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Principal-Agent Problems When Principal Allocates a Budget

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Principal-Agent Problems When Principal Allocates a Budget

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Abstract

Agents benefit from having the principal believe that they share his preferences, whereas the principal may prefer that agents reveal their types. Such incentives are explored in a model which considers a principal who sets a budget in each of two periods, that each of the two agents allocates among different services. In the second period, the principal, having observed the agents' behavior in the first period, gives a larger budget to the agent he believes more likely shares the principal's preferences. Each agent may behave strategically, spending his budget on the service he thinks the principal prefers, thereby hiding his type. The principal may induce agents to reveal their types by hiding from them his preferences, or by giving them a large budget in the initial period. Such an approach, however, may lead agents in the initial period to spend too much on services the principal little values.

Keywords: delegation, budget, hidden information

1 Introduction

A principal often wants an agent to behave in a particular way, but also wants an agent to reveal his type. The problem significantly appears when a principal delegates to agents decisions of how to spend a budget, with an agent's preferences differing from the principal's. We can think of Congress allocating a budget to the Federal Aviation Administration, with the Administration deciding where to allocate air traffic personnel, how many hours each facility should be open, and so on. A state legislature may give a budget to a state university, with the university deciding how many faculty to hire in the humanities, how many in the social sciences, and so on. Or, a central government may transfer funds to a local government, with the local government deciding how to spend the budget. Though we shall speak of monetary budgets, a similar analysis applies for other resources the principal controls. For example, a president or a prime minister can allocate his time to help different agencies.

The problem becomes interesting if an agent, who allocates any budget given him across the different services, prefers a large budget. If the principal is unsure of an agent's type, then the agent may allocate his budget in a way that induces the principal to give a large budget in the next period. The principal may then not learn an agent's type, giving a large budget to an agent who would spend the money on services the agent rather than the principal wants. Faced with such strategic behavior, the principal may prefer to hide from the agents information about his own type, inducing them to reveal their types. This paper explores the conditions inducing such behavior.

So a principal who allocates a budget must consider three effects. First, a large budget in period 1 to an agent who will likely spend that budget on the service the principal prefers allows the agent to spend much on that service. Second, in period 2 the larger the budget given to an agent whom the principal believes shares his preferences, the more of that service will be provided. Third, the expected allocation in period 2 affects the agents' decisions in period 1, with a large allocation in period 1 making it more likely that agents will reveal their types.

Note that this principal-agent problem differs from the standard principal-agent model which has the principal give incentive payments to the agent. In the standard model the size of the payments does not affect what the agent can do, only what the agent will choose to do. Under federalism, an agent's resources largely depend on what the principal gives him. Therefore, the size of the incentive payment made by the principal affects the resources available to the agent, and so affects what an agent can do, not only what he will want to do. The model, with some modification, can also apply to agents who seek grants from the principal to make an investment for a service, with the investment increasing the productivity of spending on that service in period 2. The investment can take the form of infrastructure, of devising plans, of training personnel, etc.

2 Literature

The discussion below considers how an agent's actions affect the principal's beliefs about the agent's type. A large literature examines behavior intended to affect reputation, with the principal often viewed as voters, and the agent as an elected official. Reputational concerns may lead a politician to terminate a policy that he, but not the voters, knows has failed (Beniers and Dur 2007). And reputational concerns can induce political correctness: an advisor who wishes to avoid a reputation for bias may not reveal his information (Morris 2001). A career-concerns model where the incumbent attempts to signal ability is analyzed by Canes-Wrone, Herron, and Shotts (2001). Fox (2007) shows that an agent who cares about his reputation may adopt policies commonly associated with a high-quality agent, though the state of nature would call for a different policy. He further shows that if an agent can hide his actions from the public, this distortion can be reduced.

Relatedly, a career-driven agent who knows that his action is observed has an incentive to conform (Prat 2005). The principal is hurt by such behavior, and may want to commit to keep the agent's action secret. The concern about how an agent's behavior in one period affects the principal's beliefs in future periods builds on the career-concerns model of Holmstrom (1999). The idea has been applied to politics (e.g. Persson and Tabellini 2000) to consider incumbent policy makers who have implicit incentives to perform well to appear talented to voters, and where the incentives are limited to a retain-or-fire decision. In our model, the principal's only tool is the budget allocation, where that allocation not only affects incentives, but also determines the resources an agent has. Carpenter (2004) uses a career-concerns model to argue that the U.S. Food and Drug Administration may delay approving some drugs because it wants to safeguard its reputation for protecting the public's health. The analysis below builds on Terai and Glazer (2015) in considering how reputation affects budgetary allocations made by a principal. Unlike that paper, however, the concern here is on the principal's behavior rather than of the agents, including how the principal will allocate a fixed budget over two periods, and on whether the principal benefits from hiding his preferences.

The strategic behavior of agents relates to the ratchet effect, which considers a worker who may exert little effort today: he anticipates that the employer may infer that high effort signals a low cost of effort, inducing the employer to offer a lower wage in the future. For example, in Lazear (1986) and Gibbons (1987) the worker has private information about the firm (such as the job's difficulty), which he is reluctant to reveal. In Aron (1987) and in Kanemoto and MacLeod (1992) the worker has private information about a worker-specific attribute, such as ability.

An agent's preferences can differ from the principal's because the agent is corrupt, or influenced by special interest groups. The differences can also appear when the agent is intrinsically motivated, caring about policy or outcomes, rather than only about the income he earns. Work in the public administration literature provides evidence of intrinsic motivation among public-sector employees (Guyot 1962, and Crewson, 1997). Other work studies whether individuals with greater intrinsic motivation are more often found in the public sector. For example, Gregg et al. (2011) use British survey data to investigate whether prosocial behavior (as measured by the probability of working extra, unpaid, hours) is more prevalent in the nonprofit sector than in the for-profit sector. These authors find that individuals in the nonprofit sector are more likely to work such extra hours. Survey data studied by Georgellis, Iossa, and Tabvuma (2011) also support the hypothesis that individuals are attracted to the public sector more by intrinsic than by extrinsic rewards.[1]

Our consideration of a principal allocating money among agents relates to work on the Good Samaritan Dilemma, where an altruistic donor gives more money to poor recipients (Buchanan 1977).

Related work applied to politics considers how a candidate may gain from concealing information about himself. Shepsle (1972) shows that ambiguity pays when voters are risk-loving. Glazer (1990) shows that if each candidate is uncertain about the median voter's preferred policy (and therefore faces the

[1]For a selective review of research on the existence and the effects of pro-social behavior among individuals working in public organizations, see Polidori and Teobaldelli (2013).

risk of stating an unpopular position), then in equilibrium both candidates may adopt ambiguous positions. The benefits of ambiguity rise further if the position announced by one candidate allows the other candidate to estimate more accurately the voters' preferences. Similarly, Alesina and Cukierman (1990) show that a party can increase its popularity by concealing from voters its preferences.

3 Assumptions

The principal has a fixed budget, $R > 0$. He allocates the budget to agents A and B in the two periods. In each of the two periods, each agent allocates his budget between two services. Delegation to agents can appear for several reasons, including the principal's insufficient time or skill to provide the services. Or the agents may be better informed than the principal about how to provide the services. These characteristics capture characteristics of resource allocation under federalism.

Let each of the principal and of the agents be either an H-type or an L-type. A type-j agent has utility in period t of

$$v_t^j = f^j(x_{t1}, x_{t2}), \tag{1}$$

where x_{t1} and x_{t2} are provisions of services 1 and 2 in period t. Note that the utility of a type-j agent depends on his preferences for the two services. The preferences of each type, H or L, satisfy the single-crossing property for any (x_{t1}, x_{t2}):

$$\frac{\frac{\partial f^H}{\partial x_{t1}}(x_{t1}, x_{t2})}{\frac{\partial f^H}{\partial x_{t2}}(x_{t1}, x_{t2})} > \frac{\frac{\partial f^L}{\partial x_{t1}}(x_{t1}, x_{t2})}{\frac{\partial f^L}{\partial x_{t2}}(x_{t1}, x_{t2})}. \tag{2}$$

Thus a type-H agent values service 1 more than does a type-L agent. Indicate the name of the agent by superscript $k = A$, B. The prior probability that k's type is H is called π_H^k. Assume more specifically that an agent has type H with probability 1/2; that is, $\pi_H^k = 1/2$, $k = A$, B, independently from others.

For mathematical convenience, assume further that for $t = 1$, 2, the function f^j satisfies

$$f^j(0,0) = 0; \tag{3}$$

$$\frac{\partial f^j}{\partial x_{t1}} > 0, \quad \frac{\partial f^j}{\partial x_{t2}} > 0; \tag{4}$$

$$\frac{\partial^2 f^j}{\partial x_{t1}^2}\left(\frac{\partial f^j}{\partial x_{t2}}\right)^2 + \frac{\partial^2 f^j}{\partial x_{t2}^2}\left(\frac{\partial f^j}{\partial x_{t1}}\right)^2 - 2\frac{\partial^2 f^j}{\partial x_{t1}\partial x_{t2}}\left(\frac{\partial f^j}{\partial x_{t1}}\right)\left(\frac{\partial f^j}{\partial x_{t2}}\right) < 0. \tag{5}$$

Expression (5) indicates a decreasing marginal rate of substitution, ensuring that f^j is strictly quasi-concave. Moreover we suppose:

Assumption 1 *f^j is homothetic in (x_{t1}, x_{t2}).*

That is, a type-j agent allocates a budget given to him at the constant ratio between two services. His utility over two periods is

$$V^j = v_1^j + v_2^j. \tag{6}$$

The important difference between the principal and the two agents is that the principal benefits from the services both agents provide, whereas each agent benefits only from the services he himself provides. We call the two agents A and B, and indicate the name of the agent by superscript k, so that x_{ti}^k is the provision of service i in period t by agent k. The principal's utility in each period is

$$v_t^P = f^P(x_{t1}^A + x_{t1}^B, x_{t2}^A + x_{t2}^B), \tag{7}$$

where P represents the principal. The function f^P has properties corresponding to (3), (4), and (5) and it satisfies Assumption 1, i.e., homotheticity. The principal's utility over two periods is

$$V^P = v_1^P + v_2^P. \tag{8}$$

Let R_t be the budget in period t, which the principal allocates between two agents as R_t^A and R_t^B. The allocation satisfies the constraints

$$\begin{aligned} R &= R_1 + R_2; \\ R_t &= R_t^A + R_t^B; \\ R_t^k &= x_{t1}^k + x_{t2}^k. \end{aligned} \tag{9}$$

Note that some of our results will hold when there is only one agent. But consideration of two agents does matter. First, it allows us to consider asymmetric allocations, with the principal who is uncertain the agents' types in period 1 giving different agents different allocations. Second, the principal's threat to reduce the budget he gives to an agent who does not share his preferences is more credible when there is more than one agent—the principal would want to reallocate funds from an agent who does not share his preferences to one who does.

Competitions for funding are common and important. In the United States, federal allocations to states based on agency decisions rather than on legislative formulas averaged \$677 per person, ranging from \$401 in Florida to \$2,400 in Alaska.[2] Individual states also use competitive grants to local governments. For example, in 2015 the state of New York had a competition to fund microgrids, with only 83 municipalities of 130 which had applied receiving grants.[3]

Evidence is consistent with our assumption that a political principal prefers to give money to jurisdictions that share the principal's preferences. For example, Larcinese, Rizzo, and Testa (2006) show that a state in the United States gets more money from the federal government if it heavily supported the incumbent president in past presidential elections. Such allocation can reflect electoral considerations, but that hypothesis is inconsistent with the finding that marginal and swing states are not rewarded. Other evidence consistent with the idea that a central government preferentially allocates funds to local governments which more likely share its preferences is given by Hodler and Raschky (2014). They find that subnational regions have more intense nighttime light when they are the birth region of the current political leader.

