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In this paper, I examine the effectiveness of monetary policy under the ultra-low interest rate environment in Japan through the lens of yield curve dynamics. To that end, I employ the dynamic Nelson-Siegel model with time-varying parameters, thereby computing indicators for tracing the easing effects of monetary policy. I show that the estimation performance of the yield curve models is sufficiently improved even under the ultra-low interest rate environment by extending the dynamic Nelson-Siegel model to allow a loading parameter to vary over time, in addition to three parameters of yield curve dynamics: level, slope, and curvature. However, I also demonstrate that the identification of the level and loading parameters is critical in assessing monetary policy effects based on the estimation results for the yield curve dynamics. I reveal that monetary easing effects under the Quantitative and Qualitative Monetary Easing (QQE) are produced by flattening the yield curve in the ultra-long-term maturities over 10-year while easing effects from maturities shorter than 10-year remain almost unchanged. I argue that monetary policy fails to produce sufficient easing effects within the time frame of the standard macroeconomic stabilization policy, even with the full-fledged implementation of unconventional monetary policy measures under the current ultra-low interest rate environment.

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1 Introduction

In this paper, I examine the effectiveness of monetary policy under the ultra-low interest rate environment through the lens of yield curve dynamics in Japan.

Nelson and Siegel (1987) describe yield curve dynamics by three factors: level, slope, and curvature, supported by empirical studies on yield curve dynamics. This model has simple, parsimonious functional forms but is flexible enough to capture the general property of the yield curve for monetary policy purposes. Recently, following Diebold and Li (2006) and Diebold, Rudebusch and Aruoba (2006), various empirical frameworks are proposed to extend the Nelson-Siegel model into dynamic models. Diebold, Rudebusch and Aruoba (2006) formulate the Nelson-Siegel model as a state-space model by treating three factors, level, slope, and curvature, as an unobserved vector-autoregressive process. Koopman, Mallee and Van der Wel (2010) further extend the dynamic Nelson-Siegel model by introducing time-varying loading parameters as well as time-varying volatility with the generalized autoregressive conditional heteroscedasticity (GARCH) process.[1]

I apply the empirical framework of Koopman, Mallee and Van der Wel (2010) to the Japanese yield curve data under the ultra-low interest rate environment. Based on that estimation results, I also construct monetary policy indicators by slightly modifying ones proposed in Okina and Shiratsuka (2004) and Krippner (2015).

Looking back, the Bank of Japan (BOJ) reduced its policy interest rate, uncollateralized overnight call rate, down to 0.5 percent in 1995. Since then, the Japanese economy has been facing the effective lower bound (ELB) constraint of nominal interest rates for more than 25 years (see Table 1 for major Japan's monetary policy events since the mid-1990s). The BOJ continued to implement various unconventional monetary policy measures on a large scale over that time.

There seems to be no consensus as to how to measure the effectiveness of unconventional monetary policy, including large-scale asset purchases (LSAP), since no comprehensive policy indicators are readily available. That contrasts with the conventional monetary policy assessment with short-term policy interest rates, which summarizes necessarily information, as Taylor rule tells. Thus, it is important and useful to establish a framework to assess the effectiveness of unconventional monetary policy through the lens of the yield curve dynamics comparable to conventional monetary policy assessment.[2]

To that end, it should be noted that an estimation of the yield curve model generally raises

[1] Among the various specifications of the dynamic Nelson-Siegel model, Christensen, Diebold and Rudebusch (2011) construct an empirical model with due consideration on the theoretical consistency by incorporating the no-arbitrage condition. Christensen and Rudebusch (2015) and Krippner (2016) extend the dynamic Nelson-Siegel model by incorporating the ELB constraint of nominal interest rates. Note that the dynamic Nelson-Siegel model needs to assume the constant loading parameter over time to incorporate the no-arbitrage condition.

[2] An application of shadow interest rates is another possibility to construct a monetary policy indicator to assess unconventional monetary policy under the ELB of nominal interest rates, as shown in Krippner (2015) and Wu and Xia (2016). That approach explicitly assumes the ELB constraint of nominal interest rates, thereby estimating hypothetical policy interest rates without the ELB constraint. See also Ichiue and Ueno (2015), Ichiue and Ueno (2018), and Ueno (2017) as applications to Japan.

Table 1: Monetary Policy Events

Date	Policy Events
Sep 8, 1995	Reduction of the official discount rate (ODR) (1.0%→0.5%)
Sep 9, 1998	Reduction of target of the overnight (O/N) rate (0.5%→0.25%)
Feb 12, 1999	Introduction of the Zero Interest Rate Policy (ZIRP)
Apr 13, 1999	Governor's announcement of the commitment to a zero interest rate
Aug 11, 2000	Termination of the ZIRP
Mar 19, 2001	Introduction of the Quantitive Monetary Easing (QE)
Aug 14, 2001	Raise in the target CAB (¥ 50 tril.→¥6 tril.)
Dec 19, 2001	Raise in the target CAB (above ¥ 6 tril.→¥ 10–15 tril.)
Oct 30, 2002	Raise in the target CAB (¥10–15 tril.→¥15–20 tril.)
Mar 25, 2003	Raise in the target CAB (¥15–20 tril.→¥17–22 tril.)
Apr 30, 2003	Raise in the target CAB (¥17–22 tril.→¥22–27 tril.)
May 20, 2003	Raise in the target CAB (¥22–27 tril.→¥27–30 tril.)
Oct 10, 2003	Increase in the ceiling of the target CAB (¥27–30 tril.→¥27–32 tril.) Clarification of the commitment to maintaining the QE
Mar 9, 2006	Termination of the QE and setting target of O/N rate at zero
Jul 14, 2006	Rise of the target of O/N rate (0.0%→0.25%)
Feb 21, 2007	Raise in the target of O/N rate (0.25%→0.5%)
Oct 31, 2008	Reduction of target of O/N rate (0.5%→0.3%)
Dec 19, 2008	Reduction of target of O/N rate (0.3%→0.1%)
Oct 5, 2010	Introduction of Comprehensive Monetary Easing (CE)
Jan 22, 2013	Introduction of the Price Stability Target
Apr 4, 2013	Introduction of Quantitative and Qualitative Monetary Easing (QQE)
Oct 31, 2014	Expansion of QQE
Jan 29, 2016	Introduction of QQE with Negative Interest Rates
Sep 21, 2016	Introduction of QQE with Yield Curve Control
Jul 31, 2018	Enhancing the sustainability of QQE with Yield Curve Control
March 16, 2020	Monetary easing response to the outbreak of COVID-19
March, 2021	Further effective and sustainable monetary easing

an issue on the trade-off between the estimation performance and the theoretical consistency. The affine term structure models have a theoretical foundation based on the no-arbitrage condition, while they have limitations in their estimation performance. The Nelson-Siegel model, in contrast, is a typical statistical model to focus on the cross-sectional dimension of yield curves, thereby improving the goodness of fit. The dynamic Nelson-Siegel model is extended to describe the time-series dimension of the yield curve dynamics, in addition to the cross-sectional dimension.

In quantitatively assessing the effectiveness of monetary policy under the ELB constraint of nominal interest rates with the declined and flattened yield curve, it is critical to employ the yield curve model with high estimation precision. That highlights the trade-off between the estimation performance and the theoretical consistency in selecting yield curve models. I show that the dynamic Nelson-Siegel model with a time-varying loading parameter is well suited for a quantitative assessment of the effectiveness of monetary policy through yield curve dynamics under the ultra-low interest rate environment.

More precisely, I show that the estimation results for the dynamic Nelson-Siegel model reject a hypothesis of constant loading parameters over time, thus indicating the vulnerability of the assumption of the no-arbitrage condition. That is because that the parameters for the nonlinear functional form, defined as the Nelson-Siegel model, are difficult to estimate in a robust manner, especially under the ultra-low interest rate environment. The estimates for the level and loading parameters are contaminated, making it difficult to identify these parameters precisely.[3]

I thus focus on the empirical model to estimate yield curve dynamics with high estimation precision even under the ultra-low interest rate environment since such precise estimation results provide a more robust basis for examining the effectiveness of monetary policy. Based on those estimation results, I compute monetary policy indicators to demonstrate that monetary easing effects under Quantitative and Qualitative Monetary Easing (QQE) are produced by flattening the yield curve in the ultra-long-term maturities over 10-year, while monetary easing effects from maturities shorter than 10-year remain almost unchanged. I argue that monetary policy fails to produce sufficient easing effects within the time frame of the standard macroeconomic stabilization policy, even with various and massive unconventional monetary policy measures under the current ultra-low interest rate environment.

