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**CONDITIONS WHERE THE RULED CLASS UNITES FOR THE REVOLUTION
-applicability of a game theory on social dilemmas-**

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Keywords: revolution;historical materialism;social dilemma;large group;chicken game

Abstract

Revolutions, typical cases of crucial social transformations, cannot be realized successfully without a large number of activists. Therefore, creating conditions favorable for acquiring enough participants should be an important topic of Marxist social science. In particular, this problem includes the “free-ride,” because the benefits of revolutionaries’ activities are gained not only by the activists but also by all other members. The paper analyzes problems such as this one, applying non-cooperative game theory to social dilemma problems. This leads to some interesting results. In this research, the problem of the workers’ choice between unity or freeride is first defined using numerical examples of the gain structure. It is defined again in a more generalized form using other parameters. In so doing, we express both the cost of participating in the movement and the gains from the concession of the ruling class. Because this analysis focuses on the importance of the number of participants, the concession of the ruling class is framed as a function of the number of participants. The results of this analysis revealed that the economic foundation and superstructure accurately correspond in some game structures but not in others. In other words, the social dilemma presents either as a case of prisoners’ dilemma or as a chicken game. Furthermore, this paper analyzes the influence of group size, and it was revealed that groups with a large number of members, such as a ruling class, find it particularly difficult to unite. This phenomenon is called the “large group dilemma.” In these ways, this research shows that the aforementioned type of game theory can be used to analyze the difficulties and possibilities of social movements.

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In this research, the problem of the workers’ choice between unity or freeride is first defined using numerical examples of the gain structure. It is defined again in a more generalized form using other parameters. In so doing, we express both the cost of participating in the movement and the gains from the concession of the ruling class. Because this analysis focuses on the importance of the number of participants, the concession of the ruling class is framed as a function of the number of participants.

The results of this analysis revealed that the economic foundation and superstructure accurately correspond in some game structures but not in others. In other words, the social dilemma presents either as a case of prisoners’ dilemma or as a chicken game.

Furthermore, this paper analyzes the influence of group size, and it was revealed that groups with a large number of members, such as a ruling class, find it particularly difficult to unite. This phenomenon is called the “large group dilemma.” In these ways, this research shows that the aforementioned type of game theory can be used to analyze the difficulties and possibilities of social movements.

Key words: revolution, historical materialism, social dilemma, large group, chicken game

Introduction

The social movements, whose typical movement is a revolution movement, cannot be realized successfully unless many participants can be acquired, and therefore, there are many difficulties associated with it. Because the benefits gained by revolutionaries’

activities is not limited only to the activists but also to all members of the ruled class. It is the free ride problem, and this problem is discussed as an important problem in the trade union movement. Discussions on whether to limit the benefits such as wage increases to union members or to all workers without limit are discussed in any union. Therefore, research on what kind of condition the ruled class link arms under, what conditions the free rider can reduce under, etc. has a very important meaning.

Unfortunately, however, this problem has not been analyzed analytically so far, and it is hard to say that some successful academic achievement in the field of game theory have been referred or used to analyze such cooperation / non-cooperation problems. However, it is also true that the present mainstream economics or sociology want to analyze such a political issue as a kind of taboo. For example, Muto(2015) dealt a typical cooperation / non-cooperation issue in environmental problems surrounding global warming, and Ohbayashi(2015) dealt a typical topic by modeling consumer movement and environmental movement. Therefore, in this paper, we will attempt to utilize the framework of cooperative / noncooperative game theory as a tool to find the conditions to strengthen our social movement.

Two Persons' Game to Analyze the Conditions where People Unite

By the way, since cooperation / non-cooperation game analysis adopted here basically depends on the comparison between the benefit and the cost of cooperation and non-cooperation, first I would like to set the gain structures regarding the "cooperation" as a class "unity", and the "non-cooperation" as freeride. Table 1 is the table formulated by this assumption. In this paper, first we consider the case of two players, which is the basic form of game theory. The revolution will succeed if two persons "unite", gain only partial improvement if only one person contributes the movement unilaterally, while no improvement unless anyone participates in movement.