Our model could also apply to different governmental agencies providing different services. One service could be assistance to the middle class, and another service could be assistance to the poor. The service provided may differ across agencies. The Department of Education could assist the poor by spending money on poor school districts; the Department of Health could assist the poor by assigning more facilities to serve the poor. The principal may want to give more money to the governmental agency that would assist the poor. Evidence is consistent with such behavior. A study of discretionary Department of Labor grants and Department of Defense contracts in the U.S. from 1991 to 2002 finds that these allocations are larger to states

[2] $http : //www.pr51st.com/territory - gets - less - than - any - state - in - competitive - programs/$

[3] $http : //www.pressconnects.com/story/news/local\ /2015/07/08/binghamton - endicott - get - funding - microgrid - proposals/29868599/$

with senators whose ideologies are close to the president's or to the cabinet secretary's (Bertelli and Grose 2009).

4 Budget allocations and behavior of agents

4.1 Perfect Information about agents' preferences

As a benchmark, consider behavior under perfect information: the principal and the agents all know everyone's preferences. The game proceeds as follows. In period 1:

1. Nature determines the types of the principal and of the agents.
2. The principal allocates the budget R_1 among agents A and B as R_1^A and R_1^B.
3. Each agent k simultaneously and independently allocates the budget R_1^k for the two services.

In period 2:

1. The principal allocates the budget R_2 among agents A and B as R_2^A and R_2^B.
2. Each agent k simultaneously and independently allocates the budget R_2^k for the two services.

The budget the principal gives a type-j agent in period t is called R_t^j. Without loss of generality, let the principal have type H.

In the final stage in each period t, a type-j agent allocates his fixed budget R_t^j between service 1 and service 2 to maximize his utility (1). Under (5), assuming an internal solution, the optimal allocation by an agent in period t satisfies the following first-order condition:

$$\frac{\partial f^j}{\partial x_{t1}}(x_{t1}^j, x_{t2}^j) = \frac{\partial f^j}{\partial x_{t2}}(x_{21}^j, x_{t2}^j). \tag{10}$$

In this expression, x_{ti}^j represents a type-j agent's provision of service i, in the absence of signaling considerations. The marginal rate of substitution is 1 at the optimal choice by each type. It follows from (2) that for a given budget, $x_{t1}^H > x_{t1}^L$ and $x_{t2}^H < x_{t2}^L$.

In each period t, anticipating the agents' responses, the principal will give budget R_t to the agent who shares his preferences. If the agents have the same preferences, that is, both are of type H or of type L, the principal allocates the budget equally between the two agents. This behavior by the principal is rational: with a homothetic utility function (Assumption 1), agents of the same type allocate funds at the same ratio between the two services, so that

$$x_{ti}^j(R_t^j) = (R_t^j/R_t)x_{ti}^j(R_t), \tag{11}$$

and therefore, the principal gets the same utility from any allocation of R_t among agents of the same type. Because the agents are identical, we assume equal allocations.

Taking into account the allocation of R_t^j in the first-order condition for an agent (10), the principal chooses R_1, which in turn determines R_2 as $R - R_1$. The first-order partial derivative of the principal's utility function with respect to R_1 is

$$\begin{aligned} & \frac{\partial f^P}{\partial x_{11}}(x_{11}^j, x_{12}^j)\frac{\partial x_{11}}{\partial R_1} + \frac{\partial f^P}{\partial x_{12}}(x_{11}^j, x_{12}^j)\frac{\partial x_{12}}{\partial R_1} \\ - & \frac{\partial f^P}{\partial x_{21}}(x_{21}^j, x_{22}^j)\frac{\partial x_{21}}{\partial R_2} - \frac{\partial f^P}{\partial x_{22}}(x_{21}^j, x_{22}^j)\frac{\partial x_{22}}{\partial R_2}. \end{aligned} \tag{12}$$

In this expression, j represents the type of the agent(s) to whom the principal will give the budget. Again, because the utility function is homothetic, in (12),

$$\frac{\partial x_{1i}}{\partial R_1} = \frac{\partial x_{2i}}{\partial R_2} = const., \ i = 1, \ 2. \tag{13}$$

If either agent is an H-type, the principal can allocate the whole budget to the type-H agent(s); then, the conditions for the optimal allocation by the agent in each period, and for the principal's optimal intertemporal allocation of the entire budget, (10) and (12), attain the first-best allocation. Otherwise, the principal allocates the entire budget to a type-L agent.

Example. Let an agent of type $j = H, L$, have utility $v_t^j = f^j(x_{t1}, x_{t2}) = (\alpha^j x_{t1}^\rho + (1-\alpha^j)x_{t2}^\rho)^{\frac{1}{\rho}}$, $0 < \alpha^L < \alpha^H < 1$, $0 < \rho < 1$. Then (3), (4), and (5) hold. To maximize his utility, an agent of type j with budget R_t^j spends on service 1

$$x_{t1} = \frac{\alpha^{j\left(\frac{1}{1-\rho}\right)}}{\alpha^{j\left(\frac{1}{1-\rho}\right)} + (1-\alpha^j)^{\left(\frac{1}{1-\rho}\right)}} R_t^j = \frac{1}{1 + \left(\frac{1-\alpha^j}{\alpha^j}\right)^{\left(\frac{1}{1-\rho}\right)}} R_t^j, \tag{14}$$

and on service 2

$$x_{t2} = \frac{(1-\alpha^j)^{\left(\frac{1}{1-\rho}\right)}}{\alpha^{j\left(\frac{1}{1-\rho}\right)} + (1-\alpha^j)^{\left(\frac{1}{1-\rho}\right)}} R_t^j = \frac{1}{1 + \left(\frac{\alpha^j}{1-\alpha^j}\right)^{\left(\frac{1}{1-\rho}\right)}} R_t^j. \tag{15}$$

The CES function captures substitutability between two services; a higher ρ means a higher substitutability. For instance, in the following representative cases, decisions by agent of type j with $\alpha^j > 1/2$ in period t are:

- $\rho \to 1$ (perfect substitutes): $x_{t1} \to R_t^j$, $x_{t2} \to 0$;[4]
- $\rho \to 0$ (Cobb-Douglas): $x_{t1} \to \alpha^j R_t^j$, $x_{t2} \to (1-\alpha^j) R_t^j$.

The principal's utility is

$$v_t^P = f^P(x_{t1}^A + x_{t1}^B, x_{t2}^A + x_{t2}^B) = \left(\alpha^H (x_{t1}^A + x_{t1}^B)^\rho + (1-\alpha^H)(x_{t2}^A + x_{t2}^B)^\rho\right)^{\frac{1}{\rho}}.$$

This CES function is homogeneous of degree 1 with respect to R_t^j; a principal who knows the agents' types is indifferent about the intertemporal allocation. Therefore, in equilibrium,

$$\begin{array}{lll} R_1^H + R_2^H = R, & R_1^L + R_2^L = 0, & \text{if one agent's type is } H \text{ and the other's type is } L; \\ R_1^H + R_2^H = R/2, & R_1^L + R_2^L = 0, & \text{if both agents have type } H; \\ R_1^H + R_2^H = 0, & R_1^L + R_2^L = R/2, & \text{if both agents have type } L. \end{array} \tag{16}$$

4.2 Imperfect information about agents' preferences

Let each agent's type be private information. Each agent knows that the principal's type is H, but the principal is unsure about the agents' types; moreover, each agent is uncertain about the other agent's type. The timing of the game is as follows. In stage 1:

1. Nature determines the types of the principal and of the agents.
2. The principal allocates the budget R_1 to agents A and B as R_1^A and R_1^B.
3. Each agent k simultaneously and independently allocates his budget R_1^k between the two services.

[4] To keep the quasi-concavity of f^j, we take the limit of f^j instead of substituting $q = 1$ into f^j.

In period 2:

1. The principal updates his beliefs about each agent's type.
2. The principal allocates the budget R_2 to agents A and B as R_2^A and R_2^B.
3. Each agent k simultaneously and independently allocates his budget R_2^k between the two services.

We are interested in each agent's strategic behavior in period 1 to affect the principal's beliefs, and in how the principal can discover the agents' preferences. We shall say that an agent acts sincerely if he ignores signaling, choosing to provide the service in period 1 that maximizes his utility in period 1. When an agent is known to act sincerely, the principal learns the agent's type. An agent is said to act strategically in period 1 when he cares about signaling, providing the service in period 1 that he believes the principal prefers, even if that does not maximize the agent's utility in period 1. If an agent acts strategically, and the principal knows that he does, the principal does not learn the agent's type.

In period 2, each agent acts sincerely, allocating his budget according to the first-order condition (10). Therefore, from the decreasing marginal rate of substitution in (5) and the single-crossing property given in (2), the principal allocates the budget R_2, namely $R - R_1$, to an agent whose preferences are more likely to be strictly closer to his compared to another agent, if any. Otherwise, the principal gives half of the budget R_2 to each agent. The posterior probability that agent k's type is H is $\tilde{\pi}_H^k$. Then the principal sets

$$\begin{array}{lll} R_2^k = R - R_1, & R_2^{k'} = 0, & \text{if} \;\; \tilde{\pi}_H^k > \tilde{\pi}_H^{k'}; \\ R_2^k = \frac{R-R_1}{2}, & R_2^{k'} = \frac{R-R_1}{2}, & \text{if} \;\; \tilde{\pi}_H^k = \tilde{\pi}_H^{k'}. \end{array} \tag{17}$$

Consider an agent's behavior in the final stage in period 1. Indicate an equilibrium by superscript e. We first examine an agent's choice in an equilibrium with agents acting sincerely, thereby revealing their types, satisfying $(x_{11}^{H^e}, x_{12}^{H^e}) \neq (x_{11}^{L^e}, x_{12}^{L^e})$, $\tilde{\pi}_H^k(x_{11}^{H^e}, x_{12}^{H^e}) = 1$, and $\tilde{\pi}_H^k(x_{11}^{L^e}, x_{12}^{L^e}) = 0$. Given $\tilde{\pi}_H^k(x_{11}^{L^e}, x_{12}^{L^e}) = 0$, a type-$L$ agent should choose $(x_{11}^{L^e}, x_{12}^{L^e}) = (x_{11}^L, x_{12}^L)$. Assume that the principal's beliefs at the decision nodes in the information set off the equilibrium path satisfy $\tilde{\pi}_H^k(x_{11}, x_{12}) = 0$ for $(x_{11}, x_{12}) \neq (x_{11}^{H^e}, x_{12}^{H^e})$. Then the necessary conditions for an equilibrium with agents acting sincerely (and so revealing their types) are

$$\begin{aligned} & f^H(x_{11}^{H^e}, x_{12}^{H^e}) + Ef^H(x_{21}^H(R_2^H), x_{22}^H(R_2^H)) \\ \geq \;& f^H(x_{11}^H, x_{12}^H) + Ef^H(x_{21}^H(R_2^L), x_{22}^H(R_2^L)); \end{aligned} \tag{18}$$

$$\begin{aligned} & f^L(x_{11}^{H^e}, x_{12}^{H^e}) + Ef^L(x_{21}^L(R_2^H), x_{22}^L(R_2^H)) \\ \leq \;& f^L(x_{11}^L, x_{12}^L) + Ef^L(x_{21}^L(R_2^L), x_{22}^L(R_2^L)), \end{aligned} \tag{19}$$

where E denotes the expectation operator. Expectation is taken over possible sizes of the budget the principal gives the agent, which depends on the principal's posterior beliefs about another agent's type (see (17)), and is therefore stochastic. Previously R_t^j, $j = H, L$ was defined as the budget given to a type-j agent in period t, under the assumption of perfect information. In this section, the budget allocation by the principal clearly depends on his posterior beliefs: in period 2 he gives R_2^H to the agent whose type has the posterior probability $\tilde{\pi}_H^k = 1$, and gives R_2^L to the agent for whom $\tilde{\pi}_H^k = 0$.

