

**Institute for Economic Studies, Keio University**

**Keio-IES Discussion Paper Series**

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its Effects on the Wage Phillips Curve in Japan**

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**2024年12月26日**

**2024-028**

**<https://ies.keio.ac.jp/publications/24770/>**

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26 December, 2024

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キーワード: wage Phillips curve, wage growth, unemployment, DSGE models, structural change

### 【要旨】

In this paper, I quantify the effects of the increase in part-timers on the flattening wage Phillips curve in Japan. Specifically, I formulate the New Keynesian DSGE model which explicitly incorporates two different types of labour forces, which are fulltime labour and part-time labour, and their unemployment considering the structural change of the increase in part-timers. I employ estimation with Japan's data using Bayesian techniques and obtain plausible results for structural and policy parameters. Then I do a counterfactual simulation without the structural change and compare it with baseline estimation. I find that roughly 30% of the wage Phillips curve flattening in Japan has been ascribed to the increase in part-timers.

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謝辞:

I thank Ippei Fujiwara, Nao Sudo, Jiro Yoshida, Francesco Zanetti, Daisuke Ikeda, and seminar participants for valuable comments. I also thank Mariano Kulish for providing his code.

# The Increase in Part-timers and its Effects on the Wage Phillips Curve in Japan\*

Yasutaka Ogawa<sup>†</sup>

December 26, 2024

## Abstract

In this paper, I quantify the effects of the increase in part-timers on the flattening wage Phillips curve in Japan. Specifically, I formulate the New Keynesian DSGE model which explicitly incorporates two different types of labour forces, which are full-time labour and part-time labour, and their unemployment considering the structural change of the increase in part-timers. I employ estimation with Japan's data using Bayesian techniques and obtain plausible results for structural and policy parameters. Then I do a counterfactual simulation without the structural change and compare it with baseline estimation. I find that roughly 30% of the wage Phillips curve flattening in Japan has been ascribed to the increase in part-timers.

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# I. Introduction

In the last few decades, the wage Phillips curve between wage growth and the unemployment rate (hereafter WPC) in Japan has been flattening as shown in Figure 1. A simple OLS estimation with decade dummy variables, as shown in Table I, supports the argument. Especially in the last several years, the unemployment rate has been low and has almost reached the structural unemployment rate, as Bank of Japan (2017) indicates, but labour market tightness has not increased wages. The flattening of the WPC has been widely discussed, and most of the earlier studies<sup>1</sup> ascribe it to the decrease of the inflation expectation over the medium- and long-term horizon.

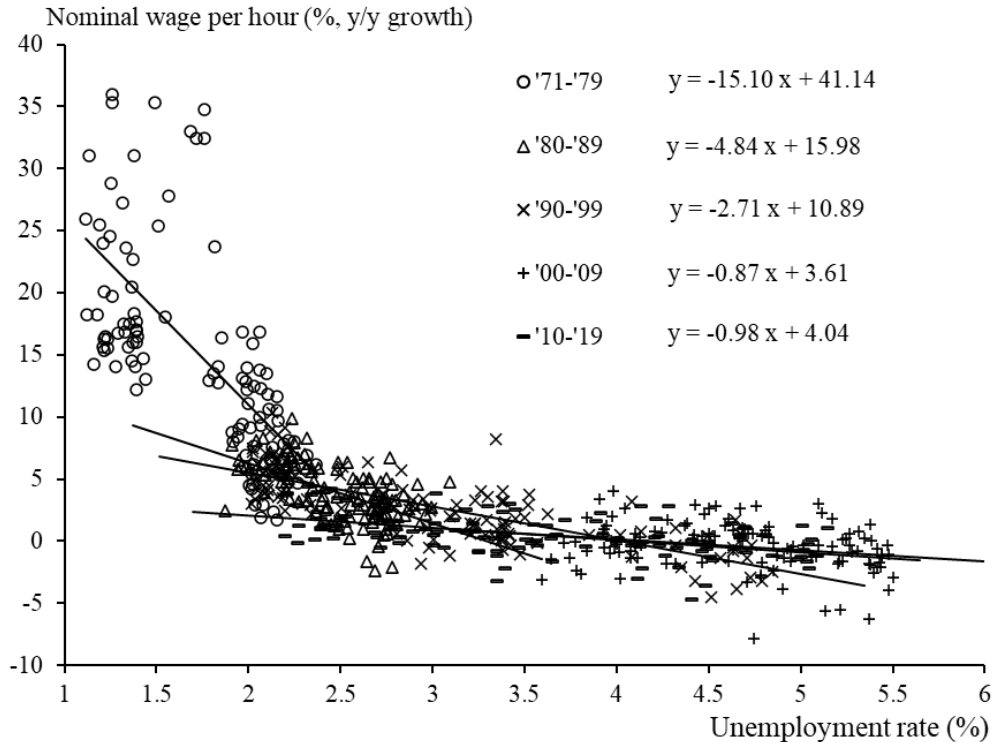


Figure 1: Nominal wage Phillips curve in Japan

Some studies ascribe it to changes in Japan's labour market itself. Yamamoto (2010) and Eguchi and Teramoto (2017) consider this question by examining the changes in deep parameters based on the New Keynesian DSGE model with search friction, and conclude

<sup>1</sup>One of the most famous studies is Kimura and Kurozumi (2010).

	coefficient	S.D.	t-value	p-value	
intercept	4.04	1.23	3.29	0.00	***
slope	-0.98	0.34	-2.93	0.00	***
slope dummy 70's	<u>-14.12</u>	0.88	-16.11	0.00	***
slope dummy 80's	<u>-3.86</u>	1.12	-3.45	0.00	***
slope dummy 90's	<u>-1.73</u>	0.48	-3.57	0.00	***
slope dummy 00's	0.12	0.64	0.18	0.86	
intercept dummy 70's	37.10	1.88	19.70	0.00	***
intercept dummy 80's	11.94	2.95	4.05	0.00	***
intercept dummy 90's	6.84	1.65	4.15	0.00	***
intercept dummy 00's	-0.43	2.82	-0.15	0.88	

Table I: OLS estimation of nominal wage Phillips curve in Japan

that the reason is the increase in (i) downward rigidity of nominal wages and (ii) real rigidity in wage. However, during the same period, significant structural change has been identified in Japan's labour market. As is the other countries, labour forces in Japan are divided into two types: one is the full-time worker<sup>2</sup> (hereafter full-timer), who works regularly under the prescribed working hours; and the other is the part-time worker (hereafter part-timer), who works less than the other ordinary workers. Although both are important labour forces, each plays a slightly different role in Japan's labour market and more importantly, the number of part-timers has been increasing and has gradually increased its presence in the labour market as shown in Figure 2<sup>3</sup>. When comparing real hourly wage and total hours worked of full-timers with those of part-timers, the standard deviations of real hourly wage and total hours worked of part-timers are greater than those of full-timers, but the standard deviation of part-timers' total hours worked is much greater than that of their real hourly wage as shown in Figure 3 and Figure 4. This implies that regarding part-timers, hours worked is

<sup>2</sup>The official definition of the two labour types (in the Monthly Labour Survey of the Ministry of Health, Labour and Welfare) is as follows: Part-time workers are the persons who satisfy either of (a) whose scheduled working hours per day is shorter than ordinary workers or (b) whose scheduled working hours per day is the same as ordinary workers, but whose number of scheduled working days per week is fewer than ordinary workers. Full-time workers are regular employees who are not a part-time worker.

<sup>3</sup>This is the share of part-time workers in the Labour Force Survey. The share of part-time workers in the Monthly Labour Survey shows almost the same development.

more inclined to be adjusted than the real hourly wage against a certain shock in labour supply and demand. Therefore, an increase in part-timers could weaken the impact of labour shortages on wage growth, leading to a flattening of the WPC.

