Fiscal Inflation in a Currency Union*

Momo Komatsu
Federal Reserve Board

David Murakami
Milan & Pavia
Bank of England

Ivan Shchapov
IP Paris
CREST

May 27, 2026

Incomplete draft
Click here for latest version.

Abstract

We show that for a determinate equilibrium in a two-country currency union model, at most one fiscal authority can issue unfunded debt. We then model an economy with partially unfunded debt by combining monetary-led and fiscal-led economies. This model shows that funded fiscal expansions cause inflation to diverge across countries, while unfunded expansions increase inflation in both countries, because the common monetary authority accommodates fiscal inflation. We bring this prediction to the data using euro area local projections: fiscal expansions in Germany and Italy raise inflation home and abroad, while fiscal expansions in France lower inflation abroad and are followed by a contractionary monetary response, consistent with German and Italian shocks being primarily unfunded and French shocks being primarily funded. Finally, we estimate a quantitative model that decomposes historical fiscal shocks into funded and unfunded components and confirm fiscal inflation in the recent crisis episodes.

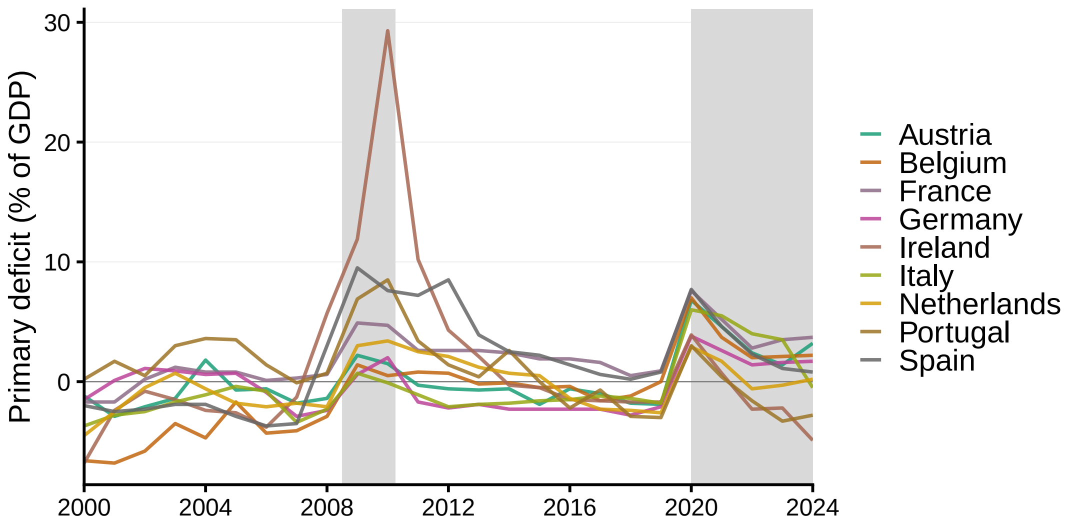
JEL codes: E62, E31, F45
Keywords: Monetary-fiscal interactions, currency union, inflation

*The authors would like to thank Francesco Bianchi, Francesco Ferrante, and Abhiprerna Smit for their useful comments. The views expressed in this manuscript are solely our own and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System, of any person associated with the Federal Reserve System, the Bank of England, or its committees.
Komatsu: `momo.komatsu@frb.gov` | Murakami: `david.murakami@bankofengland.co.uk` | Shchapov: `ivan.shchapov@ensae.fr`

1 Introduction

In the last 25 years, euro area countries have experienced episodes of large primary deficits, as shown in Figure 1. The spikes in the primary deficit coincide with the fiscal expansions during the financial crisis, the COVID-19 crisis, and the cost-of-living crisis (roughly 2007–2008, 2020–2021, and 2021–2023, respectively). A fiscal theory of the price level distinguishes between fiscal expansions that are funded by future fiscal adjustments and those that are not. In a closed economy model, unfunded fiscal expansions have an inflationary effect on the economy, because the central bank accommodates *fiscal inflation* arising from the government's intertemporal budget constraint. In the euro area, who paid for the fiscal expansions? We study the propagation of funded and unfunded fiscal expansions in a currency union and estimate the share of unfunded fiscal shocks in the euro area.

Figure 1: Primary deficit as % of GDP in the original euro area countries



Notes: The original euro area countries are the member states of the euro area that adopted the euro on January 1, 1999. We exclude Luxembourg and Finland for data availability reasons. Source: Eurostat.

The first contribution of this paper is to document price level determinacy and the propagation of funded and unfunded shocks in a currency union. We develop a two-country currency union model with bonds and goods trade, and show that for a determinate equilibrium, at most one fiscal authority can issue unfunded debt. Building on Bianchi, Faccini, and Melosi (2023), we introduce partially unfunded debt by decomposing the economy into monetary-led and fiscal-led parallel economies. In the model, funded and unfunded fiscal expansions in country A have opposite implications for inflation in country B. A funded expansion shock to lump-sum taxes in country A raises inflation at home but lowers inflation abroad through general-equilibrium effects: backed by future tax adjustments, the shock increases the government debt in country A, which increases the supply of bonds available in country B through cross-border bond markets, crowding in saving in country B. Country B's demand and inflation declines, whereas country A experiences an increase in demand and inflation, reflecting the fiscal expansion. After an unfunded fiscal expansion in country A, the common central bank accommodates the fiscal inflation, which spills over to country B through the trade in goods. Hence, inflation in both countries increases.

Second, we find new evidence on cross-country heterogeneity in fiscal spillovers within the euro area, which, through the lens of the model, we interpret as coming from different shares of unfunded shocks. Using local projections and forecast-error fiscal shocks following Auerbach and Gorodnichenko (2012, 2017), we find that in 2004–2025 fiscal expansions in Germany and Italy increase inflation domestically and in other member countries, whereas fiscal expansions in France lower inflation in other countries. Moreover, the central bank reacts in a contractionary manner to a fiscal shock in France, while after a fiscal shock in Germany and Italy, the central bank accommodates the resulting inflation. These results suggest that German and Italian fiscal expansions are primarily unfunded, whereas French ones are primarily funded.