There can be multiple equilibria. For example, the principal may believe that an agent who does the opposite of what the principal prefers has the same preferences as the principal. To obtain solutions which are comparable with solutions under perfect information, we suppose that if an agent does not spend in the way the principal prefers, then the principal believes the agent is the opposite of his type. This assumption is consistent with the principal's beliefs at the decision nodes in the information set off the equilibrium path $\tilde{\pi}_H^k(x_{11}, x_{12}) = 0$ for $(x_{11}, x_{12}) \neq (x_{11}^H, x_{12}^H)$.

In period 1, the principal, uncertain about the two agents' types, sets the budget R_1 and allocates it between agent A and agent B according to his prior beliefs. Assume that in equilibrium agents of the same type take the same actions; we can say that the equilibrium is symmetric. Under this assumption of symmetric behavior, it is natural to let the principal divide his budget in period 1 equally between the agents. As we concentrate on the equilibrium including $(x_{11}^{H^e}, x_{12}^{H^e}) = (x_{11}^H, x_{12}^H)$, (18) and (19) are rewritten as

$$\begin{aligned} & f^H\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) \\ & +\frac{1}{2}f^H\left(x_{21}^H\left(\frac{R-R_1}{2}\right), x_{22}^H\left(\frac{R-R_1}{2}\right)\right) + \frac{1}{2}f^H\left(x_{21}^H(R-R_1), x_{22}^H(R-R_1)\right) \\ \geq \;& f^H\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) \end{aligned}$$

$$+\frac{1}{2}f^H\left(x_{21}^H(0),x_{22}^H(0)\right)+\frac{1}{2}f^H\left(x_{21}^H\left(\frac{R-R_1}{2}\right),x_{22}^H\left(\frac{R-R_1}{2}\right)\right); \quad (20)$$

$$\begin{aligned}
& f^L\left(x_{11}^H\left(\frac{R_1}{2}\right),x_{12}^H\left(\frac{R_1}{2}\right)\right) \\
& +\frac{1}{2}f^L\left(x_{21}^L\left(\frac{R-R_1}{2}\right),x_{22}^L\left(\frac{R-R_1}{2}\right)\right)+\frac{1}{2}f^L\left(x_{21}^L(R-R_1),x_{22}^L(R-R_1)\right) \\
\leq\; & f^L\left(x_{11}^L\left(\frac{R_1}{2}\right),x_{12}^L\left(\frac{R_1}{2}\right)\right) \\
& +\frac{1}{2}f^L\left(x_{21}^L(0),x_{22}^L(0)\right)+\frac{1}{2}f^L\left(x_{21}^L\left(\frac{R-R_1}{2}\right),x_{22}^L\left(\frac{R-R_1}{2}\right)\right). \quad (21)
\end{aligned}$$

The second term on each side of the inequality represents an agent's utility (say, of agent A) when another agent (say, agent B) is an H-type; the third term represents an agent's utility when another agent is an L-type. In a revealing equilibrium, the principal chooses R_1 in period 1 to maximize his utility (8) subject to (20) and (21).

It is straightforward to confirm that (20) holds; under imperfect information about the agents' preferences, a type-H agent enjoys an informational rent. We assume the following relation between an agent's incentive to act sincerely and the size of his budget.

Assumption 2 *For $j=H,L$, $j'=H,L$, $j\neq j'$, and $0\leq R_1^j\leq R$,*

$$\frac{\partial}{\partial R_1^j}\left(f^j\left(x_{11}^{j'}\left(R_1^j\right),x_{12}^{j'}\left(R_1^j\right)\right)-f^j\left(x_{11}^{j}\left(R_1^j\right),x_{12}^{j}\left(R_1^j\right)\right)\right)<0. \quad (22)$$

This assumption assures that the larger an agent's budget in period 1, the more he benefits from acting sincerely, thereby revealing his type.

Definition 1 *Use the assumption that an agent's incentive to act sincerely increases with his budget (Assumption 2) to define $\bar{R}_1^r\in(0,R)$ as the critical value of the budget in period 1 that induces an L-type agent to act sincerely, thereby revealing his type. That is (21) holds if and only if $R_1\geq\bar{R}_1^r$.*

Thus the principal can induce a type-L agent, if any, to reveal his type, by giving a budget in period 1 of at least $\bar{R}_1^r$.

Moreover, $((x_{11}^{H^e},x_{12}^{H^e}),(x_{11}^{L^e},x_{12}^{L^e}))$ satisfying $(x_{11}^e,x_{12}^e)=(x_{11}^{H^e},x_{12}^{H^e})=(x_{11}^{L^e},x_{12}^{L^e})$, given $\tilde{\pi}_H^k(x_{11}^e,x_{12}^e)=\pi_H^k$, $k=A$, B, constitutes a non-revealing equilibrium, where a type-L agent acts strategically, spending in a way that makes him indistinguishable from a type-H agent. With the off-the-equilibrium beliefs such that $\tilde{\pi}_H^k(x_{11}, x_{12}) = 0$ for $(x_{11}, x_{12}) \neq (x_{11}^e, x_{12}^e)$, the necessary conditions for the existence of a non-revealing equilibrium are

$$\begin{aligned} & f^H(x_{11}^e, x_{12}^e) + f^H(x_{21}^H(R_2^H), x_{22}^H(R_2^H)) \\ \geq\ & f^H(x_{11}^H, x_{12}^H) + f^H(x_{21}^H(R_2^L), x_{22}^H(R_2^L)); \end{aligned} \tag{23}$$

$$\begin{aligned} & f^L(x_{11}^e, x_{12}^e) + f^L(x_{21}^L(R_2^H), x_{22}^L(R_2^H)) \\ \geq\ & f^L(x_{11}^L, x_{12}^L) + f^L(x_{21}^L(R_2^L), x_{22}^L(R_2^L)). \end{aligned} \tag{24}$$

In (23) and (24), the principal gives R_2^H to the agent whose type has the posterior probability $\tilde{\pi}_H^k = \pi_H^k$, and gives R_2^L to the agent for whom $\tilde{\pi}_H^k = 0$. Again, to focus on the solutions comparable with the equilibrium under perfect information, assume that the principal's beliefs at the decision nodes in the information set off the equilibrium path are confined to $\tilde{\pi}_H^k(x_{11}, x_{12}) = 0$ for $(x_{11}, x_{12}) \neq (x_{11}^H, x_{12}^H)$. So we can concentrate on the equilibrium including $(x_{11}^e, x_{12}^e) = (x_{11}^H, x_{12}^H)$. From the assumption of a symmetric equilibrium, (23) and (24) are rewritten as

$$\begin{aligned} & f^H\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) + f^H\left(x_{21}^H\left(\frac{R-R_1}{2}\right), x_{22}^H\left(\frac{R-R_1}{2}\right)\right) \\ \geq\ & f^H\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) + f^H\left(x_{21}^H(0), x_{22}^H(0)\right); \end{aligned} \tag{25}$$

$$\begin{aligned} & f^L\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) + f^L\left(x_{21}^L\left(\frac{R-R_1}{2}\right), x_{22}^L\left(\frac{R-R_1}{2}\right)\right) \\ \geq\ & f^L\left(x_{11}^L\left(\frac{R_1}{2}\right), x_{12}^L\left(\frac{R_1}{2}\right)\right) + f^L\left(x_{21}^L(0), x_{22}^L(0)\right). \end{aligned} \tag{26}$$

Thus, in a non-revealing equilibrium, in period 1 both agents provide the service the principal prefers, (x_{11}^H, x_{12}^H).

Definition 2 *Under the assumption that an agent's incentive to act sincerely increases with his budget (Assumption 2), define $\bar{R}_1^{nr} \in (0, R)$ such that R_1 satisfies (26) if and only if $R_1 \leq \bar{R}_1^{nr}$. That is, $\bar{R}_1^{nr}$ is the largest value of an L-type agent's budget in period 1 that induces him to behave strategically, thereby not revealing his type.*

We already assumed that the function f^j is homothetic (Assumption 1). We further impose homogeneity on f^j (and also on f^P), ensuring weak concavity in R_1.

Assumption 3 *The function f^j is homogeneous of degree n, $0 < n \leq 1$.*

Lemma 1 *Assumption 3 suffices for Assumptions 1 and 2 to hold.*

Proof of Lemma 1 See the Appendix.

The following lemma concerns the relation between the critical value of the budget in period 1 that induces an L-type agent in period 1 to reveal his type ($\bar{R}_1^r$), the critical value of the budget in period 1 that induces an L-type agent in period 1 to hide his type ($\bar{R}_1^{nr}$), and the principal's total budget (R).

Lemma 2 $\bar{R}_1^{nr} \geq R/2$. *Furthermore, under the assumption that the function f^j is homogeneous (Assumption 3), if it is homogeneous of degree $0 < n < 1$ then $\bar{R}_1^r < \bar{R}_1^{nr}$; if it is homogeneous of degree 1 then $\bar{R}_1^r = \bar{R}_1^{nr}$.*

Proof of Lemma 2 See the Appendix.

If both agents behave strategically, allocating their budgets as the principal prefers, each agent knows the budget he will get in period 2, and faces no risk. If the function f^j is homogeneous of degree 1, and hence, agents' indirect utility is linear in the budget, the agents are risk-neutral and then $\bar{R}_1^r = \bar{R}_1^{nr}$. If f^j is homogeneous of degree less than 1, agents' indirect utility is strictly concave in the budget, and hence, they are risk-averse. Then $\bar{R}_1^{nr}$ differs from $\bar{R}_1^r$ (see Figure 1 and Figure 2).

