Institute for Economic Studies, Keio University

Keio-IES Discussion Paper Series

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29 January, 2019
DP2019-005
https://ies.keio.ac.jp/en/publications/10818/

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Acknowledgement: The author is grateful to Anton Braun, Fumio Hayashi, and Yasuo Hirose for their helpful comments.

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JEL: E31, E52, E61, E62, E63

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1 Introduction

In the aftermath of the global financial crisis (GFC), more than a decade has passed since central banks found themseves constrained by the zero lower bound on their policy rates. Despite the recent global recovery, the inflation rate has remained low in many advanced countries and central banks have kept their policy rates virtually at zero. Such a state where policy rate is stuck at the lower bound and interest rate cannot be lowered further is often called the liquidity trap, which has become a global phenomena.

Our past experience shows that liquidity traps are observed following a severe decline in demand, which typically occurs after economic crises. As such, right after the GFC, professionals and market participants anticipated that the liquidity trap should disappear as the economy recovers and would be short-lived.[1] Such expectations, however, have turned out to be too optimistic; a number of advanced economies have been suffering from low inflation and virtually zero interest rate for over a decade.

Well before the GFC, the seminal paper of Benhabib et al. (2001) revealed that when the central bank targets a positive inflation rate and the nominal interest rate is constrained by a lower bound, multiple equilibria can arise and one of them is deflationary. Succeeding literature has often referred to this deflationary equilibria as the expectations-driven liquidity trap (ELT) since this liquidity trap arises from disanchoring of inflation expectations. Because our experience of liquidity trap has become prolonged in a number of countries, this multiplicity of equilibria has attracted a wide range of interest among academics and policy makers.

Many studies have investigated how effective policies can be implemented to avoid liquidity traps caused either by real shocks or expectations. While studies such as Sugo and Ueda (2008) and Christiano and Takahashi (2018) claim that monetary policy can play a central role in avoiding ELT, the majority of studies have emphasized the importance of fiscal policy.

Existing studies on effective fiscal policy at the liquidity trap can be divided largely into two strands. The first strand explores the effectiveness of *exogenous* policies at the liquidity trap. Correia et al. (2013) shows that tax policy can deliver stimulus when the zero lower bound is binding. Mertens and Ravn (2014) analytically studies fiscal policies at the ELT and finds that supply-side policies such as tax cuts are more effective than conventional demand-side policies.

The second strand, on the other hand, focuses on policy *rules* that prevents

[1] Earlier version of Gust et al. (2017) documented that average expectations of professional forecasts for the near-zero short rate was 3 to 4 quarters in 2009Q1 and 2010Q2.

ELT. Benhabib et al. (2002) propose a non-Ricardian fiscal policy which avoids deflationary steady state by responding to the inflation rate more in a more aggressive manner. Schmidt (2016) shows that a Ricardian fiscal policy designed to keep the real marginal cost higher than a threshold level can avoid any type of liquidity trap.

In this paper, I follow this latter strand of the literature and investigate how fiscal *rules* can be implemented to prevent ELT. This paper is motivated by the advanced economy's experience – especially Japan's – of prolonged liquidity trap, hence the goal is to investigate fiscal rules that enables the fiscal authority to prevent ELT in a simple and transparent manner.

To this end, I introduce sunspot shocks called "regime shocks" which forces the economy to move between targeted regime and unintended regime to an otherwise standard New Keynesian model. While most existing studies assume that the targeted regime is absorbing and that the ELT is merely an one-off phenomena, this paper relaxes this absorbing assumption and analyze effective fiscal policies when the ELT is assumed to be recurrent.[2]

The main findings of this paper are threefold. First, I demonstrate that a simple Ricardian tax policy which responds to the inflation rate can prevent the ELT. The proposed tax rule is analogous to the Taylor rule, therefore the rule is straightforward to implement. By affecting household's consumption demand and labor supply, either policies based on a single tax rate or a combination of taxes can rule out the ELT. Intuitively speaking, tax policies that induces households to increase labor supply when the inflation rate declines prevents the ELT to arise by raising the marginal cost.

Second, I find that the extent to which tax rates must respond to inflation to avoid the ELT is affected by the probability of remaining at the ELT. The higher the persistence of the ELT is, the larger the response of the tax rate. If the response of the tax rate is not sufficient, the government not only fails to avoid the ELT but can also make the declines in inflation and output at the ELT even larger as compared to not responding at all.

Third, I show that once the possibility of switching back to the ELT from the targeted regime is taken into account, the tax rate needs to respond to inflation by a larger amount compared to the case where the targeted regime is assumed to be absorbing. This indicates that if the fiscal authority responds to inflation without considering the expectational effects of switching back to the ELT, it fails to prevent the ELT and aggravates the drop in inflation and output. This last finding

[2] Aruoba et al. (2018) introduces a Markov regime switching structure into a New Keynesian model and allows the possibility of switching back to the ELT, while they do not discuss policies to avoid such liquidity traps.

contributes to the recent studies which emphasize that considering the probability of switching back to the ELT may affect optimal policies.[3]

The remainder of this paper is organized as follows. Section 2 describes the structure of the economy and the model. Section 3 analyzes tax rules to avoid the ELT assuming absorbing state. Section 4 analyzes tax rules to avoid the ELT assuming Markov switching structure. Section 5 concludes.

2 Model

In this section, I start with presenting the structure of the economy and the type of equilibria studied in this paper. Then I provide the details of the model and the equilibrium conditions.

2.1 Structure of the economy

Existing studies have found that liquidity traps can be caused by different reasons. The seminal paper of Eggertsson and Woodford (2003) analyzes liquidity trap arising from fundamental shocks. However, an increasing number of studies have recently focused on liquidity trapa arising from non-fundamental shocks (e.g. Mertens and Ravn (2014), Schmidt (2016), Aruoba et al. (2018), Coyle and Nakata (2018)).

To focus on how fiscal policy can be designed to avoid the liquidity trap caused by non-fundamental shocks, I abstract from fundamental shocks. It is assumed that there are only two regimes in the economy: one is the "targeted regime" where the inflation is near central bank's target, while the other is the "unintended regime" where the central bank misses its inflation target and the interest rate is stuck at zero.

There is a non-fundamental shock called the regime shocks s_t which follows a two-state Markov process. The economy is in the "targeted regime" if $s_t = T$ and in the "unintended regime" if $s_t = U$. Regime shock s_t is revealed at the beginning of the period, which is observed by the agents. Agents coordinate their decisions and therefore their information sets when forming expectations include current realization of s_t. The transition matrix P is given as

$$P = \begin{pmatrix} p_{TT} & p_{TU} \\ p_{UT} & p_{UU} \end{pmatrix}'$$

[3]Coyle and Nakata (2018) find that even a small probability of moving to the ELT can affect the optimal inflation rate significantly.

where p_{ij} is the probability of switching from regime i to regime j.

Note that the Taylor principle is not satisfied at the ELT, therefore indeterminacy may arise as an issue. In such a case, infinite number of equilibria can exist. One approach to handle indeterminacy is to introduce additional sunspot shocks as in Hirose (forthcoming). However, since the goal of this paper is to prevent the ELT and indeterminacy will not arise if that goal is achieved, I rule out such equilibria from my analysis.

