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【要旨】

This paper aims to provide a theoretical foundation of what is known as " Generally Accepted Accounting Principles (GAAP)," focusing on depreciation. It is widely accepted that a depreciation method should be rational and systematic. There are many possible depreciation methods; however, only a few are used in practice, such as the straight-line method, the declining-balance method, the sum-of-the-years' digits method, and the fair value measurement. We investigate through the axiomatic approach in what sense these depreciation methods can be considered rational and systematic. We provide a practical interpretation for each axiom examined in this paper and relate the axioms to accounting principles. Interestingly, it turns out that the straight-line method satisfies all axioms but consistency considered in the paper. Since the players are not humans in the model studied in the paper, it is not so clear whether the axiom of core selection is desirable or not in this context. In this paper, we provide a positive answer to this question. Namely, we show that three appealing axioms, population-monotonicity, the final year reasonableness, and conservatism, jointly imply core selection.

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Axioms of depreciation methods

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January 31, 2025

Abstract

This paper aims to provide a theoretical foundation of what is known as " Generally Accepted Accounting Principles (GAAP)," focusing on depreciation. It is widely accepted that a depreciation method should be rational and systematic. There are many possible depreciation methods; however, only a few are used in practice, such as the straight-line method, the declining-balance method, the sum-of-the-years' digits method, and the fair value measurement. We investigate through the axiomatic approach in what sense these depreciation methods can be considered rational and systematic. We provide a practical interpretation for each axiom examined in this paper and relate the axioms to accounting principles. Interestingly, it turns out that the straight-line method satisfies all axioms but *consistency* considered in the paper. Since the players are not humans in the model studied in the paper, it is not so clear whether the axiom of *core selection* is desirable or not in this context. In this paper, we provide a positive answer to this question. Namely, we show that three appealing axioms, *population-monotonicity*, *the final year reasonableness*, and *conservatism*, jointly imply *core selection*.

Keywords: Axiomatic approach, Cooperative games, Depreciation

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1 Introduction

This paper aims to provide a theoretical foundation of what is known as "Generally Accepted Accounting Principles." When a firm purchases a costly asset, say a machine, which will be used for a certain period of time, it is usually the case that the original cost of the asset is allocated over the expected useful life of that asset. There are various methods to achieve this allocation. However, among those "depreciation methods", only a few are used in practice, such as the straight-line method, the declining-balance method, the sum-of-the-years' digits method, and the fair value measurement. It is often said that a depreciation method should be rational and systematic (Hendricksen and van Breda, 2001, page 523). In what sense can these depreciation methods used in practice be considered rational and systematic? That is what we investigate in this paper.

Formally, this depreciation problem can be considered as a cooperative game, where the "players" are mere fiscal years without preferences of their own. We believe that the specific depreciation methods are used in practice mainly because they satisfy certain nice properties (axioms), which can be interpreted as representing some basic concepts of regarding accounting standards. Thus, we adopt the axiomatic approach in our analysis.

A few studies apply cooperative game theory to depreciation. Ben-Shahar and Sulganik (2009) and Ben-Shahar, Margalioth, and Sulganik (2009) have suggested depreciation methods that reflect how firms consume the economic benefits of their assets. Aparicio and Sánchez-Soriano (2008) have developed an innovative "depreciation game" and demonstrated that payoff vectors provided by traditional methods do not always belong to the core. They proposed a new depreciation method that gives a payoff vector belonging to the core and reflects the asset's market value. While they consider limited coalitions, Arata, Shimogawa, and Inohara (2024) have modified the model of Aparicio and Sánchez-Soriano (2008) so that the domain of the cost function is extended to all coalitions. They have shown that the straight-line method satisfies core selection under their settings, and the conditions of the core could be accepted by those accountants who engage in similar problems in practice.

Accounting researchers have applied cooperative game theory to some specific topics in their field other than depreciation problems from the late 1970s to the early 1980s. They have studied some particular allocation rules of joint costs among departments within a firm. For example, Hamlen,

Hamlen, and Tschirhar (1977) have examined the allocation of joint costs using the core, and Callen (1978), Roth and Verrecchia (1979), Hamlen, Hamlen, and Tschirhar (1980), and Balachandran and Ramakrishnan (1981) have discussed allocation using the Shapley value.

Recently, in management science, some research has focused on the relationship between allocation rules and axioms in areas such as supply chains (Zelewski, 2018) and joint orders (Saavedra-Nieves, 2018). Mueller (2018) reviews the crucial properties of some solution concepts (the Shapley value, Nucleolus, The τ-value, and Dutta-Ray solution) and provides the basis for evaluating them concerning management accounting.

In law and economics, Dehez and Ferey (2013, 2016) examine sharing damage caused jointly by several tortfeasors using cooperative game theory. Their model rationalizes the weighted Shapley value as being the legal counterpart of the process proposed by Restatement (Third) of Torts. Their research shares a similar objective as ours in the sense that it examines the correspondence between the axioms of particular allocation methods and the underlying fundamental concepts of the associated legal rules.

We extend Arata, Shimogawa, and Inohara (2024) in two ways. First, while they mainly focus on the straight-line method, we investigate other depreciation methods as well. Second, while they consider only one axiom, core selection, we study other axioms as well, such as monotonicity, additivity, population monotonicity, and consistency, and investigate the logical relations between them.

In this paper, we refer to the fundamental concepts underlying accounting rules from Accounting Principles Board Statement No. 4, "Basic Concepts and Accounting Principles Underlying Financial Statements of Business Enterprises," issued by the American Institute of Certified Public Accountants (AICPA) in 1970. Some of the axioms studied in this paper correspond to the principles described in the statement, which were inductively derived from practices and remain significant to accounting professionals today.

This paper is organized as follows: Section 2 introduces the depreciation problem and the depreciation game, and Section 3 presents the depreciation methods. Section 4 examines the characteristics of these methods within the context of the depreciation game. In Section 5, we discuss the practical implications of our findings. Finally, we conclude in Section 6.

