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石橋孝次

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Institute for Economic Studies, Keio University
2-15-45 Mita, Minato-ku, Tokyo 108-8345, Japan
ies-office@adst.keio.ac.jp
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【要旨】

この論文では、消費者が自らの選好に対して誤った信念をもつとき、企業が二部料金のプランを設計する際にどう対応するかを検討する。誤った信念をもつ消費者には、自分の真の需要を過少に予測する（過度に悲観的な）場合、そして過大に予測する（過度に楽観的な）場合がある。信念に歪みのない消費者には評価額が高いものと低いものの2タイプがあると想定する。そこで誤った信念をもつ消費者によって歪みのない消費者がどんな影響を受けるかは、市場構造に依存して異なることを示す。独占の場合、企業は過度に悲観的な消費者を教育して信念を修正したいと考えるが、過度に楽観的な消費者を教育しようとはしない可能性がある。競争の場合には、企業には歪んだ信念をもつ消費者を教育するインセンティブはない。過度に悲観的または過度に楽観的な消費者に対して信念を修正しようとする政策を行うと、競争下では社会厚生は改善するが、独占下では社会厚生が損なわれる可能性がある。

石橋孝次
慶應義塾大学経済学部
東京都港区三田二丁目１５−４５
koji@econ.keio.ac.jp

Biased Beliefs of Consumers and Two-Part Tariff Competition

Koji Ishibashi*

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Abstract

This paper explores how firms respond in designing two-part tariffs to consumers' biased beliefs about their preferences. Biased consumers could be either overpessimistic when they underestimate their true demand or overoptimistic when they overestimate. Assuming that unbiased consumers consist of two types with high and low valuations, I show that the effect of the presence of biased consumers on unbiased consumers depends on market structure. The monopolist wants to educate overpessimistic consumers while may not want to educate overoptimistic consumers. Alternatively, in competition, firms do not have the incentive to educate any biased consumers. A debiasing policy for either overpessimistic or overoptimistic consumers unambiguously improves social welfare in competition but could harm social welfare in monopoly.

JEL classification: D42; D43; D91; L12; L13

Keywords: biased belief; overoptimistic consumers; overpessimistic consumers; two-part tariff

*Department of Economics, Keio University, 2-15-45 Mita, Minato-ku, Tokyo 108-8345, Japan (Email: koji@econ.keio.ac.jp).

1 Introduction

The recent behavioral economics literature has explored many sources of consumer bias and their implications on consumer behavior.[1] One principal source is the biased beliefs of consumers about their preference or willingness-to-pay; consumers may overestimate or underestimate their true demands, particularly when they have to choose among tariff plans before they actually consume goods or services. Many works on consumer biases have investigated questions such as whether inefficiency arises or not under the presence of biased consumers, is inefficiency due to biased consumers eliminated by competition, and does learning by consumers or educating consumers mitigate inefficiency.[2]

Most studies, however, suppose that the type of consumer is binary, biased or unbiased (put differently, naive or sophisticated), and more importantly, unbiased consumers are homogeneous. When unbiased consumers are heterogeneous in their preferences, it is not apparent whether the existence of biased consumers makes each type of unbiased consumer better off or worse off. The purpose of the paper is to explore the mechanism of monopolistic and competitive screening via two-part tariffs with a high and low valuation of consumers, some of whom have biased beliefs about their own preferences, and then to investigate how the presence of biased consumers affects different types of unbiased consumers.

There are two scenarios. One is the situation where biased consumers are overpessimistic; they mistakenly believe themselves to have a low valuation when, in fact, they have a high valuation. A naive consumer would underestimate her demand for *leisure goods* with current benefits and future costs.[3] For example, credit card borrowing delivers current consumption at the expense of future consumption. Naive consumers might underpredict their desire to use credit cards for purchases they cannot afford.[4] A similar bias toward overoptimistic behavior could be seen in the consumption of addicting goods or gambling and cellular phone usage.

The second scenario is where biased consumers are overoptimistic; they mistakenly believe themselves to have a high valuation when, in fact, they have a low valuation. Consumers often overestimate their demand for *investment goods* with current costs and future benefits. For example, as to health club attendance which incurs current effort costs and provides future health benefits, they might believe that they will go to the gym more often than they actually do (DellaVigna and Malmendier 2006). Parents and children may make underinvestment in the children's education due to the present opportunity costs

[1]For surveys on behavioral industrial organization, see Ellison (2006), Huck and Zhou (2011), Spiegler (2011), and Heidhues and Kőszegi (2018).

[2]The existing literature explores various sources of consumer bias such as hyperbolic discounting or time-inconsistent preferences (DellaVigna and Malmendier 2004, Eliaz and Spiegler 2006, Heidhues and Kőszegi 2010), overconfidence (Grubb 2009), unawareness or limited attention (Gabaix and Laibson 2006). Although there are many sources of misperception, a common feature is that some consumers behave as if they mispredict the quantities they consume.

[3]I follow DellaVigna and Malmendier (2004) for the definition of leisure goods and investment goods mentioned below.

[4]Ausubel (1991) suggests that high interest rates on credit card debt can be explained by the presence of borrowers who underestimate borrowing and the importance of interest rates. See also DellaVigna and Malmendier (2004).

of schooling which could yield significant benefits in the future. In Internet service, Lambrecht and Skiera (2006) documented evidence of a "flat-rate bias," a tendency of consumers to prefer a flat rate when they would save money with a pay-per-use tariff, as a source of systematic overestimation of demand.

My model allows us to investigate how the presence of biased consumers affects unbiased consumers in these two scenarios in a unified manner. In the first scenario with overpessimistic consumers, consumers with low valuation (low type) and overpessimistic consumers cannot be screened because they have the same belief, and hence choose the same tariff. Nevertheless, they choose different quantities ex-post; overpessimistic consumers consume more than low-type because they are, in fact, high-type consumers. Therefore, the optimal two-part tariff reflects the presence of overpessimistic consumers. For consumers with high valuation (high type), firms set the marginal price equal to the marginal cost but a large fixed fee. For low-type and overpessimistic consumers, in contrast, the marginal price is set higher than the marginal cost to collect variable profit from overpessimistic consumers with high ex-post valuation. This distortion against biased consumers is consistent with the result of DellaVigna and Malmendier (2004) in the context of the self-control problem. It exists regardless of market structure, that is, monopoly or competition.

In monopoly, another source of distortion exists due to incomplete information about consumer types. Because a monopolist wants to reduce information rent to high-type consumers, it sets the marginal price for low-type and overpessimistic consumers higher than the marginal cost. A combination of this distortion from a well-known trade-off between rent and efficiency and the motive mentioned above to collect variable profit from overpessimistic consumers characterizes allocative inefficiency in monopoly under incomplete information. In competition, however, incentive constraints do not bind in the design of two-part tariffs. The full-information equilibrium remains an equilibrium under competition, and then the only distortion is from the incentive to collect variable profit. Thus, the allocative inefficiency originating from insufficient quantities for low-type and overpessimistic consumers cannot be removed through competition.

In the second scenario, overoptimistic consumers choose the same tariff as high-type consumers. However, the quantity overoptimistic consumers purchase ex-post becomes lower than that of high-type consumers because they are actually low-type. In the optimal two-part tariff with overoptimistic consumers, the marginal price for high-type and overoptimistic consumers is less than the marginal cost. This is because firms want to enlarge fixed fees by exploiting overoptimistic consumers with misperceptions, despite that this involves a loss in variable profit from high-type and overoptimistic consumers. Again, this distortion against biased consumers is consistent with DellaVigna and Malmendier (2004), and it exists under monopoly and competition.

The marginal price for low-type consumers, on the other hand, is set equal to the marginal cost under competition. Because incentive constraints are satisfied when firms compete, inefficiency does not arise concerning the quantity of low-type consumers. In monopoly, however, the marginal price for low-type consumers is distorted upward to ensure the incentive constraint for high-type consumers. Therefore, in the presence of overoptimistic consumers, there exists a distortion in both marginal prices under monopoly with incomplete information.

Given the above inefficiency due to biased consumers, I then analyze how the existence of biased consumers affects unbiased consumers. By aggregating externalities between biased and unbiased consumers, I can also infer the impacts of debiasing on consumer surplus. I further examine the incentives of firms to educate overpessimistic or overoptimistic consumers. If firms have no incentive to educate, there is a scope for debiasing policy by a third party or government agency. Combining effects on consumer surplus and profit allows investigating how a debiasing policy or activity educating biased consumers affects social welfare.

First, consider the scenario with overpessimistic consumers. In monopoly, the marginal price for low-type and overpessimistic consumers is higher than the marginal cost, and it becomes lower if more overpessimistic consumers are present. This is because the monopolist becomes more concerned with the efficiency of the quantity of overpessimistic consumers when their weight in the total population becomes large. Thus, the presence of overpessimistic consumers enlarges the information rent to high-type, yielding a positive externality to high-type consumers. Moreover, an increase in the number of overpessimistic consumers raises their net surplus because in equilibrium, overpessimistic consumers obtain the same level of surplus as high-type. In other words, a debiasing policy educating overpessimistic consumers harms high-type consumers. The net surplus of low-type consumers, in contrast, is zero because the participation constraint is binding, and therefore, the presence of overpessimistic consumers has no externality to low-type consumers. In sum, under monopoly, debiasing overpessimistic consumers reduces consumer surplus but raises profit because it yields higher fixed fees from debiased (high type) consumers. The impact on total surplus is ambiguous in general, but I show that a debiasing policy reducing the number of overpessimistic consumers harms social welfare under the linear demand and a wide range of parameter values of the iso-elastic demand.

When firms compete in two-part tariffs, as described above, the marginal price for low-type and overpessimistic consumers is higher, while that for high-type is equal to the marginal cost. Although this distortion is similar to monopoly, the marginal price for low-type and overpessimistic consumers now becomes higher if more overpessimistic consumers are present because collecting variable profit becomes more important. An increase in the number of overpessimistic consumers favors low-type consumers because a lower fixed fee is paired with a higher marginal price. This implies that, compared to monopoly, externality by the existence of overpessimistic consumers works in the opposite way between types; low-type consumers are better off, whereas high-type consumers are kept unchanged. Therefore, whether the presence of overpessimistic consumers makes unbiased consumers better or worse off crucially depends on market structure. Another contrasting result in competition is that an increase in the number of overpessimistic consumers now reduces their net surplus. To put it differently, debiasing overpessimistic consumers keeps high-type unchanged and makes low-type worse off and overpessimistic consumers better off. It increases the consumer surplus because a negative impact on low-type is dominated by a positive impact on overpessimistic consumers. The profit of each firm does not change; a reduction in variable profit from low-type and overpessimistic consumers is exactly canceled out by an increase in the fixed fee for them. Therefore, under competition, a debiasing policy educating overpessimistic consumers unambiguously raises social welfare.

Second, consider the case of overoptimistic consumers. First, in monopoly, the aforementioned inefficiency is characterized by a downward distortion in the marginal price for high-type and overoptimistic consumers and an upward distortion in the marginal price for low-type. When more overoptimistic consumers exist, the marginal price for high-type and overoptimistic consumers becomes lower because the monopolist has a stronger incentive to exploit overoptimistic consumers. At the same time, the marginal price for low-type becomes higher because reducing the information rent to high-type, who choose the same tariff as overoptimistic consumers, is more important. Then, the presence of overoptimistic consumers keeps low-type unchanged because their participation constraint is always binding, but makes high-type worse off due to smaller information rent. An increase in the number of overoptimistic consumers makes them worse off due to a larger fixed fee. In other words, high-type and overoptimistic consumers benefit from a debiasing policy for overoptimism while low-type are kept unchanged, raising consumer surplus. An effect on profit is ambiguous; because a debiasing policy implies that some overoptimistic consumers turn to low-type, it reduces the revenue from the fixed fee, but at the same time it enlarges the variable profit for low-type. Hence, the impact on total surplus is also ambiguous in general.

Next, under competition, the marginal price for high-type and overoptimistic consumers is less than the marginal cost while that for low-type is equal to the marginal cost. Now, the overoptimistic consumers yield a positive externality to high-type because high-type consumers can benefit from lower marginal price, and no externality to low-type. This means that the presence of overoptimistic consumers makes high-type better off in competition, whereas it makes them worse off in monopoly. In contrast, low-type consumers remain unchanged, independent of market structure. A debiasing policy for overoptimistic consumers protects them from exploitation, and raises consumer surplus because a negative impact on high-type is dominated by a positive impact on debiased overoptimistic consumers. It does not change profit under competition; a reduction of losses in variable profit from high-type and overoptimistic consumers is exactly canceled out by a decrease in the fixed fee. Therefore, under competition, debiasing policy unambiguously improves social welfare.

The rest of the paper is organized as follows: After the literature review in the next section, Section 3 presents the model. The optimal two-part tariff in monopoly is derived in Section 4, and two-part tariff competition is studied in Section 5. Section 6 concludes. Some proofs are in the Appendix.

2 Related Literature

In modelling consumers' types and biased beliefs, this paper is mostly related to Sandroni and Squintani (2007, 2013). Focusing on insurance markets, they study a combination of screening with respect to consumers' beliefs and with respect to consumers' preferences. They investigate competitive and monopolistic insurance markets with low-risk and high-risk consumers, some of whom are overconfident: they believe themselves to be low-risk when, in fact, they are high-risk. Low-risk and overconfident consumers cannot be screened because they share the same beliefs, and they buy the same insurance contract. Sandroni and Squintani (2013) show that whether or not overconfidence changes qualitative predictions in markets under asymmetric information depends on market structure. The introduction of overconfident agents overturns fundamental relationships between observable variables in competitive insurance markets. In monopolistic insurance markets, in contrast, overconfidence may be equivalent to changes in the risk composition in the economy. Although my model adopts a similar formulation of consumers' types and beliefs to Sandroni and Squintani (2007, 2013), there are differences in several dimensions. First, they focus on the relationship between biased beliefs and observables in insurance markets with asymmetric information, whereas I focus on how biased beliefs of consumers affect the optimal menu of two-part tariffs. Second, although overoptimism is the only bias in their model, I allow for two-way biases where biased consumers are either overoptimistic or overpessimistic. Third, unlike insurance markets, the full-information equilibrium remains an equilibrium under competition in this model.

