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Abstract

A great concern is whether there is any means of monetary policy that works for the “leaning against the wind” policy in the bubbly economy. This paper explores the scope for monetary policy that can control bubbles within the framework of the stochastic version of overlapping-generations model with rational bubbles. The policy that raises the cost of external finance, could be identified as monetary tightening, represses the boom, but appreciate bubbles.
In contrast, an open market operation using public bonds is conductive as the “leaning against the wild” policy. Selling public bonds in the open market by the central bank raises the interest rate, represses the boom, and depreciates bubbles. In conducting monetary tightening, the central bank faces the tradeoff between the loss from killing the boom and the gain from lessening the loss of the bursting of bubbles.

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Introduction

Among economists and policymakers, there are diversified opinions on whether the central bank should respond to bubbles. One strand of argument is that the central bank should focus on the inflation and the output gap, and be neutral to asset prices and bubbles.[1] The other is that the central bank should respond to asset bubbles.[2]

A primary concern is whether there is any means of monetary policy that works for the "leaning against the wind" policy. Greenspan (2002), the former governor of the FRB, addressed whether the central bank has any useful tool of controlling bubbles.

Figure 1 illustrates some controversial observation on the effectiveness of the interest rate policy. The solid graph represents net assets as a fraction of GDP that is supposedly an indicator of bubbles.[3] This figure is roughly stable in the periods without bubbles, but tends to increase in the periods of bubble episodes when assets grow faster than GDP. The thin graph illustrates the federal funds rate.

The FRB raised the federal funds rate from the bottom of 1.0 percent in 2004 to an eventual level of 5.0 percent in 2006 to repress housing bubbles. However, raising the interest rate looks to have contributed little to repressing bubbles, but rather have appreciated bubbles anymore. One might ask if raising the interest rate would be a force of stimulating bubbles.[4]

Gali (2013a) examines the general equilibrium impacts of monetary policies in the model of rational bubbles with nominal rigidities, and shows that the increase in the interest rate has a positive effect on bubble growth, enhancing the fluctuation of the economy. Some recent evidence favors this view. Kutter (2012) and Gali and Gambetti (2013b) report evidence that

[1] The literature includes Bernanke and Gertler (1999, 2001), Bernanke (2002), Greenspan (2002), Mishkin (2008), and others.

[2] The literature includes Blanchard (2000), Bordo and Jeanne (2002), Borio and Lowe (2002), Borio and White (2003), Cecchetti, Genberg, Lipsky and Wadhwani (2000), Cecchetti, Genberg and Wadhwani (2002), among others.

[3] The source of data is the flow of funds accounts of the US (FRB's). Net worth is defined as total assets less total liabilities of three sectors, households and nonprofit organizations, nonfarm nonfinancial corporate business, and nonfarm nonfinancial noncorporate business. Total assets include tangible assets and financial assets. Note that tangible assets include real estate at market value.

[4] There is also an argument addressing that the slow reaction of the FRB to bubbles is responsible to further appreciations of asset prices. Taylor (2007) criticized the interest rate policy of the FRB for that monetary tightening was not enough to repress bubbles over 2002-2007.

nominal interest rates are positively linked to stock and real estate prices over the last two decades. These arguments look to support the ineffectiveness of the monetary policy as the “leaning against the wind” policy.

The question is here is whether there is any means of monetary policy that is conductive to controlling bubbles. Bubbles are the “residuals”, that absorb the resource that has not been channeled into the demand for private and public assets. These residuals react passively to the demand for other assets. One might be tempted to imagine that any asset that is a close substitute with bubbles in the investors’ portfolio can be used as the object of open market operation.

We highlight the role of public bonds, as a means of monetary policy. Public bonds are similar to asset bubbles in that the future expectation for solvency becomes the collateral for the current evaluation. Additionally, the markets for public debt are in general so large that they could absorb part of the resource that is used to finance bubble assets. This argument suggests that the change in the amount of public bonds outstanding in the market can be promising to affect the size of bubbles.[5]

We explore the scope for the effectiveness of monetary policy in the stochastic version of overlapping-generations model with rational bubbles. There are three assets, the security issued by firms, bubbles, and one-period public bond. The security issued by firms is safe but limited by the financial constraint, bubbles are risky assets, and the public bond is safe. The interaction among these three asset features the boom-bust cycle, where emergent bubbles are followed by the investment boom, but the bursting of bubbles results in the serious recession. The developed economy is not a monetary one, but gives implications on how any policy measure is useful.

Several findings are as follows. The policy that raises the cost of external finance, could be identified as monetary tightening, represses the boom, but appreciate bubbles. This perverse policy reaction come from a different channel from Gali(2013a). The tighter borrowing

[5] There is evidence that weakly supports that public debt and bubbles are substitutes. Public debt held by the US residents decline monotonically from 30 percent (as a proportion of GDP) to 19 percent in 2008, suggesting that savings released from public debt went into buying bubble assets.

constraint on firms releases the resource that should have been used for investment to bubbles, thereby appreciating bubbles.

In contrast, an open market operation is conductive as the “leaning against the wild” policy. Selling public bonds in the open market by the central bank induces the investor’s portfolio rebalancing from bubble assets to bonds, thereby depreciating bubbles. Interestingly, the interest rate rises in the course of the operation that represses bubbles. We may conclude that raising the interest rate can contribute to repressing bubbles so long as it involves the operation.

Monetary tightening developed in this paper kills the boom as well as bubbles in almost all the parameter space. In conducting monetary tightening, the central bank faces the tradeoff between the loss from killing the boom and the gain from lessening the loss of the bursting of bubbles.

Closely related are Bernanke and Gerlter(1999) and Gali (2013a), both of which demonstrate that the policy rule stipulating the reaction of the nominal interest rate on bubbles destabilizes the economy, and has odd implications for the “leaning against the wind” policy.

Our analysis is also related to an emerging literature that explains the transmission mechanism that generates the investment boom when bubbles emerge, including work by Farhi and Tirole (2012) and Martin and Ventura (2012), among others.[6]

2. Model

Consider an overlapping-generations economy that lasts over an infinite horizon. In each period, a unit mass of agents are born, and live for three periods. There is no population growth or technological progress. Individual agents are endowed with one unit when young. Their preference is described by $\log c_{t-1}^{y} + \beta \log c_{t}^{m} + \beta^{2} \log c_{t+1}^{o}$, where c_{t-1}^{y} is consumption when

[6] There are excellent contributions that investigate the role of store of values in macroeconomics, including Tirole (1985), Woodford (1990), Kiyotaki and Moore (1997), and Holmstrom and Tirole (1998).

young, c_t^m in middle age, and c_{t+1}^0 in old age, and $\beta(\leq 1)$ is the discount factor. Each generation is indexed by the period in which it is middle-aged. As owners of the firm, middle-aged agents have access to one linear investment project that transforms one unit of a good into R^f (>1) units of the good after one period. To motivate financial market imperfections, we assume that only part of the return, $R(< R^f)$, is pledgeable to creditors. Debtors are protected by their limited liability. Assume that $R < 1$, which is necessary for the borrowing constraint to be binding at the bubbly steady state, as will be obvious below.

The government issues d units of one-period bond. Each of d is priced at unity at maturity and sells at $1/(1+r_{t+1})$, where r_{t+1} denote the rate of return on other safe asset described below. The government finances the claims on debt by rolling new bonds $d/(1+r_{t+2})$ over and raising taxation to cover the deficit $d - d/(1+r_{t+2})$. The deficit (or the subsidy) is financed by tax (or subsidy) on the old in the manner that is proportional to middle-aged savings. Let s_t^m denote the middle-aged savings and τ_t denote the proportional tax rate, the government's budget constraint is $\tau_{t+1}s_t^m = d - d/(1+r_{t+2})$. Note that under log utility, this proportional tax (or subsidy) on the savings does not affect the agent's consumption/saving decision. If investors refuse rolling over debt, public debt is honored by taxation to agents, and thus works as a non-bubbly liquidity.