2 Depreciation problems and depreciation games

In this section, we overview the definitions of depreciation problems and depreciation games, originally introduced by Aparicio and Sánchez-Soriano (2008) and generalized later by Arata, Shimogawa, and Inohara (2024). Let N be a finite set of consecutive natural numbers and $n := |N|$. We consider the depreciation procedure of an asset acquired at cost C for its useful life n years. Let R denote the residual value of the asset after use.[1] Each $i \in N$ represents a fiscal year of the asset's utilization period. We assume that the market value of the asset after k years, $k = 1, 2, \dots, n$, is given by a function f from $\{1, 2, \dots, n\}$ to $\mathbb{R}_+$ satisfying the following two conditions:

- $C > f(1) > f(2) > \cdots > f(n) = R$.
- For all $s, t \in \{1, 2, \dots, n\}$ with $s + t \leq n$,

$$C \geq f(s) + f(t) - f(s+t). \tag{1}$$

We refer to such a list (N, C, f) as a **depreciation problem** for N. As we shall see, condition (1) represents a situation in which it is less expensive for a firm to lease the asset for multiple years continuously (or to purchase it), rather than repeatedly renewing annual leases.[2] Let $\mathcal{D}^N$ denote the class of all depreciation problems for N.

A coalition is a non-empty subset of N. A **coalitional game with transferable utility** (a **TU game**, for short) is a pair (N, v) where v is a function from 2^N to $\mathbb{R}$ with $v(\emptyset) = 0$. We associate a TU game (N, d) with each depreciation problem $(N, C, f) \in \mathcal{D}^N$. Let $Seq^N \subset 2^N$ be the set of sequential coalitions, such as $\{1, 2\}$ and $\{4, 5, 6\}$. For each coalition $S \in Seq^N$, let $d^*(S)$ denote the cost the firm should pay if it uses the asset for years in S. When the firm uses the asset only for one year, for example, $S = \{2\}$, it invests in the asset through a one-year lease contract. In other cases, for example, $S = \{2, 3, 4\}$, the firm invests in the asset through a three-year lease contract. If $S = N$, the firm invests in the asset through an n-year contract, which is equivalent to purchasing the asset. In this way, the cost function $d^*(S) : Seq^N \mapsto \mathbb{R}_+$ can be defined by the amount of the corresponding lease payments.

[1] In both Aparicio and Sánchez-Soriano (2008) and Arata, Shimogawa, and Inohara (2024), R is assumed to be 0.

[2] See Arata, Shimogawa, and Inohara (2024) for a more detailed interpretation.

Let us explain how these lease payments are determined. Imagine a leasing company that leases out the asset to the firm. Assume that the leasing company can correctly predict the market value $f(k)$ of the asset used for k years because it has access to the second-hand market while the firm does not. Assume also that the leasing company sets the lease payments so as to recover the decrease of its market value $(= C - f(k))$ during the lease term. Then, the cost function on Seq^N is defined as follows: for all $S \in Seq^N$,

$$d^*(S) \;:=\; C - f(|S|) \tag{2}$$

Next, we extend the domain of d^*. Note that each $S \in 2^N$, which is not necessarily sequential, can be decomposed into maximal sequential coalitions.[3] Let $(S)^m$ denote the set of these maximal sequential coalitions in S. Then, the cost for S is defined by

$$d(S) \;:=\; \sum_{S_j \in (S)^m} d^*(S_j). \tag{3}$$

The cost function thus defined is subadditive.[4] Moreover, it is concave.[5] if the market value function is decreasing with respect to time and satisfies

$$f(0) - f(1) \geq f(1) - f(2) \geq \cdots \geq f(n-1) - f(n). \tag{4}$$

3 Depreciation methods

As mentioned above, an asset's investment cost, measured by the acquisition cost C minus the residual value $R = f(n)$, is allocated over its expected useful life through depreciation. A **depreciation method** is a function that assigns to each depreciation problem $(N, C, f) \in \mathcal{D}^N$ a vector $\varphi(N, C, f) \in \mathbb{R}^N$ such that $\sum_{i \in N} \varphi_i(N, C, f) = C - R$.

We now introduce four depreciation methods commonly used in practice. For simplicity, we assume $N = \{1, 2, \ldots, n\}$ in the following definitions.

- **The straight-line method**: For all $(N, C, f) \in \mathcal{D}^N$ and all $i \in N$,

$$SL_i(N, C, f) \;:=\; \frac{C - R}{n}.$$

[3] A sequential coalition is maximal if any superset of it is not sequential.

[4] A function g on 2^N is subadditive if, for all $S, T \in 2^N$ with $S \cap T = \emptyset$, $g(S \cup T) \leq g(S) + g(T)$.

[5] A function g on 2^N is concave if, for all $S, T \in 2^N$, $g(S \cup T) + g(S \cap T) \leq g(S) + g(T)$.

The straight-line method is the most commonly used method all over the world, and "[t]he great virtue of this method is that it is simple to apply and easy to understand" (Hendricksen and van Breda, 2001, page 528).