Concerning market interactions between biased (naive) consumers and unbiased (sophisticated) consumers, cross-subsidization from biased consumers to unbiased consumers is first emphasized by Ellison (2005) and Gabaix and Laibson (2006) in products with shrouded attributes and by Armstrong and Vickers (2012) in the context of overdraft fee in the banking industry. More extensively, Armstrong (2015) explored how naive and sophisticated consumers affect each other in market interactions and possible policy interventions using two models: a product with price dispersion and a product with add-on pricing. It is assumed that there are two kinds of consumers, naive and sophisticated, and there are no systematic differences in their preferences. In this paper, because sophisticated consumers consist of two types, the directions of externalities of the naive consumers on two types of sophisticated consumers can be different from each other.

Among the literature on mechanism design and optimal contracts, this paper is related to the articles on the optimal contracting problem with non-common priors.[5] Eliaz and Spiegler (2006) investigate monopolistic screening according to prior beliefs when dynamically inconsistent consumers differ in their ability to forecast their own demand. Eliaz and Spiegler (2008) study the monopolist's design of a menu of non-linear tariffs when consumers have optimistic beliefs regarding their future preferences. In their model, consumers are optimistic if their prior belief assigns too much weight to states characterized by large gains from trade. Grubb (2009) studies monopolistic and competitive screening for overconfident consumers who underestimate the variance of their demand forecasts in mobile phone usage, and shows that the optimal contract can be sufficiently approximated by three-part tariffs. Uthemann (2005) shows that competing firms offering menus of non-linear price schedules can screen consumers on the basis of their mistaken priors.[6] The present study is different from these papers in two respects. First, these papers formulate biased beliefs such that consumers and firms do not share the same distribution over the type of consumers. Firms try to discriminate between consumers who have the same ex-post preferences but different ex-ante beliefs. This paper, in contrast, supposes that some consumers mistakenly believe that they are different types in a deterministic sense. Because a rational consumer and a biased consumer have the same ex-ante beliefs, they choose the same option in a menu of contracts. Although they choose different quantities ex-post, ex-ante screening between them is impossible in this setting; the motive for discrimination here is screening between a group of rational and biased consumers with the same belief and another rational consumers with a different belief. Second, these papers characterize the optimal menu of non-linear tariffs to screen consumers' degrees of optimism without restrictions on the space of contracts. Although this paper restricts contracts to two-part tariffs, this simple structure allows me to study the two-way biases mentioned above and interactions between heterogeneous consumers.

There is an extensive literature on competitive non-linear pricing for rational consumers: Spulber (1989), Stole (1995), Armstrong and Vickers (2001, 2010), Rochet and Stole (2002), and Yang and Ye (2008).[7] By analyzing general non-linear pricing, these papers suggest that firms offer a single two-part tariff with marginal price equal to marginal cost when the market is fully covered, and firms are symmetric.

[5]The literature on sequential screening (Armstrong 1996, Baron and Besanko 1984, Courty and Li 2000, Miravete 1996, 2002) studies optimal non-linear pricing when consumers know only the distribution of their valuations at the time of contracting but subsequently learn their actual valuations. It is assumed, however, that consumers and the monopolist have common priors in this literature.

[6]The second-degree price discrimination based on beliefs of naive consumers is also studied by Heidhues and Kőszegi (2010) and Heidhues, Kőszegi and Murooka (2017) in the model of shrouded product attributes, and by Murooka and Schwarz (2018) and Johnen (2019) in the model of automatic renewal of contracts. I consider neither shrouded attributes nor contract renewal in this paper.

[7]For surveys on competitive non-linear pricing, see Armstrong (2016) and Stole (2007).

Focusing on two-part tariffs, Yin (2004) shows that equilibrium prices equal to marginal cost only if the marginal consumer's demand equals to the average demand in a generalized Hotelling model with variable demands. This paper, in the standard Hotelling model with inelastic demands, finds another source of distorting marginal prices in two-part tariff competition with biased consumers. Other papers on two-part tariff competition when consumers are rational include Hoernig and Valletti (2007) in the Hotelling model when consumers can buy from more than one firm, Herweg (2012) in a model of vertically differentiated duopoly, Reisinger (2014) in a model of two-sided platforms, and Griva and Vettas (2015) in a model of homogeneous product duopoly when consumers differ with respect to their usage levels.

3 The Model

I suppose there are two kinds of consumers with a higher valuation and a lower valuation of a good, and refer to high type (H) and low type (L). Type H and type L consumers (with fractions λ_H and λ_L, respectively) have a correct belief about their types. In addition to these two types, there exists the third type of consumers who have a biased belief about their own preferences. I consider two cases separately. First, the third type is overpessimistic (type P) consumers, with fraction λ_P, who mistakenly believe that they are type L when they are, in fact, type H. I denote the fraction of type H as $\lambda_H = 1 - \lambda_P - \lambda_L$ where $\lambda_P, \lambda_L \in (0, 1)$. Second, the third type is overoptimistic (type O) consumers, with fraction λ_O, who mistakenly believe that they are type H when they are, in fact, type L. I denote the fraction of type L as $\lambda_L = 1 - \lambda_H - \lambda_O$ where $\lambda_H, \lambda_O \in (0, 1)$.

The utility function of type j ($j = H, L$) is denoted by $u_j(q)$, where $u_H{}'(q) > u_L{}'(q) > 0$ and $u_j{}''(q) < 0$ for $j = H, L$. It is also assumed that $u_H(0) = u_L(0) = 0$, which implies that $u_H(q) > u_L(q)$ $\forall q$. The marginal cost is constant and is denoted by $c > 0$.

Firms cannot observe consumers' types and beliefs, but they know the fractions of each type of consumer. A menu of two-part tariffs, $T_j(q) = A_j + p_j q$ ($j = H, L$), consists of, in general, two different tariffs. Because type P (resp., type O) believes that they are type L (resp., type H) in choosing a tariff, firms cannot screen between type P and type L (resp., type O and type H) at the time of contracting.

The timing of the game is as follows:

Stage 0: Nature chooses the type of each consumer. A realized type is private information of a consumer. Type H and type L consumers correctly infer their true type, while type P and type O consumers have a wrong belief about their own types.

Stage 1: Firms offer a menu of two-part tariffs. Each consumer commits to choose a tariff.

Stage 2: Type P or type O consumers learn their true type. Then, each consumer determines a quantity under the menu she committed in Stage 1.

In Stage 1, type H chooses the tariff with (A_H, p_H) and type L chooses (A_L, p_L). The overpessimistic (type P) consumer chooses (A_L, p_L) when she is, in fact, type H, and in contrast, the overoptimistic (type O) consumer chooses (A_H, p_H) when she is, in fact, type L.

In Stage 2, the optimal quantity is determined by $u_j{}'(q_j) = p_j$, so that it is given by $q_j = (u_j{}')^{-1}(p_j) \equiv D_j(p_j)$. It follows that $D_j{}'(p_j) < 0$, because $u_j{}''(q) < 0$. Type H consumes $D_H(p_H)$ and type L consumes $D_L(p_L)$, respectively. The overpessimistic type P consumes $D_H(p_L)$, and the overoptimistic type O consumes $D_L(p_H)$.

4 Two-Part Tariff in Monopoly

In this section, I study the optimal two-part tariff for a monopolist in the presence of overpessimistic or overoptimistic consumers.

4.1 Overpessimistic Consumers

I first consider the case where biased consumers are overpessimistic. Let $S_j(p_j) = u_j(D_j(p_j)) - p_j D_j(p_j)$ denotes the consumer surplus gross of fixed fee. It follows that $S_j{}'(p_j) = -D_j(p_j)$. In Stage 1, type H and type L correctly infer their net consumer surplus, $S_H(p_H) - A_H$ and $S_L(p_L) - A_L$, respectively. Type P, who is overpessimistic, incorrectly perceives her net benefit is $S_L(p_L) - A_L$ in Stage 1, although her ex-post consumer surplus turns out to be $S_H(p_L) - A_L$.

The monopolist solves the following:

$$\max_{(A_H, p_H, A_L, p_L)} \pi = (1 - \lambda_P - \lambda_L)A_H + (\lambda_P + \lambda_L)A_L \\ + (p_H - c)(1 - \lambda_P - \lambda_L)D_H(p_H) + (p_L - c)[\lambda_P D_H(p_L) + \lambda_L D_L(p_L)]$$

subject to

$$S_H(p_H) - A_H \geq S_H(p_L) - A_L, \tag{1}$$

$$S_L(p_L) - A_L \geq S_L(p_H) - A_H, \tag{2}$$

$$S_H(p_H) - A_H \geq 0, \tag{3}$$

$$S_L(p_L) - A_L \geq 0, \tag{4}$$

where (1) and (2) are the incentive constraints, and (3) and (4) are the participation constraints, for type H and type L, respectively. For type P who believes they are type L in choosing a tariff, the incentive and the participation constraints are the same as type L.

4.1.1 Complete Information

I begin with describing the optimal two-part tariff under complete information when the monopolist can observe types of consumers. If the constraints (1) and (2) are missing, obviously the constraints (3) and (4) are binding; $A_H = S_H(p_H)$ and $A_L = S_L(p_L)$. The profit becomes

$$\pi = (1 - \lambda_P - \lambda_L)S_H(p_H) + (\lambda_P + \lambda_L)S_L(p_L)$$
$$+ (p_H - c)(1 - \lambda_P - \lambda_L)D_H(p_H) + (p_L - c)[\lambda_P D_H(p_L) + \lambda_L D_L(p_L)].$$

I denote the optimal marginal prices under complete information as (p_H^*, p_L^*). The first-order condition with respect to p_H is

$$\frac{\partial \pi}{\partial p_H} = (1 - \lambda_P - \lambda_L)[S_H{}'(p_H) + D_H(p_H) + (p_H - c)D_H{}'(p_H)] = 0. \tag{5}$$

From $S_j{}'(p_j) = -D_j(p_j)$ and $D_j{}' < 0$, the optimal marginal price for type H is given by $p_H^* = c$. Therefore, p_H^* is efficient. The first-order condition with respect to p_L is

$$\begin{aligned}\frac{\partial \pi}{\partial p_L} &= (\lambda_P + \lambda_L)S_L{}'(p_L) + \lambda_P D_H(p_L) + \lambda_L D_L(p_L) + (p_L - c)[\lambda_P D_H{}'(p_L) + \lambda_L D_L{}'(p_L)] \\ &= \lambda_P[D_H(p_L) - D_L(p_L)] + (p_L - c)[\lambda_P D_H{}'(p_L) + \lambda_L D_L{}'(p_L)] = 0, \end{aligned} \tag{6}$$

by using $S_j{}'(p_j) = -D_j(p_j)$ again. Thus, the equilibrium marginal price for type L and type P satisfies

$$p_L^* = c - \frac{\lambda_P[D_H(p_L^*) - D_L(p_L^*)]}{\lambda_P D_H{}'(p_L^*) + \lambda_L D_L{}'(p_L^*)} > c. \tag{7}$$

I assume that the profit is strictly concave in (p_H, p_L), so that the second-order conditions are satisfied.[8] The equilibrium marginal price, p_L^*, depends on the ratio λ_L/λ_P. Note that $p_L^* > c$ as long as $\lambda_P > 0$ ($p_L^* = c$ if $\lambda_P = 0$). When overpessimistic consumers are present, the monopolist raises the marginal price to collect variable profits from type P even though it reduces the revenue from the fixed fee for type L and type P.

Let w_j be the net consumer surplus of type j. It follows that $w_H = w_L = 0$, because the participation constraints are binding. Type P's ex-post (actual) consumer surplus is $w_P = S_H(p_L^*) - A_L = S_H(p_L^*) - S_L(p_L^*) > 0$, that is, type P consumers benefit from choosing the wrong tariff. This optimal two-part tariff under complete information does not satisfy the incentive constraint for type H, (1), because $S_H(p_H^*) - A_H^* = 0$ and $S_H(p_L^*) - A_L^* > 0$. Contrary, the incentive constraint for type L, (2), is satisfied with strict inequality. The optimal tariffs are described in Figure 1. Because both participation constraints are binding, the indifference curves of type H and type L cross the origin. The quantity for type H, q_H^*, is efficient, but both q_L^* and q_P^* are distorted downward.

[8] Because we have $\partial^2\pi/\partial p_H \partial p_L = 0$, the conditions, $\partial^2\pi/\partial p_H^2 < 0$ and $\partial^2\pi/\partial p_L^2 < 0$, are sufficient for strict concavity.

