Institute for Economic Studies, Keio University

Keio-IES Discussion Paper Series

Incentives and discrimination: A functional optimization approach

Tetsutaro Hatakeyama

21 March, 2025
DP2025-003
https://ies.keio.ac.jp/en/publications/24972/

Keio University

CALAMVS GLADIO FORTIOR
1858

Institute for Economic Studies, Keio University
2-15-45 Mita, Minato-ku, Tokyo 108-8345, Japan
ies-office@adst.keio.ac.jp
21 March, 2025

Incentives and discrimination: A functional optimization approach

Tetsutaro Hatakeyama

Keio-IES DP2025-003
21 March, 2025
JEL Classification: C72; D23; D62; D63; J71
Keywords: team production; discrimination; incentives; production technology

Abstract

In team production models, an optimal incentive scheme that guarantees workers' effort investment can be discriminatory (Winter, 2004). We demonstrate how this trade-off in incentives and discrimination disappears in a complementary setting. In our model, as well as payment scheme for workers, the firm chooses a production technology, mapping from workers' effort choice to the success probability of the project. Contrary to the known result, our firm-optimal incentive scheme does not discriminate any symmetric workers.

Tetsutaro Hatakeyama
Graduate School of Economics, Keio University
t.hatakeyama@keio.jp

Acknowledgement: I would like to thank the participants at the 2024 JEA Autumn Meeting for helpful comments. I acknowledge the financial support from JST SPRING Grant Number JPMJSP2123 and the Keio Economic Society.

Incentives and discrimination: A functional optimization approach*

Tetsutaro Hatakeyama†

March 21, 2025

Abstract

In team production models, an optimal incentive scheme that guarantees workers' effort investment can be discriminatory (Winter, 2004). We demonstrate how this trade-off in incentives and discrimination disappears in a complementary setting. In our model, as well as payment scheme for workers, the firm chooses a production technology, mapping from workers' effort choice to the success probability of the project. Contrary to the known result, our firm-optimal incentive scheme does not discriminate any symmetric workers.

JEL Classification: C72; D23; D62; D63; J71

Keywords: team production; discrimination; incentives; production technology

1 Introduction

Workplace discrimination is a serious issue in the society. In regions including Australia, the European Union, the United Kingdom, and the United States, anti-discrimination acts regally protect historically underrepresented groups of workers in hiring or promotion.[1] In the global capital markets, firms are required to be ethical and responsive to prevent such discrimination. Several reports show that both employees and the economy bear billions or even trillions of dollars of costs due to workplace discrimination.[2] While sociologists, psychologists, and behavioral scientists

*I would like to thank the participants at the 2024 JEA Autumn Meeting for helpful comments. I acknowledge the financial support from JST SPRING Grant Number JPMJSP2123 and the Keio Economic Society.

†Graduate School of Economics, Keio University, 2-15-45 Mita, Minato-ku, Tokyo 108-8345, Japan. E-mail: t.hatakeyama@keio.jp

[1] For the regulations in Australia, see webpage https://www.ag.gov.au/rights-and-protections/human-rights-and-anti-discrimination/australias-anti-discrimination-law (accessed March 14, 2025). For the regulations in the European Union, see the webpage https://www.europarl.europa.eu/legislative-train/theme-a-new-push-for-european-democracy/file-anti-discrimination-directive (accessed March 14, 2025).

[2] For the cost employees bear, see the webpage https://www.shrm.org/topics-tools/news/hr-magazine/ceo-racial-injustice-work-costs-us-billions (accessed March 14, 2025). For the cost the economy bears, see the webpage https://www.imf.org/en/Publications/fandd/issues/2020/09/the-economic-cost-of-racism-losavio (accessed March 14, 2025).

study the root of such discrimination, in economics, Winter (2004) provides a nontrivial source of discrimination: a firm may discriminate its workers in payments to give them incentives to work.