2.2 Household

There is a representative household who gains utility from consumption and disutility from labor supply. The household maximizes expected lifetime utility by choice of consumption c_t, labor supply l_t and bondholdings b_t given prices and subject to a budget constraint.

$$\max_{\{c_{t+s},l_{t+s},b_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Big[\frac{c_{t+s}^{1-\sigma}-1}{1-\sigma} - \frac{l_{t+s}^{\eta+1}-1}{\eta+1}\Big] \tag{1}$$

$$\text{s.t.} \quad (1+\tau_{c,t})c_t + \frac{b_t}{R_t} = (1-\tau_{w,t})w_t l_t + \frac{b_{t-1}}{\Pi_t} + (1-\tau_{d,t})d_t - \tau_t \tag{2}$$

R_t and Π_t are the gross nominal interest rate and the gross inflation rate respectively. w_t is the real wage and d_t is a dividend from intermediate goods firms. $\tau_{c,t}, \tau_{w,t}$ and $\tau_{d,t}$ are consumption tax, labor income tax and dividend tax respectively, whereas τ_t is the lump-sum tax. As we will discuss later, I allow these tax rates to vary over time.

From the first order conditions, we can derive the Euler equation and the wage equation as

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}} = \beta R_t \mathbb{E}_t \Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}} \frac{1}{\Pi_{t+1}}\Big] \tag{3}$$

$$\frac{c_t^{-\sigma}}{l_t^{\eta}} = \frac{1+\tau_{c,t}}{1-\tau_{w,t}} \frac{1}{w_t} \tag{4}$$

2.3 Firms

There are two types of firms in the economy: a continuum of intermediate goods producers and a final goods producer. The final goods producer uses intermediate goods as the only input and has CES production technology. The final goods producer is perfectly competitive and takes both output and input prices as given. The static profit maximization problem is given as follows

$$\max_{\{y_t, y_{i,t}\}} P_t y_t - \int_0^1 P_{i,t} y_{i,t} di \tag{5}$$

$$\text{s.t.} \quad y_t = \Big(\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} di \Big)^{\frac{\theta}{\theta-1}} \tag{6}$$

Perfect competition drives final good producers' profits to zero. From the first order conditions, we can derive the demand for intermediate goods and the associated price index

$$y_{i,t} = \Big(\frac{P_{i,t}}{P_{t+s}} \Big)^{-\theta} y_t \tag{7}$$

$$P_t = \Big(\int_0^1 P_{i,t}^{1-\theta} di \Big)^{\frac{1}{1-\theta}} \tag{8}$$

There are a continuum of intermediate goods producers indexed by i. They are monopolistically competitive and incur quadratic price adjustment cost as in Rotemberg (1982). Each producer uses labor as an input in production. Firm i chooses optimal price $P_{i,t}$ and labor input $l_{i,t}$ given the current aggregate output y_t and aggregate price level P_t. It maximizes the present value of discounted dividends after tax $(1-\tau_{d,t})d_{i,t}$.

$$\max_{\{y_{i,t+s}, P_{i,t+s}, l_{i,t+s}\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{c,t+s}(1-\tau_{d,t+s}) d_{i,t+s} \tag{9}$$

$$\text{s.t.} \quad d_{i,t+s} = \frac{P_{i,t+s}}{P_{t+s}} y_{i,t+s} - w_{t+s} l_{i,t+s} - \frac{\psi}{2}\Big(\frac{P_{i,t+s}}{P_{i,t+s-1}} - 1 \Big)^2 y_{t+s} \tag{10}$$

$$y_{i,t+s} = l_{i,t+s} \tag{11}$$

$$y_{i,t+s} = \Big(\frac{P_{i,t+s}}{P_{t+s}} \Big)^{-\theta} y_{t+s} \tag{12}$$

where the real stochastic discount factor is defined as

$$Q_{c,t} \equiv \beta^t \frac{c_t^{-\sigma}}{1+\tau_{c,t}} \tag{13}$$

Combining the first order conditions and imposing symmetry across firms, we can derive the following Philips Curve

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}}y_t(1-\tau_{d,t})\Big[\psi(\Pi_t-1)\Pi_t-\theta w_t+\theta-1\Big]$$
$$=\beta\mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}}y_{t+1}(1-\tau_{d,t+1})\psi(\Pi_{t+1}-1)\Pi_{t+1}\Big] \tag{14}$$

The aggregate production function and dividend payouts are

$$y_t = l_t \tag{15}$$
$$d_t = y_t - w_t l_t - \frac{\psi}{2}(\Pi_t-1)^2 y_t \tag{16}$$

2.4 Central Bank and the Fiscal Authority

The central bank sets the interest rate following the standard Taylor rule where the net nominal interest rate is bounded below by zero.

$$R_t = \max\left[1, \frac{1}{\beta}\Pi_t^{\phi_\pi}\right] \tag{17}$$

Fiscal authority's purchases of final goods are financed by taxes. In this study, I allow the tax rates on consumption goods, labor income, dividends, and aggregate output to vary over the time. The budget constraint can be expressed as

$$\frac{b_t}{R_t} + \tau_{y,t}y_t + \tau_{c,t}c_t + \tau_{w,t}w_t l_t + \tau_{d,t}d_t = \frac{b_{t-1}}{\Pi_t} + g_t \tag{18}$$

I have assumed that the lump-sum tax is a function of aggregate output $\tau_t = \tau_{y,t}y_t$. Although this assumption is not crucial to obtain the main results of this study, it allows a closed form representation at the steady state, which simplifies the algebra.

I assume that the fiscal policy is Ricardian and set $b_t = 0$ for all t without loss of generality. Then the budget constraint simplifies to

$$\tau_{y,t}y_t + \tau_{c,t}c_t + \tau_{w,t}w_t l_t + \tau_{d,t}d_t = g_t \tag{19}$$

Existing studies such as Benhabib et al. (2002) and Schmidt (2016) have shown that fiscal policies responding to inflation can avoid the ELT. Following these findings, I assume that tax rates are functions of inflation in the following form

$$\tau_{i,t} = \tau_i \Pi_t^{\lambda_i} \qquad i \in \{w, c, d, y\} \tag{20}$$

The above specification is analogous to the Taylor rule and has a simple interpretation. The fiscal authority raises or lowers its tax rate depending on the current inflation rate.

The assumption that tax rates vary depending on the inflation rate can be justified from the past observations where the tax rates have been changed depending on the economic conditions. For example, Jobs and Growth Tax Relief Reconciliation Act (JGTRRA) of 2003 in the U.S. included cutting tax rates on labor income and dividend. As a more recent example, income tax rate has been lowered after the introduction of Tax Cuts and Jobs Act (TCJA) of 2017. Therefore, tax rate responding to the inflation rate can be viewed as a natural extention of these past examples, which aimed to stimulate the economy by altering the tax rates.

2.5 Equilibrium conditions

Resource constraint of the economy is derived by combining equations (2), (16) and (18) as

$$c_t + g_t + \frac{\psi}{2}(\Pi_t - 1)^2 y_t = y_t \tag{21}$$

Equations (3), (4), (14), (15), (17), (20) and the resource constraint (21) consist the equilibrium conditions. Non-linear equilibrium conditions other than the tax rates can be summarized to the following four key equations

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}} = \beta R_t \mathbb{E}_t \Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}} \frac{1}{\Pi_{t+1}} \Big] \tag{22}$$

$$\begin{aligned} \frac{1-\tau_{d,t}}{1+\tau_{c,t}} \Big[\psi(\Pi_t - 1)\Pi_t - \theta \frac{1+\tau_{c,t}}{1-\tau_{w,t}} c_t^{\sigma} y_t^{\eta} + \theta - 1 \Big] & \\ = \beta \mathbb{E}_t \Big[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1-\tau_{d,t+1}}{1+\tau_{c,t+1}} \frac{y_{t+1}}{y_t} \psi(\Pi_{t+1} - 1)\Pi_{t+1} \Big] & \qquad (23) \\ R_t = \max \Big[1, \frac{1}{\beta} \Pi_t^{\phi_\pi} \Big] & \qquad (24) \\ (1-\tau_{d,t}) y_t \Big[1 - \frac{\psi}{2}(\Pi_t - 1)^2 \Big] = (1+\tau_{c,t}) c_t & \\ + (\tau_{w,t} - \tau_{d,t}) \frac{1+\tau_{c,t}}{1-\tau_{w,t}} c_t^{\sigma} y_t^{\eta+1} + \tau_{y,t} y_t & \qquad (25) \end{aligned}$$

The deterministic steady state values are denoted by the subscript TSS. Once the regime shocks are introduced, multiple equilibria may arise. In that case, equilibrium outcomes are denoted by T in the targeted regime and U in the unintended regime.