- **The declining-balance method**: For all $(N, C, f) \in \mathcal{D}^N$ with $R \neq 0$,

$$
\begin{aligned}
DB_1(N,C,f) &:= \left\{1 - \left(\frac{R}{C}\right)^{\frac{1}{n}}\right\} C, \\
DB_2(N,C,f) &:= \left\{1 - \left(\frac{R}{C}\right)^{\frac{1}{n}}\right\} \left(\frac{R}{C}\right)^{\frac{1}{n}} C, \\
DB_3(N,C,f) &:= \left\{1 - \left(\frac{R}{C}\right)^{\frac{1}{n}}\right\} \left(\frac{R}{C}\right)^{\frac{2}{n}} C, \\
&\vdots \\
DB_n(N,C,f) &:= \left\{1 - \left(\frac{R}{C}\right)^{\frac{1}{n}}\right\} \left(\frac{R}{C}\right)^{\frac{n-1}{n}} C.
\end{aligned}
$$

For all $(N, C, f) \in \mathcal{D}^N$ with $R = 0$,

$$
\begin{aligned}
DB_1(N,C,f) &:= \left\{1 - \left(\frac{1}{C}\right)^{\frac{1}{n}}\right\} C, \\
DB_2(N,C,f) &:= \left\{1 - \left(\frac{1}{C}\right)^{\frac{1}{n}}\right\} \left(\frac{1}{C}\right)^{\frac{1}{n}} C, \\
DB_3(N,C,f) &:= \left\{1 - \left(\frac{1}{C}\right)^{\frac{1}{n}}\right\} \left(\frac{1}{C}\right)^{\frac{2}{n}} C, \\
&\vdots \\
DB_n(N,C,f) &:= \left\{1 - \left(\frac{1}{C}\right)^{\frac{1}{n}}\right\} \left(\frac{R}{C}\right)^{\frac{n-1}{n}} C + 1.
\end{aligned}
$$

The declining-balance method is used to calculate depreciation expense by applying a constant depreciation rate to the asset's undepreciated balance. Note that there are two representations of this method depending on the value of R. The first one, which applies to the case $R \neq 0$, is a general one. However, it cannot be applied to the case when $R = 0$. In that case, we regard

R as one, calculate the depreciation rate, and add one to the depreciation expense in the final year. Firms enjoy tax benefits because this method ends up with higher depreciation expenses in the earlier years of the asset's useful life and lower expenses in the later years. Therefore, tax authorities allow this method in order to promote investments in new technology or equipment.

In fact, there are many versions of this method. The one defined above is called the fixed-percentage declining-balance method; however, in many countries, the depreciation rate for the declining-balance method is calculated using either twice or 2.5 times the depreciation rate of the straight-line method.[6] These methods are called the double declining balance method or the 250 percent declining balance method. This paper, however, will examine based on the fixed-percentage declining-balance method because it is difficult to handle fractional processing in our model with the double declining balance method or the 250 percent declining balance method.

- **The sum-of-the-years' digits method**: For all $(N, C, f) \in \mathcal{D}^N$,

$$
\begin{aligned}
SYD_1(N,C,f) &:= \frac{2n}{n(n+1)}(C-R) = \frac{2}{n+1}(C-R),\\
SYD_2(N,C,f) &:= \frac{2(n-1)}{n(n+1)}(C-R),\\
SYD_3(N,C,f) &:= \frac{2(n-2)}{n(n+1)}(C-R),\\
&\vdots\\
SYD_n(N,C,f) &:= \frac{2}{n(n+1)}(C-R).
\end{aligned}
$$

The sum-of-the-years' digits method also ends up with higher depreciation expenses in the earlier years of the asset's useful life and lower expenses in the later years. Tax authorities also allow this method for the same reason as the declining-balance method.

- **The fair value measurement**: For all $(N, C, f) \in \mathcal{D}^N$,

$$
\begin{aligned}
FV_1(N,C,f) &:= C - f(1),\\
FV_2(N,C,f) &:= f(1) - f(2),\\
&\vdots\\
FV_n(N,C,f) &:= f(n-1) - R.
\end{aligned}
$$

[6]The straight-line depreciation rate is represented as $\frac{1}{n-k+1}$ $(k \in N)$.

Note that the "market value" referred to here is an "ex-ante" value. In other words, this refers to the predicted value of an asset used for i years in the secondary market, as estimated when a firm decides on its investment (or when a leasing company determines the lease payments). In practice[7], assets are valued at their fair value as of the fiscal year-end date, namely an "ex-post" value.[8]

Let $\mathcal{V}^N$ be a class of TU games for N. Then, a **solution** on $\mathcal{V}^N$ is a function σ that assigns each TU game $(N, v) \in \mathcal{V}^N$ a vector $\sigma(N, v) \in \mathbb{R}^N$ such that $\sum_{i \in N} \sigma_i(N, v) = v(N)$. In the axiomatic analysis of TU games, desirable properties of solutions are formulated as axioms, and logical relation among them are studies.[9] In the next section, we formulate some desirable properties of depreciation methods as axioms, and investigate whether the four depreciation methods introduced above satisfies each axiom.

4 Axiomatic analysis

In this section, we examine the properties of each depreciation method. For each of them there is a corresponding axiom of solutions for TU games, but some of them are modified so as to fit the context of depreciation problems. We consider *monotonicity*, *additivity*, *population monotonicity*, *core selection*, and *consistency*. For simplicity, we define each axiom for $N = \{1, 2, \ldots, n\}$.

Monotonicity: For all $(N, C, f), (N, C', f) \in \mathcal{D}^N$, if $C < C'$, then for all $i \in N$,

$$\varphi_i(N, C', f) \geq \varphi_i(N, C, f).$$

[7]Under the current accounting system, tangible fixed assets are not revalued at fair value at the end of each fiscal year in any country. On the other hand, the International Accounting Standards No. 16 (IAS 16) allows the revaluation model for tangible fixed assets. IAS 16 permits the revaluation of an asset at fair value when its carrying amount, which is being depreciated using other methods discussed in this paper, deviates from its fair value. For simplicity, we will consider a case where costs are allocated through revaluation at fair value each fiscal year, rather than a combination of regular revaluation and depreciation by other methods as prescribed by IAS 16.

[8]Arata, Shimogawa, and Inohara (2024) have verified that the fair value measurement based on ex-post values satisfies *core selection*.

[9]See Hokari and Thomson (2015) for a survey of axiomatic analysis of cooperative games.

Monotonicity states that when the acquisition cost increases, there are no fiscal years in which the burden decreases.

Claim 1 The straight-line method satisfies *monotonicity*.

