

Revisiting Asymptotic Theory for Principal Component Estimators of Approximate Factor Models

PEIYUN JIANG[†], YOSHIMASA UEMATSU^{*}, and TAKASHI YAMAGATA[‡]

[†]*Graduate School of Management, Tokyo Metropolitan University*

^{*}*Department of Social Data Science, Hitotsubashi University*

[‡]*Department of Economics and Related Studies, University of York*

[‡]*Institute of Social and Economic Research, Osaka University*

April 30, 2024

Abstract

It is well-known that the approximate factor models have the rotation indeterminacy. It has been considered that the principal component (PC) estimators estimate some rotations of the true factors and factor loadings, but the rotation matrix commonly used in the literature depends on the PC estimator itself. This raises a question: what does the PC estimator consistently estimate? This paper aims to explore the answer. We first show that, assuming a quite general weak factor model with the r signal eigenvalues diverging possibly at different rates, there always exists a unique rotation matrix composed only of the true factors and loadings, such that it rotates the true model to the identifiable model satisfying the standard r^2 restrictions. We call the rotated factors and loadings the pseudo-true parameters. We next establish the consistency and asymptotic normality of the PC estimator for this pseudo-true parameter. The results give an answer for the question: the PC estimator consistently estimates the pseudo-true parameter. We also investigate similar problems in the factor augmented regression. Finite sample experiments confirm the excellent approximation of the theoretical results.

Keywords. Weak factor model, Rotation matrix, Consistency and asymptotic normality, Factor augmented regression.

*Correspondence: Yoshimasa Uematsu, Department of Social Data Science, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo 186-8601, Japan (E-mail: yoshimasa.uematsu@r.hit-u.ac.jp).