This paper is constructed as follows. Section 2 summarizes the original form of the Nelson-Siegel model and examines challenges for its estimation under the ultra-low interest rate environment. It then introduces the dynamic Nelson-Siegel model with a time-vary loading parameter to deal with challenges. Section 3 shows the estimation results for the dynamic Nelson-Siegel model and carries out the robustness check on estimation performance against the changes in the estimation periods. Section 4 constructs some monetary policy indicators based on the estimation results in Section 3, thereby discussing their policy implications. Finally, Section 5 concludes the paper.

2 The Nelson-Siegel Model and its Dynamic Extensions

In this section, I briefly explain the basic specification of the Nelson-Siegel model, proposed first by Nelson and Siegel (1987), which is widely used in yield curve analysis. I then discuss the challenges for estimating the Nelson-Siegel model under an ultra-low interest rate environment. I finally introduce the dynamic Nelson-Siegel model with a time-varying loading parameter to address the estimation challenges.

[3] The importance of the time-varying nature of the long-term interest rates, as an endpoint of the yield curve, is emphasized in the projection of short-term interest rates over time, as in Kozicki and Tinsley (2001) and van Dijk et al. (2014). However, the importance of the time-varying loading parameter seems to be paid less attention to in the previous literature. Koeda and Sekine (2021) share a similar concern over yield curve models with the constant loading parameter and empirically examine it with the Japanese data.

2.1 The basic specification of the Nelson-Siegel model

As preparation for explaining the dynamic Nelson-Siegel model, I first briefly explain the basic framework of the original and static Nelson-Siegel model. Nelson and Siegel (1987) describe yield curve dynamics by three factors: level, slope, and curvature, supported by empirical studies on yield curve dynamics. This model has simple, parsimonious functional forms but flexible enough to capture the general property of the yield curve for monetary policy purposes.

The original version of the Nelson-Siegel model (ONS, hereafter) specifies the instantaneous forward rate (IFR) for time-to-settlement m at period t, denoted by $r_t(m)$, is given by

$$r_t(m) = L_t + S_t e^{-\lambda_t m} + C_t \lambda_t m e^{-\lambda_t m}, \tag{1}$$

where L_t, S_t, C_t, and λ_t are parameters to be estimated from the data. L_t and λ_t are expected to be positive. [4]

The IFR curve, generated by equation (1), includes three terms, L_t, S_t, and C_t, which correspond to level, slope, and curvature factors, respectively. λ_t is the loading parameter, controlling the converging speed toward a long-term level. The model has simple, parsimonious, and smooth functional forms and is flexible enough to capture the general property of the yield curve, ensuring sufficient precision and robustness for monetary policy analysis.

The Nelson-Siegel model has a property that the limits of forward rates when maturity approaches zero and infinity, respectively, are equal to $L_t + S_t$ and L_t. In our estimation, I exploit the first feature to improve the estimation precision in the shorter maturity of the yield curve by restricting $L_t + S_t$ to the overnight uncollateralized call rate. I also use the second feature to compile monetary policy indicators since it corresponds to the restriction that forward rates for settlements very far into the future be constant.[5]

The spot rate at maturity m, denoted by $R_t(m)$, is derived by integrating equation (1) from zero to m and dividing by m.

$$R_t(m) = \frac{1}{m}\int_{s=0}^{m} r_t(s)\, ds = L_t + S_t\left(\frac{1-e^{-\lambda_t m}}{\lambda_t m}\right) + C_t\left[\frac{1-e^{-\lambda_t m}}{\lambda_t m} - e^{-\lambda_t m}\right]. \tag{2}$$

I employ equation (2) to estimate the Nelson-Siegel model by using spot rates observed in the Japanese Government Bond market, while I use the IFR curve, described as equation (1), in analyzing the yield curve dynamics. The IFR curve reflects market expectations regarding the future course of short-term interest rates, thus providing important information on market views of future monetary policymaking.

[4] SÃ¶derlind and Svensson (1997) extend the original version of the Nelson-Siegel model by considering an additional curvature term, thereby enabling to flexibly approximate more complicated shape of yield curves.

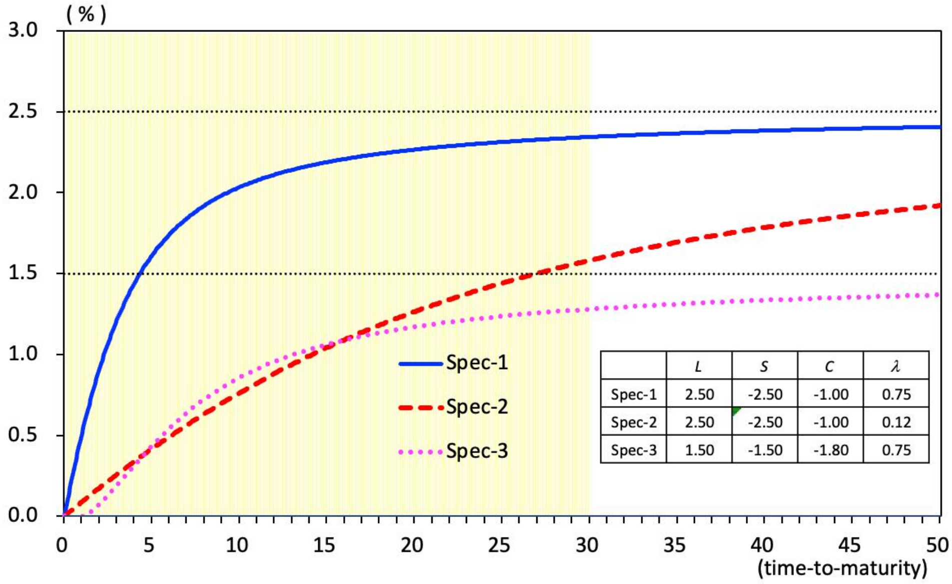
[5] As previous studies of applying the Nelson-Siegel model to Japan, Fujiki and Shiratsuka (2002) and Okina and Shiratsuka (2004) employ the Nelson-Siegel model to analyze the policy commitment effects of monetary policy under the ELB constraints of nominal interest rates in Japan.

2.2 Issues on the parameter identifications under a low interest rate environment

When applying the Nelson-Siegel model to estimate the yield curve dynamics under an extremely low interest rate environment, it is deemed important to focus on an empirical issue in identifying parameters for the nonlinear functional form.

To clarify the point mentioned above, Figure 1 plots hypothetical spot rate curves based on the original version of the Nelson-Siegel model, defined as equation (1), with parameter sets for Spec-1, 2, and 3. Spec-1 represents a standard spot rate curve with an overnight rate at zero percent and a long-term forward rate at 2.5 percent. Spec-2 and 3 show the effects of declining and flattening the yield curve by changing the parameter values for the loading parameter λ and the level, slope, and curvature parameters L, S, and C, respectively. More precisely, Spec-2 stretches the spot rate curve rightward by delaying the convergence to the long-term forward rate by lowering only λ ($0.75 \to 0.12$) while keeping L, S, and C unchanged. Spec-3 pushes the spot rate curve downward by lowering L ($2.5 \to 1.5$) and C ($-1.0 \to -1.8$) while keeping λ unchanged.[6]

Figure 1: Parameter Identification



	L	S	C	λ
Spec-1	2.50	-2.50	-1.00	0.75
Spec-2	2.50	-2.50	-1.00	0.12
Spec-3	1.50	-1.50	-1.80	0.75

Notes: Shaded area corresponds to the maturity up to 30-year, which market data on spot rates are available.

Plotted spot rate curves based on the parameter sets of Spec-2 and 3 in the figure look very similar for the maturities from zero to 20 years. However, the two parameter sets have very different endpoints of the yield curve, resulting in very different implications for monetary easing effects through the yield curve dynamics. Spec-2 pulls future monetary easing effect forward on a large scale by flattening the short- to medium-term zone since the long-term forward rate is kept high.[7] In contrast, Spec-3 fails to pull future monetary easing effect forward

[6] S automatically increases due to the parameter restriction of $L = -S$

[7] See Eggertsson and Woodford (2003) and Jung, Teranishi and Watanabe (2005) for the theoretical foundations of policy commitment effects under the ELB constraint of nominal interest rates. As discussed in Fujiki, Okina and Shiratsuka (2001), the Bank of Japan first introduced a policy commitment mechanism under the ELB in 1999.

because of the declined long-term forward rate.