Now examine the principal's choice in period 1. He allocates his budget R over two periods to maximize his utility (8). The equilibrium can have either sincere or strategic behavior by the agents. From (11), in an equilibrium where agents act sincerely, revealing their types, the principal should set R_1 to maximize his expected utility

$$\begin{aligned} V^{Pr}(R_1) &= 1/4 f^P\left(x_{11}^H(R_1), x_{12}^H(R_1)\right) \\ &+ 1/4 f^P\left(x_{11}^L(R_1), x_{12}^L(R_1)\right) \\ &+ 1/2 f^P\left((x_{11}^H + x_{11}^L)/2(R_1), (x_{12}^H + x_{12}^L)/2(R_1)\right) \\ &+ 3/4 f^P\left(x_{21}^H(R - R_1), x_{22}^H(R - R_1)\right) \\ &+ 1/4 f^P\left(x_{21}^L(R - R_1), x_{22}^L(R - R_1)\right), \end{aligned} \tag{27}$$

subject to (20) and (21). In an equilibrium with strategic behavior (so that agents do not reveal their types), the principal chooses R_1 to maximize

$$\begin{aligned} V^{Pnr}(R_1) &= f^P\left(x_{11}^H(R_1), x_{12}^H(R_1)\right) \\ &+ 1/4 f^P\left(x_{21}^H(R-R_1), x_{22}^H(R-R_1)\right) \\ &+ 1/2 f^P\left((x_{21}^H+x_{21}^L)/2(R-R_1), (x_{22}^H+x_{22}^L)/2(R-R_1)\right) \\ &+ 1/4 f^P\left(x_{21}^L(R-R_1), x_{22}^L(R-R_1)\right), \end{aligned} \tag{28}$$

subject to the conditions that make the agents behave strategically (namely (25) and (26)). The solution to each of these problems can be explored by solving an unconstrained problem maximizing (27) or (28) with respect to R_1 and then examining whether this solution satisfies the conditions that the agents behave sincerely or strategically, namely, conditions (21) or (26) (note that (20) and (25) hold for any $R_1 \in [0, R]$). Thus the following lemma is derived using Definitions 1 and 2.

Lemma 3 *Call the budget in period 1 which maximizes (27), or the budget which maximizes the principal's utility when agents act sincerely (thereby revealing their types), R_1^r; the budget in period 1 which maximizes (28), or the budget which maximizes the principal's utility when agents act strategically (thereby not revealing their types), is R_1^{nr}. Subject to the assumption that the utility function f^j is homogeneous of degree n, $0 < n \leq 1$ (Assumption 3), $R_1^r < R/2 < R_1^{nr}$. If an equilibrium exists with revelation of types, it includes $R_1^e = \max[R_1^r, \bar{R}_1^r]$; if a non-revealing equilibrium exists, it includes $R_1^e = \min[R_1^{nr}, \bar{R}_1^{nr}]$.*

Proof of Lemma 3 See the Appendix.

Lemma 3 suggests that the budget in period 1 making a type-L agent reveal his type should be large, although the principal wants to leave a large budget for period 2; he would then allocate the budget in period 2 to the agents having the same preferences as his. Also, the budget in period 1 inducing a type-L agent to act strategically (hiding his type) should be small, leaving a large budget for period 2, although the principal wanted to spend a large budget in period 1: then an agent would behave according to the principal's preferences.

The following lemma compares the principal's utility with the solutions R_1^r and R_1^{nr}.

Lemma 4 *Under imperfect information about agents' preferences, if the conditions for an L-type agent to behave sincerely or strategically were not binding, the principal has higher utility when agents act strategically than when they act sincerely, or* $V^{Pnr}(R_1^{nr}) > V^{Pr}(R_1^r)$.

Proof of Lemma 4 See the Appendix.

Lemma 4 suggests that if the solution to the unconstrained maximization problem (28) did not make the incentive-compatibility constraint (26) bind for a type-L agent, the principal would benefit from agents acting strategically, and so not revealing their types. The principal may or may not induce revelation to maximize his welfare, considering (26). Based on Lemma 4, we can now determine the principal's decision in period 1.

Proposition 1 *Assume that the principal is uncertain about the agents' types and that the utility function* f^j *is homogeneous of degree* n*, with* $0 < n \leq 1$ *(Assumption 3 which also applies to* f^P*). Assume that if an agent does not spend in the way the principal prefers, then the principal believes the agent's type is the opposite of the principal's type. Let*

- V^{Pr} *be the principal's utility when agents act sincerely (thereby revealing their types);*
- R_1^r *be the budget in period 1 which maximizes* V^{Pr}*;*
- $\overline{R}_1^r$ *be the smallest budget for the two agents in period 1 which would induce a type-L agent to act sincerely.*

Let

- V^{Pnr} *be the principal's utility when agents act strategically (thereby hiding their types);*
- R_1^{nr} *be the budget in period 1 which maximizes* V^{Pnr}*;*
- $\overline{R}_1^{nr}$ *be the largest budget given two agents in period 1 which would induce a type-L agent to act strategically.*

If $R_1^{nr} \leq \overline{R}_1^{nr}$,

- *a non-revealing equilibrium exists in which the principal chooses a budget in period 1 of* R_1^{nr}*;*

If $R_1^{nr} > \overline{R}_1^{nr}$ *and* $\overline{R}_1^{nr} = \max[R_1^r, \overline{R}_1^r]$,

- *there exist revealing and non-revealing equilibria in which the principal chooses a budget in period 1 of* $\bar{R}_1^{nr} = \max[R_1^r, \overline{R}_1^r]$*;*

If $R_1^{nr} > \overline{R}_1^{nr}$ *and* $\overline{R}_1^{nr} \neq \max[R_1^r, \overline{R}_1^r]$,

- *if* $V^{Pnr}(\overline{R}_1^{nr}) > V^{Pr}(\max[R_1^r, \overline{R}_1^r])$*, there exists a non-revealing equilibrium (in which the principal chooses a budget in period 1 of* $\overline{R}_1^{nr}$*);*
- *if* $V^{Pnr}(\overline{R}_1^{nr}) < V^{Pr}(\max[R_1^r, \overline{R}_1^r])$*, there exists an equilibrium with revelation of types (in which the principal chooses a budget of* $\max[R_1^r, \overline{R}_1^r]$*);*
- *if* $V^{Pnr}(\overline{R}_1^{nr}) = V^{Pr}(\max[R_1^r, \overline{R}_1^r])$*, two equilibria exist; one equilibrium has agents behave strategically, not revealing their types, with the budget in period 1 of* $\overline{R}_1^{nr}$*; another equilibrium has agents behaving sincerely, revealing their types, with the budget in period 1* $\max[R_1^r, \overline{R}_1^r]$.

Thus, when the agents know the principal's preferences, the principal benefits from making a type-L agent behave strategically.

Example. *Continued.* Let $v_t^j = f^j(x_{t1}, x_{t2}) = (\alpha^j x_{t1}^\rho + (1-\alpha^j)x_{t2}^\rho)^{\frac{1}{\rho}}$, $j = H$, L, and $v_t^P = f^P(x_{t1}^A + x_{t1}^B, x_{t2}^A + x_{t2}^B) = (\alpha^H (x_{t1}^A + x_{t1}^B)^\rho + (1-\alpha^H)(x_{t2}^A + x_{t2}^B)^\rho)^{\frac{1}{\rho}}$, $0 < \alpha^L < \alpha^H < 1$, $0 < \rho < 1$. The principal's utility with agents behaving sincerely ((27)) is maximized by $R_1^r = 0$; the principal's utility with agents acting strategically ((28)) is maximized by $R_1^{nr} = R$. This CES function is homogeneous of degree 1, and hence, from Lemma 2, we should have $\bar{R}_1^r = \bar{R}_1^{nr}$; in equilibrium, the principal chooses $R_1^e = \bar{R}_1^r = \bar{R}_1^{nr}$.

In particular, define

$$
\begin{aligned}
A_1^m &\equiv \frac{\alpha^{m\left(\frac{1}{1-\rho}\right)}}{\alpha^{m\left(\frac{1}{1-\rho}\right)} + (1-\alpha^m)^{\left(\frac{1}{1-\rho}\right)}};\ A_2^m \equiv \frac{(1-\alpha^m)^{\left(\frac{1}{1-\rho}\right)}}{\alpha^{m\left(\frac{1}{1-\rho}\right)} + (1-\alpha^m)^{\left(\frac{1}{1-\rho}\right)}}; \\
\Psi_m^j &\equiv \left(\alpha^j (A_1^m)^\rho + (1-\alpha^j)(A_2^m)^\rho\right)^{\frac{1}{\rho}}; \\
\Psi_{HL}^j &\equiv \left(\alpha^j \left(\frac{A_1^H + A_1^L}{2}\right)^\rho + (1-\alpha^j)\left(\frac{A_2^H + A_2^L}{2}\right)^\rho\right)^{\frac{1}{\rho}}. \qquad (29)
\end{aligned}
$$

In (29), A_i^m is associated with spending on service i by a type-m agent; Ψ_m^j is associated with a type-j agent's utility from a type-m agent's budget allocation; Ψ^j_{HL} can be interpreted analogously. Then (21) and (26) yield

$$R^e_1 = \bar{R}^r_1 = \bar{R}^{nr}_1 = \frac{\Psi^L_L}{2\Psi^L_L - \Psi^L_H} R = \frac{1}{2 - \frac{\Psi^L_H}{\Psi^L_L}} R. \tag{30}$$

Once given R^e_1, a type-L agent is indifferent between behaving sincerely and strategically. There exist multiple equilibria. From (27) and (28), the principal's expected utility is

$$\begin{aligned} V^{Pr}(R^e_1) &= \left(\frac{1}{4}\Psi^H_H + \frac{1}{4}\Psi^H_L + \frac{1}{2}\Psi^H_{HL}\right) R^e_1 + \left(\frac{3}{4}\Psi^H_H + \frac{1}{4}\Psi^H_L\right)(R - R^e_1); \\ V^{Pnr}(R^e_1) &= \Psi^H_H R^e_1 + \left(\frac{1}{4}\Psi^H_H + \frac{1}{4}\Psi^H_L + \frac{1}{2}\Psi^H_{HL}\right)(R - R^e_1). \end{aligned} \tag{31}$$

The inequalities $R^e_1 > R/2 > R - R^e_1$ and $\Psi^H_H > \Psi^H_{HL} > \Psi^H_L$ imply that $V^{Pnr} > V^{Pr}$.

For intuition, suppose $\alpha^L < 1/2 < \alpha^H$ and let $q \to 1$. We can show that (29) converges to

$$\begin{aligned} &A^H_1 \to 1;\ A^H_2 \to 0;\ A^L_1 \to 0;\ A^L_2 \to 1; \\ &\Psi^H_H \to \alpha^H;\ \Psi^H_L \to 1 - \alpha^H;\ \Psi^H_{HL} \to \frac{1}{2};\ \Psi^L_H \to \alpha^L;\ \Psi^L_L \to 1 - \alpha^L; \\ &R^e_1 \to \frac{1 - \alpha^L}{2 - 3\alpha^L} R;\ R^e_2 \to \frac{1 - 2\alpha^L}{2 - 3\alpha^L} R; \\ &V^{Pr} \to \frac{3 + 2\alpha^H - 4\alpha^L - 4\alpha^H\alpha^L}{4(2 - 3\alpha^L)} R; \\ &V^{Pnr} \to \frac{1 + 2\alpha^H - 2\alpha^L - 2\alpha^H\alpha^L}{2(2 - 3\alpha^L)} R. \end{aligned} \tag{32}$$

When two services are almost perfect substitutes, each agent spends almost all his budget on the service with greater weight in his utility function. Because of the assumption that $\alpha^H > 1/2$, the limit of V^{Pnr} is greater than the limit of V^{Pr}.

Strategic behavior in period 1 has all agents behave the way the principal prefers, even if the principal would be better off were agents to perform sincerely, revealing their types and thus allowing for better resource allocation in period 2. Such strategic behavior resembles, or offers another explanation for, herd behavior. In particular, Prendergast (1993) shows that when advisors want the principal to think highly of themselves, they have an incentive to conform to the principal's opinion, behaving as "yes men." The mechanism we discuss can yield similar outcomes.