Finally, as in Weil (1987), we allow bubbles to burst stochastically depending on the realization of a sunspot. Suppose that in each period bubbles burst with probability $1-\lambda$. Suppose further that once bubbles burst, the economy returns to the bubbleless economy forever.

3. Analysis

Let $\tilde{r}_t$ denote the rate of return on risky bubbles b_{t-1} in period t. Young agents of generation t who are born at date t-1 choose savings s^y_{t-1} to satisfy

(1a) $$(1+r_t)(s^y_{t-1}-b_{t-1})+(1+\tilde{r}_t)b_{t-1}-s^m_t=\lambda\beta(1+\tilde{r}_t)(1-s^y_{t-1}),$$

given their expected middle-aged savings s^m_t. When they become middle-aged, they choose savings

(1b) $$s^m_t=\frac{\beta}{1+\beta}\{(1+r_t)(s^y_{t-1}-b_{t-1})+(1+\tilde{r}_t)b_{t-1}\},$$

given their young savings. Under perfect foresight, young and middle-aged savings are

(2a) $$s^y_t=\{(1+\beta)\beta+\frac{1+r_t}{\lambda(1+\tilde{r}_{t+1})}\}^{-1}\{(1+\beta)\beta-\frac{(\tilde{r}_{t+1}-r_{t+1})b_t}{\lambda(1+\tilde{r}_{t+1})}\}, \text{ and}$$

(2b) $$s^m_t=\frac{\beta^2}{(1+r_t)/\lambda(1+\tilde{r}_t)+\beta+\beta^2}\{1+r_t+(\tilde{r}_t-r_t)b_{t-1}\}.$$

So long as bubbles survive, bubbles evolve as

(3) $$b_{t+1}=(1+\tilde{r}_{t+1})b_t,$$

where $\tilde{r}_{t+1}$ is a stochastic variable, satisfying $\tilde{r}_{t+1}>-1$ when bubbles persist, while $\tilde{r}_{t+1}=-1$ when bubbles burst.

We focus on the case when the borrowing constraint is binding. Otherwise, the profitability constraint should be binding with equality, where the interest rate is $1+r_{t+1}=R^f$. Bubbles never arise due to $R^f>1$. The investment realizes the first level: $i^{FB}\equiv s^y(R^f-1)+s^m(R^f-1)$.

A middle-aged agent starts the firm by investing the amount i_t in the project. Letting r_{t+1} denote the interest rate prevailing between t and t+1, the agent is willing to start the firm if

$R^f \geq 1 + r_{t+1}$, which we call the *profitability constraint*. The firm funds the investment i_t by the internal wealth s_t^m and supplying the security $(i_t - s_t^m)$, but the issued amount is limited to the present value of the pledgeable asset $Ri_t/(1+r_{t+1})$:

(4) $$i_t - s_t^m \leq Ri_t/(1+r_{t+1}) .$$

This inequality states that the debtor can borrow up to the pledgeable asset. We call this inequality the *borrowing constraint*.

The investment of borrowing constrained firms is written as a multiple of the internal wealth s_t^m and the leverage $1/\{1-R/(1+r_{t+1})\}$:

(5) $$i_t = \frac{s_t^m}{1-R/(1+r_{t+1})} .$$

This equation reveals two opposing effects of the interest rate on investment. On the one hand, a rise in the period t+1 interest rate decreases leverage $1/\{1-R/(1+r_{t+1})\}$ and represses investment of the financially constrained firms. On the other hand, a rise in the period t interest rate increases the internal wealth and stimulates investment of these firms. We call the former *the leverage effect,* and the latter *the balance sheet effect*

Savings are used for fueling investment, bubbles, and public bonds. The market clearing in the capital market requires

(6) $$s_t^y + s_t^m = i_t + b_t + \frac{d}{1+r_{t+1}} .$$

We turn to the portfolio aspect of the consumption/savings decision. Letting c_t^m $(c_t^{m,c})$ denote consumption when bubbles persist (burst), the first-order conditions on risky bubbles is

(7a) $$(c_{t-1}^y)^{-1} = \lambda\beta(1+\tilde{r}_t)(c_t^m)^{-1} ,$$

which is an alternative expression for (1a), and the first-order conditions on risky bubbles is

(7b) $$(c_{t-1}^{y})^{-1} = \lambda\beta(1+r_t)(c_t^{m})^{-1} + (1-\lambda)\beta(1+r_t)(c_t^{m,c})^{-1},$$

The beginning-of-period income of the middle-aged is $(1+\tilde{r}_t)b_{t-1} + (1+r_t)(s_{t-1}^{y} - b_{t-1})$, which is replaced, using (3), (4) with equality, and (6), by $b_t + Ri_{t-1} + d$ when bubbles persist, and $Ri_{t-1} + d$ when bubbles burst. When the preference is the log utility, the middle-aged consumption is a constant proportion of the income. Accordingly, we define the measure of risk premium from the interest rate differential as

(8) $$\delta_t \equiv \frac{1+\tilde{r}_t}{1+r_t} = \frac{1}{\lambda}\{1 + \frac{(1-\lambda)b_t}{Ri_{t-1}+d}\},$$

which exceeds $1/\lambda$, capturing that the expected return on bubbles should be higher than the return on securities, i.e., $\lambda(1+\tilde{r}_t) > 1+r_t$.[7] The risk-premium is positively related to risky assets relative to safe asset, $b_t/(Ri_{t-1}+d)$.

The equation for the evolution of bubbles (3) is accordingly replaced by

(9) $$b_{t+1} = \delta_{t+1}(1+r_{t+1})b_t .$$

Combined with (8), middle-aged savings (1b) are rewritten as

$$s_t^m = \frac{\beta^2}{1/(\lambda\delta_t) + \beta + \beta^2}\{1 + r_t + (\delta_t - 1)(1+r_t)b_{t-1}\},$$

Thus the asset demand function is written as

[7] In general, the return on bubbles could incorporate the liquidity premium that arises from the fact that when bubbles burst, the interest rate is low and the internal wealth earns high (e.g., Farhi and Tirole 2012). However, in an environment of the log-utility, the consumption/saving allocation of the middle-aged is independent of the entrepreneurs' return, and thus the return on bubbles does not reflect the liquidity premium.

$$\frac{\beta}{1/(\lambda\delta_{t+1})+\beta+\beta^2}\{1+r_{t+1}+(\delta_{t+1}-1)(1+r_{t+1})b_t\}=\frac{1}{1+\beta}(Ri_t+d+b_{t+1}), \tag{10}$$

The asset supply function implies how financially constrained firms issue the security to finance investment. Investment is constrained by the security secured by the pledgeable asset $Ri_t/(1+r_{t+1})$ and the internal wealth $\frac{\beta}{1+\beta}(Ri_{t-1}+d+b_t)$. This relation constitutes the supply function of the security;

$$i_t=\frac{Ri_t}{1+r_{t+1}}+\frac{\beta}{1+\beta}(Ri_{t-1}+d+b_t), \tag{11}$$

The competitive equilibrium of a stochastically bubbly economy that satisfies the binding borrowing constraint is defined as a sequence $\{i_t, b_t, r_t, \delta_t\}_{t=0}^{\infty}$ that satisfies four equations (8)-(11).