Last, we develop a quantitative model extension with habit formation, non-tradables, nominal rigidities, and long-term bonds. The model results preserve the stylized model's qualitative propagation patterns of funded and unfunded fiscal shocks with greater persistence. We estimate the quantitative model across various country combinations to identify episodes of fiscal inflation.

Related literature. This paper addresses monetary–fiscal interactions in a currency union and therefore bridges two strands of the literature: (i) price-level determinacy and the interaction of monetary and fiscal policy, with Sargent and Wallace (1981), Leeper (1991), Sims (1994), and Woodford (1994) as the pioneers, and (ii) monetary and fiscal policy spillovers in a currency union. Moreover, the empirical exercise contributes to the literature on fiscal policy spillovers.

The literature on price-level determinacy and the interaction of monetary and fiscal policy has been widely studied, and more recently by Cochrane (2021). Our model framework builds on Bianchi, Faccini, and Melosi (2023) (BFM), which introduced a new class of general equilibrium models that allow for partially unfunded debt. Moreover, among others, Beetsma and Jensen (2005), Galí and Monacelli (2008), Ferrero (2009), and Hjortsoe (2016) study the optimal joint conduct of monetary and fiscal policy in a currency union during asymmetric shocks. More recently, authors like Barbier-Gauchard and Betti (2021) study the spillover effects of fiscal policy in a monetary union. They study different types of fiscal instruments and their spillovers onto the rest of the union.

Our paper lies at the intersection of those two strands of literature: we extend the price determinacy literature to the currency union setting and allow for the analysis of unfunded fiscal shocks. We derive the parameter space for a monetary-led regime and a fiscal-led regime, and combine those as parallel economies following the approach of Bianchi, Faccini, and Melosi (2023) for the closed economy. Bergin (2000) also examines price level determinacy in a currency union, but abstracts from goods and bond trade and from the partially-unfunded debt framework of Bianchi, Faccini, and Melosi (2023), both of which are central to our analysis. The funded vs. unfunded distinction determines the sign of cross-border inflation spillovers, which is a prediction absent from existing currency union models. Our analysis finds that German and Italian fiscal policy has been more unfunded than French fiscal policy.

Spillovers of fiscal policy are a growing strand of the literature, with contributors like Goujard (2017), Debrun et al. (2021), Born, Müller, and Pfeifer (2020), and Ascari et al. (2024). Many such papers study the channels through which fiscal policy spillovers are transmitted, e.g. trade linkages, financial linkages, or demand spillovers. Our paper links these spillovers and transmission channels to whether fiscal shocks are funded or not and provides a structural model to identify the funded/unfunded mix from observed inflation and interest rate data.

The rest of the paper is organized as follows. Section 2 builds a stylized DSGE model, documents the price-level determinacy conditions of a currency union, and analyzes the propagation of funded and unfunded shocks. In section 3 we present the empirical analysis and interpret the heterogeneous spillovers from fiscal shocks through the lens of the model. In section 4 we estimate a quantitative model and investigate the historical share of funded and unfunded shocks in the euro area. Concluding remarks are in Section 5.

2 Stylized model

In this section, we present a stylized model of a currency union and show that for a determinate equilibrium, at most one fiscal authority in the union can pursue an active, unfunded policy. To allow for partially unfunded debt, we follow the approach by Bianchi, Faccini, and Melosi (2023) in which the economy is modelled as a mixture of monetary-led and fiscal-led equilibria. Then, we analyze the propagation mechanisms of funded and unfunded fiscal expansion shocks. The theory provides the structural interpretation of the empirical findings in Section 3.

The stylized model considers a currency union of two countries, A and B, each with a fixed endowment. The countries are inhabited by households who consume goods from both countries with a home bias. Moreover, households in each country hold bonds issued by the fiscal authority of both countries. The fiscal authority in each country collects taxes and issues bonds, and the monetary authority targets an average of the inflation rates in the two countries with an inflation-targeting rule. We allow for partially unfunded debt as in Bianchi, Faccini, and Melosi (2023). Country B variables are denoted with *, except for prices and the nominal interest rate which are the same across the union. Equations are for the economy in country A unless otherwise stated.

2.1 Households

Households derive utility from a consumption bundle and receive an endowment Y. They can smooth consumption by holding bonds issued in countries A and B, subject to portfolio adjustment costs. They face the following maximization problem:

$$\max_{C_t, B_{At}, B_{Bt}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \ln C_t, \tag{1}$$

subject to:

$$\begin{aligned} &P_t C_t + B_{At} + B_{Bt} + P_t T_t \\ &\quad = P_t Y + R_{t-1}\left(B_{A,t-1} + B_{B,t-1}\right) - \frac{\psi_b P_t \left(\overline{b}_A + \overline{b}_B\right)}{2} \left(\delta_b \frac{B_{Bt}}{B_{At}} - 1\right)^2, \end{aligned} \tag{2}$$

where β is the discount factor, C_t is a bundle of consumption goods and P_t is its price, defined as:

$$C_t = \left[(1-\alpha_I)^{1/\mu}(C_{At})^{(\mu-1)/\mu} + \alpha_I^{1/\mu}(C_{Bt})^{(\mu-1)/\mu}\right], \tag{3}$$

$$P_t = \left[(1-\alpha_I)P_{At}^{1-\mu} + \alpha_I P_{Bt}^{1-\mu}\right]^{1/(1-\mu)}, \tag{4}$$