4.3 Imperfect information about the preferences of the principal and agents

Consider next a principal who can hide his preferences. The principal's type can be H or L. The two agents are uncertain about the principal's type; they only know the prior probability π_H^P that the principal's type is H. The game proceeds as follows. In period 1:

1. Nature determines the types of the principal and of the agents.
2. The principal allocates the budget R_1 to agents A and B as R_1^A and R_1^B.
3. Each agent k simultaneously and independently allocates the budget R_1^k between the two services.

In period 2:

1. The principal updates his beliefs about each agent's type.
2. The principal allocates the budget R_2 to agents A and B as R_2^A and R_2^B.
3. Each agent k simultaneously and independently allocates his budget R_2^k between the two services.

Looking at the game backwards, we can apply the logic used in Section 4.2 up to the final stage in period 1: an agent who wants to get a large budget in period 2 may behave as if his preferences were closer to the principal's. The agents, however, are uncertain about the principal's type.

The principal gives each agent a budget $R_1/2$ in period 1, as discussed in Section 4.2. The following conditions are necessary for the existence of an equilibrium in which agents reveal their types, including $(x_{11}^{H^e}, x_{12}^{H^e}) = (x_{11}^H, x_{12}^H)$:

$$f^H\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right)$$

$$
\begin{aligned}
&+\frac{\pi_H^P}{2} f^H\left(x_{21}^H\left(\frac{R-R_1}{2}\right), x_{22}^H\left(\frac{R-R_1}{2}\right)\right)+\frac{\pi_H^P}{2} f^H\left(x_{21}^H(R-R_1), x_{22}^H(R-R_1)\right) \\
&+\frac{1-\pi_H^P}{2} f^H\left(x_{21}^H\left(\frac{R-R_1}{2}\right), x_{22}^H\left(\frac{R-R_1}{2}\right)\right)+\frac{1-\pi_H^P}{2} f^H\left(x_{21}^H(0), x_{22}^H(0)\right) \\
\geq\ & f^H\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) \\
&+\frac{\pi_H^P}{2} f^H\left(x_{21}^H(0), x_{22}^H(0)\right)+\frac{\pi_H^P}{2} f^H\left(x_{21}^H\left(\frac{R-R_1}{2}\right), x_{22}^H\left(\frac{R-R_1}{2}\right)\right) \\
&+\frac{1-\pi_H^P}{2} f^H\left(x_{21}^H(R-R_1), x_{22}^H(R-R_1)\right) \\
&+\frac{1-\pi_H^P}{2} f^H\left(x_{21}^H\left(\frac{R-R_1}{2}\right), x_{22}^H\left(\frac{R-R_1}{2}\right)\right); \qquad (33)
\end{aligned}
$$

$$
\begin{aligned}
& f^L\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) \\
&+\frac{\pi_H^P}{2} f^L\left(x_{21}^L\left(\frac{R-R_1}{2}\right), x_{22}^L\left(\frac{R-R_1}{2}\right)\right)+\frac{\pi_H^P}{2} f^L\left(x_{21}^L(R-R_1), x_{22}^L(R-R_1)\right) \\
&+\frac{1-\pi_H^P}{2} f^L\left(x_{21}^L\left(\frac{R-R_1}{2}\right), x_{22}^L\left(\frac{R-R_1}{2}\right)\right)+\frac{1-\pi_H^P}{2} f^L\left(x_{21}^L(0), x_{22}^L(0)\right) \\
\leq\ & f^L\left(x_{11}^L\left(\frac{R_1}{2}\right), x_{12}^L\left(\frac{R_1}{2}\right)\right) \\
&+\frac{\pi_H^P}{2} f^L\left(x_{21}^L(0), x_{22}^L(0)\right)+\frac{\pi_H^P}{2} f^L\left(x_{21}^L\left(\frac{R-R_1}{2}\right), x_{22}^L\left(\frac{R-R_1}{2}\right)\right) \\
&+\frac{1-\pi_H^P}{2} f^L\left(x_{21}^L(R-R_1), x_{22}^L(R-R_1)\right) \\
&+\frac{1-\pi_H^P}{2} f^L\left(x_{21}^L\left(\frac{R-R_1}{2}\right), x_{22}^L\left(\frac{R-R_1}{2}\right)\right). \qquad (34)
\end{aligned}
$$

The second, third, fourth, and fifth terms on each side of the inequality respectively represent an agent's utility (say, agent A) when the principal is an H-type and another agent (say, agent B) is an H-type; when the principal is an H-type and another agent is an L-type; when the principal is an L-type and another agent is an H-type; and when the principal is an L-type and another agent is an L-type. Thus, in this subsection, an agent considers the principal's type and considers another agent's possible type. Note that if $\pi_H^P = 1$, (33) and (34) coincide with the necessary conditions for agents revealing their types when the principal is known to have type H.

Expressions (33) and (34) show how concealing the principal's preferences affects agents' behavior. For intuition assume that

Assumption 4 *$\pi_H^P = 1/2$.*

Section 5 examines outcomes when π_H^P differs from $\pi_H^A = \pi_H^B = 1/2$.

The action of a type-H agent is described by (33). Under the assumption that $\pi_H^P = 1/2$, (33) holds. Consider next the action of a type-L agent.

Lemma 5 *Under the assumption that $\pi_H^P = 1/2$ (Assumption 4), (34) holds for $R_1 \in [0, R]$.*

Lemma 6 *Under imperfect information about the principal's and agents' preferences, subject to Assumption 4 that $\pi_H^P = 1/2$, if there exists an equilibrium with agents revealing their types, then the principal chooses a budget in period 1 of $R_1^e = R_1^r$.*

As Lemma 6 shows, to induce agents to reveal their types the principal may give a smaller budget in period 1 when he hides his type than when the agents know his type.

On the other hand, the following conditions are necessary for a non-revealing equilibrium, where a type-L agent pretends to have type H:

$$\begin{aligned}
& f^H\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) \\
& +\pi_H^P f^H\left(x_{21}^H\left(\frac{R-R_1}{2}\right), x_{22}^H\left(\frac{R-R_1}{2}\right)\right) + (1-\pi_H^P) f^H\left(x_{21}^H\left(\frac{R-R_1}{2}\right), x_{22}^H\left(\frac{R-R_1}{2}\right)\right) \\
\geq\ & f^H\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) \\
& +\pi_H^P f^H\left(x_{21}^H(0), x_{22}^H(0)\right) + (1-\pi_H^P) f^H\left(x_{21}^H(R-R_1), x_{22}^H(R-R_1)\right);
\end{aligned} \tag{35}$$

$$\begin{aligned}
& f^L\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) \\
& +\pi_H^P f^L\left(x_{21}^L\left(\frac{R-R_1}{2}\right), x_{22}^L\left(\frac{R-R_1}{2}\right)\right) + (1-\pi_H^P) f^L\left(x_{21}^L\left(\frac{R-R_1}{2}\right), x_{22}^L\left(\frac{R-R_1}{2}\right)\right) \\
\geq\ & f^L\left(x_{11}^L\left(\frac{R_1}{2}\right), x_{12}^L\left(\frac{R_1}{2}\right)\right) \\
& +\pi_H^P f^L\left(x_{21}^L(0), x_{22}^L(0)\right) + (1-\pi_H^P) f^L\left(x_{21}^L(R-R_1), x_{22}^L(R-R_1)\right).
\end{aligned} \tag{36}$$

Note that the second term on each side of the inequality is associated with the principal having type H; the third term is associated with him having type L.

Lemma 7 *Under the assumption that an agent's incentive to act sincerely increases with his budget (Assumption 2), for* $0 \leq \pi_H^P \leq 1$ *no* $R_1 > \overline{R}_1^{nr}$ *satisfies (36).*

Proof of Lemma 7 See the Appendix.

Lemma 8 *Under imperfect information about the principal's and agents' preferences, subject to Assumption 2 and Assumption 4 that* $\pi_H^P = 1/2$*, if there exists a non-revealing equilibrium, it never includes* $R_1 > \overline{R}_1^{nr}$*.*

Lemmas 5 to 8 suggest that under imperfect information about the principal's and the agents' preferences, the principal may find it easy to induce sincere behavior by agents. Note that Lemma 7 holds for any $0 \leq \pi_H^P \leq 1$. The analysis is later extended on the basis of Lemma 7.

5 Extensions

5.1 The principal may hide his preferences

Section 4.2 showed that the principal would induce the agents to provide the services he prefers in period 1 if he can do so without giving too large a budget in period 2. He can also induce agents to behave sincerely by hiding his preferences, as shown in Section 4.3. The principal then need not give a large budget in period 1, allowing him to give a larger budget in period 2. This section derives the conditions which would make the principal hide his preferences. We revise the timing of the game as follows. In period 1:

1. Nature determines the types of the principal and of the agents.
2. The principal decides whether to tell agents his type.
3. The principal allocates the budget R_1 to agents A and B as R_1^A and R_1^B.
4. Each agent k simultaneously and independently allocates the budget R_1^k between the two services.

In period 2:

1. The principal updates his beliefs about each agent's type.
2. The principal allocates the budget R_2 to agents A and B as R_2^A and R_2^B.
3. Each agent k simultaneously and independently allocates the budget R_2^k between the two services.

Looking at the game backwards, we can apply the results derived in Sections 4.2 and 4.3 up to stage 3 in period 1. In stage 2 in period 1, the principal will choose a strategy which gives him a higher utility.

Proposition 2 *Suppose that the principal is unsure about the agents' preferences, that the agents are uncertain about the principal's preferences, that the utility function f^j is homogeneous of degree n, $0 < n \leq 1$ (Assumption 3 which also applies to f^P), and that $\pi_H^P = 1/2$ (Assumption 4). Assume further that if an agent does not spend in the way the principal prefers, then the principal believes the agent is the opposite of his type. Let*

- *V^{Pr} be the principal's utility if the agents act sincerely (thereby revealing their types);*
- *R_1^r be the budget in period 1 which maximizes V^{Pr};*
- *$\overline{R}_1^r$ be the smallest budget given two agents in period 1 which would induce a type-L agent to act sincerely.*

Let

- *V^{Pnr} be the principal's utility if the agents act strategically (thereby hiding their types);*
- *R_1^{nr} be the budget in period 1 which maximizes V^{Pnr};*
- *$\overline{R}_1^{nr}$ be the largest budget for the two agents in period 1 which would induce a type-L agent to act strategically.*

Then if $R_1^{nr} > \bar{R}_1^{nr}$, $R_1^r < \bar{R}_1^r$, and $V^{Pr}(R_1^r) > V^{Pnr}(\overline{R}_1^{nr})$, the principal hides his preferences. In the equilibrium the principal chooses a budget in period 1 of R_1^r.

As Proposition 1 suggests, if $R_1^{nr} \leq \overline{R}_1^{nr}$, the principal does not benefit from hiding his preferences; by revealing his preferences he can induce the agents to act strategically, obtaining $V^{Pnr}(R_1^{nr})$. Lemma 5 shows that when the principal hides his type, the range of R_1 that can induce agents to act sincerely is wider than when agents know the principal's type. Therefore the principal can induce agents to reveal their types by giving a small budget in period 1 and a large budget in period 2, which may make the principal hide his preferences.

5.2 Generalizing the prior probability of the principal's type

The results in Section 5.1 were derived under the assumption that $\pi_H^P = 1/2$. Will these results hold even if the principal's preferences are more extreme than the preferences of the agents? Recall that Lemma 7, which says that the range of R_1 inducing a type-L agent to behave as the principal prefers, would not get wider even if the principal hides his preferences, holds with $0 \leq \pi_H^P \leq 1$. Therefore, the principal, hiding his preferences, cannot induce agents in period 1 to provide more of the service he prefers.

