0$. It means that the commercial sector does not produce value at all.

This result can be taken in the case of extended reproduction where m_1, m_2 and m_c are divided into m(c) +m(v)+m(k) respectively. In this case, this three sector reproduction scheme becomes

$$W_1=c_1+v_1+m_1(c)+m_1(v)+m_1(k)$$

$$W_2=c_2+v_2+m_2(c)+m_2(v)+m_2(k)$$

$$W_c=c_c+v_c+m_c(c)+m_c(v)+m_c(k)$$

And the intersectorial supply and demand of both goods become

Intersectoral supply = demand of the means of production

$$v_1+m_1(v)+m_1(k)=c_2+m_2(c)+c_c+m_c(c)$$

Intersectoral supply = demand of the means of consumption

$$c_2+m_2(c)=v_1+m_1(v)+m_1(k)+v_c+m_c(v)+m_c(k)$$

And substituting the second equation for the first equation leads

$$W_c=c_c+v_c+m_c(c)+m_c(v)+m_c(k)=0, \text{ that is } W_c=0 \text{ again.}$$

Of course, the commercial sector also actually spends the cost $c_c+v_c>0$, but this spending is not the 'productive' activity and therefore does not transfer and not formulate any value. Therefore, surplus value in this sector should be negative as $m_c=-(c_c+v_c)<0$. It is because these costs are spent in the non-productive sector. This fact is now reconfirmed by this calculation result as $W_c=c_c+v_c+m_c=0$.

Role of the Commercial Sector and its Optimal Weight

However, the commercial sector can receive a part of surplus value from the productive sector by shortening the time of circulation, and this relation could be formulated uniquely by Onishi (2015) in its second section of chapter five.

Onishi (2015)'s first step is to identify the contribution of commercial capital to industrial capital by reducing the time of circulation. Let us suppose Δ_p the original length of time of production, Δ_c the original length of time of circulation, Δ_c' the reduced length of time of circulation due to the activities by commercial capital, with surplus value m originally produced by industrial capital. In this case, the industrial capital can lengthen its time of production $\frac{\varDelta c-\varDelta c'}{\varDelta p}$ times longer, and additionally produce $\frac{\varDelta c-\varDelta c'}{\varDelta p}m$ amount of surplus value[2]. In this case, it also needs a certain additional production cost for its additional production, but this additional cost can be covered by its shortened turn-over.

Onishi (2015)'s second step is to identify the condition that this additional new surplus value can cover all the cost of the commercial sector and its profit. Let us suppose c_c, v_c and m_c transaction cost, wage and profit of the commercial capital[3]. In this case, this condition should be shown as

$$\frac{\varDelta c-\varDelta c'}{\varDelta p}m > c_c+v_c+m_c$$.[4]

Because this left side shows the additional surplus value produced by the industrial sectors, it also shows that the total production is also increased by them. c+v+m of the commercial sector is covered by this additional production of the surplus value. In this case, representing the constant capital and variable capital of the industrial capital after introducing the commercial capital as c_p and v_p respectively, and assuming the equalized *annual* rate of profit r between two capital, above inequality can be transformed into

$$\frac{\varDelta c-\varDelta c'}{\varDelta p}(c_p+v_p)\times annual\ rate\ of\ profit > (c_c+v_c)\{1+annual\ rate\ of\ profit\}.$$

In the calculation of the annual rate of profit, the turn-over period must be taken into consideration as long as the shortening of the circulation period is discussed, and hence

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2 Strictly speaking, following three assumptions are required for the above equation. That is, ① single-line continuous and constant production system which Marx explained in chapter 15, volume 2 of *Capital*, ② same $\frac{\varDelta c-\varDelta c'}{\varDelta p}=z$ in all the industrial sectors, ③ occupancy rate of the fixed capital is sufficiently elastic. Although this paper does not loose these assumptions here, the reader should know it.

3 Marx identified storage and transport costs as expenses that are related to the original production activity, even though they are part of the distribution process. Consequently, even if these are paid by commercial capital, in this context they need to be understood as capital investments in the productive sectors that produce value, not in the commercial sector.

