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Yasuo Hirose†

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Abstract

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models have become a popular tool in macroeconomics. In particular, following the development of Bayesian estimation and evaluation techniques, an increased number of researchers have estimated DSGE models for empirical research as well as quantitative policy analysis. These models typically consist of optimizing behavior of households and firms, and a monetary policy rule, along the lines of King (2000) and Woodford (2003). In this class of models, a central bank follows an active monetary policy rule; that is, the nominal interest rate is adjusted more than one for one when inflation deviates from a given target, and the economy fluctuates around the steady state where actual inflation coincides with the targeted inflation. In addition to such a target-inflation steady state, Benhabib et al. (2001) argue that the combination of an active monetary policy rule and the zero lower bound on the nominal interest rate gives rise to another long-run equilibrium, called a deflation steady state, where the inflation rate is negative and the nominal interest rate is very close to zero.

The primary contribution of this paper is to build and estimate a DSGE model for the Japanese economy under the assumption that the economy is near a deflation steady state during the zero interest rate period, whereas the existing studies have estimated DSGE models around a targeted-inflation steady state. The analysis in the present paper is motivated by Bullard (2010), who points out the possibility that the Japanese economy has been stuck in a deflation equilibrium. Figure 1 plots the short-term nominal interest rate and inflation in Japan during the period from 1981Q1 to 2013Q1. In the figure, an exponential curve (thick solid line) is fitted to the data in order to illustrate a nonlinear monetary policy rule. Moreover, two long-run Fisher relations are added: One (dotted line) is the Fisher equation where the real interest rate r is fixed at 3.30, which is the mean of the ex post real interest rate for the sample from 1981Q1 to 1998Q4. The other (dashed line) is the one where $r = 1.42$, which is the same mean for the sample from 1999Q1 to 2013Q1. During the latter sample period, the Bank of Japan conducted a virtually zero interest rate policy, with the exception of August 2000–March 2001 and July 2006–December 2008, and the inflation rate was almost always negative. As argued in Benhabib et al. (2001), two steady states emerge as the intersections of the nonlinear policy rule and the Fisher equations; that is, the steady states for the pre- and post-1999 period correspond to the targeted-inflation and deflation steady state, respectively. Therefore, Japan's macroeconomic fluctuations during the zero interest rate period are possibly well characterized as equilibrium dynamics near the deflation steady state.

Specifically, this paper estimates a medium-scale DSGE model, along the lines of Christiano et al. (2005), Smets and Wouters (2003, 2007), and Justiniano et al. (2010), approximated around the deflation steady state, using data from 1999 to 2013 for Japan.[1] The difficulty in estimating

[1]During the period, the Bank of Japan adopted quantitative easing (QE), which is an unconventional monetary policy that sets a target range for excess reserves held by financial institutions. However, such a policy measure is not incorporated in the present model because, as Eggertsson and Woodford (2003) argue, an increase in excess

the model around the deflation steady state is that the equilibrium is indeterminate; i.e., there are an infinite number of equilibrium trajectories that converge to the deflation steady state, because of passive monetary policy that is constrained by the zero lower bound on the nominal interest rate.[2] In this regard, following Lubik and Schorfheide (2004) and Bianchi and Nicolò (2017), a set of specific equilibrium paths is selected among an infinite number of equilibria using Bayesian methods.[3]

Through the lens of the estimated model, the characteristics of the Japanese economy during the zero interest rate period are revealed. First, positive shocks to households' preferences and wage markup, and a negative shock to monetary policy do not necessarily have an inflationary effect, in contrast to a standard model approximated around a targeted-inflation steady state. This finding about the inflation responses to these shocks provides a novel view about the flattening of the short-run Phillips curve in Japan, which has been examined in the literature by, for example, Nishizaki and Watanabe (2000) and De Veirman (2009). While the analyses in the previous studies are based on the estimation of reduced-form Phillips curves, our full-information-based estimation of the DSGE model offers a structural interpretation about their arguments. According to the estimated structural parameters, the slope of the Phillips curve itself does not become flat. Rather, the ambiguity of the inflation responses, as shown in the impulse response analysis, leads to a weak comovement between inflation and output. This weak comovement can be identified as a flattening of the Phillips curve in the estimation of reduced-form equations. Moreover, our counterfactual analysis demonstrates that a strong comovement between inflation and output could be obtained without price markup shocks. This finding suggests the importance of considering such cost-push shocks in a structural model for correctly identifying the slope of the Phillips curve as is consistent with the observed weak comovement between the two variables. Second, while an economy in the deflation equilibrium could be unexpectedly volatile because of sunspot shocks, which are nonfundamental disturbances, our estimation results show that the effect of sunspot shocks to Japan's business cycle fluctuations is quite limited. On the contrary, it turns out that the sunspot shocks contribute to stabilizing the economy after the global financial crisis. A possible

reserves, regardless of its effect on the size and composition of the central bank's balance sheet, is irrelevant in the standard DSGE model. Hayashi and Koeda (2014) quantify the effect of QE on the Japanese economy by estimating a structural vector autoregression model.

[2]According to Leeper (1991), an economy with passive monetary policy can lead to a determinate equilibrium under active fiscal policy, where inflation dynamics are determined to stabilize the process for government debt. Under such a policy regime, Japan's increasing debt-to-GDP ratio would have led to a substantial increase in inflation, which is inconsistent with the long-lasting deflation in Japan. Meanwhile, passive fiscal policy is also incompatible with the growing debt-to-GDP ratio and the unresponsive primary-surplus-to-GDP ratio in Japan. For these reasons, the model in this paper abstracts from any fiscal policy rules.

[3]Cochrane (2017) emphasizes that a New Keynesian model in a liquidity trap can exhibit completely different dynamics, depending on the choice of equilibrium.

interpretation of this result is that the sunspot shocks during the period might capture the change in expectations driven by the announcement and implementation of new unconventional policy measures, which are not explicitly specified in the estimated model.

The most closely related paper is Aruoba et al. (2018). They consider Markov switching between the targeted-inflation and deflation steady state in a small-scale New Keynesian DSGE model and estimate whether the U.S. and Japan have been in either the targeted-inflation or deflation regime.[4] Regarding the Japanese economy, they find that it shifted from a targeted-inflation regime into a deflation regime in the late 1990s and remained in the latter regime thereafter. Their finding validates our assumption that Japan has been stuck in a deflation equilibrium during our sample period; i.e., from 1999 to 2013.[5] Their primary focus is on the estimation of the timing of the regime change, given the structural parameters pre-estimated around the targeted-inflation steady state for the sample from 1981 to 1994 in Japan. In contrast, the present paper estimates parameters in a richer DSGE model around a deflation steady state using data since 1999 and investigates the economic properties during this period in more detail.[6]

The model estimated in this paper is the first benchmark model to empirically investigate a deflationary economy constrained by the zero lower bound on the nominal interest rate allowing for equilibrium indeterminacy. In the literature, Sugo and Ueda (2008), Iwata (2011), Hirose and Kurozumi (2012), Ichiue et al. (2013), Kaihatsu and Kurozumi (2014), Fueki et al. (2016), and Hirakata et al. (2016) estimate medium-scale DSGE models using Japanese data. However, these authors either exclude the zero interest rate period from their samples or ignore the zero lower bound constraint in their estimation because of computational difficulties in the treatment of nonlinearities arising from the bound.[7] The present paper avoids dealing with the nonlinearities by focusing on the equilibrium dynamics around the deflation steady state, where the nominal interest rate is assumed to be exogenously set at almost zero. In this setting, when large negative shocks against inflation and output occur, the nominal interest rate cannot be lowered, which can partly replicate a contractionary effect of being constrained by the zero lower bound.

This paper also contributes to the literature on the estimation of DSGE models under equi-

[4] Mertens and Ravn (2014) also consider Markov switching between the two steady states in order to theoretically investigate the effects of expansionary government spending and a labor tax cut in the deflation equilibrium.

[5] Mertens and Ravn (2014) argue that Japan's experience could be described as a trajectory towards a permanent deflationary state, as is considered in the present paper, whereas the U.S. and other economies should be better characterized by the Markov switching equilibrium that involves an eventual escape from the deflationary state.

[6] One might claim that the model should be estimated allowing for the regime switching as in Aruoba et al. (2018). It is, however, a quite challenging task and beyond the scope of this paper to estimate a regime switching DSGE model with possibly indeterminate equilibria in a medium-scale setting.

[7] Empirical studies that estimate nonlinear DSGE models with the zero lower bound are still scarce. Remarkable exceptions are Richter and Throckmorton (2016), Gust et al. (2017), Plante et al. (2018), and Iiboshi et al. (2018). These authors estimate fully-nonlinear New Keynesian models in which the interest-rate lower bound is occasionally binding for the U.S. or Japanese economy.

librium indeterminacy. With several exceptions such as Hirose (2007, 2008, 2013), Belaygorod and Dueker (2009), Bhattarai et al. (2012, 2016), and Zheng and Guo (2013), Doko Tchatoka et al. (2017), and Hirose et al. (2017), there have been still few papers that estimate indeterminate models. This paper is one of the first empirical work that estimates a medium-scale DSGE model under indeterminacy, whereas the previous studies estimate relatively small models.[8] Moreover, while the previous studies estimate indeterminate models using the method developed by Lubik and Schorfheide (2004), the present paper employs the approach of Bianchi and Nicolò (2017). Bianchi and Nicolò's approach has an advantage in identifying a specific equilibrium representation under indeterminacy. In their solution method, the full set of indeterminate equilibria is characterized by introducing parameters which can be mapped into the correlations of structural shocks with sunspot shocks. This feature improves the identifiability of indeterminacy-related parameters because they have a well-defined domain over the interval $[-1, 1]$.

The remainder of this paper proceeds as follows. Section 2 presents a DSGE model with a deflation steady state. Section 3 describes the solution and econometric strategy for estimating the model. Section 4 reports the estimation results. Section 5 concludes.

2 The Model

The model is a medium-scale DSGE model along the lines of Christiano et al. (2005), Smets and Wouters (2003, 2007), and Justiniano et al. (2010) but differs from these models in the following respects. First, households' preferences are specified as in Erceg et al. (2006), which ensures the existence of the balanced growth path under the constant relative risk aversion (CRRA) utility function. Second, following Greenwood et al. (1988), the model assumes that a higher utilization rate of capital leads to a higher depreciation rate of capital. This assumption is supported by Sugo and Ueda (2008) who estimate a Christiano et al. type model under the same assumption for the Japanese economy and successfully replicate a negative correlation between capital utilization and rental cost observed in the data. Finally, the equilibrium conditions are approximated around the deflation steady state, which is the main difference between our model and those in the existing studies.

In the model economy, there is a continuum of households, a representative final-good firm, a continuum of intermediate-good firms, and a central bank. Their optimization problems and equilibrium conditions are presented below.

[8] After the previous version of this study (Hirose, 2014) was released, Arias et al. (2017) estimate a medium-scale DSGE model with non-zero trend inflation using the method of Lubik and Schorfheide (2004), and Dai et al. (2017) estimate a medium-scale DSGE model with credit market frictions using the method of Bianchi and Nicolò (2017).

2.1 Households

There is a continuum of households $h \in [0,1]$, each of which purchases consumption goods $C_t(h)$ and one-period riskless bonds $B_t(h)$, and supplies one kind of differentiated labor service $l_t(h)$ to intermediate-good firms. Each household's preferences are represented by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t e^{z_t^b} \left\{ \frac{(C_t(h) - \gamma C_{t-1}(h))^{1-\sigma}}{1-\sigma} - \frac{Z_t^{1-\sigma} l_t(h)^{1+\chi}}{1+\chi} \right\},$$

where $\beta \in (0,1)$ is the subjective discount factor, $\sigma > 0$ measures the degree of risk aversion, $\gamma \in (0,1)$ represents the degree of habit persistence in consumption preferences, $\chi > 0$ is the inverse of the labor supply elasticity, Z_t represents the level of neutral technology, and z_t^b denotes a shock to the subjective discount factor. As in Erceg et al. (2006), we assume the presence of $Z_t^{1-\sigma}$ in the labor disutility, which ensures the existence of the balanced growth path in the model economy.

At the beginning of each period, each household owns capital stock $K_{t-1}(h)$ and rents utilization-adjusted capital $u_t(h)K_{t-1}(h)$ to intermediate-good firms at the real rental price $R_t^k(h)$. Then, the capital utilization rate $u_t(h)$ and investment spending $I_t(h)$ are determined subject to the capital accumulation equation

$$K_t(h) = \{1 - \delta(u_t(h))\} K_{t-1}(h) + \left\{ 1 - S\left(\frac{I_t(h)}{I_{t-1}(h)} \frac{e^{z_t^i}}{z} \right) \right\} I_t(h). \tag{1}$$

Here, following Greenwood et al. (1988), the model assumes that a higher utilization rate of capital leads to a higher depreciation rate of capital. Hence, the depreciation rate function $\delta(\cdot)$ has the properties $\delta' > 0$, $\delta'' > 0$, $\delta(u) = \delta \in (0,1)$, and $\mu = \delta'(u)/\delta''(u) > 0$, where $u = 1$ is the steady-state capital utilization rate. The function $S(\cdot)$ represents the costs involved in changing investment spending, such as financial intermediation costs as analyzed by Carlstrom and Fuerst (1997), and takes the quadratic form of $S(x) = (x-1)^2/(2\zeta)$, where ζ is a positive constant. The variable z_t^i is a shock to the investment adjustment costs. The parameter $z > 1$ represents the gross balanced growth rate.

Each household's budget constraint is given by

$$C_t(h) + I_t(h) + \frac{B_t(h)}{P_t} = W_t(h) l_t(h) + R_t^k(h) u_t(h) K_{t-1}(h) + R_{t-1}^n \frac{B_{t-1}(h)}{P_t} + T_t(h),$$

where P_t is the price of final goods, $W_t(h)$ is the real wage, R_t^n is the gross nominal interest rate and $T_t(h)$ consists of a lump-sum public transfer and profits received from firms.