Lemma 7 and Proposition 2 suggest that the principal benefits from hiding his preferences only if he thereby induces agents to reveal their types. The necessary condition for a type-H agent to reveal his preferences ((33)) never holds for $\pi_H^P < 1/2$, since the probability is small that the principal has the same type as his. The critical value of R_1 which induces a type-L agent to reveal his preferences ((34)) strictly increases with π_H^P, implying a smaller budget left for period 2. Therefore, the principal benefits from hiding his preferences if π_H^P is close to $\pi_H^A = \pi_H^B = 1/2$.

5.3 Symmetric information between two agents

In Section 4.2 we assumed that each agent is uncertain about the other agent's preferences. Now consider how the results change if either agent knows another agent's type. If another agent is an H-type, under the assumption that the utility function f^j is homogeneous of degree n, $0 < n \leq 1$ (Assumption 3), the necessary condition for an L-type agent to reveal his type (21) becomes

$$
\begin{aligned}
& f^L\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right)+\left(\frac{1}{2}\right)^n f^L\left(x_{21}^L(R-R_1), x_{22}^L(R-R_1)\right) \\
\leq\ & f^L\left(x_{11}^L\left(\frac{R_1}{2}\right), x_{12}^L\left(\frac{R_1}{2}\right)\right), \qquad (37)
\end{aligned}
$$

while the corresponding necessary condition for an H-type agent still holds. Comparing with (21), when $n = 1$, the necessary condition for an L-type agent is unchanged; for $0 < n < 1$, the principal needs a bigger budget to induce an L-type agent to reveal his type, meaning that the critical level of the budget in period 1 $\bar{R}^r$ to make an agent reveal his type increases.

On the other hand, the necessary conditions for H-type and L-type agents to hide their preferences are unchanged, because then the principal cannot know the agent's preferences so that the budget allocation in period 2 is expected to be unchanged.

Then suppose that another agent is an L-type. The necessary condition for an L-type agent to hide his preferences (21) becomes

$$
\begin{aligned}
& f^L\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right)+\left(1-\left(\frac{1}{2}\right)^n\right) f^L\left(x_{21}^L\left(R-R_1\right), x_{22}^L\left(R-R_1\right)\right) \\
\leq\ & f^L\left(x_{11}^L\left(\frac{R_1}{2}\right), x_{12}^L\left(\frac{R_1}{2}\right)\right). \qquad (38)
\end{aligned}
$$

Again, comparing with (21), when the degree of homogeneity is $n = 1$, the necessary condition for an L-type agent is unchanged; for $0 < n < 1$, $\bar{R}^r$ decreases, implying that a symmetric revealing equilibrium may be induced. Thus, when an agent with preferences differing from the principal's knows that the other agent's preferences also differ from the principal's, he may prefer to reveal his preferences because after revealing, he need not worry about getting no budget in period 2.

5.4 Asymmetric allocations in period 1

So far we assumed a symmetric allocation in period 1. Indeed, the principal may be constrained to treat the agents the same in period 1. For example, it may be considered unfair or even unconstitutional for a central government to arbitrarily discriminate among jurisdictions which appear very similar.

After the agents provide services, the agents may be seen as different; so allocating more to one agent than to another can be justified.

If the principal is not so constrained, then in period 1 he may want to give unequal budgets to the agents. Assume that the agents know the principal's type, and suppose that the principal wants to ensure that his favored service is sufficiently provided in both periods. That preference will arise if the marginal utility (when evaluated at a very small level) of his favored service is very high in each period. Then the principal may benefit from an asymmetric solution. In period 1 he gives a large budget to one of the agents. That induces the agent to reveal his type, and so the principal will give that agent a large budget in period 2 if the agent shares the principal's type. The principal gives a small budget to the other agent in period 1. That induces the agent in period 1 to do what the principal favors, because the agent loses little by doing that.

Such behavior may look like pork barrel politics or special interest politics—the principal favors one agent (say a local jurisdiction) over another. Instead, given our results, the lobbying or the campaign contributions may decide not whether, but which of, the agents gets the larger budget in period 1.

We concentrate on the equilibrium including $(x_{11}^{H^e}, x_{12}^{H^e}) = (x_{11}^H, x_{12}^H)$. Then for agent A, the necessary conditions for revealing his type ((20) and (21)) are rewritten as

$$\begin{aligned} & f^H\left(x_{11}^H\left(R_1^A\right), x_{12}^H\left(R_1^A\right)\right) + f^H\left(x_{21}^H(R-R_1), x_{22}^H(R-R_1)\right) \\ \geq\ & f^H\left(x_{11}^H\left(R_1^A\right), x_{12}^H\left(R_1^A\right)\right) + f^H\left(x_{21}^H(0), x_{22}^H(0)\right); \end{aligned} \tag{39}$$

$$\begin{aligned} & f^L\left(x_{11}^H\left(R_1^A\right), x_{12}^H\left(R_1^A\right)\right) + f^L\left(x_{21}^L(R-R_1), x_{22}^L(R-R_1)\right) \\ \leq\ & f^L\left(x_{11}^L\left(R_1^A\right), x_{12}^L\left(R_1^A\right)\right) + f^L\left(x_{21}^L(0), x_{22}^L(0)\right). \end{aligned} \tag{40}$$

For agent B, the necessary conditions for hiding his type ((25) and (26)) are rewritten as

$$\begin{aligned} & f^H\left(x_{11}^H\left(R_1^B\right), x_{12}^H\left(R_1^B\right)\right) \\ & +1/2 f^H\left(x_{21}^H(0), x_{22}^H(0)\right) + 1/2 f^H\left(x_{21}^H(R-R_1), x_{22}^H(R-R_1)\right) \\ \geq\ & f^H\left(x_{11}^H\left(R_1^B\right), x_{12}^H\left(R_1^B\right)\right) \\ & +1/2 f^H\left(x_{21}^H(0), x_{22}^H(0)\right) \\ & +1/2 f^H\left(x_{21}^H\left(\frac{R-R_1}{2}\right), x_{22}^H\left(\frac{R-R_1}{2}\right)\right); \end{aligned} \tag{41}$$

$$
\begin{aligned}
& f^L\left(x_{11}^H\left(R_1^B\right), x_{12}^H\left(R_1^B\right)\right) \\
& +1/2 f^L\left(x_{21}^L(0), x_{22}^L(0)\right)+1/2 f^L\left(x_{21}^L\left(R-R_1\right), x_{22}^L\left(R-R_1\right)\right) \\
\geq\ & f^L\left(x_{11}^L\left(R_1^B\right), x_{12}^L\left(R_1^B\right)\right) \\
& +1/2 f^L\left(x_{21}^L(0), x_{22}^L(0)\right) \\
& +1/2 f^L\left(x_{21}^L\left(\frac{R-R_1}{2}\right), x_{22}^L\left(\frac{R-R_1}{2}\right)\right), \qquad (42)
\end{aligned}
$$

where the second term in each side of the inequality represents agent B's utility when another agent A is an H-type; the third term represents agent B's utility when another agent A is an L-type. It is straightforward to confirm that the conditions (39) and (41) that induce an H-type agent to behave sincerely or strategically hold; when the principal is initially uncertain about the agents' types, a type-H agent will enjoy an informational rent. In equilibrium the posterior probability that agent B is an H-type is $\pi_H^B = 1/2$. If agent A is an H-type and reveals his type, the principal gives all the budget in period 2 to agent A. If agent A is an L-type, the principal gives all the budget in period 2 to agent B.

Lemma 9 *Under imperfect information about the agents' preferences, subject to the assumption that an agent's incentive to act sincerely increases with his budget (Assumption 2), if there exists an equilibrium in which one agent (say A) reveals his type while another agent (say B) does not, then in period 1 the principal gives agent A a larger budget than he gives agent B.*

Proof of Lemma 9 See the Appendix.

Now examine the principal's choice in period 1. Subject to (39), (40), (41), and (42), he maximizes his utility

$$
\begin{aligned}
V^{Pas}(R_1, R_1^A) &= 1/2 f^P\left(x_{11}^H(R_1^A)+x_{11}^H(R_1-R_1^A), x_{12}^H(R_1^A)+x_{12}^H(R_1-R_1^A)\right) \\
&+ 1/2 f^P\left(x_{11}^L(R_1^A)+x_{11}^H(R_1-R_1^A), x_{12}^L(R_1^A)+x_{12}^H(R_1-R_1^A)\right) \\
&+ 3/4 f^P\left(x_{21}^H(R-R_1), x_{22}^H(R-R_1)\right) \\
&+ 1/4 f^P\left(x_{21}^L(R-R_1), x_{22}^L(R-R_1)\right) \\
&= 1/2 f^P\left(x_{11}^H(R_1), x_{12}^H(R_1)\right) \\
&+ 1/2 f^P\left(x_{11}^L(R_1^A)+x_{11}^H(R_1-R_1^A), x_{12}^L(R_1^A)+x_{12}^H(R_1-R_1^A)\right) \\
&+ 3/4 f^P\left(x_{21}^H(R-R_1), x_{22}^H(R-R_1)\right) \\
&+ 1/4 f^P\left(x_{21}^L(R-R_1), x_{22}^L(R-R_1)\right). \qquad (43)
\end{aligned}
$$

Expressions (40), (42), and (43) suggest a trade-off. Strategic behavior by agent B in period 1 benefits the principal. Giving a large budget in period 1 to agent B (R_1^B), instead of to agent A (R_1^A), increases the principal's utility. With a smaller R_1^A, however, expression (40) may not hold. Indeed, the principal can satisfy (40) with a large R_1^A under the assumption that an agent's incentive to act sincerely increases with his budget (that is, under Assumption 2). Then R_1^B will be small, and hence, the principal's benefit when a type-L agent acts strategically in period 1 will be small, even if (42) holds. Moreover, with a large R_1^A, only a small budget will be left for period 2 even if the principal learns which agent more likely shares his preferences. We will examine whether the principal prefers a symmetric or an asymmetric allocation in period 1, using the specific utility function.

Example. *Continued.* Again let $v_t^j = f^j(x_{t1}, x_{t2}) = (\alpha^j x_{t1}^\rho + (1-\alpha^j)x_{t2}^\rho)^{\frac{1}{\rho}}$, $j = H$, L, and $v_t^P = f^P(x_{t1}^A + x_{t1}^B, x_{t2}^A + x_{t2}^B) = (\alpha^H(x_{t1}^A + x_{t1}^B)^\rho + (1-\alpha^H)(x_{t2}^A + x_{t2}^B)^\rho)^{\frac{1}{\rho}}$, $0 < \alpha^L < \alpha^H < 1$, $0 < \rho < 1$. Using the expression in (29), incentives to type-L agents in (40) and (42) become

$$\begin{aligned} R_1^A &\geq \frac{\Psi_L^L}{2\Psi_L^L - \Psi_H^L}(R - R_1^B); \\ R_1^B &\leq \frac{\Psi_L^L}{5\Psi_L^L - 4\Psi_H^L}(R - R_1^A). \end{aligned} \tag{44}$$

For intuition, we will examine the case of perfect substitutes ($\rho \to 1$). Let $\alpha^L < 1/2 < \alpha^H$. It is straightforward to derive that $R_1^A = 0$ and $R_1^B = R$ maximize (43). The problem is how incentives to a type-L agent ((40) and (42)) distort the principal's decision. Note that (44) leads to the solutions

$$R_1^A \to \frac{4(1-\alpha^L)(1-2\alpha^L)}{(2-3\alpha^L)(5-9\alpha^L)-(1-\alpha^L)^2}R; \; R_1^B \to \frac{(1-\alpha^L)(1-2\alpha^L)}{(2-3\alpha^L)(5-9\alpha^L)-(1-\alpha^L)^2}R. \tag{45}$$

Thus $R_1^A = 4R_1^B$. Consistent with Lemma 9, the principal gives more money to agent A in order to induce him to reveal his type.

