The Economic Implications of Mujin-ko

by

Kenichi Sakakibara

Graduate School of Humanities and Social Sciences
Chiba University

September 2014

Abstract

This paper discusses the economic implications of Mujin-ko, which is a traditional mutual financing association in Japan, and is regarded as a form of rotating savings and credit association (ROSCA). A conventional view regards ROSCA as a kind of savings. However, this paper regards it as a kind of insurance, and considers it from the aspect of welfare by using a simple multi-period model. It is proven that the allocation induced by Mujin-ko is equivalent to an allocation realized in complete market scheme, which is ex-ante Pareto Optimal. However, it is also shown that the Mujin-ko allocation is fragile in the sense of time inconsistency that the agents without the first period shock prefer to introduce loan markets rather than keeping Mujin-ko contract after the first period shock is realized, by which a sub-optimal allocation emerges in the economy. This result seems to suggest the possibility that the existence of modern banking system could be an impediment to economic welfare.

JEL Classification Number: N2, G1, G2
1 Introduction

This paper considers the implication of Mujin-ko. Mujin-ko, which is also called Tanomoshi-ko, is a traditional mutual financial institution in Japan, and is regarded as a form of rotating savings and credit association (ROSCA) observed in Japan, East and South-East Asia, and all over the world. A conventional view of ROSCA, including Mujin-ko, regards it to be a substitute for modern and efficient banks, and is used by those who cannot access them because of poverty or living in a remote rural area. However, this view seems not to explain the observation that Mujin-ko exists in Japan, where financial system is highly developed, because efficient and competitive banks force inefficient Mujin-ko to disappear from the economy in the long run. The purpose of this paper is to present another view of Mujin-ko.

Theoretical research on ROSCA in Economics was started by Besley, Coate and Loury (1993). From the viewpoint described above, regarding ROSCA as a kind of saving to purchase some expensive good, they classify it into random ROSCA and bidding ROSCA, and prove that the two ROSCAs are more efficient than autarky in a framework of a constant income stream and two goods, a consumption good and a durable good. Also, in their paper (1994), they compare the two ROSCAs with credit market, and proved that the two ROSCAs are inefficient, that bidding ROSCA is less efficient than credit market, and that, under a certain condition, random ROSCA is more efficient than credit market.

There has been a variety of Mujin-ko, and some of them can be viewed just the same as the conventional one. However, Mujin-ko is “a kind of fork mutual financing institution, and originated from an extraordinary relief system (Asajima, 1983, p.3)””. Therefore there are another type of Mujin-ko, which can be regarded as a kind of insurance, like share cropping such as Hunusan in Philippines, against unstable income or expenditure stream, rather than a kind of saving in a stable income stream. I consider the economic implication of another Mujin-ko, which is formed among people who confront random income streams.

To do this, I consider a multi-period economy in which there are a finite number of agents and a perishable consumption good. There is no production technology in the economy. Each agent is endowed with a constant amount of consumption good as income in each period. But there is a random shock in each period by which an agent is hit and his income decreases. This randomness drives him to form Mujin-ko. I assume that Mujin-ko is established by a unanimous agreement of the agents in the economy. If established, every agent contributes a certain amount of consumption good in each period, and an agent hit by the shock of that period
receives all the collected good. In this paper, the allocation of consumption good over time under Mujin-ko is derived and consider the efficiency of it by comparing with other allocations. In this paper, I also consider an allocation under a complete market and that under a competitive banking system in the form of loan market, under the assumption that banks are competitive without any reserve requirement nor any cost of lending and borrowing.

The main result of the paper is as follows. First, the Mujin-ko allocation is proven to be efficient in ex ante sense. Next, by using a numerical example, the Mujin-ko allocation is more efficient in ex ante sense than the allocation in banking system. However, the ex ante efficiency of the Mujin-ko allocation does not imply ex post efficiency. Indeed, once a shock occurred in the first period, a majority of agents prefers the banking system to the Mujin-ko. These results seem to suggest that, although the Mujin-ko brings an ex ante optimal allocation, time inconsistency arises from the introduction of competitive banking system in the economy, and it gives rise to the cancellation of the Mujin-ko and results in an inefficient allocation in the economy.\(^{5}\)

The rest of the paper is organized as follows. In the next section, the physical environment, and then the three systems, i.e., Mujin-ko, complete market and banking system, are considered. In section 3, by using a numerical example, the efficiency of the three systems is discussed. Finally, some remarks are given.
2 Economy