In the presence of complete insurance markets, the decisions are the same for all households, and hence the first-order conditions with respect to consumption, bond-holdings, investment, capital utilization, and capital stock are given by

$$\Lambda_t = e^{z_t^b} (C_t - \gamma C_{t-1})^{-\sigma} - \beta\gamma E_t e^{z_{t+1}^b} (C_{t+1} - \gamma C_t)^{-\sigma}, \tag{2}$$

$$\Lambda_t = \beta E_t \Lambda_{t+1} \frac{R_t^n}{\pi_{t+1}}, \tag{3}$$

$$R_t^k = Q_t \delta'(u_t), \tag{4}$$

$$\begin{aligned} 1 =& Q_t \left\{ 1 - S\left(\frac{I_t}{I_{t-1}} \frac{e^{z_t^i}}{z} \right) - S'\left(\frac{I_t}{I_{t-1}} \frac{e^{z_t^i}}{z} \right) \frac{I_t}{I_{t-1}} \frac{e^{z_t^i}}{z} \right\} \\ &+ \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1} S'\left(\frac{I_{t+1}}{I_t} \frac{e^{z_{t+1}^i}}{z} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \frac{e^{z_{t+1}^i}}{z}, \end{aligned} \tag{5}$$

$$Q_t = E_t\, \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ R_{t+1}^k u_{t+1} + Q_{t+1} \left(1 - \delta(u_{t+1}) \right) \right\}, \tag{6}$$

where Λ_t is the Lagrange multiplier interpreted as the marginal utility of income, $\pi_t = P_t/P_{t-1}$ is the gross inflation rate, and Q_t is the real price of capital.

In monopolistically competitive labor markets, nominal wages are set on a staggered basis à la Calvo (1983) in the face of the labor demand given by $l_t(h) = l_t(W_t(h)/W_t)^{-(1+\lambda_t^w)/\lambda_t^w}$, where $l_t = \left\{ \int_0^1 l_t(h)^{1/(1+\lambda_t^w)} dh \right\}^{1+\lambda_t^w}$, and $\lambda_t^w > 0$ is related to the substitution elasticity between differentiated labor services and represents the exogenous time-varying wage markup. In each period, a fraction $1 - \xi_w \in (0,1)$ of wages is reoptimized, while the remaining fraction ξ_w is set by indexation to the balanced growth rate z as well as a weighted average of past inflation π_{t-1} and steady-state inflation π. Then, the reoptimized wages solve the following problem

$$\max_{W_t(h)} E_t \sum_{j=0}^{\infty} (\beta\xi_w)^j \left\{ \Lambda_{t+j} l_{t+j|t}(h) \frac{P_t W_t(h)}{P_{t+j}} \prod_{k=1}^{j} \left(z \pi_{t+k-1}^{\gamma_w} \pi^{1-\gamma_w} \right) - \frac{e^{z_{t+j}^b} Z_{t+j}^{1-\sigma} l_{t+j|t}(h)^{1+\chi}}{1+\chi} \right\}$$

subject to

$$l_{t+j|t}(h) = l_{t+j} \left\{ \frac{P_t W_t(h)}{P_{t+j} W_{t+j}} \prod_{k=1}^{j} \left(z \pi_{t+k-1}^{\gamma_w} \pi^{1-\gamma_w} \right) \right\}^{-\frac{1+\lambda_{t+j}^w}{\lambda_{t+j}^w}},$$

where $\gamma_w \in [0,1]$ is the weight of wage indexation to past inflation relative to steady-state inflation. The first-order condition for the reoptimized wage W_t^o is given by

$$E_t \sum_{j=0}^{\infty} \left[\begin{array}{l} (\beta\xi_w)^j \frac{\Lambda_{t+j}}{\lambda_{t+j}^w} l_{t+j} \left[\frac{z^j W_t^o}{W_{t+j}} \prod_{k=1}^j \left\{ \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\gamma_w} \frac{\pi}{\pi_{t+k}} \right\} \right]^{-\frac{1+\lambda_{t+j}^w}{\lambda_{t+j}^w}} \\ \times \left\{ \begin{array}{l} z^j W_t^o \prod_{k=1}^j \left\{ \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\gamma_w} \frac{\pi}{\pi_{t+k}} \right\} - (1+\lambda_{t+j}^w) \frac{e^{z_{t+j}^b} Z_{t+j}^{1-\sigma}}{\Lambda_{t+j}} \\ \times \left(l_{t+j} \left[\frac{z^j W_t^o}{W_{t+j}} \prod_{k=1}^j \left\{ \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\gamma_w} \frac{\pi}{\pi_{t+k}} \right\} \right]^{-\frac{1+\lambda_{t+j}^w}{\lambda_{t+j}^w}} \right)^{\chi} \end{array} \right\} \end{array} \right] = 0. \tag{7}$$

The aggregate wage equation $W_t = \left(\int_0^1 W_t(h)^{-1/\lambda_t^w} dh \right)^{-\lambda_t^w}$ can be expressed as

$$1 = (1-\xi_w) \left(\left(\frac{W_t^o}{W_t} \right)^{-\frac{1}{\lambda_t^w}} + \sum_{j=1}^{\infty} \xi_w^j \left[\frac{z^j W_{t-j}^o}{W_t} \prod_{k=1}^{j} \left\{ \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\gamma_w} \frac{\pi}{\pi_{t+k}} \right\} \right]^{-\frac{1}{\lambda_t^w}} \right). \tag{8}$$

2.2 Firms

2.2.1 Final-good firm

The final-good firm produces output Y_t by choosing a combination of intermediate inputs $\{Y_t(f)\}$, $f \in [0,1]$, so as to maximize its profit $P_tY_t - \int_0^1 P_t(f)Y_t(f)df$ subject to the production technology $Y_t = \left\{\int_0^1 Y_t(f)^{1/(1+\lambda_t^p)}df\right\}^{1+\lambda_t^p}$, where $P_t(f)$ is the price of intermediate good f, and $\lambda_t^p > 0$ is related to the substitution elasticity between differentiated goods and corresponds to the exogenous time-varying price markup.

The first-order condition for profit maximization yields the final-good firm's demand for each intermediate good given by $Y_t(f) = Y_t(P_t(f)/P_t)^{-(1+\lambda_t^p)/\lambda_t^p}$, while perfect competition in the final-good market leads to its price P_t, given by

$$P_t = \left(\int_0^1 P_t(f)^{-\frac{1}{\lambda_t^p}}df\right)^{-\lambda_t^p}. \tag{9}$$

The market clearing condition for the final good is

$$Y_t = C_t + I_t + dZ_te^{z_t^d}, \tag{10}$$

where the term $dZ_te^{z_t^d}$ represents an external demand component that consists of government spending and net exports. z_t^d is an external demand shock, and d is a scale parameter.

2.2.2 Intermediate-good firms

Each intermediate-good firm f produces output $Y_t(f)$ by choosing a cost-minimizing pair of capital and labor services $\{u_tK_{t-1}(f), l_t(f)\}$, given their real rental prices (R_t^k, W_t) and the production function

$$Y_t(f) = (Z_tl_t(f))^{1-\alpha}(u_tK_{t-1}(f))^{\alpha} - \phi Z_t. \tag{11}$$

Here, Z_t represents the level of neutral technology and is assumed to follow the stochastic process

$$\log Z_t = \log z + \log Z_{t-1} + z_t^z, \tag{12}$$

where $z > 1$ is the steady-state gross rate of neutral technological changes, and z_t^z represents a shock to the rate of the changes. The parameter $\alpha \in (0,1)$ measures the capital elasticity of output. The last term in the production function (11), $-\phi Z_t$, is the fixed cost of producing intermediate goods, and ϕ is a positive constant.[9]

Combining cost-minimizing conditions with respect to capital and labor services shows that the real marginal cost is identical across intermediate-good firms and is given by

$$mc_t = \left(\frac{W_t}{(1-\alpha)Z_t}\right)^{1-\alpha}\left(\frac{R_t^k}{\alpha}\right)^{\alpha}. \tag{13}$$

[9] The zero profit condition for intermediate-good firms at the steady state leads to $\phi = \lambda^p$, where λ^p is the steady-state price markup.

Furthermore, combining the cost-minimizing conditions and aggregating the resulting equation over intermediate-good firms shows that the capital–labor ratio is identical across intermediate-good firms and is given by

$$\frac{u_t K_{t-1}}{l_t} = \frac{\alpha W_t}{(1-\alpha) R_t^k}, \tag{14}$$

where $K_t = \int_0^1 K_t(f) df$ and $l_t = \int_0^1 l_t(f) df$. Moreover, using this equation to aggregate the production function (11) over intermediate-good firms yields

$$Y_t d_t = (Z_t l_t)^{1-\alpha} (u_t K_{t-1})^{\alpha} - \phi Z_t, \tag{15}$$

where $d_t = \int_0^1 (P_t(f)/P_t)^{-(1+\lambda_t^p)/\lambda_t^p} df$ measures the intermediate-good price dispersion and is of second order under the staggered price setting presented below.

Facing the final-good firm's demand, each intermediate-good firm sets the price of its product on a staggered basis à la Calvo (1983). In each period, a fraction $1 - \xi_p \in (0, 1)$ of intermediate-good firms reoptimizes prices, while the remaining fraction ξ_p indexes prices to a weighted average of past inflation π_{t-1} and steady-state inflation π. Then, the firms that reoptimize prices in the current period solve the problem

$$\max_{P_t(f)} E_t \sum_{j=0}^{\infty} \xi_p^j \left(\beta^j \frac{\Lambda_{t+j}}{\Lambda_t} \right) \left\{ \frac{P_t(f)}{P_{t+j}} \prod_{k=1}^{j} \left(\pi_{t+k-1}^{\gamma_p} \pi^{1-\gamma_p} \right) - mc_{t+j} \right\} Y_{t+j|t}(f)$$

subject to

$$Y_{t+j|t}(f) = Y_{t+j} \left\{ \frac{P_t(f)}{P_{t+j}} \prod_{k=1}^{j} \left(\pi_{t+k-1}^{\gamma_p} \pi^{1-\gamma_p} \right) \right\}^{-\frac{1+\lambda_{t+j}^p}{\lambda_{t+j}^p}},$$

where $\gamma_p \in [0, 1]$ is the weight of price indexation to past inflation relative to steady-state inflation. The first-order condition for the reoptimized price P_t^o is given by

$$E_t \sum_{j=0}^{\infty} \left(\begin{array}{c} (\beta \xi_p)^j \frac{\Lambda_{t+j}}{\Lambda_t \lambda_{t+j}^p} Y_{t+j} \left[\frac{P_t^o}{P_t} \prod_{k=1}^{j} \left\{ \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\gamma_p} \frac{\pi}{\pi_{t+k}} \right\} \right]^{-\frac{1+\lambda_{t+j}^p}{\lambda_{t+j}^p}} \\ \times \left[\frac{P_t^o}{P_t} \prod_{k=1}^{j} \left\{ \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\gamma_p} \frac{\pi}{\pi_{t+k}} \right\} - \left(1 + \lambda_{t+j}^p \right) mc_{t+j} \right] \end{array} \right) = 0. \tag{16}$$

The final-good price equation (9) can be written as

$$1 = (1 - \xi_p) \left(\left(\frac{P_t^o}{P_t} \right)^{-\frac{1}{\lambda_t^p}} + \sum_{j=1}^{\infty} (\xi_p)^j \left[\frac{P_{t-j}^o}{P_{t-j}} \prod_{k=1}^{j} \left\{ \left(\frac{\pi_{t-k}}{\pi} \right)^{\gamma_p} \frac{\pi}{\pi_{t-k+1}} \right\} \right]^{-\frac{1}{\lambda_t^p}} \right). \tag{17}$$

2.3 Central bank

The central bank adjusts the nominal interest rate following a monetary policy rule of the form

$$R_t^n = R^n \left(\pi_t, \frac{Y_t}{Z_t}, R_{t-1}^n, \varepsilon_t^r \right), \tag{18}$$

where ε_t^r is a monetary policy shock that captures an unsystematic component of monetary policy. Although the functional form of $R^n(\cdot)$ is not specified at this stage, three assumptions are made regarding this monetary policy rule, as in Benhabib et al. (2001). First, $R^n(\cdot) > 1$ because of the zero lower bound constraint on the nominal interest rate. Second, $R^n(\cdot)$ is increasing, strictly convex, and differentiable. Third, around the inflation target, the monetary policy rule satisfies the so-called Taylor principle; that is, the nominal interest rate increases (decreases) by more than one percent in response to a one-percent increase (decrease) in the inflation rate.

As argued by Benhabib et al. (2001), combining the monetary policy rule (18) that satisfies the assumptions above and the Fisher equation—i.e., $R_t^n = R_t E_t \pi_{t+1}$, where R_t is the gross real interest rate—yields two steady states, which we call the targeted-inflation steady state and the deflation steady state. Analogous to Figure 1, at the targeted-inflation steady state, the gross inflation and nominal interest rates are expressed as $\pi^* > 1$ and $R^{n*} = R\pi^*$, respectively, where R is the steady-state gross real interest rate. At the deflation steady state, they are given by $\pi^D < 1$ and $R^{nD} = R\pi^D$. Notice that π^D is very close to (but not equal to) $1/R < 1$, and R^{nD} is very close to (but not equal to) unity.[10]

2.4 Fundamental shock processes

The model contains seven fundamental shocks; i.e., technology z_t^z, preference z_t^b, investment adjustment cost z_t^i, external demand z_t^d, wage markup z_t^w, price markup z_t^p, and monetary policy ε_t^r shocks. While monetary policy shock $\varepsilon_t^r \sim i.i.d.N(0, \sigma_r^2)$, the other shocks follow stationary first-order autoregressive processes

$$z_t^x = \rho_x z_{t-1}^x + \varepsilon_t^x,\ \varepsilon_t^x \sim i.i.d.N(0, \sigma_x^2), \tag{19}$$

where $\rho_x \in [0, 1)$ and $x \in \{z, b, i, d, w, p\}$.

