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February 9, 2019

Abstract

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1. Introduction

In this paper, we study the two-sector growth model in which consumption has a positive externality in production. Productive consumption was first introduced into the growth model by Steger (2000a). In low income countries, labor productivity crucially depends on a per capita consumption level that contributes to good nutrition (i.e., intake of calories and minerals, etc.), health (including medical services) and/or (basic) education. Thus, consumption not only satisfies current needs but also increases the productivity of labor.[1] The "efficiency wage hypothesis" by Leibenstein (1957a,b) is effective even today in those economies.

The efficiency wage hypothesis was analyzed in static models in the 1970s-80s, focusing on rural labor markets in developing economies (Stiglitz, 1976; Bliss and Stern, 1978; Gersovitz, 1983; Dasgupta and Ray, 1986). In the 1990s, this hypothesis was often used as one of the theories that could explain wage rigidity and involuntary unemployment in Keynesian macroeconomics. Further, the dynamic properties of the labor market have been extensively studied by several authors (Dusgupta, 1993; Ray and Streufert, 1993; Banerji and Gupta, 1997; Jellala and Zenoub, 2000).

Then, Steger (2000a, 2002) proposed two models to analyze the growth process of an entire economy with productive consumption effects.[2] In the first formulation, an increase in per capita consumption accelerates (disembodied) human capital

[1] Steger (2000a) called it the "productive consumption hypothesis". Lazear (1977) treated education as a joint product, simultaneously producing potential wage gains and utility. Strauss (1986) showed a highly significant positive effect of caloric intake on family farm labor productivity using household-level data from Sierra Leone. This study provides solid support for the nutrition-productivity hypothesis. Suen and Mo (1994) developed a microeconomic theory of productive consumption goods, demonstrating that the demand for productive goods tends to be relatively unresponsive to exogenous changes in prices and income. They all engaged in static analyses.

[2] Steger (2000a, 2002) considered the optimal path along which the representative consumer takes this effect into account and controls it. By contrast, we regard it as an externality in this paper.

accumulation. In the second formulation, an increase in per capita consumption increases workers' productivity at the same point in time (see also Gupta (2003)).

Daitoh (2010) extended Steger's first formulation to the endogenous growth model under the "productive consumption hypothesis" and provided the conditions for a unique saddle-point stable steady state or multiple steady states.[3]

The aggregated model with increasing returns-to-scale and with the externality has been used to study the indeterminacy of the equilibrium or the existence of multiple equilibrium paths by many authors since Benhabib and Farmer (1994). The indeterminacy could be the source of the volatile macroeconomic fluctuations.

In this paper, we extend Steger's second formulation to the two-sector model with a productive consumption externality and explore the possibility of indeterminacy. In low income countries where consumption increases the productivity of labor, there is insufficient physical capital stock. Thus, we assume that there is no capital externality. Following Benhabib and Nishimura (1998), we also assume the constant returns to scale of the production functions and show that indeterminacy occurs under very mild externalities.[4]

It could be useful to elucidate why indeterminacy is important for understanding the growth of low income economies in relation to the literature on development economics. Steger (2000b) refers to four stylized facts for (aggregate) economic growth primarily applied to the lower range of per capita income. He attempts to explain them by

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[3] Dinda (2008) investigates the growth process in a one-sector AK-type model with Steger's first formulation by incorporating social capital that is formed by human capital accumulation due to productive consumption.

[4] Wichmann (1997) assumed the nutrition-productivity relationship (a rise in agricultural and industrial labor productivity due to higher consumption of agricultural goods) was an externality (p.148). However, he did not consider the indeterminacy of the equilibrium, which will be the focus of the present paper.

incorporating subsistence consumption (Stone-Geary preferences) into linear (AK and Jones-Manuelli types of) growth models. A positive correlation between the savings rate and per capita income (stylized fact 2), β -divergence (stylized fact 3) and a hump-shaped pattern of growth (stylized fact 4) may be obtained in his model. Further, a broad diversity in the growth rates of per capita income (stylized fact 1) among countries with the same preferences and technologies was explained by the difference in distortive government policies. Such an explanation for the diversity of growth rates is reasonable because governments in developing countries often implement different distortive policies through the political process. The question is why the growth rates differ among countries with the same preferences and technologies whose governments implement the same distortive policies.

Indeterminacy may be a possible answer to this question, as it implies that an economy may have multiple equilibrium paths even if its preferences and technologies are uniquely given. Then, the actual equilibrium path will be chosen through the coordination of expectations, that is, when people in the society come to believe unanimously that their economy will move along that path. These expectations will be formed on the basis of conditions that are political, institutional, cultural and/or other social rather than on economic fundamentals.[5] Thus, indeterminacy could play a crucial role in determining the growth path of the economy.

In the rest of the paper, we show the diversity of growth rates among lower income countries in section 2, discuss the model in section 3, and investigate the local dynamics in section 4. Section 5 concludes the paper.

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[5] The role of political institutions in determining whether nations fail to develop has recently attracted keen interest in the development literature (Acemoglu and Robinson (2012)).