Proof. For all $i \in N$,

$$SL_i(N, C', f) = \frac{C' - R}{n} > \frac{C - R}{n} = SL_i(N, C, f). \qquad \square$$

Claim 2 The declining method satisfies *monotonicity*.

Proof. For all $i \in N$,

$$\begin{aligned} DB_i(N, C', f) &= \left\{1 - \left(\tfrac{R}{C'}\right)^{\frac{1}{n}}\right\} \left(\tfrac{R}{C'}\right)^{\frac{i-1}{n}} C' = \left\{1 - \left(\tfrac{R}{C'}\right)^{\frac{1}{n}}\right\} R^{\frac{i-1}{n}} (C')^{1-\frac{i-1}{n}}, \\ DB_i(N, C, f) &= \left\{1 - \left(\tfrac{R}{C}\right)^{\frac{1}{n}}\right\} \left(\tfrac{R}{C}\right)^{\frac{i-1}{n}} C = \left\{1 - \left(\tfrac{R}{C}\right)^{\frac{1}{n}}\right\} R^{\frac{i-1}{n}} C^{1-\frac{i-1}{n}}. \end{aligned}$$

Since $1 - \left(\frac{R}{C'}\right)^{\frac{1}{n}} > 1 - \left(\frac{R}{C}\right)^{\frac{1}{n}}$ and $1 - \frac{i-1}{n} > 0$,

$$DB_i(N, C', f) = \left\{1 - \left(\tfrac{R}{C'}\right)^{\frac{1}{n}}\right\} R^{\frac{i-1}{n}} (C')^{1-\frac{i-1}{n}} > \left\{1 - \left(\tfrac{R}{C}\right)^{\frac{1}{n}}\right\} R^{\frac{i-1}{n}} C^{1-\frac{i-1}{n}}. \ \square$$

Claim 3 The sum-of-the-years' digits method satisfies *monotonicity*.

Proof. For all $i \in N$,

$$SYD_i(N, C', f) = \tfrac{2(n-i+1)}{n(n+1)}(C' - R) > \tfrac{2(n-i+1)}{n(n+1)}(C - R) = SYD_i(N, C, f). \ \square$$

Claim 4 The fair value measurement satisfies *monotonicity*.

Proof. If $i = 1$,

$$FV_1(N, C', f) = C' - f(1) > C - f(1) = FV_1(N, C, f).$$

If $i \in \{2, 3, \ldots n\}$,

$$FV_i(N, C', f) = f(i-1) - f(i) = FV_i(N, C, f). \qquad \square$$

The above claims are summarized as follows:

Proposition 1 All of the four methods satisfy *monotonicity*.

Note that using the first three depreciation methods, if the investment amount increases, the burden amount increases accordingly in all fiscal years.

Additivity: For all $(N, C, f), (N, C', f') \in \mathcal{D}^N$ and all $i \in N$,

$$\varphi_i(N, C, f) + \varphi_i(N, C', f') = \varphi_i(N, C + C', f + f').$$

In practice, the term "group depreciation" refers to grouping multiple assets with assumed identical useful lives together and depreciating them collectively. If a depreciation method satisfies *additivity*, the result of group depreciation is the same as that of individual depreciation.

Claim 5 The straight-line method satisfies *additivity*.

Proof. For all $i \in N$,

$$\begin{aligned} SL_i(N, C, f) + SL_i(N, C', f') &= \frac{C - R}{n} + \frac{C' - R'}{n} \\ &= \frac{C + C' - R - R'}{n} \\ &= SL_i(N, C + C', f + f'). \end{aligned}$$

□

Claim 6 The declining-balance method does not satisfy *additivity*.

Proof. Note that, if $R = 10, R' = 20, C = C' = 100$, then

$$\begin{aligned} DB_1(N, C, f) + DB_1(N, C', f') &= \left\{1 - \left(\frac{R}{C}\right)^{\frac{1}{n}}\right\} C + \left\{1 - \left(\frac{R'}{C'}\right)^{\frac{1}{n}}\right\} C' \\ &= \left\{1 - \left(\frac{10}{100}\right)^{\frac{1}{n}}\right\} 100 + \left\{1 - \left(\frac{20}{100}\right)^{\frac{1}{n}}\right\} 100 \\ &= 200 - \left\{\left(\frac{1}{10}\right)^{\frac{1}{n}} + \left(\frac{1}{5}\right)^{\frac{1}{n}}\right\} 100, \\ DB_1(N, C + C', f + f') &= \left\{1 - \left(\frac{R + R'}{C + C'}\right)^{\frac{1}{n}}\right\} (C + C') \\ &= \left\{1 - \left(\frac{30}{200}\right)^{\frac{1}{n}}\right\} 200 \\ &= 200 - \left(\frac{30}{200}\right)^{\frac{1}{n}} 200. \end{aligned}$$

Thus, $DB_1(N,C,f) + DB_1(N,C',f') \neq DB_1(N,C+C',f+f')$. □

Claim 7 The sum-of-the-years' digits satisfies *additivity.*

Proof. For all $i \in N$,

$$\begin{aligned} & SYD_i(N,C,f)) + SYD_i(N,C',f') \\ = & \frac{2(n-i+1)}{n(n+1)}(C-R) + \frac{2(n-i+1)}{n(n+1)}(C'-R) \\ = & \frac{2(n-i+1)}{n(n+1)}\{C+C'-(R+R')\} \\ = & SYD_i(N,C+C',f+f'). \end{aligned}$$ □

Claim 8 The fair value measurement satisfies *additivity.*

Proof. For all $i \in N$,

$$\begin{aligned} FV_i(N,C,f) + FV_i(N,C',f') &= C - f(i) + C' - f'(i) \\ &= C + C' - (f(i) + f'(i)) \\ &= FV_i(N,C+C',f+f'). \end{aligned}$$ □

Thus, the above claims are summarized as follows:

Proposition 2 The straight-line method, the sum-of-the-years' digits method, and the fair value measurement satisfy *additivity.*

The next axiom pertains to the case when the utilization periods are shortened. It says that all remaining periods should be affected in the same direction. Let $N' := \{1,2,\ldots,n-1\}$ and f_{n-1} be the restriction of f to $\{1,2,\ldots n-1\}$. Note that, for all $(N,C,f) \in \mathcal{D}^N$, $(N',C,f_{n-1}) \in \mathcal{D}^{N'}$.