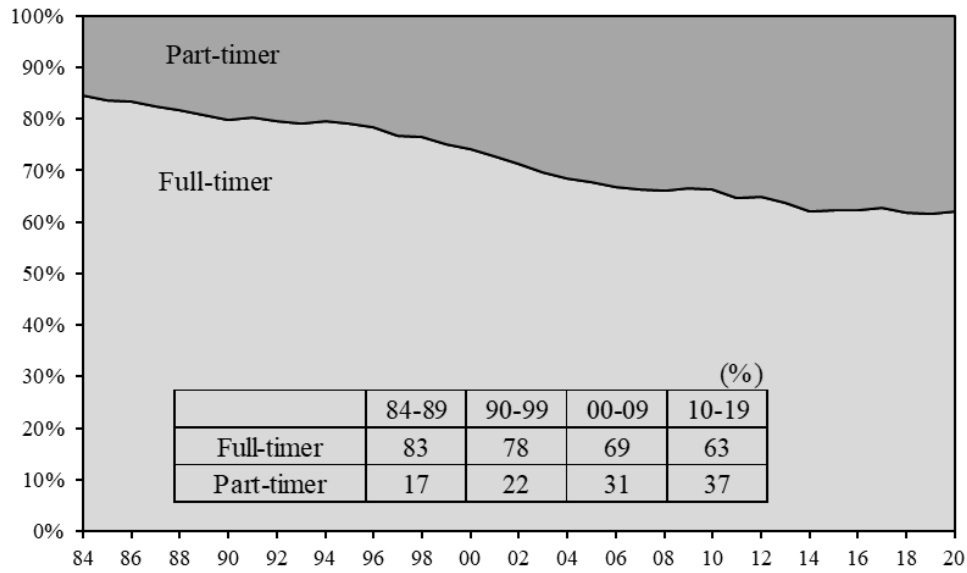


Figure 2: The share of part-timers

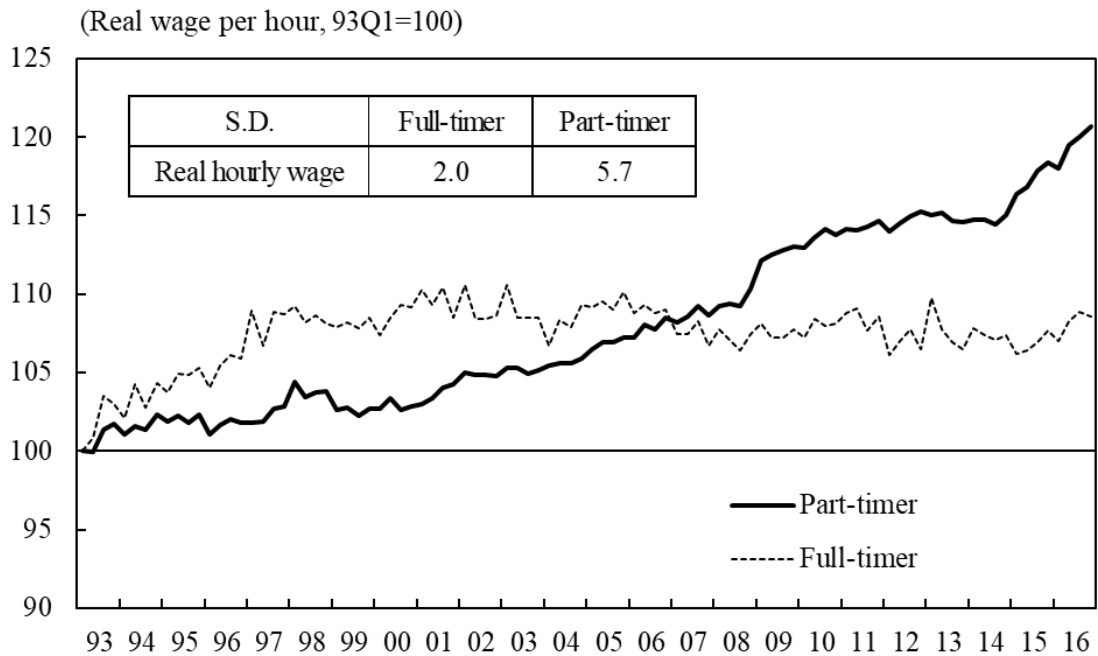


Figure 3: Real hourly wage of full-timer and part-timer

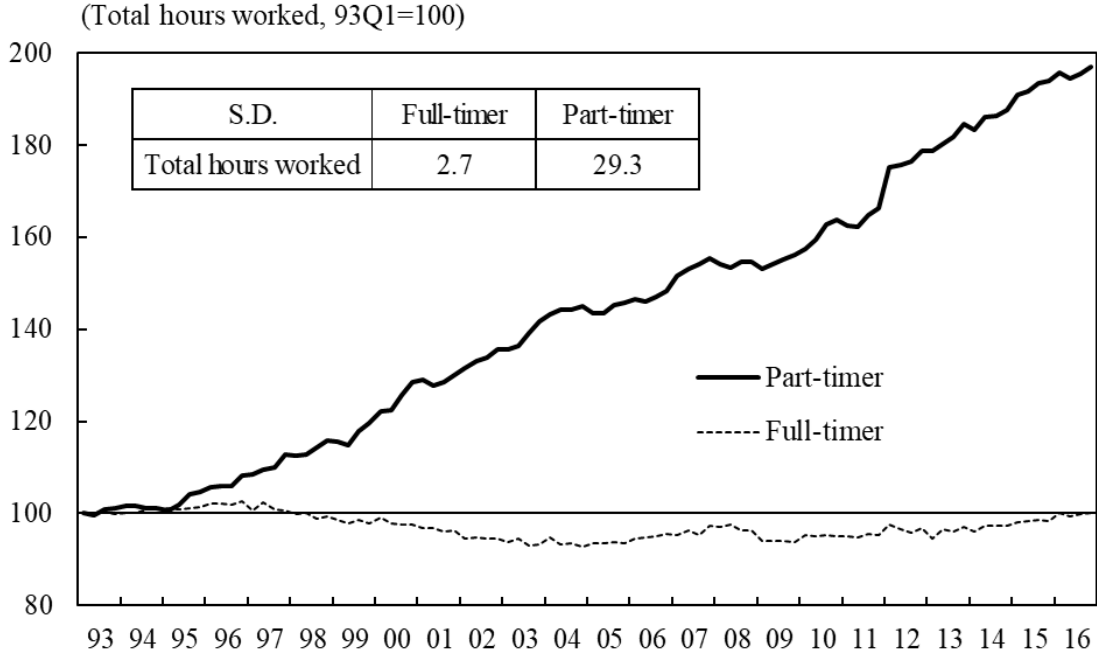


Figure 4: Total hours worked of full-timer and part-timer

A good number of studies stress the important role of part-timers in better understanding the labour market in Japan. Genda (2017) and Genda and Ozaki (2020) argue that the high labour-supply elasticity of part-timers (especially women and elderly people) explains low wage growth in Japan. In the context of the DSGE model, Mukoyama et al. (2021) finds that the procyclicality of the net flow from part-time to full-time employment is essential in accounting for countercyclical patterns of part-time employment using the U.S. data. However, there are no studies that explicitly consider the changing proportions of the two labor forces as a structural change in the context of DSGE analysis.

In this paper, I quantify the effects of the increase in part-timers on the flattening of the aggregate WPC in Japan directly. Specifically, I construct a New Keynesian DSGE model which introduces explicitly (i) two different types of labour forces, full-time labour and part-time labour, and (ii) the structural change of labour forces from full-time labour to part-time labour by utilizing the estimation method of Kulish and Pagan (2017). Then, I estimate the deep parameters for full-time labour and part-time labour, respectively, using

Japan's data. Finally, I perform a counterfactual analysis of the aggregate WPC and show that the slope would have been steeper if the structural change has not had been introduced.

We need to note that the aggregate WPC is a simple correlation between nominal wage growth and the unemployment rate, and is different from the NK-WPC, which is one of the key equations in the DSGE model. In the context of DSGE model, the aggregate WPC is determined by the following factors as (a) changes in share of part-time labour forces, (b) changes in type of shocks hitting the economy, and (c) changes in other macroeconomics conditions (including monetary policy stance, other structural changes, and so forth), so that I explicitly incorporate the structural change in Japan's labour market in the model and the try to examine its effects on the wage dynamics of full-time and part-time labour.

This paper is divided into five sections, the first being this introduction. Section II reviews the related literature, section III describes the properties of the two types of labour forces using various data, section IV explains the details of the model and the model calibration, section V presents the results, and section VI draws some conclusions.

## II. Literature

This paper contributes to the literature in four ways.

Firstly, this paper relates to the backgrounds for the flattening of the WPC typically before Covid-19. There are two types of argument about the underlying background of low wage growth. One type of argument is that structural change has occurred in the labour market and it leads to a flatter WPC. Chen et al. (2020) estimates parameters using the U.S. data and explains not only the changes in the type of shocks but also changes in other macroeconomic conditions that contribute to the observed WPC flattening. Likewise, in the U.S. and Europe, Daly and Hobijn (2014) and Stansbury and Summers (2020) ascribe the WPC flattening to structural changes in the labour market, and lots of literature mention the compositional effect for low wage growth. Regarding Japan, Genda (2017) and Genda



and Ozaki (2020) mention the high labour-supply elasticity of part-time labour as well as the compositional effect for the flatter WPC. However, there are no studies that explicitly consider the gradual structural change of the labor force in a general equilibrium model.

The other is that structural changes have not occurred in the labour market and changes in type of shocks can explain WPC flattening. This argument can be seen at Eguchi and Teramoto (2017) and Kubota et al. (2022) using Japan's data.

Secondly, this paper relates to the dual labour market model. In the context of DSGE model, Mukoyama et al. (2021), Eijffinger et al. (2020), Madeira (2014) and Miyamoto and Sasaki (2016) have dual labor or wage-setting. They focus on describing the business cycle of labour with their models (relating to the Shimer Critique) without structural changes. Specifically, Mukoyama et al. (2021) explicitly consider the transition from part-time employment to full-time by using search and matching mechanism, and Eijffinger et al. (2020) has developed a New Keynesian DSGE model wherein households are divided into four types and use monopolistic power to set wages according to four different rules (sticky prices a la Calvo, sticky information, full flexibility, and indexation), but labour itself is completely substitutable in the both studies.

Thirdly, this paper provides the deep parameter estimation of the full-timers and part-timers in Japan. In microeconomics, regarding labour supply elasticity, a few earlier studies estimate those of full-timers and part-timers in Japan. Kuroda and Yamamoto (2008) estimates those of women, but other parameters are not estimated before. Likewise, this study also shows the elasticity between the full-time and part-time labour forces in the CES production function. Yamaguchi (2011) indicates the lack of consensus on whether full-time and part-time work are substitutes or complements in Japan, but states that, at the economy-wide level, all elasticity indicators indicate a substitutive relationship between full-timers and part-timers.

Lastly, this paper is one of the applications regarding the estimation of the DSGE model with structural change as proposed in Kulish and Pagan (2017). Although there are still

few applications of this paper in Japan, this paper demonstrates that the DSGE model with structural change explains the data very well.

### III. Properties of Two Types of Labour Forces

In this section, I elaborate on the properties of the two types of labour forces in Japan's labour market. In general, full-timers are considered the main workforce whereas part-timers are considered the adjustment workforce in Japan's contemporary labour market typically before Covid-19 as Munakata and Higashi (2016) explain.