2. The Diversity of Growth Rates of Lower Income Countries

Let us see the diversity in the growth rates of per capita real gross domestic product (GDP) among "low income" and "lower middle income" countries, as defined by the World Bank. Table 1 presents the data on the levels and growth rates of real GDP per capita in 1961-2016 for the 36 "low income" and "lower middle income" countries, based on the *World Development Indicators 2017*. The first and second columns show the real GDP per capita in 1961 and 2016, respectively, and the third column shows the average annual growth rates during this period (calculated by the authors).

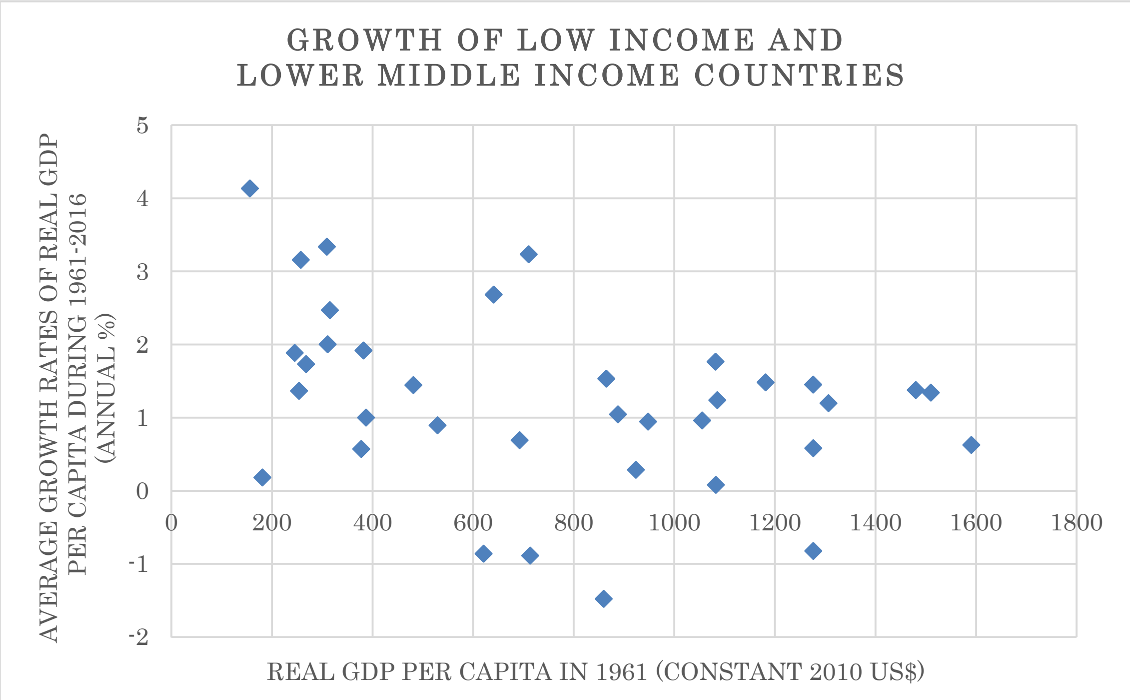
Table 1. Growth Performance of Low Income and Lower Middle Income Countries

Country Name	Real GDP per capita in 1961 (constant 2010 US$)	Real GDP per capita in 2016 (constant 2010 US$)	Average Annual Growth Rates of Real GDP per capita for 1961–2016 (%)
Myanmar	156	1408	4.13
Burundi	181	218	0.18
Burkina Faso	245	664	1.88
Malawi	254	481	1.36
Lesotho	258	1352	3.16
Nepal	268	685	1.73
India	309	1861	3.34
Rwanda	311	739	2.00
Pakistan	315	1179	2.47
Sierra Leone	378	456	0.57
Bangladesh	382	1030	1.92
Togo	387	558	1.00
Kenya	481	1143	1.44
Benin	529	837	0.90
Central African Republic	621	326	–0.86
Egypt, Arab Rep.	641	2724	2.68
Chad	692	860	0.69
Indonesia	711	3974	3.23
Madagascar	713	416	–0.89
Congo, Dem. Rep.	860	388	–1.48
Sudan	865	1924	1.53
Mauritania	888	1296	1.04
Zimbabwe	924	918	0.29
Cameroon	948	1495	0.94
Ghana	1055	1708	0.96
Philippines	1082	2753	1.77
Senegal	1083	1092	0.08
Honduras	1086	2138	1.24
Papua New Guinea	1182	2436	1.48
Nigeria	1276	2456	1.45
Liberia	1276	353	–0.82
Cote d'Ivoire	1276	1553	0.58
Bolivia	1307	2458	1.20
Guatemala	1481	3100	1.38
Congo, Rep.	1510	2798	1.34
Nicaragua	1591	1946	0.63

Source: *World Development Indicators 2017*

Figure 1 indicates that the average growth rates in 1961-2016 differ substantially among countries with the same initial levels of per capita real GDP in 1961, suggesting a diversity of growth paths starting from the same initial conditions.