Physical Environment

I consider a three period economy in which three agents exist. There is a consumption good, which is perishable, in each period. There is no production technology in the economy. Each agent \( h \) \((h = 1, 2, 3)\) has a common expected utility function \( v(c_1^h, c_2^h, c_3^h) = u(c_1^h) + \beta u(c_2^h) + \beta^2 u(c_3^h) \) \((0 < \beta < 1)\), where \( c_t^h \) is the consumption of agent \( h \) at period \( t \) \((t = 1, 2, 3)\) and \( \beta \) satisfies \( 0 < \beta < 1 \). For simplicity, below I assume that \( u(c) \) is given by \( u(c) = \sqrt{c} \).

Let \( w_{j}^{h} \) be the initial endowment of agent \( h \) at period \( t \). Each agent \( h \) is endowed with three units of consumption good over time, but he suffers a shock once in lifetime which lowers his initial endowment of that period to zero. The shocks are given to the tree agent equally as follows.

At the beginning of period one, one agent among the three suffers the shock with the probability a third. Let \( s_i \) be the state at period one in which agent \( i \) \((i = 1, 2, 3)\) suffers the shock. The probability that \( s_i \) occurs, \( P[s_i] \), is given by a third for all \( i \). Also let \( s_{ij} \) \((i, j = 1, 2, 3; i \neq j)\) be the state at period two in which agent \( i \) suffers the shock at period one and agent \( j \) suffers the shock at period two. The probability that \( s_{ij} \) occurs, \( P[s_{ij}] \), is given by one sixth \((i, j = 1, 2, 3; i \neq j)\). Finally, let \( s_{ijk} \) \((i, j, k = 1, 2, 3; i \neq j, k; j \neq i, k; k \neq i, j)\) be the state at period three in which agent \( i \) suffers the shock at period one, agent \( j \) suffers the shock at period two, and agent \( k \) suffers the shock at period three. In this case, the probability that \( s_{ijk} \) occurs, \( P[s_{ijk} | h_3, h_2, h_1] \), is given one sixth \((i, j, k = 1, 2, 3; i \neq j, k; j \neq i, k; k \neq i, j)\). The stream of initial endowments, \( \{(w^1_t, w^2_t, w^3_t)\}_{t=1}^3 \), for state \( s_{ijk} \) \((i, j, k = 1, 2, 3; i \neq j, j \neq k, k \neq i)\) is given by

\[
\{(w^1_1, w^2_1, w^3_1), (w^1_2, w^2_2, w^3_2), (w^1_3, w^2_3, w^3_3)\}
\]

\[
\begin{cases}
\{(0, 3, 3), (3, 0, 3), (3, 3, 0)\} & \text{if } s_{ijk} = s_{123} \\
\{(0, 3, 3), (3, 3, 0), (3, 0, 3)\} & \text{if } s_{ijk} = s_{132} \\
\{(3, 0, 3), (0, 3, 3), (3, 3, 0)\} & \text{if } s_{ijk} = s_{213} \\
\{(3, 0, 3), (3, 3, 0), (0, 3, 3)\} & \text{if } s_{ijk} = s_{231} \\
\{(3, 3, 0), (0, 3, 3), (3, 0, 3)\} & \text{if } s_{ijk} = s_{312} \\
\{(3, 3, 0), (3, 0, 3), (0, 3, 3)\} & \text{if } s_{ijk} = s_{321} 
\end{cases}
\]

Below I introduce two types of markets, i.e., a complete market and loan markets in the economy. Let us define “period 0” as the period before the first shock is
realized. The complete market is opened at period 0. On the other hand, a loan market is opened at each period \( t (t = 1, 2) \) after period \( t \) shock is realized.

**Complete Market**

Suppose there exists a complete market in this economy at period 0. Let \( p_i, p_{ij} \) and \( p_{ijk} \) be the price and of one unit of contingent claim of consumption good in state \( s_i, s_{ij} \) and \( s_{ijk} \), respectively \((i, j, k = 1, 2, 3; i \neq j, j \neq k, k \neq i)\). Also, let \( c_i^h \), \( c_{ij}^h \) and \( c_{ijk}^h \) be the consumption of agent \( h \) in state \( s_i, s_{ij} \) and \( s_{ijk} \), respectively \((i, j, k = 1, 2, 3; i \neq j, j \neq k, k \neq i)\).