So, look at Table 1. The first of the two figures written in each square is the gain of a member A in ruled class in each case, and the second one is that of the other member B in that class. In the case of this table 1, if both members unite, it is possible to obtain both benefits more than in the case of free ride, but under this gain structure freeride is always more beneficial for each person, whenever the other member select any choice (to free ride even if they try to unite, or to do free ride if they try to free ride). Therefore, here, the case of <free ride, free ride> out of a totally four combinations((unite × free ride) × (unite × freeride)) might be selected socially, and both members gain 60. This is smaller than the gain 68 that both sides can acquire by uniting, and in this sense this case can be regarded as a "prisoner's dilemma" case of a kind of "social dilemma". In

other words, in this situation the ruled class cannot unite and cannot get out of that condition despite such a disadvantage.

Table 1 Situations in which ruled class members cannot unite (Prisoner's Dilemma · Case)

		Choice of the member B in the ruled class	
		Unite	freeride
Choice of the member A in the ruled class	Unite	68, 68	54,81
	freeride	81,54	<u>60,60</u>

However, in fact, there is an even worse situation than the above situation in one sense, and it can be shown in Table 2 below. In this case, if the other member (player) free-rides, the loss by the non-participation to the movement (freeride) becomes more severe than the loss in the case of <unite, freeride> or <freeride, unite> combination. He should stand on the part to protect the class interest even if there are many difficulties. It is also better for him to stand on the part of the class interest (in this case, he can get 72 gains) than to free ride and get much lower gains (60). This represents the situation of the game called "chicken · game", and in this case each member wants to make the other to cooperate (unite) unilaterally. However, if he fails to make the other to do so, and therefore he is enforced to cooperate (unite) unilaterally, his resentment and envy must be accumulated. It is a situation that getting the benefit without fighting (cooperating) becomes the true "free-riding". In fact, the majority of the present Japanese trade unions is facing such a situation.

Table 2 Conditions in which ruled class members divide into revolutionaries and free riders (chicken games · cases)

		choice of the member B in the ruled class	
		Unite	freeride
choice of the member A in the ruled class	Unite	104, 104	<u>72,108</u>
	freeride	<u>108,72</u>	60,60

While the above situation has a reality, it is also true that the ruled classes had realized some revolutionary revolt successfully, and overthrown the old social system in the history. However, this type of situation needs another kind of gain structure, which is shown in Table 3 below. Here, the improvement of the gain that can be acquired by uniting becomes very large (the gain becomes 160 each), so also the side who tried to

free ride does not do so because in this case to unite is better for him here. In other words, the fact that there were many successful revolutions by the oppressive classes in the past means that there was such a certain scale of gains by the revolutions. Situations where system change is needed should be understood as situations under such a gain structure. And, if it comes to such a situation, the ruled class members which had not united until that time will also unite. In this case, it is also reasonable for the whole society to select the same choice (here, both players select unity), and take the maximum gain $160 + 160$. In this sense, this situation is out of the "social dilemma situation" and is named as "non-problematic situation".

Table 3 Situations in which all ruled class members consolidate and revolutionize (non-problematic situation)

		choice of the member B in the ruled class	
		Unite	Freeride
choice of the member A in the ruled class	unite	<u>160, 160</u>	100,150
	freeride	150,100	60,60

There are other cases where each member makes the same choice without hesitation. As shown in the following Table 4, a situation where both members unite (gains of 56, 56 in this case) is worse than the situation where they cooperate with the ruling class without "unity". That is, the combination of freeride becomes better for both members, because both gains increase to 60, 60. In the image, the current social system basically works well, and if it is overthrown by the revolution, it results much worse situation not only for the ruling class but also for the ruled class. Strictly speaking, even in this case, it is possible to obtain the maximum gain (72) personally by free-riding if the other member chooses "unity". However, it can be said for both members. So, as a result, both members select free-riding, and realize the <freeride, freeride> combination which is better than the <unite, unite> combination. This is a difference from the case of Table 1, and this state in this sense is also classified as "non-problematic situation".