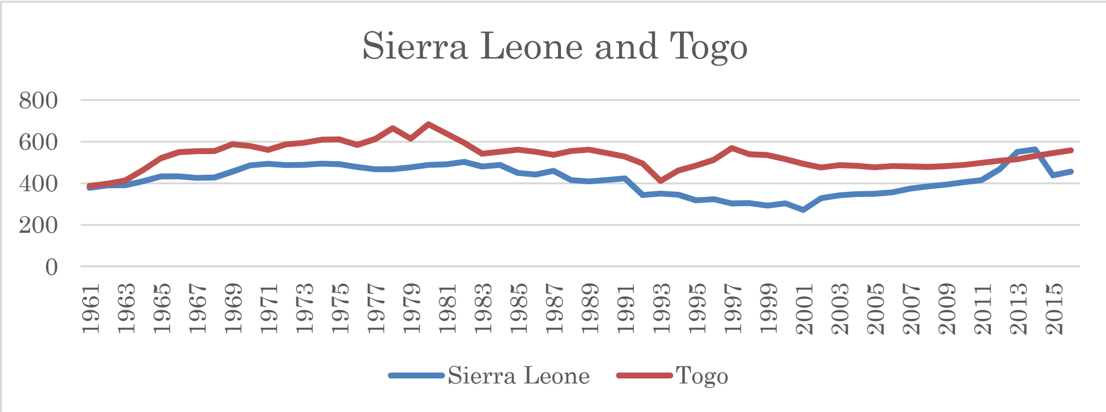
Figure 1. The Initial Levels of Real GDP Per Capita and the Average Growth Rates

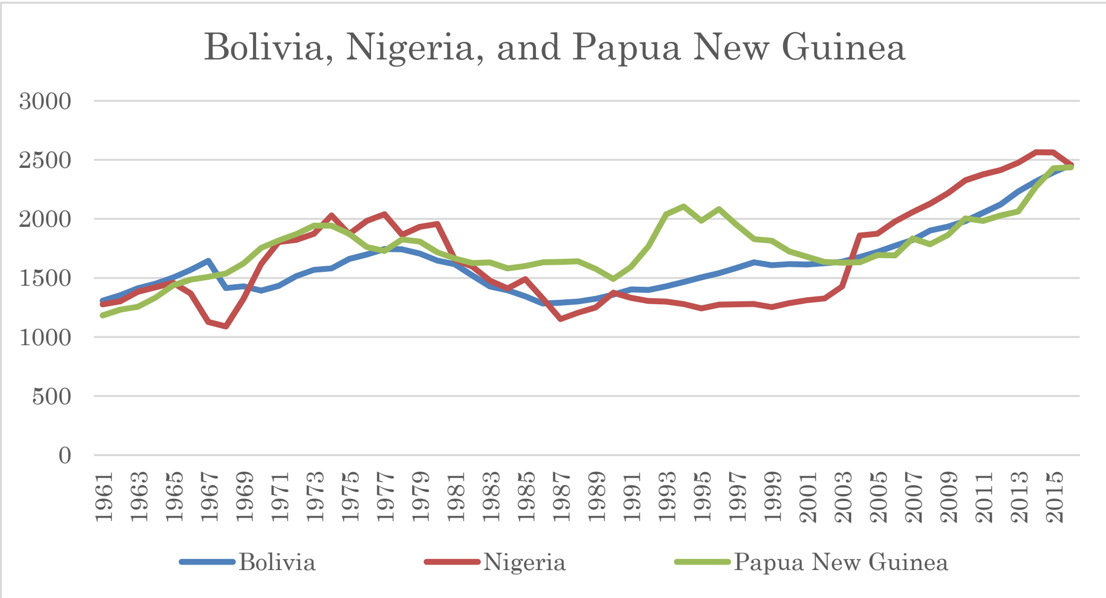


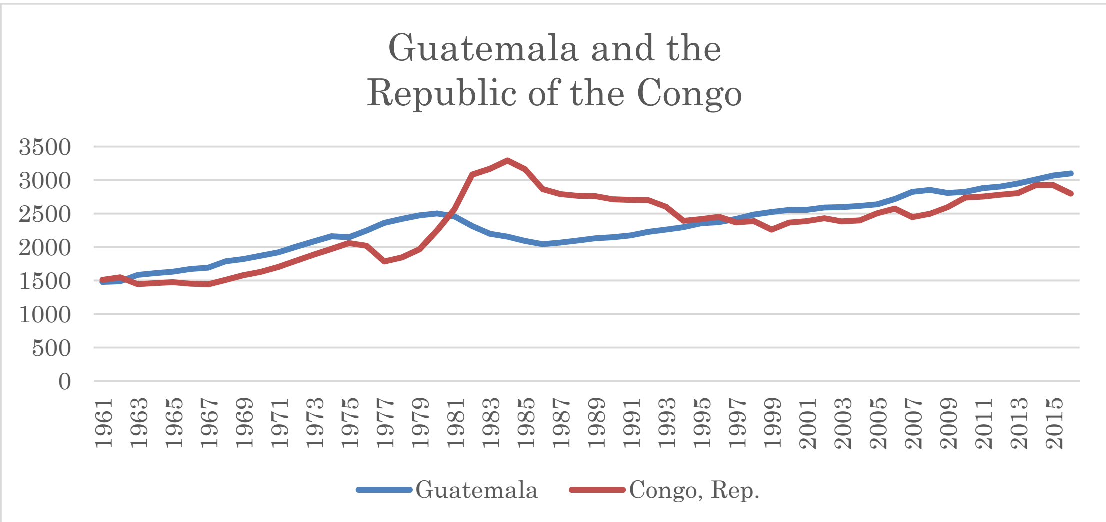
Source: *World Development Indicators 2017*

This growth diversity can be seen more clearly by looking at the time paths of per capita real GDP that countries with the same *initial* levels of income have followed during the 1961-2016 period. Figure 2 shows those time paths for countries with a per capita real GDP of approximately 400, 1300 and 1500 US dollars (constant in 2010 dollars) in 1961.