[Figure 1]

Proposition 1 *Suppose that types of consumers are observable. In the optimal two-part tariff for a monopolist under the presence of overpessimistic type P consumers, the marginal price for type H is equal to the marginal cost ($p_H^* = c$), and that for type L and type P is larger than the marginal cost ($p_L^* > c$). The fixed fee for type H is $A_H^* = S_H(c)$, and that for type L and type P is $A_L^* = S_L(p_L^*) < A_H^*$.*

4.1.2 Incomplete Information

Now, let us turn to the case of incomplete information. By the standard procedure with ignoring the incentive constraint for type L, (2), for a moment, we can show that only (1) and (4) are binding. Therefore, we have $A_L = S_L(p_L)$ and $A_H = S_H(p_H) - [S_H(p_L) - S_L(p_L)]$. Using this, the monopolist's problem is reduced to

$$\begin{aligned}\max_{(p_H, p_L)} \ \pi = &(1-\lambda_P-\lambda_L)[S_H(p_H) - S_H(p_L) + S_L(p_L)] + (\lambda_P+\lambda_L)S_L(p_L) \\ &+ (p_H - c)(1-\lambda_P-\lambda_L)D_H(p_H) + (p_L - c)[\lambda_P D_H(p_L) + \lambda_L D_L(p_L)]. \end{aligned} \tag{8}$$

I denote the optimal marginal prices under incomplete information as $(\widehat{p}_H, \widehat{p}_L)$. The first-order condition with respect to p_H is the same as (5), and we obtain $\widehat{p}_H = c$. The first-order condition with respect to p_L is

$$\begin{aligned}\frac{\partial \pi}{\partial p_L} &= (1-\lambda_P-\lambda_L)[{S_L}'(p_L) - {S_H}'(p_L)] + (\lambda_P+\lambda_L){S_L}'(p_L) \\ &\quad + \lambda_P D_H(p_L) + \lambda_L D_L(p_L) + (p_L - c)[\lambda_P {D_H}'(p_L) + \lambda_L {D_L}'(p_L)] \\ &= (1-\lambda_L)[D_H(p_L) - D_L(p_L)] + (p_L - c)[\lambda_P {D_H}'(p_L) + \lambda_L {D_L}'(p_L)] = 0. \end{aligned} \tag{9}$$

Thus, the equilibrium marginal price for type L and type P satisfies

$$\widehat{p}_L = c - \frac{(1-\lambda_L)[D_H(\widehat{p}_L) - D_L(\widehat{p}_L)]}{\lambda_P {D_H}'(\widehat{p}_L) + \lambda_L {D_L}'(\widehat{p}_L)} > c. \tag{10}$$

The net surplus of type L is $w_L = 0$, because the participation constraint is binding. In contrast, type H receives the information rent, R, that is, $w_H = R(\widehat{p}_L) \equiv S_H(\widehat{p}_L) - S_L(\widehat{p}_L) > 0$. Type P's ex-post surplus is $w_P = S_H(\widehat{p}_L) - A_L = S_H(\widehat{p}_L) - S_L(\widehat{p}_L) > 0$, which is equal to the information rent to type H; because the incentive constraint (1) is binding, the overpessimistic, type P, consumers obtain the same net surplus as type H consumers. By comparing (10) with (7), we see that $\widehat{p}_L > p_L^*$; under incomplete information, the monopolist raises the marginal price for type L more than complete information to reduce the information rent.

The ignored incentive constraint for type L, (2), is rewritten as

$$0 \geq S_L(\widehat{p}_H) - S_H(\widehat{p}_H) + S_H(\widehat{p}_L) - S_L(\widehat{p}_L) = \int_{\widehat{p}_H}^{\widehat{p}_L} [D_L(p) - D_H(p)]dp.$$

Because $\widehat{p}_H = c < \widehat{p}_L$, this is satisfied with strict inequality. This result is due to the well-known rent-efficiency tradeoff; the monopolist distorts the marginal price for type L upward to reduce the information rent to type H (and type P). The marginal price for type H is efficient because it yields the maximum fixed fee, and the marginal price for type H does not affect the information rent. The optimal two-part tariffs are depicted in Figure 2. Type H consumers are indifferent between the two tariffs; the indifference curve of type H tangent to the tariff $(\widehat{A}_H, \widehat{p}_H)$ is also tangent to the tariff $(\widehat{A}_L, \widehat{p}_L)$. Type L consumers receive zero net surplus; the indifference curve of type L tangent to the tariff $(\widehat{A}_L, \widehat{p}_L)$ crosses the origin.[9] The quantities are $\widehat{q}_H = D_H(c)$, $\widehat{q}_P = D_H(\widehat{p}_L)$, $\widehat{q}_L = D_L(\widehat{p}_L)$, and it follows that $\widehat{q}_L < \widehat{q}_P < \widehat{q}_H$. Compared to complete information, we have $\widehat{q}_L < q_L^*$, $\widehat{q}_P < q_P^*$ and $\widehat{q}_H = q_H^*$.

[Figure 2]

Proposition 2 *Suppose that types of consumers are unobservable. In the optimal two-part tariff for a monopolist under the presence of overpessimistic type P consumers, the marginal price for type H is the same as complete information ($\widehat{p}_H = p_H^* = c$). The marginal price for type L and type P is larger than complete information, $\widehat{p}_L > p_L^* > c$. The fixed fee for type H is $\widehat{A}_H = S_H(c) - S_H(\widehat{p}_L) + S_L(\widehat{p}_L)$, and that for type L and type P is $\widehat{A}_L = S_L(\widehat{p}_L)$. Type H and type P obtain the same net surplus, $w_H = w_P = S_H(\widehat{p}_L) - S_L(\widehat{p}_L) > 0$, while the net surplus of type L is $w_L = 0$.*

Now, I investigate the effects of a change in the fraction of overpessimistic consumers. In particular, we are interested in the impacts of debiasing policy toward overpessimistic consumers, namely, a decrease in λ_P given the level of λ_L, which leads to an increase in $\lambda_H = 1 - \lambda_P - \lambda_L$. First, the marginal price for type H does not depend on λ_P. The marginal price for type L and type P, $\widehat{p}_L$, is decreasing in λ_P; differentiating (9) with respect to λ_P, we have

$$\frac{\partial \widehat{p}_L}{\partial \lambda_P} = -\frac{(\widehat{p}_L - c){D_H}'(\widehat{p}_L)}{\frac{\partial^2 \pi}{\partial p_L^2}} < 0,$$

because we are assuming that $\partial^2 \pi / \partial p_L^2 < 0$. That is, the debiasing policy enlarges quantity distortion for type L and type P. When more overpessimistic consumers are debiased, the monopolist becomes less concerned with the loss in variable profit from type P by raising the marginal price, and hence it raises the marginal price to reduce the information rent.

The equilibrium profit becomes

$$\pi(\widehat{p}_H = c, \widehat{p}_L, \lambda_P, \lambda_L) = (1 - \lambda_P - \lambda_L)[S_H(c) - S_H(\widehat{p}_L)] + S_L(\widehat{p}_L) + (\widehat{p}_L - c)[\lambda_P D_H(\widehat{p}_L) + \lambda_L D_L(\widehat{p}_L)].$$

[9] Because I restrict possible contracts to two-part tariffs, the outcomes in Figure 2 are different from the optimal contract described in the standard textbook argument. At the optimal contract, type H is indifferent between the pairs of quantity and payment, (q_H, T_H) and (q_L, T_L); the indifference curve of type H across (q_H, T_H) also crosses (q_L, T_L). However, this optimal contract cannot be implemented by the two-part tariffs.

By differentiating with respect to λ_P, and applying the envelope theorem, we have

$$\frac{\partial \pi}{\partial \lambda_P} = S_H(\widehat{p}_L) - S_H(c) + (\widehat{p}_L - c)D_H(\widehat{p}_L) + \frac{\partial \pi}{\partial p_L}\frac{\partial \widehat{p}_L}{\partial \lambda_P} = -\int_c^{\widehat{p}_L} [D_H(p) - D_H(\widehat{p}_L)]dp < 0,$$

because D_H is strictly decreasing. A decrease in the overpessimistic consumers enlarges the revenue from the fixed fee while reducing the variable profit from type P. However, a gain in the fixed fee turns out to be larger than a loss in the variable profit. Therefore, the monopolist benefits from the debiasing policy of overpessimistic consumers.

A change in λ_P does not affect the net surplus of type L, but does affect the net surplus of type H and P. Because

$$\frac{\partial R(\widehat{p}_L)}{\partial \lambda_P} = -[D_H(\widehat{p}_L) - D_L(\widehat{p}_L)]\frac{\partial \widehat{p}_L}{\partial \lambda_P} > 0,$$

a decrease in the number of overpessimistic consumers reduces the information rent. In this respect, the presence of overpessimistic consumers yields a positive externality to type H. Because the monopolist reacts to the debiasing policy by raising the marginal price for type L to reduce the information rent, it makes type H and type P worse off, and keeps type L unchanged. In other words, the debiasing policy has an adverse effect not only on the debiased type P consumers (who turn into type H) but on the remaining biased (type P) consumers.

The total consumer surplus, $w = (1-\lambda_P-\lambda_L)w_H+\lambda_P w_P+\lambda_L w_L$, is now $w(\widehat{p}_L, \lambda_L) = (1-\lambda_L)[S_H(\widehat{p}_L)-S_L(\widehat{p}_L)]$, and the impact on the total consumer surplus is given by

$$\frac{\partial w}{\partial \lambda_P} = -(1-\lambda_L)[D_H(\widehat{p}_L) - D_L(\widehat{p}_L)]\frac{\partial \widehat{p}_L}{\partial \lambda_P} > 0.$$

That is, the debiasing policy reduces the total consumer surplus because it reduces the total information rent to type H and type P. However, the impact of a decrease in the overpessimistic consumers on the total surplus, $W = w + \pi$, is ambiguous.

Proposition 3 *In monopoly with overpessimistic consumers, the debiasing policy reducing the overpessimistic consumers raises the monopolist's profit. It makes type H and type P worse off, and keeps type L unchanged. It reduces the total consumer surplus, but the effect on social welfare is ambiguous.*

To investigate further the effect on social welfare, I consider two examples. The first is the quadratic utility, $u_j(q) = \theta_j q - \frac{1}{2}q^2$ where $\theta_H > \theta_L$. This yields the linear demand, $D_j(p) = \theta_j - p$, and $S_j = \frac{1}{2}(\theta_j - p)^2$.[10] The second example is $u_j(q) = \frac{\varepsilon}{\varepsilon-1}\theta_j^{\frac{1}{\varepsilon}}q^{1-\frac{1}{\varepsilon}}$ ($\varepsilon > 1$) which yields the iso-elastic demand, $D_j(p) = \theta_j p^{-\varepsilon}$ with $S_j(p) = \frac{\theta_j}{\varepsilon-1}p^{1-\varepsilon}$. In the Appendix, I show that $\partial W/\partial \lambda_P > 0$ holds under the linear demand. Therefore, the positive effect of debiasing policy on profit is dominated by the negative effect on

[10] Another expression of the quadratic utility is given by $u_j(q) = \theta_j(q - \frac{1}{2}q^2)$. However, we can obtain the same qualitative results with this alternative formulation.

the consumer surplus, so that a decrease in λ_P reduces the total surplus. Under the iso-elastic demand, we have at least $\lim_{\varepsilon \to 1} \partial W/\partial \lambda_P > 0$, and a numerical analysis reveals that $\partial W/\partial \lambda_P > 0$ holds for any $\varepsilon > 1$ in a wide range of parameter values of the iso-elastic demand.

Remark 1 *In monopoly with overpessimistic consumers, the debiasing policy reducing the overpessimistic consumers deteriorates social welfare under the linear demand, and the same is true under the iso-elastic demand when the elasticity is sufficiently small.*

4.2 Overoptimistic Consumers

Next, I consider the case where biased consumers are overoptimistic. In Stage 1, type H and L correctly infer their consumer surplus, but type O mistakenly perceives her net benefit is $S_H(p_H) - A_H$, although her ex-post surplus is $S_L(p_H) - A_H$.

4.2.1 Complete Information

Let us begin with complete information. Because the participation constraints are binding, the monopolist's problem is:

$$\max_{(p_H, p_L)} \pi = (\lambda_H + \lambda_O)S_H(p_H) + (1 - \lambda_H - \lambda_O)S_L(p_L) + (p_H - c)[\lambda_H D_H(p_H) + \lambda_O D_L(p_H)] + (p_L - c)(1 - \lambda_H - \lambda_O)D_L(p_L). \tag{11}$$

The first-order condition with respect to p_H is

$$\begin{aligned}\frac{\partial \pi}{\partial p_H} &= (\lambda_H + \lambda_O){S_H}'(p_H) + \lambda_H D_H(p_H) + \lambda_O D_L(p_H) + (p_H - c)[\lambda_H {D_H}'(p_H) + \lambda_O {D_L}'(p_H)] \\ &= \lambda_O[D_L(p_H) - D_H(p_H)] + (p_H - c)[\lambda_H {D_H}'(p_H) + \lambda_O {D_L}'(p_H)] = 0,\end{aligned} \tag{12}$$

which yields

$$p_H^* = c + \frac{\lambda_O[D_H(p_H^*) - D_L(p_H^*)]}{\lambda_H {D_H}'(p_H^*) + \lambda_O {D_L}'(p_H^*)} < c. \tag{13}$$

The equilibrium marginal price, p_H^*, depends on the ratio λ_H/λ_O and is lower than the marginal cost. Note that $p_H^* = c$ if $\lambda_O = 0$. When overoptimistic consumers are present, the monopolist reduces the marginal price for type H and type O to enlarge the fixed fee for them even though it makes a loss in variable profit. The first-order condition with respect to p_L is

$$\frac{\partial \pi}{\partial p_L} = (1 - \lambda_H - \lambda_O)[{S_L}'(p_L) + D_L(p_L) + (p_L - c){D_L}'(p_L)] = 0, \tag{14}$$

which yields $p_L^* = c$. Again, I assume that the profit is strictly concave in (p_H, p_L) to satisfy the second-order conditions.

The net surplus of type H and type L is $w_H = w_L = 0$, and type O's ex-post (actual) consumer surplus is $w_O = S_L(p_H^*) - A_H < 0$. As before, this optimal two-part tariff under complete information does not satisfy the incentive constraint for type H, (1), because $S_H(p_H^*) - A_H^* = 0$ and $S_H(p_L^*) - A_L^* = S_H(c) - S_L(c) > 0$. In contrast, the incentive constraint for type L, (2), is satisfied with strict inequality. Figure 3 describes the optimal two-part tariffs with overpessimistic consumers. Again, the indifference curves of type L and type H cross the origin because the participation constraints are binding. The quantity for type L, q_L^*, is efficient, but both q_H^* and q_O^* are distorted upward.