To sum up, the parameters for the nonlinear functional form, defined as the Nelson-Siegel model, are difficult to estimate in a robust manner, especially under an extremely low interest rate environment. The estimates for level and loading parameters are contaminated with each other, thus making it difficult to identify these parameters precisely. That suggests that the dynamic extension of the Nelson-Siegel model needs to make the loading parameter vary over time, in addition to the level, slope, and curvature parameters of yield curve dynamics. When estimating the dynamic Nelson-Siegel model with constant loading parameter, the yield curve flattening tends to be regarded just as the decline of the yield curve, potentially leading to the worsening of estimation precision.

2.3 Dynamic Nelson-Siegel model

To deal with the identification issues for the level and loading parameters, I employ the dynamic Nelson-Siegel model with the time-varying loading parameter and the GARCH process in the error term, following Koopman, Mallee and Van der Wel (2010).

The original version of Nelson-Siegel model (ONS) can be estimated by simple regression model below using cross-sectional observations of spot rates $R_t(m_i)$ for a set of N maturities $m_1 < m_2 < \cdots < m_N$ at period t,

$$R_t\left(m_i\right) = L_t + S_t\left(\frac{1-e^{-\lambda_t m_i}}{\lambda_t m_i}\right) + C_t\left[\frac{1-e^{-\lambda_t m_i}}{\lambda_t m_i} - e^{-\lambda_t m_i}\right] + \varepsilon_t(m_i). \tag{3}$$

Similarly, with pooling cross-sectional observations of spot rates $R_t(m_i)$ over the estimation period, the dynamic Nelson-Siegel model with the time-varying loading parameter is estimated by

$$\begin{pmatrix} R_t(m_1) \\ R_t(m_2) \\ \vdots \\ R_t(m_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda_t m_1}}{\lambda_t m_1} & \frac{1-e^{-\lambda_t m_1}}{\lambda_t m_1} - e^{-\lambda_t m_1} \\ 1 & \frac{1-e^{-\lambda_t m_2}}{\lambda_t m_2} & \frac{1-e^{-\lambda_t m_2}}{\lambda_t m_2} - e^{-\lambda_t m_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda_t m_N}}{\lambda_t m_N} & \frac{1-e^{-\lambda_t m_N}}{\lambda_t m_N} - e^{-\lambda_t m_N} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(m_1) \\ \varepsilon_t(m_2) \\ \vdots \\ \varepsilon_t(m_N) \end{pmatrix}. \tag{4}$$

In addition, I also assume the two types of the dynamic processes for parameter transitions: vector-autoregressive (VAR) process and random walk (RW) process.[8] In the dynamic Nelson-Siegel model with VAR process, denoted by DNS-VAR, parameter dynamics is given by

The policy mechanism was called "policy duration effect" at that time, while such mechanism is called "forward guidance" after the Global Financial Crises in 2008.

[8] Koopman, Mallee and Van der Wel (2010) employ the VAR process for the parameter dynamics, while they point out the possibility of dynamic process es, including the RW process. van Dijk et al. (2014) and Buitenhuis (2017) propose the introduction of the time-varying constant term in the state equation for the dynamic Nelson-Siegel model based on the VAR(1) process assumption for the time-varying parameters. However, I do not employ that specification since such an extension of the dynamic Nelson-Siegel model does not improve the estimation performance using Japanese data.

$$\beta_{t+1} = (I_4 - F)\,\mu + F\beta_t + \eta_t, \tag{5}$$

$$\beta_t = \begin{pmatrix} L_t \\ S_t \\ C_t \\ \lambda_t \end{pmatrix}, F = \begin{pmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} \\ \varphi_{21} & \varphi_{22} & \varphi_{23} & \varphi_{24} \\ \varphi_{31} & \varphi_{32} & \varphi_{33} & \varphi_{34} \\ \varphi_{41} & \varphi_{42} & \varphi_{43} & \varphi_{44} \end{pmatrix}, \mu = \begin{pmatrix} \mu^1 \\ \mu^2 \\ \mu^3 \\ \mu^4 \end{pmatrix}, \eta_t = \begin{pmatrix} \eta_t^1 \\ \eta_t^2 \\ \eta_t^3 \\ \eta_t^4 \end{pmatrix},$$

$$\begin{bmatrix} \eta_t \\ \varepsilon_t \end{bmatrix} \sim NID\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma_\eta' \Sigma_\eta & 0 \\ 0 & \Sigma_\varepsilon' \Sigma_\varepsilon \end{pmatrix}\right],$$

$$\Sigma_\varepsilon' \Sigma_\varepsilon = \begin{pmatrix} \delta_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_N^2 \end{pmatrix}, \Sigma_\eta' = \begin{pmatrix} \sigma_{11} & 0 & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix},$$

where $\Sigma_\varepsilon' \Sigma_\varepsilon$ and Σ_η' are diagonal matrix and lower triangular matrix, respectively.

In the dynamic Nelson-Siegel model with RW process, denoted by DNS-RW, the parameter dynamics is also given by

$$\beta_{t+1} = \beta_t + \eta_t, \tag{6}$$

$$\beta_t = \begin{pmatrix} L_t \\ S_t \\ C_t \\ \lambda_t \end{pmatrix}, \eta_t = \begin{pmatrix} \eta_t^1 \\ \eta_t^2 \\ \eta_t^3 \\ \eta_t^4 \end{pmatrix},$$

$$\begin{bmatrix} \eta_t \\ \varepsilon_t \end{bmatrix} \sim \mathrm{NID}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma_\eta' \Sigma_\eta & 0 \\ 0 & \Sigma_\varepsilon' \Sigma_\varepsilon \end{pmatrix}\right],$$

$$\Sigma_\varepsilon' \Sigma_\varepsilon = \begin{pmatrix} \delta_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_N^2 \end{pmatrix}, \Sigma_\mu' = \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix},$$

where $\Sigma_\varepsilon' \Sigma_\varepsilon$ and $\Sigma_\eta'$are diagonal matrixes.[9]

In both DNS-VAR and DNS-RW, the error terms are assumed below.

$$\varepsilon_t = \Gamma_\varepsilon \varepsilon_t^* + \varepsilon_t^+. \tag{7}$$

[9] The RW model is frequently employed in the time-varying parameter regression model, as shown in Kim and Nelson (1999), and the time-varying parameter vector autoregression (TVP-VAR) model. The RW model in this paper assumes that factors in the observation equation follow a random walk process. That assumption does not imply that factors follow the random walk process regardless of economic fundamentals but that the observation equation swiftly incorporates all incoming information, such as unexpected policy changes and external shocks. Thus, the adjustment speed of factors is much faster in the RW model than in the VAR model.

where Γ_ε and ε_t^+ are (Nx1) vectors. ε_t^* is scalar and a common error term for all maturities. Γ_ε denotes the sensitivity parameters for each maturity. It is assumed that ε_t^+ and ε_t^* follow $\text{NID}(0, \Sigma_\varepsilon^{+\prime}\Sigma_\varepsilon^+)$ and $\text{NID}(0, h_t)$, respectively, and variance for ε_t^* follows the GARCH(1,1) process below:

$$h_t = \gamma_0 + \gamma_1(\varepsilon_{t-1}^*)^2 + \gamma_2 h_{t-1}, \tag{8}$$

where $\gamma_0 > 0$, $0 < \gamma_1 < 1$, $0 < \gamma_2 < 1$, and $h_1 = \gamma_0(1-\gamma_1-\gamma_2)^{-1}$. Since the variance of ε_t^*, denoted by h_t, follows GARCH(1,1) process, the variance and covariance matrix for ε_t, denoted by $\Sigma_\varepsilon'\Sigma_\varepsilon$, also becomes time-varying.[10]

The estimation of the original version of Nelson-Siegel model requires at least ten observations from different maturities since just cross-sectional information for a specific period in time is employed. On the contrary, the estimation of the dynamic Nelson-Siegel model needs fewer cross-sectional observations since both cross-sectional and time-series information are used at the same time. As a result, the dynamic Nelson-Siegel model reduces the risk for overfitting, even though the Nelson-Siegel model is less vulnerable to the overfitting problem. [11]In estimating DNS-VAR and DNS-RW, the extended Kalman filter is applied to the nonlinear formulation of equation (4). [12]

3 Estimation Results and Their Robustness Check

In this section, I estimate two specifications of the dynamic Nelson-Siegel model, DNS-VAR and DNS-RW, and compare their estimation performance with the original version of the static Nelson-Siegel model (ONS) as a benchmark.[13] I also examine the robustness of the empirical performance regarding the changes in the estimation periods.