Table 4 Situations in which ruled class members are satisfied with the present situation and don't cause a revolution (non-problematic situation)

		choice of the member B in the ruled class	
		Unite	freeride
choice of the member A in	unite	56, 56	48,72

the ruled class	freeride	72,48	<u>60,60</u>
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In this way, it was found that the unity / freeride problem of the ruled class members is defined in the gain structure, but in order to make it clearer, let us show the gain structure not as a numerical example but as a general form by using some parameters. In doing so, we need to express both of the cost of participating the movement and the gain by the concession of the ruling class, and we have set the gain structure shown in the following Table 5. Here, we express the original gain of each ruled class member before the revolution as 'S (Status quo)', and the additional gain to each ruled class member taken by additional one participant to the movement as 'F (Fruit)'. F is assumed to increase in proportion to the number of participants in the movement; that is, both members' participation gives a large amount of benefit to each member, but only one member's participation gives a small amount of benefit. The former case can be understood as a revolution, and the latter as a reform. Furthermore, we express the "cost of participation in the revolutionary movement" as h by assuming $0 < h < 1$ to reduce the individual gain. The reason why here we use this character 'h' is that the basic cost of such participations is the loss of the working time by consuming it for the movement. As a matter of fact, the above four tables were obtained by substituting $F = 21, 48, 90, 12$ respectively for $S = 60$ and $h = 2/3$. This eventually expresses that the degree of development of the movement is determined by the balance between the cost and the benefit to each participant of the social movement. In other words, the magnitude relationship between the present gains represented by (S, S) and the gains after the revolution represented by (h (S + 2F), h (S + 2F)) both of which depend on these three parameters determines the success or failure of the revolution.

Table 5 Gain structure that determines unity / freeride problem of the ruled class members

		choice of the member B in the ruled class	
		unite	Freeride
choice of the member A in the ruled class	unite	$h(S+2F), h(S+2F)$	$h(S+F), S+F$
	freeride	$S+F, h(S+F)$	S, S

Indeed, the difference in the above four situations can be understood as the difference of the following four situations determined by S, F and h. In particular,

Situation ① when $\frac{2h}{1-h}F < S$ Non-problematic situation shown in Table 4 where

all ruled class members are satisfied

Situation ② when $\frac{h}{1-h}F < S < \frac{2h}{1-h}F$ Prisoner's dilemma case shown in Table 1

where everyone do not cooperate

Situation ③ when $\frac{2h-1}{1-h}F < S < \frac{h}{1-h}F$ Chicken game case shown in Table 2

where ruled class members divide into revolutionaries and free riders

Situation ④ when $S < \frac{2h-1}{1-h}F$ Non-problematic situation shown in Table 3 where

all ruled class members voluntarily participate the movement (This case does not exist when h is 1/2 or less)

This result is very interesting for historical materialism. It is because the economic foundation and superstructure accurately respond in the situations ① and ④ in a sense that the desired state for the whole society is acquired by the voluntary selection of the members of the whole society, but do not respond accurately in the situations ② and ③. In other words, under certain conditions, the superstructure is displaced from the economic foundation, showing the relative autonomy of the superstructure from the economic foundation. It is an expression that the subjective condition is not matured in that case, even if social change is required objectively. In this model, it has been shown that the improvement of the situation expected after the revolution is still not sufficiently above the cost of the revolution. In other words, the point is that the current situation is perceived as bad compared to the situation expected after the revolution. Of course, "the cost of the revolution" is also an important factor, so the ruling class want to raise it to suppress the movement.

Prisoner's dilemma, Chicken game and Non-problematic situation in N-person game

Thus, the conditions for unity of the revolutionary movement could be shown concretely and analytically, but the remaining problem here is that the number of the ruled class members are abstracted to two. This is, of course, unrealistic, and therefore we need much concrete analysis of the case in which the number of members is large. And, for the sake of that, let us consider the case below where the total number of members is N, and m of them freeride. The other gain structure is assumed as mostly same as the case in Table 5 above, but a new assumption on additional gains by the participation of the movement is introduced; that is, such additional gains are multiplied (N-m) times to all the ruled class members. Under these assumptions, if the number of free-riders is m, the gain to cooperate can be expressed as C(m) below, and

the gain to freeride can be expressed as $D(m)$ below;

$$\begin{aligned} C(m) &= h \{S + (N - m) F\} \\ D(m) &= S + (N - m) F^1 \end{aligned}$$