where C_{At} is a good produced in country A, C_{Bt} a good produced in country B, and P_{At} and P_{Bt} are their prices, respectively. α_I is the share of imported goods and μ is the elasticity of substitution between domestic and imported goods. B_{At} is nominal holdings of bonds issued by the government in country A and B_{Bt} is nominal holdings of bonds issued by the government in country B, and R_t is the gross nominal interest rate in the currency union. We define real bond holdings (as shares of output) as $b_{At} \equiv \frac{B_{At}}{P_t Y}$ and $b_{Bt} \equiv \frac{B_{Bt}}{P_t Y}$. Moreover, $\overline{b}_A = \frac{\overline{B}_A}{\overline{P}Y}$ and $\overline{b}_B = \frac{\overline{B}_B}{\overline{P}Y}$ are steady-state real bond holdings as shares of output, and $\delta_b = \frac{\overline{b}_A}{\overline{b}_B}$ is the desired portfolio mix.[1] Parameter ψ_b

1. We check that our baseline results are robust to changing the bond holding specification. In Appendix G we derive the model when households aggregate bonds under Constant Elasticity of Substitu-

is the cost of deviating from the desired mix. T_t are real lump-sum net taxes.

The cost minimization problem for the consumption bundle results in the per-period relative demand between country A and country B goods:

$$\frac{C_{At}}{C_{Bt}} = \frac{1-\alpha_I}{\alpha_I}\left(\frac{P_{At}}{P_{Bt}}\right)^{-\mu}, \tag{5}$$

using demand equations $C_{At} = (1-\alpha_I)\left(\frac{P_{At}}{P_t}\right)^{-\mu} C_t$ and $C_{Bt} = \alpha_I \left(\frac{P_{Bt}}{P_t}\right)^{-\mu} C_t$. The intertemporal optimization problem results in two Euler equations, one for each bond:

$$1 = \frac{\beta R_t}{1-\psi_b(\bar{b}_A+\bar{b}_B)\left(\delta_b \frac{b_{Bt}}{b_{At}}-1\right)\delta_b \frac{b_{Bt}}{b_{At}^2}} \mathbb{E}_t\left(\Pi_{t+1}^{-1}\frac{C_t}{C_{t+1}}\right), \tag{6}$$

$$1 = \frac{\beta R_t}{1+\psi_b\left(\bar{b}_A+\bar{b}_B\right)\left(\delta_b \frac{b_{Bt}}{b_{At}}-1\right)\delta_b \frac{1}{b_{At}}} \mathbb{E}_t\left(\Pi_{t+1}^{-1}\frac{C_t}{C_{t+1}}\right), \tag{7}$$

where $\Pi_t = P_t/P_{t-1}$ is gross inflation.

We define the terms of trade as the relative price of the good produced in country B to the good produced in country A:

$$S_t = \frac{P_{Bt}}{P_{At}}. \tag{8}$$

2.2 Fiscal authority

The fiscal authority of both countries issues bonds and collects taxes. The nominal budget constraint for the fiscal authority is thus the following:

$$B_t + P_t T_t = R_{t-1} B_{t-1}, \tag{9}$$

where taxes T_t coincide with the real primary surplus and B_t is the supply of bonds. The government follows a fiscal rule:

$$\frac{\tau_t}{\bar{\tau}} = \left(\frac{s_{b,t-1}}{\bar{s}_b}\right)^{\gamma} e^{\zeta_t}, \tag{10}$$

where $\tau_t = \frac{T_t}{Y}$ is the surplus-to-output ratio and $s_{bt} = \frac{B_t}{P_t Y}$ is the real market value of debt as a share of output. Parameter γ determines how strongly the government adjusts primary surpluses with debt levels. ζ_t is a lump-sum tax shock, following an AR(1) process. In Section 2.6 we explain how we introduce partially unfunded government debt.

tion (CES) and Figures A.15a and A.15b show that the results are quantitatively and qualitatively similar to the baseline case with quadratic bond adjustment costs.

2.3 Monetary authority

The central bank of the currency union follows a simple inflation-targeting rule:

$$\frac{R_t}{\overline{R}} = \left(\frac{\Pi_t^{MU}}{\overline{\Pi}^{MU}}\right)^{\phi}, \tag{11}$$

where $\Pi_t^{MU} = \Pi_t^{\Theta}\Pi_t^{*1-\Theta} = \left(\frac{P_t}{P_{t-1}}\right)^{\Theta}\left(\frac{P_t^*}{P_{t-1}^*}\right)^{1-\Theta}$ is gross union-wide inflation. Parameter ϕ determines how strongly the central bank adjusts nominal interest rate with inflation movements.

2.4 Equilibrium

The equilibrium of this economy is characterized by a sequence of prices $\{P_{At}, P_{Bt}, R_t\}$ and allocations $\{C_{At}, C_{Bt}, C_{At}^*, C_{Bt}^*, B_{At}, B_{Bt}, B_{At}^*, B_{Bt}^*\}$ such that goods markets clear for the goods produced in countries A and B, and the bond markets clear for bonds supplied by fiscal authorities in both countries.

The goods market clearing conditions for countries A and B are the following:

$$Y = C_{At} + C_{At}^*, \tag{12}$$

$$Y^* = C_{Bt} + C_{Bt}^*. \tag{13}$$

The bond markets clear when the supply of bonds in countries A and B matches demand:

$$B_t = B_{At} + B_{At}^*, \tag{14}$$

$$B_t^* = B_{Bt} + B_{Bt}^*. \tag{15}$$

Finally, we can express the dynamics of net foreign assets in country A with the current account definition:

$$CA_t \equiv P_{At}Y - P_tC_t = B_{Bt} - B_{At}^* - R_{t-1}\left(B_{B,t-1} - B_{A,t-1}^*\right) - \frac{\psi_b P_t\left(\overline{b}_A + \overline{b}_B\right)}{2}\left(\delta_b\frac{B_{Bt}}{B_{At}} - 1\right)^2. \tag{16}$$

We linearize the relevant equilibrium conditions, such that the hatted variables are log-deviations from the steady state values. See Appendix B for the full set of equilibrium conditions.