In estimating the dynamic Nelson-Siegel models, I use data on zero-coupon yield rates for nine maturities: 3- and 6-month, and 1-, 2-, 3-, 5-, 7- 10-, and 30-year, computed by Bloomberg. However, in estimating the static Nelson-Siegel model, used as a benchmark, I use data on zero-coupon yield rates for 16 maturities: overnight, 3- and 6-month, and 1-, 2-, 3-, 4-, 5-, 6-, 7-, 8-, 9-, 10-, 15-, 20-, and 30-year. The sample period is from January 1995 to June 2020 and common to all the specifications.

[10] In estimation, I follow the proposal in Koopman, Mallee and Van der Wel (2010) for fixing γ_0 at a very small value of 0.0001, instead of normalizing by $\Gamma_\varepsilon'\Gamma_\varepsilon = 1$.

[11] I impose the parameter restriction of "$L_t + S_t =$ overnight interest rate" to minimize the overfitting problem in estimating the static Nelson-Siegel model.

[12]Following Buitenhuis (2017), loading parameter λ_t is log-transformed so as to prevent λ_t from taking negative values.

[13] I estimate the ONS model without restricting "$L_t + S_t =$ overnight interest rate" after introducing negative interest rates in January 2016 since the overnight call rate constantly deviates from the target level.

3.1 Estimation results

The estimation results of the two specifications of the dynamic Nelson-Siegel models are summarized in Table 2. Comparing the estimation performance of the two models with the information criteria of AIC (Akaike's information criterion) and BIC (Bayesian information criterion), the estimation performance is slightly higher for DNS-VAR than DNS-RW. [14]

Table 2: Estimation Results

	DNS-VAR		DNS-RW	
	Coef.	S.E.	Coef.	S.E.
Likelihood	16,557.2		15,776.1	
AIC/BIC	-107.9 /-107.3		-103.0 / -102.7	
φ_{11}	1.0078	0.1932	1.0000	—
φ_{12}	0.0046	0.0259	0.0000	—
φ_{13}	0.0457	0.0376	0.0000	—
φ_{14}	-0.0003	0.0082	0.0000	—
φ_{21}	-0.0584	0.0737	0.0000	—
φ_{22}	0.8707	0.0366	1.0000	—
φ_{23}	0.0013	0.0199	0.0000	—
φ_{24}	0.0003	0.0029	0.0000	—
φ_{31}	0.0813	0.0473	0.0000	—
φ_{32}	0.0723	0.2080	0.0000	—
φ_{33}	0.9311	0.0486	1.0000	—
φ_{34}	-0.0002	0.0042	0.0000	—
φ_{41}	-0.4745	0.0960	0.0000	—
φ_{42}	0.2061	0.6477	0.0000	—
φ_{43}	0.9701	0.0386	0.0000	—
φ_{44}	0.9993	0.0323	1.0000	—
μ^1	0.0576	0.00368	0.0000	—
μ^2	-0.0374	0.00361	0.0000	—
μ^3	-0.0125	0.00754	0.0000	—
μ^4	0.0020	0.01236	0.0000	—
σ_{11}	0.0019	0.00013	0.0029	0.00013
σ_{22}	0.0006	0.00004	0.0023	0.00011
σ_{33}	0.0036	0.00029	0.0034	0.00018
σ_{44}	0.0685	0.00791	0.1040	0.00959

	DNS-VAR		DNS-RW	
	Coef.	S.E.	Coef.	S.E.
σ_{21}	-0.0018	0.00014	0.0000	—
σ_{31}	-0.0022	0.00035	0.0000	—
σ_{32}	0.0003	0.00023	0.0000	—
σ_{41}	-0.0001	0.01383	0.0000	—
σ_{42}	-0.0203	0.00965	0.0000	—
σ_{43}	0.0230	0.01143	0.0000	—
δ_1	3.07E-04	2.47E-05	3.13E-04	2.26E-05
δ_2	1.69E-04	2.77E-05	1.36E-04	3.04E-05
δ_3	4.37E-04	2.19E-05	4.70E-04	2.21E-05
δ_4	2.63E-04	2.60E-05	3.27E-04	1.66E-05
δ_5	2.86E-04	3.66E-05	4.92E-04	2.20E-05
δ_6	3.05E-04	2.47E-05	4.62E-10	2.18E-05
δ_7	6.94E-12	3.96E-05	5.72E-04	2.33E-05
δ_8	8.58E-04	3.58E-05	8.25E-11	2.81E-05
δ_9	2.12E-10	9.04E-05	7.51E-04	5.27E-05
γ_1	0.497	0.126	0.389	0.049
γ_2	0.499	0.127	0.610	0.050
Γ_1	0.0019	0.0016	0.0040	0.0026
Γ_2	0.0015	0.0018	0.0039	0.0026
Γ_3	0.0007	0.0021	0.0030	0.0023
Γ_4	0.0005	0.0030	0.0016	0.0020
Γ_5	0.0000	0.0036	0.0004	0.0018
Γ_6	0.0068	0.0051	0.0000	0.0017
Γ_7	0.0150	0.0071	0.0020	0.0022
Γ_8	0.0107	0.0060	0.0060	0.0035
Γ_9	0.0000	0.0030	0.0019	0.0020

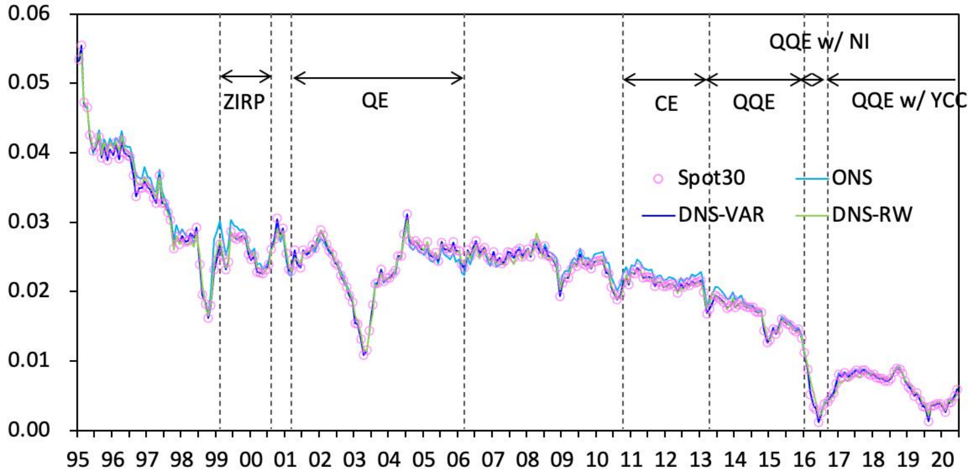
I next compute the spot rates based on the estimation results in Table 2 since it is difficult to assess and interpret the estimation results with estimated parameters alone. When plotting

[14] By comparing the estimation results for various specifications of the dynamic Nelson-Siegel model, it is confirmed that the estimation performance is improved significantly by introducing the time-varying loading parameter rather than the GARCH process in the error terms.

	Specifications					
Dynamic Process	VAR			RW		
Time-varying L, S, C	yes	yes	yes	yes	yes	yes
Time-varying λ	no	yes	yes	no	yes	yes
GARCH in error term	no	no	yes	no	no	yes
AIC	-101.4	-106.0	-107.9	-91.0	-102.0	-103.0
BIC	-101.1	-105.5	-107.3	-90.9	-101.8	-102.7

the time-series data of the estimated spot rates and the observed spot rates for the maturity of 30-year in Figure 2, the estimated rates follow almost exactly the observed rates.[15]

Figure 2: Estimated Spot Rates at 30-year Maturity



Notes: Abbreviations in the figures correspond to the monetary policy regime below:
ZIRP: Zero Interest Rate Policy (February 1999-August 2000); QE: Quantitative Monetary Easing Policy (March 2001-March 2006); CE: Comprehensive Monetary Easing Policy (October 2010-March 2013); QQE: Quantitative and Qualitative Monetary Easing Policy (April 2013-January 2006); QQE w/ NI: QQE with Negative Interest Rates (January 2016-September 2016); QQE w/ YCC: QQE with Yield Curve Control (September 2016-present).

Table 3 computes the root mean squared error (RMSE) of the estimated spot rates and the observed rates for all maturities used in estimating DNS-VAR and DNS-RW. The table shows that the RMSE is relatively small, suggesting high goodness of fit in both specifications. That result confirms that the dynamic Nelson-Siegel model with time-varying loading parameter produces good estimation performance. Comparing with the evaluation based on the information criterion, the difference in the assumptions of the dynamic process of the parameters does not make a big difference in RMSE. DNS-VAR shows significantly smaller RMSE for the maturity of 30-year, while DNS-RW for the 5- and 10-year.