The steady state values in the TSS are as follows

$$R_{TSS} = \frac{1}{\beta} \tag{26}$$

$$y_{TSS} = (1+\tau_c)^{\frac{\sigma}{\eta+\sigma}} \left[1 - \tau_d - \tau_y - (\tau_w - \tau_d)\frac{\theta-1}{\theta}\right]^{-\frac{\sigma}{\eta+\sigma}} \left(\frac{\theta-1}{\theta}\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1}{\eta+\sigma}} \tag{27}$$

$$c_{TSS} = (1+\tau_c)^{-\frac{\eta}{\eta+\sigma}} \left[1 - \tau_d - \tau_y - (\tau_w - \tau_d)\frac{\theta-1}{\theta}\right]^{\frac{\eta}{\eta+\sigma}} \left(\frac{\theta-1}{\theta}\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1}{\eta+\sigma}} \tag{28}$$

Log-linearized equilibrium conditions can be derived by log-linearizing equations (22) – (25) around the targeted deterministic steady state.

$$\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}) - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}(\hat{\tau}_{c,t} - \mathbb{E}_t\hat{\tau}_{c,t+1}) \tag{29}$$

$$\begin{aligned}\hat{\pi}_t = {} & \frac{\theta-1}{\psi}\frac{\tau_c}{1+\tau_c}\hat{\tau}_{c,t} - \frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\hat{\tau}_{w,t} \\ & + \sigma\frac{\theta-1}{\psi}\hat{c}_t + \eta\frac{\theta-1}{\psi}\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1}\end{aligned} \tag{30}$$

$$\hat{i}_t = \max[\log\beta, \phi_\pi\hat{\pi}_t] \tag{31}$$

$$\gamma_y\hat{y}_t = \gamma_c\hat{c}_t + \gamma_{\tau,c}\hat{\tau}_{c,t} + \gamma_{\tau,w}\hat{\tau}_{w,t} + \gamma_{\tau,d}\hat{\tau}_{d,t} + \gamma_{\tau,y}\hat{\tau}_{y,t} \tag{32}$$

where

$$\begin{aligned}\gamma_y &\equiv 1 - \tau_d - \tau_y - (\eta+1)(\tau_w - \tau_d)\frac{\theta-1}{\theta}, \\ \gamma_c &\equiv (1+\tau_c)\frac{c_{TSS}}{y_{TSS}} + \sigma(\tau_w - \tau_d)\frac{\theta-1}{\theta}, \\ \gamma_{\tau,c} &\equiv \left[(1+\tau_c)\frac{c_{TSS}}{y_{TSS}} + (\tau_w - \tau_d)\frac{\theta-1}{\theta}\right]\frac{\tau_c}{1+\tau_c}, \\ \gamma_{\tau,w} &\equiv \frac{\theta-1}{\theta}\frac{\tau_w}{1-\tau_w}, \quad \gamma_{\tau,d} \equiv \frac{1}{\theta}\tau_f, \quad \gamma_{\tau,y} \equiv \tau_y\end{aligned}$$

In equation (31) the zero lower bound is imposed on the nominal interest rate after log-linearization. The variables in hat are percentage deviations from the TSS values, i.e. $\hat{x}_t \equiv \log x_t - \log x_{TSS}$.

After substitution, the equilibrium conditions simplify to the following two equations

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}(\max[\log\beta, \phi_\pi\hat{\pi}_t] - \mathbb{E}_t\hat{\pi}_{t+1}) + \Big(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\Big)\hat{\tau}_{c,t}$$
$$+ \frac{\gamma_{\tau,w}}{\gamma_y}\hat{\tau}_{w,t} + \frac{\gamma_{\tau,d}}{\gamma_y}\hat{\tau}_{d,t} + \frac{\gamma_{\tau,y}}{\gamma_y}\hat{\tau}_{y,t} + \Big(\frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y} - \frac{\gamma_{\tau,c}}{\gamma_y}\Big)\mathbb{E}_t\hat{\tau}_{c,t+1}$$
$$- \frac{\gamma_{\tau,w}}{\gamma_y}\mathbb{E}_t\hat{\tau}_{w,t+1} - \frac{\gamma_{\tau,d}}{\gamma_y}\mathbb{E}_t\hat{\tau}_{d,t+1} - \frac{\gamma_{\tau,y}}{\gamma_y}\mathbb{E}_t\hat{\tau}_{y,t+1} \tag{33}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\theta-1}{\psi}\Big(\eta + \sigma\frac{\gamma_y}{\gamma_c}\Big)\hat{y}_t - \frac{\theta-1}{\psi}\Big(\sigma\frac{\gamma_{\tau,c}}{\gamma_c} - \frac{\tau_c}{1+\tau_c}\Big)\hat{\tau}_{c,t}$$
$$- \frac{\theta-1}{\psi}\Big(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\Big)\hat{\tau}_{w,t} - \sigma\frac{\theta-1}{\psi}\frac{\gamma_{\tau,d}}{\gamma_c}\hat{\tau}_{d,t} - \sigma\frac{\theta-1}{\psi}\frac{\gamma_{\tau,y}}{\gamma_c}\hat{\tau}_{y,t} \tag{34}$$

Log-linearized tax rules are

$$\hat{\tau}_{c,t} = \lambda_c\hat{\pi}_t, \quad \hat{\tau}_{w,t} = \lambda_w\hat{\pi}_t, \quad \hat{\tau}_{d,t} = \lambda_d\hat{\pi}_t, \quad \hat{\tau}_{y,t} = \lambda_y\hat{\pi}_t \tag{35}$$

Above log-linearized model allows us to derive a closed form solution under both assumptions that the targeted regime is absorbing and the unintended regime is recurrent. Therefore, in the remainder of this paper, effective tax policies are studied using this log-linearized model.

2.6 Calibration

It is assumed that a model period corresponds to a quarter. I set $\beta = 0.996$ which yields an annual real interest rate of 2 percent. The elasticity of intertemporal substitution is chosen to be $\sigma = 1.5$. Frisch elasticity of labor substitution is set to $1/\eta = 2.5$. The elasticity of substitution between intermediate goods is $\theta = 6$, which yields markup of 20 percent. Price adjustment cost is set to $\psi = 59.3$ which is calibrated to match the price adjustment probability of 0.75 using Calvo (1983) model.[4]

The target net inflation rate is set equal to zero (a stable price level) and the Taylor coefficient is set to $\phi_\pi = 1.5$. Regarding the tax rates, the benchmark calibration sets lump-sum tax rate τ_y to zero, while consumption tax rate is set to $\tau_c = 0.1$, labor income tax to $\tau_w = 0.15$ and the dividend tax rate to $\tau_d = 0.15$. This yields the consumption-to-output ratio at the TSS of $c_{TSS}/y_{TSS} = 0.77$. As an alternative specification, all distortionary tax rates are set to zero while the lump-sum tax rate is set to $\tau_y = 0.23$, which yields the same ratio of $c_{TSS}/y_{TSS} = 0.77$ in

[4]In a linearized model, setting $\psi = \frac{\omega(\theta-1)}{(1-\omega)(1-\beta\omega)}$ yields the identical New Keynesian Philips Curve between model with Rotemberg price adjustment cost and the Calvo pricing, where ω is the parameter of price stickiness.

the deterministic steady state.