2.5 Existence and uniqueness of a solution

Leeper (1991) shows that for closed economy versions of such models, there are two regions of the parameter space that deliver existence and uniqueness of a stationary solution. In the first region monetary policy is active and stabilizes inflation, and the fiscal authority keeps debt on a stable path: active monetary policy and passive fiscal policy. In the second region fiscal policy does not commit to a stable path of debt, such that monetary policy accommodates inflation driven by fiscal imbalances: passive monetary policy and active fiscal policy. We extend the logic to the currency union and characterize the determinacy regions of the general currency union model.

To characterize determinacy, we first derive the monetary and the fiscal block of the economy. For the monetary block we combine the Fisher equation with the monetary rule followed by the central bank, eq. (11). For the fiscal block, we combine the government budget constraints with the fiscal rules from each country, eq. (10) (in country A) and its counterpart (in country B).

We obtain the Fisher equation by adding the Euler equation of country A with respect to b_{At}, eq. (6), to the Euler equation of country B with respect to b^*_{At}, and applying the goods market clearing conditions (12) and (13). We use the log-linearized versions of the equations from Appendix B:

$$\hat{i}_t = \mathbb{E}_t \hat{\pi}^{MU}_{t+1} + \Theta \psi_b \frac{\bar{b}_A + \bar{b}_B}{\bar{b}_A} \left(\hat{b}_{Bt} - \hat{b}_{At} \right) + (1 - \Theta) \psi_b \frac{\bar{b}^*_A + \bar{b}^*_B}{\bar{b}^*_A} \left(\hat{b}^*_{Bt} - \hat{b}^*_{At} \right). \tag{17}$$

The equation links the nominal interest rate to the expected inflation and portfolio adjustment costs. Combining the Fisher equation with the log-linear monetary rule,

$$\hat{i}_t = \phi \hat{\pi}^{MU}_t, \tag{18}$$

leads to the monetary block of the currency union:

$$\mathbb{E}_t \hat{\pi}^{MU}_{t+1} = \phi \hat{\pi}^{MU}_t, \tag{19}$$

for which we apply the no arbitrage conditions $\hat{b}_{Bt} = \hat{b}_{At}$ and $\hat{b}^*_{Bt} = \hat{b}^*_{At}$. The coefficient in the monetary rule, ϕ, determines the eigenvalue in the monetary block. For the fiscal block, we combine the log-linear government budget constraints of country A and B,

$$\hat{s}_{bt} = \frac{1}{\beta} \left[\hat{s}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_t - (1 - \beta) \hat{\tau}_t \right], \tag{20}$$

$$\hat{s}^*_{bt} = \frac{1}{\beta} \left[\hat{s}^*_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}^*_t - (1 - \beta) \hat{\tau}^*_t \right], \tag{21}$$

with fiscal rules of each country,

$$\hat{\tau}_t = \gamma \hat{s}_{b,t-1}, \tag{22}$$

$$\hat{\tau}^*_t = \gamma^* \hat{s}^*_{b,t-1}. \tag{23}$$

The fiscal block of the currency union consists of two equations:

$$\hat{s}_{bt} = \frac{1}{\beta} \left[(1 - (1 - \beta)\gamma) \hat{s}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_t \right], \tag{24}$$

$$\hat{s}^*_{bt} = \frac{1}{\beta} \left[(1 - (1 - \beta)\gamma^*) \hat{s}^*_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}^*_t \right], \tag{25}$$

for which γ and γ^*, the debt stabilization parameters, determine the eigenvalues. Equations (19), (24), and (25) describe the currency union model.

Proposition 1 (Determinacy regions of the currency union)**.** *Consider a currency union with two endowment economies and cross-border bond holdings subject to portfolio adjustment costs, characterized by the monetary block* (19) *and the fiscal block* (24) *and* (25)*. The eigenvalues of this system are:*

$$\lambda_1 = \phi, \qquad \lambda_2 = \frac{1-(1-\beta)\gamma}{\beta}, \qquad \lambda_3 = \frac{1-(1-\beta)\gamma^*}{\beta}. \tag{26}$$

For existence and uniqueness of a bounded rational-expectations equilibrium (*Blanchard-Kahn conditions*)*, exactly one eigenvalue must be outside the unit circle. The three determinate regimes are:*

Regime	ϕ	γ	γ^*	*Interpretation*
Monetary-led	>1	≥ 1	≥ 1	*CB controls* π*; both govts fund debt*
Fiscal-led by A	≤ 1	<1	≥ 1	*Govt A: unfunded; CB accommodates*
Fiscal-led by B	≤ 1	≥ 1	<1	*Govt B: unfunded; CB accommodates*

When two or more eigenvalues are outside the unit circle, the equilibrium is non-existent. When no eigenvalues are outside the unit circle, the equilibrium is indeterminate.

Proof. Step 1: Eigenvalues. The system consists of three independent difference equations for $\hat{\pi}_t^{MU}$, $\hat{s}_{bt}$, and $\hat{s}_{bt}^*$, yielding eigenvalues as in eq. (26). Note that $|\lambda_2| > 1$ if and only if $\gamma < 1$ and analogously for λ_3: $|\lambda_2| > 1$ requires $\left|\frac{1-(1-\beta)\gamma}{\beta}\right| > 1$. Since $\beta \in (0,1)$ and $\gamma \geq 0$, we have a positive numerator $1-(1-\beta)\gamma > 0$ for $\gamma < 1/(1-\beta)$. So, the condition is $1-(1-\beta)\gamma > \beta$, which requires $\gamma < 1$.