Figure 2.







Source: *World Development Indicators 2017*

We find that among the countries with an income of 380 US dollars in 1961, Sierra Leone and Togo seem to have traced different growth paths, converging to a similar level of per capita real GDP in 2016. The same property holds for Bolivia, Nigeria and Papua New Guinea (with an income of 1300 US dollars in 1961) and Guatemala and Republic of the Congo (with an income of 1500 US dollars in 1961). The countries in the same group of per capita real GDP in 1961 and in 2016 seem to have followed different growth paths during the 1961-2016 period.

These "low income" and "lower middle income" countries around the world are likely to have different political, institutional, cultural and/or other social factors, and thus people in those economies may form different expectations about the growth path that their economy will follow in the future. In this sense, the indeterminacy in the present paper might provide a relevant explanation for these empirical data on lower income economies.

3. The Model

Consider a perfectly competitive closed economy where a pure consumption good c (numeraire) and a pure investment good x are produced with capital $k_i (i = c, x)$ and labor $n_i (i = c, x)$. The total number of workers in the entire economy is normalized to be one. In the consumption-good sector, the labor input in efficiency units is $h n_c$, where n_c is physical labor (each worker's number of working hours) and h is the level of human capital (labor productivity) embodied by each worker.

We introduce the "productive consumption hypothesis" in the form of each worker's labor productivity depending positively on the average level of consumption $\overline{c}$ in the society, i.e., $h = h(\bar{c})$ with $h'(\bar{c}) > 0$. This effect works as an externality. To make the analysis clear, we use the specification of $h(\bar{c}) = \bar{c}^{\theta}$ ($0 < \theta < 1$). The production function of the consumption good is:

$$c = k_c^{\alpha_1}(\bar{c}^{\theta} n_c)^{\alpha_2}[\bar{c}^{\theta}\bar{n}_c]^{a_2}, \quad \alpha_1 > 0, \alpha_2 > 0, \quad \alpha_1 + (1+\theta)(\alpha_2 + a_2) = 1, \quad a_2 > 0 \tag{1}$$

Here, we assume labor-input externality $[\bar{c}^{\theta}\bar{n}_c]^{a_2}$. As we consider the low income countries with little capital stock, we assume that there is no externality of capital input. The production technology (1) is assumed to exhibit *constant* returns to scale in k_c and n_c *from the social perspective*.

The investment good is produced by the Cobb-Douglas type constant-returns-to-scale production function with no externalities:

$$x = k_x^{\beta_1} n_x^{\beta_2}, \quad \beta_1 > 0, \beta_2 > 0, \quad \beta_1 + \beta_2 = 1 \tag{2}$$

The capital accumulation function at any point in time *t* is:

$$\dot{k}(t) = k_x(t)^{\beta_1} n_x(t)^{\beta_2} - \delta k(t) \tag{3}$$

where $\delta > 0$ is the depreciation rate of capital.

To focus on the possibility of indeterminacy, we assume:

Assumption 1: (i) $\dfrac{\alpha_1}{\alpha_2} > \dfrac{\beta_1}{\beta_2}$, (ii) $\dfrac{\alpha_1}{\alpha_2 + a_2} < \dfrac{\beta_1}{\beta_2}$

We follow Uzawa (1961,1963,1964) by assuming that the investment-good sector is more labor intensive from the private perspective than the consumption-good sector (Assumption 1 (i)). In addition, the labor externality is sufficiently larger and the investment-good sector is more capital intensive from the social perspective than the consumption-good sector (Assumption 1 (ii)).

The representative consumer's instantaneous utility function is $u(c) = c$, where c is per capita consumption. Given the expected time path of the consumption externality $\{\bar{c}(t)^{\theta}\}_{t=0}^{\infty}$ and the labor-input externality $\{\bar{c}(t)^{\theta}\bar{n}_c(t)\}_{t=0}^{\infty}$, the representative consumer chooses the time path of $\{n_c(t)\}_{t=0}^{\infty}$, $\{k_c(t)\}_{t=0}^{\infty}$, $\{n_x(t)\}_{t=0}^{\infty}$ and $\{k_x(t)\}_{t=0}^{\infty}$ to

$$\text{maximize} \quad \int_0^{\infty} k_c(t)^{\alpha_1}(\bar{c}^{\theta}(t)n_c(t))^{\alpha_2}[\bar{c}(t)^{\theta}\bar{n}_c(t)]^{a_2}e^{-\rho t}dt$$

$$\text{subject to} \quad \dot{k}(t) = k_x(t)^{\beta_1}n_x(t)^{\beta_2} - \delta k(t), \; n_c(t) + n_x(t) = n, \; k_c(t) + k_x(t) = k(t) \quad (4)$$

where $\rho > 0$ is a constant time discount rate. The last two equalities are, respectively, the labor and capital constraints at a point in time. In what follows, we normalize the total number of working hours endowed to each worker to one ($n = 1$).