4 Onishi (2018) put only the commercial profit in the right side of the above inequality, but it is a mistake. Right side should be $c_c+v_c+m_c$.

the annual rate of profit is set as $r\frac{1}{\Delta p+\Delta c}$ below. Here, r is a profit rate not considering the turn-over period. In this case, above inequality becomes

$$\frac{\Delta c-\Delta c'}{\Delta p}(c_p+v_p)\mathrm{r}\left(\frac{1}{\Delta p+\Delta c}\right) > (c_c+v_c)\left\{1+r\left(\frac{1}{\Delta p+\Delta c}\right)\right\},$$

and also transformed into an inequality which shows the ratio of total cost between both sectors. That is,

$$\frac{\Delta c-\Delta c'}{\Delta p}\cdot\frac{r}{(\Delta p+\Delta c)+r} > \frac{c_c+v_c}{c_p+v_p}$$

Until now we've discussed an individual industrial capital and commercial capital. However, this inequality can be changed to an equal sign if the scope of consideration is not limited to individual commercial capital, but applied to commercial capital in general. This is because when such commercial technology becomes widespread enough, less productive commercial capital will enter the field. Therefore, replacing this inequality sign with an equal sign, we can take.

$$\frac{\Delta c-\Delta c'}{\Delta p}\cdot\frac{r}{(\Delta p+\Delta c)+r} = \frac{c_c+v_c}{c_p+v_p}$$

It is very important to understand that this left side tends to rise under a given profit rate and turn-over period. It is because $\Delta c-\Delta c'$, Δp and $\Delta p+\Delta c$ tend to decrease by the technological progress. In Marxian economics, r tends to fall and therefore we cannot perceive whether the right side rises or not, but it is true that the weight of the commercial sector rises as far as under a given rate. Honestly, this conclusion is opposite from Marx's opinion that weight of the commercial sector fall due to the technological progress of the commercial sector which is expressed as the rise of $\Delta c-\Delta c'$ in this paper. Although Marx's opinion looks right intuitively, technological progress of the commercial sector makes this sector much useful, and therefore industrial sectors tend to use the commercial sector actively.

Therefore, based on this conclusion, and by simbolizing $\frac{\Delta c-\Delta c'}{\Delta p}$ part as z for the latter manipulation, we can express simply

$$c_c+v_c=z\frac{r}{(\Delta p+\Delta c)+r}(c_1+c_2+v_1+v_2).$$

Of course, under the condition where profit rates are equalized, it becomes

$$m_c=z\frac{r}{(\Delta p+\Delta c)+r}(m_1+m_2).$$

Therefore, commercial sector in the reproduction scheme can be transformed as

$$W_c = z\frac{r}{(\Delta p+\Delta c)+r}(c_1+c_2+v_1+v_2) + z\frac{r}{(\Delta p+\Delta c)+r}(m_1+m_2)$$

$$= z\frac{r}{(\Delta p+\Delta c)+r}(c_1+c_2+v_1+v_2+m_1+m_2)$$

$$= z\frac{r}{(\Delta p+\Delta c)+r}(W_1+W_2).$$

Recalculation of c and v under the Equalized Profit-rate; Shibata-Okishio's Method

Above mentioned method solved the question what weight should be taken by the commercial sector to the industrial sectors (productive sectors), but still that commercial sector is not incorporated satisfactorily in the reproduction scheme, because 'r' is not still equalized in this form. Therefore, in order to do so, we need to introduce the equalization process of the profit rates of the three sectors. In this paper, we use Shibata and Okishio's method[5] to equalize profit rates by recalculating transformed c and v. Therefore, we first introduce their method briefly for the readers.

Their method starts from the below two equation systems, that is

$$W_1 = c_1+v_1+m_1$$
$$W_2 = c_2+v_2+m_2,$$

and its first step to equalize profit rates is to transform them into

$$W_1' = (c_1+v_1)(1+r^0)$$

$$W_2' = (c_2+v_2)(1+r^0)$$

where r^o expresses the first round equatization of this transformation, and W_1' and W_2' express the first round value of both sectors of this transformation. However, take a note that the above formation does not consider the problem of turn-over. This shortcoming will be solved in the next section.