Since I construct a DSGE model with two different types of labour in the later section, I mainly focus on the following three deep parameters of the two. They are the Calvo parameter  $\rho_w$ , substitution elasticity of differentiated labour  $\psi_w$  and inverse Frisch elasticity of labour supply  $\varphi_w$ . The intuitive interpretation is that  $(1 - \rho_w)$  means the probability of each household member changing his or her wage in each period,  $\Lambda^w (= \psi_w / (\psi_w - 1))$  stands for the wage mark-up rate (premium against the marginal productivity of labour), and  $\varphi$  captures the elasticity of hours worked to a wage rate change.

First, regarding the probability of changing the wage, the wages of part-timers are considered easier to change than those of full-timers. There is no direct data or analysis which can compare the frequency of wage change between full-timers' hourly wages and part-timers' hourly wages, but the following facts would be enough to suggest its plausibility. Almost all the full-timers are employed at will, whereas two-thirds of part-timers are employed with a definite term as shown in Figure 5. The wages for employment at will are usually fixed for at least one year and changed at the beginning of the financial year. The wages for employment for a fixed term are usually changed at the beginning of a new term of employment and influenced by the minimum wage and business conditions.

Second, regarding the wage level, the hourly wages of part-timers are much lower than those of full-timers even when they do the same tasks. This is because not only the base

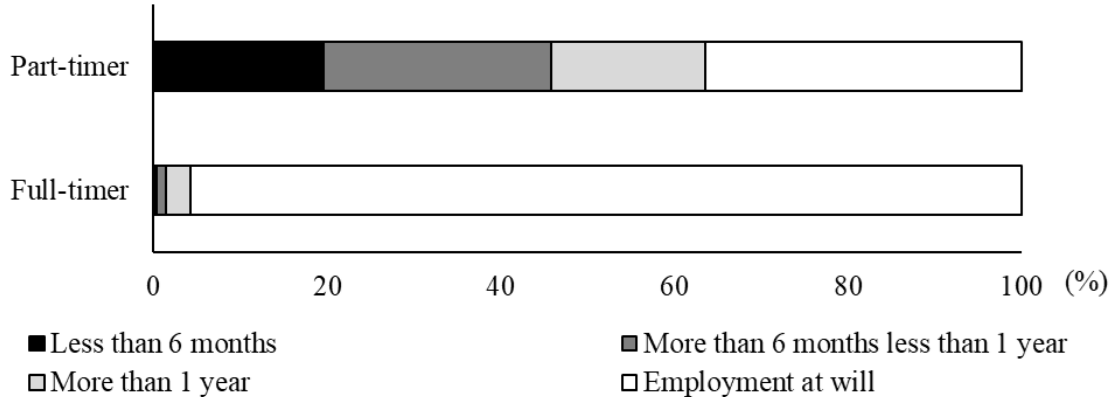


Figure 5: Average contract period of full-timers and part-timers in 2012

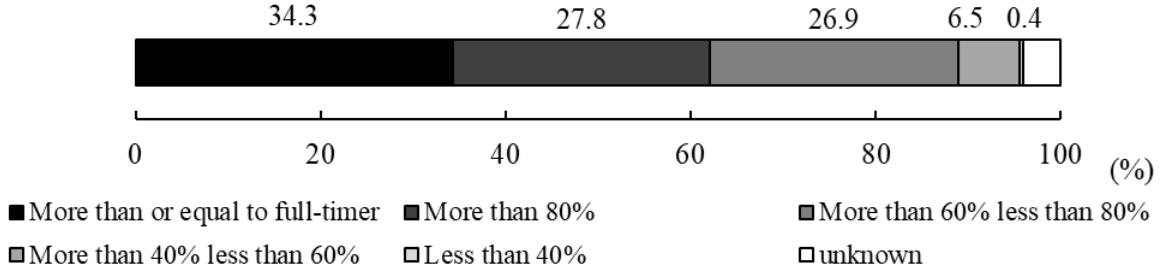
salary itself is different between them, but also special cash earnings are usually less (or not paid in some cases) for part-timers as shown in Figure 6. These facts imply that the wage markup rate for full-timers is much larger than that of part-timers. There is no consensus on the value of the wage mark-up rate in Japan, but the existence of a wage gap between full-timers and part-timers is widely accepted.

Third, regarding labour supply elasticity, a few earlier studies estimate those of full-timers and part-timers in Japan. Kuroda and Yamamoto (2008) estimates those of women and concludes that that of full-timers (0.132) is smaller than that of part-timers (0.292). In addition, Sakura et al. (2005) suggests the same fact as well. Therefore, even though there is no consensus about the values of the parameters, the Frisch elasticity of full-time labour would be smaller than that of part-time labour.

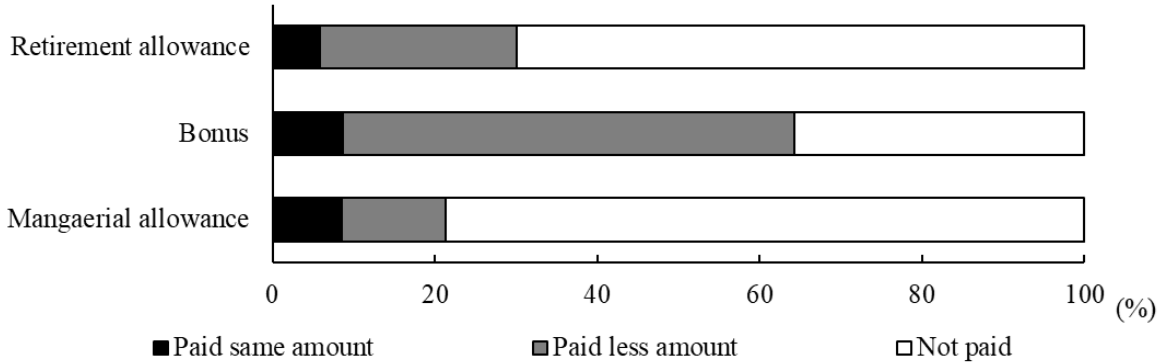
In the next section, I construct a DSGE model which explicitly incorporates the two different labour types based on the discussion in this section.

## IV. Model Setup

This section presents a DSGE model which is used for estimation. The model is mainly based on Erceg et al. (2000), or one of its applications, Christiano et al. (2005). There are three major amendments to explicitly incorporate the two different types of labour forces



(a) Hourly base salary of part-timers compared to full-timers when both do the same tasks



(b) Special cash earnings paid to part-timers compared to full-timers

Figure 6: Comparison between the wages of full-timers and part-timers in 2011

and their unemployment. One is the technology of the labour bundler, another is the utility function of household members, and the other is the assumptions of Galí (2011). In addition, I assume that the steady-state value of the two different types of labour forces continues to change over time by utilizing the estimation method of Kulish and Pagan (2017).

Regarding the technology of the labour bundler, I introduce the same setting used for goods heterogeneity in Aoki (2001). The total labour demand of the intermediate firms is divided into full-time labour demand and part-time labour demand by the technology of the labour bundler. Both wages are chosen by household members à la Calvo and each actual employment is decided by the demand of the firms. In the model, to enhance generosity, I introduce the CES function rather than the Cobb-Douglas function, which is used to incorporate goods heterogeneity in Aoki (2001). For the utility function of household members, each labour burden independently and negatively contributes to the household member's utility.

Regarding the assumptions of Galí (2011), I add two more assumptions about household members and their labour. First, the model assumes that labour is indivisible, with all variations in hired labour input taking place at the extensive margin (that is in the form of variations in employment). Second, the model assumes a large representative household with a continuum of members represented by the unit square and indexed by a pair  $(i, j) \in [0, 1] \times [0, 1]$ . The first dimension, indexed by  $i \in [0, 1]$ , represents the variety of labour services in which a given household member is specialized. The second dimension, indexed by  $j \in [0, 1]$ , determines his or her disutility from work. The individual disutility from work is given by  $j^\varphi$  if he or she is employed and zero otherwise, where  $\varphi \geq 0$  is the inverse Frisch elasticity of labour supply.

### A. Labor Bundler

The labour bundler defines  $H_t$  as the time  $t$  effective labor that is supplied to the intermediate firms with

$$H_t = [\gamma e^{z_t^\gamma} (H_t^F)^{\frac{\tau-1}{\tau}} + (1 - \gamma e^{z_t^\gamma}) (H_t^P)^{\frac{\tau-1}{\tau}}]^{\frac{\tau}{\tau-1}}, \quad (1)$$

where

$$H_t^F = \left[ \int_0^1 \left( H_t^{F(i)} \right)^{\frac{\psi_w^F - 1}{\psi_w^F}} di \right]^{\frac{\psi_w^F}{\psi_w^F - 1}}$$

$$H_t^P = \left[ \int_0^1 \left( H_t^{P(i)} \right)^{\frac{\psi_w^P - 1}{\psi_w^P}} di \right]^{\frac{\psi_w^P}{\psi_w^P - 1}}$$

and  $z_t^\gamma$  is labour bundler composition shock.

The labour bundler maximizes the profit

$$\Pi_t = H_t P_t W_t - \int_0^1 h_t^{F(i)} P_t W_t^{F(i)} di - \int_0^1 h_t^{P(i)} P_t W_t^{P(i)} di \quad (2)$$

where  $W_t$  is the aggregate real wage at time  $t$ , and  $W_t^{F(i)}$  and  $W_t^{P(i)}$  are real wages of differentiated labour of each labour type indexed by  $i$  at time  $t$ .