3 Avoiding the ELT: the absorbing case

In this section, I assume that the targeted regime is absorbing. Fiscal rules using different tax rates are assessed in terms of whether they can prevent the ELT.

3.1 Solution at the ELT

When the restriction $p_{TT} = 1$ is imposed on the transition matrix, the targeted regime is absorbing and the ELT becomes an one-off regime. In this case, deviations of output and inflation in the targeted regime are equivalent to the values in the targeted deterministic steady state, i.e. $\hat{y}_T = \hat{y}_{TSS} = 0$ and $\hat{\pi}_T = \hat{\pi}_{TSS} = 0$. Deviations of the fiscal policy variables are also zero in the targeted regime.

Under the absorbing assumption, the equilibrium outcome in the targeted regime and the unintended regime can be obtained by solving for the intersections of the Euler equation (EE) and the Philips curve (PC)

$$\hat{y}_t = -\frac{1}{\sigma}\max[\log\beta, \phi_\pi\hat{\pi}_t] + \Big(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\Big)\hat{\tau}_{c,t} + \frac{\gamma_{\tau,w}}{\gamma_y}\hat{\tau}_{w,t} + \frac{\gamma_{\tau,d}}{\gamma_y}\hat{\tau}_{d,t} + \frac{\gamma_{\tau,y}}{\gamma_y}\hat{\tau}_{y,t}$$
$$+ p_{UU}\Big[\hat{y}_t + \frac{1}{\sigma}\hat{\pi}_t + \Big(\frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y} - \frac{\gamma_{\tau,c}}{\gamma_y}\Big)\hat{\tau}_{c,t} - \frac{\gamma_{\tau,w}}{\gamma_y}\hat{\tau}_{w,t} - \frac{\gamma_{\tau,d}}{\gamma_y}\hat{\tau}_{d,t} - \frac{\gamma_{\tau,y}}{\gamma_y}\hat{\tau}_{y,t}\Big] \quad (36)$$

$$\hat{\pi}_t = p_{UU}\beta\hat{\pi}_t + \frac{\theta-1}{\psi}\Big(\eta + \sigma\frac{\gamma_y}{\gamma_c}\Big)\hat{y}_t - \frac{\theta-1}{\psi}\Big(\sigma\frac{\gamma_{\tau,c}}{\gamma_c} - \frac{\tau_c}{1+\tau_c}\Big)\hat{\tau}_{c,t}$$
$$- \frac{\theta-1}{\psi}\Big(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\Big)\hat{\tau}_{w,t} - \sigma\frac{\theta-1}{\psi}\frac{\gamma_{\tau,d}}{\gamma_c}\hat{\tau}_{d,t} - \sigma\frac{\theta-1}{\psi}\frac{\gamma_{\tau,y}}{\gamma_c}\hat{\tau}_{y,t} \quad (37)$$

In the rest of the analysis, log-linearized tax rules (35) are substituted out of the equations.

Let us first assume that the unintended regime is not excluded as an equilibrium outcome. The inflation rate is close to the target and the zero lower bound on the interest rate does not bind at the targeted regime, while the inflation is low and the interest rate is stuck at zero in the unintended regime. These assumptions can be stated as

$$\hat{\pi}_T > \frac{\log\beta}{\phi_\pi} \quad \text{and} \quad \hat{i}_T = \phi_\pi\hat{\pi}_T \quad (38)$$
$$\hat{\pi}_U \leq \frac{\log\beta}{\phi_\pi} \quad \text{and} \quad \hat{i}_U = \log\beta \quad (39)$$

When the lower bound does not bind as in inequality (38), the Taylor rule is active and the EE can be expressed as

$$\hat{y}_t = \left[\frac{1}{\sigma}\frac{p_{UU} - \phi_\pi}{(1 - p_{UU})} + \xi\right]\hat{\pi}_t \tag{40}$$

$$\text{where} \quad \xi \equiv \left(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\right)\lambda_c + \frac{\gamma_{\tau,w}}{\gamma_y}\lambda_w + \frac{\gamma_{\tau,d}}{\gamma_y}\lambda_d + \frac{\gamma_{\tau,y}}{\gamma_y}\lambda_y$$

On the other hand, when the lower bound binds as in inequality (39), the Taylor rule is inactive and the EE can be expressed as

$$\hat{y}_t = -\frac{1}{\sigma}\frac{1}{(1 - p_{UU})}\log\beta + \left[\frac{1}{\sigma}\frac{p_{UU}}{(1 - p_{UU})} + \xi\right]\hat{\pi}_t \tag{41}$$

In contrast to the EE, the PC does not have a kink

$$\hat{y}_t = (1 - \beta p_{UU} + \kappa)\frac{\psi}{\theta - 1}\left(\eta + \sigma\frac{\gamma_y}{\gamma_c}\right)^{-1}\hat{\pi}_t \tag{42}$$

$$\begin{aligned} \text{where} \quad \kappa \equiv & \frac{\theta - 1}{\psi}\left(\sigma\frac{\gamma_{\tau,c}}{\gamma_c} - \frac{\tau_c}{1+\tau_c}\right)\lambda_c + \frac{\theta - 1}{\psi}\left(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\right)\lambda_w \\ & + \sigma\frac{\theta - 1}{\psi}\frac{\gamma_{\tau,d}}{\gamma_c}\lambda_d + \sigma\frac{\theta - 1}{\psi}\frac{\gamma_{\tau,y}}{\gamma_c}\lambda_y \end{aligned}$$

Inflation rate at the unintended regime can be obtained by solving equations (41) and (42).

$$\hat{\pi}_U = \frac{\frac{\log\beta}{1 - p_{UU}}\frac{1}{\eta}\frac{1}{\sigma}\left(\eta + \sigma\frac{\gamma_y}{\gamma_c}\right)}{\Lambda - (1 - \beta p_{UU})\frac{1}{\eta}\frac{\psi}{\theta - 1} + \frac{p_{UU}}{1 - p_{UU}}\frac{1}{\eta}\frac{1}{\sigma}\left(\eta + \sigma\frac{\gamma_y}{\gamma_c}\right)} \tag{43}$$

$$\begin{aligned} \text{where} \quad \Lambda \equiv & \, \xi\frac{1}{\eta}\left(\eta + \sigma\frac{\gamma_y}{\gamma_c}\right) - \kappa\frac{1}{\eta}\frac{\psi}{\theta - 1} \\ = & \left(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\right)\lambda_c - \left(\frac{1}{\eta}\frac{\tau_w}{1-\tau_w} - \frac{\gamma_{\tau,w}}{\gamma_y}\right)\lambda_w + \frac{\gamma_{\tau,d}}{\gamma_y}\lambda_d + \frac{\gamma_{\tau,y}}{\gamma_y}\lambda_y \end{aligned}$$

Figure 1 graphically shows the kinked EE and the PC when all fiscal response parameters λ_i are set equal to zero. The PC captures the relation that an increase in output creates upward pressure on the inflation rate. In the targeted regime, the zero lower bound on the interest rate does not bind, hence the EE is downward sloping. The intersection between the PC and the downward sloping EE is the equilibrium outcome at the targeted regime.

In the unintended regime, the zero lower bound on the interest rate binds and the central bank cannot lower its policy rate even if the inflation rate declines. This causes the real interest rate to increase, which creates the upward sloping EE. The intersection between the PC and the upward sloping EE is the equilibrium outcome in the unintended regime, which I call the ELT equilibrium. Note that even in the absence of any policy interventions, if the transition probability p_{UU} is sufficiently low, the ELT equilibrium does not exist. An example where the ELT equilibrium does not exist is shown in figure 2.

Figure 3 shows more concretely the relation between transition probability p_{UU} and the inflation rate at the ELT equilibrium. When p_{UU} is close to one, the inflation rate is modestly below zero, but as the p_{UU} becomes smaller, the inflation rate declines. Since the inflation rate at the targeted regime is fixed with $\hat{\pi}_T = 0$, the inflation rate at the unintended regime requires to be well below zero in order to satisfy the rational expectations assumption.