Step 2: Blanchard-Kahn conditions. The system has one non-predetermined (forward-looking, jump) variable, $\hat{\pi}_t^{MU}$, and two predetermined (backward-looking, state) variables, $\hat{s}_{b,t-1}$ and $\hat{s}_{b,t-1}^*$. The Blanchard-Kahn conditions require the number of eigenvalues outside the unit circle to equal the number of non-predetermined variables, which is one.

Step 3: Determinate regimes.

1. **Monetary-led:** $\phi > 1$, $\gamma \geq 1$, and $\gamma^* \geq 1$, for which λ_1 is the unstable eigenvalue outside the unit circle. The central bank reacts more than one-to-one to inflation, pinning down inflation expectations. Both fiscal authorities adjust taxes with debt changes, stabilizing debt. Tax adjustments to debt changes are sufficiently large that the fiscal authorities stabilize demand by signaling that higher debt levels are matched by future taxation. In a monetary-led economy, monetary policy is *active* and fiscal policy in both countries is *passive*.

2. **Fiscal-led by A:** $\phi \leq 1$, $\gamma < 1$, and $\gamma^* \geq 1$, for which λ_2 is the unstable eigenvalue outside the unit circle. Country A's fiscal authority does not stabilize debt, generating an explosive path for $\hat{s}_{bt}$ that is resolved by inflation through the intertemporal government budget constraint. The central bank accommodates this fiscal-led inflation. Country B's authority stabilizes debt. In this fiscal-led economy, monetary policy is *passive*, fiscal policy in country A is *active*, and fiscal policy in country B is *passive*.

3. **Fiscal-led by B:** By symmetry with the regime fiscal-led by A, interchanging γ and γ^*.

Step 4: Indeterminate and non-existent equilibria. A system with two or more eigenvalues outside the unit circle is over-determined: no bounded equilibrium exists. A system with zero eigenvalues outside the unit circle is indeterminate: multiple bounded equilibria (sunspot equilibria) exist. □

Corollary 1 (At most one *active* authority in a currency union of N countries)**.** *With N fiscal authorities and one central bank, at most one fiscal authority can issue unfunded debt by setting $\gamma < 1$ while maintaining a determinate equilibrium.*

Proof. With N fiscal authorities and one central bank, there are N fiscal eigenvalues and one monetary eigenvalue, but still only one non-predetermined variable, $\hat{\pi}_t^{MU}$, the aggregate inflation rate of the currency union. Hence, at most one of the $N+1$ eigenvalues can be outside the unit circle, implying at most one fiscal authority can have $\gamma < 1$. □

2.6 Introducing partially unfunded debt

This section explains how we model the monetary-led and fiscal-led economy in the above section in one economy, allowing for debt to be *partially* unfunded. We follow the framework developed by BFM but extend it to a currency union setting: we keep track of unfunded government debt in country A, s_{bt}^F, which is a component of the total government debt, s_{bt}. The superscript F denotes that the variable is fiscal-led. To implement an economy in which country A holds unfunded debt, i.e. the fiscal-led economy, we decompose each variable in the monetary rule and fiscal rules as follows:

$$\text{Fiscal rule } A \qquad \frac{\tau_t}{\overline{\tau}} = \left(\frac{s_{b,t-1}}{s^F_{b,t-1}}\right)^{\gamma^M} \left(\frac{s^F_{b,t-1}}{\overline{s}_b}\right)^{\gamma^F} e^{\zeta^M_t + \zeta^F_t}, \tag{27}$$

$$\text{Fiscal rule } B \qquad \frac{\tau^*_t}{\overline{\tau}^*} = \left(\frac{s^*_{b,t-1}}{s^{F*}_{b,t-1}}\right)^{\gamma^{M*}} \left(\frac{s^{F*}_{b,t-1}}{\overline{s}^*_b}\right)^{\gamma^{F*}}, \tag{28}$$

$$\text{Monetary rule} \qquad \frac{R_t}{\overline{R}} = \left[\left(\frac{\Pi_t}{\Pi^F_t}\right)^{\phi^M} \left(\frac{\Pi^F_t}{\overline{\Pi}}\right)^{\phi^F}\right]^{\Theta} \left[\left(\frac{\Pi^*_t}{\Pi^{F*}_t}\right)^{\phi^{M*}} \left(\frac{\Pi^{F*}_t}{\overline{\Pi}^*}\right)^{\phi^{F*}}\right]^{1-\Theta}, \tag{29}$$

for which superscripts M and M^* denote monetary-led variables and parameters, and F and F^* the fiscal-led ones. In the fiscal-led part of the economy, country A's government is not committed to bring debt to a stable path: $\gamma^F \leq 1$. In contrast, country B's government stabilizes debt in the fiscal-led economy, s^{F*}_{bt}, because only country A holds unfunded debt in this economy: $\gamma^{F*} > 1$. The monetary authority accommodates fiscal-led inflation rates, Π^F_t and Π^{F*}_t, with $\phi^F \leq 1$ and $\phi^{F*} \leq 1$. The remaining part of the economy is monetary-led, so that both countries are on a debt stabilizing path, $\gamma^M > 1$ and $\gamma^{M*} > 1$, and the central bank reacts more than one-to-one to inflation $\phi^M > 1$ and $\phi^{M*} > 1$. ζ^M_t is monetary-led, hence the funded fiscal shock and ζ^F_t is the fiscal-led, unfunded fiscal shock in country A. Because of symmetry, we do not present the shocks in country B in the stylized model.