FOCs with respect to  $h_t^{F(i)}$  and  $h_t^{P(i)}$  are

$$h_t^{F(i)} = \left[ \frac{\gamma e^{z_t^\gamma} W_t}{W_t^{F(i)}} \left( \frac{H_t}{H_t^F} \right)^{\frac{1}{\tau}} \right]^{\psi_w^F} H_t^F \quad (3)$$

$$h_t^{P(i)} = \left[ \frac{(1 - \gamma e^{z_t^\gamma}) W_t}{W_t^{P(i)}} \left( \frac{H_t}{H_t^P} \right)^{\frac{1}{\tau}} \right]^{\psi_w^P} H_t^P \quad (4)$$

where

$$W_t = \left[ (\gamma e^{z_t^\gamma})^\tau (W_t^F)^{(1-\tau)} + (1 - \gamma e^{z_t^\gamma})^\tau (W_t^P)^{(1-\tau)} \right]^{\frac{1}{1-\tau}} \quad (5)$$

$$W_t^F = \left[ \int_0^1 \left( W_t^{F(i)} \right)^{1-\psi_w^F} di \right]^{\frac{1}{1-\psi_w^F}} \quad (6)$$

$$W_t^P = \left[ \int_0^1 \left( W_t^{P(i)} \right)^{1-\psi_w^P} di \right]^{\frac{1}{1-\psi_w^P}} \quad (7)$$

$W_t^F$  is the composite real wage of the differentiated labours of full-time labour and  $W_t^P$  is the composite real wage of the differentiated labours of part-time labour.

## B. Households

The household's period utility corresponds to the integral of its members' utility, and is thus given by

$$U \left( C_t^{(i)}, h_t^{F(i)}, h_t^{P(i)} \right) \equiv \frac{\left( C_t^{(i)} \right)^{1-\sigma}}{1-\sigma} - \chi e^{z_t^{F,l}} Z_t^{1-\sigma} \int_0^{h_t^{F(i)}} j^{\varphi_t^F} dj - (1-\chi) e^{z_t^{P,l}} Z_t^{1-\sigma} \int_0^{h_t^{P(i)}} j^{\varphi_t^P} dj$$

$$= \frac{\left(C_t^{(i)}\right)^{1-\sigma}}{1-\sigma} - \chi e^{z_t^{F,l}} Z_t^{1-\sigma} \frac{\left(h_t^{F(i)}\right)^{1+\varphi_w^F}}{1+\varphi_w^F} - (1-\chi) e^{z_t^{P,l}} Z_t^{1-\sigma} \frac{\left(h_t^{P(i)}\right)^{1+\varphi_w^P}}{1+\varphi_w^P} \quad (8)$$

where  $h_t^{F(i)} \in [0, 1]$  and  $h_t^{P(i)} \in [0, 1]$  are the fraction of members specialized in type  $i$  labour who are employed in period  $t$  regarding full-time labour and part-time labour, respectively, and  $z_t^{F,l}$  and  $z_t^{P,l}$  are labour supply shock for full-time labour and part-time labour, respectively, and  $z_t^{F,l}$  and  $z_t^{P,l}$  are labour supply shock for full-time labour and part-time labour, respectively.

Therefore, the objective of household members specialized in type  $i$  labour is to maximize

$$E_t \sum_{n=0}^{\infty} \beta^n e^{z_{t+n}^\beta} \left[ \frac{\left(C_{t+n}^{(i)}\right)^{1-\sigma}}{1-\sigma} - \frac{\chi e^{z_{t+n}^{F,l}} Z_{t+n}^{1-\sigma} \left(h_{t+n}^{F(i)}\right)^{1+\varphi_w^F}}{1+\varphi_w^F} - \frac{(1-\chi) e^{z_{t+n}^{P,l}} Z_{t+n}^{1-\sigma} \left(h_{t+n}^{P(i)}\right)^{1+\varphi_w^P}}{1+\varphi_w^P} \right] \quad (9)$$

subject to a real budget constraint,

$$C_t^{(i)} + \frac{B_t^{(i)}}{P_t} = W_t^{F(i)} h_t^{F(i)} + W_t^{P(i)} h_t^{P(i)} + \xi_t^{(i)} + R_{t-1} \frac{B_{t-1}^{(i)}}{P_{t-1}} \quad (10)$$

where  $\xi_t^i$  is the excess profits of the intermediate goods firms that are paid to household members specialized in type  $i$  in period  $t$ . The model assumes complete financial markets with no obstacles to borrowing against future income, so that each household member faces a single intertemporal budget constraint. The model further assumes that household members can insure one another against idiosyncratic income risk.

Note that there is neither a government nor government bond in the model.

## B.1. Consumption and savings of household

The household members specialized in type  $i$  labour maximize their utility with respect to  $C_t^{(i)}$  and  $B_t^{(i)}$ .

The Lagrangian is

$$\mathcal{L} = E_t \sum_{n=0}^{\infty} \beta^n \left\{ e^{z_{t+n}^\beta} \left[ \frac{\left(C_{t+n}^{(i)}\right)^{1-\sigma}}{1-\sigma} - \frac{\chi e^{z_{t+n}^{F,l}} Z_{t+n}^{1-\sigma} \left(h_{t+n}^{F(i)}\right)^{1+\varphi_w^F}}{1+\varphi_t^F} - \frac{(1-\chi) e^{z_{t+n}^{P,l}} Z_{t+n}^{1-\sigma} \left(h_{t+n}^{P(i)}\right)^{1+\varphi_w^P}}{1+\varphi_w^P} \right] - \lambda_{t+n} \left[ C_{t+n}^{(i)} + \frac{B_{t+n}^{(i)}}{P_{t+n}} - W_{t+n}^{F(i)} h_{t+n}^{F(i)} - W_{t+n}^{P(i)} h_{t+n}^{P(i)} - \xi_{t+n}^{(i)} - R_{t+n-1} \frac{B_{t+n-1}^{(i)}}{P_{t+n}} \right] \right\}. \quad (11)$$

FOCs with respect to  $C_t^{(i)}$  and  $B_t^{(i)}$  are

$$\lambda_t = e^{z_t^\beta} \left(C_t^{(i)}\right)^{-\sigma} \quad (12)$$

$$\lambda_t = \beta E_t \left[ \lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right]. \quad (13)$$

With the assumption of the insurance plan and the same initial capital stock, all household members who can optimize their wages in period  $t$  will choose the same real wage,  $W_t^{F(i)*} = W_t^{F*}$  and  $W_t^{P(i)*} = W_t^{P*}$ . Since a fraction  $(1 - \rho_w^F)$  for full-time labour and  $(1 - \rho_w^P)$  for part-time labour of the household members choose this wage and the rest modify the nominal wage referring the inflation rate in the steady state with a weight  $\nu_w^F$  for full-time labour and  $\nu_w^P$  for part-time labour, composite nominal wages follow

$$W_{t+n}^F = Z^n W_t^F \prod_k^n \left[ \left( \frac{\pi_{t+k-1}}{\pi} \right)^{\nu_w^F} \frac{\pi}{\pi_{t+k}} \right] \quad (14)$$

$$W_{t+n}^P = Z^n W_t^P \prod_k^n \left[ \left( \frac{\pi_{t+k-1}}{\pi} \right)^{\nu_w^P} \frac{\pi}{\pi_{t+k}} \right]. \quad (15)$$

## B.2. Wage setting of household

The household members are assumed to be price-setters regarding their wages. In each period, a randomly and independently chosen fraction,  $(1 - \rho_w^F)$  for full-time labour and  $(1 - \rho_w^P)$  for part-time labour, of the household members specialized in type  $i$  labour can change their wages. The others keep their nominal wages fixed. When the household mem-



bers get a chance to change their wages at time  $t$ , under the demands of the labour bundler, they choose their optimal wages,  $W_t^{F(i)*}$  and  $W_t^{P(i)*}$ , in order to maximize the following objective

$$E_t \sum_{n=0}^{\infty} (\beta \rho_w^F)^n \left\{ e^{z_{t+n}^\beta} \left[ \frac{(C_{t+n}^{(i)})^{1-\sigma}}{1-\sigma} - \frac{\chi e^{z_{t+n}^{F,l}} Z_{t+n}^{1-\sigma}}{1+\varphi_t^F} \left[ \left( \frac{W_{t+n}^F}{W_t^{F(i)*}} \right)^{\psi_w^F} \left( \frac{\gamma W_{t+n} H_{t+n}}{W_{t+n}^F} \right)^\tau \right]^{1+\varphi_w^F} \right. \right. \\ \left. \left. - \frac{(1-\chi) e^{z_{t+n}^{P,l}} Z_{t+n}^{1-\sigma} (h_{t+n}^{P(i)})^{1+\varphi_w^P}}{1+\varphi_w^P} \right] - \lambda_{t+n} \left[ C_{t+n}^{(i)} + \frac{B_{t+n}^{(i)}}{P_{t+n}} \right. \right. \\ \left. \left. - W_{t+n}^{F(i)} \left( \frac{W_{t+n}^F}{W_t^{F(i)*}} \right)^{\psi_w^F} \left( \frac{\gamma W_{t+n} H_{t+n}}{W_{t+n}^F} \right)^\tau - W_{t+n}^{P(i)} h_{t+n}^{P(i)} - \xi_{t+n}^{(i)} - R_{t+n-1} \frac{B_{t+n-1}^{(i)}}{P_{t+n}} \right] \right\} \quad (16)$$