However, because price change to W_1' and W_2' from W_1 and W_2 also changes the prices of constant capital and variable capital, we need to calculate new prices of c_1, c_2, v_1 and v_2 by a recalculation. Therefore,

$$W_1'' = (c_1\frac{W_1'}{W_1} + v_1\frac{W_2'}{W_2})(1+r^1)$$

[5] See Shibata (1935) and Okishio (1972, 1973). In the Western world, this iterative method is well-known by Shaikh(1977), but it was introduced by Shibata and developed by Okishio.

$$W_2'' = (c_2 \frac{W_1'}{W_1} + v_2 \frac{W_2'}{W_2})(1 + r^1)$$

where r^1 is a new average rate of profit. This is the second round of transformation, but, since the “redistribution” is not completed yet, we still have to look for another average rate of profit, r^2, as expressed as follows:

$$W_1''' = (c_1 \frac{W_1''}{W_1} + v_1 \frac{W_2''}{W_2})(1 + r^2)$$

$$W_2''' = (c_2 \frac{W_1''}{W_1} + v_2 \frac{W_2''}{W_2})(1 + r^2)$$

This is the step by step recalculation, and what is finally obtained are the equal rate of profit, r^*, and each sector’s sales, W_1^* and W_2^*. That is,

$$W_1^* = (c_1 \frac{W_1^*}{W_1} + v_1 \frac{W_2^*}{W_2})(1 + r^*)$$

$$W_2^* = (c_2 \frac{W_1^*}{W_1} + v_2 \frac{W_2^*}{W_2})(1 + r^*)$$

Shibata (1935) shows that this iterative calculation will converge, but take a note that all these transformation are just the redistribution which does not add any new value. Therefore, $W_1+W_2=W_1'+W_2'=W_1''+W_2''=W_1^*+W_2^*$. This identification and the above two equations determines three variables: W_1, W_2 and r.

Reproduction Scheme with Commercial Sector

Based on the above results, this final section finalizes the transformation of the three sector reproduction scheme with the commercial sector. For this purpose, first, we must set the original three equation system of the reproduction before the transformation. That is,

$$W_1=c_1+v_1+m_1$$

$$W_2=c_2+v_2+m_2$$

$$W_c=z\frac{r}{(\Delta p+\Delta c)+r}(c_1+c_2+v_1+v_2+m_1+m_2).$$

Here, as seen in the second section, Wc amount of value is additionally produced by the industrial sectors. The form of the Wc sector comes from the result of the weight of the commercial sector calculated at the same section, and sets the profit rate '*r*' as the annual rate of profit considering the periods of turn-over which are assumed equal between two industrial sectors. Furthermore, take a note of a strangeness that only the last equation includes '*r*' but '*r*' does not appear in the first two equations. That's why we need a transformation of this three equation system including the commercial sector.

The first step of this transformation change the above three equations into

$$W_1' = (c_1 + v_1)\left(1 + \frac{r^0}{\Delta p + \Delta c}\right)$$

$$W_2' = (c_2 + v_2)\left(1 + \frac{r^0}{\Delta p + \Delta c}\right)$$

$$W_c' = z\frac{r^0}{(\Delta p + \Delta c) + r^0}\,(c_1 + c_2 + v_1 + v_2)\left(1 + \frac{r^0}{\Delta p + \Delta c}\right) = z(c_1 + c_2 + v_1 + v_2)\,\frac{r^0}{\Delta p + \Delta c}.$$

As mentioned above, because Wc' is the value produced by the industrial sector additionally after the introduction of the commercial sector, still $W_1' + W_2'$ is same with $W_1 + W_2$. Therefore, $W_1' + W_2' = W_1 + W_2$ and the first two equations can solve three variables: W_1'、W_2' and r^0, and later this r^0 determines Wc'. In other words, the first two equations are independent from the last equation.