2.5 Approximation around the deflation steady state

The equations (1)$-$(8), (10), (12)$-$(18), are the equilibrium conditions for the model economy. In the model, the real variables are nonstationary because the level of neutral technology has a unit root with drift as shown in (12). Thus, we rewrite the equilibrium conditions in terms of stationary variables detrended by Z_t; i.e., $y_t = Y_t/Z_t$, $c_t = C_t/Z_t$, $w_t = W_t/Z_t$, $\lambda_t = \Lambda_t Z_t^\sigma$, $i_t = I_t/Z_t$, and $k_t = K_t/Z_t$, so that we can compute the steady states for the detrended variables. The steady-state equilibrium conditions are presented in Appendix A.

A remarkable feature of our analysis is that the model is approximated around the deflation steady state, whereas the estimated DSGE models in the existing studies are approximated around the targeted-inflation steady state. Taking account of the fact that the nominal interest rate setting is constrained by the zero lower bound, the monetary policy can be no longer specified by a Taylor-type monetary policy rule. Instead, the nominal interest rate is assumed to follow the exogenous

[10] Aruoba et al. (2018) consider monetary policy rules that are kinked at zero, and hence their deflation steady-state values are given by $\pi^D = 1/R$ and $R^{nD} = 1$.

process

$$\tilde{R}^n_t = \psi_r \tilde{R}^n_{t-1} + \varepsilon^r_t, \tag{20}$$

where $\tilde{R}^n_t$ represents the percentage deviation of the nominal interest rate from its steady state and $\psi_r \in [0, 1)$ is the degree of policy rate smoothing.[11] Then, the monetary policy does not react to inflation any more and hence cannot satisfy the Taylor principle.

The other log-linearized equilibrium conditions are presented in Appendix B.

3 Model Solution and Econometric Methodology

The log-linearized equilibrium conditions, together with the stochastic processes of fundamental shocks, constitute a linear rational expectations system. It is well known that sticky price monetary DSGE models have multiple equilibria, often referred to as indeterminacy, if the Taylor principle is not satisfied.[12] As addressed in the preceding section, the monetary policy approximated around the deflation steady state does not satisfy the Taylor principle, and hence the present system exhibits equilibrium indeterminacy. In this section, we describe how to solve and estimate the model under indeterminacy.

3.1 Solution under indeterminacy

In solving the linear rational expectations system under indeterminacy, we follow the approach of Bianchi and Nicolò (2017), who provide a solution method that accommodates both the case of determinacy and indeterminacy with the same augmented system of equations.[13]

Following Sims (2002) and Lubik and Schorfheide (2003), the log-linearized system is written in the canonical form

$$\Gamma_0(\theta) s_t = \Gamma_1(\theta) s_{t-1} + \Psi_0(\theta) \varepsilon_t + \Pi_0(\theta) \eta_t, \tag{21}$$

where $\Gamma_0(\theta)$, $\Gamma_1(\theta)$, $\Psi_0(\theta)$ and $\Pi_0(\theta)$ are the conformable matrices of coefficients that depend on the structural parameters θ, s_t is a vector of endogenous variables including those expected at t, and ε_t is a vector of disturbances to fundamental shocks. η_t is a vector of endogenous forecast errors, defined as

$$\eta_t = s^E_t - E_{t-1} s^E_t,$$

[11] Another possible specification for the nominal interest around the deflation steady state is to exogenously fix it at zero or its effective lower bound. However, this paper employs the stochastic process for the nominal interest rate because of the fact that the Bank of Japan changed the policy rate from virtually zero during the periods of August 2000–March 2001 and July 2006–December 2008.

[12] See, for instance, Bullard and Mitra (2002) or Woodford (2003).

[13] Bianchi and Nicolò (2017) extend the solution method proposed by Farmer et al. (2015), which requires to rewrite the canonical system depending on the degree of indeterminacy.

where s_t^E is a subvector of s_t whose expectational variables appear in the system. In the present model, $s_t^E = [\tilde{c}_t, \tilde{\lambda}_t, \tilde{\pi}_t, \tilde{\imath}_t, \tilde{R}_t^k, \tilde{q}_t, \tilde{w}_t]'$, where the variables with $\tilde{}$ represent percentage deviations from their steady-state values.

Our prior investigation has confirmed that our baseline model is characterized by at most one degree of indeterminacy. Thus, the solution method of Bianchi and Nicolò (2017) augments the canonical form (21) with the auxiliary equation

$$\omega_t = \frac{1}{\varrho}\omega_{t-1} + \nu_t - \eta_{x,t}, \tag{22}$$

where ω_t is an auxiliary variable, ν_t is a sunspot shock, which is a nonfundamental stochastic disturbance, and $\eta_{x,t}$, $x \in \{c, \lambda, \pi, i, R^k, q, w\}$ is one of the elements in the vector of forecast errors η_t. We set $|\varrho| < 1$ to obtain an equilibrium representation under indeterminacy. Then, the augmented system can be written as

$$\hat{\Gamma}_0(\theta)\,\hat{s}_t = \hat{\Gamma}_1(\theta)\,\hat{s}_{t-1} + \hat{\Psi}_0(\theta)\,\hat{\varepsilon}_t + \hat{\Pi}_0(\theta)\,\eta_t, \tag{23}$$

where $\hat{\Gamma}_0(\theta)$, $\hat{\Gamma}_1(\theta)$, $\hat{\Psi}_0(\theta)$ and $\hat{\Pi}_0(\theta)$ are the augmented matrices of coefficients, $\hat{s}_t = [s_t', \omega_t]'$, and $\hat{\varepsilon}_t = [\varepsilon_t', \nu_t]'$.

The augmented system of equations (23) delivers a solution of the following form using a standard solution algorithm, as if it were under determinacy:

$$\hat{s}_t = \Phi_1(\theta)\,\hat{s}_{t-1} + \Phi_\varepsilon(\theta)\hat{\varepsilon}_t.$$

Bianchi and Nicolò (2017) establish that their equilibrium representation under indeterminacy can map into a full set of nonunique solutions characterized by Lubik and Schorfheide (2003) if the correlations of fundamental shocks with sunspot shocks are parameterized. Therefore, in the subsequent empirical analysis, we estimate the correlation coefficients between the fundamental shocks and the sunspot shock as well as other structural parameters in order to pin down a specific equilibrium representation under indeterminacy.

3.2 Bayesian inference

The model is estimated using Bayesian methods. Seven quarterly time series of Japan's economy are used as observed variables: the log difference of real GDP, real consumption, real investment, and real wage, the log of hours worked, the log difference of the GDP deflator, and the overnight call rate. Real GDP, real consumption, and real investment are on a per capita basis, divided by the population over 15 years old. The real series of consumption and investment are respectively obtained by dividing the nominal private consumption expenditure and gross private domestic investment expenditure series by the GDP deflator. The series of real wage and hours worked are constructed following Sugo and Ueda (2008).

The data are related to model-implied variables by the following measurement equations

$$\begin{bmatrix} 100\Delta\log Y_t \\ 100\Delta\log C_t \\ 100\Delta\log I_t \\ 100\Delta\log W_t \\ 100\log l_t \\ 100\Delta\log P_t \\ 100\log R_t^n \end{bmatrix} = \begin{bmatrix} \bar{z} \\ \bar{z} \\ \bar{z} \\ \bar{z} \\ \bar{l} \\ \bar{\pi} \\ \bar{r}+\bar{\pi} \end{bmatrix} + \begin{bmatrix} \tilde{y}_t - \tilde{y}_{t-1} + z_t^z \\ \tilde{c}_t - \tilde{c}_{t-1} + z_t^z \\ \tilde{\imath}_t - \tilde{\imath}_{t-1} + z_t^z \\ \tilde{w}_t - \tilde{w}_{t-1} + z_t^z \\ \tilde{l}_t \\ \tilde{\pi}_t \\ \tilde{R}_t^n \end{bmatrix},$$

where $\bar{z} = 100\log z$, $\bar{l} = 100\log l$, $\bar{\pi} = 100\log\pi$, and $\bar{r} = 100\log R$.

The sample period is from 1999Q1 to 2013Q1, when the Bank of Japan conducted the virtually zero interest rate policy, with the exception of August 2000–March 2001 and July 2006–December 2008, and the inflation rate was almost always negative. These observations are consistent with a deflation equilibrium argued by Benhabib et al. (2001). Moreover, the choice of the sample period is supported by Aruoba et al. (2018). They consider Markov switching between the targeted-inflation and deflation steady state in a New Keynesian model and find that the Japanese economy shifted from the targeted-inflation regime into the deflation regime in the late 1990s and has remained there ever since.

Before estimation, some parameters are fixed to avoid identification issues. Following Sugo and Ueda (2008), we set the steady-state depreciation rate at $\delta = 0.06/4$, the capital elasticity of output at $\alpha = 0.37$, and the steady-state wage markup at $\lambda^w = 0.2$. The steady-state ratio of external demand to output is set at the sample mean; i.e., $d/y = 0.248$.

Table 1 summarizes the prior distributions of parameters. Most of the priors for the structural parameters $(\sigma, \gamma, \chi, 1/\zeta, \mu, \gamma_w, \xi_w, \gamma_p, \xi_p, \lambda^p, \psi_r)$ are taken from Justiniano et al. (2010), except for the parameters that determine the degree of relative risk aversion σ, the inverse elasticity of the utilization adjustment costs μ, and the monetary policy smoothing ψ_r. The priors for σ, μ , and ψ_r are set according to Smets and Wouters (2007) and Sugo and Ueda (2008).

The priors for the steady-state values for the balanced growth rate, the hours worked, the inflation rate, and the real interest rate $(\bar{z}, \bar{l}, \bar{\pi}, \bar{r})$ are set using a normal distribution with a mean based on the sample average of the corresponding data. Notice that the prior mean for $\bar{\pi}$ is negative, which is consistent with the deflation steady state considered in the present model.

The priors for the shock persistence parameters $(\rho_x,\ x \in \{z, b, i, d, w, p\})$ are set using the beta distribution with a mean of 0.5 and a standard deviation of 0.15, and the priors for the standard deviations of the shock innovations $(\sigma_x,\ x \in \{z, b, i, d, w, p, r, \nu\})$ are set using the inverse gamma distribution with a mean of 0.5 and a standard deviation of infinity. For the correlations of the fundamental shocks with the sunspot shock $(\rho_{x\nu},\ x \in \{z, b, i, d, w, p, r\})$, we assign the uniform distribution over the interval $[-1, 1]$ to let the data select a specific equilibrium representation.

The likelihood function is evaluated using the Kalman filter. Draws from the posterior distribution of the model parameters are generated with the Metropolis–Hastings algorithm.[14] Based on the posterior draws, we make inferences on the parameters, impulse response functions, and variance decompositions.

4 Empirical Results

This section presents the estimation results. Based on the estimates of the parameters, impulse response functions, and shock decompositions, we reveal some remarkable features of the Japanese economy during the zero interest rate period.

4.1 Model selection

In the approach of Bianchi and Nicolò (2017), we need to specify the auxiliary equation (22) with the choice of one of the forecast errors $\eta_{x,t}$, $x \in \{c, \lambda, \pi, i, R^k, q, w\}$. Farmer et al. (2015) prove that this choice is irrelevant as long as the variance covariance matrix of shocks is unrestricted. In our estimation procedure, however, the fundamental shocks are assumed to be *a priori* uncorrelated with one another, as is common in the literature, and thus the choice does matter. For this reason, we investigate which choice gives rise to a better fit of the model to the data by comparing marginal data densities.

Let $\mathcal{M}_x$ denote the model including the auxiliary equation with the forecast errors $\eta_{x,t}$, $x \in \{c, \lambda, \pi, i, R^k, q, w\}$ and $p\left(\mathcal{Y}^T|\mathcal{M}_x\right)$ denote the marginal data density given the corresponding model, where $\mathcal{Y}^T$ is the sample of observations. The resulting log marginal data densities are $\log p\left(\mathcal{Y}^T|\mathcal{M}_c\right) = -377.25$, $\log p\left(\mathcal{Y}^T|\mathcal{M}_\lambda\right) = -372.54$, $\log p\left(\mathcal{Y}^T|\mathcal{M}_\pi\right) = -407.30$, $\log p\left(\mathcal{Y}^T|\mathcal{M}_i\right) = -408.46$, $\log p\left(\mathcal{Y}^T|\mathcal{M}_{R^k}\right) = -370.99$, $\log p\left(\mathcal{Y}^T|\mathcal{M}_q\right) = -373.05$, and $\log p\left(\mathcal{Y}^T|\mathcal{M}_w\right) = -433.37$.[15] Therefore, in the subsequent analysis, we focus on the results from the model $\mathcal{M}_{R^k}$ with the forecast error of the rental rate of capital η_{R^k}.

4.2 Parameter estimates

The second and third columns of Table 2 report the posterior mean and 90-percent credible intervals for parameters of the model selected above.[16] For comparison, the last two columns of the table show the posterior estimates when a similar model is estimated for the period from 1983Q2 to

[14] In the estimation procedure, 500,000 draws are generated, and the first half of the draws are discarded. The scale factor for the jumping distribution in the Metropolis–Hastings algorithm is adjusted so that the acceptance rate is about 25 percent.

[15] The log marginal data densities are approximated using the harmonic mean estimator proposed by Geweke (1999).

[16] We also conducted the local identification analysis proposed by Iskrev (2010) and found that all the estimated parameters were identified at the posterior mean.