Team production models capture a situation where a team of workers engages in a project. Each worker chooses whether to exert costly effort, which increases the success probability of the project. The mapping from the workers' effort choice to the success probability is named the *production technology* and fixed exogenously. Since workers may avoid exerting effort to save the costs (Holmstrom, 1982), the literature has investigated how to control the situation (Segal 2003; Winter 2004; Agastya and Birulin 2023). The firm chooses a success-dependent bonus amount for each worker so that (i) each worker exerts effort in the *unique* Nash equilibrium (full implementation) and (ii) given (i), the total bonus amount is the smallest (optimality). As the workers being symmetric, discrimination is purely defined as the gap in bonus amounts for different workers. Winter's (2004) findings are on the relationship between discrimination and production technology: The firm chooses a (fully) discriminatory bonus profile if and only if the production technology is increasing returns to scale (IRS).

Our question is the following: How will the firm's (profit-maximizing or cost-minimizing) technology choice affect the level of discrimination? While the literature considers a prefix technology as primitive, it is also natural to consider that the firm sets a technology as its optimal choice.The next example shows a case in which multiple production technologies are possible.[3]

Example 1. Two consultants, Alice and Bob, have two complementary tasks, preparing analytical results (A) and preparing a reporting document (R). When x percent of task A and y percent of task R are completed, the probability that the project succeeds is the *minimum* of x and y. each of Alice and Bob has two choices: to complete her/his task ("work") and not to complete ("shirk").
Case 1: Alice does analysis A and Bob does report R. The success probability is (i) 100% if both work and (ii) 0% otherwise.
Case 2: Each of Alice and Bob does a half of analysis and a half of report. The success probability is (i) 100% if both work, (ii) 50% if only one of them works, and (iii) 0% if both shirk. ◇

Precisely, we search for the combination of production technology and bonus profile, under which (i') each worker exerts effort in the unique Nash and (ii') given (i'), the total bonus amount is the smallest. We assume a rich domain of production technologies (Assumption 1).[4] Compared to the prefix technology case, the firm can reduce the bonus amount in total but it is ambiguous whether the variance is increased or decreased. For this functional optimization problem, a set of simple logic provides a clear answer.

[3]We consider our problem as the firm's "long-run" problem, compared to the standard (short-run) problem with a fixed technology. Long- and short-run problems are known to lead to different conclusions (Kreps and Scheinkman 1983; Davidson and Deneckere 1986).

[4]Another plausible scenario is that the firm can choose a technology from a narrowly restricted domain or it is costly to choose specific technologies. This scenario is in between Winter's and our cases.

The answer is quite positive. The firm chooses a symmetric bonus profile under which all workers are offered the same amount (Theorem 1). This result is in sharp contrast to Winter's results, which characterize discriminatory optimal bonus scheme by the shape of prefix technology. The proof sketch is as follows. We first show that, to make workers exert effort, technologies where optimal schemes are known to be discriminatory cost relatively higher total bonus amount. Under other complex technologies, it is tough to find optimal bonus profiles (thus we cannot directly compare the amounts). Instead, we show that for any such technology, as long as it is symmetric, we can construct another technology under which the total bonus amount is smaller (thus, such complex technologies cannot be optimal).

Our finding makes clear that the objective of a profit-maximizing firm described in (i') and (ii') above is aligned with fair treatment among workers. Hence, the finding provides a relatively optimistic view that, as long as the technology is freely chosen, a firm will not choose a discriminatory incentive scheme. In other words, if a firm has a discriminatory taste and sets a discriminatory bonus profile, it loses potential benefit from reduced total bonus payment.