Population monotonicity[10]: For all $(N,C,f) \in \mathcal{D}^N$, either

(i) for all $i \in N'$,

$$\varphi_i(N',C,f_{n-1}) \geq \varphi_i(N,C,f),$$

or

[10]This axiom was first introduced by Thomson (1983) in a model of fair allocation. See Thomson (1995) for a survey of this axiom applied in various situations.

(ii) for all $i \in N'$,

$$\varphi_i(N', C, f_{n-1}) \quad \leq \quad \varphi_i(N, C, f).$$

Next, we consider a variant of this axiom, which says that the condition (i) above should hold always:

Population monotonicity$_+$: For all $(N, C, f) \in \mathcal{D}^N$ and all $i \in N'$,

$$\varphi_i(N', C, f_{n-1}) \quad \geq \quad \varphi_i(N, C, f).$$

To see the logical relation of these two variants, consider the following axiom:

The final year reasonableness: For all $(N, C, f) \in \mathcal{D}^N$,

$$\varphi_n(N, C, f) \quad \geq \quad f(n-1) - f(n).$$

Proposition 3 *Population monotonicity* and *the final year reasonableness* jointly imply *population monotonicity$_+$*.

Proof. Note that

$$\begin{aligned} & \sum_{i=1}^{n-1} \varphi_i(N', C, f_{n-1}) \geq \sum_{i=1}^{n-1} \varphi_i(N, C, f) \\ \leftrightarrow \quad & C - f(n-1) \geq C - f(n) - \varphi_n(N, C, f) \\ \leftrightarrow \quad & \varphi_n(N, C, f) \geq f(n-1) - f(n). \end{aligned}$$

□

Claim 9 The straight-line method satisfies *population monotonicity$_+$*.

Proof.

$$\begin{aligned} SL_i(N', C, f_{n-1}) - SL_i(N, C, f) &= \frac{C - f(n-1)}{n-1} - \frac{C - f(n)}{n} \\ &= \frac{C - f(n) + f(n) - f(n-1)}{n-1} - \frac{C - f(n)}{n} \\ &= \frac{C - f(n)}{n(n-1)} - \frac{f(n-1) - f(n)}{n-1} \\ &= \frac{1}{n-1}\left(\frac{C - f(n)}{n} - (f(n-1) - f(n))\right). \end{aligned}$$

By (1),

$$\begin{aligned} C - f(1) &\geq f(n-1) - f(n) \\ f(1) - f(2) &\geq f(n-1) - f(n) \\ &\vdots \\ f(n-1) - f(n) &\geq f(n-1) - f(n) \end{aligned}$$

Summing up these inequalities,

$$C - f(n) \geq n(f(n-1) - f(n)) \tag{5}$$

Thus, $SL_i(N', C, f_{n-1}) - SL_i(N, C, f) \geq 0$. □

Claim 10 The declining-balance method does not satisfy *population monotonicity.*

Proof. When $|N| = 3, C = 160, f(1) = 70, (2) = 30,$ and $R = 10$,

$$\begin{aligned} DB_1(N', C, f_2) - DB_1(N, C, f) &= \left\{1 - \left(\frac{f(2)}{C}\right)^{\frac{1}{2}}\right\} C - \left\{1 - \left(\frac{f(3)}{C}\right)^{\frac{1}{3}}\right\} C \\ &= \left\{\left(\frac{10}{160}\right)^{\frac{1}{3}} - \left(\frac{30}{160}\right)^{\frac{1}{2}}\right\} 160 < 0 \end{aligned}$$

and

$$\begin{aligned} &DB_2(N', C, f_2) - DB_2(N, C, f) \\ &= \left\{1 - \left(\frac{f(2)}{C}\right)^{\frac{1}{2}}\right\} \left(\frac{f(2)}{C}\right)^{\frac{1}{2}} C - \left\{1 - \left(\frac{f(3)}{C}\right)^{\frac{1}{3}}\right\} \left(\frac{f(3)}{C}\right)^{\frac{1}{3}} C \\ &= \left[\left\{1 - \left(\frac{30}{160}\right)^{\frac{1}{2}}\right\} \left(\frac{30}{160}\right)^{\frac{1}{2}} - \left\{1 - \left(\frac{10}{160}\right)^{\frac{1}{3}}\right\} \left(\frac{10}{160}\right)^{\frac{1}{3}}\right] 160 > 0. \end{aligned}$$

□

Claim 11 The sum-of-the-years' digits method does not satisfy *population monotonicity.*

Proof. When $|N| = 3, C = 160, f(1) = 70, (2) = 30,$ and $R = 10$,

$$\begin{aligned} SYD_1(N', C, f_2) - SYD_1(N, C, f) &= \frac{2(C - f(2))}{3} - \frac{C - f(3)}{2} \\ &= (260/3) - (150/2) > 0 \end{aligned}$$

and

$$\begin{aligned} SYD_2(N', C, f_2) - SYD_2(N, C, f) &= \frac{(C - f(2))}{3} - \frac{C - f(3)}{3} \\ &= (130/3) - (150/3) < 0. \end{aligned}$$ □

Claim 12 The fair value measurement satisfies both types of *population monotonicity.*

Proof.

$$FV_i(N', C, f_{n-1}) - FV_i(N, C, f) = 0$$ □

The fair value measurement can be considered as a "marginal contribution solution" in TU games. In the context of TU games, *population monotonicity* compares what each player gets in a given TU game and what she gets in its subgames. It is well known that if a game is concave or convex, given any ordering of players, the corresponding marginal contribution solution satisfies *population monotonicity.* Here, our *pupulation-monotonicity* axiom considers only on particular subgame, the one with respect to $\{1, 2, \ldots, n - t\}$. Also, the fair value measurement uses a particular ordering of years. These facts partially explain why this method satisfies *population monotonicity*$_+$ without any extra assumptions on f, such as concavity.