4. Deterministic Economy

It is useful to study the deterministic version of this economy before going into the stochastic version of the model. If there is no uncertainty, savings of young and middle-aged, (2a) and (2b), reduce to

$$s_t^y=\frac{\beta+\beta^2}{1+\beta+\beta^2}, \qquad \text{and } s_t^m=\frac{\beta^2}{1+\beta+\beta^2}(1+r_t). \tag{12}$$

Young savings are constant, and middle-aged savings are proportional to the beginning-of-the income with a constant saving rate. Since $\delta_t=1$ when there is no uncertainty, bubbles evolve as

$$b_{t+1}=(1+r_{t+1})b_t \tag{13}$$

The asset demand and supply functions are written as

$$\frac{\beta+\beta^2}{1+\beta+\beta^2}(1+r_{t+1})=Ri_t+d+b_{t+1}, \text{ and} \tag{14}$$

(15) $$i_t = \frac{Ri_t}{1+r_{t+1}} + \frac{\beta}{1+\beta}(Ri_{t-1} + d + b_t),$$

We define the competitive equilibrium of a deterministic version of the bubbly economy as a sequence $\{i_t, b_t, r_t\}_{t=0}^{\infty}$ that satisfies (13), (14), and (15).

By imposing $b_t = 0$ on (14) and (15), we can describe the competitive equilibrium of a bubbleless economy as

(16) $$\frac{\beta+\beta^2}{1+\beta+\beta^2}(1+r_{t+1}) = Ri_t + d, \text{ and}$$

(17) $$i_t = \frac{Ri_t}{1+r_{t+1}} + \frac{\beta}{1+\beta}(Ri_{t-1} + d).$$

Proposition 1: Suppose that

(#1) $$\beta(1+\beta) - \frac{\beta^2 RR^f}{R^f - R} - \frac{(1+\beta+\beta^2)d}{R^f} > 0$$

The competitive equilibrium of a bubbleless economy that satisfies the binding borrowing constraint is dynamically stable. The steady state interest rate denoted $r^D(R,d)$ is increasing in R and d.

The first concern is if public bonds and investment are complementary or substitutes. Public bonds push up the interest rate. A rise in the interest rate contributes to an enhancement of the internal wealth of middle-aged entrepreneurs, thereby stimulating investment through the balance sheet effect, while it leads to the decline in the debtor's leverage, thereby repressing investment.

Proposition 2: Suppose that (#1) holds. There exists a threshold level of public bonds $d_0 \geq 0$ above (below) which public bonds and investment are complementary (substitutes).

Proof: We use (16) and (17) to write $i = \frac{\beta^2}{1+\beta+\beta^2}\frac{(1+r)}{1-R/(1+r)}$. It is easy to check that $\partial i/\partial r > 0$ if and only if $1+r > 2R$. Proposition 1 says that $r^D(.)$ is increasing in d. Note that $r^D(0) = \frac{R(1+\beta)}{1+\beta-\beta R}$. If $1+\beta-2\beta R > 0$, there exists a threshold level of public bonds $d_0 > 0$ above (below) which public bonds and investment are complementary. If $1+\beta-2\beta R < 0$, public bonds and investment are complementary for any $d > 0$. Q.E.D.

When the liquidity effect dominates the leverage effect, public bonds operate to crowd investment in, while otherwise, it crowds investment out.

We turn to the bubbly equilibrium. The steady state of the bubbly equilibrium is represented as $\{i^B, b^B, r^B\}$, satisfying

(18) $$i^B = \frac{\beta^2}{(1+\beta+\beta^2)(1-R)},\quad b^B = \frac{\beta(1+\beta)}{1+\beta+\beta^2} - \frac{\beta^2 R}{(1+\beta+\beta^2)(1-R)} - d, \text{ and } r^B = 0.$$

Bubbles are positively valued only if

(#2) $$\beta(1+\beta) - \frac{\beta^2 R}{1-R} - (1+\beta+\beta^2)d > 0$$

Condition (#2) is equivalent to the property that the bubbleless interest rate is negative.

The next concern is whether bubbles and investment are complementary or substitutes. We rewrite the savings of the middle-aged as $s_t^m = (1+r_t)B$, where $B = \frac{\beta^2}{1+\beta+\beta^2}$. The steady state of the bubbleless economy satisfies $s^y + (1+r^D)B = i^D + \frac{d}{1+r^D}$ and $1+r^D = \frac{Ri^D}{s^y - d/(1+r^D)}$, and thus $i^D = \frac{s^y - d/(1+r^D)}{1 - RB/\{s^y - d/(1+r^D)\}}$. On the other hand, the steady state of the bubbly economy satisfies $s^y + B = i^B + b^B + d$ and $s^y - b^B - d = Ri^B$, and thus $i^B = \frac{B}{1-R}$. We have

$$i^B - i^D = \frac{\{B - s^y + d/(1+r^D)\}\{(1-R)\{s^y - d/(1+r^D) - RB\}}{(1-R)\{s^y - d/(1+r^D) - RB\}}.$$

Combining above equations, we rewrite the interest rate as $1+r^D = \frac{R\{s^y - d/(1+r^D)\}}{s^y - RB}$. If the bubbly economy exists, we should have $r^D < 0$, and thus $(1-R)\{s^y - d/(1+r^D)\} > RB$. Next the positively valued investment guarantees $s^y - d/(1+r^D) > RB$. Therefore, crowding-in occurs if and only if $B + d/(1+r^D) > s^y$.

Proposition 3: Suppose that Condition (#2) holds.[8] Bubbles and investment are complementary if and only if

(*) $$\beta\{1 + r^D(R,d)\} < (1+\beta+\beta^2)d ,$$ [9]

which has the following properties:

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[8] Condition (#1) is sufficient if (#2) holds.

[9] If $d^* < d_0$, bubbles and investment are complementary whenever public debt and investment are complementary, but the converse is not always true.

(i) There is a threshold $d^* > 0$, above which crowding-in occurs and below which crowding-out occurs, for any given $R > 0$.

(ii) There is a threshold $R^* (> 0)$, below which crowding-in occurs and above which crowding-out occurs, for any given $d > 0$.

(iii) Bubbles crowd investment out when $d = 0$, for any given $R > 0$.

Proof: See the Appendix.

When there is no public debt, the leverage effect dominates the balance sheet effect, and bubbles crowd investment out. As the public bonds are abundant, bubbles are more likely to crowd investment in. When bubbles arise, the interest rate goes up, and the bond price goes down. The released savings are channel to financing investment.

In this deterministic version we examine impacts of the interest rate policy. In this nonmonetary model we cannot conduct monetary policy directly, but the experiment of raising the cost of external finance could capture impacts of monetary tightening. Actually, raising the nominal interest rate is intended to repress economic activities by raising the cost of external funds. We capture this by imposing tax on the firm's cost of financing at rate $1+\tau$. The tax revenue is refunded to the old in the manner that is proportional to middle-aged savings. The determination of investment in (5) is replaced by

$$i_t = \frac{s_t^m}{1 - R/(1+r_{t+1})(1+\tau)}.$$

Any other conditions are unchanged. The following is straightforward from (14) and

$$i_t = \frac{Ri_t}{(1+r_{t+1})(1+\tau)} + \frac{\beta}{1+\beta}(Ri_{t-1} + d + b_t).$$

Proposition 4: Suppose that (#2) holds. The interest rate policy that raises the cost of external finance represses investment and appreciates bubbles.

As expected, raising the interest rate represses investment. However, it exerts no force of repressing bubbles, but rather appreciates bubbles; savings that have been released from purchasing the private security go into buying bubbles, thereby appreciating bubbles.