2 Model

We consider a team production model with $n \geq 2$ agents who are engaged in a project. The set of agents is $N = \{1, 2, \ldots, n\}$. Each agent chooses whether to exert effort ("work") or not ("shirk"). The cost $c > 0$ of working is the same for all agents. The project succeeds with a probability that depends on subset $J \subseteq N$ of agents who exert effort: A **(production) technology** $P : 2^N \to [0, 1]$ is a function mapping J to success probability $P(J)$. The firm, principal, cannot observe each agent's choice on exerting effort. The firm sets reward ("bonus") only depending on whether the project succeeds. Each agent $i \in N$ receives amount B_i if the project succeeds and amount zero if the project fails. A **bonus profile** is $B = (B_i)_{i \in N}$.

While the literature focuses on fixed technology P, our model allows the firm to choose P. We call pair (P, B) an **incentive scheme.** Let $\mathcal{P}$ be the collection of possible technologies.

Assumption 1. $\mathcal{P}$ is the collection of *any* functions satisfying 1, 2, and 3.
1. If all agents work, the project succeeds for sure: $P(N) = 1$.
2. Strictly increasing: for any $J, J' \subseteq N$ with $J \subsetneq J'$, $P(J) < P(J')$.
3. Either symmetric, supermodular, or submodular:
symmetric: for any $J, J' \subseteq N$ with $|J| = |J'|$, $P(J) = P(J')$.
supermodular: for any $J, J' \subseteq N$ with $J \nsubseteq J'$ and $J' \nsubseteq J$, $P(J \cup J') + P(J \cap J') \geq P(J) + P(J')$.
submodular: for any $J, J' \subseteq N$ with $J \nsubseteq J'$ and $J' \nsubseteq J$, $P(J \cup J') + P(J \cap J') \leq P(J) + P(J')$.

The domain $\mathcal{P}$ is rich enough to contain technologies studied in the literature. Winter (2004) considers symmetric technologies while recent studies such as Halac et al. (2021) and Boyarchenko et al. (2025) require supermodularity instead. Each incentive scheme (P, B) induces a normal form game among agents: Each agent $i \in N$ has two strategies, to work ($s_i = 1$) or to shirk ($s_i = 0$) where agents choose strategies simultaneously. Slightly abusing the notation, for each strategy profile $s = (s_i)_{i \in N}$, the success probability is $P(s) := P(\{j \in N : s_j = 1\})$. Agent i's payoff is $P(s)B_i - c$ if she works ($s_i = 1$) and $P(s)B_i$ if she shirks ($s_i = 0$).

The firm's goal is to achieve the least-cost incentive scheme that induces a unique Nash equilibrium in which all agents work. That is, the firm wishes to choose (P^*, B^*) where (i') $s^* = (1, 1, \ldots, 1)$ is the unique Nash and (ii') among incentive schemes satisfying (i'), (P^*, B^*) minimizes the total bonus payment $\sum_{i \in N} B_i$. We say that an incentive scheme (P, B) **uniquely implements work** if for each $\varepsilon > 0$, $(P, (B_i + \varepsilon)_{i \in N})$ induces a unique Nash equilibrium $s^* = (1, 1, \ldots, 1)$.[5] An incentive scheme (P, B) is P**-optimal** if B minimizes the total bonus payment $\sum_{i \in N} B_i$ among all incentive schemes that uniquely implement work and whose technologies are P. We say that an incentive scheme is **globally optimal** if it minimizes the total bonus payment $\sum_{i \in N} B_i$ among all incentive schemes that uniquely implement work.

Since agents are symmetric, it is fair to set a symmetric bonus profile. An incentive scheme is **symmetric** if $B_1 = B_2 = \cdots = B_n$. An incentive scheme is **discriminatory** if it is not symmetric, or equivalently, there are $i, j \in N$ with $B_i \neq B_j$.

