Institute for Economic Studies, Keio University

Keio-IES Discussion Paper Series

Connecting Exchange Rates to Fundamentals Under Indeterminacy

Ippei Fujiwara, Yasuo Hirose

19 December, 2024
2024-024
https://ies.keio.ac.jp/en/publications/24713/

Institute for Economic Studies, Keio University
2-15-45 Mita, Minato-ku, Tokyo 108-8345, Japan
ies-office@adst.keio.ac.jp
19 December, 2024

Connecting Exchange Rates to Fundamentals Under Indeterminacy
Ippei Fujiwara, Yasuo Hirose
Keio-IES 2024-024
19 December, 2024
JEL Classification: C62, E32, F31, F41
Keywords: Exchange rate disconnect, UIP puzzle, Indeterminacy, Bayesian estimation

Abstract

This paper establishes the connection of exchange rates to macroeconomic fundamentals by estimating a small open-economy model for Canada. The model incorporates an endogenous interest rate spread on foreign bond holdings, enabling the modified uncovered interest rate parity (UIP) condition to exhibit a negative relationship between expected exchange rate depreciation and interest rate differentials, as observed in the data. Given the model's susceptibility to equilibrium indeterminacy, we estimate it using Bayesian methods that allow for both determinacy and indeterminacy of equilibrium. The results reveal that preference shocks to the household utility function are the primary drivers of exchange rate fluctuations, highlighting the connection between exchange rates and macroeconomic fundamentals. We further demonstrate that both allowing for indeterminacy and selecting a specific equilibrium representation from the data are essential for achieving this finding.

Ippei Fujiwara
Keio University
ippei.fujiwara@keio.jp

Yasuo Hirose
Keio University
yhirose@econ.keio.ac.jp

Acknowledgement: The authors would like to thank George Alessandria, Kosuke Aoki, Guido Ascari, Philippe Bacchetta, Paul Beaudry, Paul Bergin, Lawrence Christiano, Chun-Che Chi, Michael Devereux, Martin Ellison, Refet Gürkaynak, Qazi Haque, Hibiki Ichiue, Munechika Katayama, Jinill Kim, Mariano Kulish, Takushi Kurozumi, Amartya Lahiri, Moritz Lenel, Zheng Liu, Emile Marin, James Morley, Masao Ogaki, Bruce Preston, Marco Ratto, Tim Robinson, Kozo Ueda, Donghoon Yoo, and Francesco Zanetti, along with seminar participants at Academia Sinica and National Taiwan University, and conference attendees at BOJ--CEPR IMFC, EDMM, CEF, SETA, Melbourne Macro Policy Meeting, SED, and WAMS+LAEF, for their comments and discussions. This study is supported by JSPS KAKENHI (Fujiwara: 22H00058; Hirose: 21H04397, 24K00238) and a grant-in-aid from Zengin Foundation for Studies on Economics and Finance.

Connecting Exchange Rates to Fundamentals Under Indeterminacy*

Ippei Fujiwara† Yasuo Hirose‡

This version: December 2024

Abstract

This paper establishes the connection of exchange rates to macroeconomic fundamentals by estimating a small open-economy model for Canada. The model incorporates an endogenous interest rate spread on foreign bond holdings, enabling the modified uncovered interest rate parity (UIP) condition to exhibit a negative relationship between expected exchange rate depreciation and interest rate differentials, as observed in the data. Given the model's susceptibility to equilibrium indeterminacy, we estimate it using Bayesian methods that allow for both determinacy and indeterminacy of equilibrium. The results reveal that preference shocks to the household utility function are the primary drivers of exchange rate fluctuations, highlighting the connection between exchange rates and macroeconomic fundamentals. We further demonstrate that both allowing for indeterminacy and selecting a specific equilibrium representation from the data are essential for achieving this finding.

JEL codes: C62; E32; F31; F41

Keywords: Exchange rate disconnect; UIP puzzle; Indeterminacy; Bayesian estimation

*The authors would like to thank George Alessandria, Kosuke Aoki, Guido Ascari, Philippe Bacchetta, Paul Beaudry, Paul Bergin, Lawrence Christiano, Chun-Che Chi, Michael Devereux, Martin Ellison, Refet Gürkaynak, Qazi Haque, Hibiki Ichiue, Munechika Katayama, Jinill Kim, Mariano Kulish, Takushi Kurozumi, Amartya Lahiri, Moritz Lenel, Zheng Liu, Emile Marin, James Morley, Masao Ogaki, Bruce Preston, Marco Ratto, Tim Robinson, Kozo Ueda, Donghoon Yoo, and Francesco Zanetti, along with seminar participants at Academia Sinica and National Taiwan University, and conference attendees at BOJ–CEPR IMFC, EDMM, CEF, SETA, Melbourne Macro Policy Meeting, SED, and WAMS+LAEF, for their comments and discussions. This study is supported by JSPS KAKENHI (Fujiwara: 22H00058; Hirose: 21H04397, 24K00238) and a grant-in-aid from Zengin Foundation for Studies on Economics and Finance.

†Keio University; Australian National University; CEPR. Email: ippei.fujiwara@keio.jp

‡Keio University. Email: yhirose@econ.keio.ac.jp

1 Introduction

The uncovered interest rate parity (UIP) condition is one of the fundamental building blocks in modern open-economy models. It postulates that countries with relatively high interest rates should experience subsequent currency depreciation to ensure zero expected excess returns from cross-border financial investments. However, this theoretical relationship between currency exchange rates and interest rate differentials is not supported empirically. Instead, exchange rate fluctuations often show little to no discernible connection to underlying macroeconomic fundamentals—a phenomenon commonly referred to as the *exchange rate disconnect*. Indeed, as first documented by Fama (1984), a broad range of international data has empirically rejected the UIP condition and frequently exhibit near-zero or negative correlations between expected exchange rate depreciation and interest rate differentials, giving rise to the UIP puzzle. As a result, estimations of open-economy models often find that fluctuations in nominal exchange rates are largely disconnected from macroeconomic fundamentals and instead attribute them to a wedge in the standard UIP condition, referred to as a UIP shock.[1]

The purpose of this paper is to establish a connection between exchange rates and macroeconomic fundamentals by estimating a fully specified dynamic stochastic general equilibrium (DSGE) model for the Canadian economy, augmented with a modified UIP condition that can exhibit a negative relationship between expected exchange rate depreciation and interest rate differentials, as observed in the data. Specifically, we incorporate an endogenous interest rate spread on foreign bond holdings *à la* Adolfson et al. (2008) into a small open-economy version of the standard New Keynesian model. The spread is specified as a function of an expected change in the exchange rate, a net foreign asset position, and a shock. In this setup, the coefficient on interest rate differentials in the modified UIP equation can become negative if the parameter on the spread associated with exchange rate changes is sufficiently large.[2] This modified UIP condition is in accordance with the empirical findings of the

[1] See Lubik and Schorfheide (2006), Hirose (2013), and Chen et al. (2021). Gabaix and Maggiori (2015), Valchev (2020), Itskhoki and Mukhin (2021), and Kekre and Lenel (2024a) provide microfoundations for a direct shock to the UIP condition. Notably, Itskhoki and Mukhin (2021) quantitatively demonstrate the importance of this shock in explaining aggregate variables, including exchange rates.

[2] Christiano et al. (2011) specify the spread as a function of interest rate differentials instead of expected exchange rate changes, deriving a modified UIP equation similar to ours, where the coefficient on interest rate differentials can also be negative. However, their analysis is limited to the determinacy region of the

Fama regression and thereby enhances the connection between exchange rates and interest rate differentials, which represent macroeconomic fundamentals. As a result, the role of the UIP shock—as a residual term in the modified UIP equation—is attenuated, resolving the exchange rate disconnect.

Based on a small open-economy version of a simple continuous-time sticky-price model, Beaudry and Lahiri (2019) analytically show that its perfect foresight equilibrium is indeterminate if the UIP condition fails and exhibits a negative relationship between the expected exchange rate depreciation and interest rate differentials, as posited by the UIP puzzle. This indeterminacy property carries over to our model with a richer dynamic structure in a stochastic setting.

A notable feature of our analysis is that we estimate the model over the parameter space in which the coefficient on interest rate differentials in the modified UIP equation can be either positive or negative, resulting in either determinacy or indeterminacy of equilibrium.[3] Restricting the parameter space of the model to only allow equilibrium determinacy would make it unlikely for the estimated model to replicate the observed relationship between the exchange rate and interest rate differentials. Therefore, we estimate the model using full-information Bayesian methods that account for both determinacy and indeterminacy of equilibrium. Specifically, following Lubik and Schorfheide (2004), we construct the model's likelihood function for both the determinacy and indeterminacy regions of the parameter space. While Lubik and Schorfheide (2004) perform model estimation separately for each region, we estimate the model for both regions in a single step by employing a sequential Monte Carlo (SMC) algorithm, as implemented by Hirose et al. (2020). Unlike the widely used Metropolis–Hastings algorithm, the SMC algorithm can handle discontinuities in the likelihood function at the boundaries of each region and enables us to explore the full posterior distribution of the model's parameters.

Based on the estimated model, we examine the sources of macroeconomic fluctuations, with a particular focus on Canada-US exchange rate dynamics. In the context of estimated open-economy models, fluctuations in nominal exchange rates are often found to be largely

---

parameter space and does not consider the possibility of indeterminacy.

[3]Equilibrium indeterminacy does not arise in the model with Adolfson et al. (2008)'s original specification of the endogenous interest rate spread on foreign bond holdings, where the spread depends on expected *consecutive* currency depreciation (*i.e.*, changes from $t-1$ to $t+1$).

unrelated to macroeconomic fundamentals and are attributed to a UIP shock. However, our empirical analysis challenges this prevailing view. The modified UIP condition in our model strengthens the connection between expected exchange rate depreciation and interest rate differentials by allowing for a negative relationship between the two, consistent with observed data. Consequently, the influence of UIP shocks can be diminished. Additionally, by incorporating equilibrium indeterminacy, our model allows sunspot shocks—nonfundamental disturbances to agents' expectations—to affect aggregate fluctuations. Under indeterminacy, the contribution of UIP shocks to exchange rate movements may be partially replaced by that of sunspot shocks.

The main results of this paper are as follows. First, the posterior distributions of the model parameters indicate that the parameter on the interest rate spread for foreign bond holdings associated with the exchange rate is large enough to render the coefficient on interest rate differentials in the modified UIP equation negative, resulting in equilibrium indeterminacy. Comparing the baseline estimation results with those obtained by estimating the model only within the determinacy region of its parameter space, we find that the baseline model, which allows for indeterminacy, fits the data significantly better than the model restricted to determinacy.

Second, the propagation of shocks differs remarkably between the baseline model and the model estimated only under determinacy. These differences arise from the estimated arbitrary components—which act as an equilibrium selection device—in the solution under indeterminacy. Specifically, we demonstrate that only the selected equilibrium solution under indeterminacy can replicate the delayed overshooting, an empirical pattern in which a currency tends to appreciate with a lag following a positive monetary policy shock.

Third, forecast error variance decompositions based on the estimated model reveal that, rather than the UIP shock, a preference shock—interpreted as a demand shock—is the primary driver of exchange rate fluctuations.[4] Consequently, the conventional wisdom in the literature is overturned, establishing that exchange rate fluctuations are connected to macroeconomic fundamentals. We demonstrate that allowing for indeterminacy and using the data to select a specific equilibrium representation are both essential for obtaining this

[4] This finding aligns with the recent results of Kekre and Lenel (2024b), who show that demand shocks account for most of the variability in the US dollar-G10 exchange rate based on a calibrated two-country model featuring shocks to demand, supply, and the UIP condition.

result.

Finally, sunspot shocks play a negligible role in explaining fluctuations in exchange rates and other aggregate variables. This is because sunspot shocks merely induce nonfundamental revisions in agents' expectations, with limited pass-through to aggregate fluctuations. Moreover, as sunspot shocks are by definition i.i.d., their contribution to aggregate dynamics is inherently smaller than that of persistent fundamental shocks.

The remainder of this paper proceeds as follows. Section 2 provides a brief overview of the related literature. Section 3 presents the small open-economy DSGE model used for our empirical analysis and examines the determinacy and indeterminacy regions in the model's parameter space. Section 4 outlines the estimation strategy and data. Section 5 presents and discusses the results of the empirical analysis. Section 6 conducts a robustness analysis across several estimation settings. Section 7 concludes.

2 Related Literature

While the exchange rate disconnect is often related to various anomalies in the context of structural open-economy macroeconomic models, namely, the Meese and Rogoff (1983) puzzle, the purchasing power parity (PPP) puzzle (Rogoff, 1996), the Backus and Smith (1993) puzzle, and the UIP puzzle (Fama, 1984), this paper revolves around the UIP puzzle. In the field of international finance, this paper also contributes to the strands of literature on accounting for exchange rate dynamics and equilibrium indeterminacy in open economies.

UIP puzzle When agents are risk-neutral and there are no frictions in international financial markets, a no-arbitrage condition for home and foreign bond holdings yields the UIP condition:

$$\log \mathbb{E}_t S_{t+1} - \log S_t = \log i_t - \log i_t^*,$$

where S_t denotes the nominal exchange rate (price of foreign currency in terms of domestic currency), and i_t and i_t^* are respectively the home and foreign (gross) nominal interest rates. Assuming the rational expectations, this UIP condition can be empirically tested by conducting so-called UIP regression:

$$\log S_{t+1} - \log S_t = \alpha_0 + \alpha_1(\log i_t - \log i_t^*) + v_t,$$

with the null hypothesis $H_0 : \alpha_0 = 0$ and $\alpha_1 = 1$, where α_0 and α_1 are regression coefficients and v_t is an error term. Since the seminar paper by Fama (1984), numerous papers using a wide variety of international data have rejected this null hypothesis and have found the estimated slope coefficient α_1 to be significantly less than unity, and often negative.

In line with these findings, Lustig and Verdelhan (2007) and Burnside et al. (2008) find sizable gains from the *carry trade*, an investment strategy undertaken by investing in high interest rate currencies with funding from low interest rate currencies. Eichenbaum and Evans (1995) and Scholl and Uhlig (2008) provide the empirical pattern called *delayed overshooting*: *i.e.*, a country's currency tends to appreciate for a while after a positive monetary policy shock. Grilli and Roubini (1996) and Kim and Roubini (2000) reveal the same pattern for Canadian data.

There have been many attempts to replicate this failure of the UIP condition using a theoretical framework. Backus et al. (2001), Duarte and Stockman (2005), Verdelhan (2010), Colacito and Croce (2011), Bansal and Shaliastovich (2012), Benigno et al. (2011), Backus et al. (2010), Gourio et al. (2013), Engel (2016), and Chen et al. (2021) aim to solve the UIP puzzle through risk corrections, namely the covariance between the stochastic discount factor and payoffs. Bacchetta and van Wincoop (2021) show that delayed portfolio adjustment can account for the UIP puzzle. In these studies, structural or macroeconomic fundamental shocks can raise interest rates and appreciate nominal exchange rates simultaneously. Gourinchas and Tornell (2004), Chakraborty and Evans (2008), Burnside et al. (2011), Ilut (2012), and Candian and De Leo (2023) explain the UIP puzzle by deviating from the rational expectations.

Accounting for exchange rate dynamics Because the standard UIP condition fails to replicate the observed pattern between expected exchange rate depreciation and interest rate differentials, the literature finds that exchange rate dynamics are mostly explained by a wedge to the UIP condition, referred to as a UIP shock. Gabaix and Maggiori (2015), Valchev (2020), Itskhoki and Mukhin (2021), and Kekre and Lenel (2024a) provide microfoundations for a direct shock to the UIP condition. Especially, Itskhoki and Mukhin (2021) quantitatively illustrate its importance in accounting for aggregate fluctuations, including exchange rate dynamics.

Lubik and Schorfheide (2006) and Hirose (2013) estimate a two-country open-economy model for the US and the Euro economies in a linear setting and find that a shock to the PPP condition, which works in a similar way to the UIP shock, explains more than 80% of exchange rate fluctuations. Chen et al. (2021) extend their analysis by allowing for stochastic volatilities in fundamental shocks and estimate their model approximated up to the third order. Albeit with the consideration of nonlinearity and risk components, they report that the UIP shock continues to play a significant role in accounting for exchange rate dynamics.

In contrast to these findings, recent studies emphasize the importance of structural shocks other than the UIP shock in explaining exchange rate fluctuations. Kekre and Lenel (2024b) calibrate a two-country model featuring shocks to demand, supply, and the UIP condition to match data on the US and G10 countries and demonstrate that the demand shock contributes to 75% of the variance in the US dollar-G10 exchange rate. Chahrour et al. (2024) show that anticipated news shocks about US productivity explain more than half of the US dollar-G7 exchange rate fluctuations, based on a structural VAR model. Engel and Wu (2024) estimate a reduced-form model for the US dollar against the other G10 currencies and find that the explanatory power of macroeconomic fundamentals, including global risk and liquidity measures, increases markedly in the post-1999 data.

Regarding the dynamic behavior of exchange rates, Adolfson et al. (2008) introduce an endogenous interest rate spread on foreign bond holdings in a small open-economy model, which depends on the aggregate net foreign asset position of domestic households and expected currency depreciation. They show that their estimated model better replicates the observed properties of Swedish macroeconomic data, including the exchange rate.

Indeterminacy in open economies While Kareken and Wallace (1981) discuss steady-state indeterminacy in an open economy, we focus on dynamic indeterminacy, *i.e.*, the possibility of multiple equilibrium paths toward a unique steady state.

In a small open-economy setting, Carlstrom and Fuerst (2002) report that determinacy conditions in a closed economy framework carry over to a small open economy, whereas De Fiore and Liu (2005) point out the importance of trade openness for a determinate equilibrium. In a multicountry setting, Bullard and Singh (2008) and Bullard and Schaling (2009) demonstrate that the worldwide equilibrium can be indeterminate when one country satisfies the determinacy condition but when one of the others does not. Hirose (2013) estimates a two-country model for the US and the Euro area that allows for both the determinacy and indeterminacy of equilibrium.