To characterize the determinacy of the model, we construct the monetary block and the fiscal block of the economy, as in the previous section. We linearize the above three equations to obtain:

$$\hat{\tau}_t = \gamma^M \left(\hat{s}_{b,t-1} - \hat{s}^F_{b,t-1}\right) + \gamma^F \hat{s}^F_{b,t-1} + \zeta^M_t + \zeta^F_t, \tag{30}$$

$$\hat{\tau}^*_t = \gamma^{M*} \left(\hat{s}^*_{b,t-1} - \hat{s}^{F*}_{b,t-1}\right) + \gamma^{F*} \hat{s}^{F*}_{b,t-1}, \tag{31}$$

$$\hat{i}_t = \Theta \left[\phi^M \left(\hat{\pi}_t - \hat{\pi}^F_t\right) + \phi^F \hat{\pi}^F_t\right] + (1-\Theta) \left[\phi^{M*} \left(\hat{\pi}^*_t - \hat{\pi}^{F*}_t\right) + \phi^{F*} \hat{\pi}^{F*}_t\right]. \tag{32}$$

Substituting the linear monetary rule that allows for inflation coming from unfunded debt, equation (32), into the Fisher equation (19), we obtain the monetary block of this economy:

$$\begin{aligned} \mathbb{E}_t \hat{\pi}^{MU}_{t+1} &= \phi^M \hat{\pi}^{MU}_t + \Theta \left(\phi^F - \phi^M\right) \hat{\pi}^F_t + (1-\Theta) \left(\phi^{F*} - \phi^M\right) \hat{\pi}^{F*}_t \\ &- \Theta \psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A} \left(\hat{b}_{Bt} - \hat{b}_{At}\right) - (1-\Theta) \psi_b \frac{\overline{b}^*_A + \overline{b}^*_B}{\overline{b}^*_A} \left(\hat{b}^*_{Bt} - \hat{b}^*_{At}\right), \end{aligned} \tag{33}$$

where we use $\phi^{M*} = \phi^M$. Similarly to the closed-economy case, the parameter on the monetary rule, ϕ^M, determines the eigenvalue in the monetary block. For the fiscal block, we substitute the two fiscal rules allowing for unfunded debt, equations (30) and (31), into the government budget constraints (20) and (21):

$$\hat{s}_{bt} = \frac{1}{\beta}\begin{bmatrix} \left(1-(1-\beta)\gamma^{M}\right)\hat{s}_{b,t-1} + (1-\beta)\gamma^{M}\hat{s}^{F}_{b,t-1} \\ +(1-\beta)\gamma^{F}\hat{s}^{F}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_{t} - (1-\beta)\left(\zeta^{M}_{t} + \zeta^{F}_{t}\right) \end{bmatrix}, \tag{34}$$

$$\hat{s}^{*}_{bt} = \frac{1}{\beta}\begin{bmatrix} \left(1-(1-\beta)\gamma^{M*}\right)\hat{s}^{*}_{b,t-1} + (1-\beta)\gamma^{M*}\hat{s}^{F*}_{b,t-1} \\ +(1-\beta)\gamma^{F*}\hat{s}^{F*}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}^{*}_{t} \end{bmatrix}. \tag{35}$$

Again, the debt stabilization parameters γ^{M} and γ^{M*} characterize the eigenvalues of the fiscal block.

To close the model, we need to pin down the dynamics of fiscal-led inflation $\hat{\pi}^{F}_{t}$ and $\hat{\pi}^{F*}_{t}$ and the fiscal-led government debt $\hat{s}^{F}_{bt}$ and $\hat{s}^{F*}_{bt}$. To do so, as in BFM we construct a shadow economy which keeps track of the fiscal-led variables. The monetary block of this shadow economy is:

$$\mathbb{E}_{t}\hat{\pi}^{MU,F}_{t+1} = \phi^{F}\hat{\pi}^{MU,F}_{t} - \Theta\psi_{b}\frac{\overline{b}^{F}_{A} + \overline{b}^{F}_{B}}{\overline{b}^{F}_{A}}\left(\hat{b}^{F}_{Bt} - \hat{b}^{F}_{At}\right) - (1-\Theta)\psi_{b}\frac{\overline{b}^{F*}_{A} + \overline{b}^{F*}_{B}}{\overline{b}^{F*}_{A}}\left(\hat{b}^{F*}_{Bt} - \hat{b}^{F*}_{At}\right). \tag{36}$$

obtained by aggregating the monetary block in each country:

$$\mathbb{E}_{t}\hat{\pi}^{F}_{t+1} = \phi^{F}\hat{\pi}^{F}_{t} - \psi_{b}\frac{\overline{b}^{F}_{A} + \overline{b}^{F}_{B}}{\overline{b}^{F}_{A}}\left(\hat{b}^{F}_{Bt} - \hat{b}^{F}_{At}\right), \tag{37}$$

$$\mathbb{E}_{t}\hat{\pi}^{F*}_{t+1} = \phi^{F*}\hat{\pi}^{F*}_{t} - \psi_{b}\frac{\overline{b}^{F*}_{A} + \overline{b}^{F*}_{B}}{\overline{b}^{F*}_{A}}\left(\hat{b}^{F*}_{Bt} - \hat{b}^{F*}_{At}\right), \tag{38}$$

and using $\phi^{F} = \phi^{F*}$. This parameter governs how the central bank reacts to fiscal-led inflation and determines the eigenvalue of the monetary block in the shadow economy. The fiscal block of the shadow economy is characterized by the government budget constraints combined with the fiscal rules:

$$\hat{s}^{F}_{bt} = \frac{1}{\beta}\left[\hat{s}^{F}_{b,t-1} + \hat{i}^{F}_{t-1} - \hat{\pi}^{F}_{t} - (1-\beta)\tau^{F}_{t}\right], \tag{39}$$

$$\hat{s}^{F*}_{bt} = \frac{1}{\beta}\left[\hat{s}^{F*}_{b,t-1} + \hat{i}^{F*}_{t-1} - \hat{\pi}^{F*}_{t} - (1-\beta)\tau^{F*}_{t}\right], \tag{40}$$

where

$$\hat{\tau}^{F}_{t} = \gamma^{F}\hat{s}^{F}_{b,t-1} + \zeta^{F}_{t}, \tag{41}$$

$$\hat{\tau}^{F*}_{t} = \gamma^{F*}\hat{s}^{F*}_{b,t-1} \tag{42}$$

and where $\hat{i}^{F}_{t-1}$ is the part of the nominal interest rate that responds to fiscal-led inflation. Following the monetary rule (32), this component can be written as $\hat{i}^{F}_{t} \equiv \Theta\phi^{F}\hat{\pi}^{F}_{t} + (1-\Theta)\phi^{F*}\hat{\pi}^{F*}_{t}$. The debt stabilization parameters, γ^{F} and γ^{F*} determine the eigenvalues in the fiscal block of the shadow economy.