3 Results

3.1 Supermodular or submodular technology

Our first two propositions extend Winter's (2004) results to asymmetric technology cases. To simplify the notation, we introduce an **order** $\pi : N \to \{1, 2, \ldots, n\}$ on agents, which is a one-to-one mapping. For each π, we define the associated **set order** $\Pi : N \to 2^N$ such that for each $i \in N$, $\Pi(i) := \{j \in N : \pi(j) \leq \pi(i)\}$. When $N = \{1, 2, 3\}$ and $(\pi(1), \pi(2), \pi(3)) = (2, 3, 1)$, $(\Pi(1), \Pi(2), \Pi(3)) = (\{1, 3\}, \{3\}, N)$ and $\Pi(\pi^{-1}(3)) \subsetneq \Pi(\pi^{-1}(2)) \subsetneq \Pi(\pi^{-1}(1))$.

Proposition 1. *If the technology P is submodular, a P-optimal incentive scheme B satisfies*

$$\text{for each } i \in N,\ B_i = \frac{c}{P(N) - P(N \backslash \{i\})}. \tag{1}$$

Proof. Let P be submodular and (P, B) be P-optimal. Since (P, B) uniquely implements work, (i) $s^* = (1, 1, \ldots, 1)$ is a Nash and (ii) any other strategy profile $s \neq s^*$ is not a Nash. By (i), for each i, B_i satisfies $P(N)B_i - c \geq P(N \backslash i)B_i$, i.e., $B_i \geq c/[P(N) - P(N \backslash i)]$...(*1). Fix any profile $s \neq s^*$ and let $J = \{j \in N : s_j = 1\}$. By (ii), there is $i \in J$ such that $P(J)B_i - c \geq P(J \backslash i)B_i$, i.e., $B_i \geq c/[P(J) - P(J \backslash i)]$...(*2). Since P is submodular, $P(J) + P(N \backslash i) \geq P([J] \cup [N \backslash i]) + P([J] \cap [N \backslash i]) = P(N) + P(J \backslash i)$, i.e., $P(J) - P(J \backslash i) \geq P(N) - P(N \backslash i)$ and thus (*1) implies (*2). To be an optimum, (*1) should hold with equality. We obtain the relation (1). □

[5] This definition follows the one in Halac et al. (2021).

Proposition 2. *If the technology P is supermodular, a P-optimal incentive scheme B satisfies*[6]

$$\text{for some } \pi \text{ and each } i \in N,\ B_i = \frac{c}{P(\Pi(i)) - P(\Pi(i)\backslash\{i\})}. \tag{2}$$

Proof. Let P be supermodular. Fix π and rename agents so that for each $i \in N$, $\pi(i) = i$. Let $B^\pi = (c/[P(\Pi(i)) - P(\Pi(i)\backslash i)])_{i\in N}$. We see by induction of $i = 1, 2, \ldots, n$ that (P, B^π) uniquely implements work. For $i = 1$, since P is supermodular, for any $J \subseteq N$, $P(J)B_1^\pi - c - P(J\backslash 1)B_1^\pi = [[P(J) + P(\emptyset) - P(J\backslash 1) - P(1)]] \times c/[P(1) - P(\emptyset)] \geq 0$. That is, for any s_{-1}, $i = 1$ chooses $s_1 = 1$. For $i > 1$, assume that each $j \in J = \{1, \ldots, i-1\}$ chooses $s_j = 1$. For any $J' \supseteq J$, $P(J')B_i^\pi - c - P(J'\backslash i)B_i^\pi = [[P(J') + P(J\backslash i) - P(J'\backslash i) - P(J)]] \times c/[P(J) - P(J\backslash i)] \geq 0$. That is, for any s_{-J}, i chooses $s_i = 1$. We have $s^* = (1, \ldots, 1)$ is a unique Nash. Fix B so that for any π, $\sum_{i\in N} B_i^\pi \leq \sum_{i\in N} B_i$ and $B \neq B^\pi$. We show this B does not uniquely implements work. Because there are no π with $B = B^\pi$, for some $J \subsetneq N$, (i) for each $j \in J$, $P(J)B_j - c \geq P(J\backslash j)B_j$ and (ii) for each $i \in N\backslash J$, $P(J \cup i)B_i - c < P(J)B_i$. Clearly, profile s where $s_j = 1$ for each $j \in J$ and $s_i = 0$ for each $i \in N\backslash J$ is also a Nash. The P-optimum is a selection from $\{B^\pi\}$ and the relation (2) holds. □