[Figure 3]

Proposition 4 *Suppose that types of consumers are observable. In the optimal two-part tariff for a monopolist under the presence of overoptimistic type O consumers, the marginal price for type H and type O is lower than the marginal cost ($p_H^* < c$), and that for type L is equal to the marginal cost ($p_L^* = c$). The fixed fee for type H and type O is $A_H^* = S_H(p_H^*)$, and that for type L is $A_L^* = S_L(c)$.*

4.2.2 Incomplete Information

Next, I turn to incomplete information. Among the constraints (1)−(4), only (1) and (4) are binding as before. The monopolist's problem is:

$$\max_{(p_H, p_L)} \pi = (\lambda_H + \lambda_O)\left[S_H(p_H) - S_H(p_L) + S_L(p_L)\right] + (1 - \lambda_H - \lambda_O) S_L(p_L) \\ + (p_H - c)[\lambda_H D_H(p_H) + \lambda_O D_L(p_H)] + (p_L - c)(1 - \lambda_H - \lambda_O) D_L(p_L) \tag{15}$$

The first-order condition with respect to p_H is the same as (12), and therefore we have $\widehat{p}_H = p_H^*$. As before, the marginal price for type H is the same as complete information because it does not affect the information rent. The first-order condition with respect to p_L is

$$\frac{\partial \pi}{\partial p_L} = (\lambda_H + \lambda_O)[S_L{}'(p_L) - S_H{}'(p_L)] + (1 - \lambda_H - \lambda_O)[S_L{}'(p_L) + D_L(p_L) + (p_L - c) D_L{}'(p_L)] \\ = (\lambda_H + \lambda_O)[D_H(p_L) - D_L(p_L)] + (1 - \lambda_H - \lambda_O)(p_L - c) D_L{}'(p_L) = 0, \tag{16}$$

which yields

$$\widehat{p}_L = c - \frac{(\lambda_H + \lambda_O)[D_H(\widehat{p}_L) - D_L(\widehat{p}_L)]}{(1 - \lambda_H - \lambda_O) D_L{}'(\widehat{p}_L)} > c. \tag{17}$$

The equilibrium marginal price, $\widehat{p}_L$, depends on $\lambda_L = 1 - \lambda_H - \lambda_O$. It is larger than $p_L^* = c$ because the monopolist wishes to reduce the information rent.

As we have shown, the marginal price for type H and type O is smaller, and that for L is larger than the marginal cost. Thus, the ignored incentive constraint for type L, (2), is satisfied with strict inequality. The net surplus of type L is zero, $w_L = 0$, whereas type H receives the information rent, $w_H = R(\widehat{p}_L) = S_H(\widehat{p}_L) - S_L(\widehat{p}_L) > 0$. Type O's ex-post (actual) consumer surplus is

$$\begin{aligned} w_O &= S_L(\widehat{p}_H) - A_H = S_L(\widehat{p}_H) - S_H(\widehat{p}_H) + S_H(\widehat{p}_L) - S_L(\widehat{p}_L) \\ &= \int_{\widehat{p}_H}^{\widehat{p}_L} [{S_H}'(p) - {S_L}'(p)]dp = \int_{\widehat{p}_H}^{\widehat{p}_L} [D_L(p) - D_H(p)]dp < 0, \end{aligned}$$

which means that overoptimistic consumers suffer a loss by choosing the wrong tariff. Figure 4 depicts the optimal tariffs under incomplete information. As before, type H consumers are indifferent between the two tariffs, and type L consumers receive zero net surplus. In contrast to the case of overpessimistic consumers, the monopolist sets the marginal price for type H and type O below the marginal cost to enlarge a gain in the fixed fee, although it suffers from a loss in the variable profit. Under incomplete information, the marginal price for type L becomes larger than the marginal cost to reduce the information rent. The quantities are $\widehat{q}_H = D_H(\widehat{p}_H)$, $\widehat{q}_O = D_L(\widehat{p}_H)$, $\widehat{q}_L = D_L(\widehat{p}_L)$, and we have $\widehat{q}_L < \widehat{q}_O < \widehat{q}_H$. Compared to complete information, it follows that $\widehat{q}_L < q_L^*$, $\widehat{q}_O = q_O^*$ and $\widehat{q}_H = q_H^*$.

[Figure 4]

Proposition 5 *Suppose that types of consumers are unobservable. In the optimal two-part tariff for a monopolist under the presence of overoptimistic type O consumers, the marginal price for type H and type O is the same as complete information ($\widehat{p}_H = p_H^* < c$). The marginal price for type L is larger than the marginal cost ($\widehat{p}_L > c$). The fixed fee for type H and type O is $A_H^* = S_H(\widehat{p}_H) - S_H(\widehat{p}_L) + S_L(\widehat{p}_L)$, and that for type L is $A_L^* = S_L(\widehat{p}_L)$. The net surpluses are $w_H > 0$, $w_L = 0$, $w_O < 0$.*

Now, let us consider the effect of a change in the fraction of overoptimistic consumers under incomplete information. In particular, we are interested in the impact of debiasing overoptimistic consumers, a decrease in λ_O given the level of λ_H, which leads to an increase in λ_L.

First, consider the marginal price for type H and type O in (13). By differentiating (12) with respect to λ_O, we have

$$\frac{\partial \widehat{p}_H}{\partial \lambda_O} = \frac{\lambda_H {D_H}'(\widehat{p}_H)[D_H(\widehat{p}_H) - D_L(\widehat{p}_H)]}{\frac{\partial^2 \pi}{\partial p_H^2} \cdot [\lambda_H {D_H}'(\widehat{p}_H) + \lambda_O {D_L}'(\widehat{p}_H)]} < 0,$$

because we are assuming that $\partial^2 \pi / \partial p_H^2 < 0$. Thus, the marginal price for type H and type O is decreasing in λ_O. Similarly, as to the marginal price for type L in (17), by differentiating (16) with respect to λ_O, we have

$$\frac{\partial \widehat{p}_L}{\partial \lambda_O} = -\frac{D_H(\widehat{p}_L) - D_L(\widehat{p}_L)}{\frac{\partial^2 \pi}{\partial p_L^2} \cdot (1 - \lambda_H - \lambda_O)} > 0,$$

because $\partial^2 \pi / \partial p_L^2 < 0$. That is, the marginal price for type L is increasing in λ_O. When more overoptimistic consumers are debiased, the marginal price for type H and type O rises while the marginal price for type

L falls. Therefore, the debiasing policy leads to a more efficient quantity for each type. The more overoptimistic consumers are debiased, the less incentive the monopolist has to reduce the marginal price for type H and type O in order to enlarge revenue from the fixed fee. Moreover, because debiasing leads to more type L consumers, the monopolist becomes more concerned with the efficient quantity for type L rather than reducing information rent for type H.

By differentiating (15) with respect to λ_O, and applying the envelope theorem, we have

$$\begin{aligned}\frac{\partial \pi}{\partial \lambda_O} &= S_H(\widehat{p}_H) - S_H(\widehat{p}_L) + (\widehat{p}_H - c)D_L(\widehat{p}_H) - (\widehat{p}_L - c)D_L(\widehat{p}_L) + \frac{\partial \pi}{\partial p_H}\frac{\partial \widehat{p}_H}{\partial \lambda_O} + \frac{\partial \pi}{\partial p_L}\frac{\partial \widehat{p}_L}{\partial \lambda_O} \\ &= \int_{\widehat{p}_H}^{\widehat{p}_L} D_H(p)dp + (\widehat{p}_H - c)D_L(\widehat{p}_H) - (\widehat{p}_L - c)D_L(\widehat{p}_L) \\ &= \left[\int_c^{\widehat{p}_L} D_H(p)dp - (\widehat{p}_L - c)D_L(\widehat{p}_L)\right] + \left[\int_{\widehat{p}_H}^{c} D_H(p)dp - (c - \widehat{p}_H)D_L(\widehat{p}_H)\right]. \end{aligned} \tag{18}$$

In general, the sign of (18) is ambiguous. As to the first term of (18), it follows that

$$\int_c^{\widehat{p}_L} D_H(p)dp - (\widehat{p}_L - c)D_L(\widehat{p}_L) > \int_c^{\widehat{p}_L} D_H(p)dp - (\widehat{p}_L - c)D_H(\widehat{p}_L) = \int_c^{\widehat{p}_L} [D_H(p) - D_H(\widehat{p}_L)]dp > 0.$$

The first inequality follows from $D_H(p) > D_L(p)$ and $\widehat{p}_L > c > \widehat{p}_H$, and the second follows because D_H is strictly decreasing. About the second term of (18), if $D_L(\widehat{p}_H) < D_H(c)$ holds, then we have

$$\int_{\widehat{p}_H}^{c} D_H(p)dp - (c - \widehat{p}_H)D_L(\widehat{p}_H) > \int_{\widehat{p}_H}^{c} D_H(p)dp - (c - \widehat{p}_H)D_H(c) = \int_{\widehat{p}_H}^{c} [D_H(p) - D_H(c)]dp > 0,$$

and thus (18) is positive. Because a decrease in the overoptimistic consumers implies that some of them turn to type L, the debiasing policy reduces the revenue from the fixed fee, and at the same time it enlarges the variable profit from overoptimistic consumers who changed to type L. The sign of the total effect is ambiguous in general. However, if the optimal marginal price satisfies $D_L(\widehat{p}_H) < D_H(c)$, a loss in the fixed fee is larger than a gain in the variable profit. Therefore, a decrease in overoptimistic consumers reduces the monopolist's profit.

Next, consider the effect on consumer surplus. A change in λ_O does not affect the net surplus of type L, but does affect the net surplus of type H and type O. Because

$$\frac{\partial R(\widehat{p}_L)}{\partial \lambda_O} = -[D_H(\widehat{p}_L) - D_L(\widehat{p}_L)]\frac{\partial \widehat{p}_L}{\partial \lambda_O} < 0,$$

A decrease in the number of overoptimistic consumers enlarges the information rent to type H. Put differently, the presence of overoptimistic consumers yields a negative externality to type H. The monopolist reacts to the debiasing policy reducing λ_O (raising λ_L) by lowering the marginal price for type L, and hence the information rent to type H becomes large. The total consumer surplus, $w = \lambda_H w_H + \lambda_O w_O + (1 - \lambda_H - \lambda_O)w_L$, is expressed as

$$w(\widehat{p}_H, \widehat{p}_L, \lambda_H, \lambda_O) = (\lambda_H + \lambda_O)[S_H(\widehat{p}_L) - S_L(\widehat{p}_L)] + \lambda_O[S_L(\widehat{p}_H) - S_H(\widehat{p}_H)].$$

It follows that

$$\frac{\partial w}{\partial \lambda_O} = S_H(\widehat{p}_L) - S_L(\widehat{p}_L) + S_L(\widehat{p}_H) - S_H(\widehat{p}_H)$$
$$+ \lambda_O[D_H(\widehat{p}_H) - D_L(\widehat{p}_H)]\frac{\partial \widehat{p}_H}{\partial \lambda_O} - (\lambda_H + \lambda_O)[D_H(\widehat{p}_L) - D_L(\widehat{p}_L)]\frac{\partial \widehat{p}_L}{\partial \lambda_O} < 0,$$

That is, a decrease in the number of overoptimistic consumers increases the total consumer surplus. When more overoptimistic consumers are debiased, not only a debiased consumer does not suffer from a loss, but the information rent to type H increases. Moreover, the debiasing also mitigates the loss of (remaining) type O's surplus by changing both marginal prices, $\widehat{p}_H$ and $\widehat{p}_L$, in the favorable direction. Therefore, the debiasing policy makes type H and type O better off, and keeps type L unchanged. In contrast to the case of overpessimistic consumers, it is beneficial to both the debiased type O consumers (who turn into type L) and the remaining biased (type O) consumers.

The impact on the total surplus, $W = w + \pi$, is given by

$$\frac{\partial W}{\partial \lambda_O} = S_L(\widehat{p}_H) - S_L(\widehat{p}_L) - (c - \widehat{p}_H)D_L(\widehat{p}_H) - (\widehat{p}_L - c)D_L(\widehat{p}_L)$$
$$+ \lambda_O[D_H(\widehat{p}_H) - D_L(\widehat{p}_H)]\frac{\partial \widehat{p}_H}{\partial \lambda_O} - (\lambda_H + \lambda_O)[D_H(\widehat{p}_L) - D_L(\widehat{p}_L)]\frac{\partial \widehat{p}_L}{\partial \lambda_O}. \tag{19}$$

Again, the sign of this expression is ambiguous.

Proposition 6 *In monopoly with overoptimistic consumers, the effect of the debiasing policy reducing the overoptimistic consumers on the monopolist's profit is ambiguous, but it reduces the profit if $D_L(\widehat{p}_H) < D_H(c)$ holds. It makes type H and type O better off, and keeps type L unchanged. It raises the total consumer surplus, but the effect on social welfare is ambiguous.*

As before, I consider two examples to see the impact on the total surplus. In the Appendix, I show that $\partial W/\partial \lambda_O < 0$ in the linear demand example. Therefore, the negative impact of the debiasing policy on profit is dominated by the positive effect on the total consumer surplus, and hence a decrease in λ_O raises the total surplus. Under the iso-elastic demand, we have at least $\lim_{\varepsilon \to 1} \partial W/\partial \lambda_O < 0$, and a numerical analysis reveals that $\partial W/\partial \lambda_O < 0$ holds for any $\varepsilon > 1$ in a wide range of parameter values of the iso-elastic demand.