One might wonder if this result owes to the fact that investors who are young hold bubbles, and the middle-aged financially constrained firms do not hold bubbles. We can show that this is not true. Assume additionally that bubbles are perfectly pledgeable to investors. Then in the absence of any tax, there is an equilibrium where the middle-aged hold bubbles by borrowing. Any resource allocation is unchanged. However, once the tax is imposed, any equilibrium where the middle-aged hold bubbles is not sustainable because if so, no arbitrage condition would require $b_{t+1} = (1 + r_{t+1})(1 + \tau)b_t$, but young agents who are willing to buy bubbles would face the different condition, $b_{t+1} = (1 + r_{t+1})b_t$. Given the next period price b_{t+1}, the young could bid at a higher price. If bubbles are preserved, young agents hold all the bubbles. The imposition of tax yields the same result as Proposition 4.

We turn to the dynamics. We impose a technical assumption.

($) $$(1 + \beta + \beta^2)d < \beta R\{1 + (1 + \beta + \beta^2)d\}$$

This assumption is a sufficient condition for the dynamic behavior to be well defined.

Proposition 5: Suppose that (#2) holds, and additionally that either (*) holds or ($) holds unless (*) holds. There exists a bubbly steady state of the competitive equilibrium that satisfies the binding borrowing constraint. Given $i_{t-1} > 0$, there exist maximum feasible bubbles $b(i_{t-1})$, for which the competitive equilibrium converges to the bubbly steady state.

Figure 2 illustrates a typical phase diagram when bubbles crowd investment in. Given i^D, there exist bubbles $b(i^D)$, for which investment approaches monotonically the one at the bubbly steady state W. Along the saddle-path, emergent bubbles are followed by the investment boom.

5. Stochastic Bubbles and Boom-Bust Cycle

We turn to the stochastic version of this economy. We combine (8) and $\tilde{r} = 0$ with the asset supply function (11) to write the asset supply function as a continuously increasing function $b = \Theta^s(i)$ that is implicitly derived from

$$1 - \frac{\beta R}{1+\beta} - \frac{\beta(b+d)}{(1+\beta)i} = \frac{R}{\lambda}\{1 + \frac{(1-\lambda)b}{Ri+d}\}$$

We turn to the asset demand function. We combine (1a), (1b), and $\tilde{r} = 0$ to link young and middle-aged savings by

(19) $$s^y = 1 - \frac{1}{\beta^2 \lambda} s^m,$$

We rewrite middle-aged savings (1b) by combining (4), (6), and $\tilde{r} = 0$ as

(20) $$s^m = \frac{\beta}{1+\beta}(b + Ri + d),$$

We combine (19), (20), and $\tilde{r} = 0$ with the market clearing (6) to derive the asset demand function as a continuously function $b = \Theta^d(i)$ that is implicitly derived from

(21) $$\{(\frac{1}{\lambda\beta^2} - 1)\frac{\beta}{1+\beta} + 1 + \frac{d}{\lambda}\frac{(1-\lambda)}{Ri+d}\}b$$

$$= -\{(\frac{1}{\lambda\beta^2}-1)\frac{\beta}{1+\beta}R+1\}i + 1 - \{(\frac{1}{\lambda\beta^2}-1)\frac{\beta}{1+\beta}+\frac{1}{\lambda}\}d \ ,$$

where $\lambda\beta^2 < 1$ and both parentheses are positive. When $\Theta^d(0) > 0$, there exists a threshold investment $\hat{i}(>0)$ below (above) which bubbles are positively (negatively) valued.

Figure 3 illustrates the steady state equilibrium. The two functions are described to intersect at the first quadrant. Letting $i^{\#}$ denote the investment that satisfies $0 = \Theta^s(i^{\#})$, $i^{\#}$ should be positively valued because $\Theta^s(0) = -d$.

The necessary and sufficient condition for the existence of the bubbly steady state is $b = \Theta^d(i^{\#}) > 0$, where $i^{\#} = \dfrac{\lambda\beta d}{(\lambda - R)(1+\beta) - \lambda\beta R}$ because then there should be an intersection on $(i^{\#}, \hat{i})$.[10]

We use (21) to derive the latter condition as

(#3) $$\{(\frac{1}{\lambda\beta^2}-1)\frac{\beta}{1+\beta}R+1\}\ \frac{\lambda\beta d}{(\lambda - R)(1+\beta)-\lambda\beta R} + (\frac{1}{\lambda\beta^2}-1)\frac{\beta d}{1+\beta} + \frac{d}{\lambda} < 1$$

The remarkable feature for Condition (#3) is that the bubbly equilibrium is more likely to arise as public bonds are not abundant. The abundant bonds push the interest rate up, making bubbles less viable.

Proposition 6: Suppose that (#3) holds. There exists a steady state equilibrium of the stochastic version of the bubbly economy that satisfies the borrowing constraint. For sufficiently small 1-λ, there exists a unique equilibrium that converges to the bubbly steady of the competitive equilibrium, given i_{t-1}.

Proof: We make the proof in three steps. First, as Proposition 3 demonstrates, in the

[10] We cannot exclude the multiplicity of the bubbly steady states because the asset demand curve may not be monotonically decreasing.

deterministic case (λ =1), there exists a unique equilibrium that converges to the bubbly steady state. Second, it is obvious that the steady state equilibrium is continuous in λ near λ =1. Finally, we show that for λ <1 the dynamic system is described as a two dimensional system of (i_{t-1}, b_t) as the case for λ =1.

Two primary channels work through which public bonds repress investment. First, the public bond competes with the private security in the investor's portfolio, and operates to substitute it, thereby repressing investment. Second, public bonds can contribute to the soundness of the asset portfolio, lessening the risk premium by pushing up the interest rate, thereby repressing investment.

6. Open Market Operation

The ongoing analysis suggests that the policy using the interest rate is not effective to repress bubbles. We have to look for other means of controlling bubbles. Interestingly, bubbles and public bonds are imperfectly substitutes in the investors' portfolio. This property could motivate us to expect if any policy using some quantitative measure may be promising.

We consider an open market policy where the central bank conducts the open market operation using public bonds . Consider the following experiment. Central bank buys public bonds $(1-\theta)d/(1+r_{t+1})$ in the market, and the bond at maturity is transferred to the old. The net deficit is again covered by a tax on the old in the manner that is proportional to the middle-aged savings. The integrated government's budget constraint is transfer is also made in the manner that is proportional to middle-aged savings. $\tau_{t+1}s_t^m = \theta d - \theta d/(1+r_{t+2})$. A rise in θ is identified as monetary tightening, and a decline in θ as monetary easing. Note that this tax scheme does not distort the consumption/savings behavior of the middle-aged.

Accordingly, Condition (#3) is replaced by

(#4) $$\{(\frac{1}{\lambda\beta^2}-1)\frac{\beta}{1+\beta}R+1\}\frac{\lambda\beta\theta d}{(\lambda-R)(1+\beta)-\lambda\beta R}+(\frac{1}{\lambda\beta^2}-1)\frac{\beta\theta d}{1+\beta}+\frac{\theta d}{\lambda}<1,$$

which is more likely to hold when public bonds are scarce or the survival probability of bubbles is high. It is important to note that the monetary stance influences the viability of bubbles.

The specification of the open market operation is different from the ordinary one in that it does not involve the exchange of the government's liabilities. If central bank issues fiat money in exchange for public bonds, the substitutability between these two assets plays a crucial role in the policy effect. If fiat money and public bonds are perfect substitutes, the operation changes nothing, as the famous irrelevance theorem by Wallace (1981) shows. In contrast, if the liquidity service makes fiat money a different asset from public bonds, main results below will continue to hold.[12]

In this environment, the income of the middle-aged is $b_t + Ri_{t-1} + \theta d$, and (14) and (15) are changed. Furthermore, private agents hold only a fraction θ of public bonds, and the market clearing (6) is rewritten as

(22) $$s_t^y + s_t^m = i_t + b_t + \frac{\theta d}{1+r_{t+1}}.$$

The asset demand and supply functions are finally rewritten as

[12] The simplest model with liquidity service of fiat money will be that the young have to hold some minimum level of fiat money. When the constraint binds with equality (which will arise with positive fiat money growth), repurchasing fiat money in exchange for selling nominal public bonds yields the decline in the price level, which appreciates the real value of public debt. The appreciation of real public bonds as well as the exchange of the operation becomes a force of depreciating bubbles.