Beaudry and Lahiri (2019) indicate that a perfect foresight equilibrium in a small open economy becomes indeterminate when the UIP condition fails and exhibits a negative relationship between expected exchange rate depreciation and interest rate differentials consistently with the UIP puzzle. Their finding is quite novel and relevant to the empirical regularities. However, their analysis is based on a small open-economy version of a simple (continuous-time) sticky-price model, where the modified UIP equation is specified in a reduced-form manner, and the empirical validity of their model is not examined formally through model estimation. Moreover, we demonstrate that the negative relationship is not a sufficient condition for equilibrium indeterminacy in a more general stochastic setting.

3 The Model

The model estimated in this paper is a small open-economy version of the standard New Keynesian model as in Galí and Monacelli (2005), but incorporates an endogenous interest rate spread on foreign bond holdings *à la* Adolfson et al. (2008); therefore, the resulting modified UIP condition can exhibit the negative relationship between expected exchange rate depreciation and interest rate differentials observed in the data. A representative household gains utility from aggregate consumption composed of home and foreign goods, and trades both home and foreign bonds in domestic and international asset markets. Monopolistically competitive firms produce differentiated goods and are subject to a price adjustment cost. The central bank adjusts the nominal interest rate in response to inflation, output growth, and nominal exchange rate depreciation. For a better fit to the macroeconomic data, the model features habit persistence in consumption preferences, price indexation to past inflation, and monetary policy smoothing.

3.1 Household

A representative household in the home country maximizes the utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \exp(d_t) \left[\log\left(C_t - b\bar{C}_{t-1}\right) - \frac{h_t^{1+\eta}}{1+\eta} \right],$$

where C_t is aggregate consumption, $b\bar{C}_{t-1}$ is an external habit taken as given by the household, h_t is labor supply, β is the subjective discount factor, and η is the inverse of the labor supply elasticity. d_t is a preference shock, broadly interpreted as a demand shock. Aggregate consumption C_t is a composite of home- and foreign-produced goods, $C_{H,t}$ and $C_{F,t}$, given by

$$C_t = \left(\frac{C_{H,t}}{\lambda}\right)^{\lambda} \left(\frac{C_{F,t}}{1-\lambda}\right)^{1-\lambda},$$

with

$$C_{H,t} = \left[\int_0^1 C_{H,t}\left(i\right)^{1-\frac{1}{\epsilon}} \mathrm{d}i\right]^{\frac{\epsilon}{\epsilon-1}},$$

$$C_{F,t} = \left[\int_0^1 C_{F,t}\left(i^*\right)^{1-\frac{1}{\epsilon}} \mathrm{d}i^*\right]^{\frac{\epsilon}{\epsilon-1}},$$

where $C_{H,t}(i)$ and $C_{F,t}(i^*)$ are differentiated consumption goods produced by home and foreign firms, each of which is indexed by j and j^*, respectively. λ is the relative weight on the domestically produced goods in aggregate consumption, and ϵ is the elasticity of substitution among the differentiated products in each country.

The household's utility maximization is subject to the budget constraint:

$$P_t C_t + A_t + S_t A_t^* = W_t h_t + i_{t-1} A_{t-1} + S_t \Phi_{t-1} i_{t-1}^* A_{t-1}^* + T_t,$$

where P_t is the consumer price index, A_t and A_t^* are respectively the holding of home and foreign bonds, S_t denotes the nominal exchange rate (price of foreign currency in terms of domestic currency), W_t is the nominal wage, i_t and i_t^* are respectively the home and foreign nominal interest rates, Φ_t is the interest rate spread on foreign bond holdings, and T_t is the net transfers from firms and the government.

Following Adolfson et al. (2008), the interest rate spread depends on the net foreign asset position of the domestic household, the expected change in the exchange rate, and a shock.[5] Specifically, Φ_t is of the form

$$\Phi_t := \exp\left[-\phi_a\left(\frac{S_t A_t^*}{P_t Z_t} - a^*\right) - \phi_s\left(\frac{\mathbb{E}_t S_{t+1}}{S_t} - \frac{\pi}{\pi^*}\right) + \psi_t\right], \tag{1}$$

where Z_t is a nonstationary trend component as explained below. a^*, π, and π^* are the steady-state values of detrended real net foreign assets in the home currency and home and foreign inflation. ϕ_a and ϕ_s are parameters. ψ_t is a shock to the spread. We refer to this shock as a UIP shock because it will appear as a direct shock to the resulting modified UIP condition. The first term $-\phi_a\left(S_t A_t^*/(P_t Z_t) - a^*\right)$ is needed to ensure the stationarity of the small-open economy model with incomplete asset markets. The second term $-\phi_s\left(\mathbb{E}_t S_{t+1}/S_t - \pi/\pi^*\right)$ is based on the empirical regularity that interest rate premia are negatively correlated with expected currency depreciations (e.g., Fama, 1984; Duarte and Stockman, 2005).[6]

3.2 Firms

In the home country, each firm, indexed by j, produces one kind of differentiated good $Y_t(j)$ by choosing a cost-minimizing labor input $h_t(j)$, given the wage, subject to the production function:

$$Y_t(j) = \exp(z_t) Z_t h_t(j),$$

where z_t is a stationary technology shock, and Z_t is a nonstationary trend component that grows at a constant rate γ, *i.e.*,

$$\frac{Z_t}{Z_{t-1}} = \gamma.$$

In a monopolistically competitive market, each firm sets the price of its products in the presence of a Rotemberg (1982)-type adjustment cost and indexation to a weighted average of the past inflation rate for the domestically produced goods $\pi_{H,t-1} := P_{H,t-1}/P_{H,t-2}$ and

[5] This spread can be interpreted as a convenience yield, similar to its treatment in Valchev (2020) and Kekre and Lenel (2024a).

[6] Assuming that $\pi = \pi^*$, Adolfson et al. (2008) specify the second term as $-\phi_s\left[(\mathbb{E}_t S_{t+1}/S_t)(S_t/S_{t-1}) - 1\right]$, in which the spread depends on expected *consecutive* depreciation, or expected changes in the exchange rate from two periods ago. Our specification is simpler than theirs but straightforwardly captures the empirical regularity found in the literature.

the steady-state inflation rate π to maximize the present discounted value of its profit:

$$\mathbb{E}_t \sum_{n=0}^{\infty} m_{t,t+n} \left[\frac{P_{H,t+n}(j)}{P_{H,t+n}} - \frac{W_{t+n}}{\exp(z_t) Z_t P_{H,t+n}} - \frac{\phi}{2} \left(\frac{P_{H,t+n}(j)}{\pi_{H,t+n-1}^{\omega} \pi^{1-\omega} P_{H,t+n-1}(j)} - 1 \right)^2 \right] Y_{t+n}(j),$$

subject to the firm-level resource constraint

$$Y_t(j) = C_{H,t}(j) + C_{H,t}^*(j) + \frac{\phi}{2} \left(\frac{P_{H,t}(j)}{\pi_{H,t-1}^{\omega} \pi^{1-\omega} P_{H,t-1}(j)} - 1 \right)^2 Y_t(j),$$

and the downward sloping demand curves, which are obtained from the household's optimization problem in each country,

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\epsilon} C_{H,t},$$
$$C_{H,t}^*(j) = \left[\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right]^{-\epsilon} C_{H,t}^*,$$

where $m_{t,t+n}$ is the stochastic discount factor, $C_{H,t}^*$ is the demand for the domestically produced goods in the foreign country, $P_{H,t}^*$ is the export price of the domestically produced goods in terms of the foreign currency, ϕ is the adjustment cost parameter, and $\omega \in [0, 1]$ is the weight of price indexation to past inflation relative to steady-state inflation.

Assuming the law of one price and a symmetric equilibrium, we obtain

$$P_{H,t} = S_t P_{H,t}^*.$$

Let $p_{H,t} := P_{H,t}/P_t$. Then, $\pi_{H,t} := P_{H,t}/P_{H,t-1}$ can be expressed as

$$\pi_{H,t} = \frac{p_{H,t} \pi_t}{p_{H,t-1}},$$

where $\pi_t := P_t/P_{t-1}$. The real exchange rate e_t is defined as

$$e_t := \frac{S_t P_t^*}{P_t}.$$

Aggregating the firm-level resource constraint leads to

$$Y_t = C_{H,t} + C^*_{H,t} + \frac{\phi}{2}\left(\frac{\pi_t}{\pi^{\omega}_{t-1}\pi^{1-\omega}} - 1\right)^2 Y_t.$$

The balance of payments identity is given by

$$P_{H,t}Y^*_{H,t} - P_{F,t}Y_{F,t} = S_t\left(A^*_t - \Phi_{t-1}i^*_{t-1}A^*_{t-1}\right),$$

where $Y^*_{H,t}$ is exports of the domestically produced goods, $P_{F,t}$ is the import price of the foreign goods expressed in foreign currency, and $Y_{F,t}$ is imports of the foreign goods.

3.3 Monetary policy

The central bank in the home country adjusts the nominal interest rate in response to deviations of inflation, output growth, and nominal exchange rate depreciation from their steady-state values with policy smoothing:[7]

$$i_t = i^{\rho}_{t-1}\left[i\left(\frac{\pi_t}{\overline{\pi}}\right)^{\alpha_\pi}\left(\frac{Y_t}{\gamma Y_{t-1}}\right)^{\alpha_y}\left(\frac{S_t\pi^*}{S_{t-1}\pi}\right)^{\alpha_s}\right]^{1-\rho}\exp\left(u_t\right),$$

where ρ is the degree of interest rate smoothing, and α_π, α_y, and α_s are the degrees of monetary policy responses to their target variables. u_t is a monetary policy shock. We include exchange rate depreciation in the policy rule following Justiniano and Preston (2010), who estimate a small open-economy model like ours for the Canadian economy.

3.4 Equilibrium conditions and detrending

The equilibrium conditions of the model are presented in Appendix A. To ensure the stationarity of the system of equations, real variables are detrended by the nonstationary trend component Z_t as follows: $y_t := Y_t/Z_t$, $y_{H,t} := Y_{H,t}/Z_t$, $y_{F,t} := Y_{F,t}/Z_t$, $y^*_t := Y^*_t/Z_t$, and $c_t := C_t/Z_t$. The steady-state conditions in terms of detrended variables are presented in

[7] Our model does not explicitly consider the zero lower bound on the nominal interest rate because the policy interest rate in Canada was constrained at its effective lower bound only for four quarters after the global financial crisis.

Appendix B, whereas the log-linearized version of the detrended system of equations is shown in Appendix C.

3.5 Exogenous shock processes

In addition to the fundamental shocks mentioned above, we treat foreign output $\hat{y}_t^*$, inflation $\hat{\pi}_t^*$, and the nominal interest rate $\hat{i}_t^*$ as exogenous shocks.[8] We assume that all the shocks except for the monetary policy shock follow stationary AR(1) processes:

$$
\begin{aligned}
\hat{\psi}_t &= \rho_\psi \hat{\psi}_{t-1} + \varepsilon_{\psi,t}, \\
z_t &= \rho_z z_{t-1} + \varepsilon_{z,t}, \\
d_t &= \rho_d d_{t-1} + \varepsilon_{d,t}, \\
u_t &= \varepsilon_{u,t}, \\
\hat{y}_t^* &= \rho_{y^*} \hat{y}_{t-1}^* + \varepsilon_{y^*,t}, \\
\hat{\pi}_t^* &= \rho_{\pi^*} \hat{\pi}_{t-1}^* + \varepsilon_{\pi^*,t}, \\
\hat{i}_t^* &= \rho_{i^*} \hat{i}_{t-1}^* + \varepsilon_{i^*,t},
\end{aligned}
$$

where ρ_x, $x \in \{\psi, z, d, y^*, \pi^*, i^*\}$ are the autoregressive parameters and $\varepsilon_{x,t} \sim$ i.i.d. $N(0, \sigma_x^2)$, $x \in \{\psi, z, d, u, y^*, \pi^*, i^*\}$.

3.6 Modified UIP condition and equilibrium indeterminacy

3.6.1 Modified UIP condition

From the optimality conditions for home and foreign bond holdings, we can derive the modified UIP condition as follows:

$$
i_t = \mathbb{E}_t \left\{ s_{t+1} \exp\left[-\phi_a \left(a_t^* - a^* \right) - \phi_s \left(s_{t+1} - \frac{\pi}{\pi^*} \right) + \psi_t \right] i_t^* \right\},
$$

[8] Justiniano and Preston (2010) endogenize these foreign variables by adding a small-scale New Keynesian model into the system of equations. We treat them as exogenous because our estimation sample includes the zero lower bound periods in the US, which cannot be well captured by linearized equations.

where $s_t := S_t/S_{t-1}$ represents the depreciation of the nominal exchange rate and $a_t^* := S_t A_t^*/P_t Z_t$ is detrended real net foreign assets in the home currency.

Assuming that $\pi = \pi^*$, log-linearizing this equation around the steady state gives

$$\mathbb{E}_t \hat{s}_{t+1} = \frac{1}{1-\phi_s}(\hat{i}_t - \hat{i}_t^*) + \frac{1}{1-\phi_s}(\phi_a a^* \hat{a}_t^* - \hat{\psi}_t), \tag{2}$$

where hatted variables denote the percentage deviation from their corresponding steady-state values. Note that setting $\phi_s = \phi_a = 0$ leads to the standard UIP condition:

$$\mathbb{E}_t \hat{s}_{t+1} = \hat{i}_t - \hat{i}_t^* - \hat{\psi}_t. \tag{3}$$

As addressed in Section 2, the UIP relationship given by (3) has been frequently rejected for various international data by conducting the UIP regression:

$$\log S_{t+1} - \log S_t = \alpha_0 + \alpha_1(\log i_t - \log i_t^*) + v_t, \tag{4}$$

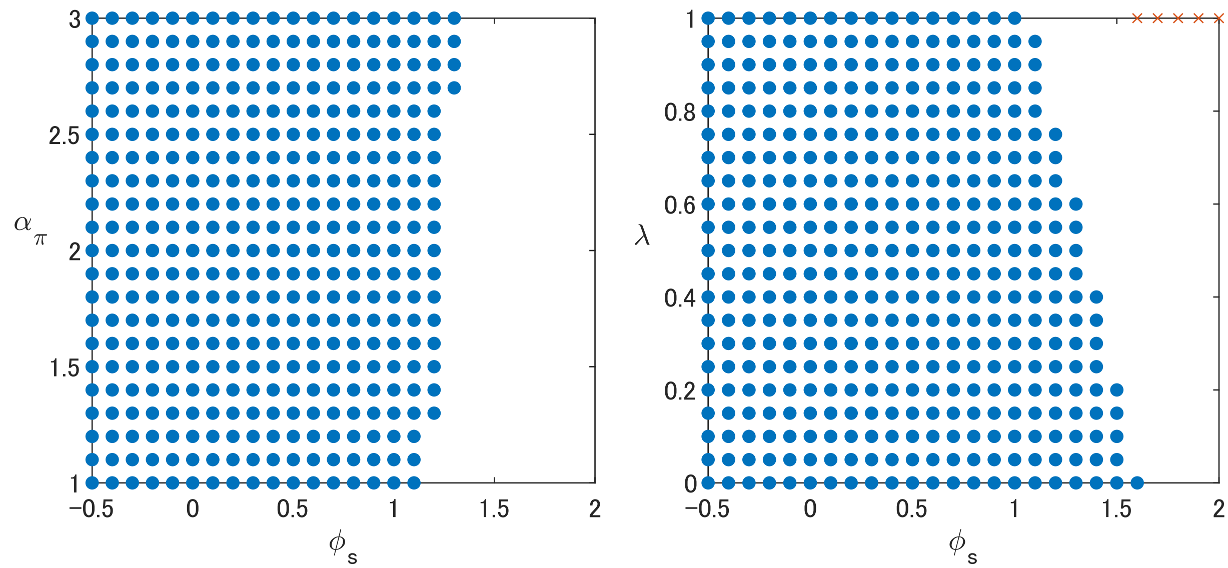
with the null hypothesis $H_0 : \alpha_0 = 0$ and $\alpha_1 = 1$. Indeed, the OLS estimator of α_1 is -0.012 with the standard error of 0.702 according to our preliminary estimation of (4) using the Canada-US data set, described in Section 4.3, ranging from 1984:Q1 to 2019:Q4. Assuming the rational expectations and ignoring the endogeneity of $\hat{a}_t^*$,[9] the regression coefficient α_1 is consistent with $1/(1-\phi_s)$ in the modified UIP condition given by (2). A novel feature of our analysis is that we allow for values of ϕ_s greater than one; hence, the modified UIP condition given by (2) can be consistent with negative coefficients in the UIP regressions. Besides, we permit ϕ_s to be negative, considering the fact that some international data posit that the UIP regression coefficient α_1 lies between zero and one.

Because of the empirical failure of the standard UIP relationship, the literature that estimates open-economy DSGE models with a UIP equation, such as (3), finds very large contributions of the UIP shocks $\hat{\psi}_t$ to exchange rate fluctuations.[10] This conventional wisdom in the literature may be overturned once we embed the modified UIP condition (2) into the

[9] We can ignore the endogeneity of $\hat{a}_t^*$ because this term is needed to ensure the stationarity of the small-open economy model with incomplete asset markets and because ϕ_a is commonly set at very small values less than 0.01.

[10] See Lubik and Schorfheide (2006), Hirose (2013), and Chen et al. (2021).

Figure 1: Determinacy regions in the parameter space



Notes: Each panel of this figure displays the equilibrium determinacy region in the parameter space for (ϕ_s, α_π) and (ϕ_s, λ), respectively, given the prior mean of the other parameters reported in Table 1. '×' denotes the region for nonexistence of equilibrium.

model and allow for the observed negative relationship between expected exchange rate depreciation $\mathbb{E}_t \hat{s}_{t+1}$ and the interest rate differential $\hat{i}_t - \hat{i}_t^*$.