Once the p_{UU} is below the threshold level, the inflation rate turns positive. Our initial assumption $\hat{\pi}_U \leq \log \beta / \phi_\pi$ is violated in this case, which indicates that the ELT equilibrium does not exist and only $\hat{\pi}_T$ is supported as an equilibrium. Hence the ELT equilibrium exists only if the transition probability is high and falls in region right to the bifurcation point.

Japan's prolonged experience of zero interest rates suggests that the probability of remaining at the zero lower bound is high. For example, Boneva et al. (2016) assume $p_{UU} = 0.92$ for their baseline parameterization while Aruoba et al. (2018) choose $p_{UU} = 0.95$ for their estimation. Therefore, it is natural to assume that the relevant cases in our study are situations shown in figure 1, where the probability of remaining at the unintended regime is high and policy intervention is necessary to prevent the ELT.

Suppose that the following inequality obtains by suitable choice of the fiscal response parameters λ_i

$$\hat{\pi}_U > \frac{\log \beta}{\phi_\pi} \tag{44}$$

This contradicts our initial assumption stated in inequality (39). If inequality (44) holds, only the targeted regime is supported as a rational expectations equilibrium. Therefore, setting fiscal response parameters λ_i in the region that satisfies inequality (44) enables the fiscal authority to prevent the ELT.

The Λ in equation (43) is a parameter which summarizes the fiscal response parameters. Given the observation that $\hat{\pi}_U$ is affected by the deep parameters, we can conjecture that $\hat{\pi}_U$ also varies depending on the value of Λ. Figure 4 illustrates the relation between $\hat{\pi}_U$ and Λ. The hyperbola in blue lines represents the equation (43). The right hand side of inequality (44), which is the threshold, is shown in red lines. The horizontal asymptote of the hyperbola ($\hat{\pi}_U = 0$) is shown in black dashed lines.

We can observe that many of the properties confirmed in figure 3 carries over to figure 4. $\hat{\pi}_U$ approaches the asymptote as Λ becomes larger, while the inflation rate turns positive once Λ is below the threshold. Since the asymptote is higher than the cut-off value ($\log \beta/\phi_\pi < 0$), the region of Λ that satisfies inequality (44) is

$$\Lambda \leq (1-\beta p_{UU})\frac{1}{\eta}\frac{\psi}{\theta-1} - \frac{p_{UU}}{1-p_{UU}}\frac{1}{\eta}\frac{1}{\sigma}\Big(\eta + \sigma\frac{\gamma_y}{\gamma_c}\Big) \equiv \Psi^L \tag{45}$$

$$\Lambda > (1-\beta p_{UU})\frac{1}{\eta}\frac{\psi}{\theta-1} + \frac{\phi_\pi - p_{UU}}{1-p_{UU}}\frac{1}{\eta}\frac{1}{\sigma}\Big(\eta + \sigma\frac{\gamma_y}{\gamma_c}\Big) \equiv \Psi^U \tag{46}$$

In other words, as long as the condition (45) or (46) is satisfied by a suitable choice of the fiscal response parameters λ_i, the fiscal authority can avoid the ELT equilibrium.

Note that the parameter Ψ^L is always larger than Ψ^U. Ψ^L is decreasing and Ψ^U is increasing in the persistence of unintended regime p_{UU}

$$\frac{\partial \Psi^L}{\partial p_{UU}} < 0 \quad \text{and} \quad \frac{\partial \Psi^U}{\partial p_{UU}} > 0 \tag{47}$$

I now examine how changing each tax rate can eliminate the ELT equilibrium.

3.2 Supply-side policies

Consider the labor income tax first. Changes in the labor income tax rate alter the effective real wage level that households face and affect the labor supply. Thus changes in the labor income tax affects the equilibrium outcome from the supply-side.

If all tax response parameters are set to zero other than λ_w, the inequalities (45) and (46) simplify to

$$\lambda_w \geq -\Psi^L\Big(\frac{1}{\eta}\frac{\tau_w}{1-\tau_w} - \frac{\gamma_{\tau,w}}{\gamma_y}\Big)^{-1} \equiv \lambda_w^L \tag{48}$$

$$\lambda_w < -\Psi^U\Big(\frac{1}{\eta}\frac{\tau_w}{1-\tau_w} - \frac{\gamma_{\tau,w}}{\gamma_y}\Big)^{-1} \equiv \lambda_w^U \tag{49}$$

Figure 5 shows the EE and PC when λ_w is set equal to λ_w^L and λ_w^U respectively. The shape of the EE and the PC are significantly different between the two cases. Both graphs in figure 5 display that the slopes are parallel for the EE and the PC when λ_w is set equal to λ_w^L or λ_w^U. However, setting $\lambda_w = \lambda_w^U$ makes the EE and PC perfectly identical at the targeted regime, which is neither desirable nor realistic.

Based on this observation, I focus on the case of $\lambda_w = \lambda_w^L$ in the rest of the analysis.

Under standard parameterization, λ_w^L takes positive values, which indicates that labor income tax rate increases as the inflation rate rises. $\partial \lambda_w^L / \partial p_{UU} > 0$ shows that the response parameter must be set larger as the probability of remaining at the ELT gets larger.

How does the labor tax rate affect the equilibrium? When the λ_w^L is positive, an increase in the inflation rate raises the labor income tax rate, which discourages the households to supply labor. This mitigates the inflationary pressure caused by an increase in the marginal cost, which causes the PC to flatten compared to the case where the tax rate does not respond to the inflation rate at all.

Labor income tax also affects the households' consumption behavior. In the targeted regime, an increase in the inflation rate raises the real interest rate, which decreases households' consumption. The decline in labor supply caused by the increase in the labor tax rate further reduces household's consumption. Therefore, positive λ_w^L causes the EE to steepen in the region where the zero lower bound does not bind.

The effect is different when the zero lower bound binds. At the unintended regime, the Taylor rule is not active and an increase in the inflation rate decreases the real interest rate, which induces the household to increase consumption. This increase in consumption is partly offset by the increase in the labor tax rate. Therefore, positive λ_w^L flattens the EE in the region where the zero lower bound binds.

3.3 Demand-side policies

Next, consider the role of taxes which operate through the demand-side, i.e. the consumption tax, the dividend tax, and the lump-sum tax.

First, all tax response parameters are set to zero except the consumption tax. Whether consumption tax can affect the equilibrium outcome depends on the parameter of substitution in the utility function.

(i) When $\sigma = 1$

Changing consumption tax affects the equilibrium through income effect. When the utility function takes the form of log, income effect and substitution effect perfectly offset each other. This is reflected in the coefficients

$$\frac{1}{\sigma} \frac{\tau_c}{1+\tau_c} \frac{\gamma_c}{\gamma_y} - \frac{\gamma_{\tau,c}}{\gamma_y} = 0$$

in Λ. Altering λ_c does not affect the equilibrium, hence whether the ELT equilibrium exists or not depends solely on the probability p_{UU}.

(ii) When $\sigma > 1$
Inequalities (46) and (45) simplify to

$$\lambda_c \le -\Psi^L\Big(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\Big)^{-1} \equiv \lambda_c^L \tag{50}$$

$$\lambda_c > -\Psi^U\Big(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\Big)^{-1} \equiv \lambda_c^U \tag{51}$$

Figure 5 shows the EE and PC when λ_c is set equal to λ_c^L and λ_c^U respectively. As we have seen in the case of the labor income tax, the EE and the PC become parallel under these parameterization.