(23S) $$1-\frac{\beta R}{1+\beta}-\frac{\beta(b+\theta d)}{(1+\beta)i}=\frac{R}{\lambda}\{1+\frac{(1-\lambda)b}{Ri+\theta d}\}$$, and

(23D) $$\{(\frac{1}{\lambda\beta^2}-1)\frac{\beta}{1+\beta}+1+\frac{\theta d(1-\lambda)}{(Ri+\theta d)\lambda}\}b$$
$$=-\{(\frac{1}{\lambda\beta^2}-1)\frac{\beta}{1+\beta}R+1\}i+1-\{(\frac{1}{\lambda\beta^2}-1)\frac{\beta}{1+\beta}+\frac{1}{\lambda}\}\theta d.$$

It is useful to examine first the non-stochastic version of the bubbly economy, with $\lambda = 1$. These two equations determine investment i and the sum of bubbles and public bonds $b+\theta d$, with

$$i=[\{(\frac{1}{\beta^2}-1)+\frac{1+\beta}{\beta}\}(1-\frac{\beta R}{1+\beta}-R)+\{(\frac{1}{\beta^2}-1)\frac{\beta R}{1+\beta}+1\}]^{-1}$$, and

$$b+\theta d=(1-\frac{\beta R}{1+\beta}-R)\frac{(1+\beta)i}{\beta}.$$

Investment is independent of the monetary stance, and also the interest rate. An change in θ can only affect the composition of bubbles and public bonds , both of which are perfect substitutes.

Turn to the stochastic case when the risk premium channel is at work. It is useful to examine first the case when R=0. Two equations, (23S) and (23D), reduce to

(24S) $$i=\frac{\beta(b+\theta d)}{1+\beta}$$, and

(24D) $$\{(\frac{1}{\lambda\beta^2}-1)\frac{\beta}{1+\beta}+\frac{1}{\lambda}\}b=-i+1-\{(\frac{1}{\lambda\beta^2}-1)\frac{\beta}{1+\beta}+\frac{1}{\lambda}\}\theta d$$

Equation (23S) says that a rise in θ enhances the internal wealth and thus investment, given b. On the other hand, (24D) says that a rise in θ crowds the resource for investment out, given b. The investment is finally $i=[\{(\frac{1}{\lambda\beta^2}-1)\frac{\beta}{1+\beta}+\frac{1}{\lambda}\}\frac{1+\beta}{\beta}+1]^{-1}$, which is independent of θ; these two effects offset completely.

The interest rate reacts to the change in θ. The steady state relation of (8) captures the general mechanism behind the endogenous change in the interest rate;

$$\frac{1}{1+r}=\frac{1}{\lambda}\{1+(1-\lambda)\frac{b}{Ri+\theta d}\}.$$

Reacting to a rise in θ, investors tend to hold more safe bonds and less risky bubbles. This portfolio rebalancing requires the price of public bond to decline, or equivalently the interest rate to rise. This mechanism is obvious when R=0. In other words, the decline in the price of bond exerts the substitution from bubbles to public bonds, thereby depreciating bubbles.

Consider next the case when R>0. The endogenous change in the interest rate impacts on investment additionally through the leverage channel. So long as monetary tightening leads to the rise in the interest rate, it is expected to repress investment.

Table 1 provides some simulation findings. We identify several key features. As the monetary stance is tightened: that is, θ goes up, investment declines and bubbles depreciate. Interestingly, the tighter monetary policy is accompanied by the rise in the interest rate. This finding is contrasted with the effects of the interest rate policy as examine in the previous section. We may conclude that raising the interest rate can repress bubbles only if it involves the asset rebalancing, made through the market operation.

Tables 2A and 2B illustrate the policy effects on investment and bubbles in the parameter space $(R,\theta d)$. Both investment and bubbles both move monotonically in terms of R and θd.

The next concern is how the size of bubbles affects the recession once bubbles have burst. Suppose that bubbles burst at the timing when at date T. Once bubbles burst, the middle-aged who hold bubbles as assets lose part of their assets. At the bursting period T, their internal wealth declines from $\beta(Ri_{T-1}+b_T+\theta d)/(1+\beta)$ to $\beta(Ri_{T-1}+\theta d)/(1+\beta)$. Thus at t=T, the supply and demand functions are written as

(25) $$\frac{\beta+\beta^2}{1+\beta+\beta^2}(1+r_{T+1}) = Ri_T + \theta d \text{, and}$$

(26) $$i_T = \frac{Ri_T}{1+r_{T+1}} + \frac{\beta}{1+\beta}(Ri_{T-1}+\theta d) .$$

The term of bubbles disappears from either equation. Equations (25) and (26) are part of the bubbleless equilibrium defined by (16) and (17). When i_{T-1} lies above the bubbleless steady state, which typically arises when there is crowding-in, i_T falls below than i_{T-1}, that is, the bursting of bubbles results in recession.

Figure 4 illustrates the typical boom-bust cycle. Starting from i^D, the emergence of bubbles is followed by the investment boom. Investment monotonically approaches i^B, but the bursting of bubbles represses investment at i^C. Investment follows the dynamics of the bubbleless equilibrium, and converges monotonically to i^D. Over the cycle, investment and GDP, $R^f i_t + 1$, are higher than the bubbleless economy.

The magnitude of the recession is expected to be serious when bubbles that have burst are large. Bubbles tend to be negatively related to the amount of public bonds, implying that when the public bonds are abundant, the recession is mild. Therefore, monetary tightening has a negative effect of repressing investment during the boom, but has a positive effect of mitigating the recession.

Table 3 illustrates some simulation results.[13] It shows when the economy is at the bubbly steady state until T-1, at T bubbles burst. As the monetary stance is tightened, the investment at the boom declines, but the investment at the bust does not move monotonically. As θ changes from 0.2 to 0.4, the recession is more serious, but as θ goes down further, the recession becomes milder. Central bank can stabilize the boom-bust cycle using an open market operation.

[13] R=0.2, β=0.7, d=0.5, λ=0.9.

The central bank faces the trade-off between the gain from weakening the recession and the loss from killing the boom, and is guided to respond to bubbles if the former gain is larger than the latter loss.

It is important to note that in this experiment the change in the policy stance did not change the market's expectation for bubbles to survive, expressed by the survival probability of bubbles λ. Actually, the policy change could influence the survival probability of bubbles, which works for the further force of influencing the boom-bust cycle. Or central bank could change the market expectation on bubbles by revealing the policy stance. For example, if central bank induces the market believe that central bank crash bubbles with some probability (by selling public bonds at large scale to violate Condition (#4)), the market expectation for the survival probability of bubbles goes down, thereby influencing the cycle.

This analysis gives a scope for examining an operation that exchanges public bonds with bubbles. There are at least two impacts; the first is how are the aggregate bubbles that are the sum of bubbles held by private investors and the government. The second is if the credibility of bubbles improves; investors may believe that bubbles that are now held by the government are implicitly backed by the future tax.