The equations that characterize the currency union model with partially unfunded debt are: the monetary and fiscal block of the actual economy, (33)–(35) and the monetary and fiscal block of the fiscal-led, shadow economy, (36), (39), and (40). There are two non-predetermined variables, $\hat{\pi}_t^{MU}$ and $\hat{\pi}_t^{MU,F}$, and two eigenvalues outside the unit circle in equations (33) and (39) such that Blanchard-Kahn conditions are satisfied and there is a unique and stable equilibrium.

As in BFM, we exploit the linearity of the model to prove that the non-fiscal-led components of inflation and debt—so $\hat{\pi}_t^M = \hat{\pi}_t - \hat{\pi}_t^F$, $\hat{\pi}_t^{M*} = \hat{\pi}_t^* - \hat{\pi}_t^{F*}$, $\hat{s}_{bt}^M = \hat{s}_{bt} - \hat{s}_{bt}^F$, and $\hat{s}_{bt}^{M*} = \hat{s}_{bt}^* - \hat{s}_{bt}^{F*}$—are indeed the inflation and debt of a fully monetary-led economy. See Appendix C for details.

2.7 Calibration

The calibration of the stylized model is straightforward and follows the literature. The discount factor β is 0.99, the degree of trade openness α_I is 0.25 to allow for home bias according to Eurostat (2023), and the elasticity of substitution between goods produced in country A and B μ is 1. The countries are of equal size, which means that $\Theta = 0.5$. All shocks have a persistence of 0.5.

Debt adjustment cost ψ_b is 0.001. We calibrate the steady-state real market value of debt as share of output, $\bar{s}_b$, as 0.80, which is the average debt-to-GDP ratio of France and Germany for the period 2004–2024, the main countries and period we focus on in the empirical section that follows (Eurostat Data, 2025). The steady-state ratio of domestic versus foreign bond holdings is $\delta_b = 0.44/0.56$, to match the data for France and Germany for the period 2004-2019 (Bruegel, 2025). The countries are symmetric such that the net foreign asset position is 0 in steady state.

We follow BFM for the parameters specific to the model with partially unfunded debt. The weight on the monetary-led part of inflation, $\phi^M = \phi^{M*} = 2$, whereas the central bank disregards the fiscal-led part of inflation, $\phi^F = \phi^{F*} = 0$. The fiscal governments stabilize the monetary-led part of government debt with $\gamma^M = \gamma^{M*} = 20$ and disregard the fiscal-led part of debt with $\gamma^F = \gamma^{F*} = 0$.

A summary of the calibration of the quantitative model is in Table 4, which includes parameters from the stylized model.

2.8 Results

The results of the stylized model provide intuition on how funded and unfunded fiscal shocks propagate through a currency union and generate testable predictions for the empirical analysis that follows. We perform the quantitative exercises, including model estimation and impulse response matching, with the quantitative model in Section 4.

Figure 2a shows the impulse responses to a funded fiscal expansion in country A. The fiscal expansion is in the form of a decline in the primary surplus, or taxes. In a closed economy or in a currency union model when the net foreign asset (NFA) position is always 0, a funded fiscal expansion increases debt and has no effect on inflation. The debt increases because it is backed by future tax adjustments.[2] In the currency union model with bonds trade, the increase in domestic debt creates general equilibrium effects through the current account equation (16). The increase in government debt in country A raises country B households' holdings of bonds issued by country A. So, net savings in country A, as expressed by the current account or the NFA position, decreases, and consumption increases. The increased demand pushes inflation upward. On the other hand, the net savings in country B increases. Hence, demand slows down and consumption and inflation decreases.

The central bank targets the average inflation rate across the two countries. Inflation moves in opposite directions, creating divergent real conditions across the two countries. The real interest rate reveals that the monetary policy is expansionary for home and contractionary for country B on impact. Hence, sharing a central bank exacerbates the divergence between the two countries.

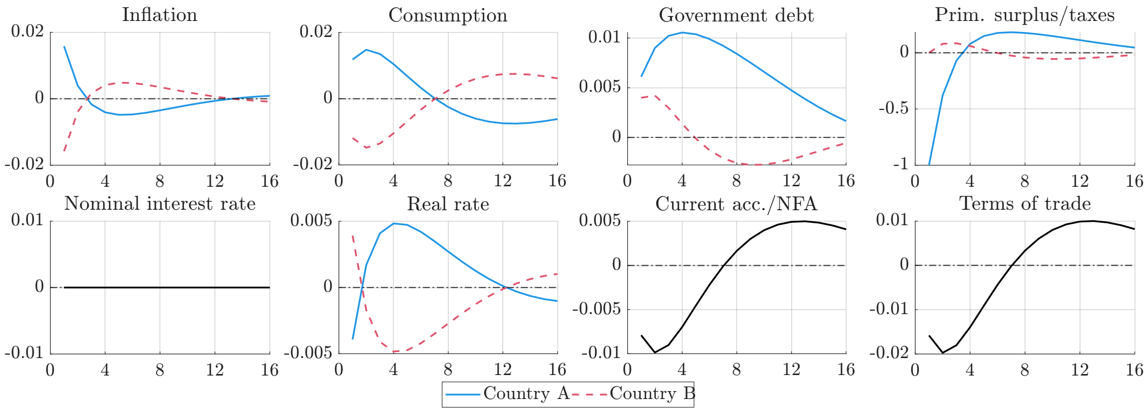
Figure 2b shows the impulse responses to an unfunded fiscal expansion in country A. In a closed economy or in a currency union model without bonds trade, an unfunded fiscal expansion adjusts through an inflation increase, decreasing the real value of government debt.[3] In the currency union, we see similarly that inflation in country A increases more than in the funded case and that the real government debt decreases. Since the inflation is fiscal-led, the monetary authority does not react. Households in country B also experience higher inflation, because part of their consumption basket is produced in country A. Hence, we see an increase in inflation in country B too, as a spillover from the unfunded fiscal expansion in country A, and therefore also a decrease in government debt.