The same argument holds for both dividend tax and lump-sum taxes.

$$\lambda_d \le -\Psi^L\frac{\gamma_y}{\gamma_{\tau,d}} \equiv \lambda_d^L, \quad \lambda_d > -\Psi^L\frac{\gamma_y}{\gamma_{\tau,d}} \equiv \lambda_d^U \tag{52}$$

$$\lambda_y \le -\Psi^L\frac{\gamma_y}{\gamma_{\tau,y}} \equiv \lambda_y^L, \quad \lambda_y > -\Psi^L\frac{\gamma_y}{\gamma_{\tau,y}} \equiv \lambda_y^U \tag{53}$$

Setting λ_i equal to λ_i^U for $i = \{c, d, y\}$ makes the EE and PC perfectly identical at the targeted regime. We exclude these cases from the rest of the analysis.

Under the baseline parameterization, λ_i^L takes negative values. This means that the fiscal authority should raise the tax rate when the inflation is below the target. Besides, $\partial\lambda_i^L/\partial p_{UU} < 0$ for $i \in \{c, d, y\}$ shows that the size of response parameter is also declining in p_{UU}.

The mechanism how demand-side policies affect the equilibrium outcome is as follows. When λ_d^L is negative, household's income decreases as the inflation rate increases. To compensate for this decrease in income, household increases labor supply, which puts further inflationary pressure and steepens the PC. When the zero lower bound does not bind, this increase in labor supply increases the output, which causes the EE to flatten. On the other hand, when the zero lower bound binds, the upward sloping EE steepens.

3.4 Combining different taxes

We have confirmed that supply-side and demand-side policies affect consumption demand and labor supply in different directions. Although we have examined each tax one by one, different taxes can be combined to achieve our goal to avoid the ELT; any combination of λ_i that satisfies inequality (46) can prevent the ELT.

Since labor income taxes and dividend taxes have been adjusted in the past period of low economic activity, I focus on these two taxes as policy instruments. Both λ_c and λ_y are set equal to zero. Then inequality (46) can be rearranged as

$$\lambda_d \leq \frac{\gamma_y}{\gamma_{\tau,d}}\Big(\frac{1}{\eta}\frac{\tau_w}{1-\tau_w} - \frac{\gamma_{\tau,w}}{\gamma_y}\Big)\lambda_w + \frac{\gamma_y}{\gamma_{\tau,d}}\Psi^L \tag{54}$$

The left-hand graph in figure 7 displays the area given in inequality (54). Both the frontier and the area in grey shows the parameter space where the ELT equilibrium does not arise. Any linear combination $\mu\lambda_w^L + (1-\mu)\lambda_d^L$ lies in the frontier and therefore satisfies (54).

Right-hand side of figure 7 shows the case where each of the parameters are set to $\lambda_w = 0.5\bar{\lambda}_w$ and $\lambda_d = 0.5\bar{\lambda}_d$. Unlike the cases where only a single tax rate was chosen as the policy instrument, combining labor income tax and dividend tax raises results similar to that in figure 1. That is to say, the PC is upward sloping while the EE is downward sloping when the zero lower bound does not bind. Thus, combining different taxes preserves the usual model property while preventing the ELT.

The mechanism through which inflation sensitive tax rates prevent the ELT equilibrium can be summarized as follows. Increase in the dividend tax rate reduces the real income of the household, which induces the household to supply more labor. Decrease in the labor income tax rate also induces the household to supply more labor. This increase in labor supply raises the marginal cost. Since inflation rate is the discounted sum of the future marginal costs, avoiding the marginal cost to fall prevents the ELT to emerge. This mechanism is consistent with the findings in Schmidt (2016) that a government spending rule which keeps the marginal cost over a threshold level prevents the ELT.

3.5 When the response is insufficient

We have seen that $\partial\Psi^L/\partial p_{UU} < 0$ and $\partial|\lambda_i^L|/\partial p_{UU} > 0$ for all $i \in \{w, c, d, y\}$. This indicates that the extent to which tax rates respond to inflation must become larger as the probability of remaining at the ELT becomes higher. If the response parameters λ_i are not set large enough, the fiscal authority not only fails to prevent the ELT but aggravates the drop in inflation and output at the ELT.

Figure 8 shows the case where policy parameters are set to $\lambda_w = 0.4\lambda_w^L$ and $\lambda_d = 0.4\lambda_d^L$, which does not satisfy the condition (54). Since the tax rates do not respond to inflation sufficiently, the fiscal authority fails to avoid the ELT. What is worse, the inflation rate and the output are even lower than in the case where fiscal authority has no policy response as is shown in the right-hand graph in figure 1.

4 Avoiding the ELT: the recurrent case

In the previous section, I have assumed that the targeted inflation regime is absorbing. In this section, I assume that both “targeted regime” and the “unintended regime” are recurrent.

4.1 Solution at the ELT

Let us first assume that the ELT equilibrium exists. The zero lower bound on the nominal interest rate does not bind at the targeted regime, while the interest rate is stuck at zero at the unintended regime. These assumptions can be stated formally as

$$\hat{\pi}_T > \frac{\log\beta}{\phi_\pi} \quad \text{and} \quad \hat{i}_T = \phi_\pi \hat{\pi}_T \tag{55}$$

$$\hat{\pi}_U \leq \frac{\log\beta}{\phi_\pi} \quad \text{and} \quad \hat{i}_U = \log\beta \tag{56}$$

Equilibrium conditions can be expressed by the following four equations

$$\hat{y}_T = \hat{y}_U - \frac{1}{\sigma}\frac{\phi_\pi - 1}{1 - p_{TT}}\hat{\pi}_T + (\hat{\pi}_T - \hat{\pi}_U)\Big(\xi - \frac{1}{\sigma}\Big) \tag{57}$$

$$(1 - \beta p_{TT} + \kappa)\hat{\pi}_T = \beta(1 - p_{TT})\hat{\pi}_U + \frac{\theta - 1}{\psi}\Big(\eta + \sigma\frac{\gamma_y}{\gamma_c}\Big)\hat{y}_T \tag{58}$$

$$\hat{y}_U = \hat{y}_T - \frac{1}{\sigma}\frac{1}{1 - p_{UU}}\log\beta + \frac{1}{\sigma}\frac{1}{1 - p_{UU}}\hat{\pi}_U + (\hat{\pi}_U - \hat{\pi}_T)\Big(\xi - \frac{1}{\sigma}\Big) \tag{59}$$

$$(1 - \beta p_{UU} + \kappa)\hat{\pi}_U = \beta(1 - p_{UU})\hat{\pi}_T + \frac{\theta - 1}{\psi}\Big(\eta + \sigma\frac{\gamma_y}{\gamma_c}\Big)\hat{y}_U \tag{60}$$

Let us assume $p_{TT} > p_{UU}$ that the probability of remaining at the targeted regime is higher than that of remaining at the unintended regime.

Whether the above linear system given by equations (57) - (60) has equilibria that satisfies both assumptions (55) and (56) depends on the parameterization. As we have seen in the absorbing case, in general, when the expected duration of remaining in the liquidity trap is low, no multiple rational expectations equilibria exist.