for  $W_t^{F(i)*}$  and

$$E_t \sum_{n=0}^{\infty} (\beta \rho_w^P)^n \left\{ e^{z_{t+n}^\beta} \left[ \frac{(C_{t+n}^{(i)})^{1-\sigma}}{1-\sigma} - \frac{\chi e^{z_{t+n}^{F,l}} Z_{t+n}^{1-\sigma} (h_{t+n}^{F(i)})^{1+\varphi_w^F}}{1+\varphi_t^F} \right. \right. \\ \left. \left. - \frac{(1-\chi) e^{z_{t+n}^{P,l}} Z_{t+n}^{1-\sigma}}{1+\varphi_w^P} \left[ \left( \frac{W_{t+n}^P}{W_t^{P(i)*}} \right)^{\psi_w^P} \left( \frac{(1-\gamma) W_{t+n} H_{t+n}}{W_{t+n}^P} \right)^\tau \right]^{1+\varphi_w^P} \right] - \lambda_{t+n} \left[ C_{t+n}^{(i)} + \frac{B_{t+n}^{(i)}}{P_{t+n}} \right. \right. \\ \left. \left. - W_{t+n}^{F(i)} h_{t+n}^{F(i)} - W_{t+n}^{P(i)} \left( \frac{W_{t+n}^P}{W_t^{P(i)*}} \right)^{\psi_w^P} \left( \frac{(1-\gamma) W_{t+n} H_{t+n}}{W_{t+n}^P} \right)^\tau - \xi_{t+n}^{(i)} - R_{t+n-1} \frac{B_{t+n-1}^{(i)}}{P_{t+n}} \right] \right\} \quad (17)$$

for  $W_t^{P(i)*}$ .

FOC with respect to  $W_t^{F(i)*}$  is

$$0 = E_t \sum_{n=0}^{\infty} (\beta \rho_w^F)^n \left\{ \frac{\psi_w^F}{W_t^{F(i)*}} \left[ \left( \frac{W_{t+n}^F}{W_t^{F(i)*}} \right)^{\psi_w^F} \left( \frac{\gamma W_{t+n} H_{t+n}}{W_{t+n}^F} \right)^\tau \right]^{1+\varphi_w^F} - \lambda_{t+n} \frac{(1 - \psi_w^F)}{P_{t+n}} \left( \frac{W_{t+n}^F}{W_t^{F(i)*}} \right)^{\psi_w^F} \left( \frac{\gamma W_{t+n} H_{t+n}}{W_{t+n}^F} \right)^\tau \right\} \quad (18)$$

which becomes, after substituting for the Lagrange multipliers,

$$0 = -\frac{(\psi_w^F - 1)}{W_t^{F(i)*}} \sum_{n=0}^{\infty} (\beta \rho_w^F)^n E_t \left[ h_{t+n}^{F(i)} (C_{t+n}^{(i)})^{-\sigma} \left( \frac{W_t^{F(i)*}}{P_{t+n}} - \Lambda_w^F MRS_{t+n}^{F(i)} \right) \right] \quad (19)$$

where

$$\Lambda_w^F = \frac{\psi_w^F}{\psi_w^F - 1} \quad (20)$$

and

$$MRS_{t+n}^{F(i)} = (C_{t+n}^{(i)})^\sigma (h_{t+n}^{F(i)})^{\varphi_w^F} = (C_{t+n}^{(i)})^\sigma \left[ \left( \frac{W_{t+n}^F}{W_t^{F(i)*}} \right)^{\psi_w^F} \left( \frac{\gamma W_{t+n} H_{t+n}}{W_{t+n}^F} \right)^\tau \right]^{\varphi_w^F} \quad (21)$$

Likewise, FOC with respect to  $W_t^{P(i)*}$  is

$$0 = -\frac{(\psi_w^P - 1)}{W_t^{P(i)*}} \sum_{n=0}^{\infty} (\beta \rho_w^P)^n E_t \left[ h_{t+n}^{P(i)} (C_{t+n}^{(i)})^{-\sigma} \left( \frac{W_t^{P(i)*}}{P_{t+n}} - \Lambda_w^P MRS_{t+n}^{P(i)} \right) \right] \quad (22)$$

where

$$\Lambda_w^P = \frac{\psi_w^P}{\psi_w^P - 1} \quad (23)$$

and

$$MRS_{t+n}^{P(i)} = (C_{t+n}^{(i)})^\sigma (h_{t+n}^{P(i)})^{\varphi_w^P} = (C_{t+n}^{(i)})^\sigma \left[ \left( \frac{W_{t+n}^P}{W_t^{P(i)*}} \right)^{\psi_w^P} \left( \frac{(1 - \gamma) W_{t+n} H_{t+n}}{W_{t+n}^P} \right)^\tau \right]^{\varphi_w^P} \quad (24)$$

### B.3. Work choice of household members

Now I consider household member  $(i, j)$ , specialized in type  $i$  labour and with disutility of full-time work  $j^{\varphi_w^F}$  and part-time work  $j^{\varphi_w^P}$ . He or she will find it optimal to participate in each labour market in period  $t$  if and only if

$$W_t^{F(i)} \left( C_t^{(i)} \right)^{-\sigma} \geq \chi e^{z_t^{F,l}} Z_t^{1-\sigma} j^{\varphi_t^F} \quad (25)$$

$$W_t^{P(i)} \left( C_t^{(i)} \right)^{-\sigma} \geq (1 - \chi) e^{z_t^{P,l}} Z_t^{1-\sigma} j^{\varphi_t^P} \quad (26)$$

Thus, the marginal supplier of type  $i$  labour (employed or unemployed), which is denoted by  $L_t^{F(i)}$  and  $L_t^{P(i)}$  respectively, is implicitly given by

$$W_t^{F(i)} \left( C_t^{(i)} \right)^{-\sigma} = \chi e^{z_t^{F,l}} Z_t^{1-\sigma} \left( L_t^{F(i)} \right)^{\varphi_t^F} \quad (27)$$

$$W_t^{P(i)} \left( C_t^{(i)} \right)^{-\sigma} = (1 - \chi) e^{z_t^{P,l}} Z_t^{1-\sigma} \left( L_t^{P(i)} \right)^{\varphi_t^P}. \quad (28)$$

### C. Final Goods Firm

The final goods production technology is

$$Y_t = \left[ \int_0^1 \left( Y_t^{(k)} \right)^{\frac{\psi-1}{\psi}} dk \right]^{\frac{\psi}{\psi-1}} \quad (29)$$

for  $\psi > 1$ .

The profit-maximizing final goods firm chooses  $Y_t^{(k)}$  to maximize

$$\Pi_t = P_t Y_t - \int_0^1 P_t^{(k)} Y_t^{(k)} dk, \quad (30)$$

where  $P_t$  is the aggregate price index at time  $t$  and  $P_t^{(k)}$  is the price of a differentiated good indexed by  $k$  at time  $t$ , subject to the above production technology.

FOC with respect to  $Y_t^{(k)}$  is

$$Y_t^{(k)} = Y_t \left( \frac{P_t}{P_t^{(k)}} \right)^\psi \quad (31)$$

where

$$P_t = \left[ \int_0^1 \left( P_t^{(k)} \right)^{1-\psi} dk \right]^{\frac{1}{1-\psi}} \quad (32)$$

Since all intermediate goods firms that can fix their prices have the same mark-up over the same marginal costs, in every period  $t$ ,  $P_t^{(k)*}$  is the same for all of the  $(1 - \rho_p)$  firms that adjust their prices. Combining the above final goods pricing rule — all the adjusting firms set the same price — and our assumption that all non-adjusting firms keep their price as it was in the previous period, one gets the price level updating expression of

$$P_t = \left[ (1 - \rho_p) (P_t^*)^{1-\psi} + \rho_p (P_{t-1})^{1-\psi} \right]^{\frac{1}{1-\psi}} \quad (33)$$

#### D. Intermediate Goods Firm

The intermediate goods firm maximizes the market value of the firm subject to the demand for their output by the final goods firm. In each period, a randomly and independently chosen fraction,  $(1 - \rho_p)$ , of the firms can change their prices. The others keep their prices fixed.

The production function for intermediate goods firm  $k$  is

$$Y_t^{(k)} = Z_t \left( H_t^{(k)} \right)^{1-\alpha} - \Phi Z_t \quad (34)$$

where all firms are subject to the same technology  $Z_t$ .

An intermediate goods firm,  $k$ , which can choose prices in period  $t$ , chooses the price,  $P_t^{(k)*}$ , to maximize the nominal profit with

$$E_t \sum_{n=0}^{\infty} (\beta \rho_p)^n \left[ P_t^{(k)*} Y_{t+n}^{(k)} - W_{t+n} H_{t+n}^{(k)} \right] \quad (35)$$

subject to the production technology

$$Y_{t+n}^{(k)} = Z_{t+n} \left( H_{t+n}^{(k)} \right)^{1-\alpha} - \Phi Z_t \quad (36)$$

and demand from the final goods firm

$$Y_{t+n}^{(k)} = Y_{t+n} \left( \frac{P_{t+n}}{P_t^{(k)*}} \right)^\psi. \quad (37)$$

A firm that is maximizing the above expression is simultaneously minimizing total costs in each period. The real cost minimization problem for intermediate goods firm  $k$  in period  $t+n$  is

$$\min_{H_{t+n}^{(k)}} \frac{W_{t+n}}{P_{t+n}} H_{t+n}^{(k)} \quad (38)$$

subject to the production technology (36).