Next, the initial endowment of agent \( h \) \((h = 1, 2, 3)\) is given as follows.

\[
(w_{1h}^h, w_{2h}^h, w_{3h}^h, w_{12h}^h, w_{13h}^h, w_{23h}^h, w_{132h}^h, w_{123h}^h, w_{312h}^h, w_{321h}^h)
= \begin{cases} 
(0, 3, 3, 3, 0, 3, 0, 3, 3, 0, 3, 0) & \text{for } h = 1 \\
(3, 0, 3, 0, 3, 3, 0, 3, 0, 3, 0, 3) & \text{for } h = 2 \\
(3, 3, 0, 3, 0, 3, 0, 3, 0, 3, 3, 3) & \text{for } h = 3
\end{cases}
\]

Under the above setup, the definition of competitive equilibrium is described in the usual manner.\(^6\)

**Proposition 1.** There exists a competitive equilibrium in the complete market such that

\[
c_i^{h*} = 2, c_{ij}^{h*} = 2, c_{ijk}^{h*} = 2 \ (h = 1, 2, 3; i, j, k = 1, 2, 3; i \neq j, j \neq k, k \neq i)
\]

\[
p_i^* = 1, p_{ij}^* = \beta/2, p_{ijk}^* = \beta^2/2 \ (i, j, k = 1, 2, 3; i \neq j, j \neq k, k \neq i)
\]

**Proof of Proposition 1.** Trivial.

Note that the above equilibrium allocation is apparently Pareto Optimal. Also, the real gross interest rate from period \( t \) to \( t + 1 \) \((t = 1, 2)\) is given by \(1/\beta\).

**Loan Market**

Let us introduce banks in the economy. The banks play the role of financial intermediaries by taking deposits in terms of consumption good from people at
real interest and lending them to borrowers at real interest. For simplicity, I assume that banks act competitively. Also I assume that no reserve requirements are imposed on bank deposits and that banks are not imposed and there are no cost for bank activities. Under these assumptions, the banks make no profits or loss in equilibrium, and can be regarded as just the auctioneer of the loan markets. Therefore below I do not consider the bank explicitly in the analysis of loan markets.

Now suppose that there exists a loan market in each period $t$ ($t = 1, 2$) in which a unit of consumption good at period $t$ is exchanged for $r(t)$ units of consumption good at period $t + 1$, where $r(t)$ is the real gross interest rate at period $t$ and depends on the state at period $t$. I assume that agents are rational and behave competitively in the market.

**Utility Maximization**

For each period $t$ ($t = 1, 2$) after period $t$ shock is observed, each agent $h$ ($h = 1, 2, 3$) maximizes his expected utility subject to his budget constraint, which is given by

$$c^h(1) + l^h(1) \leq w^h(1)$$
$$c^h(2) + l^h(2) \leq w^h(2) + r(1)l^h(1)$$
$$c^h(3) \leq w^h(3) + r(2)l^h(2)$$

where $c^h(t)$, $w^h(t)$ and $l^h(t)$ are his consumption, endowment at period $t$ and loan supply ($l^h(t) < 0$ implies the demand for loan) at period $t$, respectively.

In the first period, the second period shock is not realized, and $w^h(2)$ and $r(2)$ are uncertain. Therefore, in general, agent $h$ is to choose optimal $c^h(1)$ and $l^h(1)$. However, in the model, the number of agents who is hit by the shock in each period $t$ is one irrelevant to the realized state at period $t$. In the first period, one agent is hit by shock and the remaining two agents are not. Because the three agents are symmetric, the aggregate loan supply and demand in the economy at period 1 becomes the same irrelevant to the realized state. This implies that the market clearing interest rate at period 1 becomes the same irrelevant to the realized state. In period 2, the agent who was hit by the first period shock is not hit by the second period shock, and one of the remaining two agents who was not hit by the period one shock is hit by the period two shock. Therefore the aggregate loan supply and demand in the economy at period 2 also becomes the same irrelevant to the realized state at period 2, which implies the market clearing interest rate $r(2)$
is the same irrelevant to the realized state at period 2. For this reason, below I consider only such an equilibrium in which \( r(1) \) and \( r(2) \) are deterministic. Under this condition, each agent maximizes his expected utility under given deterministic \( r(t) \)'s and random \( w^h(t) \)'s, and his loan supply becomes a function of \( r(1) \) and \( r(2) \).