3.6.2 Equilibrium indeterminacy

In dynamic general equilibrium economies, equilibrium can be indeterminate, depending on model structures and parameters, and in such a case, sunspot shocks, which are nonfundamental disturbances, can affect economic fluctuations. In what follows, we demonstrate that the equilibrium can be indeterminate, depending on the parameters in the modified UIP condition (2):

$$\mathbb{E}_t \hat{s}_{t+1} = \frac{1}{1-\phi_s}(\hat{i}_t - \hat{i}_t^*) + \frac{1}{1-\phi_s}(\phi_a a^* \hat{a}_t^* - \hat{\psi}_t). \tag{5}$$

Figure 1 presents the equilibrium determinacy regions in the parameter space for (ϕ_s, α_π) and (ϕ_s, λ), respectively, given the prior means of the other parameters reported in Table 1 (Section 4). The first panel illustrates that sufficiently large values of ϕ_s lead to equilibrium indeterminacy and that its threshold value for indeterminacy increases as α_π (monetary policy reaction to inflation) increases. Likewise, the second panel shows that large values of ϕ_s

give rise to indeterminacy, although decreases in λ (relative weight on domestically produced goods in aggregate consumption) expand the determinacy region. A notable finding from these two panels is that, regardless of the values of α_π and λ, indeterminacy arises for such large values of ϕ_s that the modified UIP condition exhibits a negative relationship between expected exchange rate depreciation $\mathbb{E}_t \hat{s}_{t+1}$ and interest rate differential $\hat{i}_t - \hat{i}_t^*$ as observed in international data.

To understand why large values of ϕ_s generate equilibrium indeterminacy, consider the case in which the monetary policy rule is of the simple form:

$$\hat{i}_t = \alpha_\pi \hat{\pi}_t. \tag{6}$$

In the model, the aggregate price is given by

$$P_t = P_{H,t}^{\lambda} \left(S_t P_t^* \right)^{1-\lambda},$$

which can be written in log-linearized form in terms of rates of change and solved for exchange rate depreciation $\hat{s}_t$:

$$\hat{s}_t = \frac{1}{1-\lambda} \hat{\pi}_t - \frac{\lambda}{1-\lambda} \hat{\pi}_{H,t} - \hat{\pi}_t^*. \tag{7}$$

Substituting (6) and (7) into the modified UIP condition (2), we obtain

$$\frac{1}{1-\lambda} \mathbb{E}_t \hat{\pi}_{t+1} - \frac{\lambda}{1-\lambda} \mathbb{E}_t \hat{\pi}_{H,t+1} - \mathbb{E}_t \hat{\pi}_{t+1}^* = \frac{1}{1-\phi_s} (\alpha_\pi \hat{\pi}_t - \hat{i}_t^*) + \frac{1}{1-\phi_s} (\phi_a a^* \hat{a}_t^* - \hat{\psi}_t). \tag{8}$$

Isolating the relationship between $\mathbb{E}_t \hat{\pi}_{t+1}$ and $\hat{\pi}_t$ from this equation under the assumption that the stability of the other variables is guaranteed by other equations in the system, we have

$$\mathbb{E}_t \hat{\pi}_{t+1} = \varrho \hat{\pi}_t,$$

where $\varrho = \alpha_\pi (1-\lambda)/(1-\phi_s)$. In the analogy of Blanchard and Kahn (1980) conditions, expected inflation is uniquely pinned down (*i.e.*, determinate) if the composite parameter ϱ is outside the unit circle.[11] In a limiting case where $\alpha_\pi = 1$ and $\lambda = 0$, this condition is

[11] Conditions for determinacy depend on eigenvalues in the autoregressive coefficient matrix of the whole system. In the present setting, however, we cannot characterize them analytically because of the high dimensionality of the system. Thus, we focus on the univariate relationship between $\mathbb{E}_t \hat{\pi}_{t+1}$ and $\hat{\pi}_t$ to

not satisfied for $\phi_s \geq 2$. Thus, sufficiently large values of ϕ_s lead to the equilibrium being indeterminate. As each of α_π and λ increases from the limiting case, the threshold value of ϕ_s for $|\varrho| \leq 1$ increases and decreases, respectively, which is consistent with the determinacy regions illustrated in Figure 1.

To gain an intuition about why the negative relationship between $\mathbb{E}_t \hat{s}_{t+1}$ and $\hat{i}_t - \hat{i}_t^*$ in the modified UIP condition gives rise to equilibrium indeterminacy, suppose that agents believe there will be a rise in future inflation without any changes in fundamentals. Then, taking one-period-ahead expectations for both sides of (7), the future exchange rate is expected to depreciate (*i.e.*, $\mathbb{E}_t \hat{s}_{t+1}$ increases). If the coefficient on $\hat{i}_t - \hat{i}_t^*$ in the modified UIP condition is negative, expected future depreciation must coincide with a decrease in the domestic nominal interest rate, given the foreign nominal interest rate. The low interest rate stimulates the demand side of the economy and increases both output and inflation. Therefore, the nonfundamental belief on inflation can be self-fulfilling as an equilibrium. Alternatively, if the modified UIP equation exhibits a positive relationship between $\mathbb{E}_t \hat{s}_{t+1}$ and $\hat{i}_t - \hat{i}_t^*$, expected future depreciation is accompanied by an increase in the domestic nominal interest rate, which dampens output and inflation. Thus, the nonfundamental belief about inflation does not materialize and cannot be an equilibrium.

If we allow for indeterminacy, sunspot shocks, which are nonfundamental disturbances to the economy, can arise and affect the equilibrium dynamics. As addressed above, the literature estimating open-economy DSGE models has found that almost all fluctuations in exchange rates are driven by wedges to the UIP condition, *i.e.*, UIP shocks, reflecting the empirical failure of the standard UIP equation. Under indeterminacy, however, some portion of its contribution might be replaced with that from sunspot shocks. This point is also examined in subsequent empirical analysis.

It should be noted that equilibrium indeterminacy does not occur for any ϕ_s if we employ Adolfson et al. (2008)'s original specification for the interest rate spread on foreign bond holdings:

$$\Phi_t := \exp\left[-\phi_a\left(\frac{S_t A_t^*}{P_t Z_t} - a^*\right) - \phi_s\left(\frac{\mathbb{E}_t S_{t+1}}{S_t}\frac{S_t}{S_{t-1}} - \frac{\pi^2}{\pi^{*2}}\right) + \psi_t\right],$$

where, in contrast to our specification (1), the spread depends on expected *consecutive*

illustrate the source of indeterminacy.

depreciation, or expected changes in the exchange rate from $t-1$ to $t+1$. Under this specification, the modified UIP condition is of the log-linearized form:

$$\mathbb{E}_t \hat{s}_{t+1} = \frac{1}{1-\phi_s}(\hat{i}_t - \hat{i}_t^*) + \frac{\phi_s}{1-\phi_s}\hat{s}_t + \frac{1}{1-\phi_s}(\phi_a a^* \hat{a}_t^* - \hat{\psi}_t).$$

Our preliminary estimation of the model with this modified UIP condition has confirmed that the model fits the data significantly worse than our baseline model, as reported in Appendix D.

4 Estimation Strategy

As shown in the previous section, equilibrium indeterminacy can occur when the modified UIP condition exhibits a negative relationship between expected exchange rate depreciation $\mathbb{E}_t \hat{s}_{t+1}$ and the interest rate differential $\hat{i}_t - \hat{i}_t^*$, often found in the UIP regressions. If we restrict the model's parameter space to the determinacy region in estimation, it would be unlikely that the estimated model could replicate the observed pattern between them.

To overcome this, we estimate the model by allowing for indeterminacy, using full-information Bayesian methods based on Lubik and Schorfheide (2004). Specifically, the model's likelihood function is constructed not only for the determinacy region of its parameter space but also for the indeterminacy region. While Lubik and Schorfheide (2004) conduct model estimation separately for each region, we estimate the model for both the determinacy and indeterminacy regions in one step by adopting a SMC algorithm, as implemented by Hirose et al. (2020). This algorithm can deal with discontinuities in the likelihood function at the boundaries of each region and help us capture the entire posterior distribution of the model's parameters.

In this section, we begin by presenting solutions to linear rational expectations models, then explain how Bayesian inference over both the determinacy and indeterminacy regions of the model parameter space are made with the SMC algorithm, and lastly describe the data and prior distributions used in the model estimation.

4.1 Rational expectations solutions under indeterminacy

Lubik and Schorfheide (2003) derive a full set of solutions to the models under indeterminacy of the form

$$\mathbf{s}_t = \mathbf{\Phi}_1^I(\boldsymbol{\theta})\,\mathbf{s}_{t-1} + \mathbf{\Phi}_\varepsilon^I(\boldsymbol{\theta}, \tilde{\mathbf{M}})\,\boldsymbol{\varepsilon}_t + \mathbf{\Phi}_\zeta^I(\boldsymbol{\theta})\,\zeta_t, \tag{9}$$

where $\mathbf{\Phi}_1^I(\boldsymbol{\theta})$, $\mathbf{\Phi}_\varepsilon^I(\boldsymbol{\theta}, \tilde{\mathbf{M}})$, and $\mathbf{\Phi}_\zeta^I(\boldsymbol{\theta})$ are coefficient matrices that depend on the vector $\boldsymbol{\theta}$ of model parameters and an arbitrary matrix $\tilde{\mathbf{M}}$; $\mathbf{s}_t$ is a vector of endogenous variables; $\boldsymbol{\varepsilon}_t$ is a vector of fundamental shocks; and $\zeta_t \sim$ i.i.d. $N(0, \sigma_\zeta^2)$ is a reduced-form sunspot shock.[12] The indeterminacy solution (9) displays three characteristics. First, the equilibrium dynamics are driven not only by the fundamental shocks $\boldsymbol{\varepsilon}_t$ but also by the sunspot shock ζ_t. Second, the solution is not unique because of the presence of the arbitrary matrix $\tilde{\mathbf{M}}$. Third, the coefficient matrix $\mathbf{\Phi}_1^I(\boldsymbol{\theta})$ in the solution induces more persistent dynamics than its counterpart $\mathbf{\Phi}_1^D(\boldsymbol{\theta})$ in the determinacy solution presented below, because fewer autoregressive roots (*i.e.*, eigenvalues) in the matrix $\mathbf{\Phi}_1^I(\boldsymbol{\theta})$ are being suppressed to zero.[13]

In the case of determinacy, the solution form is reduced to

$$\mathbf{s}_t = \mathbf{\Phi}_1^D(\boldsymbol{\theta})\,\mathbf{s}_{t-1} + \mathbf{\Phi}_\varepsilon^D(\boldsymbol{\theta})\,\boldsymbol{\varepsilon}_t, \tag{10}$$

where the coefficient matrices $\mathbf{\Phi}_1^D(\boldsymbol{\theta})$ and $\mathbf{\Phi}_\varepsilon^D(\boldsymbol{\theta})$ depend only on the model parameters $\boldsymbol{\theta}$. Thus, the solution is uniquely determined and driven by only the fundamental shocks $\boldsymbol{\varepsilon}_t$.

Under indeterminacy, the matrix $\tilde{\mathbf{M}}$ must be determined to specify the law of motion of the endogenous variables $\mathbf{s}_t$. Following Lubik and Schorfheide (2004), we estimate the components of $\tilde{\mathbf{M}}$ along with the other parameters in the model. The prior distribution for $\tilde{\mathbf{M}}$ is set so that it is centered around the matrix $\mathbf{M}^*(\boldsymbol{\theta})$ given in a particular solution. That is, $\tilde{\mathbf{M}}$ is replaced with $\mathbf{M}^*(\boldsymbol{\theta}) + \mathbf{M}$, and the components of $\mathbf{M}$ are estimated with prior mean zero. As proposed by Lubik and Schorfheide (2004), the matrix $\mathbf{M}^*(\boldsymbol{\theta})$ is selected so that the contemporaneous impulse responses of the endogenous variables to the funda-

[12] Instead of the term $\mathbf{\Phi}_\zeta^I(\boldsymbol{\theta})\,\zeta_t$ in the indeterminacy solution (9), Lubik and Schorfheide (2003) originally consider $\mathbf{\Phi}_\zeta^I(\boldsymbol{\theta}, \mathbf{M}_\zeta)\,\boldsymbol{\zeta}_t$, where $\mathbf{M}_\zeta$ is an arbitrary matrix and $\boldsymbol{\zeta}_t$ is a vector of sunspot shocks. For identification, however, Lubik and Schorfheide (2004) impose normalization on $\mathbf{M}_\zeta$ with the dimension of the sunspot shock vector being unity. Such a normalized shock is referred to as a "reduced-form sunspot shock" in that it contains beliefs associated with all the expectational variables.

[13] For details, see Lubik and Schorfheide (2003, 2004).

mental shocks (*i.e.*, $\partial \mathbf{s}_t / \partial \boldsymbol{\varepsilon}_t$) are continuous at the boundary between the determinacy and indeterminacy regions of the model parameter space, which is called a "continuity solution." More specifically, for each set of $\boldsymbol{\theta}$, the procedure searches for a vector $\boldsymbol{\theta}^*$ that lies on the boundary of the determinacy region, and selects $\mathbf{M}^*(\boldsymbol{\theta})$ that minimizes the discrepancy between $\partial \mathbf{s}_t / \partial \boldsymbol{\varepsilon}_t(\boldsymbol{\theta}, \mathbf{M}^*(\boldsymbol{\theta}))$, and $\partial \mathbf{s}_t / \partial \boldsymbol{\varepsilon}_t(\boldsymbol{\theta}^*)$ using a least-squares criterion. In searching for $\boldsymbol{\theta}^*$, the procedure finds $\boldsymbol{\theta}^*$ numerically by perturbing the parameter ϕ_s in the modified UIP condition (2), which is crucial for determinacy or indeterminacy, given the other parameters in $\boldsymbol{\theta}$.[14]

4.2 Bayesian inference

To conduct Bayesian inference over both the determinacy and indeterminacy regions of the model parameter space, we construct the likelihood function for a sample of observations $\mathbf{Y}^T = [\mathbf{Y}_1, ..., \mathbf{Y}_T]'$ as

$$p(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M}) = \mathbf{1}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D\}\, p^D(\mathbf{Y}^T | \boldsymbol{\theta}) + \mathbf{1}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^I\}\, p^I(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M}),$$

where $\boldsymbol{\Theta}^D$ and $\boldsymbol{\Theta}^I$ are the determinacy and indeterminacy regions of the model parameter space; $\mathbf{1}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^i\}$, $i \in \{D, I\}$ is an indicator function that is equal to one if $\boldsymbol{\theta} \in \boldsymbol{\Theta}^i$ and zero otherwise; and $p^D(\mathbf{Y}^T | \boldsymbol{\theta})$ and $p^I(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M})$ are the likelihood functions of the state-space models that consist of observation equations and either the determinacy solution (10) or the indeterminacy solution (9). Then, by Bayes' theorem, updating a prior distribution $p(\boldsymbol{\theta}, \mathbf{M})$ with the sample observations $\mathbf{Y}^T$ leads to the posterior distribution

$$p(\boldsymbol{\theta}, \mathbf{M} | \mathbf{Y}^T) = \frac{p(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M}) p(\boldsymbol{\theta}, \mathbf{M})}{p(\mathbf{Y}^T)} = \frac{p(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M}) p(\boldsymbol{\theta}, \mathbf{M})}{\int p(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M}) p(\boldsymbol{\theta}, \mathbf{M}) d\boldsymbol{\theta} d\mathbf{M}}.$$

To approximate the posterior distribution, we adopt a generic SMC algorithm with likelihood tempering, as described in Herbst and Schorfheide (2014, 2015). The details of the algorithm are provided in Appendix E. Based on particles from the final importance sampling

[14] In estimating a closed-economy New Keynesian model with nonzero trend inflation under both determinacy and indeterminacy, Hirose et al. (2020) employ a similar procedure, which finds $\boldsymbol{\theta}^*$ by perturbing a monetary policy reaction parameter on inflation. In this paper, the prior distribution for this parameter is truncated so that it does not cause indeterminacy (see Section 4.3), and hence the parameter ϕ_s is the primary source of indeterminacy.

in the algorithm, we make inferences about the parameters and approximate the marginal data densities.

4.3 Data and prior distributions

The data used in the model estimation are seven quarterly time series on $\Delta \log Y_t$, $\log \pi_t$, $\log i_t$,$\Delta \log S_t$, $\hat{y}_t^*$, $\hat{\pi}_t^*$, and $\hat{i}_t^*$. The first four series are constructed from Canadian data: real GDP per capita, the GDP deflator, the Bank of Canada's policy (overnight) rate, and the Canadian to US dollar exchange rate. The other series are proxied by the US data. In the model, foreign variables are treated as exogenous shocks that follow AR(1) processes. To identify these exogenous processes directly without considering the differences in trends and the steady states between Canada and the US, the data on $\hat{y}_t^*$, $\hat{\pi}_t^*$, and $\hat{i}_t^*$ are constructed by detrending per capita real GDP using the Hodrick–Prescott (HP) filter and demeaning the GDP deflator inflation and federal funds rates. Thus, the observation equations that relate the data to the corresponding variables in the model are given by

$$\begin{bmatrix} 100\Delta \log Y_t \\ 100 \log \pi_t \\ 100 \log i_t \\ 100\Delta \log S_t \\ \hat{y}_t^*, \\ \hat{\pi}_t^* \\ \hat{i}_t^* \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\pi} \\ \overline{i} \\ \overline{\pi} - \overline{\pi}^* \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{\pi}_t \\ \hat{i}_t \\ \hat{s}_t \\ \hat{y}_t^* \\ \hat{\pi}_t^* \\ \hat{i}_t^* \end{bmatrix},$$

where $\overline{\gamma} := 100(\gamma - 1)$, $\overline{\pi} := 100(\pi - 1)$, $\overline{i} := 100(i - 1)$, and $\overline{\pi}^* := 100(\pi^* - 1)$. The sample period is from 1984:Q1 to 2019:Q4; therefore, we exclude the Great Inflation and the COVID-19 pandemic periods from the sample.