Given this gain function, the above four cases are realized under the following conditions, respectively. That is,

i) N-person prisoners' dilemma case

Here, $S + (N-m) F > h \{S + (N - m + 1) F\}$ is led from the $D(m) > C(m - 1)$ condition which means that free-riding is much better for each member. On the other hand, $S < h(S + NF)$ is derived from the condition of $D(N) < C(0)$ which means that each member's gain in the case of no one's cooperation is smaller than the gain in the case of whole members' unity (Pareto inefficiency). These two conditions lead

$$\frac{h}{1-h} F < S < \frac{h}{1-h} NF .$$

ii) N-person chicken game case

In this case, first, it is necessary to satisfy the condition $D(1) > C(0)$ that at least one person free rides and the condition $D(N) < C(N-1)$ that at least one person stands on a side to unite. From the former, $S + (N - 1) F > h(S + NF)$ is obtained, and from the latter, $S < h(S + F)$ is obtained, and summing up all these results, we can introduce

$$\frac{hN - N + 1}{1-h} F < S < \frac{h}{1-h} F .$$

Here, as same as the case (i), we also need the Pareto inefficiency condition that each member's gain in the case of no one's cooperation is smaller than the gain in the case of whole members' unity, but it is already satisfied by the right side of above condition

$$\left(S < \frac{h}{1-h} NF \right).$$

iii) N persons non-problematic situation in which all ruled class members consolidate and revolutionize.

In this case, necessary condition $D(m) < C(m - 1)$ can be transformed into

$$\begin{aligned} S + (N - m) F &< h \{S + (N - m + 1) F\} \\ \Leftrightarrow (1 - h) S &< \{(N - m + 1) h - (N - m)\} F. \end{aligned}$$

Furthermore, since this condition must hold for all m , substitute $m = 1$ which gives the minimum value of this right side, and introduce

$$(1 - h) S < (Nh - N + 1) F$$

¹ $C(m) > C(m + 1)$ and $D(m) > D(m + 1)$ also hold here since the increase of free riders is supposed to increase social loss. This is said to be a co-benefit condition.

$$\Leftrightarrow S < \frac{1-(1-h)N}{1-h} F .$$

In this case as well, each member's gain in the case of no one's cooperation should be smaller than the gain in the case of whole members' unity (Pareto inefficiency), but this condition has been covered by the above inequality.

iv) N-person non-problematic situation where no one's cooperation is rational choice
Here, unlike all the above cases, since the gain D (N) in a situation where all members are not united exceeds the gain C (0) when all members are united, $S > h(S + NF)$ should

be held, and therefore $S > \frac{h}{1-h} NF$.

On the other hand, the condition that free ride is beneficial for each person also should be held, and therefore $S > \frac{h}{1-h} F$ as same as the former condition of the prisoners' dilemma. However, this condition is already covered by $S > \frac{h}{1-h} F$ when $N > 1$.

Therefore, summing up the above conditions, these four conditions from i to iv become

Situation ① when $S > \frac{h}{1-h} NF$ N-person Non-problematic situation where all ruled class

Situation ② when $\frac{h}{1-h} F < S < \frac{h}{1-h} NF$ N-person prisoners' dilemma case

Situation ③ when $\frac{hN - N + 1}{1-h} F < S < \frac{h}{1-h} F$ N-person chicken game case²

² In a chicken game, a certain number "unites" and a certain number "free ride", so we calculated the equilibrium number of the free-riders ($m^* = N -$ the rational number of participants to the movement). Its first condition is $D(m^* + 1) \leq C(m^*)$ indicating that to unite is beneficial for the m^*+1 th member of the ruled class, and the second condition is $D(m^*) \geq C(m^*)$ indicating that free-ride is beneficial for the m^* th member, and both conditions make the next inequality;

$$\frac{S}{F} + N - \frac{1}{1-h} \leq m^* \leq \frac{S}{F} + N - \frac{h}{1-h} .$$

This result shows that the number of free-riders m^* increases when the current situation S and the number of whole members N rise and decreases as the width F of the result of the participation and the cost h of the participation increase. Although participants tend to be irritated at times with a small number of followers, they could be a little calm if they understand that the number of followers ($N - m^*$) is also determined by these objective situations.