1998Q4. During the period, the economy was in a normal state in the sense that the Bank of Japan was able to adjust the nominal interest rate to achieve an implicit inflation target. Thus, the model for the pre-1999 period differs from the baseline model in two respects. First, the model is approximated around the targeted-inflation steady state, as is standard in the literature.[17] Second, the monetary policy rule is given by

$$\tilde{R}_t^n = \psi_r \tilde{R}_{t-1}^n + (1-\psi_r)\left(\psi_\pi \tilde{\pi}_t + \psi_y \tilde{y}_t\right) + \varepsilon_t^r, \tag{24}$$

where $\psi_\pi > 0$ and $\psi_y > 0$ are the degrees of policy responses to inflation and output respectively, and satisfies the Taylor principle.[18] Then, the equilibrium is determinate, and hence the sunspot shock ν_t no longer affects the equilibrium dynamics.

Some of the households' preference parameters for the post-1999 sample are substantially different from those for the pre-1999 sample. The post-1999 estimate of relative risk aversion σ is much smaller than the pre-1999 estimate. The habit persistence parameter γ is also smaller for the post-1999 sample, implying less internal persistence in consumption dynamics during the period.

The adjustment cost parameters related to investment $1/\zeta$ and capital utilization μ are not much different across the samples. Regarding wage and price setting behavior, the parameters for wage stickiness ξ_w and indexation of wage and price $(\gamma_w, \gamma_p,)$ are somewhat lower for the post-1999 period.

The estimate of the policy smoothing parameter ψ_r is larger than its prior mean for both samples, whereas the estimates of the steady-state balanced growth rate, the hours worked, the inflation rate, and the real interest rate $(\bar{z}, \bar{l}, \bar{\pi}, \bar{r})$ are in line with the priors.

Some of the shocks' persistence parameters $(\rho_b, \rho_i, \rho_d, \rho_p)$ are lower for the post-1999 sample than for the pre-1999 sample. A straightforward explanation for this result is that the observed data exhibit less persistent dynamics during the zero interest rate period. Another possible explanation is that, as addressed in Lubik and Schorfheide (2003, 2004), Beyer and Farmer (2007), and Fujiwara and Hirose (2014), the model under indeterminacy can generate internally persistent dynamics as observed in the data without relying on the persistency of exogenous shocks.

The standard deviations of the shocks $(\sigma_x,\ x \in \{z, b, i, d, w, p, r\})$ are not much different for both periods, except for those of the preference shock σ_b and the monetary policy shocks σ_r. The small standard deviation of the preference shock implies that the consumption Euler equation in the present model can accurately capture the consumption dynamics without any wedges for the

[17]The prior mean for steady-state inflation is set at $\bar{\pi} = 0.214$, which is the sample mean for the pre-1999 period. The prior mean for the steady-state balanced growth rate, the hours worked, and the real interest rate $(\bar{z}, \bar{l}, \bar{R})$ are also set at their corresponding sample mean; i.e., 0.476, 1.172, 0.778, respectively.

[18]The priors for the policy response parameters, ψ_π and ψ_y, are the gamma distributions with mean of 1.5 and 0.125, and standard deviations of 0.15 and 0.1, respectively. These mean values follow from the coefficients in the original Taylor (1993) rule, adapted to a quarterly frequency. For the policy smoothing parameter ψ_r, the beta distribution is set with a mean of 0.5 and a standard deviation of 0.15.

post-1999 sample. On the contrary, the large standard deviation for the pre-1999 sample can be explained by the increased volatility in consumption because of the consumption tax increases in 1989 and 1997. The standard deviation of the monetary policy shock is less than half for the post-1999 sample, compared with that for the pre-1999 sample, reflecting the fact that the Bank of Japan kept the nominal interest rate at virtually zero for most of the post-1999 period.

As for the indeterminacy-related parameters, the correlations of the investment adjustment cost and price markup shocks with the sunspot shock ($\rho_{i\nu}, \rho_{p\nu}$) are far different from zero. This finding indicates the importance of considering multiplicity of the equilibrium representation under indeterminacy. As shown in the following subsection, the uncertainty about the correlations of the fundamental shocks with the sunspot shock, which is measured by the dispersion of the posterior estimates, generates ambiguous responses of inflation to several fundamental shocks.

4.3 Impulse responses

Figures 2–9 present the Bayesian impulse responses of the growth rates of output, consumption, investment, and wage, the level of hours worked, the inflation rate, and the nominal interest rate to the shocks to technology, preferences, investment adjustment costs, external demand, wage markup, price markup, monetary policy, and sunspot, expressed in quarterly terms. In each panel, the solid line and dashed lines respectively show the posterior mean and 90-percent credible interval for the estimated responses to each one-standard-deviation shock, in terms of percentage deviation from the steady state. Each figure compares the responses estimated for the post-1999 sample around the deflation steady state (thick lines) with those for the pre-1999 sample around the targeted-inflation steady state (thin lines).

Remarkable differences are found in the estimated impulse responses to the shocks about preferences (Figure 3), wage markup (Figure 6), and monetary policy (Figure 8). According to the pre-1999 estimates, the preferences and wage markup shocks have an inflationary effect, and the monetary policy shock has a deflationary effect. For the post-1999 sample, however, the effects on inflation to these shocks are ambiguous; that is, the 90-percent intervals contain both positive and negative values.[19] This result comes from the estimated correlations of the fundamental shocks with the sunspot shock and their parameter uncertainty, which is an inherent feature of the solution multiplicity under indeterminacy. This finding suggests that there is a substantial uncertainty about a specific equilibrium representation around the deflation steady state during the zero interest rate period.

As for the propagation of technology (Figure 2), investment adjustment cost (Figure 4), external

[19] One might wonder whether the preference shock and the monetary policy shock are identified with each other for the post-1999 sample because the impulse responses to these two shocks exhibit very similar patterns but in the opposite directions. However, as addressed in footnote 16, we confirmed that all the estimated parameters were identified including the standard deviations of the shocks.

demand (Figure 5), and price markup shocks (Figure 7), the impulse responses estimated for the post-1999 sample are similar to those for the pre-1999 sample, except that the external demand shock has a negative effect on inflation around the deflation steady state. The responses to these shocks are qualitatively in line with those in Christiano et al. (2005), Smets and Wouters (2003, 2007), and Justiniano et al. (2010) because our model shares many similarities with theirs.[20]

The sunspot shock affects equilibrium dynamics only for the post-1999 period. The identified sunspot shock has positive effects on all the observables, as presented in Figure 9. The sunspot shock in the present model is constructed as a belief shock regarding the rental rate of capital, and hence has a positive effect on its expectations, irrelevant to fundamentals. Such nonfundamental belief is self-fulfilling under indeterminacy and has an expansionary effect on other variables.

4.4 Implication for flattening of Japan's Phillips curve

The finding about the ambiguous responses of inflation to the shocks about preferences, wage markup, and monetary policy for the post-1999 period provides a novel view about the flattening of Japan's short-run Phillips curve. In the literature, Nishizaki and Watanabe (2000) show that the slope of Japan's Phillips curve became flatter as the inflation rate approached zero. De Veirman (2009) also provides evidence of a gradual flattening of the Phillips curve since the late 1990s and examines the reason why the large negative output gap did not accelerate deflation during the period. While their analyses are based on the estimates of reduced-form equations, our analysis provides a structural interpretation for their arguments. According to the Phillips curve in our model (presented in Appendix B) that relates inflation to real marginal cost, its slope is expressed as $(1-\xi_p)(1-\xi_p\beta z^{1-\sigma})/\xi_p$. Evaluating this slope based on the posterior mean estimates in Table 2 gives 0.020 for the post-1999 sample and 0.019 for the pre-1999 sample. Thus, the slope itself did not become flat. Rather, our estimated model suggests that the ambiguity of the inflation responses to the aforementioned shocks leads to a weak comovement between inflation and output, which can be identified as a flattening of the Phillips curve in the estimation of reduced-form equations.

Another possible explanation for the flattening of the reduced-form Phillips curve is that a particular shock in the economy caused a weak comovement between inflation and output, albeit with the parameters associated with the slope of the structural Phillips curve unchanged. To examine this possibility, we simulate artificial data from the baseline model and those from the same model but excluding each shock, given the posterior mean estimates of parameters, and then compare the model-implied correlation between inflation $\tilde{\pi}_t$ and output gap measures. The results are shown in Table 3, where real marginal cost $\tilde{mc}_t$ and detrended output $\tilde{y}_t$ are considered as the measures of

[20] While a positive shock to investment adjustment costs decreases investment and puts downward pressure on inflation, a rise in the rental rate of capital because of a reduction in capital has an inflationary effect through an increase in real marginal cost. The latter effect dominates the former in our estimated model.

output gap. According to the baseline model, the correlations of inflation with real marginal cost and detrended output are 0.336 and 0.240, respectively. These correlations substantially increase to 0.843 and 0.515 when the price markup shock is excluded from the model. The correlations decrease or remain almost unchanged when the other shocks are excluded. Therefore, the price markup shock plays a crucial role in generating a weak comovement between inflation and output gap measures. This finding suggests the importance of considering such cost-push shocks in a structural model for correctly identifying the slope of the Phillips curve as is consistent with the observed weak comovement between the two variables.

4.5 Variance and historical decompositions

Table 4 presents the posterior mean estimates of asymptotic forecast error variance decompositions of output growth, consumption growth, investment growth, wage growth, hours worked, inflation, and the nominal interest rate for the post- and pre-1999 samples. Each number shows the relative contribution of technology, preferences, investment adjustment costs, external demand, wage markup, price markup, monetary policy, and sunspot shocks, in percentage terms.

The business cycle characteristics during the zero interest rate period are analyzed by focusing on the decomposition of output growth for the post-1999 sample. The primary source of output fluctuations is the technology shock, which accounts for more than half of output volatility. This finding is consistent with the conventional wisdom in the business cycle research for the U.S. economy (e.g., King and Rebelo, 1999) and the results in the existing studies on Japan's business cycles in the 1980s and 1990s (e.g., Hayashi and Prescott, 2002; Sugo and Ueda, 2008; Hirose and Kurozumi, 2012; Kaihatsu and Kurozumi, 2014). The second largest contribution to output fluctuations is the investment adjustment cost shock. The same result is obtained by Hirose and Kurozumi (2012) for the Japanese economy before 1999. The external demand shock also plays a substantial role in explaining output volatility, which is compatible with a common view that Japanese economic expansion in the mid-2000s was largely dependent on export demand. A distinctive feature of our analysis is that we can assess the extent to which sunspot shock affects the macroeconomic fluctuations. In general, an economy in the deflation equilibrium could be unexpectedly volatile because of sunspot fluctuations. However, the estimated contribution of the sunspot shock to output growth turns out to be very small. Thus, Japan's output fluctuations in the zero interest period are mainly driven by fundamental shocks rather than nonfundamental changes in expectations.

These findings are also confirmed by the historical decomposition. Figure 10 shows the historical decomposition of the output growth rate in terms of percentage deviation from the steady state, evaluated at the posterior mean estimates of parameters for the post-1999 sample. Consistent with the results in the variance decomposition, the shocks to technology, investment adjustment costs, and external demand are the main driving forces of output fluctuations. In particular, the contribution of the technology shock and output growth itself fluctuate in the same direction for most of the sample period. It is worth noting that the sunspot shock positively contributes to output growth after the global financial crisis. A possible interpretation of this result is that the sunspot shock might capture the changes in expectations driven by the announcement and implementation of new unconventional policy measures, which are not explicitly specified in the estimated model, and hence contributes to stabilizing the economy during the period.

As for the variance decompositions of the other observed variables, the price markup shock has a substantial effect in addition to the technology shock. This finding is interpreted as follows. Once the deflation steady state is taken into account, inflation fluctuations are largely explained by the exogenous shocks rather than endogenous feedback mechanism in the model. The effects of this shock are broadly transmitted to the other macroeconomic variables because, around the deflation steady state, the monetary policy is not able to react to the movements of inflation and output because of the interest-rate lower bound.

In the pre-1999 period (the lower half of Table 4), the preference shock has larger effects on all the observed variables, compared with those in the post-1999 period. This result is explained by the difference in the estimates of its standard deviation across the samples, as mentioned in Section 4.2. Another finding is that the contributions of the price markup shock become smaller to most of the variables for the pre-1999 sample. This is because, around the targeted-inflation steady state, the central bank's adjustment of the nominal interest rate is not constrained and can mitigate the effect of the shock.

5 Concluding Remarks

In this paper, we have estimated a medium-scale DSGE model approximated around a deflation steady state for the Japanese economy during the zero interest rate period. Although the equilibrium of the model is indeterminate because of the zero lower bound constraint, a specific equilibrium representation is selected by applying the Bayesian methods developed by Lubik and Schorfheide (2004) and Bianchi and Nicolò (2017). The estimated model differs from a standard model with a targeted-inflation steady state in that positive shocks to households' preferences and wage markup, and a negative shock to monetary policy do not necessarily have an inflationary effect. This finding provides a novel interpretation about the flattening of the short-run Phillips curve observed in Japan. Moreover, the estimated model has demonstrated the possibility that the price markup shock plays a crucial role in generating the weak comovement between inflation and output gap. According to the variance decompositions, Japan's business cycle fluctuations are mainly driven by the shocks about technology, investment adjustment costs, and external demand. In contrast, the effect of the sunspot shock on macroeconomic volatilities is very small. We have, however, found that the sunspot shocks helped to stabilize the economy after the global financial crisis.