To avoid any identification issues, we fix two parameters in the model. The steady-state ratio of net foreign assets to GDP is fixed at $a^*/y = -1.2$, which is calculated from the steady-state relationship on the balance of payments identity and the sample average of net exports in Canada. The elasticity of substitution among differentiated goods is fixed at $\epsilon = 8$, following Justiniano and Preston (2010). We assume $\overline{\pi}^* = \overline{\pi}$ and estimate $\overline{\pi}$ as a parameter.[15]

All other parameters in the model are estimated, and their prior distributions are presented in Table 1.[16]

The novelty of this study is that we allow one of the spread parameters ϕ_s to exceed one so that the modified UIP condition exhibits a negative relationship between the expected exchange rate depreciation and the interest rate differential. Moreover, we allow for negative values of ϕ_s to make it consistent with the UIP regression coefficients ranging from zero to one. Thus, we impose a normal distribution with mean zero and standard deviation 1.5, whereas Adolfson et al. (2008) use a beta distribution with mean 0.5 and standard deviation 0.15. Note that our prior is centered at $\phi_s = 0$, which corresponds to the standard UIP equation shown in (3). Hence, our prior strongly favors the conventional parameter values for ϕ_s, so that the equilibrium is determinate.[17] For another spread parameter ϕ_a, we adopt the same prior as Adolfson et al. (2008).

The prior mean for price adjustment cost is set at $\phi = 20$ so that the slope of the Phillips curve is nearly 0.2 when $\epsilon = 8$ and $\omega = 0.5$. The priors for the other structural parameters for the household, firms, and central bank follow Justiniano and Preston (2010). We impose a gamma prior on the degree of monetary policy response to inflation α_π, but it is truncated at one so that this parameter itself cannot be the source of indeterminacy.

The prior means for the steady-state (quarterly) rates of GDP growth $\overline{\gamma}$, inflation $\overline{\pi}$, and nominal interest $\overline{i}$ are set at the respective averages of the data used in the estimation.

Regarding the shock parameters, the priors for persistence parameters in the AR(1) processes ρ_x, $x \in \{\psi, z, d, y^*, \pi^*, i^*\}$ are the same as those in Smets and Wouters (2007). For the shock standard deviations σ_x, $x \in \{\psi, z, d, u, y^*, \pi^*, i^*, \zeta\}$, we set $\nu = 0.15$ and $s = 4$ in the inverse gamma distribution of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$.

For the components $M_x, x \in \{\psi, z, d, u, y^*, \pi^*, i^*\}$ of the arbitrary matrix $\mathbf{M}$ in the indeterminacy solution, we use the standard normal distribution, following Lubik and Schorfheide (2004).

[15]This assumption is supported by the data. That is, in our sample period, the quarterly inflation rates in Canada and the US are 0.557% and 0.543%, respectively.

[16]As for the subjective discount factor β, the steady-state condition $\beta = \pi\gamma/i$ is used in estimation.

[17]Indeed, the prior distributions presented here lead to the prior probability of determinacy of 0.801.

Table 1: Prior distributions of parameters

Parameter	Distribution	Mean	S.D.
ϕ_s	Normal	0.000	1.500
ϕ_a	Inverse Gamma	0.010	2.000
b	Beta	0.500	0.100
η	Gamma	1.000	0.300
ϕ	Gamma	20.00	5.000
ω	Beta	0.500	0.100
λ	Beta	0.710	0.020
ρ	Beta	0.600	0.200
α_π	Gamma	1.800	0.300
α_y	Gamma	0.300	0.200
α_s	Gamma	0.300	0.200
$\overline{\gamma}$	Normal	0.290	0.050
$\overline{\pi}$	Normal	0.557	0.050
$\bar{i}$	Gamma	1.194	0.050
ρ_ψ	Beta	0.500	0.200
ρ_z	Beta	0.500	0.200
ρ_d	Beta	0.500	0.200
ρ_{y^*}	Beta	0.500	0.200
ρ_{π^*}	Beta	0.500	0.200
ρ_{i^*}	Beta	0.500	0.200
σ_ψ	Inverse Gamma	0.150	4.000
σ_z	Inverse Gamma	0.150	4.000
σ_d	Inverse Gamma	0.150	4.000
σ_u	Inverse Gamma	0.150	4.000
σ_{y^*}	Inverse Gamma	0.150	4.000
σ_{π^*}	Inverse Gamma	0.150	4.000
σ_{i^*}	Inverse Gamma	0.150	4.000
σ_ζ	Inverse Gamma	0.150	4.000
M_ψ	Normal	0.000	1.000
M_z	Normal	0.000	1.000
M_d	Normal	0.000	1.000
M_u	Normal	0.000	1.000
M_{y^*}	Normal	0.000	1.000
M_{π^*}	Normal	0.000	1.000
M_{i^*}	Normal	0.000	1.000

Notes: The prior probability of equilibrium determinacy is 0.801. Inverse gamma distributions are of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$, where ν and s are respectively set at the mean and standard deviation (S.D.) values in the table.

5 Estimation Results

In this section, we begin by presenting the parameter estimates for our baseline model and its empirical performance and compare them with those obtained by estimating the same model only in the determinacy region of the parameter space. Next, we analyze how the dynamic properties of the model differ between the two cases of determinacy and indeterminacy by comparing the impulse response functions. Lastly, we conduct variance decompositions to investigate the sources of macroeconomic fluctuations in the estimated model, particularly exchange rate fluctuations.

5.1 Parameter estimates and empirical performance of the model

Table 2 compares the posterior estimates of the parameters in the baseline model with those in the model estimated only in the determinacy region of the parameter space. This table presents four key features.

First, in the baseline model, the posterior mean estimate of ϕ_s (spread parameter associated with exchange rate depreciation in the modified UIP condition) is 3.930, which gives the negative slope coefficient $1/(1-\phi_s) = -0.341$ in the modified UIP equation (2). Thus, our modified UIP condition exhibits a negative relationship between exchange rate depreciation and the interest rate differentials. Because the estimate of ϕ_s is far above one, the equilibrium is indeterminate. Indeed, the posterior probability of determinacy $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D|\mathbf{Y}^T\}$ shown in the last row of the table is zero, indicating that all posterior draws of the model parameters lie in the indeterminacy region. In contrast, when the model is estimated only under determinacy, ϕ_s is estimated to be negative (-3.666 at the posterior mean), implying the slope coefficient $1/(1-\phi_s) = 0.214$.

Second, the second last row shows that the log marginal data density $\log p(\mathbf{Y}^T)$ is much larger in the baseline estimation than the case in which we restrict the parameter space to its determinacy region. This huge difference in the marginal data density indicates that the data strongly favor indeterminacy over determinacy.

Third, both the UIP shock's persistence parameter ρ_ψ and standard deviation σ_ψ are remarkably smaller in the baseline estimation in contrast to the determinacy case. This reflects that the explanatory power of the interest differentials on exchange rate depreciation

Table 2: Posterior estimates of parameters

	Baseline		Determinacy	
Parameter	Mean	90% interval	Mean	90% interval
ϕ_s	3.930	[3.078, 4.869]	-3.666	[-4.723, -2.643]
ϕ_a	0.015	[0.006, 0.023]	0.084	[0.060, 0.108]
b	0.307	[0.240, 0.371]	0.321	[0.249, 0.400]
η	1.503	[0.969, 2.065]	1.486	[0.904, 2.026]
ϕ	11.888	[7.415, 16.583]	13.858	[7.083, 19.930]
ω	0.198	[0.082, 0.304]	0.178	[0.080, 0.274]
λ	0.812	[0.791, 0.829]	0.815	[0.790, 0.841]
ρ	0.878	[0.855, 0.904]	0.893	[0.864, 0.921]
α_π	2.133	[1.696, 2.503]	2.017	[1.545, 2.428]
α_y	0.438	[0.062, 0.765]	0.375	[0.036, 0.710]
α_s	0.157	[0.039, 0.257]	0.110	[0.018, 0.189]
$\overline{\gamma}$	0.274	[0.210, 0.331]	0.266	[0.213, 0.319]
$\overline{\pi}$	0.546	[0.477, 0.620]	0.575	[0.503, 0.654]
$\overline{i}$	1.186	[1.099, 1.268]	1.175	[1.082, 1.260]
ρ_ψ	0.538	[0.221, 0.892]	0.860	[0.797, 0.922]
ρ_z	0.981	[0.965, 0.998]	0.970	[0.947, 0.993]
ρ_d	0.911	[0.881, 0.941]	0.903	[0.878, 0.930]
ρ_{y^*}	0.854	[0.780, 0.924]	0.849	[0.782, 0.914]
ρ_{π^*}	0.576	[0.465, 0.667]	0.532	[0.423, 0.642]
ρ_{i^*}	0.959	[0.940, 0.981]	0.972	[0.954, 0.989]
σ_ψ	0.355	[0.201, 0.508]	1.477	[1.006, 1.960]
σ_z	1.029	[0.887, 1.154]	1.044	[0.898, 1.202]
σ_d	2.986	[2.402, 3.509]	3.190	[2.604, 3.781]
σ_u	0.228	[0.191, 0.259]	0.211	[0.177, 0.243]
σ_{y^*}	0.504	[0.447, 0.563]	0.484	[0.431, 0.536]
σ_{π^*}	0.196	[0.174, 0.216]	0.195	[0.173, 0.217]
σ_{i^*}	0.126	[0.112, 0.140]	0.123	[0.110, 0.137]
σ_ζ	0.150	[0.085, 0.221]	-	-
M_ψ	0.832	[-0.677, 2.464]	-	-
M_z	-0.697	[-1.274, -0.109]	-	-
M_d	-0.719	[-0.980, -0.461]	-	-
M_u	2.695	[1.176, 4.128]	-	-
M_{y^*}	0.709	[0.031, 1.491]	-	-
M_{π^*}	0.492	[-0.910, 1.926]	-	-
M_{i^*}	0.167	[-1.294, 1.913]	-	-
$\log p(\mathbf{Y}^T)$	-816.099		-945.929	
$\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D \mid \mathbf{Y}^T\}$	0.000		1.000	

Notes: This table reports the posterior mean and 90% highest posterior density intervals based on $10,000$ particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathbf{Y}^T)$ represents the SMC-based approximation of log marginal data density and $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D|\mathbf{Y}^T\}$ denotes the posterior probability of equilibrium determinacy.

has increased substantially in the baseline model by allowing for the negative slope coefficient in the modified UIP condition. If the model is estimated only under determinacy, the slope coefficient is restricted to be positive, at odds with the data, giving rise to very persistent deviation from the UIP relationship.

Finally, regarding the indeterminacy-related parameters, some of the components (M_z, M_d, M_u, and M_{y^*}) in the arbitrary matrix $\mathbf{M}$ are substantially different from zero in that the 90% posterior intervals do not include zero. This finding suggests the importance of considering the multiplicity of the equilibrium representation under indeterminacy. As shown in the following subsection, these estimates considerably alter the impulse response functions under indeterminacy.

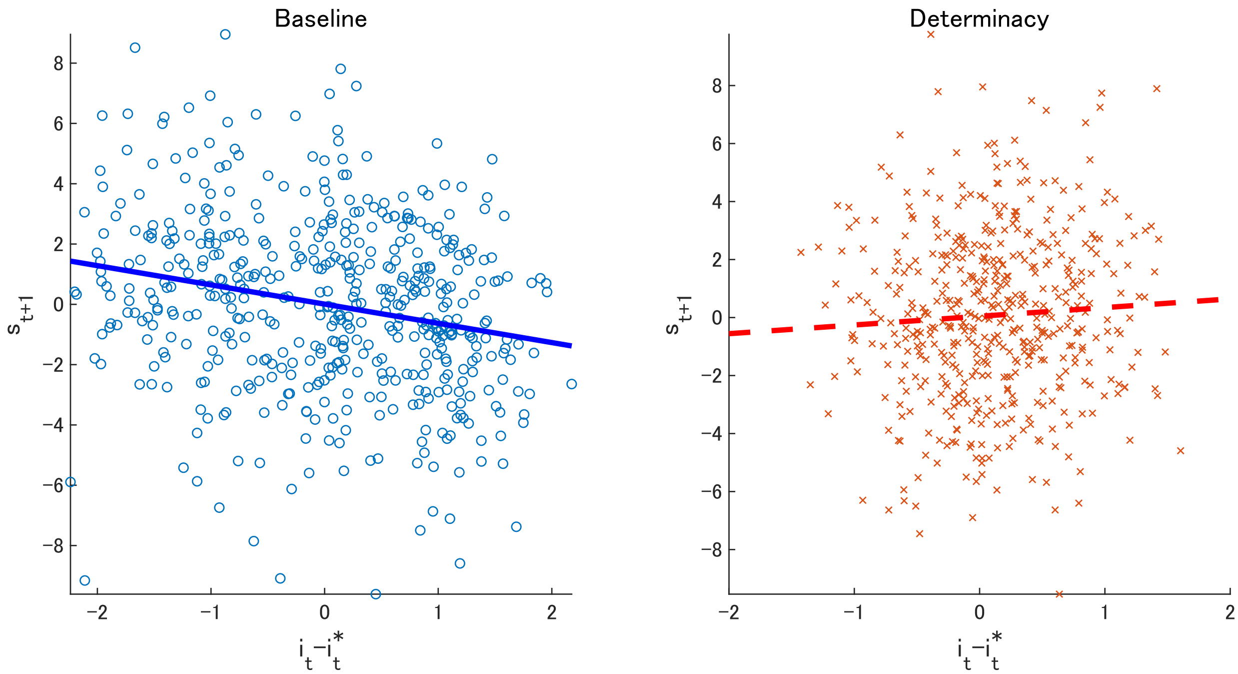
To examine whether our model can replicate a negative relationship between the expected exchange rate depreciation and the interest rate differentials, we conduct UIP regressions using simulated data. Figure 2 depicts the scatter plots and regression lines of exchange rate depreciation $\hat{s}_{t+1}$ and the interest rate differential $\hat{i}_t - \hat{i}_t^*$ simulated by the baseline model and its counterpart estimated only under determinacy.[18] This figure illustrates that the baseline model generates a negative correlation between the two, unlike its determinacy counterpart. Thus, our baseline model can replicate a negative coefficient in the UIP regression, as reported in numerous studies using a variety of international data.

5.2 Impulse response functions

Figure 3 depicts the impulse responses of output growth, inflation, interest rate, and exchange rate depreciation in terms of percentage deviations from the steady-state values, to a one-standard-deviation shock to the modified UIP condition, technology, preference, monetary policy, US output, US inflation, US interest rate, and sunspot, given the posterior mean estimates of the parameters in the baseline model allowing for indeterminacy (solid lines) and its counterpart estimated only in the determinacy region of the parameter space (dashed lines). To examine how and to what extent the estimated arbitrary matrix $\mathbf{M}$ alter the propagation of shocks, the figure also presents the responses in the baseline model with $\mathbf{M} = 0$ (dotted lines), with the other parameters fixed at the same values as the baseline

[18] Given the posterior mean estimates of the parameters, each model is simulated for 250 periods, and the first 50 observations are discarded.

Figure 2: UIP regressions based on simulated data driven by all shocks



Notes: This figure shows the UIP regressions based on the simulated data of the exchange rate depreciation and the nominal interest rate differentials driven by all shocks in the baseline model and in its counterpart estimated only under determinacy.

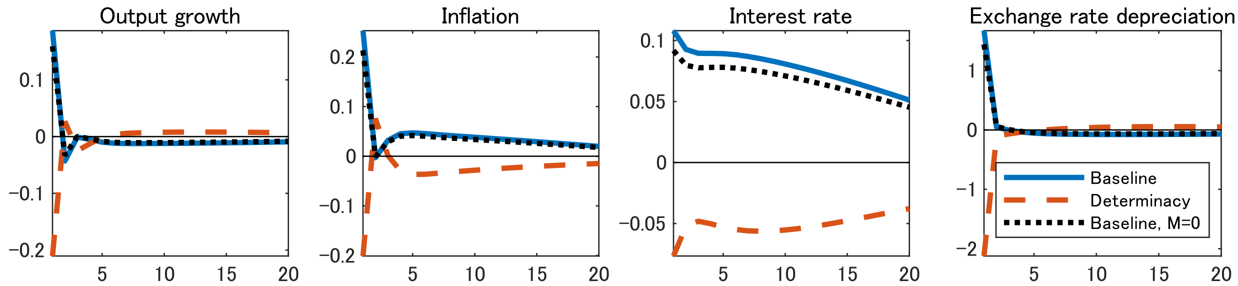
estimation.

For all shocks, most of the impulse responses differ remarkably between the baseline and determinacy cases, although our solution under indeterminacy is centered at the continuity solution as addressed in Section 4.1. In particular, the exchange rate responds in the opposite direction in response to the UIP, monetary policy, and US interest rate shocks. Also, regarding the shocks to technology, preference, monetary policy, US output, and US inflation, the responses are substantially altered by the estimated components in the arbitrary matrix $\mathbf{M}$, which cause the indeterminacy solution to deviate from the continuity solution.