Now let \( L_t[r(1), r(2)] \) be the aggregate loan supply at period \( t \) \((t = 1, 2)\) such that

\[
L_t[r(1), r(2)] = \sum_{h=1}^{3} l^h(t)
\]

Then the market clearing condition is given as follows.

\[
L_t[r(1), r(2)] = 0 \quad \text{for } t = 1, 2.
\]

**Definition.** A loan market equilibrium is \((r^*(1), r^*(2))\) such that

\[
L_t[r^*(1), r^*(2)] = 0 \quad \text{for } t = 1, 2.
\]

**Proposition 2.** There exists a unique loan market equilibrium with \( r^*(2) = 1/\beta \).

**Proof of Proposition 2.** Trivial.

**Mujin-ko**

Let us consider a basic and simple Mujin-ko. In this economy, the three agents may form an organization, which we call Mujin-ko, at period 0 conditional on unanimous agreement. Under Mujin-ko, in each period \( t \) \((t = 1, 2, 3)\), every agent \( h \) with \( w^h_t = 3 \) contributes one unit of consumption good, and agent \( j \) who suffers the shock at period \( t \) and his initial endowment at period \( t \), \( w^j_t \) is zero, receives all the contribution at period \( t \).

Then, the allocation by Mujin-ko is given by

\[
(c^h_i, c^h_{ij}, c^h_{ijk}) = (2, 2, 2)
\]
for any $h$ and any state $s_i$, $s_{ij}$ and $s_{ijk}$ ($i,j,k = 1,2,3; i \neq j, j \neq k, k \neq i$). I call this allocation “Mujin-ko Allocation”.

**Remark.** This Mujin-ko allocation is equivalent to that in complete market equilibrium, which is Pareto Optimal. If loan market is introduced to the economy under the Mujin-ko allocation, then apparently the equilibrium allocation is the same as the Mujin-ko allocation. In the equilibrium, it is easily shown that the gross real interest rate $r(t)$ is given by $r(t) = 1/\beta$.

**Numerical Example**

Here I consider the case that $\beta = 0.9$. Then each equilibrium is given as follows.

**Complete market**

First, for complete market, the equilibrium is given by

\[
\begin{align*}
  c_{ih}^* &= 2, \quad c_{ijh}^* = 2, \quad c_{ijkh}^* = 2 \quad (h = 1,2,3; i,j,k = 1,2,3; i \neq j, j \neq k, k \neq i) \\
  p_i^* &= 1, \quad p_{ij}^* = 0.45, \quad p_{ijk}^* = 0.405 \quad (i,j,k = 1,2,3, i \neq j, j \neq k, k \neq i)
\end{align*}
\]

**Loan market**

The equilibrium in loan market is given by $r(1)^* = 1.53$ and $r(2)^* = 1/\beta = 1.11$. For the allocation, it is given as follows.

\[
\begin{align*}
  (c^h(1), c^h(2), c^h(3)) &= (2.42, 1.05, 1.05) \quad \text{if } h \text{ is hit by the shock at period 1} \\
  (c^h(1), c^h(2), c^h(3)) &= (1.79, 2.39, 2.39) \quad \text{if } h \text{ is hit by the shock at period 2} \\
  (c^h(1), c^h(2), c^h(3)) &= (1.79, 2.55, 2.55) \quad \text{if } h \text{ is hit by the shock at period 3.}
\end{align*}
\]

**Mujin-ko**

Mujin-ko allocation is equivalent to that in complete market equilibrium.
3 Time Inconsistency

In this section, I will consider whether Mujin-ko is formed at period 0. So far, three systems, i.e., complete market, loan market and Mujin-ko, have been considered. Among them, because complete market is an imaginary one and its allocation is the same as that in Mujin-ko, below I consider the allocation in loan market and that in Mujin-ko. I assume that loan market exists. Also I assume that the formation decision of each agent is based on his expected utility. For simplicity, I employ the values of numerical example in the previous section.