Our analysis assumed that Japan has been stuck in a deflation equilibrium since the Bank of Japan adopted its zero interest rate policy in 1999. However, the Japanese economy will possibly return to the targeted-inflation steady state at some time in the future. In order to consider such a steady-state change, regime switching between the two steady states, as in Aruoba et al. (2018), must be incorporated into the present model. Estimating such a regime switching DSGE model with indeterminate equilibria is left for future research.[21]

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Appendix

A Steady-State Equilibrium Conditions

The steady-state equilibrium conditions are given by

$$\beta = \frac{z^{\sigma}}{R},$$

$$R^k = R - 1 + \delta,$$

$$w = (1-\alpha)\left(\frac{1}{1+\lambda^p}\right)^{\frac{1}{1-\alpha}}\left(\frac{R^k}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}},$$

$$\frac{k}{l} = \frac{\alpha z w}{(1-\alpha)R^k},$$

$$\phi = \lambda^p,$$

$$\frac{k}{y} = (1+\phi)z^{\alpha}\left(\frac{k}{l}\right)^{1-\alpha},$$

$$\frac{i}{y} = \left(1 - \frac{1-\delta}{z}\right)\frac{k}{y},$$

$$\frac{c}{y} = 1 - \frac{i}{y} - \frac{d}{y}.$$

B Log-Linearized Equilibrium Conditions

Log-linearizing the equilibrium conditions represented in terms of the detrended variables and rearranging the resulting equations with the steady-state conditions leads to

$$\begin{aligned}\left(1 - \frac{\beta\gamma}{z^{\sigma}}\right)\tilde{\lambda}_t = &- \frac{\sigma z}{z-\gamma}\left\{\tilde{c}_t - \frac{\gamma}{z}\left(\tilde{c}_{t-1} - z_t^z\right)\right\} + z_t^b \\ &+ \frac{\beta\gamma}{z^{\sigma}}\left\{\frac{\sigma z}{z-\gamma}\left(E_t\tilde{c}_{t+1} + E_t z_{t+1}^z - \frac{\gamma}{z}\tilde{c}_t\right) - E_t z_{t+1}^b\right\},\end{aligned}$$

$$\tilde{\lambda}_t = E_t\tilde{\lambda}_{t+1} - \sigma E_t z_{t+1}^z + \tilde{R}_t^n - E_t\tilde{\pi}_{t+1},$$

$$\tilde{u}_t = \mu\left(\tilde{r}_t^k - \tilde{q}_t\right),$$

$$\frac{1}{\zeta}\left(\tilde{\imath}_t - \tilde{\imath}_{t-1} + z_t^z + z_t^i\right) = \tilde{q}_t + \frac{\beta z^{1-\sigma}}{\zeta}\left(E_t\tilde{\imath}_{t+1} - \tilde{\imath}_t + E_t z_{t+1}^z + E_t z_{t+1}^i\right),$$

$$\tilde{q}_t = E_t\tilde{\lambda}_{t+1} - \tilde{\lambda}_t - \sigma E_t z_{t+1}^z + \frac{\beta}{z^{\sigma}}\left\{R^k E_t\tilde{R}_{t+1}^k + (1-\delta)E_t\tilde{q}_{t+1}\right\},$$

$$\tilde{k}_t = \frac{1-\delta}{z}\left(\tilde{k}_{t-1} - z_t^z\right) + \frac{R^k}{z}\tilde{u}_t + \left(1 - \frac{1-\delta}{z}\right)\tilde{\imath}_t,$$

$$\begin{aligned}\tilde{w}_t - \tilde{w}_{t-1} + \tilde{\pi}_t - \gamma_w\tilde{\pi}_{t-1} - z_t^z = &\beta z^{1-\sigma}\left(E_t\tilde{w}_{t+1} - \tilde{w}_t + E_t\tilde{\pi}_{t+1} - \gamma_w\tilde{\pi}_t + E_t z_{t+1}^z\right) \\ &+ \frac{(1-\xi_w)(1-\xi_w\beta z^{1-\sigma})\lambda^w}{\xi_w\{\lambda^w + \chi(1+\lambda^w)\}}\left(\chi\tilde{l}_t - \tilde{\lambda}_t - \tilde{w}_t + z_t^b\right) + z_t^w,\end{aligned}$$

$$\tilde{y}_t = \frac{c}{y}\tilde{c}_t + \frac{i}{y}\tilde{\imath}_t + \frac{d}{y}z_t^d,$$

$$\tilde{mc}_t = (1-\alpha)\tilde{w}_t + \alpha\tilde{R}_t^k,$$

$$\tilde{w}_t - \tilde{R}_t^k = \tilde{u}_t + \tilde{k}_{t-1} - \tilde{l}_t - z_t^z,$$

$$\tilde{y}_t = (1+\lambda^p)\left\{(1-\alpha)\tilde{l}_t + \alpha\left(\tilde{u}_t + \tilde{k}_{t-1} - z_t^z\right)\right\},$$

$$\tilde{\pi}_t - \gamma_p\tilde{\pi}_{t-1} = \beta z^{1-\sigma}\left(E_t\tilde{\pi}_{t+1} - \gamma_p\tilde{\pi}_t\right) + \frac{(1-\xi_p)(1-\xi_p\beta z^{1-\sigma})}{\xi_p}\tilde{mc}_t + z_t^p,$$

$$\tilde{R}_t^n = \psi_r\tilde{R}_{t-1}^n + (1-\psi_r)\left(\psi_\pi\tilde{\pi}_t + \psi_y\tilde{y}_t\right) + z_t^r,$$

where $z_t^w = \frac{(1-\xi_w)\left(1-\beta\xi_w z^{1-\sigma}\right)\lambda^w}{\xi_w\{\lambda^w+\chi(1+\lambda^w)\}}\tilde{\lambda}_t^w$ is a shock relevant to the wage markup, $z_t^p = \frac{(1-\xi_p)\left(1-\beta\xi_p z^{1-\sigma}\right)}{\xi_p}\tilde{\lambda}_t^p$ is a shock associated with the price markup, the variables with ˜ denote percentage deviations from their (detrended) steady-state values, and the variables without time subscripts represent their steady-state values.

Table 1: Prior distributions of parameters

Parameter		Distribution	Mean	S.D.
σ	Relative risk aversion	Gamma	1.500	0.375
γ	Habit persistence	Beta	0.500	0.100
χ	Inverse elasticity of labor supply	Gamma	2.000	0.750
$1/\zeta$	Elasticity of the investment adjustment cost	Gamma	4.000	1.000
μ	Inverse elasticity of the utilization rate adjustment cost	Gamma	1.000	0.500
γ_w	Wage indexation	Beta	0.500	0.150
ξ_w	Wage stickiness	Beta	0.660	0.100
γ_p	Price indexation	Beta	0.500	0.150
ξ_p	Price stickiness	Beta	0.660	0.100
λ_p	Steady-state price markup	Gamma	0.150	0.050
$\overline{z}$	Steady-state output growth rate	Normal	0.145	0.050
$\bar{l}$	Steady-state hours worked	Normal	0.000	0.100
$\overline{\pi}$	Steady-state inflation rate	Normal	-0.332	0.100
$\overline{r}$	Steady-state real interest rate	Normal	0.361	0.100
ψ_r	Interest rate smoothing	Beta	0.750	0.150
ρ_z	Persistence of technology shock	Beta	0.500	0.150
ρ_b	Persistence of preference shock	Beta	0.500	0.150
ρ_i	Persistence of investment shock	Beta	0.500	0.150
ρ_d	Persistence of external demand shock	Beta	0.500	0.150
ρ_w	Persistence of wage markup shock	Beta	0.500	0.150
ρ_p	Persistence of price markup shock	Beta	0.500	0.150
σ_z	Standard deviation of technology shock	Inverse gamma	0.500	∞
σ_b	Standard deviation of preference shock	Inverse gamma	0.500	∞
σ_i	Standard deviation of investment shock	Inverse gamma	0.500	∞
σ_d	Standard deviation of external demand shock	Inverse gamma	0.500	∞
σ_w	Standard deviation of wage markup shock	Inverse gamma	0.500	∞
σ_p	Standard deviation of price markup shock	Inverse gamma	0.500	∞
σ_r	Standard deviation of monetary policy shock	Inverse gamma	0.500	∞
σ_ν	Standard deviation of sunspot shock	Inverse gamma	0.500	∞
$\rho_{z\nu}$	Correlation betw. technology and sunspot shocks	Uniform	0.000	0.577
$\rho_{b\nu}$	Correlation betw. preference and sunspot shocks	Uniform	0.000	0.577
$\rho_{i\nu}$	Correlation betw. investment and sunspot shocks	Uniform	0.000	0.577
$\rho_{d\nu}$	Correlation betw. external demand and sunspot shocks	Uniform	0.000	0.577
$\rho_{w\nu}$	Correlation betw. wage markup and sunspot shocks	Uniform	0.000	0.577
$\rho_{p\nu}$	Correlation betw. price markup and sunspot shocks	Uniform	0.000	0.577
$\rho_{r\nu}$	Correlation betw. monetary policy and sunspot shocks	Uniform	0.000	0.577

Note: S.D. denotes standard deviation.

Table 2: Posterior distributions of parameters

	Post-1999		Pre-1999	
Parameter	Mean	90% interval	Mean	90% interval
σ	0.853	[0.661, 1.051]	1.819	[1.212, 2.378]
γ	0.358	[0.252, 0.460]	0.561	[0.421, 0.703]
χ	2.270	[1.074, 3.467]	2.872	[1.510, 4.194]
$1/\zeta$	5.160	[3.543, 6.847]	4.709	[2.947, 6.335]
μ	2.246	[1.394, 3.076]	1.911	[1.207, 2.623]
γ_w	0.295	[0.141, 0.454]	0.343	[0.174, 0.500]
ξ_w	0.689	[0.591, 0.792]	0.872	[0.822, 0.924]
γ_p	0.225	[0.070, 0.380]	0.412	[0.132, 0.703]
ξ_p	0.868	[0.818, 0.920]	0.873	[0.805, 0.948]
λ_p	0.220	[0.118, 0.317]	0.159	[0.079, 0.234]
$\bar{z}$	0.127	[0.060, 0.200]	0.432	[0.362, 0.500]
$\bar{l}$	-0.027	[-0.185, 0.134]	1.168	[1.002, 1.331]
$\bar{\pi}$	-0.382	[-0.501, -0.267]	0.163	[0.028, 0.295]
$\bar{r}$	0.452	[0.341, 0.562]	0.962	[0.825, 1.100]
ψ_r	0.825	[0.680, 0.983]	0.887	[0.850, 0.924]
ψ_π	-	-	1.328	[1.102, 1.545]
ψ_y	-	-	0.317	[0.160, 0.471]
ρ_z	0.431	[0.298, 0.563]	0.323	[0.179, 0.472]
ρ_b	0.488	[0.239, 0.743]	0.697	[0.506, 0.893]
ρ_i	0.353	[0.189, 0.510]	0.535	[0.425, 0.643]
ρ_d	0.877	[0.812, 0.944]	0.941	[0.910, 0.972]
ρ_w	0.252	[0.087, 0.401]	0.165	[0.055, 0.270]
ρ_p	0.268	[0.087, 0.444]	0.494	[0.250, 0.720]
σ_z	1.641	[1.352, 1.938]	1.705	[1.353, 2.047]
σ_b	0.351	[0.128, 0.596]	5.158	[3.064, 7.202]
σ_i	4.269	[3.466, 5.011]	5.895	[4.490, 7.199]
σ_d	3.486	[2.875, 4.082]	2.832	[2.329, 3.330]
σ_w	0.338	[0.269, 0.408]	0.402	[0.333, 0.471]
σ_p	0.394	[0.302, 0.483]	0.350	[0.211, 0.486]
σ_r	0.063	[0.059, 0.067]	0.129	[0.107, 0.150]
σ_ν	0.642	[0.489, 0.789]	-	-
$\rho_{z\nu}$	-0.038	[-0.298, 0.215]	-	-
$\rho_{b\nu}$	-0.003	[-0.321, 0.333]	-	-
$\rho_{i\nu}$	0.259	[0.005, 0.510]	-	-
$\rho_{d\nu}$	0.029	[-0.183, 0.253]	-	-
$\rho_{w\nu}$	-0.006	[-0.141, 0.129]	-	-
$\rho_{p\nu}$	-0.779	[-0.910, -0.650]	-	-
$\rho_{r\nu}$	-0.181	[-0.525, 0.155]	-	-

Note: Each posterior mean and 90% credible interval are calculated from the draws generated using the Metropolis-Hastings algorithm.