Let us define the Lagrangean function as:

$$L = k_c(t)^{\alpha_1}(\bar{c}(t)^{\theta}n_c(t))^{\alpha_2}[\bar{c}(t)^{\theta}\bar{n}_c(t)]^{a_2} + p(t)\{k_x(t)^{\beta_1}n_x(t)^{\beta_2} - \delta k(t)\}$$

$$+w(t)\{n - n_c(t) - n_x(t)\} + r(t)\{k(t) - k_c(t) - k_x(t)\} \quad (5)$$

where $p(t)$ is a costate variable; that is, the imputed price of the investment good, and $w(t)$ and $r(t)$ are Lagrangean multipliers. From the first-order conditions, the input coefficients $a_{ij}(i = k, n; j = c, x)$, e.g., $a_{nc} = \frac{n_c}{c}$ may be obtained:

$$\text{(i)} \quad \frac{\alpha_2 c}{n_c} = w, \quad \text{or} \quad a_{nc} = \frac{\alpha_2}{w} \quad (6)$$

$$\frac{\alpha_1 c}{k_c} = r, \quad \text{or} \quad a_{kc} = \frac{\alpha_1}{r} \quad (7)$$

$$p\beta_2 \frac{x}{n_x} = w, \quad \text{or} \quad a_{nx} = \frac{p\beta_2}{w} \tag{8}$$

$$p\beta_1 \frac{x}{k_x} = r, \quad \text{or} \quad a_{kx} = \frac{p\beta_1}{r} \tag{9}$$

(ii) $\dot{p}(t) = \rho p(t) - \dfrac{\partial L}{\partial k(t)} = (\rho+\delta) p(t) - r(k(t), p(t))$ (10)

$$\dot{k}(t) = k_x(t)^{\beta_1} n_x(t)^{\beta_2} - \delta k(t)$$

Because the Lagrangean function (5) is concave in the control variables (n_c, k_c, n_x, k_x) and the state variable (k), the time paths that satisfy the first-order conditions and the transversality condition are the solution path.

We consider the market equilibrium path on which the derived consumption path coincides with the expected path of the consumption externality. Setting $c = \overline{c}$ in (1) and solving the resulting equation, we obtain:

$$c = k_c^{\frac{\alpha_1}{1-\theta(\alpha_2+a_2)}} n_c^{\frac{\alpha_2+a_2}{1-\theta(\alpha_2+a_2)}} \tag{11}$$

The market equilibrium path is characterized by the autonomous dynamical system of capital k and its imputed price p:

$$\dot{k}(t) = x(k(t), p(t)) - \delta k(t) \tag{12}$$

$$\dot{p}(t) = (\rho+\delta) p(t) - r(k(t), p(t)) \tag{13}$$

Defining a steady state (k^*, p^*) by $\dot{k}(t) = \dot{p}(t) = 0$, we obtain, from the appendix, the steady state values in the following:

$$w^* = (\alpha_1^{\alpha_1} \alpha_2^{\alpha_2+a_2})^{1/\tau} \left[\left(\frac{\beta_1^{\beta_1}}{\rho+\delta} \right)^{1/\beta_2} \beta_2 \right]^{\alpha_1/\tau},$$

$$r^{*}=(\alpha_1^{\alpha_1}\alpha_2^{\alpha_2+a_2})^{\frac{1}{\tau}}\left[\left(\frac{\beta_1^{\beta_1}}{\rho+\delta}\right)^{1/\beta_2}\beta_2\right]^{-\frac{\alpha_2+a_2}{\tau}},$$

$$p^{*}=\frac{1}{\rho+\delta}(\alpha_1^{\alpha_1}\alpha_2^{\alpha_2+a_2})^{\frac{1}{\tau}}\left[\left(\frac{\beta_1^{\beta_1}}{\rho+\delta}\right)^{1/\beta_2}\beta_2\right]^{-\frac{\alpha_2+a_2}{\tau}}, \text{ and}$$

$$k^{*}=\left[\left(\frac{\beta_1^{\beta_1}}{\rho+\delta}\right)^{1/\beta_2}\beta_2\right]\frac{(\rho+\delta)n\alpha_1}{(\rho+\delta)\alpha_2+\delta(\alpha_1\beta_2-\alpha_2\beta_1)}, \tag{14}$$

where $\tau=1-\theta(\alpha_2+a_2)$ represents the labor-input and productive consumption externalities.

4. Local Dynamics

We will show that the steady state is locally indeterminate in the present model or that there exists a continuum of equilibrium paths converging to the steady state. We focus on the indeterminacy of the equilibrium in this sense in the present paper.