Remark 2 *In monopoly with overoptimistic consumers, the debiasing policy reducing the overoptimistic consumers improves social welfare under the linear demand, and the same is true under the iso-elastic demand when the elasticity is sufficiently small.*

5 Two-Part Tariff Competition

Following Armstrong and Vickers (2001), I consider a variation of the Hotelling duopoly with allowing variable consumption quantity for consumers. Consumers are uniformly distributed on $[0,1]$ interval, with the total mass normalized to one. Firm 1 is located at 0, and firm 2 at 1. At each location, there exist three types of consumers, type H, type L, and the type with a biased belief (type P or type O). It is assumed that consumers make all their purchases from a single firm ("one-stop-shopping" assumption).

Firm i ($i = 1, 2$) offers a menu of two-part tariffs, $T_H^i = A_H^i + p_H^i q_H$ for type H, and $T_L^i = A_L^i + p_L^i q_L$ for type L. Each firm has the same marginal cost, c. The utility function is $u_j(q)$ ($j = H, L$), and I maintain the same assumptions about $u_j(q)$. In Stage 2, the quantity consumed by consumers who purchased from firm i is determined by $u_j'(q_j) = p_j^i$, and hence it is given by $q_j = (u_j{}')^{-1}(p_j^i) \equiv D_j(p_j^i)$. Type H consumes $D_H(p_H^i)$ and type L consumes $D_L(p_L^i)$, respectively. Overpessimistic type P consumes $D_H(p_L^i)$, and overoptimistic type O consumes $D_L(p_H^i)$.

Let $S_j(p_j^i) = u_j(D_j(p_j^i)) - p_j^i D_j(p_j^i)$ be the consumer surplus of type j gross of fixed fee. We have $S_j{}'(p_j^i) = -D_j(p_j^i)$. Denote the net consumer surplus of type j buying from firm i as $w_j^i = S_j(p_j^i) - A_j^i$; w_H^i and w_L^i are true net surplus for type H and type L, respectively. The perceived net surplus for type P at the time of contracting is w_L^i, and that for type O is w_H^i. I assume that each consumer receives a benefit r from buying a good independent of quantity and r is sufficiently large that all consumers would buy a good. The consumer located at x incurs transportation cost tx if she buys from firm 1 and $t(1-x)$ if she buys from firm 2. The location of the marginal type j consumer is determined by

$$r - t\widetilde{x}_j + w_j^1 = r - t(1 - \widetilde{x}_j) + w_j^2 \quad \Longleftrightarrow \quad \widetilde{x}_j = \frac{1}{2t}(t + w_j^1 - w_j^2).$$

5.1 Overpessimistic Consumers

Overpessimistic type P mistakenly believe they are type L when they are type H. First, I consider two-part tariff competition under complete information where firms can observe types of consumers.

The profit of firm 1 is given by

$$\begin{aligned}
\pi_1 &= (1 - \lambda_P - \lambda_L)\widetilde{x}_H[A_H^1 + (p_H^1 - c)D_H(p_H^1)] \\
&\quad + \lambda_P \widetilde{x}_L[A_L^1 + (p_L^1 - c)D_H(p_L^1)] + \lambda_L \widetilde{x}_L[A_L^1 + (p_L^1 - c)D_L(p_L^1)] \\
&= (1 - \lambda_P - \lambda_L)\frac{1}{2t}(t + w_H^1 - w_H^2)[S_H(p_H^1) - w_H^1 + (p_H^1 - c)D_H(p_H^1)] \\
&\quad + (\lambda_P + \lambda_L)\frac{1}{2t}(t + w_L^1 - w_L^2)[S_L(p_L^1) - w_L^1] + \frac{1}{2t}(t + w_L^1 - w_L^2)(p_L^1 - c)[\lambda_P D_H(p_L^1) + \lambda_L D_L(p_L^1)].
\end{aligned}$$

Firm 1 chooses $(p_H^1, w_H^1, p_L^1, w_L^1)$ to maximize π_1. The first-order condition for p_H^1 is

$$\frac{\partial \pi_1}{\partial p_H^1} = (1 - \lambda_P - \lambda_L)\frac{1}{2t}(t + w_H^1 - w_H^2)[S_H{}'(p_H^1) + D_H(p_H^1) + (p_H^1 - c)D_H{}'(p_H^1)] = 0.$$

Since $S_j{}'(p_j^i) = -D_j(p_j^i)$, this is reduced to $(p_H^1 - c)D_H{}'(p_H^1) = 0$. From $D_j{}' < 0$, we obtain $p_H^1 = c$. The profit of firm 2 is expressed similarly, and we also obtain $p_H^2 = c$. Therefore, the same as monopoly, the equilibrium marginal price satisfies $p_H^1 = p_H^2 \equiv p_H^* = c$. The first-order condition for p_L^1 is

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_L^1} &= (\lambda_P + \lambda_L)\frac{1}{2t}(t + w_L^1 - w_L^2)S_L{}'(p_L^1) \\ &\quad + \frac{1}{2t}(t + w_L^1 - w_L^2)[\lambda_P D_H(p_L^1) + \lambda_L D_L(p_L^1) + (p_L^1 - c)\{\lambda_P D_H{}'(p_L^1) + \lambda_L D_L{}'(p_L^1)\}] \\ &= \frac{1}{2t}(t + w_L^1 - w_L^2)\left[\lambda_P D_H(p_L^1) - \lambda_P D_L(p_L^1) + (p_L^1 - c)\{\lambda_P D_H{}'(p_L^1) + \lambda_L D_L{}'(p_L^1)\}\right] = 0, \end{aligned} \tag{20}$$

by using $S_j{}'(p_j^i) = -D_j(p_j^i)$ again. Thus, p_L^1 satisfies

$$p_L^1 = c - \frac{\lambda_P[D_H(p_L^1) - D_L(p_L^1)]}{\lambda_P D_H{}'(p_L^1) + \lambda_L D_L{}'(p_L^1)} > c. \tag{21}$$

Therefore, the marginal price for type L and type P is larger than the marginal cost. Again, the profit of firm 2 is expressed similarly, and we can derive the same expression about p_L^2. The equilibrium marginal price is given by $p_L^1 = p_L^2 \equiv p_L^* > c$, which is equal to the optimal marginal price in monopoly under complete information, (7). As before, the marginal price for type L is also efficient if type P is absent. This is because duopolists maintain the efficient marginal price to enlarge the fixed fee for each type. When type P is present, however, duopolists want to raise the marginal price for type L and type P to collect variable profits from type P.

As to the optimal fixed fee, the first-order condition for w_H^1 is

$$\frac{\partial \pi_1}{\partial w_H^1} = (1 - \lambda_P - \lambda_L)\frac{1}{2t}[S_H(p_H^1) - w_H^1 + (p_H^1 - c)D_H(p_H^1) - (t + w_H^1 - w_H^2)] = 0.$$

From $p_H^1 = c$, evaluating at the symmetric equilibrium in which $w_H^1 = w_H^2$, we obtain $w_H^1 = w_H^2 = S_H(c) - t$, which implies $A_H^1 = A_H^2 = t$. Finally, the condition for w_L^1 is

$$\frac{\partial \pi_1}{\partial w_L^1} = (\lambda_P + \lambda_L)\frac{1}{2t}[S_L(p_L^1) - w_L^1 - (t + w_L^1 - w_L^2)] + \frac{1}{2t}(p_L^1 - c)[\lambda_P D_H(p_L^1) + \lambda_L D_L(p_L^1)] = 0.$$

Again, evaluating at the symmetric equilibrium in which $w_L^1 = w_L^2$, we obtain

$$w_L^1 = w_L^2 = S_L(p_L^1) - t + \frac{1}{\lambda_P + \lambda_L}(p_L^1 - c)[\lambda_P D_H(p_L^1) + \lambda_L D_L(p_L^1)].$$

The optimal fixed fee for type L and type P is

$$A_L^1 = A_L^2 = t - \frac{1}{\lambda_P + \lambda_L}(p_L^1 - c)[\lambda_P D_H(p_L^1) + \lambda_L D_L(p_L^1)] < t, \tag{22}$$

which implies $A_L^i < A_H^i$.

In choosing the fixed fee for type H, duopolists face the trade-off between enlarging revenue from fixed fees and shrinking market share. Competition leads to the fixed fee equal to t; the firms can enjoy some profits because their products are differentiated. As to the fixed fee for type L and type P, in addition to this trade-off, duopolists also take into account the gain in variable profit by expanding market share; this leads to the fixed fee for type L and type P below t. Note that $A_L^1 = A_L^2 = t$ if $\lambda_P = 0$. Therefore, the optimal menu of two-part tariffs becomes a single two-part tariff when type P is not present. This also implies that both incentive constraints are satisfied with equality under incomplete information.

Plugging the above solutions into the profit of firm 1, we can see that the equilibrium profit is equal to $t/2$, independent of λ_P and λ_L. This is because the gain in variable profit from type L and type P is exactly canceled out by the loss in a reduction of the fixed fee. Thus, the profit amounts to $t/2$ for any consumer type distributions.

Proposition 7 *When firms compete in two-part tariffs under complete information with overpessimistic type P consumers,*

(i) *the marginal price for type H is equal to the marginal cost ($p_H^* = c$), and that for type L and type P is larger than the marginal cost ($p_L^* > c$). These marginal prices are the same as those in monopoly under complete information. The fixed fee for type H is $A_H^* = t$, and that for type L and type P is $A_L^* < t$.*

(ii) *The net surplus of type H and type L are $w_H = S_H(c) - t$, $w_L = S_L(p_L^*) - A_L^*$. The actual surplus of type P is $w_P = S_H(p_L^*) - A_L^* > w_L$.*

(iii) *The equilibrium profit of each firm is $t/2$, independent of λ_P and λ_L.*

(iv) *When the overpessimistic consumer is not present, the optimal menu of two-part tariffs degenerates to a single two-part tariff in which $(p_H^*, A_H^*) = (p_L^*, A_L^*) = (c, t)$.*

[Figure 5]

The equilibrium menu of contracts is depicted in Figure 5. The benchmark is the single two-part tariff when type P is not present; $T = t + cq$. Both firms offer the same tariff, and type H choose $D_H(c)$ (point H) while type L chooses $D_L(c)$. When type P is present, the menu for type H remains the same, and type H continues to choose point H. As to the menu for type L and type P, $T_L = A_L^* + p_L^* q$, the marginal price is larger, and the fixed fee is smaller. Type L chooses point L, and type P chooses point P, respectively. Then, type L consumers are better off with the presence of the overpessimistic consumers.[11] This contrasts to monopoly in which type L receives no externality by the overpessimistic consumers. In addition, as we will see formally in the next proposition, the actual net surplus of type P is smaller than that of type H, that is, $w_H > w_P$; type P consumers suffer from being biased under competition. This result is also opposite to monopoly where type P consumers do not suffer from being biased.

[11] Figure 5 depicts the case where the lines of the two-part tariffs cross at $D_L(c)$. Although this happens under the quadratic utility, it is not true in general. However, as we see in the next proposition, type L consumers strictly prefer choosing (p_L^*, A_L^*) to $(p_H^*, A_H^*) = (c, t)$, so that they are better off with the presence of the overpessimistic consumers.

Now, let us turn to incomplete information in which firms cannot observe types of consumers. Under incomplete information, the menu offered by firm i, $(p_H^i, A_H^i, p_L^i, A_L^i)$, should satisfy the following incentive constraints:

$$S_H(p_H^i) - A_H^i \geq S_H(p_L^i) - A_L^i, \tag{23}$$

$$S_L(p_L^i) - A_L^i \geq S_L(p_H^i) - A_H^i, \tag{24}$$

for $i = 1, 2$. The participation constraints are not necessary because it is always satisfied under duopoly competition. We have the following:

Proposition 8 *When firms compete in two-part tariffs under incomplete information with overpessimistic type P consumers, the optimal two-part tariff under complete information in Proposition 7 always satisfies both* (23) *and* (24) *with strict inequality.*

Proof: See Appendix.

Now, let us turn to the welfare effects of the debiasing policy. The marginal price for type H does not depend on λ_P. As to the marginal price for type L and type P, by differentiating (20) we have

$$\frac{\partial p_L^*}{\partial \lambda_P} = -\frac{1}{2}\frac{D_H(p_L^*) - D_L(p_L^*)}{\frac{\partial^2 \pi_1}{\partial (p_L^1)^2}} \frac{\lambda_L D_L'(p_L^*)}{\lambda_P D_H'(p_L^*) + \lambda_L D_L'(p_L^*)} > 0,$$

because we are assuming that $\partial^2 \pi_1/\partial(p_L^1)^2 < 0$. In contrast to the monopoly under incomplete information, the debiasing policy leads to lower marginal price for type L and type P. The duopolists are less concerned with collecting variable profits when there are fewer overpessimistic consumers.

As we have seen, each firm's profit is $t/2$, and the producer surplus is t. In Figure 5, the debiasing policy changes the tariff for type L and type P towards the tariff for type H, and hence type L is worse off while type P is better off. The total consumer surplus, $w = (1 - \lambda_P - \lambda_L)w_H + \lambda_P w_P + \lambda_L w_L$, is exressed as

$$w = -t + (1 - \lambda_P - \lambda_L)S_H(c) + \lambda_P S_H(p_L^*) + \lambda_L S_L(p_L^*) + (p_L^* - c)[\lambda_P D_H(p_L^*) + \lambda_L D_L(p_L^*)].$$

The total surplus, $W = w + \pi_1 + \pi_2$, amounts to

$$W = (1 - \lambda_P - \lambda_L)S_H(c) + \lambda_P S_H(p_L^*) + \lambda_L S_L(p_L^*) + (p_L^* - c)[\lambda_P D_H(p_L^*) + \lambda_L D_L(p_L^*)].$$

By differentiating with respect to λ_P, we have

$$\begin{aligned}\frac{\partial W}{\partial \lambda_P} = \frac{\partial w}{\partial \lambda_P} &= S_H(p_L^*) - S_H(c) + (p_L^* - c)D_H(p_L^*) + (p_L^* - c)[\lambda_P D_H'(p_L^*) + \lambda_L D_L'(p_L^*)]\frac{\partial p_L^*}{\partial \lambda_P} \\ &= \int_c^{p_L^*} [D_H(p_L^*) - D_H(p)]dp - \lambda_P[D_H(p_L^*) - D_L(p_L^*)]\frac{\partial p_L^*}{\partial \lambda_P} < 0,\end{aligned}$$

where we have used (21) and $\partial p_L^*/\partial \lambda_P > 0$. Therefore, a negative impact of the debiasing policy against type L is dominated by a positive impact against type P.