Then, the above claims are summarized as follows:

Proposition 4 The straight-line method and the fair value measurement satisfy *population monotonicity.*

Now, let us consider the following axioms:

Core selection: For all $(N, C, f) \in \mathcal{D}^N$,

$$\sum_{i \in S} \varphi_i(N, C, f) \leq d(S) \text{ for all } S \subset N. \tag{6}$$

The first-year individual rationality: For all $(N, C, f) \in \mathcal{D}^N$,

$$\varphi_1(N, C, f) \leq C - f(1) = d(\{1\}). \tag{7}$$

The first-year individual rationality is weaker than *core selection.*

Claim 13 The straight-line method satisfies *core selection.*

The proof of this statement can be found in Arata, Shimogawa, and Inohara (2024).

Claim 14 The declining-balance method does not satisfy *core selection.*

Proof. If $R > 0$, $C(\frac{R}{C})^{\frac{1}{n}} < f(1)$, and $S = \{1\}$,

$$\sum_{1 \in S} \varphi_i(N, C, f) = DB_1(N, C, f) = \left\{1 - \left(\frac{R}{C}\right)^{\frac{1}{n}}\right\} C > C - f(1) = d(\{1\}).$$

Thus, it does not satisfy *the first-year rationality*, therefore it does not satisfy *core selection.* □

Claim 15 The sum-of-the-years' digits method does not satisfy *core selection.*

Proof. If $C - \frac{2}{n+1}(C - R) < f(1)$ and $S = \{1\}$,

$$\sum_{1 \in S} \varphi_i(N, C, f) = SYD_1(N, C, f) = \frac{2}{n+1}(C - R) > C - f(1) = d(\{1\}). \quad \square$$

Thus, it does not satisfy *the first-year rationality*, therefore it does not satisfy *core selection.*

Claim 16 The fair value measurement satisfies *core selection.*

The proof of this statement can also be found in Arata, Shimogawa, and Inohara (2024).

The above claims are summarized as follows:

Proposition 5 The straight-line method and the fair value measurement satisfy *core selection.*

Next, we show that *core selection* is implied by combining other axioms. Here, we introduce a new axiom.

Conservatism: For all $(N, C, f) \in \mathcal{D}^N$ and all $i, j \in N$ with $i < j$,

$$\varphi_i(N, C, f) \geq \varphi_j(N, C, f).$$

This axiom implies the early recognition of expenses in this context. The convention known as "conservatism" exists in accounting practice. This axiom can be regarded as an embodiment of the convention. We discuss this in more detail in the next section.

Proposition 6 *Population-monotonicity, the final year reasonableness*, and *conservatism* jointly imply *core selection*.

Proof. By Proposition 3, *population monotonicity* and *final year reasonableness* jointly imply *population monotonicity*$_+$. By *population monotonicity*$_+$,

$$\sum_{i=1}^{n-1} \varphi_i(N', C, f_{n-1}) \;\; \geq \;\; \sum_{i=1}^{n-1} \varphi_i(N, C, f).$$

By the definition of depreciation method,

$$\sum_{i=1}^{n-1} \varphi_i(N', C, f_{n-1}) = C - f_{n-1}(n-1) = C - f(n-1) = d^*(\{1, 2, \ldots, n-1\})$$

where $d^* \colon Seq^N \to \mathbb{R}$ is the restricted cost function associated with (N, C, f). Thus,

$$d^*(\{1, 2, \ldots, n-1\}) \;\; \geq \;\; \sum_{i=1}^{n-1} \varphi_i(N, C, f). \tag{8}$$

By the definition of d^*,

$$d^*(\{2, 3, \ldots, n\}) = C - f(n-1) = d^*(\{1, 2, \ldots, n-1\}). \tag{9}$$

By *conservatism*,

$$\sum_{i=1}^{n-1} \varphi_i(N, C, f) \geq \sum_{i=2}^{n} \varphi_i(N, C, f) \tag{10}$$

From (8), (9) and (10),

$$d^*(\{2, 3, \ldots, n\}) \geq \sum_{i=2}^{n} \varphi_i(N, C, f). \tag{11}$$

From (8) and (11), we can see that (6) holds for all $S \in Seq^N$ with $|S| = n-1$.

Similarly, by *population monotonicity*$_+$, for $N'' := \{1, 2, \ldots, n-2\}$,

$$\sum_{i=1}^{n-2} \varphi_i(N'', C, f_{n-2}) \geq \sum_{i=1}^{n-2} \varphi_i(N', C, f_{n-1}) \geq \sum_{i=1}^{n-2} \varphi_i(N, C, f_n)$$

where f_{n-2} is the restriction of f_{n-1} to $\{1, 2, \ldots, n-2\}$, which is equivalent to the restriction of f to $\{1, 2, \ldots, n-2\}$. By the definition of depreciation method,

$$\sum_{i=1}^{n-2} \varphi_i(N'', C, f_{n-2}) = C - f_{n-2}(n-2) = C - f(n-2).$$

Thus,

$$d^*(\{1, 2, \ldots, n-2\}) \geq \sum_{i=1}^{n-2} \varphi_i(N, C, f).$$

By *conservatism*,

$$\sum_{i=1}^{n-2} \varphi_i(N, C, f) \geq \sum_{i=2}^{n-1} \varphi_i(N, C, f) \geq \sum_{i=3}^{n} \varphi_i(N, C, f).$$

Then, by the definition of d^*, we can induce for any $S \in Seq^N$ and $|S| = n-2$,

$$d^*(S) \geq \sum_{i \in S} \varphi_i(N, C, f).$$

Repeating this procedure proves that Proposition 6 holds for all $S \in Seq^N$.