Let us derive the Jacobian matrix for the linearized system of (12) and (13) evaluated at the steady state:

$$J^{*}=\begin{bmatrix}\frac{\partial x^{*}}{\partial k}-\delta & \frac{\partial x^{*}}{\partial p}\\ -\frac{\partial r^{*}}{\partial k} & \rho+\delta-\frac{\partial r^{*}}{\partial p}\end{bmatrix} \tag{15}$$

Full employment conditions for capital and labor hold on the market equilibrium path:

$$a_{kc}c+a_{kx}x=k \tag{16}$$

$$a_{nc}c+a_{nx}x=n=1 \tag{17}$$

Totally differentiating (16) and (17) and using Shephard's lemma (the sum of indirect effects through changes in $a_{ij}(i=k,n;j=c,x)$ is zero), we obtain $a_{kc}dc+a_{kx}dx=dk$

and $a_{nc}dc + a_{nx}dx = 0$. Eliminating dc and rearranging the terms yield:

$$\frac{\partial x^*}{\partial k} = \frac{a_{nc}}{a_{kx}a_{nc} - a_{kc}a_{nx}} \tag{18}$$

Substituting (6) through (9) into (18), we obtain under Assumption 1:

$$\frac{\partial x^*}{\partial k} = \frac{\alpha_2 / w}{(p\beta_1 / r)(\alpha_2 / w) - (\alpha_1 / r)(p\beta_2 / w)} = \frac{\alpha_2 r}{p(\alpha_2\beta_1 - \alpha_1\beta_2)} < 0 \tag{19}$$

An increase in capital stock k decreases the production of investment good x, which is more labor intensive from the private perspective. Thus, we conclude:

$$\frac{\partial x^*}{\partial k} - \delta < 0 \tag{20}$$

For price relationships, let us define $\hat{a}_{ij}(i = k,n; j = c,x)$ in the following:

$$\hat{a}_{nc} = \frac{(\alpha_2 + a_2)}{\alpha_2 \tau} a_{nc} = \frac{\alpha_2 + a_2}{\tau w} \tag{21}$$

$$\hat{a}_{kc} = \frac{1}{\tau} a_{kc} = \frac{\alpha_1}{\tau r} \tag{22}$$

$$\hat{a}_{nx} = a_{nx} \tag{23}$$

$$\hat{a}_{kx} = a_{kx} \tag{24}$$.

Using these definitions, simple calculations yield:

$$\hat{a}_{kc} r + \hat{a}_{nc} w = 1 \tag{25}$$

$$\hat{a}_{kx} r + \hat{a}_{nx} w = p \tag{26}$$

Thus, factor price equalization holds, that is, factor prices (r, w) are uniquely determined by output prices $(1, p)$, independent of factor endowments (k, n). Then, we obtain:

$$\frac{\partial r^*}{\partial k} = 0$$

Therefore, the roots of J^* are $(\partial x^* / \partial k) - \delta$ and $\rho + \delta - (\partial r^* / \partial p)$.

We have $1 = (a_{kc})^{\frac{\alpha_1}{\tau}} (a_{nc})^{\frac{\alpha_2 + a_2}{\tau}}$ from (11). Substituting (6) and (7) into this and rearranging the terms, we get $(r / \alpha_1)^{\frac{\alpha_1}{\tau}} (w / \alpha_2)^{\frac{\alpha_2 + a_2}{\tau}} = 1$. The total differentiation yields:

$$\hat{a}_{kc} dr + \hat{a}_{nc} dw = 0 \tag{27}$$

Similarly, we have $1 = (a_{kx})^{\beta_1} (a_{nx})^{\beta_2}$ from (2). Substituting (8) and (9) into this and rearranging the terms, we get $p = (r / \beta_1)^{\beta_1} (w / \beta_2)^{\beta_2}$. The total differentiation yields:

$$\hat{a}_{kx} dr + \hat{a}_{nx} dw = dp \tag{28}$$

By simultaneously solving (27) and (28), we obtain the change in the rental rate of capital corresponding to a change in the price of investment good. In addition, by Assumption 1,

$$\frac{\partial r^*}{\partial p} = \frac{-\hat{a}_{nc}}{\hat{a}_{kc}\hat{a}_{nx} - \hat{a}_{kx}\hat{a}_{nc}} = \frac{-(\alpha_2 + a_2)/\tau w}{(\alpha_1 / \tau r)(p\beta_2 / w) - (p\beta_1 / r)(\alpha_2 + a_2)/\tau w}$$

$$= -\frac{r}{p}\left[\frac{\alpha_2 + a_2}{\alpha_1\beta_2 - (\alpha_2 + a_2)\beta_1}\right] > 0 \tag{29}$$

Because $p^*(\rho + \delta) = r(k^*, p^*)$ holds at the steady state, we obtain:

$$\rho + \delta - \frac{\partial r^*}{\partial p} = (\rho + \delta)\left(1 - \frac{p^*}{r^*}\frac{\partial r^*}{\partial p}\right) = (\rho + \delta)\frac{\alpha_1\beta_2 + (\alpha_2 + a_2)\beta_2}{\alpha_1\beta_2 - (\alpha_2 + a_2)\beta_1} < 0 \tag{30}$$

Therefore, both roots of J^* are negative, and thus, the steady state is indeterminate. This establishes our main theorem.