However, as we see, this transformation is not enough, and then go to the next three equations as

$$W_1'' = \left(c_1\frac{W_1''}{W_1} + v_1\frac{W_2''}{W_2}\right)\left(1 + \frac{r^1}{\Delta p + \Delta c}\right)$$

$$W_2'' = \left(c_2\frac{W_1''}{W_1} + v_2\frac{W_2''}{W_2}\right)\left(1 + \frac{r^1}{\Delta p + \Delta c}\right)$$

$$W_c'' = z\left(c_1\frac{W_1'}{W_1} + c_2\frac{W_1'}{W_1} + v_1\frac{W_2'}{W_2} + v_2\frac{W_2'}{W_2}\right)\frac{r^1}{\Delta p + \Delta c}$$

And then, after some steps, we can finalized this transformation process as the result of

$$W_1^* = \left(c_1\frac{W_1^*}{W_1} + v_1\frac{W_2^*}{W_2}\right)\left(1 + \frac{r^*}{\Delta p + \Delta c}\right)$$

$$W_2^* = \left(c_2\frac{W_1^*}{W_1} + v_2\frac{W_2^*}{W_2}\right)\left(1 + \frac{r^*}{\Delta p + \Delta c}\right)$$

$$W_c^* = z\left(c_1\frac{W_1^*}{W_1} + c_2\frac{W_1^*}{W_1} + v_1\frac{W_2^*}{W_2} + v_2\frac{W_2^*}{W_2}\right)\frac{r^*}{\Delta p + \Delta c}$$

This result shows several important points. These are,

1) As seen in the first step of this transformation, the first two equations and supposed $W_1^* + W_2^* = W_1 + W_2$ can determine three variables: W_1^*、W_2^* and r^* independent from the commercial sector. In other words, the commercial sector does not commit to the

determination of the equal profit rate at all but just accepts.

2) Total sale of the commercial sector W_c^* can be rewritten as $z\frac{r^*}{(\Delta p+\Delta c)+r^*}$ $(W_1^*+W_2^*)=$ $z\frac{r^*}{(\Delta p+\Delta c)+r^*}$ (W_1+W_2) which shows that $z\frac{r^*}{(\Delta p+\Delta c)+r^*}$ times larger than the total production of the industrial sectors.

3) Of course, profit of the commercial sector also should be $z\frac{r^*}{(\Delta p+\Delta c)+r^*}$ times larger than the total profit of the industrial sectors, and $z\frac{r^*}{(\Delta p+\Delta c)+r^*}$ is determined partly by the existing technologies Δp, Δc and Δc' and partly by r* which is determined by the industrial sectors.

We can translate the above three equations into c+v+m type of formation as

$$W_1^* = c_1\frac{W_1^*}{W_1} + v_1\frac{W_2^*}{W_2} + \left(c_1\frac{W_1^*}{W_1} + v_1\frac{W_2^*}{W_2}\right)\frac{r^*}{\Delta p + \Delta c}$$

$$W_2^* = c_2\frac{W_1^*}{W_1} + v_2\frac{W_2^*}{W_2} + \left(c_2\frac{W_1^*}{W_1} + v_2\frac{W_2^*}{W_2}\right)\frac{r^*}{\Delta p + \Delta c}$$

$$W_c^* = c_c\frac{W_1^*}{W_1} + v_c\frac{W_2^*}{W_2} + \left\{z\left(c_1\frac{W_1^*}{W_1} + c_2\frac{W_1^*}{W_1} + v_1\frac{W_2^*}{W_2} + v_2\frac{W_2^*}{W_2}\right)\frac{r^*}{\Delta p + \Delta c} - \left(c_c\frac{W_1^*}{W_1} + v_c\frac{W_2^*}{W_2}\right)\right\}$$

The last equation of this system can show the complexity to determine the profit rate in the commercial sector.

In conclusion, we could show that 1) surplus value in the commercial sector is not produced in this sector but just redistributed from the industrial sector, but 2)the commercial sector can improve the production of the surplus value by shortening the circulation period for the industrial sector, and 3)the commercial sector can take a certain weight in the whole economy based on some conditions such as period of production (Δp), period of circulation (Δc) and shortening ratio of period of circulation (Δc- Δc'), and finally 4)average profit rate is determined only by the industrial sectors independent of the commercial sectors.

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