Appendix

Proof of Proposition 1

It follows from (16) and (17) that

(A1) $$\Gamma(i_t) \equiv i_t[1 - \frac{(\beta+\beta^2)R}{(1+\beta+\beta^2)(Ri_t+d)}] = \frac{\beta}{1+\beta}(Ri_{t-1}+d).$$

The RHS is positive and increasing for $i_{t-1} > 0$. Define a threshold investment by $\underline{i} \equiv \frac{\beta(1+\beta)}{1+\beta+\beta^2} - \frac{d}{R}$. The LHS is convex and satisfies $\Gamma(0) = 0$. When $\underline{i} < 0$, it is increasing for $i_t > 0$, while when $\underline{i} > 0$, it is increasing for $i_t > \underline{i}$, with $\Gamma(\underline{i}) = 0$. There exists the unique steady state $i^D(<i^{FB})$ if $\Gamma(i^{FB}) > \frac{\beta}{1+\beta}(Ri^{FB}+d)$, or

(#1) $$\beta(1+\beta) - \frac{\beta^2 RR^f}{R^f - R} - \frac{(1+\beta+\beta^2)d}{R^f} > 0,$$

while $i^D = i^{FB}$ otherwise. The inequality $i^D < i^{FB}$ holds if and only if $1+r < R^f$ from (3), which implies that when (#1) holds, the borrowing constraint should be binding. For $i_{t-1} < i^D$,

$\Gamma(i_t) = \frac{\beta}{1+\beta}(Ri_{t-1}+d) > \Gamma(i_{t-1})$, and i_t is increasing, while for $i_{t-1} > i^D$,

$\Gamma(i_t) = \frac{\beta}{1+\beta}(Ri_{t-1}+d) < \Gamma(i_{t-1})$, and i_t is decreasing. Investment is dynamically stable.

Incorporating (12a), (12b), (5) into (6) yields

(A2) $$\beta^2 R(1+r_t) = (1+r_{t+1} - R)\ \{\beta(1+\beta) - \frac{(1+\beta+\beta^2)d}{1+r_{t+1}}\} \equiv \Lambda(1+r_{t+1}).$$

The LHS is positive and increasing, while the RHS satisfies $\lim_{1+r\to 0} \Lambda(1+r_{t+1}) = \infty$,

$\Lambda(R) = \Lambda(\frac{(1+\beta+\beta^2)d}{\beta(1+\beta)}) = 0$, and is increasing and convex for $\max\{R, \frac{(1+\beta+\beta^2)d}{\beta(1+\beta)}\} < 1+r_{t+1}$.

There exists two solutions, $1+r^-$ and $1+r^+$, that satisfy $\beta^2 R(1+r) = \Lambda(1+r)$, and satisfies $0 < 1+r^- < R < 1+r^+$. Only $1+r^+$ satisfies the binding borrowing constraint. At $r = r^+$, $\partial r^+ / \partial d > 0$ holds . Q.E.D.

Proof of Proposition 3

Use (16) and (17) to solve the interest rate in terms of d; $1+r(\omega) = \frac{B(d)+\sqrt{D(d)}}{2\beta\{1+\beta(1-R)\}}$, where

$B(d) \equiv R\beta(1+\beta) + (1+\beta+\beta^2)d$ and

$D(d) \equiv \{R\beta(1+\beta)+(1+\beta+\beta^2)d\}^2 - 4\beta\{1+\beta(1-R)\}(1+\beta+\beta^2)Rd$. We define a new function,

$1+\hat{r}(d) = \frac{R\beta(1+\beta)+(1+\beta+\beta^2)d}{\beta\{1+\beta(1-R)\}}$, which is greater than $1+r(d)$ for any $d>0$ and equal only at $d=0$. $\beta\{1+\hat{r}(d)\}$ is increasing, with the slope being less than $(1+\beta+\beta^2)$ unity, and satisfies $\beta\{1+\hat{r}(d)\}=\dfrac{R(1+\beta)}{1+\beta(1-R)}$. On the other hand, $\beta\{1+r(d)\}$ is also increasing and less than $\beta\{1+\hat{r}(d)\}$ for any $d>0$.There exists a threshold $d^*(>0)$ only below which $\beta\{1+r^D(d^*)\}<(1+\beta+\beta^2)d^*$. Q.E.D.

Proof of Proposition 5

We use (13) to rewrite (15) as

(B1) $$1+r_{t+1}=\frac{(1+\beta+\beta^2)(Ri_t+d)}{\beta(1+\beta)-(1+\beta+\beta^2)b_t}\equiv\Phi(i_t,b_t),$$

where the denominator is positive from (18), with $\Phi_i\equiv\dfrac{\partial\Phi}{\partial i_t}=\dfrac{(1+\beta+\beta^2)R}{\beta(1+\beta)-(1+\beta+\beta^2)b_t}>0$ and $\Phi_b\equiv\dfrac{\partial\Phi}{\partial b_t}=\dfrac{(1+\beta+\beta^2)\Phi(.)}{\beta(1+\beta)-(1+\beta+\beta^2)b_t}>0$. Incorporating (B1) into (13) yields,

(B2) $$b_{t+1}=\Phi(i_t,b_t)b_t$$

On the other hand, incorporating (B1) into (15) yields

(B3) $$i_t\{1-\frac{R}{\Phi(i_t,b_t)}\}=\frac{\beta}{1+\beta}(Ri_{t-1}+d+b_t),$$

Then we implicitly derive

(B4) $$i_t=\Psi(i_{t-1},b_t),$$

with $\Psi_i\equiv\dfrac{\partial\Psi}{\partial i_t}=\dfrac{\beta R/(1+\beta)}{1-R/\Phi+i_tR\Phi_i/\Phi^2}>0$ and $\Psi_b\equiv\dfrac{\partial\Psi}{\partial b_t}=\dfrac{\beta/(1+\beta)-Ri_t\Phi_b}{1-R/\Phi+i_tR\Phi_i/\Phi^2}$.

Two equations (B2) and (B4) constitute the two dimensional system consisting of (i_{t-1},b_t). We first show the local stability of the bubbly steady state. Letting ρ denote an eigenvalue and

$M(\rho)=0$ denote the characteristic equation, we can write

$M(\rho)=\rho^2-\{\Psi_i+\Phi_i\Psi_b b+\Phi_b b+1\}\rho+\Psi_i(\Phi_b b+1)$. This function has the following properties, $M(0)=\Psi_i(\Phi_b b+1)>0$, and $M(1)=-b(\Phi_i\Psi_b+\Phi_b-\Psi_i\Phi_b)$

$=-\dfrac{(1+\beta+\beta^2)(1-R)(1+\beta)b}{\beta(1+\beta)-(1+\beta+\beta^2)b}<0$. Both eigenvalues are positive, and one is larger than unity and the other is less than unity. Therefore there exists the local stable manifold that is saddle-path stable. Furthermore, there exists a unique global stable manifold obtained through backward iteration of the local stable manifold that converges to the bubble steady state.

Proof of Proposition 6

When 1-λ is sufficiently small, the system composed of (8)-(11) is continuous in λ, and approaches the deterministic case as λ →1. Thus it is sufficient to prove that the two-dimensional system of (i_{t-1}, b_t) fully describes the equilibrium. The following lemma is useful for later analysis.

Lemma $\qquad (1+\beta)\beta > b_{t-1}\{(1+\beta)\beta+\dfrac{1}{\lambda\delta_t}\}$,

Proof: Young investors only hold bubbles and from (C-3), we have

$s^y_{t-1}=\{(1+\beta)\beta+\dfrac{1+r_t}{\lambda(1+\tilde{r}_t)}\}^{-1}\{(1+\beta)\beta-\dfrac{\omega+(\tilde{r}_t-r_t)b_{t-1}}{\lambda(1+\tilde{r}_t)}\}>b_{t-1}$, Rearranging yields $(1+\beta)\beta-$ $b_{t-1}\{(1+\beta)\beta+\dfrac{1}{\lambda\delta_t}\}>\dfrac{\omega+(\tilde{r}_t-r_t)b_{t-1}}{\lambda(1+\tilde{r}_t)}$ >0. Q.E.D.