3.2 Symmetric technology

For a symmetric technology P, we derive a lower bound of the total payment in P-optimal incentive scheme. Given that technology P is symmetric, we abuse the notation as $P(J) =: p(j)$ where $j = |J|$. Let $\tilde{\pi}$ be such that $\tilde{\pi}^{-1}(1) = \arg\min_{\ell\in\{1,2,\ldots,n\}}\{p(\ell) - p(\ell-1)\}$ and for each $j = 2, 3, ..., n$, $\tilde{\pi}^{-1}(j) = \arg\min_{\ell\in\{1,2,\ldots,n\}}\{p(\ell) - p(\ell-1) : \ell \notin \{\tilde{\pi}^{-1}(1), ..., \tilde{\pi}^{-1}(\ell-1)\}\}$. Let $\tilde{P}$ be such that for each $j = 1, 2, ..., n$, $\tilde{p}(j) = p(\tilde{\pi}(j))$. By the construction, $\tilde{P}$ is IRS.

Proposition 3. *If the technology P is symmetric, a P-optimal incentive scheme B satisfies*

$$\sum_{i\in N} B_i \geq \sum_{j=1}^{n} \frac{c}{p(j) - p(j-1)} = \sum_{j=1}^{n} \frac{c}{\tilde{p}(j) - \tilde{p}(j-1)}. \tag{3}$$

Proof. When P is IRS, relation (3) holds with equality by Proposition 2. Suppose that P is not IRS (thus $P \neq \tilde{P}$). Let B be a P-optimal scheme such that $B_1 \geq B_2 \geq ... \geq B_n$. Choose any $j \in \{1, 2, ..., n\}$. To exclude $s^{[j]}$ where $s_i^{[j]} = 1$ if and only if $i \leq j$ from Nash, B must satisfy either of the following.
Case 1: $p(j)B_j - c < p(j-1)B_j$. Since $s^{[0]} = (0, 0, ..., 0)$ should not be a Nash, there must be $j' \leq j$ such that $p(j')B_{j'} - c < p(j'-1)B_{j'}$ and $p(j'-1)B_{j'} < p(j')B_{j'} - c$. These relations are required to exclude $s^{[j']}$ ($s_i^{[j']} = 1$ if and only if $i \leq j'$) and $s^{[j'-1]}$ ($s_i^{[j'-1]} = 1$ if and only if $i \leq j'-1$) from Nash. Since these conditions contradict each other, this case should not hold.

[6] Similar results are known in recent papers (e.g., Halac et al. (2021)).

Case 2: $p(j+1)B_{j+1} - c \geq p(j)B_{j+1}$. From above, this should hold for any $j = 1, 2, ..., n$. We have, for each $i \in N$, $B_i \geq \max_{\ell \leq i}\{c/[p(\ell) - p(\ell - 1)]\} \geq c/[p(i) - p(i-1)]$. By summing up for all i, we obtain the inequality in relation (3). The equality in relation (3) also holds as $\tilde{\pi}$ is one-to-one. □

3.3 Global optimum

Propositions 1 and 2 show that, given arbitrarily fixed technology P, a P-optimal scheme can be discriminatory in line with Winter (2004). Proposition 3 shows that IRS technologies, under which P-optimal schemes are discriminatory, are candidates for the global optimum. In contrast, the following result, our main theorem, shows that the global optimum never be discriminatory.