Table 3: Correlation between inflation and output gap measures

	Correlation of inflation with:	
	Real marginal cost	Detrended output
Estimated model	0.336	0.240
Estimated model excluding:		
Technology shock	0.041	0.018
Preference shock	0.336	0.241
Investment shock	0.335	0.401
External demand shock	0.333	0.263
Wage markup shock	0.337	0.239
Price markup shock	0.843	0.515
Monetary policy shock	0.338	0.243
Sunspot shock	0.228	0.189

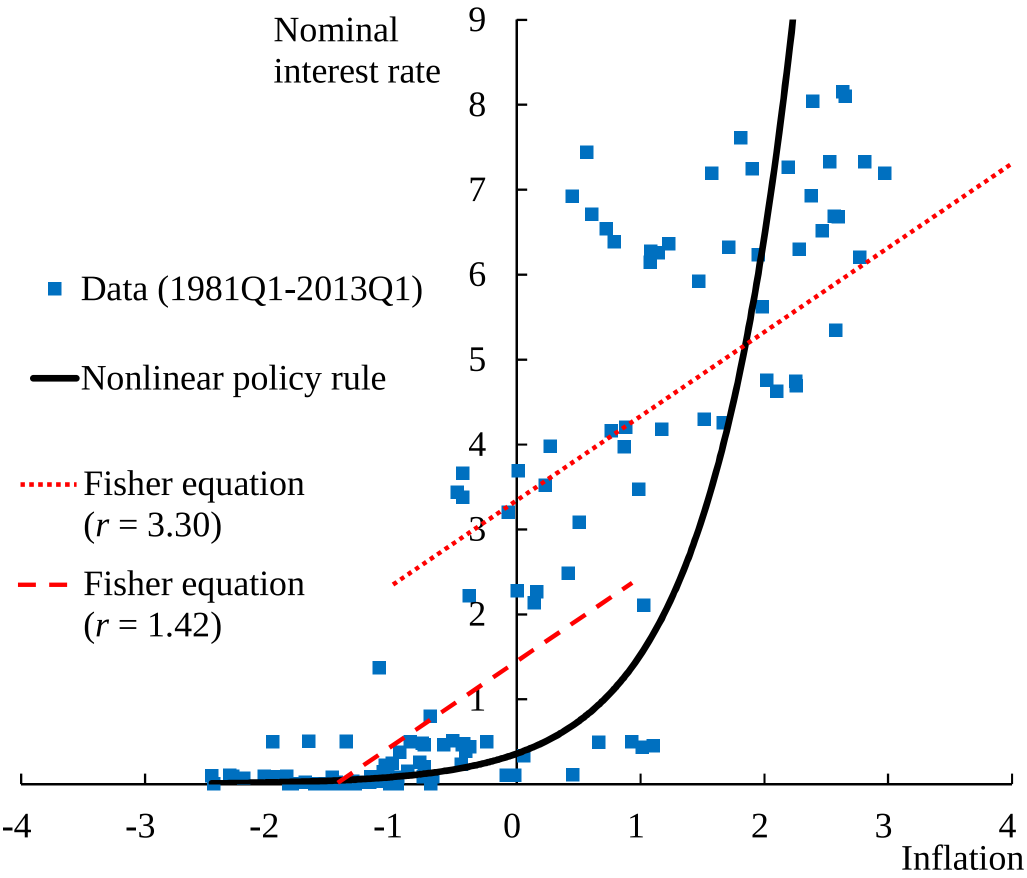
Note: The table shows the correlations of inflation with real marginal cost and detrended output implied by the baseline model and those implied by the model excluding each shock, evaluated at the posterior mean estimates of parameters for the post-1999 sample.

Table 4: Variance decompositions

	$\Delta \log Y_t$	$\Delta \log C_t$	$\Delta \log I_t$	$\Delta \log W_t$	$\log l_t$	$\Delta \log P_t$	$\log R_t^n$
Post-1999							
Technology	63.9	74.4	24.2	63.8	70.4	18.4	0.0
Preference	0.1	0.2	0.1	0.0	0.3	0.2	0.0
Investment	17.7	9.1	67.9	0.3	7.4	5.8	0.0
External demand	11.8	4.2	1.4	0.1	2.6	1.3	0.0
Wage markup	0.4	0.6	0.5	9.4	3.5	0.3	0.0
Price markup	4.2	7.2	4.2	25.7	9.3	66.9	0.0
Monetary policy	0.2	0.2	0.3	0.1	1.3	3.8	100.0
Sunspot	1.7	4.0	1.5	0.5	5.1	3.3	0.0
Pre-1999							
Technology	48.9	45.2	12.5	35.7	76.0	28.1	40.4
Preference	5.7	45.8	0.8	0.1	2.1	0.5	4.0
Investment	33.7	6.2	81.6	0.4	11.4	1.8	23.0
External demand	8.1	1.3	1.1	0.0	3.3	0.5	3.4
Wage markup	0.0	0.0	0.1	18.4	0.7	2.6	2.0
Price markup	2.7	0.9	3.1	45.3	4.2	66.5	17.5
Monetary policy	0.9	0.6	0.8	0.0	2.3	0.1	9.8

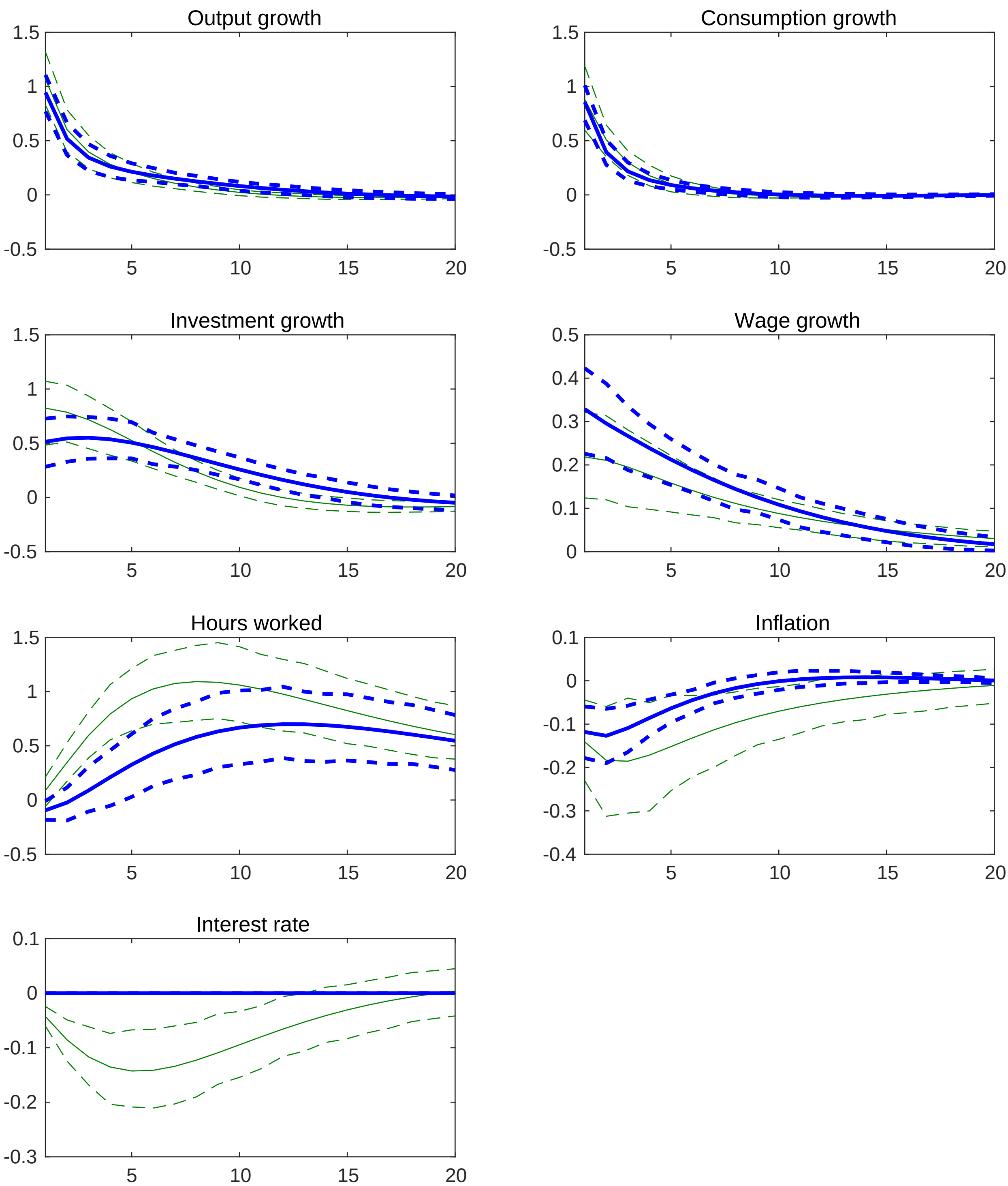
Note: The table shows the posterior mean estimates of the asymptotic forecast error variance decompositions of output growth, consumption growth, investment growth, wage growth, hours worked, inflation, and the nominal interest rate for the post-1999 and pre-1999 samples.

Figure 1: Interest rate and inflation in Japan



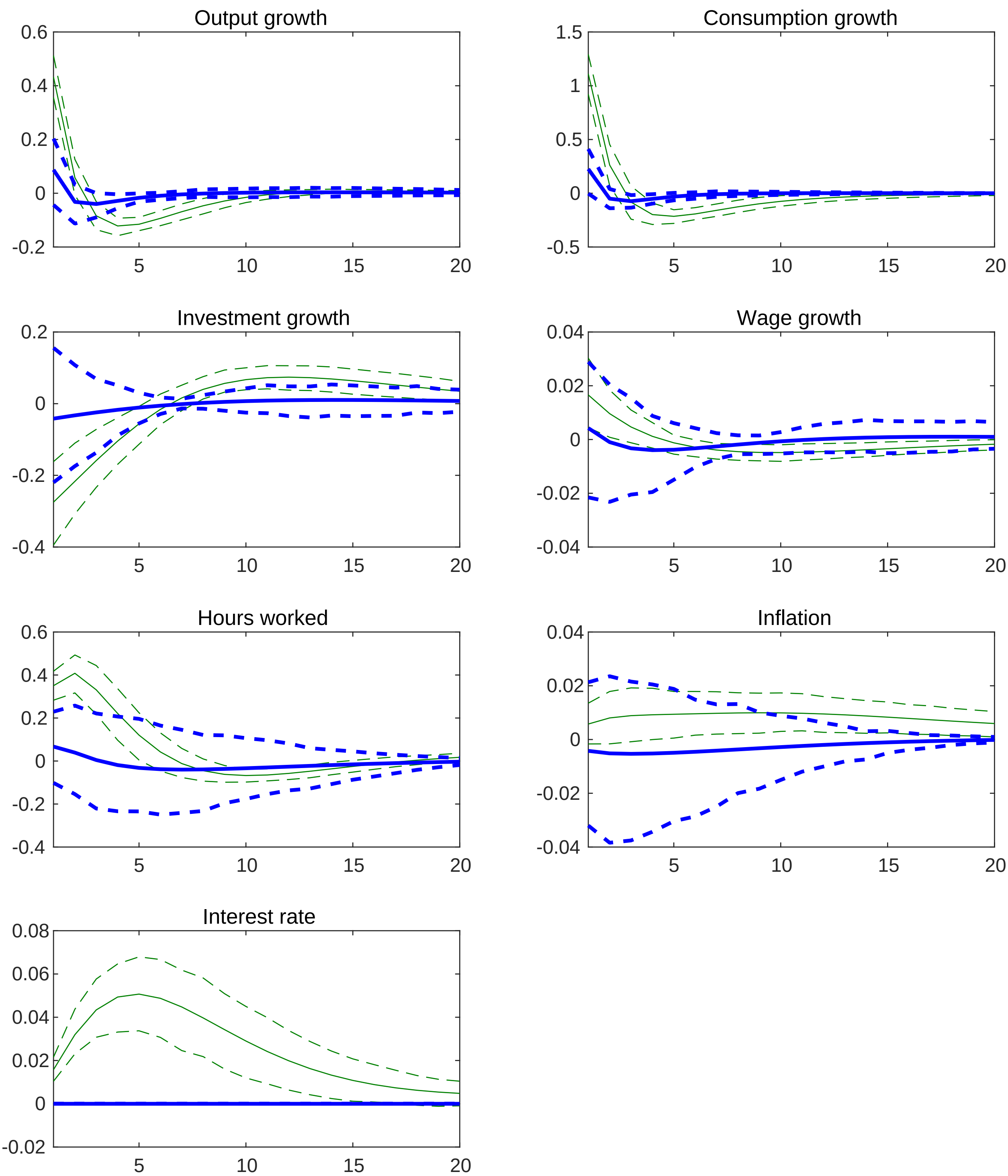
Notes: This figure plots the overnight call rate and the percentage change in the GDP deflator from one year earlier for the sample period from 1981Q1 to 2013Q1. The thick solid line is a nonlinear monetary policy rule fitted to the data. The dotted and dashed lines represent long-run Fisher relations with the real interest rates fixed at 3.30 and 1.42, respectively.