Using (8) and (9), totally differentiation of (10) yields

$$\delta_{t+1}(\frac{\lambda\beta}{1+\lambda\beta\delta_{t+1}+\lambda\beta^2\delta_{t+1}}-\frac{b_t}{1+\beta})d(1+r_{t+1})=\frac{R}{1+\beta}di_t+\frac{\delta_{t+1}(1+r_{t+1})}{1+\beta}db_t$$
$$+\Delta(1+r_{t+1},i_t,b_t)\{\delta_r d(1+r_t)+\delta_i di_{t-1}+\delta_b db_{t-1}\}-\frac{(\delta_{t+1}-1)\lambda\beta\delta_{t+1}\{b_t d(1+r_{t+1})+(1+r_{t+1})db_t\}}{1+\lambda\beta\delta_{t+1}+\lambda\beta^2\delta_{t+1}},$$

where $\Delta(.)$ is a continuous function, $\delta_r \equiv \frac{\partial \delta}{\delta(1+r_{t+1})} = (1-\lambda)b_t \, \mathrm{I}(1+r_{t+1,}i_t, b_t)$,

$\delta_b \equiv \frac{\partial \delta}{\delta b_t} = (1-\lambda)(1+r_{t+1}) \, \mathrm{I}(1+r_{t+1,}i_t, b_t)$, and $\delta_i \equiv \frac{\partial \delta}{\delta i_t} = \frac{(1-\lambda)R(1+r_{t+1})b_t}{(Ri_t+\omega)} \, \mathrm{I}(1+r_{t+1,}i_t, b_t)$, with

$\mathrm{I}(1+r_{t+1,}i_t, b_t) \equiv \frac{\delta_{t+1}}{\lambda(Ri_t+\omega)-(1-\lambda)(1+r_{t+1})b_t}$.

As $1-\lambda \to 0$, $\delta_r \to 0$, $\delta_i \to 0$, $\delta_b \to 0$, and $\delta_{t+1} \to 1$ so that for sufficiently small $1-\lambda$, under Lemma, there exists a well-defined function,

(E1) $$1+r_{t+1} = \hat{\Phi}^r(i_t, b_t),$$

where $\hat{\Phi}^r_i \equiv \partial\hat{\Phi}^r / \partial i_t > 0$ and $\hat{\Phi}^r_b \equiv \partial\hat{\Phi}^r / \partial b_t > 0$. Incorporating (8) and (E1) into (9) and rearranging yield

(E2) $$b_{t+1} = [\frac{\lambda}{\hat{\Phi}^r(i_t, b_t)b_t} - \frac{1-\lambda}{Ri_t+\omega}]^{-1} \equiv \tilde{\Phi}^r(i_t, b_t),$$

with $\tilde{\Phi}^r_i \equiv \partial\tilde{\Phi}^r / \partial i_t$ and $\tilde{\Phi}^r_b \equiv \partial\tilde{\Phi}^r / \partial b_t > 0$.

On the other hand, incorporating (E1) into (11) yields a well-defined function,

(E3) $$i_t = \tilde{\Phi}^i(i_{t-1}, b_t),$$

for sufficiently small $1-\lambda$, with $\tilde{\Phi}^i_i \equiv \partial\tilde{\Phi}^i / \partial i_t > 0$ and $\tilde{\Phi}^i_b \equiv \partial\tilde{\Phi}^i / \partial b_t$, which is implicitly derived from

$$i_t = \frac{Ri_t}{\hat{\Phi}^r(i_t, b_t)} + \frac{\beta}{1+\beta}(Ri_{t-1}+\omega+b_t).$$

Two equations (E2) and (E3) constitute the dynamic system of (i_{t-1}, b_t). Q.E.D.

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Figure 1: Federal Fund Rate and Bubbles