Theorem 1. *An incentive scheme* (P, B) *is globally optimal if and only if*

$$P: \quad P \text{ is supermodular where for some } \pi \text{ and each } i \in N,\ P(\Pi(i)) - P(\Pi(i)\backslash\{i\}) = 1/n. \tag{4.1}$$

$$B: \quad \text{for each } i \in N,\ B_i = nc. \tag{4.2}$$

Proof. Denote the scheme satisfying (4.1) and (4.2) by (P^*, B^*). It is immediate from Proposition 2 that (P^*, B^*) uniquely implements work. We show below that (P^*, B^*) is the unique global optimum. Choose $(P, B) \neq (P^*, B^*)$ that uniquely implements work arbitrarily. Without loss of generality, we assume that (P, B) is P-optimal. We show $\sum_{i \in N} B_i > \sum_{i \in N} B_i^*$.
Case 1: P is submodular (and not supermodular). Fix π so that for each i, $\pi(i) = i$. Submodularity of P implies $P(\Pi(i)) + P(N\backslash i) \geq P(N) + P(\Pi(i)\backslash i)$. By summing up for all i, we have $\sum_{i \in N}[P(N) - P(N\backslash i)] \leq \sum_{i \in N}[P(\Pi(i)) - P(\Pi(i)\backslash i)] = P(N) - P(\emptyset)$. Since for each i, $P(N) - P(N\backslash i) \in (0, 1)$, taking inverse and multiplying $c > 0$ to each term, $\sum_{i \in N} c/[P(N) - P(N\backslash i)] > n \times c/[(P(N) - P(\emptyset))/n] \geq n \times c/(1/n) = n^2 c$. LHS and RHS are $\sum_{i \in N} B_i$ and $\sum_{i \in N} B_i^*$. Strict inequality holds because LHS is more diverse.
Case 2: P is supermodular (and not submodular). Fix π so that for each i, $\pi(i) = i$. We have $\sum_{i \in N}[P(\Pi(i)) - P(\Pi(i)\backslash i)] = P(N) - P(\emptyset) = \sum_{i \in N}[(P(N) - P(\emptyset))/n]$. Since for each i, $P(\Pi(i)) - P(\Pi(i)\backslash i) \in (0, 1)$, taking inverse and multiplying $c > 0$ to each term, $\sum_{i \in N} c/[P(\Pi(i)) - P(\Pi(i)\backslash i)] > n \times c/[(P(N) - P(\emptyset))/n] \geq n \times c/(1/n) = n^2 c$. LHS and RHS are $\sum_{i \in N} B_i$ and $\sum_{i \in N} B_i^*$. Strict inequality holds because LHS is more diverse.
Case 3: P is symmetric (and neither IRS nor DRS). Since $\tilde{P}$ is IRS (i.e., supermodular), Case 2 implies that RHS of relation (3) is greater than $\sum_{i \in N} B_i^*$. We obtain $\sum_{i \in N} B_i > \sum_{i \in N} B_i^*$. □

Corollary 1. *The global optimum is symmetric.*

The next example illustrates our results.

Example 2. Agents are $N = \{1, 2, 3\}$ with working cost $c = 10$. The production technologies are on Table 1. For $\varepsilon = 1$, payoffs induced by each scheme $(P, B + \varepsilon)$ are on Table 2.

Table 1: Production technologies and their least cost bonuses

J	$\emptyset$	$\{1\},\{2\},\{3\}$	$\{1,2\},\{1,3\}$	$\{2,3\}$	N
$P^{I}(J)$	0.2	0.6	0.8	0.9	1
$P^{II}(J)$	0.4	0.5	0.75	0.7	1
$P^{III}(J)$	0	0.5	0.6	0.6	1
$P^{*}(J)$	0	1/3	2/3	2/3	1

i	1	2	3
B_i^{I}	100	50	50
B_i^{II}	100	40	40
B_i^{III}	100	100	25
B_i^{*}	30	30	30