The Lagrangian is

$$\mathcal{L}_{t+n} = \frac{W_{t+n}}{P_{t+n}} H_{t+n}^{(k)} + mc_{t+n}^{(k)} \left[ Z_{t+n} \left( H_{t+n}^{(k)} \right)^{1-\alpha} - \Phi Z_t - Y_{t+n}^{(k)} \right] \quad (39)$$

FOC with respect to  $H_{t+n}^{(k)}$  is

$$\frac{W_{t+n}}{P_{t+n}} + mc_{t+n}^{(k)} (1-\alpha) Z_{t+n} \left( H_{t+n}^{(k)} \right)^{-\alpha} = 0. \quad (40)$$

Substituting the factor demand and production function in the cost equation gives

$$mc_{t+n}^{(k)} = -\frac{W_{t+n}}{(1-\alpha) Z_{t+n} P_{t+n}} \left( H_{t+n}^{(k)} \right)^\alpha = -\frac{W_{t+n}}{(1-\alpha) Z_{t+n} P_{t+n}} \left[ \frac{Y_{t+n}}{Z_{t+n}} \left( \frac{P_{t+n}}{P_t^{(k)*}} \right)^\psi + \Phi \right]^{\frac{\alpha}{1-\alpha}} \quad (41)$$

Since in any period, all firms face the same wages, rentals, and technology, the period

$t + n$  real marginal unit costs are the same for all firms and are equal to

$$MC_{t+n} = \frac{W_{t+n}}{(1-\alpha)Z_{t+n}P_{t+n}} \left[ \frac{(1-\alpha)P_{t+n}}{\alpha W_{t+n}} \right]^\alpha. \quad (42)$$

Therefore, intermediate goods firm  $k$ 's nominal profit maximization problem in period  $t$  can be written as

$$\max_{P_t^{(k)*}} E_t \sum_{n=0}^{\infty} (\beta \rho_p)^n \left[ P_t^{(k)*} Y_{t+n} \left( \frac{P_{t+n}}{P_t^{(k)*}} \right)^\psi - P_{t+n} MC_{t+n} Y_{t+n} \left( \frac{P_{t+n}}{P_t^{(k)*}} \right)^\psi \right]. \quad (43)$$

FOC with respect to  $P_t^{(k)*}$  is

$$0 = E_t \sum_{n=0}^{\infty} (\beta \rho_p)^n Y_{t+n} \left( 1 - \psi - \frac{\psi P_{t+n}}{P_t^{(k)*}} MC_{t+n} \right). \quad (44)$$

### *E. The equilibrium Conditions*

Since household members have the same Euler equation and real budget constraints, all household members will end up identical in terms of income and preferences. Then, in equilibrium, the equation for consumption becomes simply

$$Y_t = C_t = C_t^{(i)} \quad (45)$$

for all household members from the market clearance.

Likewise, in equilibrium, the equations for the labour supply of the household members are

$$L_t^F = L_t^{F(i)} \quad (46)$$

$$L_t^P = L_t^{P(i)} \quad (47)$$

Aggregating the real budget constraints for all household members in period  $t$  gives

$$C_t + \frac{B_t}{P_t} = W_t^F h_t^F + W_t^P h_t^P + \xi_t + R_{t-1} \frac{B_{t-1}}{P_t} \quad (48)$$

which becomes

$$C_t + \frac{B_t}{P_t} = W_t H_t + \xi_t + R_{t-1} \frac{B_{t-1}}{P_t}. \quad (49)$$

From the assumption that the labour bundler is also perfectly competitive, in equilibrium, they make neither profits nor losses and in each period

$$W_t H_t = W_t^F H_t^F + W_t^P H_t^P \quad (50)$$

where

$$W_t^F H_t^F = \int_0^1 W_t^{F(i)} h_t^{F(i)} di \quad (51)$$

$$W_t^P H_t^P = \int_0^1 W_t^{P(i)} h_t^{P(i)} di. \quad (52)$$

Equilibrium in the factor markets implies that supply equals demand for labour. In the labour market, equilibrium requires

$$H_t = \int_0^1 H_t^{(k)} dk = \frac{1}{Z_t} \left[ \frac{(1-\alpha) R_t P_t}{\alpha W_t} \right]^\alpha \int_0^1 Y_t^{(k)} dk \quad (53)$$

$$K_t = \int_0^1 K_t^{(k)} dk = \frac{1}{Z_t} \left[ \frac{(1-\alpha) R_t P_t}{\alpha W_t} \right]^{\alpha-1} \int_0^1 Y_t^{(k)} dk. \quad (54)$$

Therefore, the aggregation of output of the intermediate firms are

$$\int_0^1 Y_t^{(k)} dk = Z_t (H_t)^{1-\alpha} \left( \neq Y_t = \left[ \int_0^1 \left( Y_t^{(k)} \right)^{\frac{\psi-1}{\psi}} dk \right]^{\frac{\psi}{\psi-1}} \right). \quad (55)$$

### *F. Aggregation of labour and unemployment*

The aggregate employment and labour supply are derived as the simple sum of employment and labour supply. This is expressed as

$$AH_t = H_t^F + H_t^P \quad (56)$$

$$AL_t = L_t^F + L_t^P \quad (57)$$

Unemployment is derived as excess labour supply over employment. This gives the unemployment of aggregate labour as

$$U_t = \frac{AL_t}{AH_t} \quad (58)$$

### *G. Log-Linearized Model and Steady State Assumption*

From the above setup, I detrend with  $Z_t$  and derive a log-linearized form. I use logarithmic deviations from steady states with lower-case letters, except that  $\pi_t$ ,  $\pi_t^{w,F}$  and  $\pi_t^{w,P}$  represent deviations in level. In addition to the equations presented until the last subsection, here I add one equation which is the monetary policy reaction function of the central bank. The detrended and log-linearized version of the model is described in Appendix A and equation (A11) is the monetary policy function. I also add nine persistent exogenous shocks and seven observation equations.

### *H. Estimation Methodology*

Before estimating the model using Japan's data, some parameters should be fixed to focus on the parameters of interest. Following the earlier studies, such as Sugo and Ueda (2008), I set the values of some variables as Table II.

I estimate the model using Japan's quarterly data over the period from 1994Q2 to 2016Q1. I do not utilize the data after 2016Q2 because the BOJ introduced the Yield Curve Control



Parameters	Value	Description
$\rho_p$	0.7	Calvo parameter of price
$\nu$	0.198	Weight to refer to inflation one period ago for the households that cannot change prices
$\sigma$	1.8	Substitution elasticity of consumption
$\iota$	0.067	Fixed cost parameter for production

Table II: Values for pre-determined parameters

policy in September 2016 and the effect of the monetary policy needs to be modelled more complexly. The data set consists of 7 variables: real GDP, total hours worked (of full-timers and part-timers), real hourly wage (of full-timers and part-timers), the inflation rate and the overnight call rates. A notable difference from many of the earlier studies is that I utilize the data of total hours worked and real hourly wages of full-timers and part-timers. The details of the data sources are given in Appendix B.

In the estimation of the model, I use the backward-forward algorithm as indicated in Kulish and Pagan (2017) to consider the structural change in the labour market. I assume that the steady-state levels of full-time labour and part-time labour change over the estimation period as shown in Figure 7. Specifically, the steady-state levels of each labour are fixed at the average of the first 10 quarters and the last 10 quarters of the respective periods, and those in the periods in between change gradually. The steady-state levels of the other variables are fixed over time. Please note that the estimation process is less complicated compared to the original estimation of Kulish and Pagan (2017) as I assume the pattern of the structural change and do not estimate the period of the structural change.

## V. Results

### A. Parameter Estimation

Table III shows the prior distributions, the posterior mode and means, and 90% confidence intervals for the parameters. Most importantly, all the estimated values are inside the

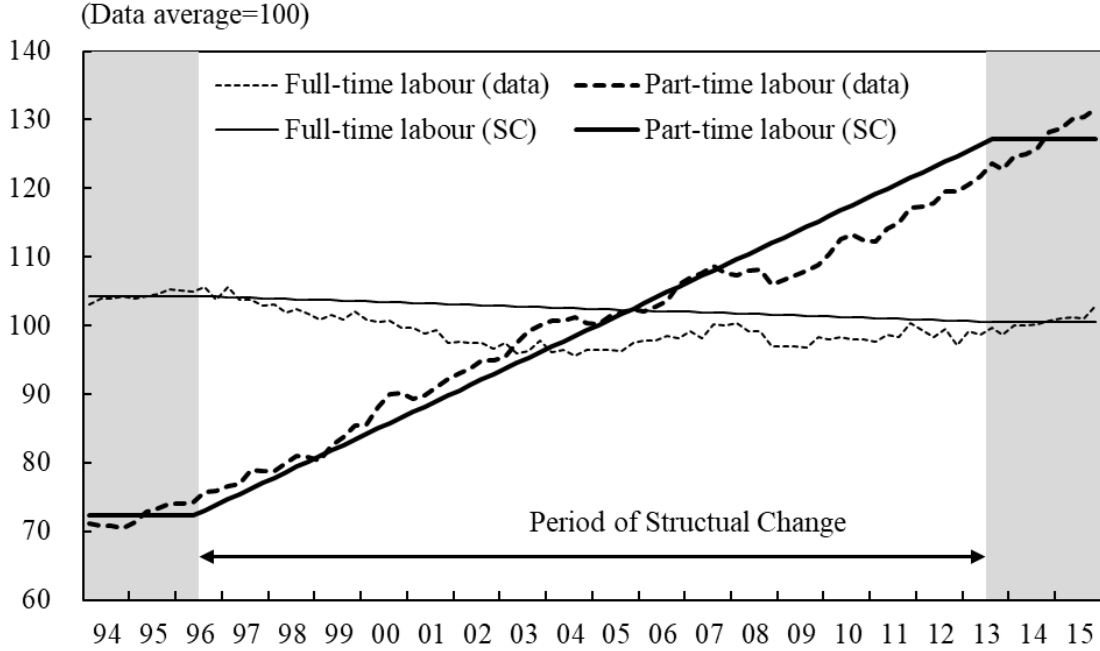


Figure 7: Assumption of the structural change in full-time labour and part-time labour

reasonable ranges of the earlier studies, such as Sugo and Ueda (2008). Several important parameters are worth mentioning.