Using Lemma 2, we showed that with this CES function that is homogeneous of degree 1, there exist symmetric revealing and non-revealing equilibria in which the principal chooses a budget in period 1 of $\bar{R}_1^{nr} = \overline{R}_1^r$. With $\rho \to 1$, in the asymmetric equilibrium, the principal gets a higher utility than in the symmetric revealing equilibrium for each of the combinations of

$0 < \alpha^L < 1/2$ and $1/2 < \alpha^H < 1$ (Figure 3), but a lower utility than in the symmetric non-revealing equilibrium for the same range of α^L and α^H (Figure 4). If the principal subjectively anticipates that the symmetric revealing equilibrium occurs with a high probability, he may prefer the asymmetric equilibrium.

5.5 Increased number of agents

The analysis so far treated the number of agents as exogenously fixed, at two. The outcomes, and the principal's welfare, can, however, depend on the number of agents. Note first that the principal does better by having two agents than having only one. If there is only one agent who acts sincerely in period 1, in period 2 the principal cannot reallocate the budget to an agent who shares the principal's preferences. Moreover, with one agent instead of two, there is a smaller probability that at least one agent shares the principal's preferences.

Now consider a very large number of agents, so that with high probability about half of the agents share the principal's preferences in period 1; that is the same as with two agents. If the agents act symmetrically and strategically, knowing the principal's preferences, then in period 1 all agents provide the services the principal prefers. The principal then learns nothing about the agents, and so in period 2 can do no better than to allocate the budget equally among the agents. So both when there are two agents, and when there are many agents, in period 2 the principal expects half the agents to provide the service he prefers. Thus, if agents act strategically, a risk-neutral principal does no better with many agents than with two agents.

Consider next an equilibrium in which agents act sincerely rather than strategically in period 1, thereby revealing their types. Now the principal does better if there are many agents, because with many agents there is a higher probability that at least one of them shares the principal's preferences, allowing the principal to be provided in period 2 with the service he prefers. The question remains whether agents have a greater incentive to act sincerely when the number of agents is large. Consider a putative symmetric equilibrium in which agents are expected to act sincerely. The necessary condition for an L-type agent to reveal his type (21) is rewritten for n agents, $n \geq 2$, as

$$f^L\left(x_{11}^H\left(\frac{R_1}{n}\right), x_{12}^H\left(\frac{R_1}{n}\right)\right)$$

$$
\begin{aligned}
&+\sum_{k=0}^{n-1}\frac{(n-1)!}{(n-1-k)!k!}\left(\frac{1}{2}\right)^{(n-1)} f^L\left(x_{21}^L\left(\frac{R-R_1}{k+1}\right), x_{22}^L\left(\frac{R-R_1}{k+1}\right)\right)\\
\leq\ & f^L\left(x_{11}^L\left(\frac{R_1}{n}\right), x_{12}^L\left(\frac{R_1}{n}\right)\right)\\
&+\left(\frac{1}{2}\right)^{(n-1)} f^L\left(x_{21}^L\left(\frac{R-R_1}{n}\right), x_{22}^L\left(\frac{R-R_1}{n}\right)\right), \qquad (46)
\end{aligned}
$$

where $\frac{(n-1)!}{(n-1-k)!k!}(1/2)^{(n-1)}$ on the left-hand side represents the probability that k agents among other $n-1$ agents have H-type; $(1/2)^{(n-1)}$ on the right-hand side represents the probability that other $n-1$ agents have L-type.

For intuition, employ the utility function $\lim_{\rho\to 1}(\alpha^j x_{t1}^\rho + (1-\alpha^j)x_{t2}^\rho)^{\frac{1}{\rho}}$ with $0 < \alpha^j < 1$. Then a j-type agent's utility in one period proportionally increases with a budget given to him. With $n = 2$, in period 1 each agent gets a budget of $R_1/2$, and so deviating (i.e., behaving strategically), he loses the benefit of spending $R_1/2$ as he likes. But by deviating, he increases his expected budget in period 2 from $(1/2)(R-R_1)/2$ to $(1/2)(R-R_1)+(1/2)(R-R_1)/2$; that is, if the other agent does not share the principal's preferences, then in period 2 the deviating agent gets the whole budget instead of half. With n agents where n is large, the agent who deviates in period 1 by acting strategically loses the benefit of spending R_1/n as he likes. In period 2 he increases his expected budget from $\left(\frac{1}{2}\right)^{n-1}\frac{R-R_1}{n}$ to $\left(2-\left(\frac{1}{2}\right)^{n-1}\right)\frac{R-R_1}{n}$ (see the Appendix). With a larger n, $(1/2)^{(n-1)}$ (the probability that other agents have L-type) will be smaller, so that the principal should set a larger budget (R_1) in period 1 to induce agents to reveal their types. In summary, the principal may prefer a large number of agents if the principal can induce the agents to behave sincerely without giving too large a budget in period 1.

6 Conclusion

This paper considered three effects of an increased budget given to an agent in the initial period. First, an increased budget in one period reduces the budget the principal could give in future periods, thereby reducing the provision of services the principal favors. Second and relatedly, an increased budget in the initial period increases the level of services agents can provide in that period, to the principal's benefit. These effects are standard. Third, an increased budget in the initial period increases an agent's incentive to spend money on the services he prefers, thereby revealing to the principal his type. Such revelation in turn allows the principal to allocate budgets in a later period to an agent who would spend the budget on the services in the way the principal prefers. This consideration, which does not appear in standard models of the principal-agent relationship, can be important in federalism, where a central government allocates funds to many jurisdictions. Furthermore, we showed that under plausible conditions a principal may benefit from hiding his preferences, thereby inducing agents to reveal their types without giving them large budgets in the initial period.

Some leaders have recognized the benefits of competition among agencies, coupled, perhaps, with some ambiguity about what he wants. A historian writes of President Franklin Roosevelt that "whatever Roosevelt's impatience with public brawling, he essentially did not mind—he even welcomed—competition. 'A little rivalry is stimulating, you know.'" And when Roosevelt's advisors, and editorial writers, called for a centralized agency to manage mobilization, he refused, "driving his jostling horses with a loose bit and a nervous but easy rein."[5]

The analysis has implications for how the principal's performance changes over the two periods. If agents act sincerely, then in the first period the principal learns each agent's type, and so can better allocate resources in the second period, thereby better achieving his goals. If, instead, agents act strategically, in period 1, providing the services the principal prefers, then the principal's performance is better in the first period than in the second; in the second period each agent pursues his own objectives, with the principal ignorant of which agents would make better use of resources he gave them.

Contrast these outcomes to predictions that would appear if the principal knew the agents' preferences, but not their ability. The strategic behavior we considered would then not appear, the principal would learn in the first period the agents' abilities, and in the second period he could improve his performance by allocating resources to agents revealed to have high ability. Poor performance in a president's second term would then be consistent with some implications of our model.

[5]See James MacGregor Burns, *Roosevelt: The Soldier of Freedom.* New York: Harcourt Brace Jovanovich, 1970.

Appendix

Definition 1

Note that (21) is rearranged as

$$\begin{aligned} & f^L\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) - f^L\left(x_{11}^L\left(\frac{R_1}{2}\right), x_{12}^L\left(\frac{R_1}{2}\right)\right) \\ + \ & \frac{1}{2} f^L\left(x_{21}^L(R-R_1), x_{22}^L(R-R_1)\right) \leq 0. \end{aligned} \tag{47}$$

Under the assumption that an agent's incentive to act sincerely increases with his budget (Assumption 2), the sum of the first and the second terms on the left-hand side of (47) strictly decreases with R_1. Also, the third term there strictly decreases with R_1. Furthermore, for $R_1 = 0$, the sum of the first to the third terms on the left-hand side of (47) is strictly positive; for $R_1 = R$, it is strictly negative. These results enable us to set Definition 1.

Definition 2

We can rearrange (26) as

$$\begin{aligned} & f^L\left(x_{11}^H\left(\frac{R_1}{2}\right), x_{12}^H\left(\frac{R_1}{2}\right)\right) - f^L\left(x_{11}^L\left(\frac{R_1}{2}\right), x_{12}^L\left(\frac{R_1}{2}\right)\right) \\ + \ & f^L\left(x_{21}^L\left(\frac{R-R_1}{2}\right), x_{22}^L\left(\frac{R-R_1}{2}\right)\right) \geq 0. \end{aligned} \tag{48}$$

Under the assumption that an agent's incentive to act sincerely increases with his budget (Assumption 2), the sum of the first and the second terms on the left-hand side of (48) strictly decreases with R_1. Also, the third term there strictly decreases with R_1. Furthermore, for $R_1 = 0$, the sum of the first to the third terms on the left-hand side of (48) is strictly positive; for $R_1 = R$, it is strictly negative. These results enable us to set Definition 2.

Proof of Lemma 1

It is straightforwardly shown that a homogeneous function f^j with degree n, $0 < n \leq 1$, is homothetic. The value of $f^j(sx_{t1}, sx_{t2})$, $s > 0$, is $f^j(x_{t1}, x_{t2})$ times s^n. If $f^j(x'_{t1}, x'_{t2}) = f^j(x''_{t1}, x''_{t2})$ so that (x'_{t1}, x'_{t2}) and (x''_{t1}, x''_{t2}) are on the same indifference curve, $f^j(sx'_{t1}, sx'_{t2}) = f^j(sx''_{t1}, sx''_{t2})$, indicating that (sx'_{t1}, sx'_{t2}) and (sx''_{t1}, sx''_{t2}) are also on the same indifference curve. Thus the indifference curves constructed from f^j are radial expansions, and therefore, f^j is homothetic.

Furthermore, if f^j is homogeneous of degree n, $0 < n \leq 1$, and therefore, homothetic,

$$\begin{aligned} & f^j\left(x^{j'}_{11}\left(R^j_1\right), x^{j'}_{12}\left(R^j_1\right)\right) - f^j\left(x^j_{11}\left(R^j_1\right), x^j_{12}\left(R^j_1\right)\right) \\ = \ & f^j\left(x^{j'}_{11}(1), x^{j'}_{12}(1)\right)(R^j_1)^n - f^j\left(x^j_{11}(1), x^j_{12}(1)\right)(R^j_1)^n, \end{aligned} \tag{49}$$

which strictly decreases with R^j_1 since

$$f^j\left(x^{j'}_{11}(1), x^{j'}_{12}(1)\right) - f^j\left(x^j_{11}(1), x^j_{12}(1)\right) < 0. \tag{50}$$

Proof of Lemma 2

Regarding the first assertion, for $R_1 \leq \frac{R}{2}$,

$$f^L\left(x^L_{21}\left(\frac{R-R_1}{2}\right), x^L_{22}\left(\frac{R-R_1}{2}\right)\right) \geq f^L\left(x^L_{11}\left(\frac{R_1}{2}\right), x^L_{12}\left(\frac{R_1}{2}\right)\right), \tag{51}$$

which means that (48) holds since

$$f^L\left(x^H_{11}\left(\frac{R_1}{2}\right), x^H_{12}\left(\frac{R_1}{2}\right)\right) \geq 0. \tag{52}$$

Therefore the first assertion is verified.