By the definition of d, for all $S \subset N$, we have

$$d(S) = \sum_{S_j \in (S)^m} d^*(S_j) \geq \sum_{S_j \in (S)^m} \sum_{i \in S_j} \varphi_i(N, C, f) \geq \sum_{i \in S} \varphi_i(N, C, f). \qquad \square$$

Proposition 6 provides a reason why the straight-line method satisfies *core selection*.

The next axiom pertains to the situation that arises after the first year. It says that even after the first year if the remaining problem is regarded as a new game starting from the second year, the burden amount for other years should remain the same as in the original game. Let $N'' := \{2, 3, \ldots, n\}$ and $g\colon \{1, \ldots, n-1\} \to \mathbb{R}_+$ be defined by setting $g(k) := f(k+1)$ for all $k \in \{1, \ldots, n-1\}$.

Consistency[11]: For all $(N, C, f) \in \mathcal{D}^N$ and all $i \in N''$,

$$(N'', C - \varphi_1(N, C, f), g) \in \mathcal{D}^{N''} \tag{12}$$

and

$$\varphi_i(N'', C - \varphi_1(N, C, f), g) \;\; = \;\; \varphi_i(N, C, f). \tag{13}$$

We refer to $(N'', C - \varphi_1(N, C, f), g)$ as a reduced problem.

Let $C'' = C - \varphi_1(N, C, f)$. Then note that $(N'', C'', g) \in \mathcal{D}^{N''}$ if and only if the acquisition cost $C'' = C - \varphi_1(N, C, f)$ and the revised market value function g defined above satisfy

$$C'' = C - \varphi_1(N, C, f) > g(1) > \ldots > g(n-1) = R, \tag{14}$$

and for all $s, t \in \{1, 2, \ldots, n-1\}$ with $s + t \leq n - 1$,

$$C'' - g(s) \geq g(t) - g(t+s). \tag{15}$$

We check whether these conditions are satisfied for each method.

Claim 17 The straight-line method satisfies (14).
Proof. From (1),

$$\begin{array}{rcl} C - f(1) & \geq & f(1) - f(2) \\ C - f(1) & \geq & f(2) - f(3) \\ & \vdots & \\ C - f(1) & \geq & f(n-1) - f(n), \end{array}$$

Summing up these inequalities,

$$(n-1)(C - f(1)) \geq f(1) - f(n).$$

[11]See Thomson (2011) for a survey of this axiom applied in various situations.

Then,

$$
\begin{aligned}
& n(C - f(1)) \geq C - f(1) + f(1) - f(n) \\
\leftrightarrow \quad & C - f(1) \geq \frac{C - R}{n}.
\end{aligned}
$$

By this inequality and (1),

$$C - SL_1(N, C, f) = C - \frac{C - R}{n} \geq f(1) > f(2) = g(1). \qquad \square$$

Claim 18 The straight-line method does not satisfy (15), consequently, it does not satisfy *consistency*.

Proof. When $|N| = 3, C = 160, f(1) = 100, f(2) = 65, \text{and } f(3) = R = 10,$

$$C'' - g(1) = 110 - 65 \leq g(1) - g(2) = 65 - 10.$$

Therefore, the straight-line method does not satisfy (15). □

Claim 19 The declining-balance method does not satisfy (15), consequently, it does not satisfy *consistency*.

Proof. When $|N| = 3, C = 160, f(1) = 100, f(2) = 65, \text{and } f(3) = R = 10,$

$$C'' - DB_1(N, C, f) = 160 - 96.5039\ldots < f(1) = 100.$$

Then, the declining-balance method does not satisfy (14). □

Claim 20 The declining-balance method does not satisfy (15).

Proof. When $|N| = 3, C = 160, f(1) = 100, f(2) = 65, \text{and } f(3) = R = 10,$

$$C'' - g(1) < 0$$

Then, the declining-balance method does not satisfy (15). □

Claim 21 The sum-of-the-years' digits method satisfies (14).

Proof. By the definition of the sum-of-the-years' digits method,

$$SYD_1(N, C, f) = \frac{2}{n+1}(C - R).$$

From Claim 13,

$$C - f(2) \geq \frac{2}{n}(C - R).$$

Then,

$$C - f(2) \geq \frac{2}{n}(C - R) > SYD_1(N, C, f)$$

Therefore,

$$C - SYD_1(N, C, f) > C - (C - f(2)) = g(1). \qquad \square$$

Claim 22 The sum-of-the-years' digits method does not satisfy (15), consequently, it does not satisfy *consistency*.
Proof. When $|N| = 3, C = 160, f(1) = 100, f(2) = 65,$ and $f(3) = R = 10,$

$$C'' - g(1) = 75 - 65 \leq g(1) - g(2) = 55$$

Therefore, the straight-line method does not satisfy (15). $\square$

Claim 23 The fair value measurement satisfies (14).
Proof.

$$C - FV_i(N, C, f) = f(1) \geq f(2) = g(1). \qquad \square$$

Claim 24 The fair value measurement does not satisfy (15), consequently, it does not satisfy *consistency*.
Proof. When $|N| = 3, C = 160, f(1) = 100, f(2) = 65,$ and $f(3) = R = 10,$

$$C'' - g(1) = 100 - 65 \leq g(1) - g(2) = 55$$

Therefore, the fair value measurement does not satisfy (15). $\square$

Then, the above claims are summarized as follows:

Proposition 7 None of the four methods satisfies *consistency*.

Now, let us consider a weaker version:

Conditional consistency: For all $(N, C, f) \in \mathcal{D}^N$, if

$$(N'', C'', g) \in \mathcal{D}^{N''}, \tag{16}$$

then for all $i \in N$",

$$\varphi_i(N'', C'', g) \;\; = \;\; \varphi_i(N, C, f) \tag{17}$$

where N'', C'' and g are defined in the same manner as before.