Regarding wage inflation dynamics, the calvo parameter of full-time wage  $\rho_w^F$  is greater than that of part-time wage  $\rho_w^P$ , and their 90% confidence intervals do not overlap. Since  $(1 - \rho_w)$  stands for the probability that a household member can change his or her wage, this result shows that the wage of part-timers is more often changed. This is consistent with the discussion in Section 3. As for substitution elasticity, substitution elasticity for full-time labour  $\psi_w^F$  is smaller than that for part-time labour  $\psi_w^P$  although they are largely designated by the prior distributions. However, the prior distributions are consistent with the huge gap between full-timers' and part-timers' wages as shown in Section 3. From the posterior means, the markup of full-time wage is 20% and that of part-time wage is 5%. Regarding labour supply elasticity, the inverse Frisch elasticity of labour supply of full-time labour  $\psi_w^F$  is larger than that of part-time labour  $\psi_w^P$ . This implies that with a wage increase of 1%, a household member is more willing to increase part-time labour than full-time labour.

Parameter	Posterior mode	Posterior mean	90%CI		Prior distribution	Prior mean
Standard deviations						
$\sigma_r$	0.0298	0.0293	[ 0.0249	0.0329 ]	Inv. gamma ( $a.b$ )	0.001
$\sigma_z$	1.0621	1.1469	[ 1.0143	1.2718 ]	Inv. gamma ( $a.b$ )	0.001
$\sigma_b$	2.2008	2.2883	[ 1.9648	2.6437 ]	Inv. gamma ( $a.b$ )	0.001
$\sigma_p$	0.2203	0.2036	[ 0.1656	0.2412 ]	Inv. gamma ( $a.b$ )	0.001
$\sigma_w^F$	1.1399	1.1348	[ 0.9951	1.3457 ]	Inv. gamma ( $a.b$ )	0.001
$\sigma_w^P$	0.395	0.3871	[ 0.2848	0.487 ]	Inv. gamma ( $a.b$ )	0.001
$\sigma_l^F$	0.0523	0.0968	[ 0.024	0.3081 ]	Inv. gamma ( $a.b$ )	0.001
$\sigma_l^P$	1.7378	1.7089	[ 1.3288	2.1612 ]	Inv. gamma ( $a.b$ )	0.001
$\sigma_\gamma$	0.2863	0.2805	[ 0.2005	0.3815 ]	Inv. gamma ( $a.b$ )	0.001
Structural parameters						
$\theta_r$	0.9573	0.9561	[ 0.9493	0.9613 ]	Normal (0.7, 0.1)	0.7
$\theta_\pi$	1.7441	1.7062	[ 1.5399	1.8534 ]	Normal (1.7, 0.1)	1.7
$\theta_y$	0.039	0.0494	[ -0.0067	0.112 ]	Normal (0.2, 0.1)	0.2
$\gamma$	0.8054	0.8173	[ 0.7803	0.8536 ]	Beta (2, 2)	0.5
$\tau$	1.2393	1.2335	[ 1.1322	1.3327 ]	Normal (1.6, 0.2)	1.6
$\rho_w^F$	0.3994	0.3828	[ 0.3495	0.4137 ]	Beta (2, 2)	0.5
$\psi_w^F$	5.9445	6.0391	[ 4.3974	7.6307 ]	Normal (6, 1)	6
$\varphi_w^F$	1.8631	1.8979	[ 1.61	2.1798 ]	Normal (2, 0.2)	2
$\nu_w^F$	0.066	0.0847	[ 0.0181	0.1677 ]	Beta (2, 2)	0.5
$\rho_w^P$	0.2878	0.2864	[ 0.2717	0.2949 ]	Beta (2, 2)	0.5
$\psi_w^P$	21.4094	19.7097	[ 16.1563	22.1378 ]	Normal (21, 2)	21
$\varphi_w^P$	0.9934	1.0001	[ 0.7509	1.1661 ]	Normal (1, 0.2)	1
$\nu_w^P$	0.7223	0.7237	[ 0.6266	0.8284 ]	Beta (2, 2)	0.5
$\eta_z$	0.0164	0.0192	[ 0.0036	0.0486 ]	Beta (2, 2)	0.5
$\eta_b$	0.8983	0.8932	[ 0.8457	0.9347 ]	Beta (2, 2)	0.5
$\eta_p$	0.9694	0.968	[ 0.9438	0.9873 ]	Beta (2, 2)	0.5
$\eta_w^F$	0.0135	0.0617	[ 0.0108	0.1482 ]	Beta (2, 2)	0.5
$\eta_w^P$	0.0789	0.1091	[ 0.019	0.2658 ]	Beta (2, 2)	0.5
$\eta_l^F$	0.7123	0.7609	[ 0.7006	0.8379 ]	Beta (2, 2)	0.5
$\eta_l^P$	0.9589	0.9356	[ 0.8851	0.9759 ]	Beta (2, 2)	0.5
$\eta_\gamma$	0.9358	0.9337	[ 0.9036	0.9562 ]	Beta (2, 2)	0.5
$\eta_r$	0.458	0.4653	[ 0.3455	0.5774 ]	Beta (2, 2)	0.5
$z^*$	0.3588	0.3515	[ 0.1883	0.5286 ]	Gamma (3, 1)	0.3
$r^*$	0.1106	0.1199	[ 0.0352	0.2533 ]	Gamma (3, 1)	0.3
$\pi^*$	0.0368	0.0971	[ 0.027	0.2007 ]	Gamma (2, 1)	0.5

\*  $a = 2.0025$ ,  $b = 0.10025$

Table III: Posterior estimates

Although the posterior means are largely designated by the prior distributions, the result is consistent with the discussion in Section 3. As for substitution elasticity between full-time labour and part-time labour  $\tau$ , it converges to the posterior mean 1.2335 and the two types of labour are substitutive. This value could be a benchmark for future research.

### B. *Estimated Coefficients of the WPC*

Using the posterior means of each parameter, I calculate the coefficients of the WPC regarding full-timers and part-times from equation (A3) and (A4) in Appendix A as

$$\kappa_w^F = \frac{(1 - \rho_w^F) (1 - \beta z^{1-\alpha} \rho_w^F) \varphi_w^F}{\rho_w^F (1 + \psi_w^F \varphi_w^F)} = 0.160065$$

$$\kappa_w^P = \frac{(1 - \rho_w^P) (1 - \beta z^{1-\alpha} \rho_w^P) \varphi_w^P}{\rho_w^P (1 + \psi_w^P \varphi_w^P)} = 0.076172$$

The relationship between the coefficient and each parameter is explained as

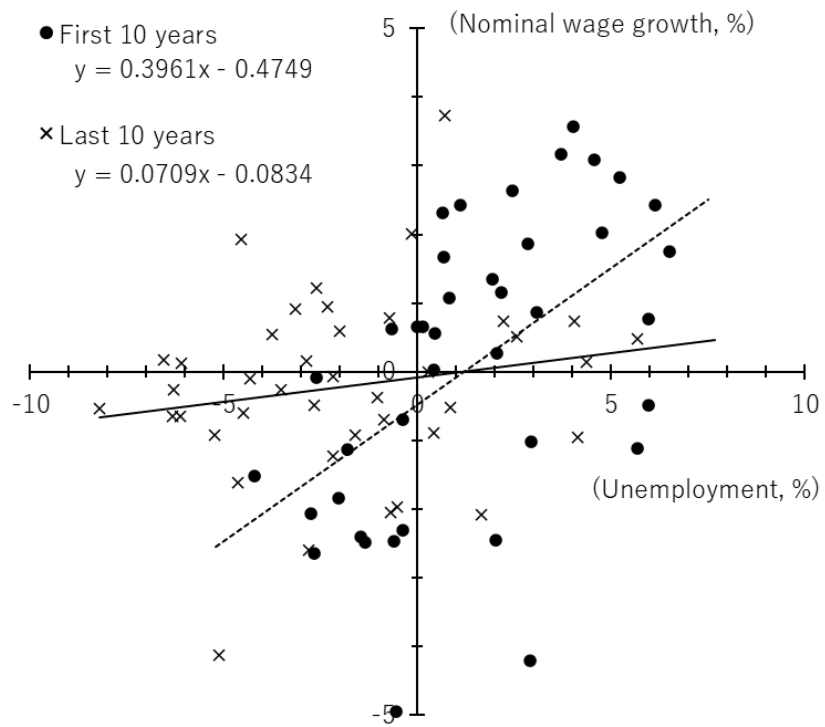
$$\kappa_w \downarrow \quad \text{if } \rho_w \uparrow, \psi_w \uparrow, \text{ or } \varphi_w \downarrow.$$

Therefore, although a smaller  $\rho_w^P$  effect makes  $\kappa_w^P$  greater, the sum of the larger  $\psi_w^P$  effect and smaller  $\varphi_w^P$  effect exceeds the opposite impact and makes  $\kappa_w^P$  smaller than  $\kappa_w^F$ .