Table 3: Root Mean Squared Errors between Estimates and Observed Data

Maturity	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y	30Y	Average
DNS-VAR	0.0259	0.0134	0.0436	0.0226	0.0263	0.0307	0.0080	0.0860	0.0066	0.0292
DNS-RW	0.0285	0.0090	0.0451	0.0340	0.0513	0.0094	0.0582	0.0077	0.0549	0.0331
ONS	0.0763	0.0605	0.0565	0.0398	0.0435	0.0402	0.0538	0.0513	0.0962	0.0576

Note: Average is the average of RMSE for all maturities shown in the table, which correspond to the maturities used in the estimation of DNS-VAR and DNS-RW.

The estimation results suggest that the dynamic Nelson-Siegel model with the time-varying loading parameter shows good estimation precisions even under the ultra-low nominal interest

[15] Contrastingly, the estimated spot rates based on the dynamic Nelson-Siegel model with the fixed loading parameter show significant downward deviations from observed spot rates, especially after the Global Financial Crisis. That is because the estimated loading parameters are fixed at a higher level in the second half of the estimation period, and thus the estimated long-term forward rates are biased downward.

rate environment. That indicates that the time-varying loading parameter enables more robust identification for the level and loading parameters.

3.2 Estimated parameters over time

Next, I compare the time-series movements of the estimated parameters for the two specifications of the dynamic Nelson-Siegel model, DNS-VAR and DNS-RW, and the benchmark static Nelson-Siegel model, ONS. Figure 3 plots the estimates for ONS with their 95% confidence intervals and the estimates for DNS-VAR and DNS-RW for level, slope, curvature, and loading parameters: L, S, C, and λ, respectively.

The figure shows that estimated parameters for DNS-VAR and DNS-RW are close to ONS until the early 2010s and generally stay within the 95% confidence intervals for ONS. DNS-RW appears to deviate first from the ONS in the early 2010s, while it becomes closer to ONS again in the mid-2010s, especially after introducing negative interest rates in the early-2016. In contrast, DNS-VAR shows continued and large deviations from ONS from early-2016.

DNS-VAR seems to succeed in producing relatively stable estimates over time by assuming a VAR process for the parameter dynamics, while DNS-RW strongly reflects the influence of the more recent data and is thus able to replicate ONS, which estimates using data only for that day, by assuming a random walk process for the parameter dynamics.

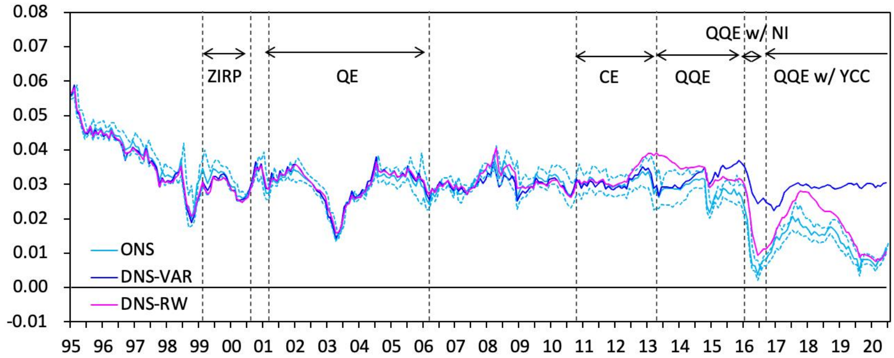
Focusing on the level and loading parameters, DNS-VAR and DNS-RW produce constructing results after introducing negative interest rates in early-2016. DNS-VAR estimates the level parameter at a relatively high level in a stable manner, around 3 percent, and the loading parameter with a declining trend even at a very low level. DNS-RW estimates the level and loading parameters to follow ONS estimates: the level parameter declines rapidly soon after introducing negative interest rates in early-2016, rebounds to the level before introducing negative interest rates toward the end of 2017, but declines again after that to around 1 percent. The loading parameter does not respond so much when introducing negative interest rates but starts increasing from the end of 2017, in line with the decline in the level parameter.

The above contrasting estimation results of DNS-VAR and DNS-RW on the level parameters indicate very different implications on the effectiveness of monetary policy. A higher level parameter implies larger room for additional monetary easing. However, it is difficult to determine which estimation results are more appropriate. Especially, there remains a concern about the robustness of the estimation results for the dynamic Nelson-Siegel model with the time-varying loading parameters over time. I thus conduct a more in-depth study below through estimations by changing the estimation periods. [16]

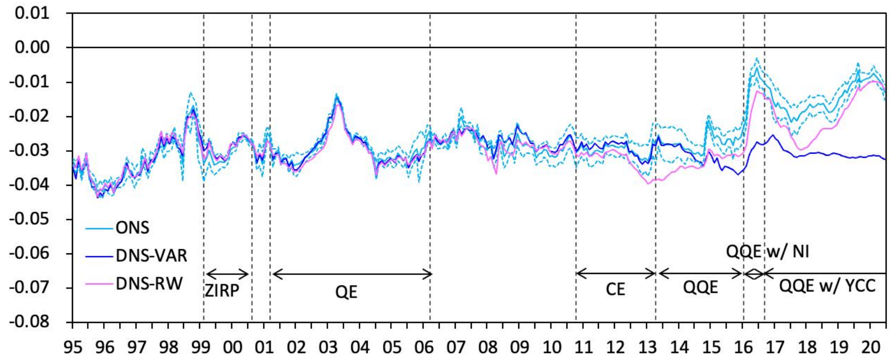
[16] As pointed out in footnote 14, the estimates for the level parameter L with the fixed loading parameter tend to be biased downward, as the estimates for the fixed loading parameter deviate from the time-varying estimates. In particular, focusing on the estimation results from 2016, when negative interest rates were introduced, the level L stays very close to zero, and the slope S and curvature C converge to near zero. That implies that the dynamic Nelson-Siegel models with the fixed loading parameter identify the yield curve as perfectly flattened at zero. The dynamic Nelson-Siegel model with the fixed loading parameter is unable to grasp the shape of the yield curve with a slightly positive slope beyond the maturity of 10-year.

Figure 3: Estimated Parameters over Time

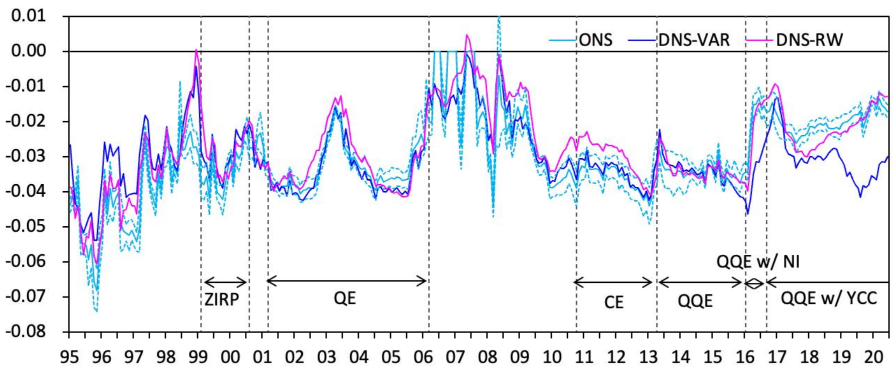
(1) Level parameter L



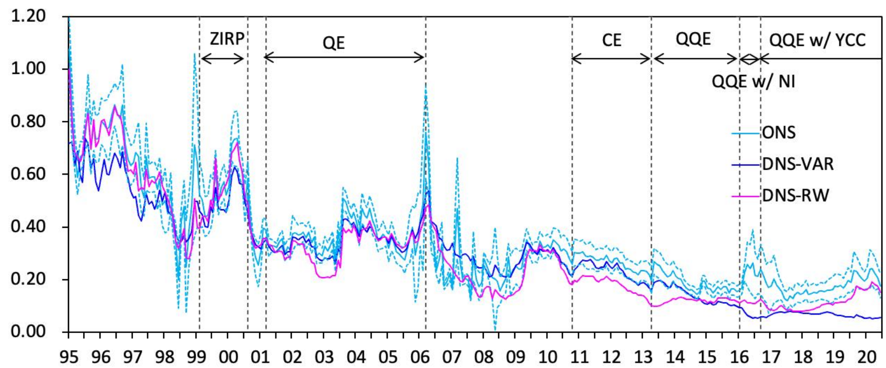
(2) Slope parameter S



(3) Curvature parameter C



(4) Loading parameter λ



Notes: Light blue dotted lines are 95% confidence intervals for ONS estimates.