Situation ④ when $S < \frac{1-(1-h)N}{1-h} F$ N-person Non-problematic situation

where all ruled class members voluntarily participate the movement

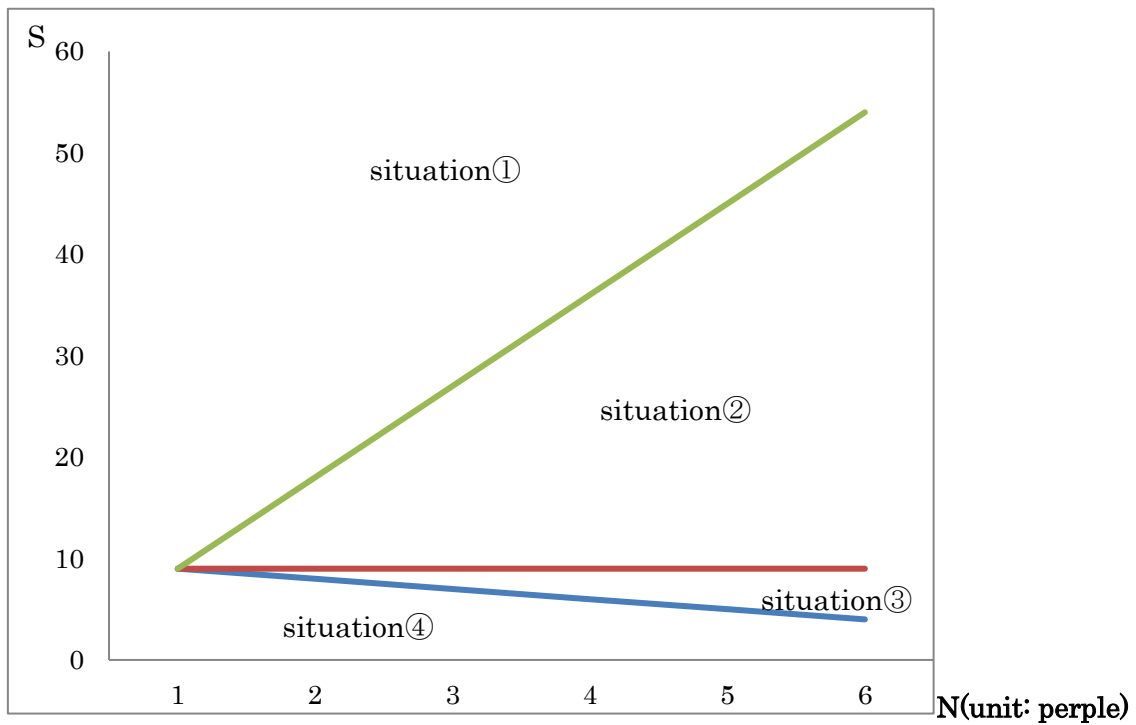
Large Groups Are Hard to Unite

Thus, it turned out that the consequences of the game will change depending on various circumstances such as S, F, h, N. Compared to the two-player game above, this N-person game is superior to analyze the size of the group members N which determines the consequences. In order to clarify this fact, we draw Figure 1 by substituting various numbers for N. Here, it is assumed that $F = 1$ and $h = 0.9$.

So, looking at N in this graph, we can see that the non-problematic situation of situation ① or ④ is shrinking due to the increase of N, and the situation of social dilemmas represented by situation ② or ③ is expanding. Especially in this situation ② and ③, it is interesting because the social transformation by "unity" is desirable for the whole society but all the members or at least a certain part of the society remain in the freeride side. In other words, in a group with a small number of members, it is easy to "unite", but in a group with a large number of members, it is not easy to "unite". I think that such a "large group dilemma" analyzed by Olson (1965) and Kimura (2002) clarified why it is difficult for the ruled class to unite compared with the ruling class, and why small and medium enterprises cannot unite easily compared with the big businesses. This may also answer the question why social change in large countries is difficult.

In these senses, I think, above type of game theory can be used to analyze the difficulties and possibilities of our social movements.

Figure 1 Bifurcation condition of N-person revolution game



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