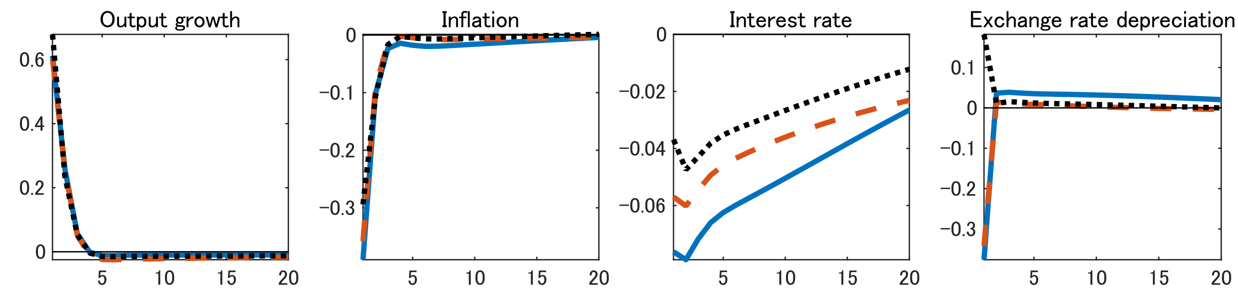
In comparison with the empirical findings in the literature on the Canadian economy (Grilli and Roubini, 1996; Kim and Roubini, 2000), our baseline model can generate the reasonable response of the exchange rate to the monetary policy shock, as shown in panel (d) of Figure 3. While the exchange rate reacts excessively to the monetary policy shock in both the determinate model (dashed line) and the baseline model with $\mathbf{M} = 0$ (dotted line), the baseline model (solid line) exhibits a mild response. Moreover, the baseline model

Figure 3: Impulse response functions

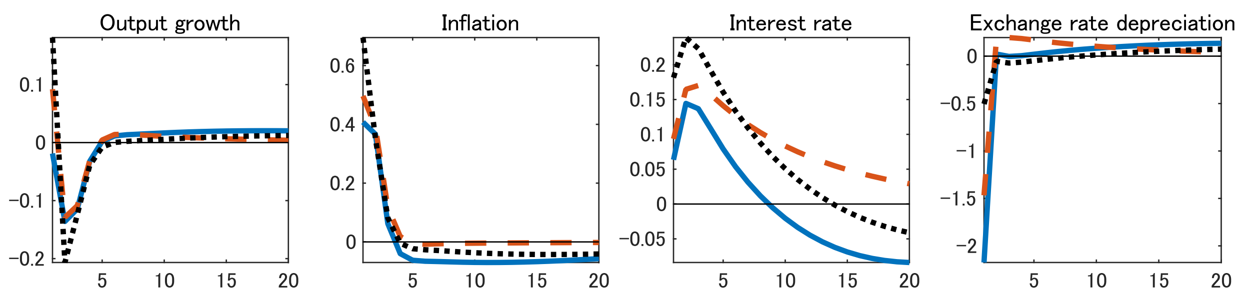
(a) UIP shock



(b) Technology shock



(c) Preference shock



(d) Monetary policy shock

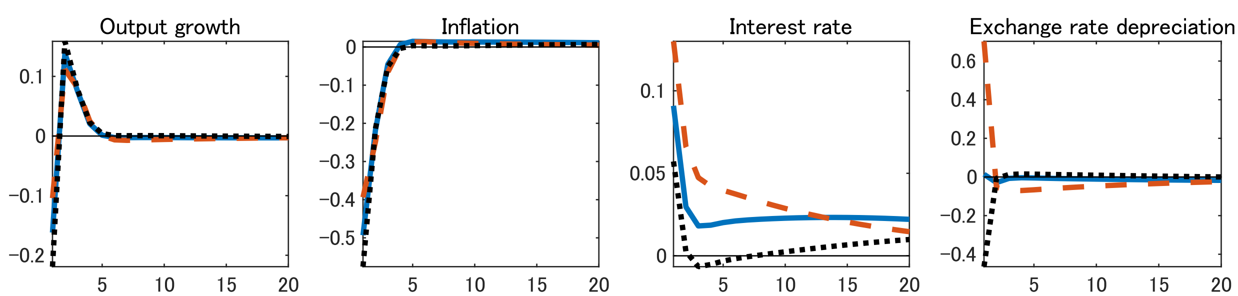
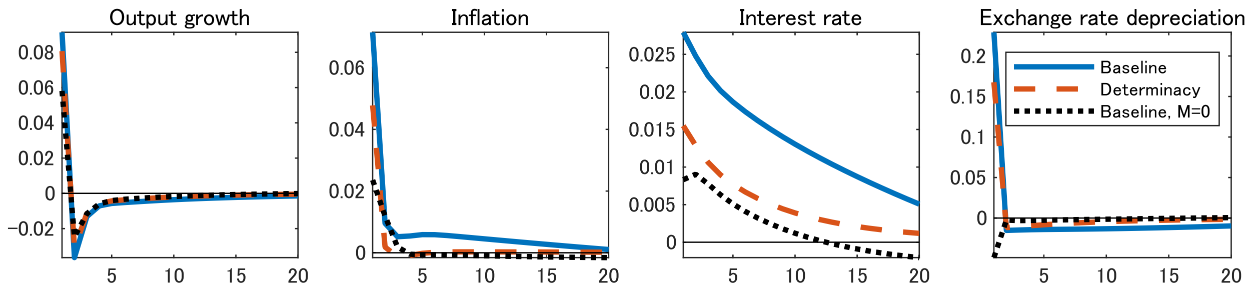
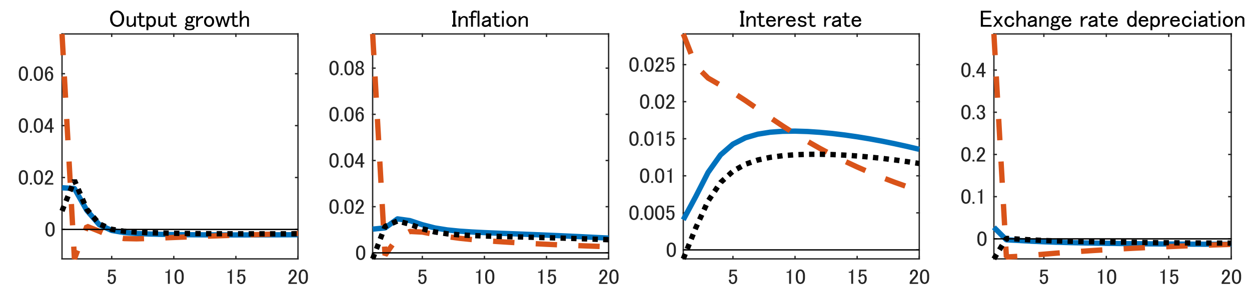


Figure 3: Impulse response functions (continued)

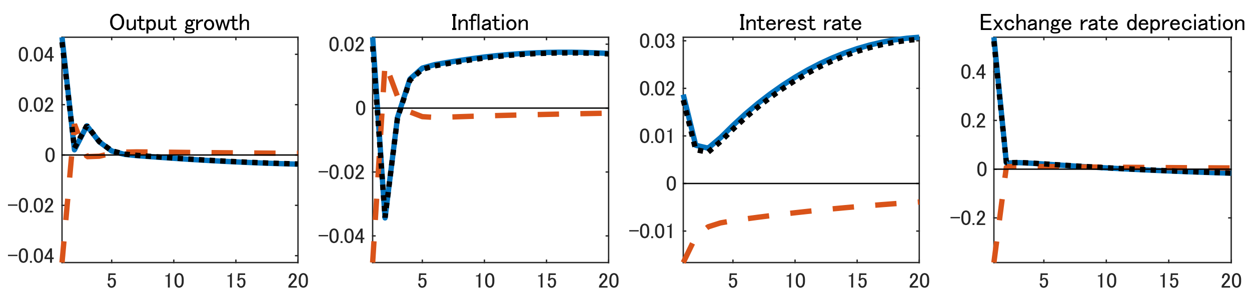
(e) US output shock



(f) US inflation shock

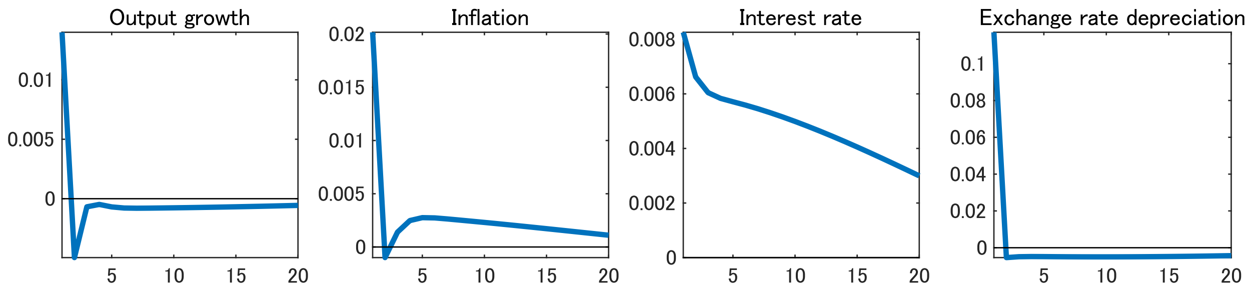


(g) US interest rate shock



Notes: This figure depicts the impulse responses of output growth, inflation, interest rate, and exchange rate depreciation in terms of percentage deviations from steady-state values to a one-standard-deviation shock to the modified UIP condition, technology, preference, monetary policy, US output, US inflation, and US interest rate, given the posterior mean estimates of parameters in the baseline model, in its counterpart estimated only under determinacy, and in the baseline model with $\mathbf{M} = 0$.

Figure 4: Impulse responses to sunspot shock



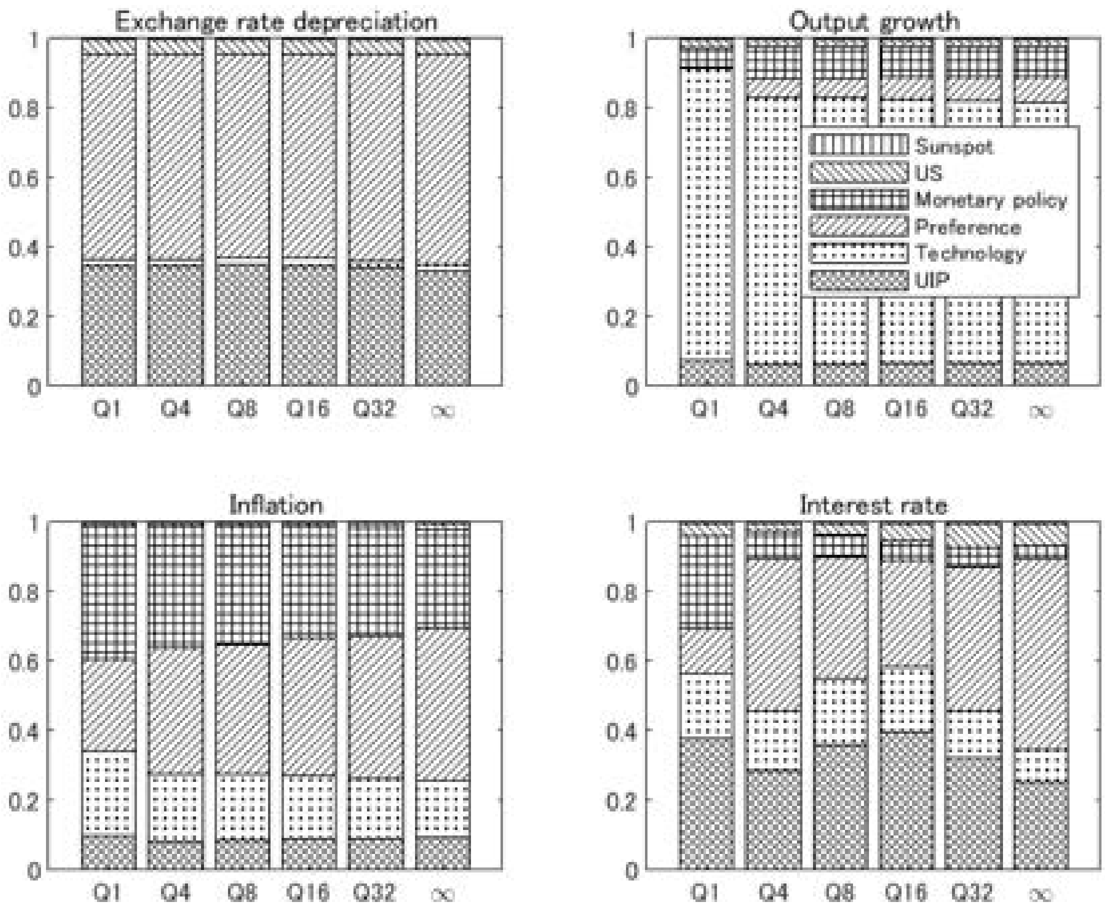
Notes: This figure depicts the impulse responses of output growth, inflation, interest rate, and exchange rate depreciation in terms of percentage deviations from steady-state values to a one-standard-deviation sunspot shock, given the posterior mean estimates of parameters in the baseline model.

can replicate the delayed overshooting: a positive monetary policy shock causes exchange depreciation with lags. These dynamic properties of the exchange rate result from the estimated components in the arbitrary matrix $\mathbf{M}$, which work as an equilibrium selection device under indeterminacy.

As shown in Figure 4, the sunspot shock affects the equilibrium dynamics only in the baseline model, which exhibits equilibrium indeterminacy. Upon impact, the identified sunspot shock has positive effects on all observables. The sunspot shock in the present model is constructed as a reduced-form sunspot shock following Lubik and Schorfheide (2004) and hence has positive effects on all expectational variables, which are irrelevant to the fundamentals. Such nonfundamental beliefs are self-fulfilling under indeterminacy and have expansionary effects on their realizations. The rise in the interest rate, however, dampens these effects in subsequent periods.

5.3 Variance decompositions

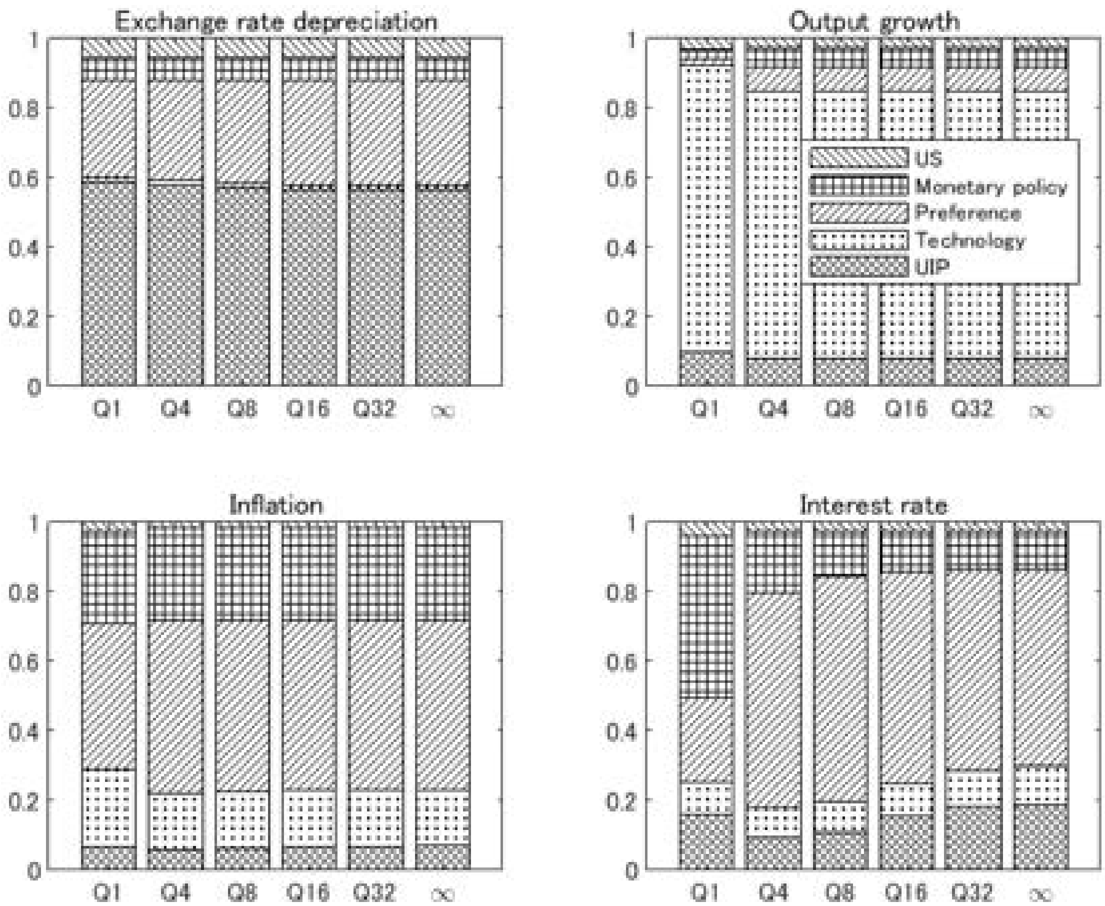
To assess the relative contribution of each shock to aggregate fluctuations, we conduct variance decompositions. In particular, we focus primarily on the sources of fluctuations in the nominal exchange rate. The literature has documented that estimations of open-economy DSGE models typically find that fluctuations in nominal exchange rates relate little to macroeconomic fundamentals and are mostly attributable to a direct shock to the exchange rate such as the UIP shock specified in the standard UIP condition (3). Such an



Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of the parameters in the baseline model. "US" denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

established view might be overturned if the model incorporates the modified UIP equation that replicates the empirical regularities between exchange rate depreciation and the interest rate differentials and allows for sunspot fluctuations, as considered in this paper.

Figure 5 shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and the interest rate at various forecast horizons: 1, 4, 8, 16, and 32 quarters, and infinity, given the posterior mean estimates of the parameters in the baseline model. For comparison, Figure 6 presents the same decompositions based on the model estimated only in the determinacy region of the parameter space.



Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of the parameters in the model estimated only under determinacy. "US" denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

The upper-left panel in Figure 6 illustrates that the UIP shock is the main driving force of exchange rate fluctuations in the model estimated only under determinacy, as is consistent with the findings in Lubik and Schorfheide (2006), Hirose (2013), and Chen et al. (2021). By contrast, the same panel in Figure 5 uncovers that the preference shock, rather than the UIP shock, plays the primary role in explaining the exchange rate dynamics for all forecast horizons. Thus, the conventional wisdom in the literature is overturned, establishing that the exchange rate is "connected" to the fundamentals.

In our approach, the UIP shock is identified as residuals in the modified UIP condition.

The residuals become small by allowing for the negative relationship between the exchange rate depreciation and interest rate differentials in the modified UIP equation. Indeed, as addressed in Section 5.1, both the UIP shock's persistence parameter ρ_ψ and standard deviation σ_ψ becomes small in the baseline estimation, and thereby the contribution of the UIP shock decreases substantially.