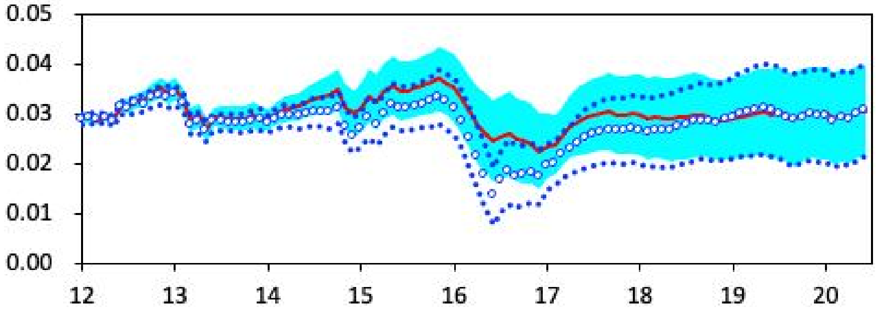
3.3 Robustness on changes in sample periods

I carry out robustness checks on DNS-VAR and DNS-RW against the changes in estimation periods after introducing negative interest rates in early-2016. The level and loading parameters for DNS-VAR and DNS-RW are repeatedly estimated by extending the end of the estimation period from January 2012 by one month and compare the estimates with full-sample period estimates.

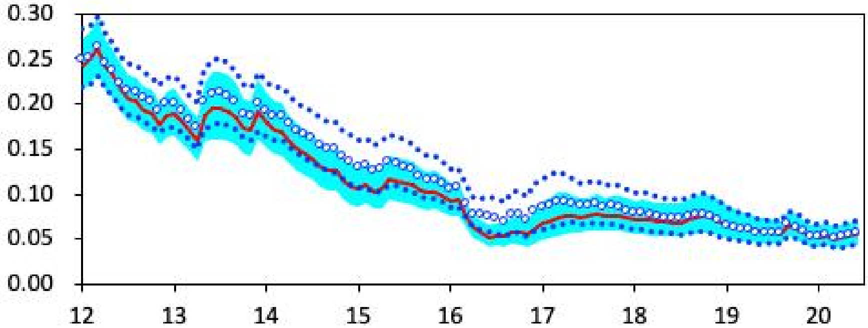
Figure 4 plots the estimation results: the estimates for the end of the subsample periods with their 95-percent confidence intervals (circles and blue dotted lines), along with the estimates for the full sample period with their 95-percent confidence intervals (red bold line and blue shaded area). The estimated level and loading parameters for DNS-VAR and DNS-RW generally move parallel with the estimates for full sample periods. However, DNS-RW tends to produce a jump in the parameters when extending the estimation period by just one month. As mentioned above, it seems consistent with the interpretation that by assuming a random walk as the dynamic process of the parameters, DNS-RW strongly reflects the influence of the more recent data and thus is able to replicate ONS estimates using data only for that day.

Figure 4: Inch-warm Estimation Results

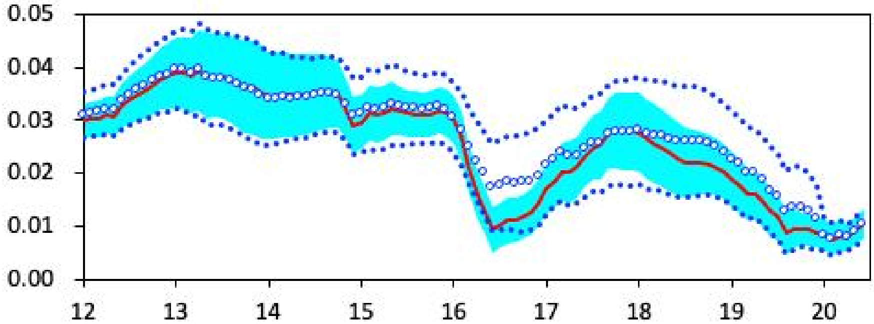
(1-a) Level parameters for DNS-VAR



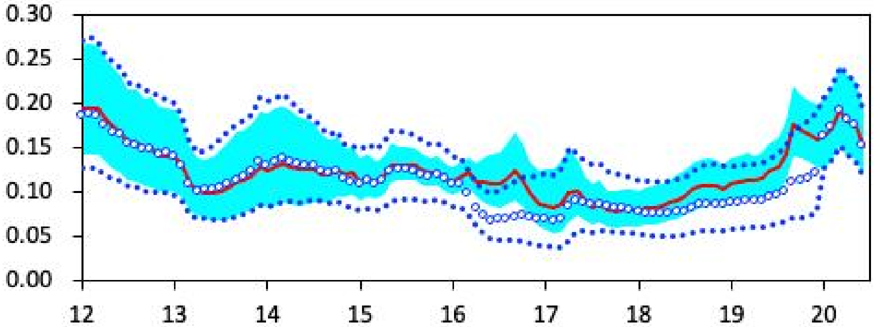
(1-b) Loading parameters for DNS-VAR



(2-a) Level parameters for DNS-RW



(2-b) Loading parameters for DNS-RW



Notes: Red bold line and light blue shaded area are estimates and their 95-percent confidence intervals based on the full-sample estimation, respectively. Circles and blue dashed lines are estimates and their 95-percent confidence intervals based on the subsample estimations ending at each month on the horizontal axis, respectively.

I also compute the root mean squared error (RMSE) of the estimated spot rates and the observed rates for the selected maturities, same as Table 3, for subsample periods before and after the introduction of negative interest rates in January 2016, in Table 4. DNS-VAR and DNS-RW generally produce smaller RMSEs than ONS. Moreover, DNS-VAR produces smaller RMSEs than DNS-RW in the ultra-long-term maturity of 30-year, while DNS-RW produces smaller RMSEs in medium to long-term maturities of 5- and 10-year.

Table 4: Root Mean Squared Errors between Estimates and Observed Data

Maturity	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y	30Y	Average
From January 1995 to December 2015										
DNS-VAR	0.0245	0.0142	0.0478	0.0243	0.0285	0.0335	0.0088	0.0928	0.0067	0.0312
DNS-RW	0.0274	0.0094	0.0494	0.0364	0.0560	0.0104	0.0636	0.0084	0.0531	0.0349
ONS	0.0764	0.0642	0.0614	0.0431	0.0465	0.0436	0.0589	0.0556	0.1056	0.0617
From January 2016 to June 2020										
DNS-VAR	0.0315	0.0084	0.0085	0.0121	0.0106	0.0090	0.0023	0.0416	0.0060	0.0145
DNS-RW	0.0332	0.0064	0.0122	0.0199	0.0160	0.0023	0.0182	0.0028	0.0626	0.0193
ONS	0.0760	0.0383	0.0215	0.0179	0.0246	0.0162	0.0148	0.0186	0.0209	0.0276
Full sample										
DNS-VAR	0.0259	0.0134	0.0436	0.0226	0.0263	0.0307	0.0080	0.0860	0.0066	0.0292
DNS-RW	0.0285	0.0090	0.0451	0.0340	0.0513	0.0094	0.0582	0.0077	0.0549	0.0331
ONS	0.0763	0.0605	0.0565	0.0398	0.0435	0.0402	0.0538	0.0513	0.0962	0.0576

Note: Figures for the full sample are identical to ones in Table 3.

Based on the estimation results so far, it is difficult to reach clear conclusions on which assumption of the parameter dynamics for the dynamic Nelson-Siegel model is more reliable, DNS-VAR or DNS-RW. Both DNS-VAR and DNS-RW show a relatively high estimation precision for the unprecedentedly declined and flattened yield curve after introducing negative interest rates.[17] Nevertheless, DNS-VAR and DNS-RW produce very contrasting estimates for the level and loading parameters. That observation implies that the identification of the level and loading parameters becomes extraordinarily difficult in a robust manner under the ultra-low interest rate environment. That, in turn, casts doubt on the information content of the yield curve for monetary policy, especially the long-term forward rate or the long-run equilibrium level of the yield curve, considering the massive monetary policy intervention to the JGB market.

4 Monetary Policy Indicators and Their Policy Implications

Based on estimates for the Nelson-Siegel models, I next compute monetary policy indicators, thereby examining how assessments on monetary policy effects differ depending on the specifications.

[17] Comparing likelihood values of DNS-VAR and DNS-RW over subsample periods, they smoothly converge to likelihood values for the full sample period, and the relationship between the size of likelihood values is also stable. That observation indicates that the general estimation performances of DNS-VAR and DNS-RW are fairly robust against the changes in the sample periods, even though estimates sometimes show jumps a little bit.

4.1 Formulation of monetary policy indicators

I define three monetary policy indicators with reference to those proposed by Okina and Shiratsuka (2004) and Krippner (2015).