As evident from the definition of each depreciation method in Section 3, if the reduced problem is well-defined, then, (17) is automatically satisfied by all depreciation methods. Then, we obtain the following proposition as follows:

Proposition 8 All four methods satisfy *conditional consistency.*

5 Axioms in practice

This section will discuss the practical implications of the results presented in the previous section.

5.1 Core selection

Arata, Shimogawa, and Inohara (2024) interpret this axiom as follows: "The grand coalition minimizes the overall cost of the investment on the asset and, moreover, that the burden for each fiscal year is also smaller than the burden when using it through other coalitions." Recall that "players" in a depreciation game are fiscal years. So, what they are assuming is that there are some agents, such as managers and employees, who are evaluated by annual profits in the background of each fiscal year. In such a context, their interpretation of core selection seems to be reasonable.

However, one might object to this interpretation because there are not always such agents in the background of this model. As shown above, the declining-balance method and the sum-of-the-years' digits method don't satisfy *core selection.* As we have described in Section 3, these two methods end up with higher expenses, and hence lower taxable income, in the earlier years of the asset's useful life. This is the main reason why tax authorities in many countries allow these two methods to promote investments. This suggests that the depreciation problem is not merely a cost-sharing game but rather a more complex one in which some positive benefits of being charged more, such as tax-saving effects, should also be considered.

On the other hand, Proposition 6 says that *core selection* is implied by a combination of a neutral axiom *population monotonicity* with two particular axioms, *final year reasonableness* and *conservatism*. Thus, even if you feel that *core selection* is not so desirable, you cannot avoid accepting it if you accept these other axioms.

5.2 Conservatism

Proposition 6 states that core selection is implied when population monotonicity$_+$ is combined with conservatism. As described in Section 4, conservatism can also be seen as embodying one of the accounting conventions known as conservatism.

AICPA (1970) is the statement concerning basic concepts and accounting principles underlying financial statements of business enterprises. It shows pervasive principles which specify the general approach accountants should take to recognition and measurement of events that affect the financial position and results of operations of enterprises. According to this statement, the pervasive principles are divided into (1) pervasive measurement principles and (2) modifying conventions. The pervasive measurement principles mainly establish the basis for the recognition of earnings (revenue and expense). For example, "systematic and rational allocation" is one of the pervasive measurement principles described in the statement. The modifying conventions are applied when rigid adherence to the pervasive measurement principles produce unsatisfactory results. They describe conservatism as follows:

> "Historically, managers, investors, and accountants have generally preferred that possible errors in measurement be in the direction of understatement rather than overstatement of net income and net assets. This has led to the convention of conservatism, which is expressed in rules adopted by the profession as a whole such as the rules that inventory should be measured at the lower of cost and market and that accrued net losses should be recognized on firm purchase commitments for goods for inventory. These rules may result in stating net income and net assets at amounts lower than would otherwise result from applying the

pervasive measurement principles."

However, no clear criteria were ever provided for when conservatism as a modifying convention should be applied. As a result, the interpretation and treatment of conservatism within accounting rules remained ambiguous and controversial.

On the other hand, our result (Proposition 6) can be interpreted as suggesting that *conservatism* modifies or restricts *population monotonicity*$_+$, which might be regarded as a type of pervasive principle. The discussion here may offer new insights into the current, ambiguous positioning of conservatism.

5.3 Consistency

Finally, we discuss the practical implications of this axiom. AICPA (1970) lists the qualitative objectives of financial statements, and one of them is comparability. It considers "consistency" as one of the important factors supporting comparability. AICPA (1970) describes that "[a]lthough financial accounting practices and procedures are largely conventional, consistency in their use permits comparisons over time" and "consistency of treatment over time are important factors in determining the appropriate expense recognition principle." As mentioned above, a depreciation method should be rational and systematic. Therefore, we can consider that the axiom, *consistency*, examined here, corresponds to this accounting concept.

6 Conclusion

In this paper, we have studied through the axiomatic approach in what sense depreciation methods used in practice, such as the straight-line method, the declining-balance method, the sum-of-the-years' digits method, and the fair value measurement, can be considered rational and systematic.

First, we have examined the axioms of the depreciation methods. Table 1 summarizes our results. As Table 1 shows, the straight-line method satisfies all axioms but *consistency* discussed in this paper. It's been commonly said, as described in Section 3, that the great virtue of this method is that it is simple to apply and easy to understand. However, our results indicate that the advantages of the straight-line method are not only in its simplicity but

Table 1: Properties of depreciation methods

Axiom	SL	DB	SYD	FV
Monotonicity	Yes	Yes	Yes	Yes
Additivity	Yes	No	Yes	Yes
Population monotonicity	Yes	No	No	Yes
The Final year reasonableness	Yes	No	No	Yes
Population monotonicity$_+$	Yes	No	No	Yes
Conservatism	Yes	Yes	Yes	No
Core selection	Yes	No	No	Yes
Consistency	No	No	No	No
Conditional consistency	Yes	Yes	Yes	Yes

also in the many other favorable characteristics it possesses, as examined here.

Second, we have provided a practical interpretation for each axiom examined in this paper and associate the two axioms, *consistency* and *conservatism*, with the accounting principles bearing the same names.

Since the players are not humans in the model studied in the paper, it is not so clear whether the axiom of *core selection* is desirable or not in this context. We have also provided a positive answer to this question. Namely, we show that three appealing axioms, *population monotonicity*, *the final year reasonableness*, and *conservatism*, jointly imply core selection. Meanwhile, the two depreciation methods that do not satisfy core selection are allowed by tax law. This suggests that the depreciation problem is not merely a cost-sharing game but rather a more complex one in which some positive benefits of being charged more, such as tax-saving effects, should also be considered.

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