It is straightforward why the preference shock can be a major source of exchange rate fluctuations. In the model, the modified UIP condition can be written in terms of the marginal utility of consumption:

$$S_t \exp(d_t) \left(C_t - bC_{t-1}\right)^{-\sigma}$$
$$= \beta \mathbb{E}_t \left\{ S_{t+1} \exp\left[-\phi_a \left(\frac{S_t A_t^*}{P_t Z_t} - \bar{a}^* \right) - \phi_s \left(\frac{S_{t+1}}{S_t} - \frac{\bar{\pi}}{\pi^*} \right) + \psi_t \right] i_t^* \frac{\exp(d_{t+1}) \left(C_{t+1} - bC_t\right)^{-\sigma}}{\pi_{t+1}} \right\},$$

wherein the preference shocks, d_t and d_{t+1}, can directly affect the exchange rate.

Contrary to our initial conjecture, the contribution of the sunspot shock is invisible for all observables including the exchange rate. This is because the sunspot shock can affect economic fluctuations through nonfundamental revisions in expectations, so that its pass-through to the aggregate variables is limited when compared with the direct shocks to the equilibrium conditions. Besides, the sunspot shock has no persistency, *i.e.*, i.i.d. by its construction. The contribution of such an i.i.d. shock is, *ceteris paribus*, smaller than that of persistent fundamental shocks which follow AR(1) processes.

The decompositions of output growth, inflation, and the nominal interest rate are not very different between the baseline model and its determinacy counterpart. Output fluctuations are mainly explained by the technology shock, in line with the prevailing views in the business cycle literature. On inflation and the interest rate, we find notable contributions of the preference and monetary policy shocks. For all these observables, the relative contribution of the foreign (US) shocks is marginal, as shown by Justiniano and Preston (2010).

6 Robustness Analysis

In this section, we investigate whether the results obtained from our baseline estimation are robust to alternative solution methods under indeterminacy and subsamples before and after the global financial crisis.

6.1 Alternative solutions under indeterminacy

6.1.1 Belief shock specification

In the baseline estimation, we follow the approach of Lubik and Schorfheide (2004) to derive the full set of solutions for the linear rational expectations system under indeterminacy, in which we construct a reduced-form sunspot shock in that it contains beliefs associated with all the expectational variables. Lubik and Schorfheide (2003) propose another approach for constructing a sunspot shock called "a belief shock." In this approach, sunspots trigger a belief shock ζ_t^b that leads to a revision of the forecast of a specific expectational variable, say, $\mathbb{E}_t x_{t+1}$. Then, the definition of the rational expectations forecast error gives

$$x_t = (\mathbb{E}_{t-1} x_t + \zeta_t^b) + \tilde{\eta}_t^x,$$

where $\mathbb{E}_{t-1} x_t + \zeta_t^b$ is the revised forecast and $\tilde{\eta}_t^x$ is the error of this revised forecast. Lubik and Schorfheide (2003) show that such a belief shock affects equilibrium dynamics under indeterminacy and works like a sunspot shock. In what follows, we replace the reduced-form sunspot shock in the baseline estimation with a belief shock to the forecast of exchange rate depreciation:

$$\hat{s}_t = (\mathbb{E}_{t-1} \hat{s}_t + \zeta_t^b) + \tilde{\eta}_t^s,$$

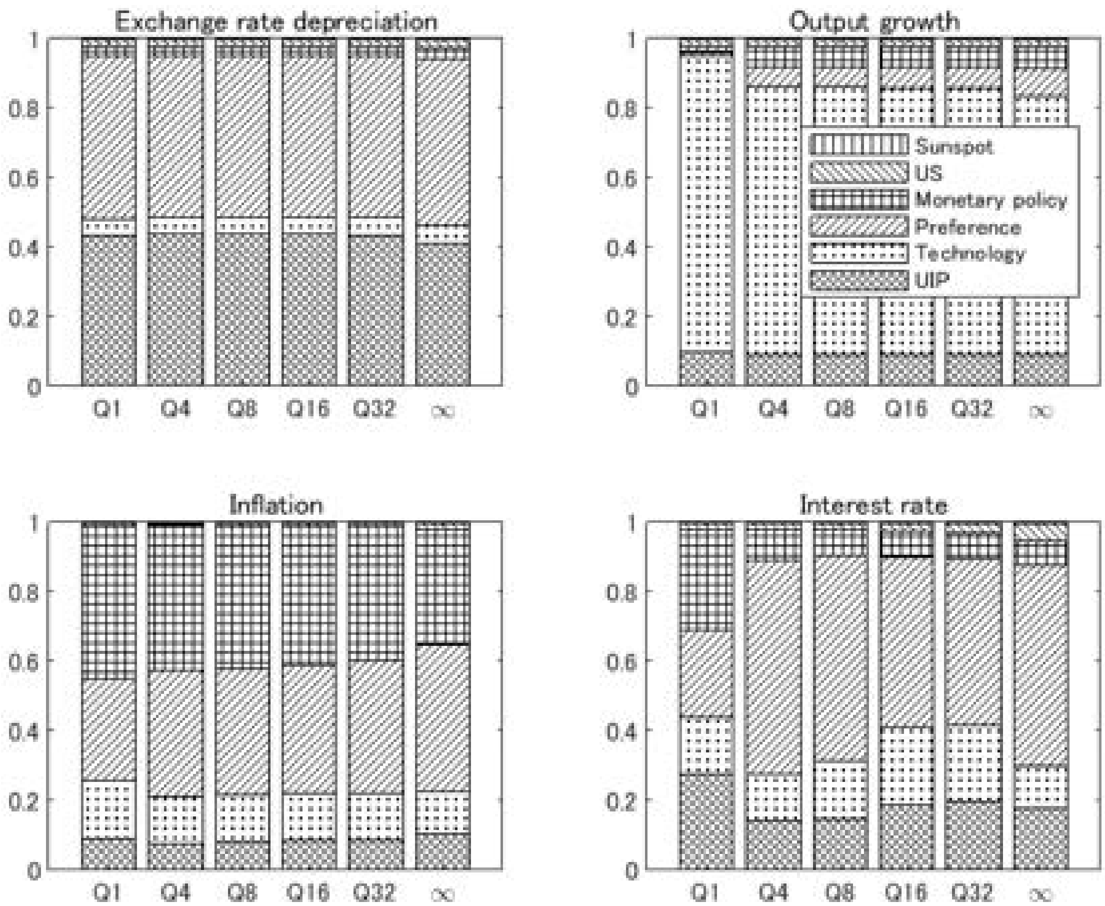
and estimate the model with this belief-shock specification. We assume $\zeta_t^b \sim$ i.i.d. $N(0, \sigma_\zeta^2)$ and set the same inverse gamma prior for its standard deviation σ_ζ as in the baseline estimation.

The left part of Table 3 presents the estimation results of the model with the belief-shock specification. No remarkable differences are found in the parameter estimates, although the estimate of ϕ_s (spread parameter associated with exchange rate depreciation in the modified UIP condition) becomes somewhat larger than that in the baseline specification. In terms of the model fit, the marginal data density (−892.9) deteriorates, compared with the baseline estimation (−816.1, see Table 2). Therefore, the belief shock specification does not improve the empirical performance of the model.

Table 3: Posterior estimates of parameters with alternative solutions under indeterminacy

	Belief shock		$\mathbf{M}=0$	
Parameter	Mean	90% interval	Mean	90% interval
ϕ_s	5.341	[4.026, 6.805]	3.906	[3.233, 4.532]
ϕ_a	0.009	[0.004, 0.013]	0.011	[0.006, 0.016]
b	0.272	[0.200, 0.342]	0.273	[0.211, 0.335]
η	1.307	[0.782, 1.871]	2.188	[1.518, 2.842]
ϕ	8.369	[4.529, 11.562]	10.957	[7.400, 14.874]
ω	0.213	[0.085, 0.331]	0.187	[0.089, 0.284]
λ	0.817	[0.794, 0.841]	0.808	[0.786, 0.833]
ρ	0.871	[0.839, 0.904]	0.875	[0.851, 0.899]
α_π	2.301	[1.830, 2.762]	2.276	[1.815, 2.701]
α_y	0.300	[0.017, 0.579]	0.434	[0.119, 0.730]
α_s	0.098	[0.006, 0.196]	0.273	[0.179, 0.365]
$\overline{\gamma}$	0.217	[0.153, 0.275]	0.271	[0.223, 0.318]
$\overline{\pi}$	0.559	[0.481, 0.644]	0.569	[0.494, 0.638]
$\overline{i}$	1.164	[1.070, 1.250]	1.188	[1.103, 1.264]
ρ_ψ	0.598	[0.341, 0.873]	0.968	[0.936, 0.998]
ρ_z	0.982	[0.967, 0.996]	0.972	[0.951, 0.994]
ρ_d	0.891	[0.850, 0.930]	0.933	[0.900, 0.963]
ρ_{y^*}	0.830	[0.755, 0.895]	0.868	[0.808, 0.926]
ρ_{π^*}	0.585	[0.483, 0.698]	0.584	[0.478, 0.678]
ρ_{i^*}	0.963	[0.942, 0.984]	0.961	[0.941, 0.979]
σ_ψ	0.467	[0.343, 0.585]	0.659	[0.495, 0.827]
σ_z	0.982	[0.864, 1.118]	0.931	[0.817, 1.040]
σ_d	2.079	[1.655, 2.485]	2.364	[1.806, 2.922]
σ_u	0.250	[0.203, 0.294]	0.239	[0.205, 0.268]
σ_{y^*}	0.475	[0.427, 0.525]	0.485	[0.434, 0.533]
σ_{π^*}	0.195	[0.169, 0.217]	0.189	[0.170, 0.209]
σ_{i^*}	0.124	[0.109, 0.138]	0.124	[0.111, 0.136]
σ_ζ	0.154	[0.079, 0.224]	0.205	[0.081, 0.330]
M_ψ	-4.467	[-5.665, -3.083]	-	-
M_z	-0.935	[-1.465, -0.433]	-	-
M_d	-1.130	[-1.441, -0.813]	-	-
M_u	2.692	[1.223, 4.151]	-	-
M_{y^*}	0.870	[-0.021, 1.738]	-	-
M_{π^*}	0.465	[-0.978, 2.075]	-	-
M_{i^*}	1.382	[-0.568, 3.191]	-	-
$\log p(\mathbf{Y}^T)$	-892.933		-824.717	
$\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D \vert \mathbf{Y}^T\}$	0.000		0.000	

Notes: This table reports the posterior mean and 90% highest posterior density intervals based on $10,000$ particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathbf{Y}^T)$ represents the SMC-based approximation of log marginal data density and $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D|\mathbf{Y}^T\}$ denotes the posterior probability of equilibrium determinacy.



Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of the parameters in the model with a belief shock. "US" denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

Figure 7 shows the variance decompositions based on the posterior mean estimates of parameters in the model under the belief shock specification. The contribution of the sunspot shock to exchange rate fluctuations remains invisible even though we assume that the belief shock directly affects the revision of the forecast of the exchange rate. As a consequence, the results are very similar to those in the baseline estimation.

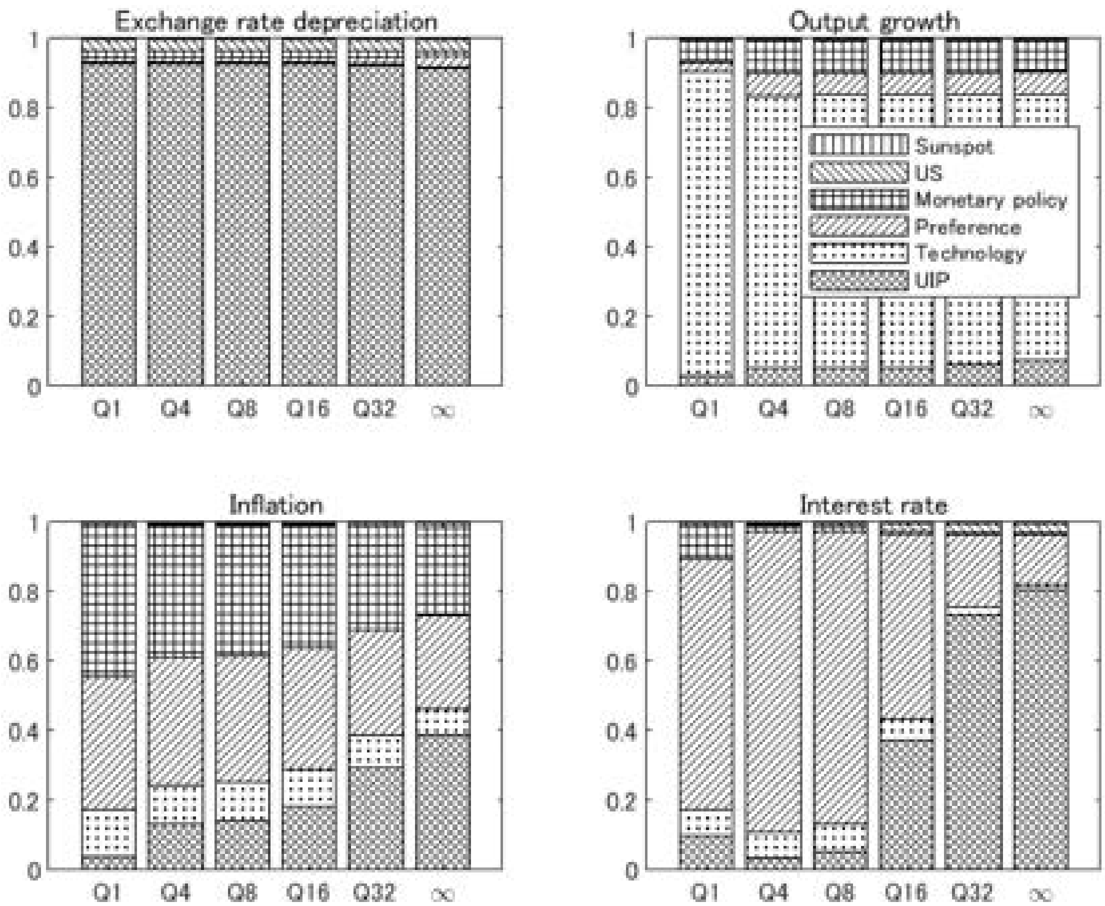
6.1.2 $\mathbf{M} = 0$

An intrinsic feature of the full set of linear rational expectations solution given by (9) is that the arbitrary matrix $\mathbf{M}$ appears in the solution under indeterminacy, and $\mathbf{M}$ consists of purely free parameters. One might argue that the introduction of such free parameters has improved the fit of the model dramatically and that the remarkable increase in the marginal data density in the baseline model, reported in Section 5.1, is attributable to these free parameters. To investigate this point, we estimate the baseline model with all the components of $\mathbf{M}$ fixed at zero.

The last two columns of Table 3 show the estimation results when $\mathbf{M} = 0$. The marginal data density (-824.7) is lower than that in the baseline estimation (-816.1, see Table 2), but the difference is very marginal. Thus, the existence of the free parameters in $\mathbf{M}$ plays a minor role in the improved fit of the model.

Because most of the parameter estimates, including the spread parameter ϕ_s, are in line with those in the baseline estimation, the modified UIP equation (2) still exhibits a negative relationship between the expected exchange rate depreciation and interest rate differentials, leading all the posterior draws into the indeterminacy region ($\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D | \mathbf{Y}^T\} = 0$). However, both the UIP shock's AR(1) parameter ρ_ψ and standard deviation σ_ψ becomes larger than in the baseline estimation, which causes the distinct result for the variance decomposition shown in Figure 8.

Figure 8 presents the variance decompositions based on the posterior mean estimates of parameters in the model with $\mathbf{M} = 0$. In contrast to the baseline result, the direct shock to the modified UIP condition explains almost all the exchange rate volatility, in line with the findings in previous studies. Under this specific equilibrium representation ($\mathbf{M} = 0$), the size and persistence of the UIP shock are identified much larger, enhancing its contribution to exchange rate fluctuations. These changes are due to the restriction on $\mathbf{M}$ that alters the initial impact of the fundamental shocks as shown in Section 5.2 (Figure 3). Therefore, the selection of a specific equilibrium representation under indeterminacy, *i.e.*, the estimation of arbitrary parameters in $\mathbf{M}$, is of crucial importance for obtaining our main result that the exchange rate is connected to the fundamentals.



Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of the parameters in the model with $\mathbf{M} = 0$. "US" denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

6.2 Subsample analysis

In contrast to earlier findings, Bussière et al. (2022) argue that the coefficient on the interest rate differential in the UIP regression has become large and positive for several currencies, including the Canadian dollar during and in the decade after the global financial crisis, which they term "the new Fama puzzle." To investigate whether their argument based on a single-equation estimation approach using monthly data carries over to our system estimation approach allowing for indeterminacy using quarterly data, we estimate the model for two subsamples: before and after the global financial crisis. More specifically, we split the full sample used in the baseline estimation into 1984:Q1–2007:Q2 and 2007:Q3–2019:Q4 samples.[19] The prior means for the steady-state (quarterly) rates of GDP growth $\overline{\gamma}$, inflation $\overline{\pi}$, and nominal interest $\overline{i}$ are set at the averages of the corresponding data for each subsample: 0.385, 0.653, and 1.634 for the former; and 0.111, 0.377, and 0.367 for the latter, respectively. The remaining priors are the same as in the baseline estimation (see Table 1).

Table 4 presents the parameter estimates for the two subsamples. The posterior estimates of ϕ_s (spread parameter associated with exchange rate depreciation in the modified UIP condition) are very close to each other for the two subsamples and are both much larger than one. Thus, the slope coefficient in the modified UIP equation (2) remains negative for both subsamples, leading the model to exhibit equilibrium indeterminacy. Consequently, the posterior probabilities of determinacy $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D | \mathbf{Y}^T\}$ shown in the last row are both zero. Therefore, there is no evidence concerning the new Fama puzzle from our system estimation of the fully specified structural model for the Canadian economy.