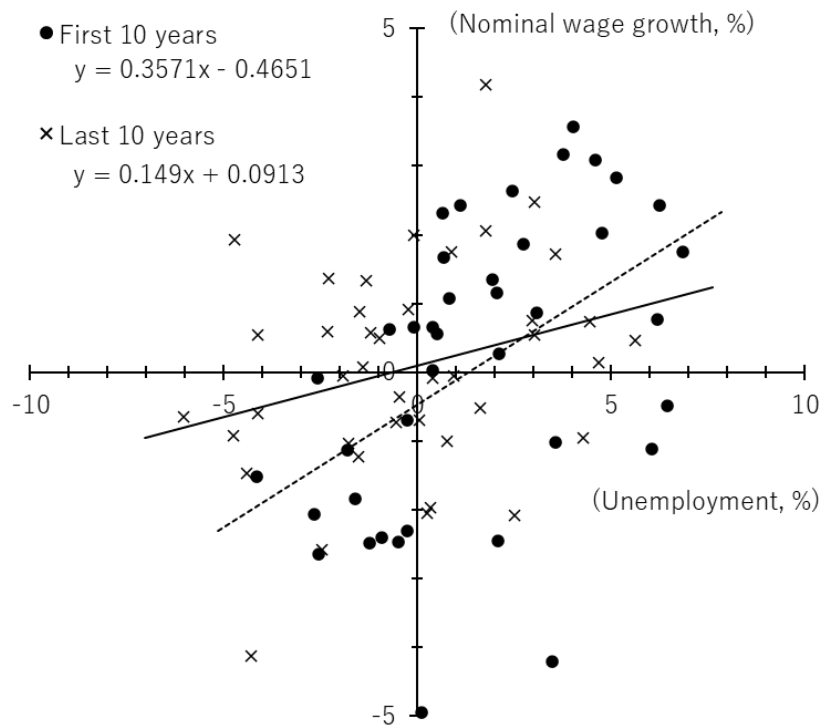
### C. *Comparison with Counterfactual Simulation*

In order to identify the WPC flattening effect of the structural change in the labour force from the effects of shocks and other changes, I do a counterfactual simulation with the case that the structural change does not happen, and compare it with the outcome of the baseline simulation. Then I pick the first 10 years and the last 10 years from the simulation results and check the development of the observed WPC over the two periods. Figure 8 shows the results of each estimation with the scattered plots. Table IV indicates the slopes.

Comparing the changes in the slopes of each estimate, both flatten over the two periods,



(a) Baseline simulation



(b) Counterfactual simulation

Figure 8: Comparison of the estimations: with/without structural change

slope of the WPC	Baseline simulation	Counterfactual simulation
First 10 years	(i) 0.396	(ii) 0.357
Last 10 years	(iii) 0.071	(iv) 0.149

Table IV: Slope of the WPC in stochastic simulation

but the change in the counterfactual simulation is smaller than that in the baseline simulation. We can consider that the difference ratio from (ii) to (iv) divided by the difference ratio from (i) to (iii) is the effect of the shock profile change in the observed WPC, and the rest is the effect of the structural change to the observed WPC, which is 29.0%.

## VI. Conclusions

This paper formulates the New Keynesian DSGE model which explicitly incorporates two different types of labour forces, which are full-time labour and part-time labour and their unemployment considering the structural change of the increase in part-timers. It also employs estimation with Japan's data using Bayesian techniques and obtains plausible results for structural and policy parameters. In particular, regarding the deep parameters of part-timers, its Calvo parameter is smaller, its substitution elasticity is larger, and its inverse Frisch elasticity of labour supply is smaller compared to those of full-timers. It then performs a counterfactual analysis of the aggregate WPC with the case that the part-time labour ratio did not increase, and compares the slopes with those of the baseline estimate. The results show that roughly 30% of the recent flattening of the WPC in Japan is associated with the increase in part-time labour. To the best of my knowledge, this is the inaugural analysis to incorporate a structural change in Japan's labour market explicitly and examine its effects on the wage dynamics.

There are several remaining tasks in my analysis. First and foremost I need to incorporate a zero lower bound in the model. This would allow me to utilize the samples more and analyze the impact of monetary policy. However, this requires a more complex modelling approach as well as the modeling of structural changes.

Secondly, in this paper, I did not derive the optimal monetary policy, but instead formulated it by referring to other significant papers. To conduct a more precise analysis, I need to define the loss function and derive the optimal monetary policy. It is important to know how much attention should be paid to each wage as the optimal monetary policy in the model setting.

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## Appendix A. Log-linearized Equations

The following is the log-linearized equations of the model.

$$\sigma \tilde{y}_t + r_t - z_t^\beta = E_t \widetilde{\pi_{t+1}} + \sigma E_t \widetilde{y_{t+1}} - E_t z_{t+1}^\beta + \sigma E_t z_{t+1}^z \quad (\text{A1})$$

$$\tilde{\pi}_t - \nu_p \widetilde{\pi_{t-1}} = \beta z^{1-\sigma} (E_t \widetilde{\pi_{t+1}} - \nu_p \tilde{\pi}_t) + \frac{(1-\rho_p)(1-\beta z^{1-\sigma} \rho_p)}{\rho_p} \tilde{w}_t + z_t^P \quad (\text{A2})$$

$$\begin{aligned} \widetilde{w}_t^F - \widetilde{w}_{t-1}^F + \tilde{\pi}_t - \nu_w^F \widetilde{\pi_{t-1}} + z_t^z &= \beta z^{1-\sigma} \left( E_t \widetilde{w_{t+1}^F} - \widetilde{w}_t^F + E_t \widetilde{\pi_{t+1}} - \nu_w^F \tilde{\pi}_t + E_t z_{t+1}^z \right) \\ &+ \frac{(1-\rho_w^F)(1-\beta z^{1-\sigma} \rho_w^F)}{\rho_w^F (1+\psi_w^F \varphi_t^F)} \varphi_t^F \widetilde{u}_t^F + z_t^{F,w} \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} \widetilde{w}_t^P - \widetilde{w}_{t-1}^P + \tilde{\pi}_t - \nu_w^P \widetilde{\pi_{t-1}} + z_t^z &= \beta z^{1-\sigma} \left( E_t \widetilde{w_{t+1}^P} - \widetilde{w}_t^P + E_t \widetilde{\pi_{t+1}} - \nu_w^P \tilde{\pi}_t + E_t z_{t+1}^z \right) \\ &+ \frac{(1-\rho_w^P)(1-\beta z^{1-\sigma} \rho_w^P)}{\rho_w^P (1+\psi_w^P \varphi_t^P)} \varphi_t^P \widetilde{u}_t^P + z_t^{P,w} \end{aligned} \quad (\text{A4})$$

$$\widetilde{h}_t^F = \tau \left( \tilde{w}_t - \widetilde{w}_t^F + z_t^\gamma \right) + \frac{1}{1+\iota} \tilde{y}_t \quad (\text{A5})$$

$$\widetilde{h}_t^P = \tau \left( \tilde{w}_t - \widetilde{w}_t^P - \frac{\gamma}{1-\gamma} z_t^\gamma \right) + \frac{1}{1+\iota} \tilde{y}_t \quad (\text{A6})$$

$$\widetilde{w}_t^F = \frac{1}{\varphi_t^F} \left( \widetilde{w}_t^F - \sigma \tilde{y}_t - z_t^{F,l} \right) - \widetilde{h}_t^F \quad (\text{A7})$$

$$\widetilde{w}_t^P = \frac{1}{\varphi_t^P} \left( \widetilde{w}_t^P - \sigma \tilde{y}_t - z_t^{P,l} \right) - \widetilde{h}_t^P \quad (\text{A8})$$

$$\tilde{w}_t = \gamma^\tau \left( \frac{w^F}{w} \right)^{1-\tau} \left( \widetilde{w}_t^F + \frac{\tau}{1-\tau} z_t^\gamma \right) + (1-\gamma)^\tau \left( \frac{w^P}{w} \right)^{1-\tau} \left( \widetilde{w}_t^P + \frac{\tau}{1-\tau} \frac{\gamma}{1-\gamma} z_t^\gamma \right) \quad (\text{A9})$$

$$\tilde{u}_t = \left( \frac{l^F}{al} - \frac{h^F}{ah} \right) \widetilde{h}_t^F + \frac{l^F}{al} \widetilde{u}_t^F + \left( \frac{l^P}{al} - \frac{h^P}{ah} \right) \widetilde{h}_t^P + \frac{l^P}{al} \widetilde{u}_t^P \quad (\text{A10})$$

$$\tilde{r}_t = \theta_r \widetilde{r_{t-1}} + (1-\theta_r) (\theta_\pi \tilde{\pi}_t + \theta_y \tilde{y}_t) + z_t^r \quad (\text{A11})$$

## Appendix B. Data description

The date is quarterly and the period is from 1994Q2 to 2016Q1 with taking log (except inflation rate and interest rate).

Table V shows the data sources for the model estimation.

Variable	Sources	For
Real GDP	Cabinet Office, "national Accounts"	$\tilde{y}_t$
Total hours worked of full-timers and part-timers	Ministry of Health, Labour and Welfare, "Monthly Labour Survey"	$\tilde{h}_t^F, \tilde{h}_t^P$
Real hourly wage of full-timers and part-timers	Ministry of Health, Labour and Welfare, "Monthly Labour Survey" Ministry of Internal Affairs and Communications, "Consumer Price Index"	$\tilde{w}_t^F, \tilde{w}_t^P$
Inflation rates (excluding fresh food)	Ministry of Internal Affairs and Communications, "Consumer Price Index"	$\tilde{\pi}_t$
Overnight call rates	Bank of Japan	$\tilde{r}_t$

Table V: Data sources