Theorem: *Consider the two-sector growth model with a productive consumption externality in the pure consumption-good sector. Under Assumption 1, the steady state is locally indeterminate.*

Let us illustrate the relevance of Assumption 1. Eliminating β_2, conditions (i) and (ii) are both satisfied if and only if

$$\frac{\alpha_1}{\alpha_1+\alpha_2+a_2} < \beta_1 < \frac{\alpha_1}{\alpha_1+\alpha_2} \tag{31}$$

holds. Recalling the standard view in the growth and macroeconomics literature that the relative share of capital is 1/3, consider an example with $\alpha_1 = 0.3$ in the consumption-good sector. If, in addition, $\alpha_2 = 0.6$ and $a_2 = 0.05$ hold, then (31) will be $0.316 < \beta_1 < 0.333$. Then, when the relative share of capital in the investment-good sector is $\beta_1 = 0.32$, which is also empirically relevant, Assumption 1 holds. The strength of the productive consumption externality in this example is $\theta \doteq 0.077$ (by $\alpha_1 + (1+\theta)(\alpha_2 + a_2) = 1$). Another example is $\alpha_1 = 0.33$, $\alpha_2 = 0.6$ and $a_2 = 0.04$. Then, we have $\theta \doteq 0.047$, which implies a weaker productive consumption externality. Note that (31) is $0.340 < \beta_1 < 0.354$.

We will intuitively explain the mechanism generating indeterminacy in the present model with the productive consumption externality. Consider a capital stock level k higher than and sufficiently close to the steady state value k^*. Because the investment good is more labor intensive from the private perspective, its output x decreases by (19). This corresponds to the meaning of the Rybczynski theorem. Because this leads to a net decrease in capital stock by (20), the capital stock k decreases, implying the stability of the quantity side of the system.

For the price side, consider a price level p higher than and sufficiently close to the steady state value p^*. Because the investment good is more capital intensive from the social perspective, by (30), the return to capital r increases more than proportionally, implying the stability of the price side of the system. This corresponds to the meaning of the Stolper-Samuelson theorem. Note that the duality between the Rybczynski and Stolper-Samuelson theorems does not hold in the presence of external effects.

We have extended the model of Steger (2002) to the two-sector dynamic model and shown that indeterminacy occurs under a very mild productive consumption externality. With indeterminacy, an economy may have multiple equilibrium paths converging to the steady state. Thus, if people in each country form an expectation on the growth path that their economy will follow based on the political, institutional, cultural and/or other social fundamentals, then the economy will actually trace that growth path. Indeterminacy can thus explain why lower income countries starting from the same initial levels of income follow different growth paths.

5. Concluding Remarks

The growth path of low income and lower middle income countries varies widely from country to country, even if they were at similar income levels in 1961 and seem to converge to similar income levels in 2016. In this paper, we have extended the model of Steger (2002) and shown that indeterminacy can occur in a two-sector constant-return-to-scale dynamic model with a productive consumption externality. The productive consumption effect is based on a substantial contribution of good nutrition, health and education to higher labor productivity. Indeterminacy occurs when this effect works as an externality. Our model provides a relevant explanation for why the growth rates differ among developing countries with the same preferences and technologies whose governments implement the same distortive policies.

Appendix: Derivation of Steady State Values

In this appendix, we explicitly derive the steady state values. First, we derive the values of factor and commodity prices in the steady state. By substituting the first-order conditions into the investment-good production function $x = k_x^{\beta_1} n_x^{\beta_2}$, using $\beta_1 + \beta_2 = 1$ and inserting the steady state condition $p = r/(\rho+\delta)$, we obtain:

$$r = \left(\frac{w}{\beta_2}\right)\left(\frac{\rho+\delta}{\beta_1^{\beta_1}}\right)^{1/\beta_2}. \tag{A.1}$$

By substituting the first-order conditions into the consumption-good production function $c = k_c^{\alpha_1} n_c^{\alpha_2+a_2} \bar{c}^{\theta(\alpha_2+a_2)}$, we obtain:

$$r^{\alpha_1} w^{\alpha_2+a_2} = \alpha_1^{\alpha_1} \alpha_2^{\alpha_2+a_2}. \tag{A.2}$$

Substituting (A.1) into (A.2), we get:

$$w^* = (\alpha_1^{\alpha_1} \alpha_2^{\alpha_2+a_2})^{1/\tau} \left[\left(\frac{\beta_1^{\beta_1}}{\rho+\delta}\right)^{1/\beta_2} \beta_2\right]^{\alpha_1/\tau}. \tag{A.3}$$

where $\tau = 1 - \theta(\alpha_2 + a_2) = \alpha_1 + \alpha_2 + a_2$. Because $r^* = w^* \left[\left(\frac{\beta_1^{\beta_1}}{\rho+\delta}\right)^{1/\beta_2} \beta_2\right]^{-1}$ (or $w^* = r^* \left[\left(\frac{\beta_1^{\beta_1}}{\rho+\delta}\right)^{1/\beta_2} \beta_2\right]$) holds from (A.1), the rental rate of capital in the steady state is:

$$r^* = (\alpha_1^{\alpha_1}\alpha_2^{\alpha_2+a_2})^{\frac{1}{\tau}}\left[\left(\frac{\beta_1^{\beta_1}}{\rho+\delta}\right)^{1/\beta_2}\beta_2\right]^{-\frac{\alpha_2+a_2}{\tau}} . \tag{A.4}$$

Then, the price of the consumption good in the steady state $p^* = r^*/(\rho+\delta)$ is:

$$p^* = \frac{1}{\rho+\delta}(\alpha_1^{\alpha_1}\alpha_2^{\alpha_2+a_2})^{\frac{1}{\tau}}\left[\left(\frac{\beta_1^{\beta_1}}{\rho+\delta}\right)^{1/\beta_2}\beta_2\right]^{-\frac{\alpha_2+a_2}{\tau}} . \tag{A.5}$$