Figure 2: Impulse responses to technology shock



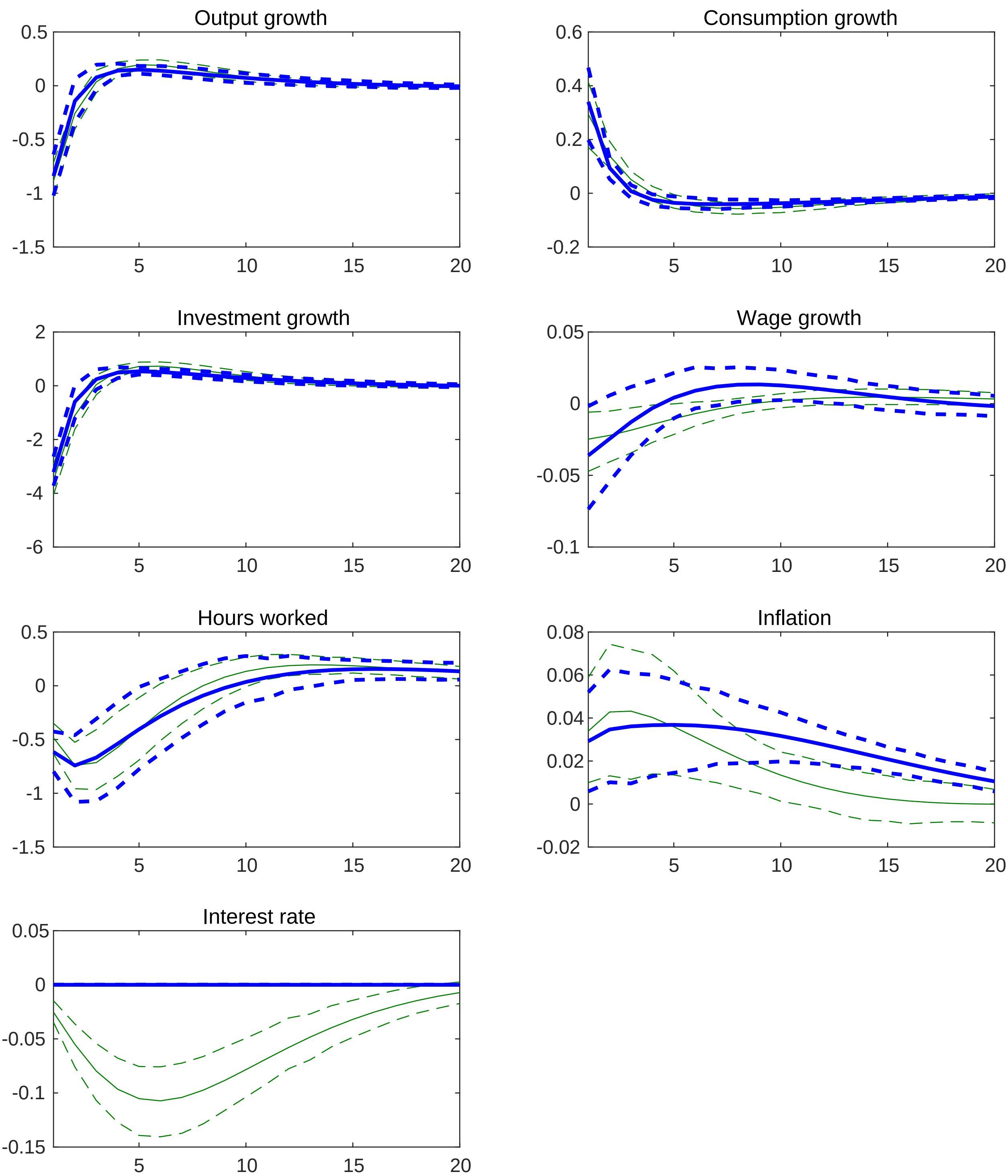
Notes: The figure shows the Bayesian impulse responses of the growth rates of output, consumption, investment, and wage, the level of hours worked, the inflation rate, and the nominal interest rate (in terms of percentage deviations from the steady state) to a one-standard-deviation technology shock. Each panel compares the responses estimated for the post-1999 sample (thick lines) with those estimated for the pre-1999 sample (thin lines). The solid and dashed lines show the posterior mean and 90-percent credible intervals respectively.

Figure 3: Impulse responses to preference shock



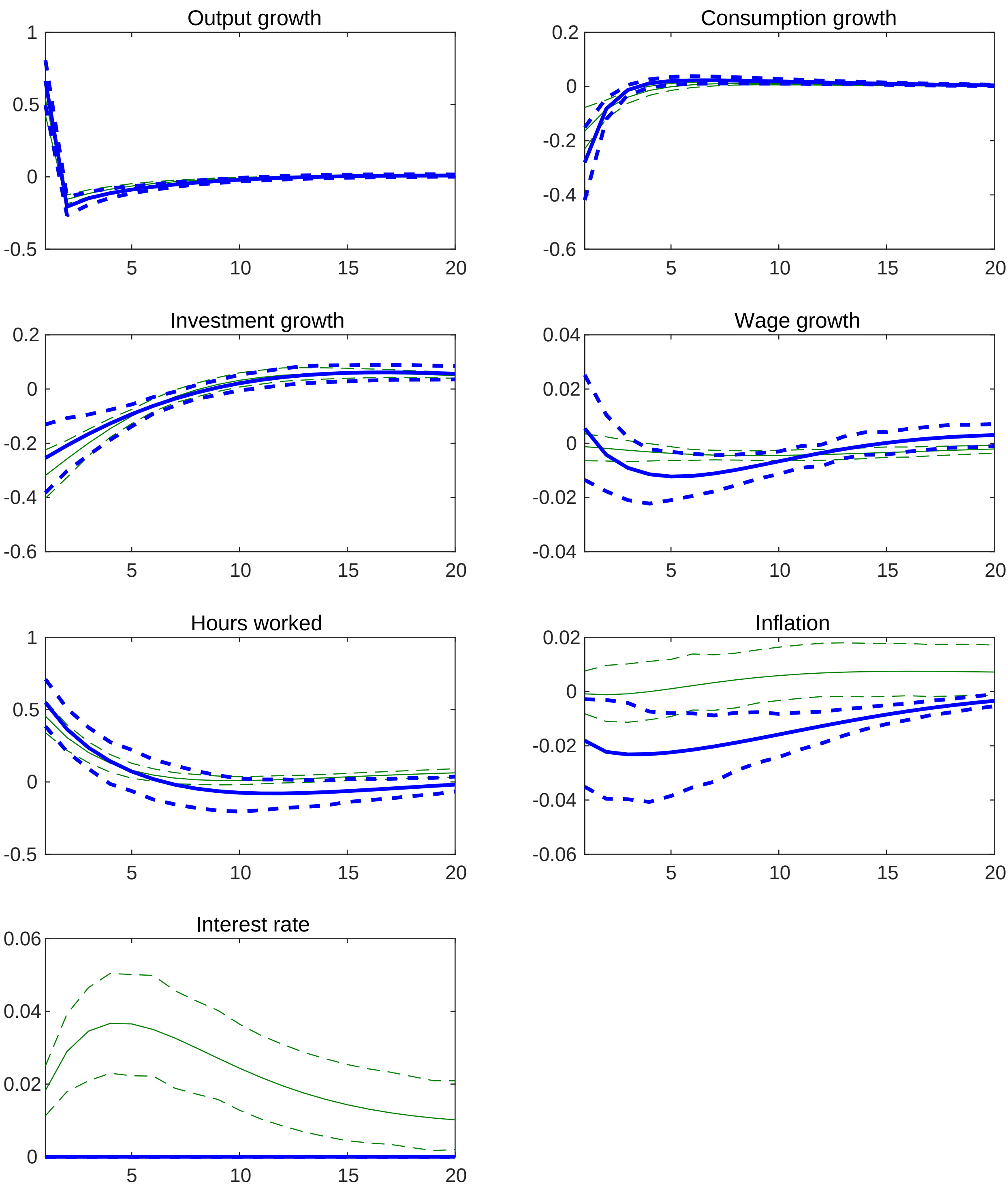
Notes: The figure shows the Bayesian impulse responses of the growth rates of output, consumption, investment, and wage, the level of hours worked, the inflation rate, and the nominal interest rate (in terms of percentage deviations from the steady state) to a one-standard-deviation preference shock. Each panel compares the responses estimated for the post-1999 sample (thick lines) with those estimated for the pre-1999 sample (thin lines). The solid and dashed lines show the posterior mean and 90-percent credible intervals respectively.

Figure 4: Impulse responses to investment shock



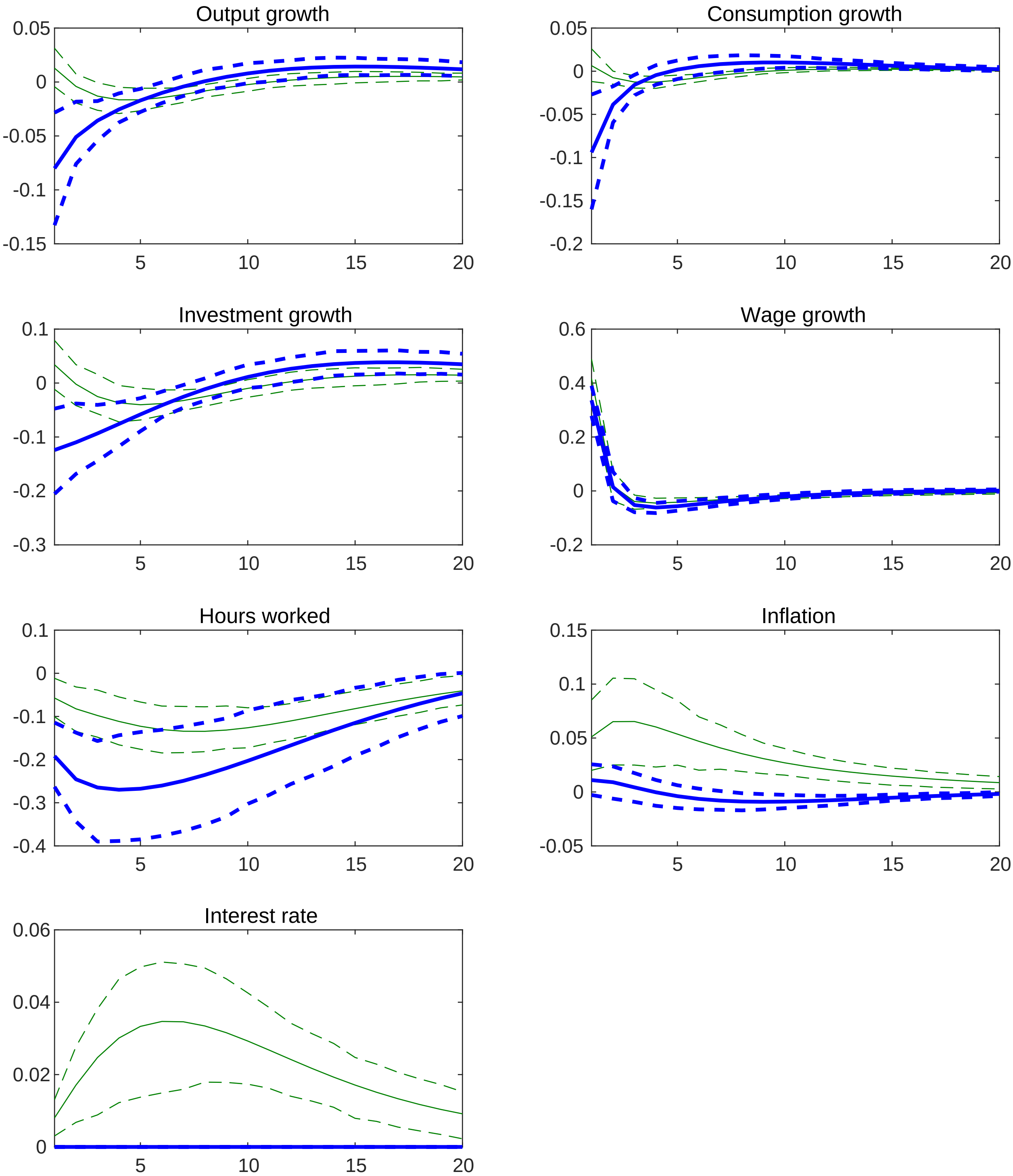
Notes: The figure shows the Bayesian impulse responses of the growth rates of output, consumption, investment, and wage, the level of hours worked, the inflation rate, and the nominal interest rate (in terms of percentage deviations from the steady state) to a one-standard-deviation investment adjustment cost shock. Each panel compares the responses estimated for the post-1999 sample (thick lines) with those estimated for the pre-1999 sample (thin lines). The solid and dashed lines show the posterior mean and 90-percent credible intervals respectively.

Figure 5: Impulse responses to external demand shock



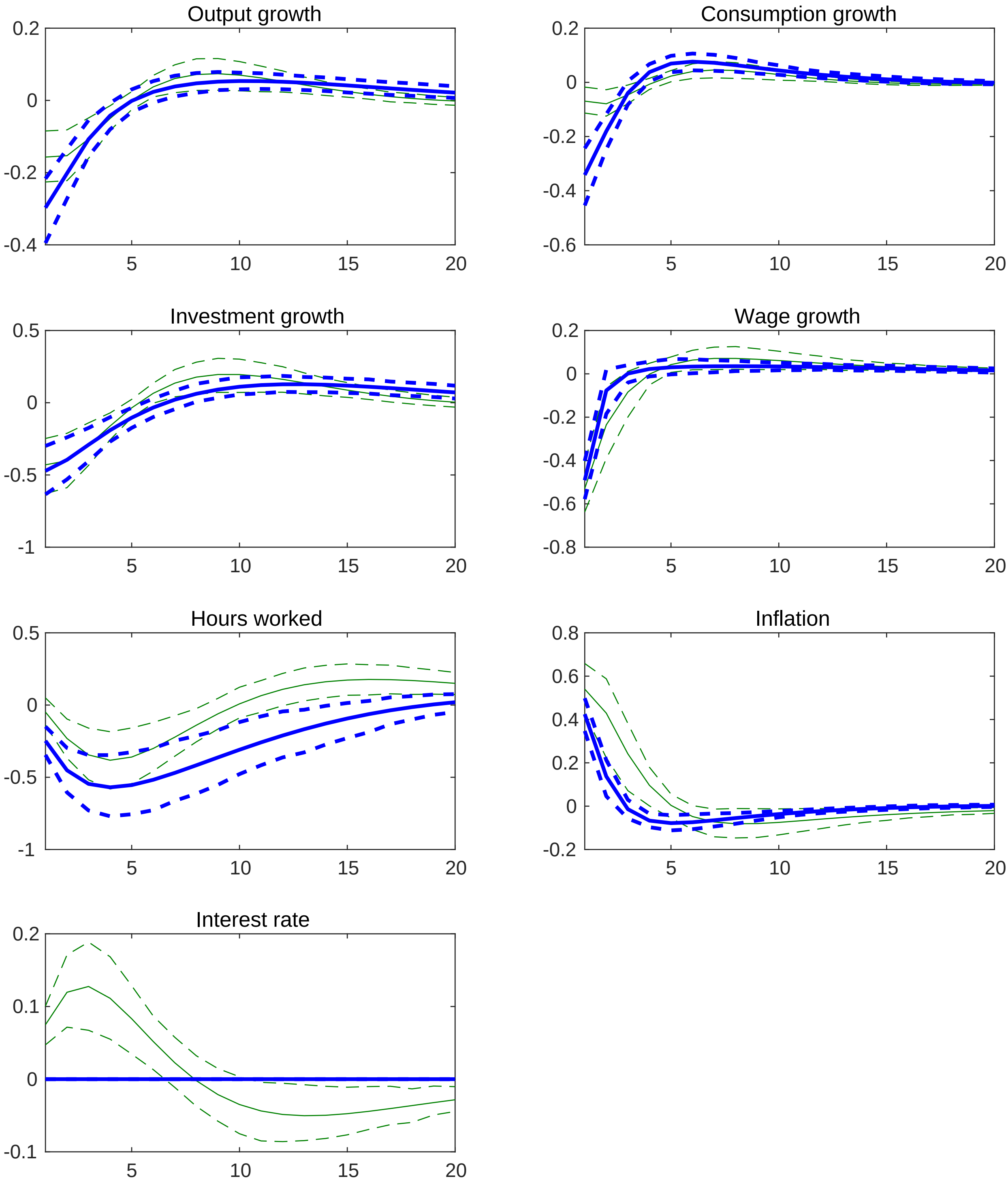
Notes: The figure shows the Bayesian impulse responses of the growth rates of output, consumption, investment, and wage, the level of hours worked, the inflation rate, and the nominal interest rate (in terms of percentage deviations from the steady state) to a one-standard-deviation external demand shock. Each panel compares the responses estimated for the post-1999 sample (thick lines) with those estimated for the pre-1999 sample (thin lines). The solid and dashed lines show the posterior mean and 90-percent credible intervals respectively.