Table 2: Payoffs induced by each scheme

	work	shirk		work	shirk
work	**$\underline{91}$, $\underline{41}$, $\underline{41}$**	$\underline{70.8}$, 40.8, $\underline{30.8}$	work	$\underline{70.8}$, $\underline{30.8}$, 40.8	$\underline{50.6}$, 30.6, 30.6
shirk	90.9, $\underline{35.9}$, $\underline{35.9}$	60.6, 30.6, $\underline{20.6}$	shirk	60.6, $\underline{20.6}$, 30.6	20.2, 10.2, 10.2

$(P^{I}, B^{I}+1)$

	work	shirk		work	shirk
work	**$\underline{91}$, $\underline{31}$, $\underline{31}$**	$\underline{65.8}$, 30.8, $\underline{20.8}$	work	$\underline{65.8}$, $\underline{20.8}$, 30.8	$\underline{40.5}$, 20.5, 20.5
shirk	70.7, 18.7, 18.7	50.5, $\underline{20.5}$, 10.5	shirk	50.5, 10.5, $\underline{20.5}$	40.4, $\underline{16.4}$, $\underline{16.4}$

$(P^{II}, B^{II}+1)$

	work	shirk		work	shirk
work	**$\underline{91}$, $\underline{91}$, $\underline{16}$**	$\underline{50.6}$, 60.6, 5.6	work	$\underline{50.6}$, $\underline{50.6}$, 15.6	$\underline{40.5}$, 50.5, $\underline{13}$
shirk	60.6, $\underline{50.6}$, 5.6	50.5, 50.5, $\underline{3}$	shirk	50.5, $\underline{40.5}$, $\underline{13}$	0, 0, 0

$(P^{III}, B^{III}+1)$

	work	shirk		work	shirk
work	**$\underline{21}$, $\underline{21}$, $\underline{21}$**	$\underline{10.7}$, 20.7, $\underline{10.7}$	work	$\underline{10.7}$, $\underline{10.7}$, 20.7	$\underline{0.3}$, 10.3, 10.3
shirk	20.7, $\underline{10.7}$, $\underline{10.7}$	10.3, 10.3, $\underline{0.3}$	shirk	10.3, $\underline{0.3}$, 10.3	0, 0, 0

$(P^{*}, B^{*}+1)$

Note: Agents 1 and 2 are row and column players. Agent 3 chooses to work in the left tables and to shirk in the right ones. Underline indicates best response. Bold is Nash.

P^{I} is submodular, P^{II} is supermodular, and P^{III} is symmetric. (P^{*}, B^{*}) is a global optimum ($\sum_{i\in N} B_i^{*} = 90$ is smaller than $\sum_{i\in N} B_i^{I} = 200$, $\sum_{i\in N} B_i^{II} = 180$, or $\sum_{i\in N} B_i^{III} = 225$) and it is symmetric ($B_1^{*} = B_2^{*} = B_3^{*}$). $\diamond$

References

Agastya, M. and O. Birulin (2023): "Optimal Task Scheduling under Adverse Selection and Hidden Actions," *American Economic Journal: Microeconomics*, 15, 660–698.

Boyarchenko, S., D. Machowska, and I. Topolyan (2025): "Supermodularity and incentive reversal in teams," *Games and Economic Behavior*, 150, 93–105.

Davidson, C. and R. Deneckere (1986): "Long-run competition in capacity, short-run competition in price, and the Cournot model," *The RAND Journal of Economics*, 404–415.

Halac, M., E. Lipnowski, and D. Rappoport (2021): "Rank uncertainty in organizations," *American Economic Review*, 111, 757–786.

Holmstrom, B. (1982): "Moral hazard in teams," *The Bell Journal of Economics*, 324–340.

Kreps, D. M. and J. A. Scheinkman (1983): "Quantity precommitment and Bertrand competition yield Cournot outcomes," *The Bell Journal of Economics*, 326–337.

Segal, I. (2003): "Coordination and discrimination in contracting with externalities: Divide and conquer?" *Journal of Economic Theory*, 113, 147–181.

Winter, E. (2004): "Incentives and discrimination," *American Economic Review*, 94, 764–773.