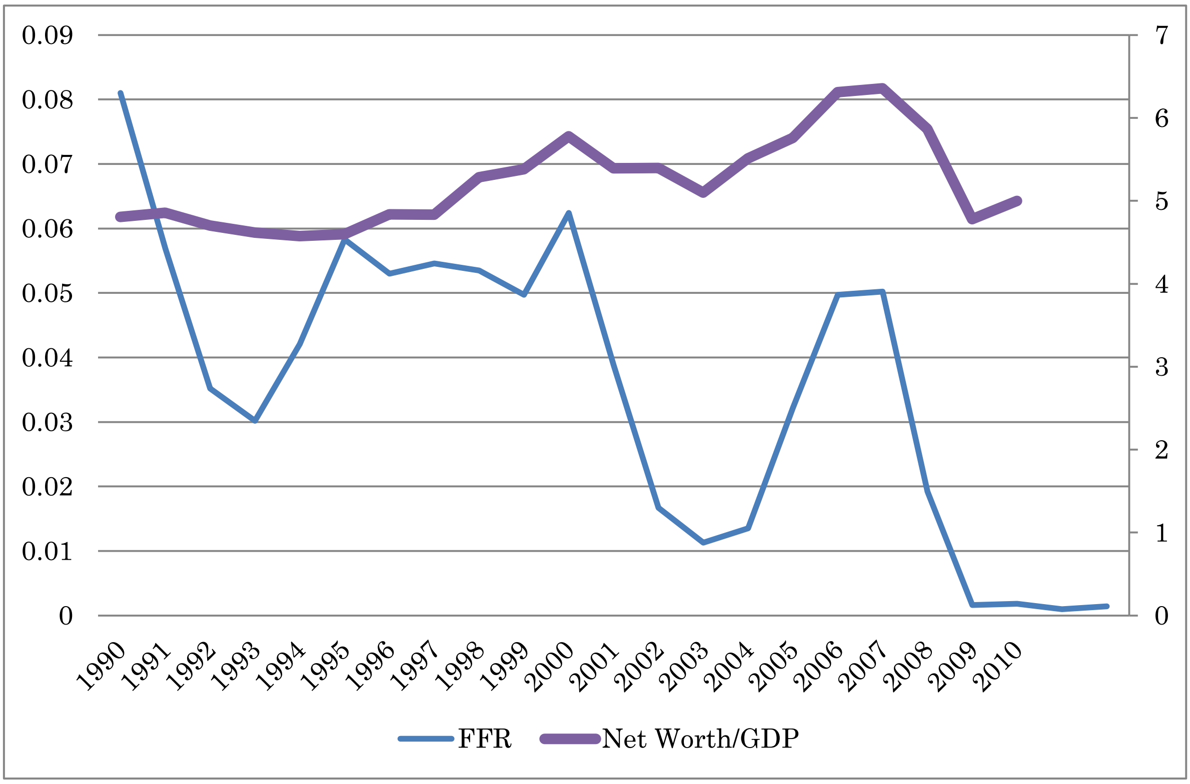

Figure 2: Phase Diagram for Crowding-In

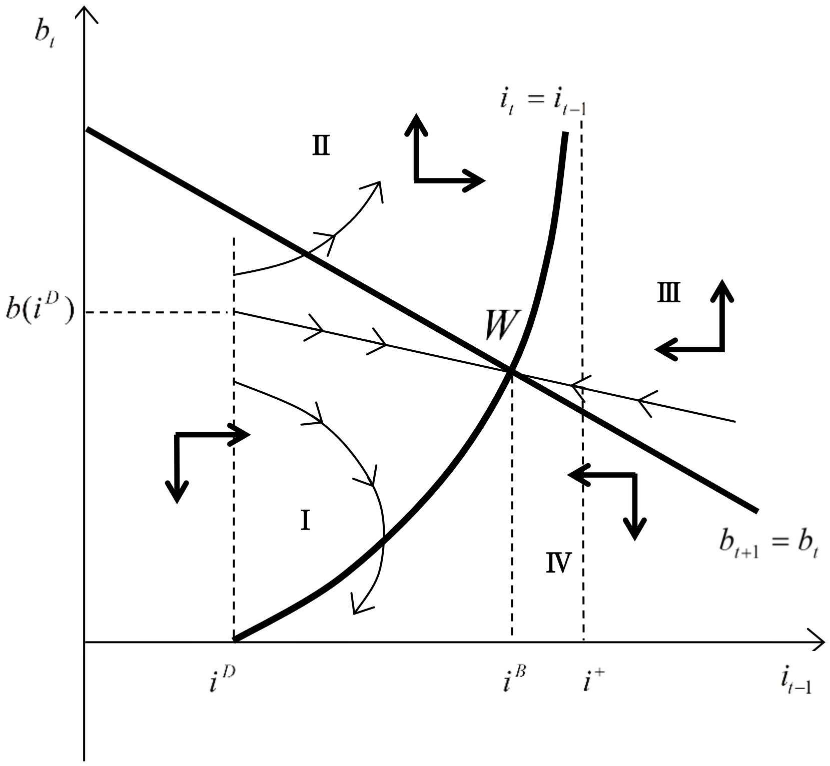

Figure 3: Steady State Equilibrium

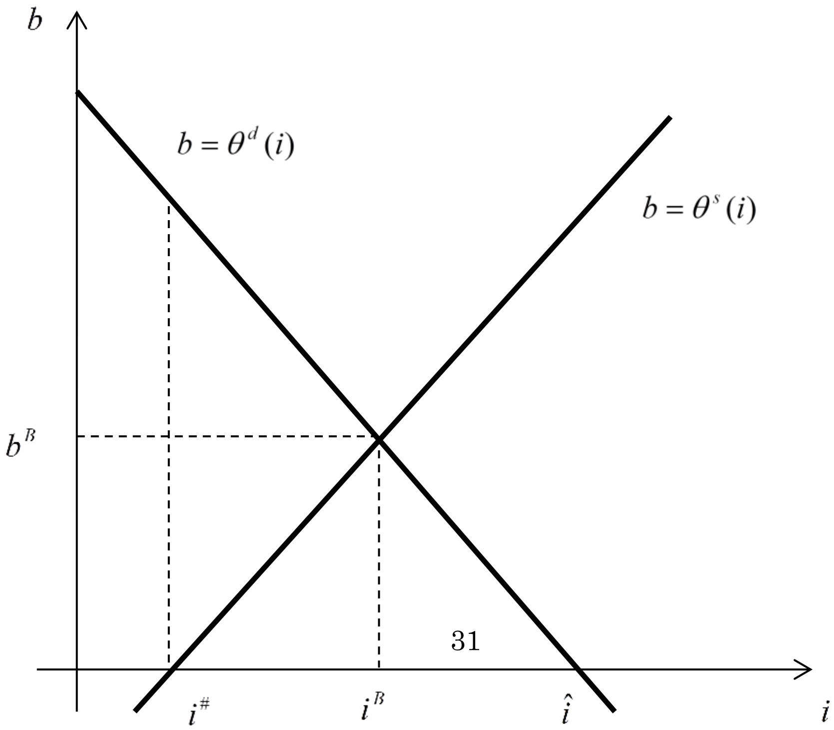

Figure 4: Boom-Bust Cycle

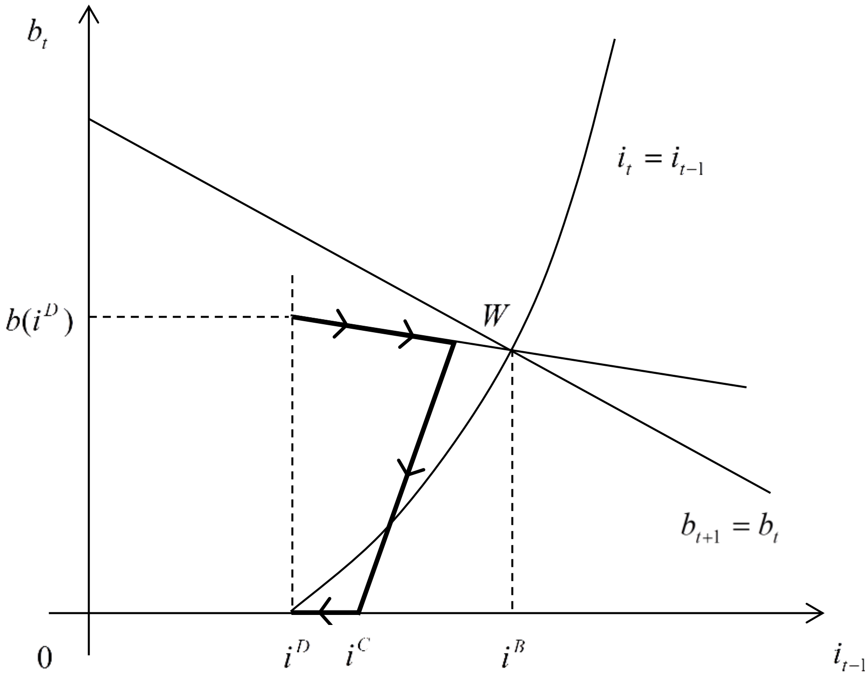

Table 1: Effects of Monetary Tightening

θ	Investment	bubbles	bubbles and public debt	Interest rate	
0.1	0.43	0.29	0.34	0.74	
0.2	0.40	0.25	0.35	0.79	
0.3	0.38	0.21	0.36	0.82	
0.4	0.35	0.18	0.38	0.84	
0.5	0.33	0.14	0.39	0.86	
0.6	0.31	0.10	0.40	0.88	
0.7	0.29	0.07	0.42	0.89	
0.8	0.27	0.03	0.43	0.89	
0.9	-	-	-	-	
1	-	-	-	-	

R=0.2, β =0.2,

Table 2A: Effects on Investment on space $(R, \theta d)$

θa	R= 0.1	R= 0.2	R=0.3	R=0.4	R= 0.5
0.05	0.38	0.43	0.47	0.52	0.58
0.1	0.36	0.40	0.44	0.49	0.55
0.15	0.34	0.38	0.42	0.46	0.52
0.2	0.32	0.35	0.39	0.44	0.49
0.25	0.30	0.33	0.37	0.41	0.46
0.3	0.28	0.31	0.35	0.38	-
0.35	0.27	0.29	0.32	0.36	-
0.4	0.25	0.27	0.30	-	-
0.45	0.23	-	-	-	-
0.5	-	-	-	-	-

β =0.7, λ =0.9

Table 2B: Effects on Bubbles on space $(R,\theta d)$

θa	R= 0.1	R= 0.2	R=0.3	R=0.4	R= 0.5
0.05	0.32	0.29	0.24	0.19	0.12
0.1	0.29	0.25	0.21	0.16	0.09
0.15	0.25	0.21	0.17	0.12	0.06
0.2	0.21	0.18	0.14	0.09	0.04
0.25	0.17	0.14	0.10	0.06	0.01
0.3	0.13	0.10	0.07	0.03	-
0.35	0.09	0.07	0.03	0.00	-
0.4	0.05	0.03	0.00	-	-
0.45	0.01	-	-	-	-
0.5	-	-	-	-	-

Table 3: Killing the Boom or Weakening the Recession