Formation of Mujin-ko

First, By using the values of the numerical example, the expected utility of each agent $h$ ($h = 1, 2, 3$) for Mujin-ko allocation at period 0 is given by

$$E[\ln c^h(1) + \beta \ln c^h(2) + \beta^2 \ln c^h(3)]$$

$$= \ln 2 + (0.9) \ln 2 + (0.9)^2 \ln 2 = 1.88.$$  

On the other hand, his expected utility for loan market at period 0 is given by

$$E[\ln c^h(1) + \beta \ln c^h(2) + \beta^2 \ln c^h(3)]$$

$$= (1/3)(\ln(2.42) + (0.9) \ln(1.05) + (0.9)^2 \ln(1.05))$$

$$+(1/3)(\ln(1.79) + (0.9) \ln(2.39) + (0.9)^2 \ln(2.39))$$

$$+(1/3)(\ln(1.79) + (0.9) \ln(2.55) + (0.9)^2 \ln(2.55)) = 1.74$$

Apparently, the expected utility of each agent $h$ ($h = 1, 2, 3$) for Mujin-ko is larger than that for loan market. Therefore the agents agree to form Mujin-ko at period 0.

**Proposition 3.** Suppose $\beta = 0.9$. Then Mujin-ko is formed by unanimous agreement of the agents.

Mujin-ko at Period One

Now let us consider the expected utility of each agent after the first shock hits an agent at the beginning of period 1. First, if the agents keep Mujin-ko, then the allocation after the shock is the same as that at period 0. Therefore the expected utility of each agent $h$ ($h = 1, 2, 3$) is given by 1.88.
On the other hand, in loan market, the expected utility of agent \( h \) who is hit by the shock and that who is not are different. The former is given by

\[
E[\ln c^h(1) + \beta \ln c^h(2) + \beta^2 \ln c^h(3)]
= (\ln(2.42) + (0.9) \ln(1.05) + (0.9)^2 \ln(1.05)) = 0.967.
\]

And the latter is given by

\[
E[\ln c^h(1) + \beta \ln c^h(2) + \beta^2 \ln c^h(3)]
= (1/2)(\ln(1.79) + (0.9) \ln(2.39) + (0.9)^2 \ln(2.39))
+ (1/2)(\ln(1.79) + (0.9) \ln(2.55) + (0.9)^2 \ln(2.55)) = 2.1275.
\]

The former is smaller than the expected utility on Mujin-ko, but the latter is larger than that. The two allocations are Pareto non-comparable, though, for those who are not hit by the first period shock, the cancellation of the agreement on Mujin-ko will make them better off. Therefore, if Mujin-ko formation is not binding, it will be cancelled, and the agents will go to banks instead of Mujin-ko. To sum up the above, we have the following proposition.

**Proposition 4.** Suppose that \( \beta = 0.9 \). Then Mujin-ko is formed at period 0 under unanimous agreement of agents. But if the agreement is not binding, the Mujin-ko is cancelled after the shock is realized at period 1.

The above proposition states the existence of time inconsistency on Mujin-ko formation if the agreement on the formation is not binding. Of course, if the agreement is regarded as a legally protected contract, then the Mujin-ko is not cancelled at period 1, and an ex ante Pareto Optimal allocation will be realized. However, if the agreement is not binding enough, then it is cancelled and ex ante non Pareto Optimal allocation is realized in the economy. Indeed, the agreement may not be considered to be a contract, because no one contributes the good just after the first shock is realized.\(^7\)
4 Concluding Remarks

In this paper, I have considered the implication of Mujin-ko that plays the role of insurance in the economy. As it has been shown, even if Mujin-ko is efficient in ex ante sense, the existence of banking system, or loan market, might destroy the efficient allocation and an inefficient allocation could emerge. This seems to suggest another view of Mujin-ko, which regards it as an efficient device for the economy, rather than as a pre-modern and inefficient substitute for modern and efficient banking system. This view also seems to suggest another view of banking system that there might be a better device for us to improve our society than banking system or loan markets.\footnote{2}

Finally, the model discussed here is a very simple one, and needs generalization such as the introduction of production technologies or another type of shocks, which should be investigated in the future.
Footnotes

1. In their paper of an empirical study in Taiwan, Levenson and Besley (1996) found that the participation of ROSCA is highest among high-income households.

2. van den Brink and Chavas (1997) studied, by using the data in Africa, the effect of loan markets on ROSCA.

3. For the recent history of Mujin-ko, see, for example, Asajima (1983) and Dekle and Hamada (2000).

4. For Hunusan, see, for example, Hayami and Kikuchi (2000).

5. For Time Inconsistency, see, for example, Ljungqvist and Sargent (2004).

6. For the definition, see Debreu (1959).

7. Asajima (1983) gives an example of the original form of Mujin-ko in which it was formed to relieve a villager after the person had suffered an accident or disease.

8. For a similar viewpoint, Diamond and Dybvig (1983) discusses a defect of banking system from the aspect of bank run.
References