Proposition 9 *Under competition with overpessimistic type P consumers, the debiasing policy reducing the overpessimistic consumers does not change the profit of each firm. It keeps type H unchanged, and makes type L worse off while type P better off. The total consumer surplus is raised by the debiasing policy. Therefore, the debiasing policy improves social welfare.*

It turns out that the effect of debiasing policy against overpessimistic consumers depends on market structure. As we have seen in Proposition 3, under monopoly it is bad for both the debiased type P consumers who turn into type L and the remaining type P consumers. Under competition, in contrast, it is good for both the debiased and the remaining biased consumers. Moreover, it makes type H worse off and type L unchanged in monopoly, while type H unchanged and type L worse off in competition. Debiasing overpessimism is harmful in monopoly but beneficial in competition, not only for overall consumers but for social welfare.

5.2 Overoptimistic Consumers

Overoptimistic type O mistakenly believe that they are type H when they are type L. First, I consider two-part tariff competition under complete information. The profit of firm 1 is given by

$$\begin{aligned}\pi_1 &= \lambda_H \widetilde{x}_H[A_H^1 + (p_H^1 - c)D_H(p_H^1)] + \lambda_O \widetilde{x}_H[A_H^1 + (p_H^1 - c)D_L(p_H^1)] \\ &\quad + (1-\lambda_H-\lambda_O)\widetilde{x}_L[A_L^1 + (p_L^1 - c)D_L(p_L^1)] \\ &= (\lambda_H+\lambda_O)\frac{1}{2t}(t + w_H^1 - w_H^2)[S_H(p_H^1) - w_H^1] + \frac{1}{2t}(t + w_H^1 - w_H^2)(p_H^1 - c)[\lambda_H D_H(p_H^1) + \lambda_O D_L(p_H^1)] \\ &\quad + (1-\lambda_H-\lambda_O)\frac{1}{2t}(t + w_L^1 - w_L^2)[S_L(p_L^1) - w_L^1 + (p_L^1 - c)D_L(p_L^1)].\end{aligned}$$

Firm 1 chooses $(p_H^1, w_H^1, p_L^1, w_L^1)$ to maximize π_1. The first-order condition for p_H^1 is

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_H^1} &= (\lambda_H+\lambda_O)\frac{1}{2t}(t + w_H^1 - w_H^2){S_H}'(p_H^1) \\ &\quad + \frac{1}{2t}(t + w_H^1 - w_H^2)\left[\lambda_H D_H(p_H^1) + \lambda_O D_L(p_H^1) + (p_H^1 - c)\{\lambda_H {D_H}'(p_H^1) + \lambda_O {D_L}'(p_H^1)\}\right] \\ &= \frac{1}{2t}(t + w_H^1 - w_H^2)\left[\lambda_O(D_L(p_H^1) - D_H(p_H^1)) + (p_H^1 - c)\{\lambda_H {D_H}'(p_H^1) + \lambda_O {D_L}'(p_H^1)\}\right] = 0, \quad (25)\end{aligned}$$

by using ${S_j}'(p_j^i) = -D_j(p_j^i)$ again. Thus, p_H^1 satisfies

$$p_H^1 = c + \frac{\lambda_O[D_H(p_H^1) - D_L(p_H^1)]}{\lambda_H {D_H}'(p_H^1) + \lambda_O {D_L}'(p_H^1)} < c. \quad (26)$$

The profit of firm 2 is defined similarly, and the equilibrium marginal price for type H and type O, $p_H^1 = p_H^2 = p_H^*$, is smaller than the marginal cost. This marginal price is the same as that in monopoly, (13). In particular, the markup is zero if $\lambda_O = 0$. When type O is present, as in monopoly under complete information, duopolists want to reduce the marginal price for type H and type O to enlarge revenue from the fixed fee, sacrificing a loss in the variable profit.

The condition for p_L^1 is

$$\frac{\partial \pi_1}{\partial p_L^1} = (1 - \lambda_H - \lambda_O)\frac{1}{2t}(t + w_L^1 - w_L^2)[S_L{}'(p_L^1) + D_L(p_L^1) + (p_L^1 - c)D_L{}'(p_L^1)] = 0.$$

Since $S_j{}'(p_j^i) = -D_j(p_j^i)$, this is reduced to $(p_L^1 - c)D_L{}'(p_L^1) = 0$. From $D_j{}' < 0$, we obtain $p_L^1 = c$. Again, the profit of firm 2 is defined similarly, and we can derive that $p_L^2 = c$. Therefore, similar to monopoly, each firm chooses the efficient price for type L.

As to the optimal fixed fee, the first-order condition for w_H^1 is

$$\frac{\partial \pi_1}{\partial w_H^1} = (\lambda_H + \lambda_O)\frac{1}{2t}[S_H(p_H^1) - w_H^1 - (t + w_H^1 - w_H^2)] + \frac{1}{2t}(p_H^1 - c)[\lambda_H D_H(p_H^1) + \lambda_O D_L(p_H^1)] = 0.$$

Evaluating at the symmetric equilibrium in which $w_H^1 = w_H^2$, we obtain

$$w_H^1 = w_H^2 = S_H(p_H^1) - t + \frac{1}{\lambda_H + \lambda_O}(p_H^1 - c)[\lambda_H D_H(p_H^1) + \lambda_O D_L(p_H^1)],$$

which implies that the optimal fixed fee is given by

$$A_H^1 = A_H^2 = t - \frac{1}{\lambda_H + \lambda_O}(p_H^1 - c)[\lambda_H D_H(p_H^1) + \lambda_O D_L(p_H^1)] > t. \tag{27}$$

Finally, the condition for w_L^1 is

$$\frac{\partial \pi_1}{\partial w_L^1} = (1 - \lambda_H - \lambda_O)\frac{1}{2t}[S_L(p_L^1) - w_L^1 + (p_L^1 - c)D_L(p_L^1) - (t + w_L^1 - w_L^2)] = 0.$$

From $p_L^1 = c$, evaluating at the symmetric equilibrium in which $w_L^1 = w_L^2$, we obtain $w_L^1 = w_L^2 = S_L(c) - t$, which implies $A_L^1 = A_L^2 = t$. Thus, the fixed fee for type H and O is larger than that for type L. Again, we can confirm that $A_H^1 = A_H^2 = t$ if $\lambda_O = 0$; the optimal menu of two-part tariffs becomes a single two-part tariff when type O is not present.

Proposition 10 *When firms compete in two-part tariffs under complete information with overoptimistic type O consumers,*

(i) *the marginal price for type H and type O is less than the marginal cost ($p_H^* < c$), and that for type L is equal to the marginal cost ($p_L^* = c$). These marginal prices are the same as those in monopoly under complete information. The fixed fee for type H and type O is $A_H^* > t$, and for type L is $A_L^* = t$.*

(ii) *The net surplus of type H and type L are $w_H = S_H(p_H^*) - A_H^*$, $w_L = S_L(c) - t$. The actual surplus of type O is $w_O = S_L(p_H^*) - A_H^* < w_H$.*

(iii) *The equilibrium profit of each firm is $t/2$, independent of λ_H and λ_O.*

[Figure 6]

The equilibrium menu of contracts is depicted in Figure 6. Again, the benchmark is the single two-part tariff when type O is absent; $T = t + cq$. Both firms offer the same tariff, and type H choose $D_H(c)$ while type L chooses $D_L(c)$ (point L). When type O is present, the menu for type L remains the same, and type L continues to choose point L. As to the menu for type H and type O, $T_H = A_H^* + p_H^* q$, in contrast, the marginal price is smaller and the fixed fee is larger. Type H chooses point H, and type O chooses point O, respectively.[12] Then, type H obtains a larger net surplus with the presence of overoptimistic consumers. This result is in contrast to the case of overpessimistic consumers in which type L benefits from positive externality by the presence of biased consumers. In addition, as we will see formally in the next proposition, the net surplus of type O is smaller than that of type L, namely $w_O < w_L$; similar to monopoly, type O suffers from being biased under competition.

Now, let us turn to the case of incomplete information in which firms cannot observe types of consumers. Similar to the case of overpessimistic consumers, we have the following:

Proposition 11 *When firms compete in two-part tariffs under incomplete information with overoptimistic type O consumers, the optimal two-part tariff under complete information in Proposition 10 always satisfies both* (23) *and* (24) *with strict inequality.*

Proof: See Appendix.

Now, I study the welfare effects of debiasing policy. The marginal price for type L does not depend on λ_O. As to the marginal price for type H and O, by differentiating (25) we have

$$\frac{\partial p_H^*}{\partial \lambda_O} = \frac{1}{2} \frac{D_H(p_H^*) - D_L(p_H^*)}{\frac{\partial^2 \pi_1}{\partial (p_H^1)^2}} \frac{\lambda_H D_H'(p_H^*)}{\lambda_H D_H'(p_H^*) + \lambda_O D_L'(p_H^*)} < 0,$$

since we are assuming that $\partial^2 \pi_1 / \partial (p_H^1)^2 < 0$. Like monopoly, the debiasing policy leads to higher marginal price for type H and type O. The duopolists are less concerned with sacrificing variable profits to boost fixed fees when there are less overoptimistic consumers.

As we have seen, each firm's profit is $t/2$, so the producer surplus is t. In Figure 6, the debiasing policy changes the tariff for type H and type O towards the tariff for type L, and hence type H is worse off while type O is better off. The total consumer surplus, $w = \lambda_H w_H + \lambda_O w_O + (1 - \lambda_H - \lambda_O) w_L$, is expressed as

$$w = -t + \lambda_H S_H(p_H^*) + \lambda_O S_L(p_H^*) + (1 - \lambda_H - \lambda_O) S_L(c) + (p_H^* - c)[\lambda_H D_H(p_H^*) + \lambda_O D_L(p_H^*)].$$

[12] Figure 6 describes the case where the two-part tariffs cross at $D_H(c)$. Although this happens under quadratic utility, it is not true in general. However, as we see in the next proposition, type H consumers strictly prefer choosing (p_H^*, A_H^*) to $(p_L^*, A_L^*) = (c, t)$, so that they are better off with the presence of the overoptimistic consumers.

The total surplus, $W = w + \pi_1 + \pi_2$, amounts to

$$W = \lambda_H S_H(p_H^*) + \lambda_O S_L(p_H^*) + (1 - \lambda_H - \lambda_O) S_L(c) + (p_H^* - c)[\lambda_H D_H(p_H^*) + \lambda_O D_L(p_H^*)].$$

By differentiating with respect to λ_O, we have

$$\begin{aligned}\frac{\partial W}{\partial \lambda_O} = \frac{\partial w}{\partial \lambda_O} &= S_L(p_H^*) - S_L(c) + (p_H^* - c) D_L(p_H^*) + (p_H^* - c)[\lambda_H D'_H(p_H^*) + \lambda_O D'_L(p_H^*)] \frac{\partial p_H^*}{\partial \lambda_O} \\ &= \int_{p_H^*}^{c} [D_L(p) - D_L(p_H^*)] dp + \lambda_O [D_H(p_H^*) - D_L(p_H^*)] \frac{\partial p_H^*}{\partial \lambda_O} < 0,\end{aligned}$$

where the we have used (26) and $\partial p_H^* / \partial \lambda_O < 0$. Therefore, a negative impact of the debiasing policy against type H is dominated by a positive impact against type O.

Proposition 12 *Under competition with overoptimistic type O consumers, the debiasing policy reducing the overoptimistic consumers does not change the profit of each firm. It keeps type L unchanged, and makes type H worse off while type O better off. The total consumer surplus is raised by the debiasing policy. Therefore, the debiasing policy improves social welfare.*

From Propositions 6 and 12, we can see that the debiasing policy against overoptimism has similar effect on consumers under any market structure. In monopoly as well as in competition, it is good not only for the debiased type O consumers who turn into type H but for still biased type O consumers. The only difference is that it makes type H better off in monopoly while makes them worse off in competition; it keeps type L unchanged in monopoly and competition. Debiasing overoptimism is beneficial not only for overall consumers but for social welfare, regardless of market structure.

6 Conclusion

In this article, I examined how a monopolist and competitive firms respond in designing two-part tariffs when some consumers either underestimate or overestimate their true demand, and how the existence of biased consumers affects unbiased consumers. Incomplete information about consumer types matters in monopoly but not in competition, because full-information equilibrium remains an equilibrium in competition. When unbiased consumers are heterogeneous in their usage levels, consumers with biased beliefs could have different impacts on different types of consumers, depending on market structure. I have shown that the presence of overpessimistic consumers makes high-type consumers better off and keeps low type unchanged in monopoly, while keeps high type unchanged and makes low type better off in competition. Alternatively, the presence of overoptimistic consumers makes high type worse off in monopoly and better off in competition, while always keeps low type unchanged.