Solving equations (57) – (60) gives the following solution for $\hat{\pi}_U$ and $\hat{\pi}_T$.

$$\hat{\pi}_U = \log\beta\frac{\Phi - \Omega\Lambda}{(1 - \Omega)\Lambda + \Upsilon} \tag{61}$$

$$\hat{\pi}_T = \frac{\hat{\pi}_U - \log\beta}{\phi_\pi - 1}\frac{1 - p_{TT}}{1 - p_{UU}} \tag{62}$$

where

$$
\begin{aligned}
\Phi &\equiv \frac{1}{\eta}\frac{\psi}{\theta-1}\Big[\beta(1-p_{TT})+\frac{1-\beta p_{TT}}{\phi_\pi-1}\frac{1-p_{TT}}{1-p_{UU}}\Big]+\\
&\qquad\qquad \frac{1}{\sigma}\frac{1}{\eta}\Big(\eta+\sigma\frac{\gamma_y}{\gamma_c}\Big)\frac{1}{1-p_{UU}}\Big(1+\frac{1-p_{TT}}{\phi_\pi-1}\Big)\\
\Omega &\equiv \frac{1}{\phi_\pi-1}\frac{1-p_{TT}}{1-p_{UU}}\\
\Upsilon &\equiv \frac{1}{\eta}\frac{1}{\sigma}\Big(\eta+\sigma\frac{\gamma_y}{\gamma_c}\Big)\frac{1}{1-p_{UU}}\Big[p_{UU}+\frac{1-p_{TT}}{\phi_\pi-1}\Big]\\
&\quad-\frac{1}{\eta}\frac{\psi}{\theta-1}\Big[\beta(1-p_{TT})-\frac{1-\beta p_{TT}}{\phi_\pi-1}\frac{1-p_{TT}}{1-p_{UU}}+(1-\beta p_{UU})-\frac{\beta(1-p_{TT})}{\phi_\pi-1}\Big]
\end{aligned}
$$

Solutions (61) and (62) shows that when both regimes are recurrent, the equilibrium outcome at the targeted regime is affected by the inflation rate at the unintended regime. When we substitute $p_{TT}=1$, above equation becomes identical to the solution under absorbing case as in equation (43).

I follow the same procedure as in the previous section. Suppose that following inequality obtains by suitable choice of the fiscal response parameters λ_i

$$\hat{\pi}_U > \frac{\log\beta}{\phi_\pi} \tag{63}$$

This contradicts with our initial assumption (56). In this case only the targeted regime is supported as the rational expectations equilibrium. Therefore setting fiscal response parameters λ_i in the region defined by (63) enables the fiscal authority to prevent the ELT.

From our assumption that $p_{TT} > p_{UU}$, $0<\Omega<1$ holds as long as the Taylor principle is satisfied. We can rearrange the hyperbola given in (61) as follows

$$\hat{\pi}_U + \frac{\Omega}{1-\Omega}\log\beta = \frac{\Omega}{1-\Omega}\log\beta\frac{\Upsilon+\frac{1-\Omega}{\Omega}\Phi}{(1-\Omega)\Lambda+\Upsilon} \tag{64}$$

The horizontal asymptote of the hyperbola is larger than the cut-off level in (63)

$$\frac{\log\beta}{\phi_\pi} < 0 < -\frac{\Omega}{1-\Omega}\log\beta \tag{65}$$

Therefore, the region of Λ which satisfies the inequality (63) is as follows

$$\Lambda \leq -\frac{\Upsilon}{1-\Omega} \equiv \tilde{\Psi}^L \tag{66}$$

$$\Lambda > \frac{\phi_\pi \Phi - \Upsilon}{(1-\Omega)+\Omega\phi_\pi} \equiv \tilde{\Psi}^U \tag{67}$$

The conditions to avoid the ELT in (66) and (67) are very similar to the conditions in (45) and (46). Especially, $\tilde{\Psi}^L \to \Psi^L$ and $\tilde{\Psi}^U \to \Psi^U$ holds as $p_{TT} \to 1$. Therefore, qualitative results in the absorbing state case carry over to the case of regime switching.

4.2 The possibility of switching back to the ELT

Although the qualitative results on the conditions to avoid the ELT are similar between the absorbing case and the regime switching case, whether the policy maker is actually aware of the probability of switching back to the ELT makes a large difference in the equilibrium outcome.

From our assumption $p_{UU} < p_{TT}$, $\Psi^L < \tilde{\Psi}^L$ holds if $p_{TT} < 1$. This indicates that tax rates must respond more to the inflation rate once the expectational effects of moving back to the ELT is taken into account. Figure 9 shows the ratio $\tilde{\Psi}^L/\Psi^L$ for different values of p_{TT}. When $p_{TT} < 1$, $\tilde{\Psi}^L/\Psi^L$ is larger than 1 for all $p_{UU} < p_{TT}$. Especially, $\tilde{\Psi}^L \to \infty$ as $p_{UU} \to p_{TT}$ indicates that tax rates must respond infinitely large as the probability of remaining at the unintended regime approaches the probability of remaining at the targeted regime.

What happens if the fiscal authority ignores the possibility of switching back to the ELT and chooses policy parameters simply assuming that the targeted regime is absorbing? Figure 9 shows that setting tax response parameters λ_i to satisfy inequality (45) will always be insufficient to prevent the ELT if the true structure allows moving back to the ELT. In this case, the fiscal authority fails to prevent the ELT and faces the situation similar to figure 8 making the inflation and output even worse.

5 Conclusion

In this paper, I argued that a simple Ricardian tax rule which responds to inflation rate can prevent the economy from falling into expectations-driven liquidity trap. The qualitative properties of the conditions to prevent ELT are unchanged between the case where inflation regime is assumed to be absorbing and the case with regime switching. However, I show that if the fiscal authority chooses policy parameters without knowing the true structure of the economy and ignores the expectational effects, introducing the policy can aggravate the equilibrium outcome.

Although distortionary taxes are chosen as policy instruments, the efficiency of the allocation is not investigated in this study. Since the conditions represented by inequality (45) and (66) allows various choices on λ_i, we could further explore what parameter choice is most desirable in terms of welfare once the ELT has been eliminated. Such in depth investigation on efficiency is left for future work.

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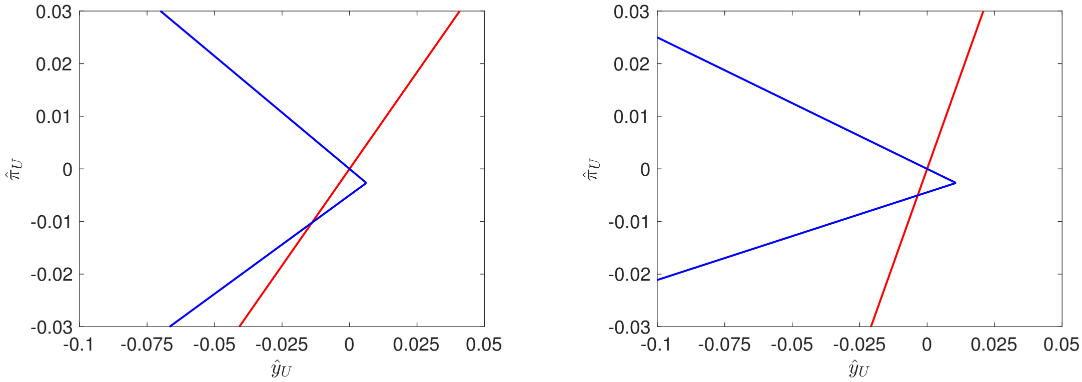


(a) $p_{UU} = 0.80$ (b) $p_{UU} = 0.90$

Figure 1: Log-linearized EE and PC under absorbing assumption. All fiscal response parameters are set equal to zero.

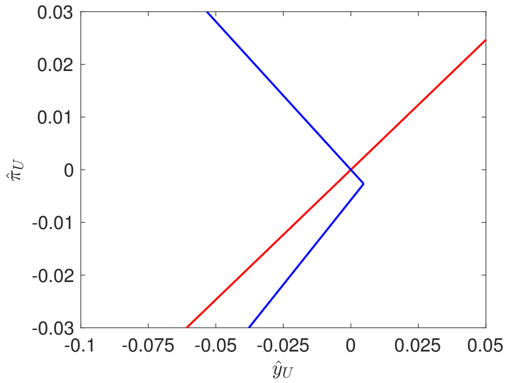


Figure 2: An example where the ELT does not exist ($p_{UU} = 70$).