The first indicator is the long-term forward rate, or LFR (same unit as interest rates), corresponding to estimated parameter L_t.

$$LFR_t = L_t \tag{9}$$

LFR measures the steady-state nominal interest rates, implying the convergence level of the yield curve in the long run. Using Fisher's equation, LFR is decomposed into a steady-state real interest rate, a steady-state inflation rate, and a term premium. That is deemed to reflect market expectations for long-term economic performance.

The second indicator is policy duration, or PD, computed as the inverse of the loading parameter (unit is years). [18]

$$PD_t = 1/\lambda_t \tag{10}$$

PD corresponds to the maturity that minimizes the third term in equation (1), implying the maximum point in the yield curve with the largest downward effects from the curvature factor. PD expresses the converging speed of the yield curve toward the long-term equilibrium level of LFR, reflecting market expectations regarding how long easy monetary conditions persist into the future.

The third indicator is effective monetary stimulus, or EMS (same unit as interest rates), a slightly revised indicator proposed in Krippner (2015).

$$EMS_t(\tau) = LFR_t - R_t(\tau) = -\frac{1}{\tau}\int_{m=0}^{\tau}(S_t + C_t\lambda_t m)e^{-\lambda_t m}dm. \tag{11}$$

EMS quantitatively shows how much the yield curve is lowered from the long-term equilibrium level of LFR up to the maturity on average.

Following Krippner (2015), I set the parameter τ at 30 years, while making two revisions. First, I include the negative region of the yield curve for the integral interval in equation (11). I need to assess the downward effects of the yield curve by including the negative interest rate regions since the ELB constraints become slightly negative after introducing the negative interest rate policy. Second, I divide the area between LFR and the estimated IFR curve by maturity, thereby converting the indicator into a spot rate equivalent unit to makes the quantitative implication clearer.[19]

[18] It should be noted that confidence intervals expand significantly after introducing the QQE since the policy duration is defined as the inverse of the loading parameter.

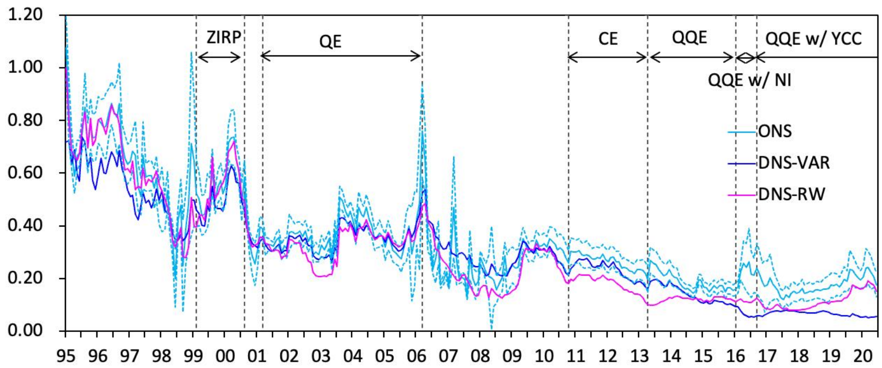
[19] Krippner (2015) employs the specification of the crude are size between LFR and IFR curve without dividing by maturity. I revise to normalize EMS by dividing with maturity, thereby enabling to decompose EMS into the contributions across maturities.

4.2 Estimates for monetary policy indicators

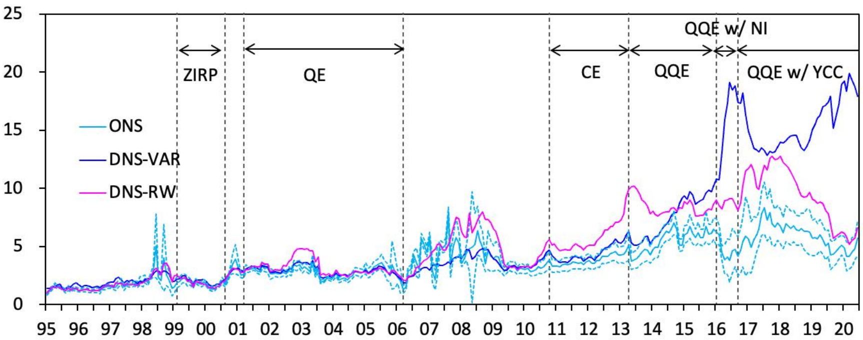
Now let me compare the time-series fluctuations of monetary policy indicators based on the estimation results of DNS-VAR and DNS-RW. Figure 5 plots ONS estimates with their 95% confidence intervals and estimates for the two dynamic Nelson-Siegel models.

Figure 5: Monetary Policy Indicators

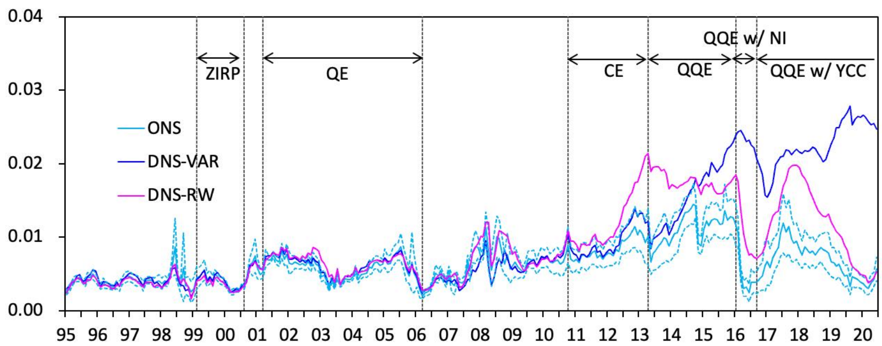
(1) Long-term forward rate (*LFR*)



(2) Policy duration (*PD*)



(3) Effective monetary stimulus (*EMS*)



Notes: Light blue dotted lines are the 95-percent confidence intervals for ONS estimates. Estimates for *LFR* are replication of level parameter in Figure 3 by definition of equation (9).

Focusing on the period after introducing the negative interest rate policy, monetary policy indicators for ONS and DNS-RW, and DNS-VAR show the different movements, producing contrasting monetary policy implications. Based on ONS and DNS-RW, the decline in *EMS* indicates the weakened downward effects on the entire yield curve since both *LFR* and *PD* decline, implying the declined yield curve and the shortened expectations about the duration of the ultra-low interest rate environment. In contrast, based on DNS-VAR, EMS remains relatively high since LFR remains at a level close to that before introducing the negative interest rate policy, and PD follows an upward trend. It is still difficult to judge which estimation result is more appropriate in tracing the yield curve dynamics under the ultra-low interest rate environment. [20]

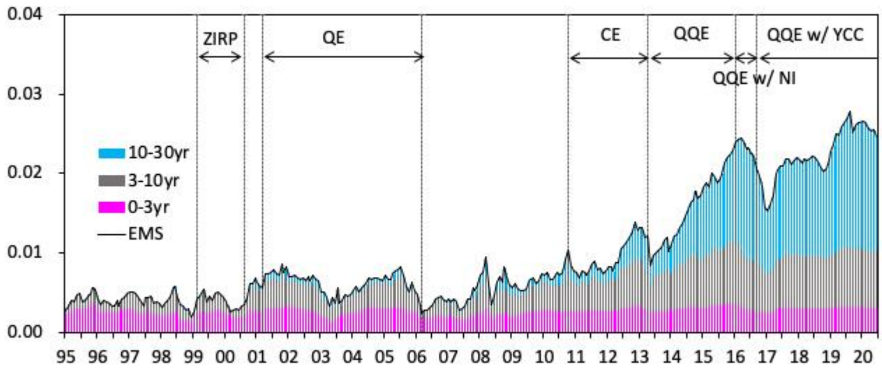
In any event, the above results clearly show that the estimates for LFR, which correspond to the long-run equilibrium level of the yield curve, play a critical role in assessing monetary easing effects through yield curve dynamics. In fact, the forward guidance of keeping interest rates at a low level is expected to produce a greater easing effect as the level of LFR, the converging level of the yield curve in the long run, becomes higher.

4.3 Discussion on monetary policy implications

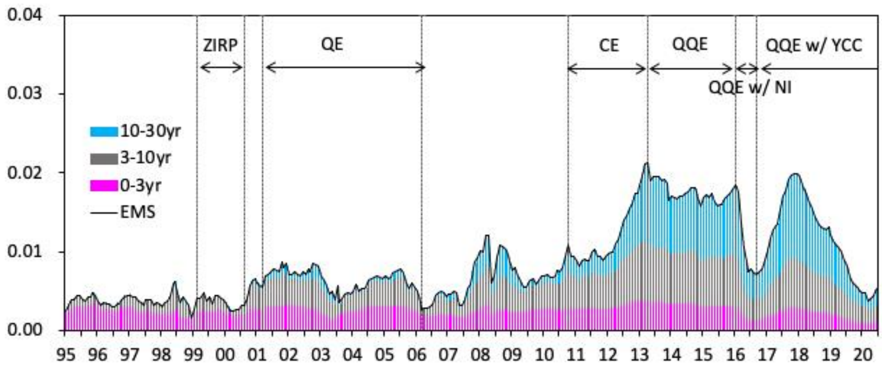
I decompose the estimated EMS by maturity in Figure 6. The maturity is divided into three ranges: 0- to 3-year, 3- to 10-year, and 10- to 30-year.