Overall, the existence of overpessimistic consumers raises the total consumer surplus in monopoly, but reduces in competition. In contrast, the existence of overoptimistic consumers always reduces the total consumer surplus. From the sellers' viewpoint, the monopolist wants to educate overpessimistic consumers while may not want to educate overoptimistic consumers. The competitive firms, however, do not have the incentive to educate any biased consumers. I find that the monopolist's desire to debias overpessimism is harmful to overall consumers and may deteriorate social welfare. On the other hand, educating overoptimism by the monopolist is beneficial for overall consumers and may improve social welfare. In competition, however, my result suggests that firms do not provide effort which leads to socially desirable debiasing for either overpessimistic or overoptimistic consumers, and therefore some intervention that helps biased consumers to correct their expectations about their own demands would be necessary.

I focused on second-degree price discrimination by two-part tariffs without repeated interaction between consumers and firms. In an intertemporal setting, however, firms can learn about private information or naiveté of consumers by observing their past usage so that they can engage in third-degree (naiveté-based) discrimination. A possible extension would be to incorporate the aspect of third-degree price discrimination by additional information about consumers (Heidhues and Kőszegi 2017) or by usage information on past purchases in dynamic contracting (Johnen 2020).

I do not consider the aspect of learning by consumers (Goettler and Clay 2011, Grubb and Osborne 2015). In this sense, the model of this paper is more prevalent to markets with an inflow of new and unsophisticated consumers or markets where switching costs could make consumers maintain the same tariff plan (Miravete 2003). Incorporating consumer learning with endogenizing fractions of biased consumers would yield further implications about dynamic tariff and usage choices by consumers and evolutions of tariff menus as a screening mechanism among heterogeneous consumers. I leave these issues for future research.

Appendix

Proof of Remark 1: First, under the linear demand, it follows that

$$\widehat{p}_L = c + \frac{1-\lambda_L}{\lambda_P+\lambda_L}(\theta_H-\theta_L),$$

and the quantity for each type of consumers are:

$$\widehat{q}_H = \theta_H - c, \quad \widehat{q}_P = \theta_H - c - \frac{1-\lambda_L}{\lambda_P+\lambda_L}(\theta_H-\theta_L), \quad \widehat{q}_L = \theta_L - c - \frac{1-\lambda_L}{\lambda_P+\lambda_L}(\theta_H-\theta_L).$$

The impact on the total surplus is derived as

$$\frac{\partial W}{\partial \lambda_P} = \frac{1}{2}\left[\frac{1-\lambda_L}{\lambda_P+\lambda_L}(\theta_H-\theta_L)\right]^2 > 0.$$

Next, in the iso-elastic demand, we obtain

$$\widehat{p}_L = \frac{\varepsilon(\lambda_P\theta_H+\lambda_L\theta_L)}{A}c,$$

where $A \equiv \varepsilon(\lambda_P\theta_H+\lambda_L\theta_L)-(1-\lambda_L)(\theta_H-\theta_L)$. From the second-order condition, $\partial^2\pi/\partial p_L^2 < 0$, it follows that $A > 0$. We also have

$$\frac{\partial \widehat{p}_L}{\partial \lambda_P} = -\frac{\varepsilon(1-\lambda_L)\theta_H(\theta_H-\theta_L)c}{A^2} < 0.$$

The effect on the total surplus can be derived as

$$\begin{aligned}
\frac{\partial W}{\partial \lambda_P} &= -(1-\lambda_L)[D_H(\widehat{p}_L)-D_L(\widehat{p}_L)]\frac{\partial \widehat{p}_L}{\partial \lambda_P} + S_H(\widehat{p}_L) - S_H(c) + (\widehat{p}_L - c)D_H(\widehat{p}_L) \\
&= \frac{\theta_H[\varepsilon(\lambda_P\theta_H+\lambda_L\theta_L)]^{-\varepsilon}c^{1-\varepsilon}}{A^{2-\varepsilon}}[K + (1-\lambda_L)(\theta_H-\theta_L)\{\varepsilon(\lambda_P\theta_H+\lambda_L\theta_L)+(\varepsilon-1)(1-\lambda_L)(\theta_H-\theta_L)\}],
\end{aligned}$$

where

$$K = \frac{\varepsilon}{\varepsilon-1}(\lambda_P\theta_H+\lambda_L\theta_L)A\left(1-\left\{\frac{\varepsilon(\lambda_P\theta_H+\lambda_L\theta_L)}{A}\right\}^{\varepsilon-1}\right).$$

Applying l'Hôpital's rule yields

$$\begin{aligned}
\lim_{\varepsilon\to 1} K &= \lim_{\varepsilon\to 1}\frac{d}{d\varepsilon}\left[\varepsilon(\lambda_P\theta_H+\lambda_L\theta_L)A\left(1-\left\{\frac{\varepsilon(\lambda_P\theta_H+\lambda_L\theta_L)}{A}\right\}^{\varepsilon-1}\right)\right] \\
&= \lim_{\varepsilon\to 1}\left[\begin{array}{l}(\lambda_P\theta_H+\lambda_L\theta_L)\left(1-\left\{\frac{\varepsilon(\lambda_P\theta_H+\lambda_L\theta_L)}{A}\right\}^{\varepsilon-1}\right)\{2\varepsilon(\lambda_P\theta_H+\lambda_L\theta_L)-(1-\lambda_L)(\theta_H-\theta_L)\} \\ -\{\varepsilon(\lambda_P\theta_H+\lambda_L\theta_L)\}^{\varepsilon}A^{2-\varepsilon}\left\{\frac{1-\varepsilon}{A}+\frac{\varepsilon-1}{\varepsilon}+\ln\left(\frac{\varepsilon(\lambda_P\theta_H+\lambda_L\theta_L)}{A}\right)\right\}\end{array}\right] \\
&= -(\lambda_P\theta_H+\lambda_L\theta_L)A|_{\varepsilon=1}\cdot\ln\left(\frac{\lambda_P\theta_H+\lambda_L\theta_L}{A|_{\varepsilon=1}}\right),
\end{aligned}$$

where $A|_{\varepsilon=1} = \lambda_P\theta_H+\lambda_L\theta_L-(1-\lambda_L)(\theta_H-\theta_L) > 0$. Therefore, we have

$$\lim_{\varepsilon\to 1}\frac{\partial W}{\partial \lambda_P} = \frac{\theta_H}{A|_{\varepsilon=1}}\left[(1-\lambda_L)(\theta_H-\theta_L) - A|_{\varepsilon=1}\cdot\ln\left(\frac{\lambda_P\theta_H+\lambda_L\theta_L}{A|_{\varepsilon=1}}\right)\right].$$

Because the logarithmic function is strictly concave, we have $\ln(1+x) < x$. Thus, we obtain

$$A|_{\varepsilon=1} \cdot \ln\left(\frac{\lambda_P\theta_H + \lambda_L\theta_L}{A|_{\varepsilon=1}}\right) < A|_{\varepsilon=1}\left(\frac{\lambda_P\theta_H + \lambda_L\theta_L}{A|_{\varepsilon=1}} - 1\right) = (1-\lambda_L)(\theta_H - \theta_L)$$

which implies that $\lim_{\varepsilon\to 1} \partial W/\partial\lambda_P > 0$. Because $\partial W/\partial\lambda_P$ is continuous in ε, $\partial W/\partial\lambda_P > 0$ holds if ε is close to unity. ■

Proof of Remark 2: In the linear demand example, the marginal prices are

$$\widehat{p}_H = c - \frac{\lambda_O}{\lambda_H + \lambda_O}(\theta_H - \theta_L), \quad \widehat{p}_L = c + \frac{\lambda_H + \lambda_O}{1 - \lambda_H - \lambda_O}(\theta_H - \theta_L),$$

and the quantity for each type of consumers are:

$$\widehat{q}_H = \theta_H - c + \frac{\lambda_O}{\lambda_H + \lambda_O}(\theta_H - \theta_L),\ \widehat{q}_O = \theta_L - c + \frac{\lambda_O}{\lambda_H + \lambda_O}(\theta_H - \theta_L),\ \widehat{q}_L = \theta_L - c - \frac{\lambda_H + \lambda_O}{1 - \lambda_H - \lambda_O}(\theta_H - \theta_L).$$

The quantity of type H is larger than the efficient quantity, $\theta_H - c$. The condition $D_L(\widehat{p}_H) < D_H(c)$ is satisfied; the debiasing policy reduces the profit. The impact on the total surplus is now given by

$$\frac{\partial W}{\partial \lambda_O} = -\frac{(\theta_H - \theta_L)^2}{2(\lambda_H + \lambda_O)^2(1 - \lambda_H - \lambda_O)^2}\left[(\lambda_H + \lambda_O)^3(2 - \lambda_H - \lambda_O) + \lambda_O(1 - \lambda_H - \lambda_O)^2(2\lambda_H + \lambda_O)\right] < 0.$$

Next, under the iso-elastic demand, the marginal prices are

$$\widehat{p}_H = \frac{\varepsilon(\lambda_H\theta_H + \lambda_O\theta_L)}{B_1}c, \quad \widehat{p}_L = \frac{\varepsilon(1 - \lambda_H - \lambda_O)\theta_L}{B_2}c$$

where $B_1 \equiv \varepsilon(\lambda_H\theta_H + \lambda_O\theta_L) + \lambda_O(\theta_H - \theta_L)$ and $B_2 \equiv \varepsilon(1 - \lambda_H - \lambda_O)\theta_L - (\lambda_H + \lambda_O)(\theta_H - \theta_L)$. The second-order condition, $\partial^2\pi/\partial p_H^2 < 0$, amounts to $B_1 > 0$, and $\partial^2\pi/\partial p_L^2 < 0$ to $B_2 > 0$. Because the sign of

$$D_L(\widehat{p}_H) - D_H(c) = c^{-\varepsilon}{B_1}^{\varepsilon}\left[\theta_L\{\varepsilon(\lambda_H\theta_H + \lambda_O\theta_L)\}^{-\varepsilon} - \theta_H\{\varepsilon(\lambda_H\theta_H + \lambda_O\theta_L) + \lambda_O(\theta_H - \theta_L)\}^{-\varepsilon}\right]$$

is ambiguous, the condition $D_L(\widehat{p}_H) < D_H(c)$ may not hold in general, but note that it does hold when $\varepsilon = 1$. Also, we have

$$\frac{\partial \widehat{p}_H}{\partial \lambda_O} = -\frac{\varepsilon\lambda_H\theta_H(\theta_H - \theta_L)c}{{B_1}^2} < 0, \quad \frac{\partial \widehat{p}_L}{\partial \lambda_O} = \frac{\varepsilon\theta_L(\theta_H - \theta_L)c}{{B_2}^2} > 0.$$

The impact on the total surplus in (19) now amounts to

$$\begin{aligned}
\frac{\partial W}{\partial \lambda_O} &= (\widehat{p}_H)^{-\varepsilon}\left[\frac{\theta_L}{\varepsilon - 1}\widehat{p}_H - \theta_L(c - \widehat{p}_H) - \frac{\varepsilon\lambda_H\lambda_O\theta_H(\theta_H - \theta_L)^2c}{{B_1}^2}\right] \\
&\quad - (\widehat{p}_L)^{-\varepsilon}\left[\frac{\theta_L}{\varepsilon - 1}\widehat{p}_L + \theta_L(\widehat{p}_L - c) + \frac{\varepsilon(\lambda_H + \lambda_O)\theta_L(\theta_H - \theta_L)^2c}{{B_2}^2}\right] \\
&= \frac{\{\varepsilon(\lambda_H\theta_H + \lambda_O\theta_L)\}^{-\varepsilon}\{\varepsilon(1 - \lambda_H - \lambda_O)\theta_L\}^{-\varepsilon}c^{1-\varepsilon}\theta_L\varepsilon^{\varepsilon}}{{B_1}^{1-\varepsilon}{B_2}^{1-\varepsilon}(\varepsilon - 1)}L_1 \\
&\quad - \frac{\{\varepsilon(\lambda_H\theta_H + \lambda_O\theta_L)\}^{-\varepsilon}\{\varepsilon(1 - \lambda_H - \lambda_O)\theta_L\}^{-\varepsilon}c^{1-\varepsilon}(\theta_H - \theta_L)^2\varepsilon^{1+\varepsilon}}{{B_1}^{2-\varepsilon}{B_2}^{2-\varepsilon}}L_2,
\end{aligned} \tag{A.1}$$

where

$$L_1 = {B_2}^{1-\varepsilon}\{(1-\lambda_H-\lambda_O)\theta_L\}^{\varepsilon}[B_1-\varepsilon\lambda_O(\theta_H-\theta_L)] - {B_1}^{1-\varepsilon}(\lambda_H\theta_H+\lambda_O\theta_L)^{\varepsilon}[B_2+\varepsilon(\lambda_H+\lambda_O)(\theta_H-\theta_L)],$$
$$L_2 = {B_2}^{2-\varepsilon}\{(1-\lambda_H-\lambda_O)\theta_L\}^{\varepsilon}\lambda_H\lambda_O\theta_H + {B_1}^{2-\varepsilon}(\lambda_H\theta_H+\lambda_O\theta_L)^{\varepsilon}(\lambda_H+\lambda_O)\theta_L.$$

By evaluating L_1 at $\varepsilon = 1$, we have

$$L_1|_{\varepsilon=1} = (1-\lambda_H-\lambda_O)\theta_L[B_1|_{\varepsilon=1}-\lambda_O(\theta_H-\theta_L)] - (\lambda_H\theta_H+\lambda_O\theta_L)[B_2|_{\varepsilon=1}+(\lambda_H+\lambda_O)(\theta_H-\theta_L)] = 0,$$

and therefore, from (A.1), we obtain

$$\lim_{\varepsilon\to 1}\frac{\partial W}{\partial\lambda_O} = -\frac{(\theta_H-\theta_L)^2[B_2|_{\varepsilon=1}(1-\lambda_H-\lambda_O)\lambda_H\lambda_O\theta_H + B_1|_{\varepsilon=1}(\lambda_H\theta_H+\lambda_O\theta_L)(\lambda_H+\lambda_O)]}{(\lambda_H\theta_H+\lambda_O\theta_L)(\lambda_H+\lambda_O)B_1|_{\varepsilon=1}B_2|_{\varepsilon=1}} < 0.$$