The second assertion of Lemma 2 comes from

$$\begin{aligned} & f^L \text{ is homogeneous of degree } n \leq 1 \\ \Leftrightarrow \ & f^L\left(x^L_{21}\left(\frac{R-R_1}{2}\right), x^L_{22}\left(\frac{R-R_1}{2}\right)\right) = f^L\left(\frac{x^L_{21}}{2}(R-R_1), \frac{x^L_{22}}{2}(R-R_1)\right) \\ = \ & \left(\frac{1}{2}\right)^n f^L\left(x^L_{21}(R-R_1), x^L_{22}(R-R_1)\right) \\ \geq \ & \frac{1}{2} f^L\left(x^L_{21}(R-R_1), x^L_{22}(R-R_1)\right), \end{aligned} \tag{53}$$

in (47) and (48).

Proof of Lemma 3

Because f^P is homogeneous of degree n, (27) is

$$\begin{aligned} V^{Pr}(R_1) &= 1/4 f^P\left(x_{11}^H(1), x_{12}^H(1)\right) R_1^n \\ &+ 1/4 f^P\left(x_{11}^L(1), x_{12}^L(1)\right) R_1^n \\ &+ 1/2 f^P\left((x_{11}^H + x_{11}^L)/2(1), (x_{12}^H + x_{12}^L)/2(1)\right) R_1^n \\ &+ 3/4 f^P\left(x_{21}^H(1), x_{22}^H(1)\right)(R - R_1)^n \\ &+ 1/4 f^P\left(x_{21}^L(1), x_{22}^L(1)\right)(R - R_1)^n, \end{aligned} \tag{54}$$

and (28) is

$$\begin{aligned} V^{Pnr}(R_1) &= f^P\left(x_{11}^H(1), x_{12}^H(1)\right) R_1^n \\ &+ 1/4 f^P\left(x_{21}^H(1), x_{22}^H(1)\right)(R - R_1)^n \\ &+ 1/2 f^P\left((x_{21}^H + x_{21}^L)/2(1), (x_{22}^H + x_{22}^L)/2(1)\right)(R - R_1)^n \\ &+ 1/4 f^P\left(x_{21}^L(1), x_{22}^L(1)\right)(R - R_1)^n. \end{aligned} \tag{55}$$

If $0 < n < 1$ expression (54) is strictly concave in R_1; if $n = 1$, from (2), it is proportionally decreasing in R_1, meaning that $R_1^r = 0$. Also, if $0 < n < 1$ expression (55) is strictly concave in R_1; if $n = 1$ it is proportionally increasing in R_1, meaning that $R_1^{nr} = R$.

If $0 < n < 1$, for $R_1 = R/2$, the first-order partial derivative of (54) with respect to R_1 reduces to

$$\begin{aligned} &\frac{\partial V^{Pr}}{\partial R_1}(R/2) = -1/2 f^P\left(x_{11}^H(1), x_{12}^H(1)\right) n(R/2)^{n-1} \\ &+1/2 f^P\left((x_{11}^H + x_{11}^L)/2(1), (x_{12}^H + x_{12}^L)/2(1)\right) n(R/2)^{n-1} < 0, \end{aligned} \tag{56}$$

which means $R_1^r < 1/2$. Similarly, the first-order partial derivative of (55) with respect to R_1 reduces to

$$\begin{aligned} \frac{\partial V^{Pnr}}{\partial R_1}(R/2) &= 3/4 f^P\left(x_{11}^H(1), x_{12}^H(1)\right) n(R/2)^{n-1} \\ &- 1/2 f^P\left((x_{11}^H + x_{11}^L)/2(1), (x_{12}^H + x_{12}^L)/2(1)\right) n(R/2)^{n-1} \\ &- 1/4 f^P\left(x_{11}^L(1), x_{12}^L(1)\right) n(R/2)^{n-1} > 0, \end{aligned} \tag{57}$$

which means $R_1^{nr} > 1/2$.

Proof of Lemma 4

Comparing (27) to (28) shows that the principal's utility in period 1 in (27) takes the same value as his utility in period 2 in (28) if R_1 in (27) equals $R - R_1$ in (28). Therefore,

$$V^{Pr}(R_1^r) < V^{Pnr}(R - R_1^r) \leq V^{Pnr}(R_1^{nr}). \tag{58}$$

Proof of Lemma 7

Definition 2 says that under Assumption 2, (26) never holds with $R_1 > \bar{R}_1^{nr}$. Note that (26) corresponds to (36) with $\pi_H^P = 1$. This means that neither satisfies (36) with $R_1 > \bar{R}_1^{nr}$.

Proof of Lemma 9

By summing the terms on the left-hand (right-hand) side of (40) and the terms on the right-hand (left-hand) side of (42), we obtain

$$\begin{aligned} & f^L(x_{11}^H(R_1 - R_1^A), x_{12}^H(R_1 - R_1^A)) - f^L(x_{11}^L(R_1 - R_1^A), x_{12}^L(R_1 - R_1^A)) \\ > \; & f^L(x_{11}^H(R_1^A), x_{12}^H(R_1^A)) - f^L(x_{11}^L(R_1^A), x_{12}^L(R_1^A)), \end{aligned} \tag{59}$$

which means Lemma 9 under Assumption 2.

Expected budgets in period 2 with n agents

Given that $n-1$ other agents behave sincerely, the expected budget given an L-type agent in period 2 when he also reveals his type is

$$\left(\frac{1}{2}\right)^{(n-1)} \frac{R - R_1}{n}, \tag{60}$$

where $(1/2)^{(n-1)}$ represents the probability that $n-1$ other agents also have L-type.

The corresponding expected budget, with the L-type agent behaving strategically, is

$$\sum_{k=0}^{n-1} \frac{(n-1)!}{(n-1-k)!k!} \left(\frac{1}{2}\right)^{(n-1)} \frac{R - R_1}{k+1}$$

$$
\begin{aligned}
&= \sum_{k=0}^{n-1} \frac{n!}{(n-1-k)!(k+1)!}\left(\frac{1}{2}\right)^n \frac{R-R_1}{n/2} \\
&= \sum_{k=-1}^{n-1} \frac{n!}{(n-(k+1))!(k+1)!}\left(\frac{1}{2}\right)^n \frac{R-R_1}{n/2} - \left(\frac{1}{2}\right)^n \frac{R-R_1}{n/2} \\
&= \left(2-\left(\frac{1}{2}\right)^{n-1}\right) \frac{R-R_1}{n}, \qquad (61)
\end{aligned}
$$

where $\frac{(n-1)!}{(n-1-k)!k!}(1/2)^{(n-1)}$ represents the probability that k agents among other $n-1$ agents have H-type. Analogously, $\sum_{k=-1}^{n-1}\frac{n!}{(n-(k+1))!(k+1)!}(1/2)^n$ represents the probability that the number of H-type agents among total n agents is equal to or less than n, which is equal to 1.

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7 Notation

$f(\cdot)$ Utility from consumption of service 1 and service 2

R Total budget given to the agents

R_t Budget given to the agents in period t

R_t^j Budget given to a type-j agent in period t

R_1^r Budget in period 1 which maximizes the principal's utility when agents act sincerely

R_1^{nr} Budget in period 1 which maximizes the principal's utility when agents act strategically

$\bar{R}_1^r$ Critical value of the budget in period 1 that induces an L-type agent in period 1 to reveal his type

$\bar{R}_1^{nr}$ Critical value of the budget in period 1 that induces an L-type agent in period 1 to hide his type

v_t^j A type-j agent's utility in period t

V^j Utility of an agent of type j over two periods

v_t^P Principal's utility in period t

V^P Principal's utility over two periods

V^{Pr} Principal's utility when agents act sincerely, revealing their types

V^{Pnr} Principal's utility when agents act strategically, not revealing their types

x_{ti} Quantity of service i provided in period t

x_{ti}^j Choice of service i in period t by a type-j agent in the absence of signaling considerations

$x_{ti}^{j^e}$ Equilibrium choice of service i in period t by an agent of type j

α^j Parameter describing preferences of j-type

π_H^k Prior probability that agent k's type is H

$\tilde{\pi}_H^k$ Posterior probability that agent k's type is H

Figure 1: Incentives by budget allocation in period 1 (f^j is homogeneous of degree n, $0 < n < 1$)

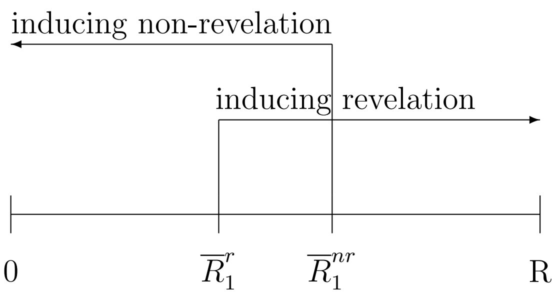

Figure 2: Incentives by budget allocation in period 1 (f^j is homogeneous of degree 1)

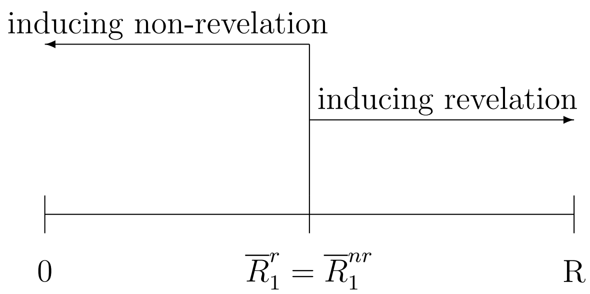

Figure 3: The difference of the principal's utility under asymmetric equilibrium and under symmetric revealing equilibrium

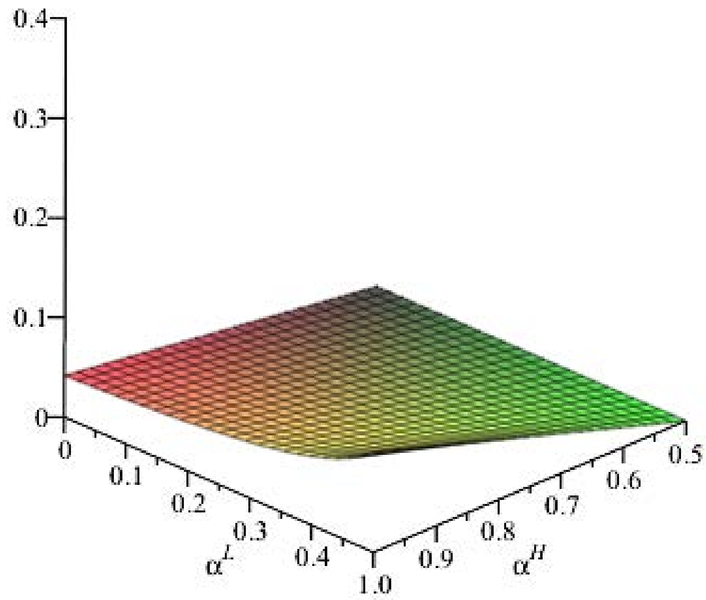

Figure 4: The difference of the principal's utility under asymmetric equilibrium and under symmetric non-revealing equilibrium