Figure 6: Maturity Decomposition of EMS

(1) DNS-VAR



(2) DNS-RW



[20] As observed by the upward trend of PD at least before introducing QQE, the yield curve tends to converge more slowly toward the long-run level. That suggests that the time scale in the financial markets becomes longer under the ultra-low interest rate environment. PD seems to be a good indicator for understanding the financial cycle time, as discussed in Filardo, Lombardi and Raczko (2018).

Looking at the figure, the overall downward pressure on the yield curve measured by EMS up to the maturity of 30-year shows a contrasting movement for the estimation result of DNS-VAR and DNS-RW after introducing negative interest rates. Such differences, however, become much smaller when focusing on the estimates for the maturity of 0- to 3-year, which is considered as strongly influencing the effective aggregate demand. The contributions to EMS from the maturity of 0- to 3-year remain almost unchanged around 0.3 to 0.5 percentage points over the entire estimation period. Monetary easing effects of additional policy actions are realized by lowering the yield curve with the maturity of 3- to 10-year from ZIRP, QE to CE, but after introducing QQE, they mainly come from the decline in the yield curve at longer maturity of 10- to 30-year. After introducing negative interest rates, it can be confirmed that the evaluation of this part is in contrast with the estimation results for DNS-VAR and DNS-RW. DNS-VAR indicates that EMS continues to expand in the maturity of 10- to 30-year with the relatively high estimates of LFR. DNS-RW shows that EMS peaks around the end-2017 then starts declining due to the decline in LFR.

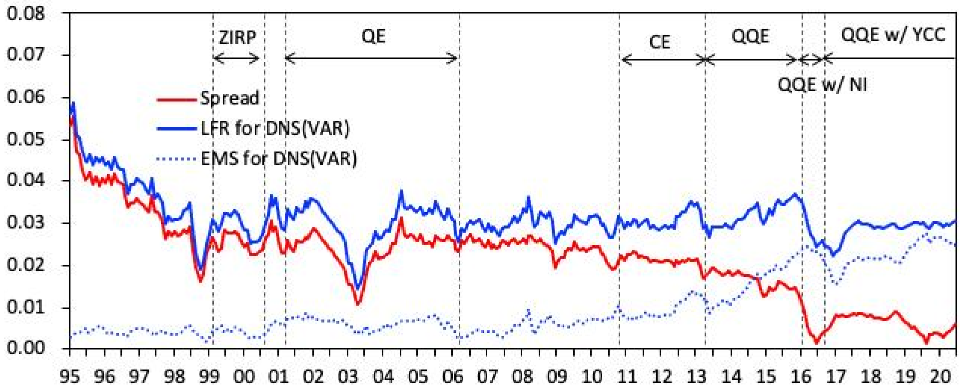
Figure 7 compares the estimates for EMS and LFR of DNS-VAR and DNS-RW. Although the EMS estimates move differently in DNS-VAR and DNS-RW after introducing negative interest rates, EMS comes close to LFR in both estimation results. By definition, the estimates for EMS have an upper bound at LFR, as shown in equation (11). Thus, it should be noted that smaller spreads between LFR and EMS imply that monetary policy now has very limited room for producing further easing effects through yield curve dynamics, even considering the ultra-long maturity zone of 0- to 30-year.

To sum up, the evaluation of additional monetary easing effects solely depends on the nuanced parameter identification of the dynamic Nelson-Siegel model under the ultra-low interest rate environment. The unprecedentedly declined and flattened yield curve in the ultra-long maturity zone of 10- to 30-year is estimated as the decline in the convergence speed toward the long-run equilibrium level of nominal interest rates or the decline in the long-run equilibrium level itself. However, regardless of the estimated level of the long-run equilibrium level, monetary policy produces relatively limited and relatively constant easing effects within a time frame of standard macroeconomic stabilization policy of 0- to 3-year over the last two decades. In addition, monetary policy now has very limited room for further easing effects even considering the yield curve dynamics for ultra-long-term maturity zone of 10- to 30-year. Based on the estimation results in this paper, under the financial condition with ultra-low interest rates, even full-fledged employment of unconventional monetary policy measures fails to deliver sufficient easing effects to escape from the ELB constraint, thus just entrenching market expectations about the continued ultra-low interest rate environment. [21]

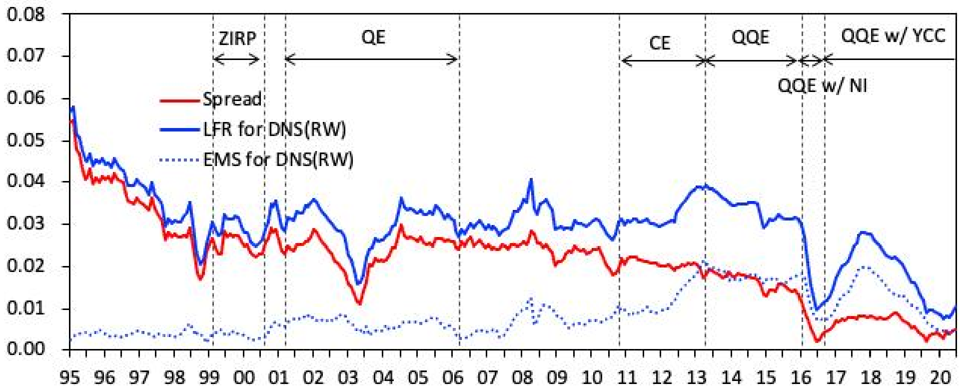
[21] Okina and Shiratsuka (2004) point out that the policy commitment to keeping the policy interest rate low for an extended period into the future is unable to produce sufficient easing effects since LFR in Japan already stays low. They argue that the policy commitment results in containing the policy interest rates at the ELB constraint.

Figure 7: Spreads between *EMS* and *LFR*

(1) DNS-VAR



(2) DNS-RW



5 Conclusions

In this paper, I employed the dynamic Nelson-Siegel model with the time-varying loading parameter to estimate the yield curve dynamics under the ultra-low interest rate environment using Japanese data from the mid-1990s. Based on the estimation results, I then constructed monetary policy indicators to examine the effectiveness of monetary policy.

In estimating the Nelson-Siegel model under the ultra-low interest rate environment, parameters for the nonlinear functional form, especially the level and loading parameters, are difficult to identify in a robust manner. In that context, it is shown that the introduction of the time-varying loading parameter for controlling the convergence speed toward the long-run level of the yield curve is very effective in improving the estimation precision even under the ultra-low interest rate environment.

At the same time, however, contrasting estimates for the level and loading parameters are produced depending on the assumption of the dynamic process for time-varying parameters, vector autoregressive (VAR) process or random walk (RW) process after introducing negative interest rates. The level and loading parameters are estimated at a relatively high level in the VAR process, while they decline significantly in the RW process.

The contrasting estimates for the level and loading parameters play a critical role in assessing monetary policy effects after introducing negative interest rates. The estimated monetary policy indicators exhibited contrasting movements, reflecting the difference in the estimated level parameters from different assumptions of the dynamic process of time-varying parameters for the dynamic Nelson-Siegel model: maintained relatively high level of easing effects in the VAR process, while significant decline in the RW process.

After introducing Quantitative and Qualitative Monetary Easing (QQE) in April 2013, monetary policy produces easing effects by lowering the yield curve in the ultra-long-term maturity zone of 10- to 30-year. That implies that monetary policy is unable to enhance easing effects within the time frame of standard macroeconomic stabilization policy, regardless of the assumption of the dynamic process of the time-varying parameters. The estimated monetary policy indicators show that easing effects from shorter maturity than 10-year remain almost unchanged over the last two decades. I argue that monetary policy fails to produce sufficient easing effects within the time frame of the standard macroeconomic stabilization policy, even with various and massive unconventional monetary policy measures under the current ultra-low interest rate environment. Even full-fledged employment of unconventional monetary policy measures fails to deliver sufficient easing effects to escape from the ELB constraint, thus entrenching market expectations about the continued ultra-low interest rate environment.

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