Finally, we will derive the steady state level of capital stock. Full employment conditions and the first-order conditions imply:

$$n = n_x + n_c = \left(\frac{p\beta_2}{w}\right)x + \left(\frac{\alpha_2}{w}\right)c \quad \text{i.e.,} \quad wn = p\beta_2 x + \alpha_2 c ,$$

$$k = k_x + k_c = \left(\frac{p\beta_1}{r}\right)x + \left(\frac{\alpha_1}{r}\right)c \quad \text{i.e.,} \quad rk = p\beta_1 x + \alpha_1 c .$$

Solving them simultaneously by eliminating c, we get:

$$k^* = \frac{wn\alpha_1}{r\alpha_2 + p\delta(\alpha_1\beta_2 - \alpha_2\beta_1)} = \frac{(w/p)n\alpha_1}{(r/p)\alpha_2 + \delta(\alpha_1\beta_2 - \alpha_2\beta_1)} .$$

Using $(r^*/p^*) = \rho+\delta$ and $(w^*/p^*) = (w^*/r^*)(r^*/p^*) = \left(\frac{\beta_1^{\beta_1}}{\rho+\delta}\right)^{1/\beta_2}\beta_2(\rho+\delta)$, we obtain:

$$k^* = \left[\left(\frac{\beta_1^{\beta_1}}{\rho+\delta}\right)^{1/\beta_2}\beta_2\right]\frac{(\rho+\delta)n\alpha_1}{(\rho+\delta)\alpha_2 + \delta(\alpha_1\beta_2 - \alpha_2\beta_1)} . \tag{A.6}$$

Under Assumption 1, the steady state level of capital stock is positive.

References

Acemoglu, D. and J.A. Robinson, 2012. Why nations fail: Origins of power, prosperity, and poverty. Crown Business. (Japanese translation)

Banerji, S., Gupta, M.R., 1997. The efficiency wage given long-run employment and concave labor constraint. Journal of Development Economics 53, 185-195.

Benhabib, J., Farmer, R.E.A., 1994, Indeterminacy and increasing returns. Journal of Economic Theory 63, 19-41.

Benhabib, J., Nishimura, K., 1998, Indeterminacy and sunspot with constant returns. Journal of Economic Theory 81, 58-96.

Bliss, C., Stern, N., 1978. Productivity, wages and nutrition part I: theory. Journal of Development Economics 5, 331-362.

Daitoh, I., 2010. Productive consumption and population dynamics in an endogenous growth model: demographic trends and human development aid in developing economies. Journal of Economic Dynamics and Control 34, 696-709.

Dasgupta, P., 1993. *An inquiry into well-being and destitution*. Oxford University Press.

Dasgupta, P., Ray, D., 1986. Inequality as a determinant of malnutrition and underemployment: theory. Economic Journal 96, 1011-1034.

Dinda, S., 2008. Social capital in the creation of human capital and economic growth: A productive consumption approach. Journal of Socio-Economics 37, 2020-2033.

Gersovitz, M., 1983. Savings and nutrition at low income. Journal of Political Economy 91, 841-855.

Gupta, M.R., 2003. Productive consumption and endogenous growth: a theoretical analysis. Keio Economic Studies 40, 45-57.

Jellala, M., Zenoub, Y., 2000. A dynamic efficiency wage model with learning by doing. Economics Letters 66, 99-105.

Lazear, E., 1977. Education: Consumption or production? Journal of Political Economy 85, 569-597.

Leibenstein, H., 1957a. Economic Backwardness and Economic Growth. Wiley, New York.

Leibenstein, H., 1957b. The theory of underemployment in backward economies. Journal of Political Economy 65, 91-103.

Ray, D., Streufert, P., 1993. Dynamic equilibria with unemployment due to undernourishment. Economic Theory 3, 61-85.

Steger, T.M., 2000a. Productive consumption and growth in developing countries. Review of Development Economics 4, 365-375.

Steger, T.M., 2000b. Economic growth with subsistence consumption. Journal of Development Economics 62, 343-361.

Steger, T.M., 2002. Productive consumption, the intertemporal consumption trade-off and growth. Journal of Economic Dynamics and Control 26, 1053-1068.

Stiglitz, J., 1976. The efficiency wage hypothesis, surplus of labor and distribution of income in the LDCs. Oxford Economic Papers 28, 185-207.

Strauss, J., 1986. Does better nutrition raise farm productivity? Journal of Political Economy 94, 297-320.

Suen, W., P.H. Mo, 1994. Simple analytics of productive consumption. Journal of Political Economy 102, 372-383.

Uzawa, H. 1961. On a Two-Sector Model of Economic Growth. Review of Economic Studies 29, 40-47.

Uzawa, H. 1963. On a Two-Sector Model of Economic Growth II. Review of Economic Studies 30, 105-118.

Uzawa, H. 1964. Optimal Growth in a Two-Sector Model of Capital Accumulation. Review of Economic Studies 31, 1-24.

Wichmann, Thorsten, 1997. Agricultural technical progress and the development of a dual economy. Physica-Verlag, Heidelberg.

World Bank, 2017. http://databank.worldbank.org/data/reports.aspx?source=wdi-database-archives-%28beta%29 (March 25, 2018).