While the other structural parameters for the household, firms, and central bank are not substantially different, several shock-related parameters vary across the two subsamples. In particular, from the precrisis period to the postcrisis period, the posterior estimate of the standard deviation of the UIP shock σ_ψ decreases, and that of the preference shock σ_d increases, while their persistence parameters (ρ_ψ and ρ_d) change little. The differences in these shock parameters produce different results in the variance decompositions of the exchange rate, as shown in Figures 9 and 10. Compared with the baseline result for the full sample, the contribution of the preference shock to exchange rate fluctuations decreases in the first subsample and increases thereafter. Thus, our main result on the *exchange rate connect* is more pronounced after the global financial crisis.

7 Conclusion

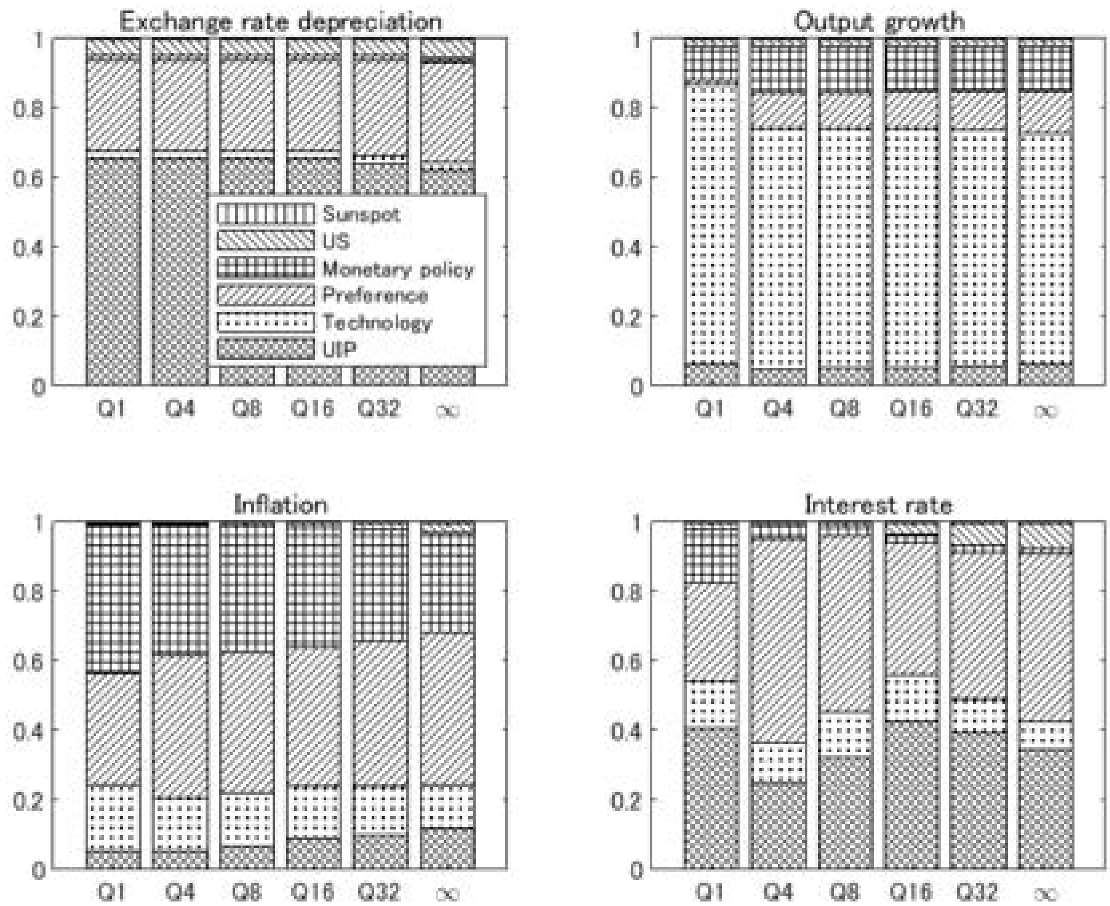
Using data for Canada and the US, we have estimated a small open-economy model with an endogenous interest rate spread on foreign bond holdings so that the modified UIP condition can replicate a negative relationship between expected exchange rate depreciation and the

[19] Bussière et al. (2022) separate the monthly data into three subsamples: 1999:M01–2006:M04, 2006:M05–2018:M04, and 2018:M05–2021:M09. If we considered the same subsamples for our quarterly data, the first and third subsamples would be too short to estimate the model.

Table 4: Posterior estimates of parameters in two subsamples

	Precrisis period		Postcrisis period	
Parameter	Mean	90% interval	Mean	90% interval
ϕ_s	3.995	[2.957, 5.033]	4.157	[3.087, 5.191]
ϕ_a	0.009	[0.005, 0.014]	0.007	[0.003, 0.010]
b	0.272	[0.214, 0.340]	0.311	[0.232, 0.396]
η	1.508	[0.968, 2.023]	1.954	[1.337, 2.558]
ϕ	13.336	[8.236, 17.880]	13.675	[8.194, 18.832]
ω	0.252	[0.130, 0.376]	0.234	[0.109, 0.356]
λ	0.781	[0.756, 0.805]	0.780	[0.756, 0.804]
ρ	0.840	[0.805, 0.876]	0.950	[0.934, 0.965]
α_π	2.498	[2.023, 2.978]	2.090	[1.530, 2.734]
α_y	0.545	[0.127, 0.947]	0.586	[0.130, 0.995]
α_s	0.238	[0.080, 0.374]	0.167	[0.025, 0.283]
$\overline{\gamma}$	0.364	[0.301, 0.424]	0.107	[0.056, 0.162]
$\overline{\pi}$	0.647	[0.575, 0.722]	0.415	[0.335, 0.495]
$\overline{i}$	1.630	[1.541, 1.718]	0.387	[0.302, 0.470]
ρ_ψ	0.500	[0.183, 0.864]	0.545	[0.238, 0.869]
ρ_z	0.978	[0.957, 0.999]	0.796	[0.652, 0.939]
ρ_d	0.909	[0.872, 0.946]	0.889	[0.843, 0.940]
ρ_{y^*}	0.874	[0.805, 0.943]	0.790	[0.674, 0.903]
ρ_{π^*}	0.646	[0.530, 0.750]	0.368	[0.167, 0.550]
ρ_{i^*}	0.925	[0.890, 0.962]	0.934	[0.891, 0.982]
σ_ψ	0.571	[0.317, 0.843]	0.183	[0.087, 0.277]
σ_z	0.886	[0.772, 0.992]	1.112	[0.920, 1.303]
σ_d	2.319	[1.708, 2.889]	3.801	[2.991, 4.566]
σ_u	0.284	[0.232, 0.337]	0.109	[0.081, 0.135]
σ_{y^*}	0.466	[0.411, 0.530]	0.526	[0.442, 0.604]
σ_{π^*}	0.156	[0.136, 0.175]	0.230	[0.187, 0.274]
σ_{i^*}	0.141	[0.123, 0.159]	0.104	[0.085, 0.122]
σ_ζ	0.175	[0.085, 0.267]	0.218	[0.083, 0.354]
M_ψ	0.565	[-1.045, 2.075]	0.210	[-1.539, 2.048]
M_z	-1.035	[-1.813, -0.148]	0.220	[-0.312, 0.819]
M_d	-0.635	[-1.169, -0.184]	-0.941	[-1.467, -0.460]
M_u	1.110	[-0.564, 2.739]	-0.400	[-2.088, 1.226]
M_{y^*}	0.629	[-0.459, 1.645]	0.531	[-0.494, 1.473]
M_{π^*}	0.578	[-0.943, 2.161]	-0.123	[-1.521, 1.271]
M_{i^*}	0.260	[-1.530, 1.848]	-0.317	[-1.805, 1.399]
$\log p(\mathbf{Y}^T)$	-502.222		-350.752	
$\mathbb{P}\{\boldsymbol{\theta} \in \mathbf{\Theta}^D \mid \mathbf{Y}^T\}$	0.000		0.000	

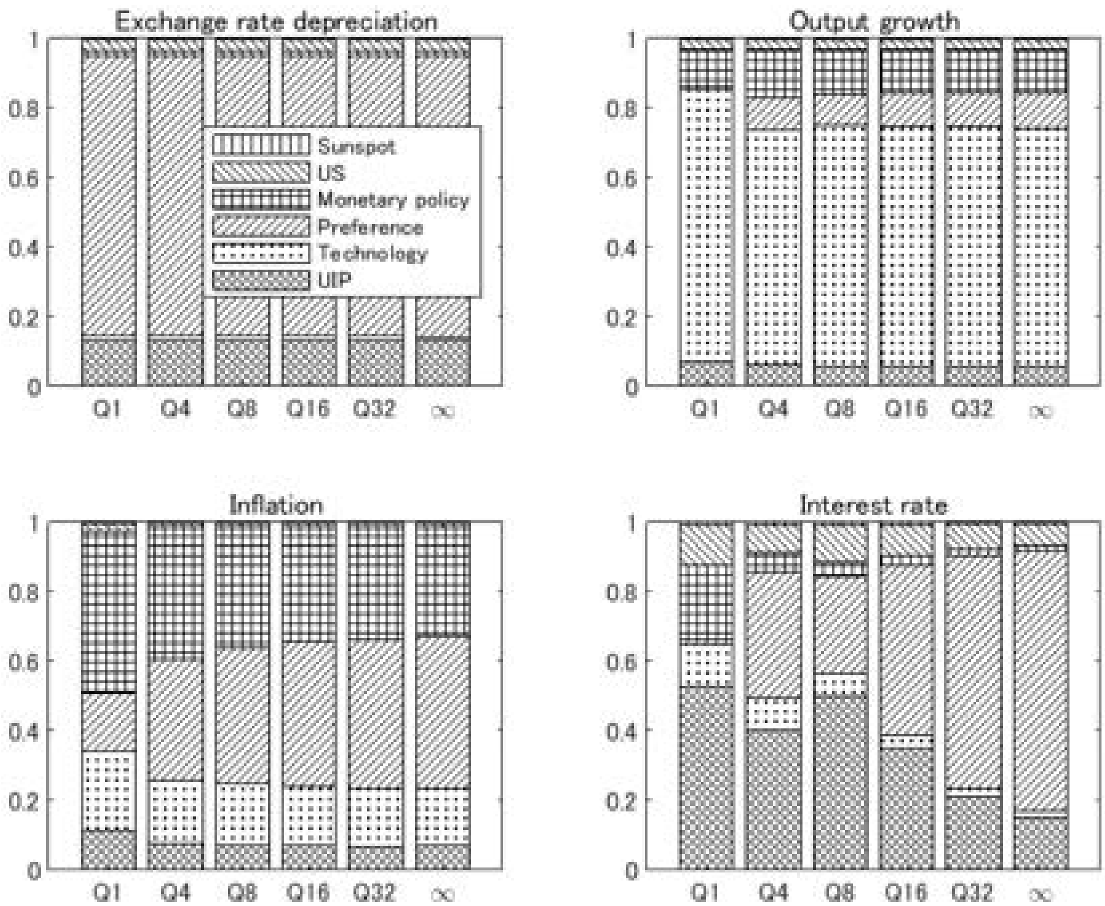
Notes: This table reports the posterior mean and 90% highest posterior density intervals based on $10,000$ particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathbf{Y}^T)$ represents the SMC-based approximation of log marginal data density and $\mathbb{P}\{\boldsymbol{\theta} \in \mathbf{\Theta}^D|\mathbf{Y}^T\}$ denotes the posterior probability of equilibrium determinacy.



Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of parameters in the model for the precrisis period. "US" denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

interest rate differentials, as observed in the actual data. Because the negative coefficient in the modified UIP condition is likely to generate equilibrium indeterminacy, we estimate the model using Bayesian methods allowing for both the determinacy and indeterminacy of equilibrium.

According to the estimation results, the data strongly favor indeterminacy over determinacy, and hence the modified UIP condition exhibits the observed negative correlation between the expected exchange rate depreciation and the interest rate differentials. The propagation of shocks differs markedly between determinacy and indeterminacy, as a specific



Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of parameters in the model for the postcrisis period. "US" denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

equilibrium is selected from multiple equilibria under indeterminacy. Forecast error variance decompositions based on the estimated model show that the preference shock, rather than the UIP shock, is the main driving force of the exchange rate dynamics, establishing the novel finding that the exchange rate is indeed connected to the macroeconomic fundamentals.

Appendix

A Equilibrium Conditions

The equilibrium conditions of the model are given by the following equations:

$$\exp(d_t)\left(C_t - bC_{t-1}\right)^{-\sigma} = \beta\mathbb{E}_t\left[i_t \frac{\exp(d_{t+1})\left(C_{t+1} - bC_t\right)^{-\sigma}}{\pi_{t+1}}\right],$$

$$Z_t^{1-\sigma} h_t^{\eta} = \left(C_t - bC_{t-1}\right)^{-\sigma} \frac{W_t}{P_t},$$

$$S_t \exp(d_t)\left(C_t - bC_{t-1}\right)^{-\sigma}$$
$$= \beta\mathbb{E}_t\left\{S_{t+1}\exp\left[-\phi_a\left(\frac{S_t A_t^*}{P_t Z_t} - \bar{a}^*\right) - \phi_s\left(\frac{S_{t+1}}{S_t} - \frac{\bar{\pi}}{\pi^*}\right) + \psi_t\right] i_t^* \frac{\exp(d_{t+1})\left(C_{t+1} - bC_t\right)^{-\sigma}}{\pi_{t+1}}\right\},$$

$$(1-\epsilon) + \epsilon\frac{W_t}{\exp(z_t) Z_t P_{H,t}} - \phi\left(\frac{\pi_{H,t}}{\pi_{H,t-1}^{\omega}\bar{\pi}^{1-\omega}} - 1\right)\frac{\pi_{H,t}}{\pi_{H,t-1}^{\omega}\bar{\pi}^{1-\omega}}$$
$$= -\beta\mathbb{E}_t\left[\phi\frac{\exp(d_{t+1})\left(C_{t+1} - bC_t\right)^{-\sigma}}{\exp(d_t)\left(C_t - bC_{t-1}\right)^{-\sigma}\pi_{t+1}}\left(\frac{\pi_{H,t+1}}{\pi_{H,t}^{\omega}\bar{\pi}^{1-\omega}} - 1\right)\frac{Y_{t+1}}{Y_t}\frac{\pi_{H,t+1}^2}{\pi_{H,t}^{\omega}\bar{\pi}^{1-\omega}}\right],$$

$$Y_{H,t} = \lambda\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t,$$

$$Y_{F,t} = (1-\lambda)\left(\frac{S_t P_{F,t}^*}{P_t}\right)^{-1} C_t,$$

$$P_t = P_{H,t}^{\lambda}\left(S_t P_{F,t}^*\right)^{1-\lambda},$$

$$i_t = i_{t-1}^{\rho}\left[i\left(\frac{\pi_t}{\bar{\pi}}\right)^{\alpha_\pi}\left(\frac{Y_t}{\gamma Y_{t-1}}\right)^{\alpha_y}\left(\frac{S_t\pi^*}{S_{t-1}\bar{\pi}}\right)^{\alpha_s}\right]^{1-\rho}\exp\left(u_t\right),$$

$$Y_{H,t} + Y_{H,t}^* + \frac{\phi}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\omega}\bar{\pi}^{1-\omega}} - 1\right)^2 Y_t = Y_t,$$

$$P_{H,t}Y_{H,t}^* - P_{F,t}Y_{F,t} = S_t\left\{A_t^* - \exp\left[-\phi_a\left(\frac{S_{t-1}A_{t-1}^*}{P_{t-1}Z_{t-1}} - \bar{a}^*\right) - \phi_s\left(\frac{S_t}{S_{t-1}} - \frac{\bar{\pi}}{\pi^*}\right) + \psi_{t-1}\right] i_{t-1}^* A_{t-1}^*\right\},$$

$$Y_t = \exp(z_t) Z_t h_t,$$

$$Y^*_{H,t} = \lambda^* \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-1} Y^*_t,$$

$$P_{H,t} = S_t P^*_{H,t},$$

$$P_{F,t} = S_t P^*_t,$$

$$\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}.$$

B Steady-State Conditions

Let χ denote the ratio of net foreign assets over GDP in the steady state, *i.e.*, $\chi := a^*/y$. Then, we can derive the following steady-state conditions analytically:

$$s = \frac{\pi}{\pi^*},$$

$$\pi_H = \pi,$$

$$i = \frac{\gamma\pi}{\beta},$$

$$p_H = 1,$$

$$w = \frac{\epsilon - 1}{\epsilon},$$

$$h = \left(\frac{\frac{\gamma}{\gamma-b}\frac{\epsilon-1}{\epsilon}}{1 + \frac{1-\beta}{\beta}\chi}\right)^{\frac{1}{1+\eta}},$$

$$c = \left(1 + \frac{1-\beta}{\beta}\chi\right)^{\frac{\eta}{1+\eta}} \left(\frac{\gamma}{\gamma - b}\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\eta}},$$

$$y_H = \lambda \left(1 + \frac{1-\beta}{\beta}\chi\right)^{\frac{\eta}{1+\eta}} \left(\frac{\gamma}{\gamma - b}\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\eta}},$$

$$y_F = (1-\lambda) \left(1 + \frac{1-\beta}{\beta}\chi\right)^{\frac{\eta}{1+\eta}} \left(\frac{\gamma}{\gamma - b}\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\eta}},$$

$$y = \left(\frac{\frac{\gamma}{\gamma-b}\frac{\epsilon-1}{\epsilon}}{1 + \frac{1-\beta}{\beta}\chi}\right)^{\frac{1}{1+\eta}},$$

$$a^* = \chi \left(\frac{\frac{\gamma}{\gamma-b}\frac{\epsilon-1}{\epsilon}}{1+\frac{1-\beta}{\beta}\chi} \right)^{\frac{1}{1+\eta}},$$

$$\lambda^* y^* = \left(\frac{\gamma}{\gamma-b}\frac{\epsilon-1}{\epsilon} \right)^{\frac{1}{1+\eta}} \left(1+\frac{1-\beta}{\beta}\chi \right)^{\frac{\eta}{1+\eta}} \left(\frac{1}{1+\frac{1-\beta}{\beta}\chi} - \lambda \right).$$

The last equation implies that the parameters satisfy the following condition:

$$\lambda < \frac{1}{1+\frac{1-\beta}{\beta}\chi}.$$

As $0 < \lambda < 1$ and $\frac{1-\beta}{\beta}\chi \approx 0$, it must be satisfied unless λ is very close to unity.