Because $\partial W/\partial\lambda_O$ is continuous in ε, $\partial W/\partial\lambda_O < 0$ holds if ε is close to unity. ■

Proof of Proposition 8: First, we consider the incentive constraint for type H, (23). Plugging p_H^*, A_H^*, A_L^*, we obtain

$$\begin{aligned}
&S_H(p_H^*) - A_H^* - [S_H(p_L^*) - A_L^*] \\
&= S_H(c) - S_H(p_L^*) - \frac{1}{\lambda_P+\lambda_L}(p_L^*-c)[\lambda_P D_H(p_L^*) + \lambda_L D_L(p_L^*)] \\
&= \int_c^{p_L^*} D_H(p)dp - (p_L^*-c)\left[\frac{\lambda_P}{\lambda_P+\lambda_L}D_H(p_L^*) + \frac{\lambda_L}{\lambda_P+\lambda_L}D_L(p_L^*)\right] \\
&> \int_c^{p_L^*} D_H(p)dp - (p_L^*-c)D_H(p_L^*) = \int_c^{p_L^*}[D_H(p) - D_H(p_L^*)]dp > 0,
\end{aligned}$$

so that (23) is satisfied with strict inequality. As to the incentive constraint for type L and type P, (24), we have

$$S_L(p_L^*) - A_L^* - [S_L(p_H^*) - A_H^*] = S_L(p_L^*) - S_L(c) + (p_L^*-c)\left[\frac{\lambda_P}{\lambda_P+\lambda_L}D_H(p_L^*) + \frac{\lambda_L}{\lambda_P+\lambda_L}D_L(p_L^*)\right].$$

Defining

$$F(p_L) = S_L(p_L) - S_L(c) + (p_L-c)\left[\frac{\lambda_P}{\lambda_P+\lambda_L}D_H(p_L) + \frac{\lambda_L}{\lambda_P+\lambda_L}D_L(p_L)\right]$$

as a function of p_L, this incentive constraint amounts to $F(p_L^*) \geq 0$.

From (20) we obtain

$$\begin{aligned}
F'(p_L) &= \frac{\lambda_P}{\lambda_P+\lambda_L}[D_H(p_L) - D_L(p_L)] + (p_L-c)\left[\frac{\lambda_P}{\lambda_P+\lambda_L}D_H'(p_L) + \frac{\lambda_L}{\lambda_P+\lambda_L}D_L'(p_L)\right] \\
&= \frac{1}{\lambda_P+\lambda_L}\frac{2t}{t+w_L^i-w_L^j}\frac{\partial\pi_i}{\partial p_L^i}.
\end{aligned}$$

Because the optimal marginal price satisfies $\partial\pi_i/\partial p_L^i = 0$, we have $F'(p_L^*) = 0$. Moreover, we have $F(c) = 0$ and $F'(c) > 0$. Because $F''(p_L) < 0$ follows from $\partial^2\pi_i/\partial(p_L^i)^2 < 0$, we must have $F(p_L^*) > 0$, and therefore (24) is satisfied with strict inequality. ■

Proof of Proposition 11: First, I consider the incentive constraint for type L, (24). Plugging p_L^*, A_H^*, A_L^*, we obtain

$$
\begin{aligned}
& S_L(p_L^*) - A_L^* - [S_L(p_H^*) - A_H^*] \\
= & S_L(c) - S_L(p_H^*) - \frac{1}{\lambda_H + \lambda_O}(p_H^* - c)[\lambda_H D_H(p_H^*) + \lambda_O D_L(p_H^*)] \\
= & -\int_{p_H^*}^{c} D_L(p)dp - (p_H^* - c)\left[\frac{\lambda_H}{\lambda_H + \lambda_O} D_H(p_H^*) + \frac{\lambda_O}{\lambda_H + \lambda_O} D_L(p_H^*)\right] \\
> & -\int_{p_H^*}^{c} D_L(p)dp - (p_H^* - c)D_L(p_H^*) = \int_{p_H^*}^{c} [D_L(p_H^*) - D_L(p)]dp > 0,
\end{aligned}
$$

and hence (24) is satisfied with strict inequality. As to the incentive constraint for type H and type O, (23), we have

$$
S_H(p_H^*) - A_H^* - [S_H(p_L^*) - A_L^*] = S_H(p_H^*) - S_H(c) - (c - p_H^*)\left[\frac{\lambda_H}{\lambda_H + \lambda_O} D_H(p_H^*) + \frac{\lambda_O}{\lambda_H + \lambda_O} D_L(p_H^*)\right].
$$

Defining

$$
G(p_H) = S_H(p_H) - S_H(c) - (c - p_H)\left[\frac{\lambda_H}{\lambda_H + \lambda_O} D_H(p_H) + \frac{\lambda_O}{\lambda_H + \lambda_O} D_L(p_H)\right]
$$

as a function of p_H, this incentive constraint amounts to $G(p_H^*) \geq 0$.

From (25) we obtain

$$
\begin{aligned}
G'(p_H) &= \frac{\lambda_O}{\lambda_H + \lambda_O}[D_L(p_H) - D_H(p_H)] - (c - p_H)\left[\frac{\lambda_H}{\lambda_H + \lambda_O} D'_H(p_H) + \frac{\lambda_O}{\lambda_H + \lambda_O} D'_L(p_H)\right] \\
&= \frac{1}{\lambda_H + \lambda_O} \frac{2t}{t + w_H^i - w_H^j} \frac{\partial \pi_i}{\partial p_H^i}.
\end{aligned}
$$

Because the optimal marginal price satisfies $\partial \pi_i / \partial p_H^i = 0$, we have $G'(p_H^*) = 0$. Moreover, we have $G(c) = 0$ and $G'(c) < 0$. Because $G''(p_H) < 0$ follows from $\partial^2 \pi_i / \partial (p_H^i)^2 < 0$, we must have $G(p_H^*) > 0$, and therefore (24) is satisfied with strict inequality. ■

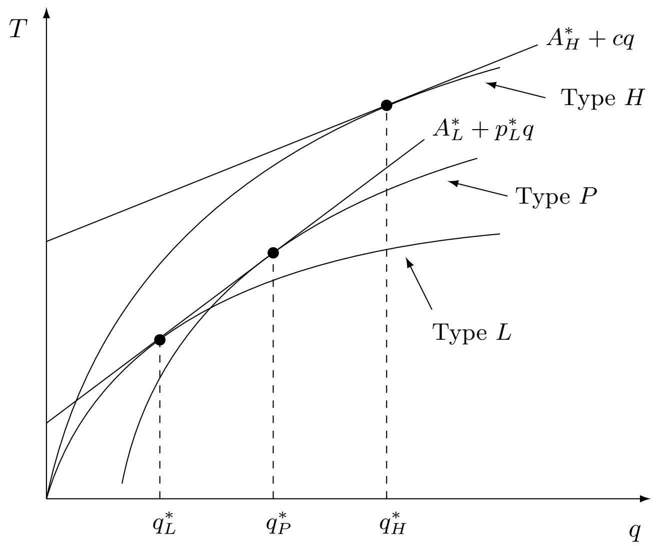

Figure 1: Monopoly with Overpessimistic Consumers; Complete Information

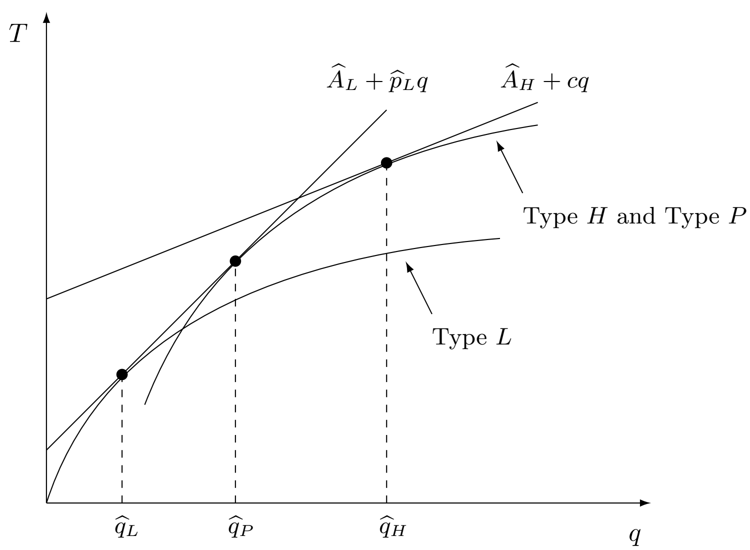

Figure 2: Monopoly with Overpessimistic Consumers; Incomplete Information

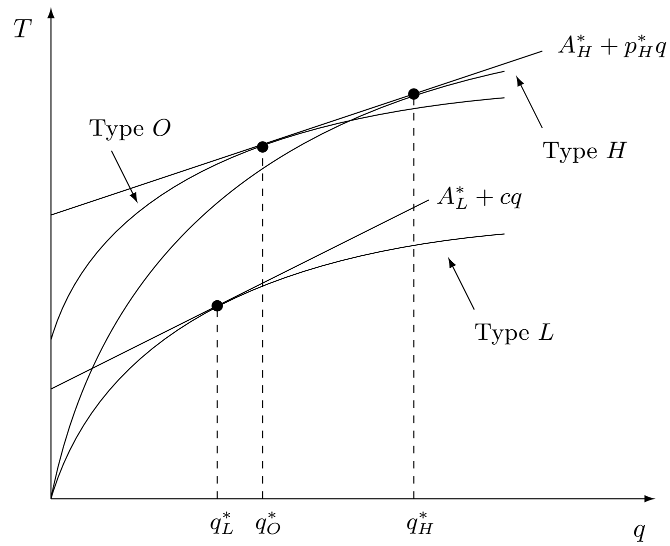

Figure 3: Monopoly with Overoptimistic Consumers; Complete Information

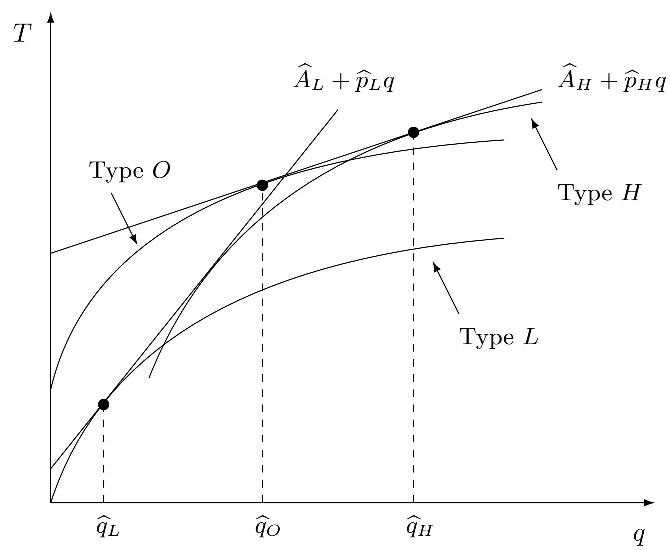

Figure 4: Monopoly with Overoptimistic Consumers; Incomplete Information

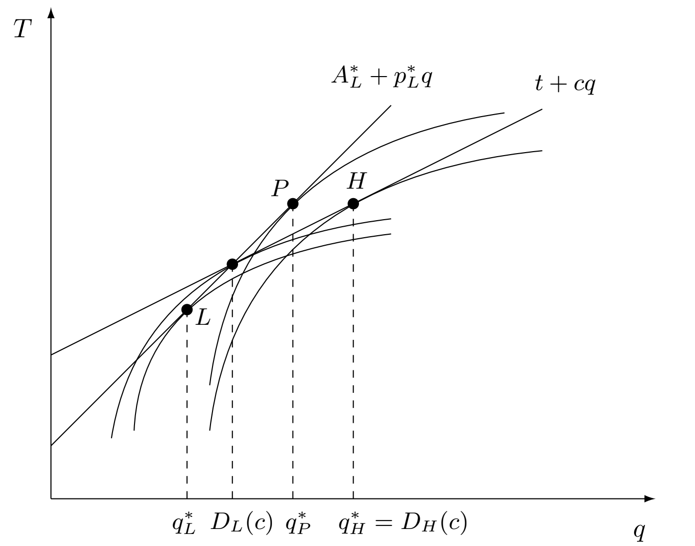

Figure 5: Competition with Overpessimistic Consumers

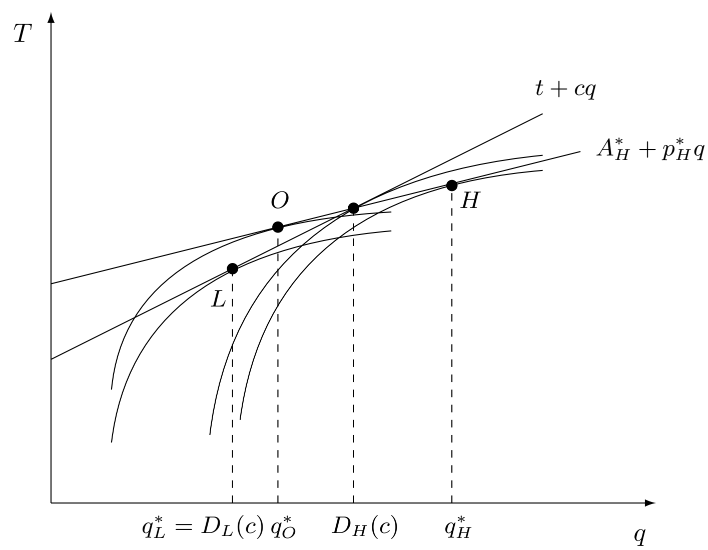

Figure 6: Competition with Overoptimistic Consumers

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