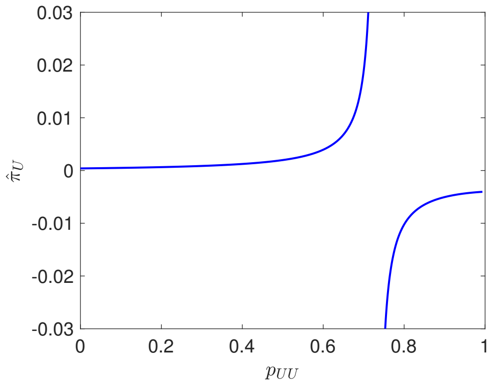


Figure 3: Relation between the transition probability and the inflation rate at the ELT.

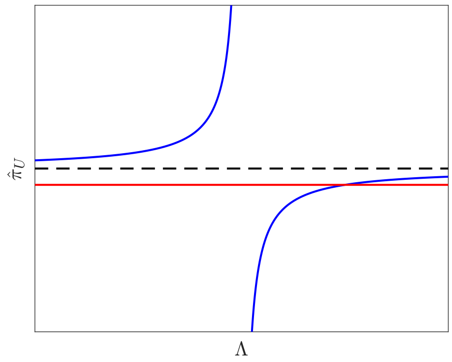
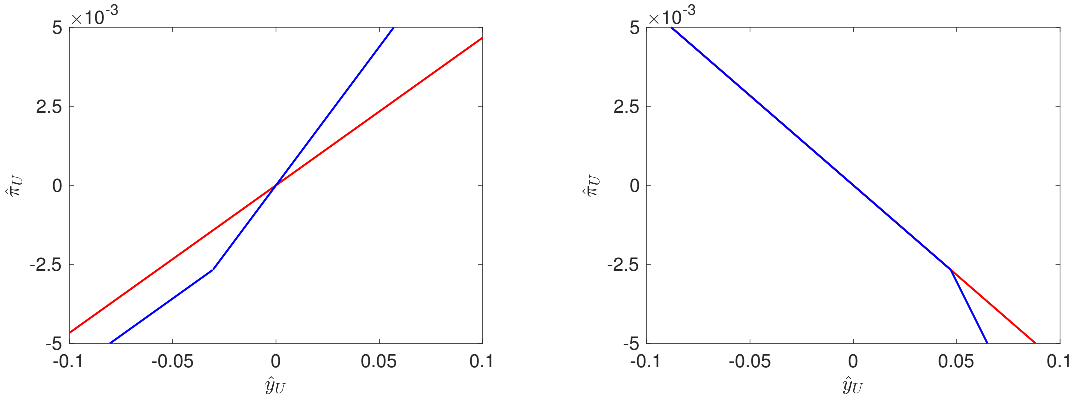


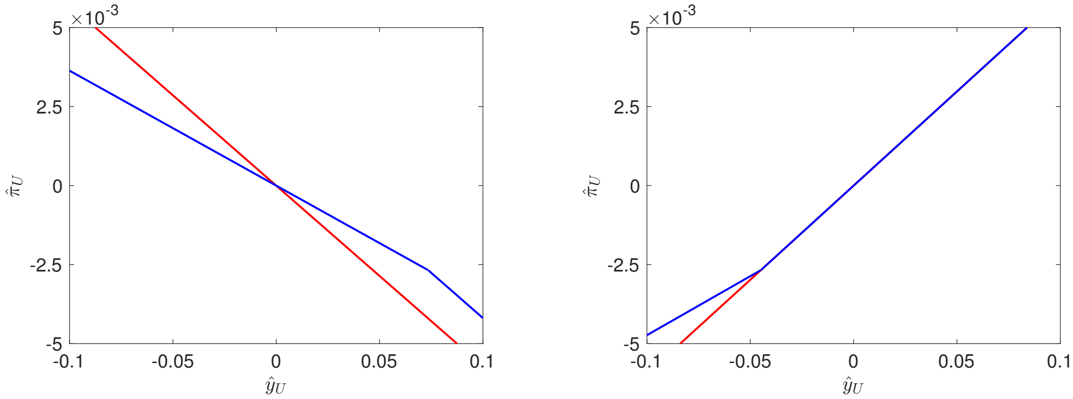
Figure 4: The blue line shows the hyperbola, the black dashed line shows the horizontal asymptote, and the red line shows the threshold.



(a) $\lambda_w = \lambda_w^L$

(b) $\lambda_w = \lambda_w^U$

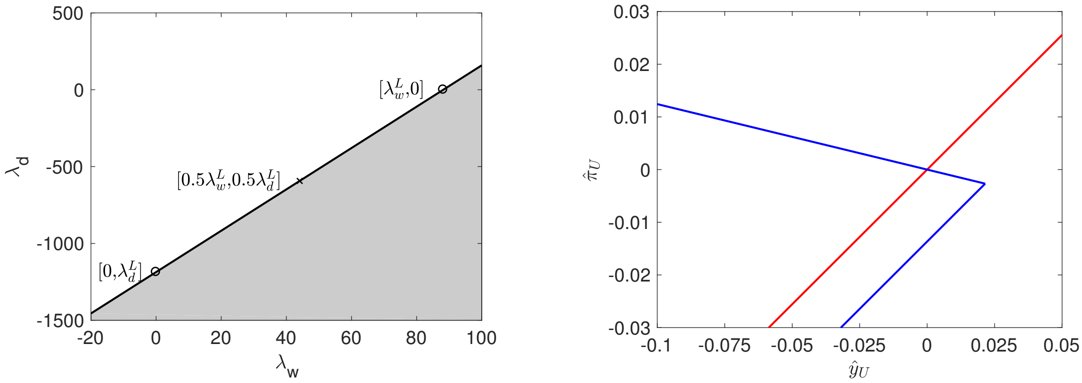
Figure 5: Supply-side policy and the existence of the ELT equilibrium ($p_{UU} = 0.90$)



(a) $\lambda_c = \lambda_c^L$

(b) $\lambda_c = \lambda_c^U$

Figure 6: Demand-side policy and the existence of the ELT equilibrium ($p_{UU} = 0.90$)



(a) Parameter space where the ELT does not exist. Both the frontier and the area in grey.

(b) $\lambda_w = 0.5\lambda_w^L$ and $\lambda_d = 0.5\lambda_d^L$

Figure 7: Combining different taxes.

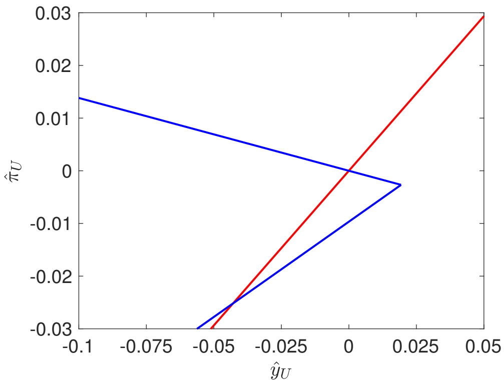


Figure 8: An example where the responses of the tax rates are insufficient ($\lambda_w = 0.4\lambda_w^L$ and $\lambda_d = 0.4\lambda_d^L$)

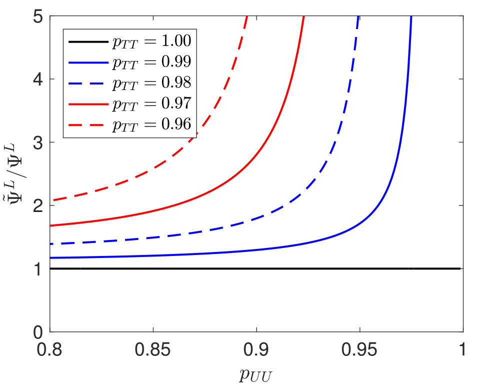


Figure 9: Differences in the response parameters when the unintended regime is recurrent.