C Log-Linearized Equilibrium Conditions

Log-linearizing the detrended equilibrium conditions around the steady state and rearranging the resulting equations yield

$$\hat{c}_t = \frac{b}{\gamma+b}\hat{c}_{t-1} + \frac{\gamma}{\gamma+b}\mathbb{E}_t\hat{c}_{t+1} - \frac{\gamma-b}{\gamma+b}\left(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t d_{t+1} - d_t \right),$$

$$\eta\hat{h}_t = \hat{w}_t - \frac{\gamma}{\gamma-b}\hat{c}_t + \frac{b}{\gamma-b}\hat{c}_{t-1},$$

$$\hat{i}_t - \hat{i}_t^* = (1-\phi_s s)\,\mathbb{E}_t s_{t+1} - \phi_a a^* \hat{a}_t^* + \hat{\psi}_t,$$

$$\hat{\pi}_{H,t} = \frac{\omega}{1+\beta\omega}\hat{\pi}_{H,t-1} + \frac{\beta}{1+\beta\omega}\mathbb{E}_t\hat{\pi}_{H,t+1} + \frac{\epsilon-1}{\phi\,(1+\beta\omega)}\left(\hat{w}_t - \hat{p}_{H,t} - z_t\right),$$

$$\hat{y}_{H,t} = -\hat{p}_{H,t} + \hat{c}_t,$$

$$\hat{y}_{F,t} = -\hat{e}_t + \hat{c}_t,$$

$$\lambda\hat{p}_{H,t} = -\left(1-\lambda\right)\hat{e}_t,$$

$$\hat{i}_t = \rho\hat{i}_{t-1} + (1-\rho)\left[\alpha_\pi\hat{\pi}_t + \alpha_y\left(\hat{y}_t - \hat{y}_{t-1}\right) + \alpha_S\hat{s}_t\right] + u_t,$$

$$\frac{y}{\lambda^* y^*}\hat{y}_t = \frac{y_H}{\lambda^* y^*}\hat{y}_{H,t} + \hat{y}_t^* - \hat{p}_{H,t} + \hat{e}_t,$$

$$\hat{a}_t^* = \frac{\lambda^* y^*}{a^*}\hat{y}_t^* - \frac{y_F}{a^*}\hat{y}_{F,t} + \frac{\lambda^* y^* - y_F}{a^*}\hat{e}_t + \frac{1}{\beta}\left[(1-\phi_a a^*)\hat{a}_{t-1}^* + (1-\phi_s s)\hat{s}_t + \hat{i}_{t-1}^* - \hat{\pi}_t + \hat{\psi}_{t-1} \right],$$

$$\hat{y}_t = z_t + \hat{h}_t,$$

$$\hat{s}_t = \hat{e}_t - \hat{e}_{t-1} + \hat{\pi}_t - \hat{\pi}_t^*,$$

$$\hat{\pi}_{H,t} = \hat{p}_{H,t} - \hat{p}_{H,t-1} + \hat{\pi}_t,$$

where hatted variables denote the percentage deviation from their corresponding steady-state values.

D Preliminary Estimation Results

Table 5 presents the posterior estimates of parameters in the model with Adolfson et al. (2008)'s specification of the endogenous interest rate spread on foreign bond holdings.

The log marginal data density (−1135.6) is significantly lower than that in the baseline estimation (−816.1, see Table 2), indicating that the data strongly favor our specification over that of Adolfson et al. (2008).

Table 5: Posterior estimates of parameters under Adolfson et al. (2008)'s specification for interest rate spread

Parameter	Mean	90% interval
ϕ_s	0.245	[0.193, 0.300]
ϕ_a	0.006	[0.003, 0.008]
b	0.299	[0.221, 0.376]
η	2.645	[1.902, 3.411]
ϕ	9.599	[5.547, 13.952]
ω	0.292	[0.125, 0.462]
λ	0.815	[0.790, 0.840]
ρ	0.857	[0.817, 0.897]
α_π	2.858	[2.313, 3.354]
α_y	0.389	[0.088, 0.671]
α_s	0.349	[0.207, 0.474]
$\overline{\gamma}$	0.262	[0.208, 0.314]
$\overline{\pi}$	0.535	[0.463, 0.608]
$\overline{i}$	1.233	[1.144, 1.324]
ρ_ψ	0.991	[0.982, 1.000]
ρ_z	0.973	[0.952, 0.996]
ρ_μ	0.890	[0.860, 0.921]
ρ_{y^*}	0.876	[0.806, 0.946]
ρ_{π^*}	0.612	[0.487, 0.734]
ρ_{i^*}	0.942	[0.922, 0.962]
σ_ψ	0.181	[0.137, 0.225]
σ_z	0.874	[0.765, 0.980]
σ_d	2.796	[2.334, 3.262]
σ_u	0.308	[0.245, 0.372]
σ_{y^*}	0.488	[0.431, 0.540]
σ_{π^*}	0.190	[0.167, 0.211]
σ_{i^*}	0.128	[0.113, 0.144]
σ_ζ	0.231	[0.096, 0.355]
M_ψ	1.525	[-0.418, 3.290]
M_z	0.816	[-1.211, 2.696]
M_d	-0.221	[-2.037, 1.560]
M_u	-0.772	[-2.759, 1.068]
M_{y^*}	3.495	[1.431, 5.545]
M_{π^*}	-0.453	[-2.253, 1.232]
M_{i^*}	0.842	[-1.091, 2.790]
$\log p(\mathbf{Y}^T)$	-1135.605	
$\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D \| \mathbf{Y}^T\}$	1.000	

Notes: This table reports the posterior mean and 90% highest posterior density intervals based on $10,000$ particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathbf{Y}^T)$ represents the SMC-based approximation of log marginal data density and $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D|\mathbf{Y}^T\}$ denotes the posterior probability of equilibrium determinacy.

E Sequential Monte Carlo Algorithm

To approximate the posterior distribution of model parameters, we employ a generic SMC algorithm with likelihood tempering, as described in Herbst and Schorfheide (2014, 2015). In the algorithm, a sequence of tempered posteriors is defined as

$$\varpi_n(\boldsymbol{\theta}) = \frac{[p(\mathbf{Y}^T|\boldsymbol{\theta}, \mathbf{M})]^{\tau_n} p(\boldsymbol{\theta}, \mathbf{M})}{\int [p(\mathbf{Y}^T|\boldsymbol{\theta}, \mathbf{M})]^{\tau_n} p(\boldsymbol{\theta}, \mathbf{M}) d\boldsymbol{\theta} d\mathbf{M}}, \quad n = 0, ..., N_\tau,$$

where N_τ denotes the number of stages and is set at $N_\tau = 200$. The tempering schedule $\{\tau_n\}_{n=0}^{N_\tau}$ is determined by $\tau_n = (n/N_\tau)^\mu$, where μ is a parameter that controls the shape of the tempering schedule and is set at $\mu = 2$, following Herbst and Schorfheide (2014, 2015). The SMC algorithm generates parameter draws $\{\boldsymbol{\theta}_n^{(i)}, \mathbf{M}_n^{(i)}\}$ and associated importance weights $w_n^{(i)}$, called particles, from the sequence of posteriors $\{\varpi_n\}_{n=1}^{N_\tau}$; *i.e.*, at each stage, $\varpi_n(\boldsymbol{\theta})$ is represented by a swarm of particles $\{\boldsymbol{\theta}_n^{(i)}, \mathbf{M}_n^{(i)}, w_n^{(i)}\}_{i=1}^N$, where N denotes the number of particles. In the subsequent estimation, the algorithm uses $N = 10,000$ particles. For $n = 0, ..., N_\tau$, the algorithm sequentially updates the swarm of particles $\{\boldsymbol{\theta}_n^{(i)}, \mathbf{M}_n^{(i)}, w_n^{(i)}\}_{i=1}^N$ through importance sampling.[20]

Posterior inferences on model parameters are made based on the particles $\{\boldsymbol{\theta}_{N_\tau}^{(i)}, \mathbf{M}_{N_\tau}^{(i)}, w_{N_\tau}^{(i)}\}_{i=1}^N$ from the final importance sampling. The SMC-based approximation of the marginal data density is given by

$$p(\mathbf{Y}^T) = \prod_{n=1}^{N_\tau} \left(\frac{1}{N} \sum_{i=1}^{N} \tilde{w}_n^{(i)} w_{n-1}^{(i)} \right),$$

where $\tilde{w}_n^{(i)}$ is the incremental weight defined as $\tilde{w}_n^{(i)} = [p(\mathbf{Y}^T|\boldsymbol{\theta}_{n-1}^{(i)}, \mathbf{M}_{n-1}^{(i)})]^{\tau_n - \tau_{n-1}}$. The posterior probability of equilibrium determinacy can be calculated as

$$\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D|\mathbf{Y}^T\} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\{\boldsymbol{\theta}_{N_\tau}^{(i)} \in \boldsymbol{\Theta}^D\}.$$

Likewise, the prior probability of equilibrium determinacy can be computed using prior draws.

[20] This process includes one step of a single-block random-walk Metropolis–Hastings algorithm.

References

Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2008): "Evaluating an estimated new Keynesian small open economy model," *Journal of Economic Dynamics and Control*, 32, 2690–2721.

Bacchetta, P. and E. van Wincoop (2021): "Puzzling exchange rate dynamics and delayed portfolio adjustment," *Journal of International Economics*, 131.

Backus, D. K., S. Foresi, and C. I. Telmer (2001): "Affine Term Structure Models and the Forward Premium Anomaly," *Journal of Finance*, 56, 279–304.

Backus, D. K., F. Gavazzoni, C. Telmer, and S. E. Zin (2010): "Monetary Policy and the Uncovered Interest Parity Puzzle," NBER Working Papers 16218, National Bureau of Economic Research, Inc.

Backus, D. K. and G. W. Smith (1993): "Consumption and real exchange rates in dynamic economies with non-traded goods," *Journal of International Economics*, 35, 297–316.

Bansal, R. and I. Shaliastovich (2012): "A long-run risks explanation of predictability puzzles in bond and currency markets," *The Review of Financial Studies*, 26, 1–33.

Beaudry, P. and A. Lahiri (2019): "The Unsettling Behavior of Exchange Rates Under Inflation Targeting," Unpublished manuscript, University of British Columbia.

Benigno, G., P. Benigno, and S. Nistico (2011): "Risk, Monetary Policy and the Exchange Rate," in *NBER Macroeconomics Annual 2011, Volume 26*, National Bureau of Economic Research, Inc, NBER Chapters, 247–309.

Blanchard, O. J. and C. M. Kahn (1980): "The Solution of Linear Difference Models under Rational Expectations," *Econometrica*, 48, 1305–1311.

Bullard, J. and E. Schaling (2009): "Monetary Policy, Determinacy, and Learnability in a Two-Block World Economy," *Journal of Money, Credit and Banking*, 41, 1585–1612.

Bullard, J. and A. Singh (2008): "Worldwide macroeconomic stability and monetary policy rules," *Journal of Monetary Economics*, 55, 34–47.

Burnside, C., M. Eichenbaum, I. Kleshchelski, and S. Rebelo (2011): "Do Peso Problems Explain the Returns to the Carry Trade?" *Review of Financial Studies*, 24, 853–891.

Burnside, C., M. Eichenbaum, and S. Rebelo (2008): "Carry Trade: The Gains of Diversification," *Journal of the European Economic Association*, 6, 581–588.

Bussière, M., M. Chinn, L. Ferrara, and J. Heipertz (2022): "The New Fama Puzzle," *IMF Economic Review*, 70, 451–486.

Candian, G. and P. De Leo (2023): "Imperfect Exchange Rate Expectations," *The Review of Economics and Statistics*, forthcoming.

Carlstrom, C. T. and T. S. Fuerst (2002): "Optimal Monetary Policy in a Small, Open Economy: A General Equilibrium Analysis," in *Monetary Policy: Rules and Transmission Mechanisms*, ed. by N. Loayza, K. Schmidt-Hebbel, N. Loayza, and K. Schmidt-Hebbel, Central Bank of Chile, vol. 4 of *Central Banking, Analysis, and Economic Policies Book Series*, chap. 10, 275–298.

Chahrour, R., V. Cormun, P. D. Leo, P. A. Guerrón-Quintana, and R. Valchev (2024): "Exchange Rate Disconnect Revisited," NBER Working Papers 32596, National Bureau of Economic Research, Inc.

Chakraborty, A. and G. W. Evans (2008): "Can perpetual learning explain the forward-premium puzzle?" *Journal of Monetary Economics*, 55, 477–490.

Chen, Y.-C., I. Fujiwara, and Y. Hirose (2021): "Exchange Rate Disconnect and the General Equilibrium Puzzle," CEPR Discussion Papers 16555, C.E.P.R. Discussion Papers.

Christiano, L. J., M. Trabandt, and K. Walentin (2011): "Introducing financial frictions and unemployment into a small open economy model," *Journal of Economic Dynamics and Control*, 35, 1999–2041.

Colacito, R. and M. M. Croce (2011): "Risks for the Long Run and the Real Exchange Rate," *Journal of Political Economy*, 119, 153–181.

De Fiore, F. and Z. Liu (2005): "Does trade openness matter for aggregate instability?" *Journal of Economic Dynamics and Control*, 29, 1165–1192.

Duarte, M. and A. C. Stockman (2005): "Rational speculation and exchange rates," *Journal of Monetary Economics*, 52, 3–29.

Eichenbaum, M. and C. L. Evans (1995): "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates," *The Quarterly Journal of Economics*, 110, 975–1009.

Engel, C. (2016): "Exchange Rates, Interest Rates, and the Risk Premium," *American Economic Review*, 106, 436–474.

Engel, C. and S. P. Wu (2024): "Exchange Rate Models are Better than You Think, and Why They Didn't Work in the Old Days," NBER Working Papers 32808, National Bureau of Economic Research, Inc.

Fama, E. F. (1984): "Forward and spot exchange rates," *Journal of Monetary Economics*, 14, 319–338.

Gabaix, X. and M. Maggiori (2015): "International liquidity and exchange rate dynamics," *The Quarterly Journal of Economics*, 130, 1369–1420.

Galí, J. and T. Monacelli (2005): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72, 707–734.

Gourinchas, P.-O. and A. Tornell (2004): "Exchange rate puzzles and distorted beliefs," *Journal of International Economics*, 64, 303–333.

Gourio, F., M. Siemer, and A. Verdelhan (2013): "International risk cycles," *Journal of International Economics*, 89, 471–484.

Grilli, V. and N. Roubini (1996): "Liquidity models in open economies: Theory and empirical evidence," *European Economic Review*, 40, 847–859.

Herbst, E. P. and F. Schorfheide (2014): "Sequential Monte Carlo Sampling For DSGE Models," *Journal of Applied Econometrics*, 29, 1073–1098.

——— (2015): *Bayesian Estimation of DSGE Models*, no. 10612 in Economics Books, Princeton University Press.

Hirose, Y. (2013): "Monetary policy and sunspot fluctuations in the United States and the euro area," *Macroeconomic Dynamics*, 17, 1–28.

Hirose, Y., T. Kurozumi, and W. Van Zandweghe (2020): "Monetary policy and macroeconomic stability revisited," *Review of Economic Dynamics*, 37, 255–274.

Ilut, C. (2012): "Ambiguity Aversion: Implications for the Uncovered Interest Rate Parity Puzzle," *American Economic Journal: Macroeconomics*, 4, 33–65.

Itskhoki, O. and D. Mukhin (2021): "Exchange Rate Disconnect in General Equilibrium," *Journal of Political Economy*, 129, 2183–2232.

Justiniano, A. and B. Preston (2010): "Can structural small open-economy models account for the influence of foreign disturbances?" *Journal of International Economics*, 81, 61–74.

Kareken, J. and N. Wallace (1981): "On the Indeterminacy of Equilibrium Exchange Rates," *The Quarterly Journal of Economics*, 96, 207–222.

Kekre, R. and M. Lenel (2024a): "The Flight to Safety and International Risk Sharing," *American Economic Review*, 114, 1650–91.

——— (2024b): "Exchange Rates, Natural Rates, and the Price of Risk," NBER Working Papers 32976, National Bureau of Economic Research, Inc.

Kim, S. and N. Roubini (2000): "Exchange rate anomalies in the industrial countries: A solution with a structural VAR approach," *Journal of Monetary Economics*, 45, 561–586.

Lubik, T. and F. Schorfheide (2006): "A Bayesian Look at the New Open Economy Macroeconomics," in *NBER Macroeconomics Annual 2005, Volume 20*, National Bureau of Economic Research, Inc, NBER Chapters, 313–382.

Lubik, T. A. and F. Schorfheide (2003): "Computing sunspot equilibria in linear rational expectations models," *Journal of Economic Dynamics and Control*, 28, 273–285.

——— (2004): "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review*, 94, 190–217.

Lustig, H. and A. Verdelhan (2007): "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk," *American Economic Review*, 97, 89–117.

Meese, R. A. and K. Rogoff (1983): "Empirical exchange rate models of the seventies : Do they fit out of sample?" *Journal of International Economics*, 14, 3–24.

Rogoff, K. (1996): "The Purchasing Power Parity Puzzle," *Journal of Economic Literature*, 34, 647–668.

Rotemberg, J. J. (1982): "Sticky Prices in the United States," *Journal of Political Economy*, 90, 1187–1211.

Scholl, A. and H. Uhlig (2008): "New evidence on the puzzles: Results from agnostic identification on monetary policy and exchange rates," *Journal of International Economics*, 76, 1–13.

Smets, F. and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.

Valchev, R. (2020): "Bond Convenience Yields and Exchange Rate Dynamics," *American Economic Journal: Macroeconomics*, 12, 124–166.

Verdelhan, A. (2010): "A Habit-Based Explanation of the Exchange Rate Risk Premium," *Journal of Finance*, 65, 123–146.