Figure 6: Impulse responses to wage markup shock



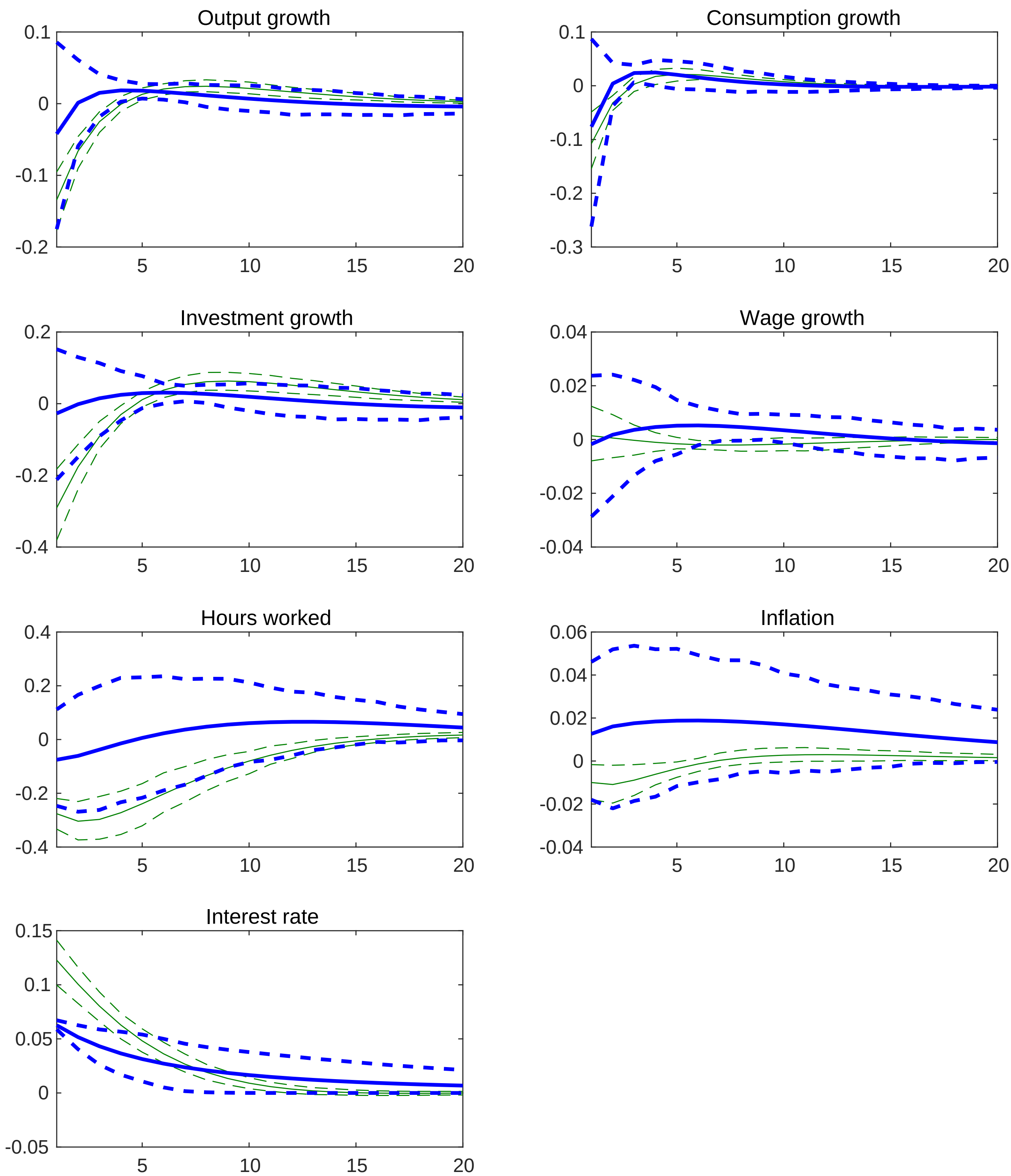
Notes: The figure shows the Bayesian impulse responses of the growth rates of output, consumption, investment, and wage, the level of hours worked, the inflation rate, and the nominal interest rate (in terms of percentage deviations from the steady state) to a one-standard-deviation wage markup shock. Each panel compares the responses estimated for the post-1999 sample (thick lines) with those estimated for the pre-1999 sample (thin lines). The solid and dashed lines show the posterior mean and 90-percent credible intervals respectively.

Figure 7: Impulse responses to price markup shock



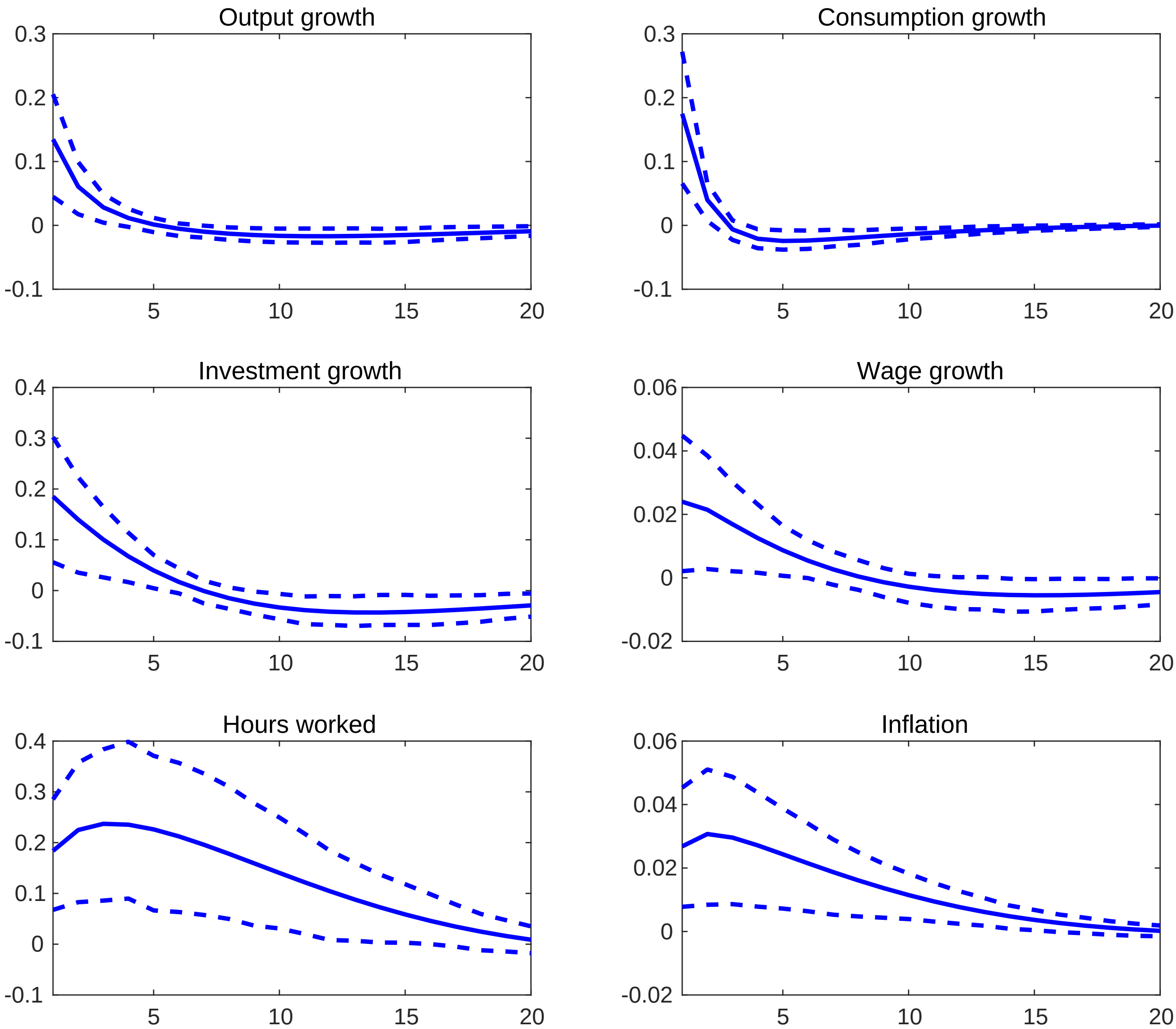
Notes: The figure shows the Bayesian impulse responses of the growth rates of output, consumption, investment, and wage, the level of hours worked, the inflation rate, and the nominal interest rate (in terms of percentage deviations from the steady state) to a one-standard-deviation price markup shock. Each panel compares the responses estimated for the post-1999 sample (thick lines) with those estimated for the pre-1999 sample (thin lines). The solid and dashed lines show the posterior mean and 90-percent credible intervals respectively.

Figure 8: Impulse responses to monetary policy shock



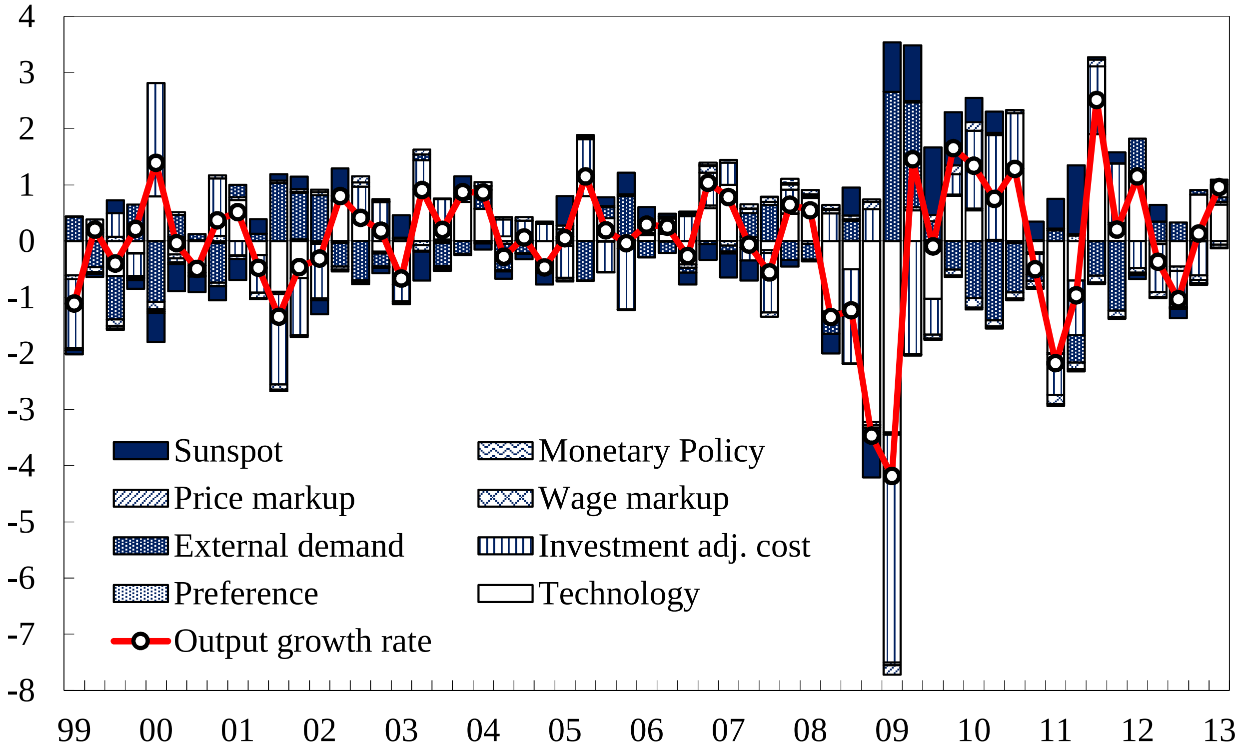
Notes: The figure shows the Bayesian impulse responses of the growth rates of output, consumption, investment, and wage, the level of hours worked, the inflation rate, and the nominal interest rate (in terms of percentage deviations from the steady state) to a one-standard-deviation monetary policy shock. Each panel compares the responses estimated for the post-1999 sample (thick lines) with those estimated for the pre-1999 sample (thin lines). The solid and dashed lines show the posterior mean and 90-percent credible intervals respectively.

Figure 9: Impulse responses to sunspot shock



Notes: The figure shows the Bayesian impulse responses of the growth rates of output, consumption, investment, and wage, the level of hours worked, and the inflation rate (in terms of percentage deviations from the steady state) to a one-standard-deviation sunspot shock. The solid and dashed lines show the posterior mean and 90-percent credible intervals respectively.

Figure 10: Historical decomposition of output growth



Note: The figure depicts the output growth rate in terms of percentage deviation from the steady state and the contribution of each shock, evaluated at the posterior mean estimates of parameters for the post-1999 sample.