Since the central bank does not respond to the increase in fiscal-led inflation, real interest rates in both countries decline. The decline in real interest rate stimulates demand in both countries, magnifying the inflation increases. This result is consistent with the findings by BFM in the closed economy, in which the real interest rate re-

2. See BFM for more details on the closed economy. Figure A.12a in Appendix A show the responses in a currency union model in which the net foreign asset position is always 0. This setup is isomorphic to imposing bonds adjustment cost only on the foreign bond.

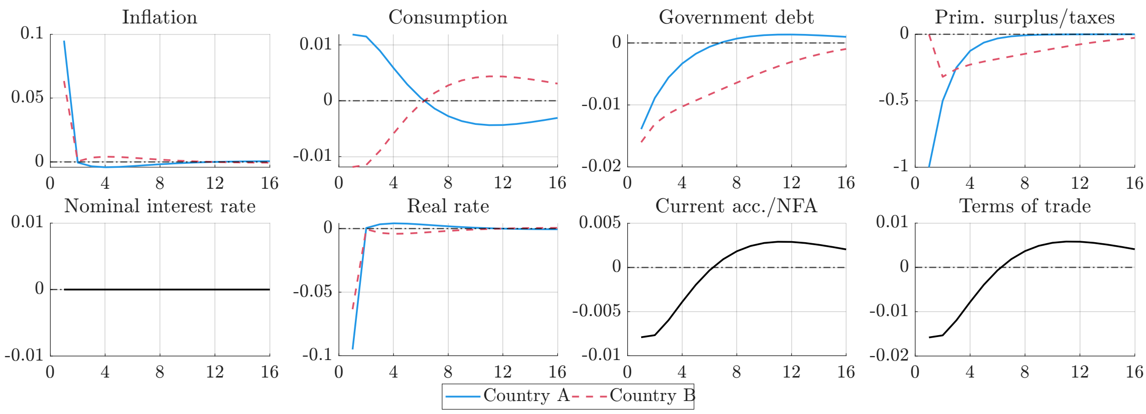
3. See BFM for more details on the closed economy. Figure A.12b in Appendix A show the responses in a currency union model in which the NFA position is always 0. In country A, inflation increases and decreases real government debt. In country B, the inflation from country A spills over since the central bank response is absent. Hence, real government debt in country B decreases too.

Figure 2: Impulse responses to funded and unfunded fiscal expansions in country A



(a) Funded fiscal expansion in country A

Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The fiscal expansion is funded, i.e. monetary-led, so a shock to ζ_t^M. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.



(b) Unfunded fiscal expansion in country A

Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The fiscal expansion is unfunded, i.e. fiscal-led, so a shock to ζ_t^F. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

sponds negatively to unfunded fiscal shocks.

All in all, under a funded fiscal expansion in country A, we see that inflation rates diverge within the currency union, whereas under an unfunded fiscal expansion, the inflation rates move together. These results are not merely numerical: Proposition 2 in Appendix D provides an analytical proof that funded shocks generate inflation divergence and unfunded shocks generate co-movement under general parameter conditions. Moreover, the qualitative results also hold for government spending shocks, instead of lump-sum tax shocks. In Figure A.14a and A.14b in Appendix A, we show the impulse responses to fiscal expansions in the form of government spending. Through similar transmission channels, a funded government spending increase causes inflation divergence, whereas an unfunded government spending increase raises inflation in both countries.

The model thus delivers a testable prediction: if a country's fiscal shocks generate divergent inflation responses across countries, those shocks are primarily funded, whereas if those shocks generate co-movement in inflation across countries, those are primarily unfunded. We test this prediction in Section 3 and find that German and Italian fiscal shocks are qualitatively similar to an unfunded shock, whereas French shocks are similar to a funded shock. In Section 4 we build and estimate a medium-scale DSGE model to quantify the mix of funded and unfunded shocks in those countries and explaining the different inflation spillovers with those shocks.

Relevance of the currency union setting. We build a version of the model with flexible nominal exchange rates to examine the role of the currency union setting. The equations that depart from the currency union setting are in Appendix F. Figure A.13 shows the results for a funded fiscal shock in country A under flexible exchange rates. The shock propagates in the same way as a closed economy: the funded fiscal expansion decreases the primary surplus in country A, and government debt increases, backed by future taxes. All other variables, including those of country B, do not respond in reaction to the shock. The central bank in country A controls inflation and hence stabilizes demand. Country B has its own central bank that independently stabilizes its inflation and the flexible exchange rate absorbs the cross-country relative price adjustments. These mechanisms together insulate country B from fiscal developments in country A. As explained thoroughly in Dupor (2000), under flexible nominal exchange rates, the nominal exchange rate constitutes a second nominal variable. The government's intertemporal budget constraint can pin down only one, the price level. Hence, the equilibrium is indeterminate under an unfunded fiscal shock. The currency union is therefore not merely a convenient but a necessary framework for studying cross-country propagation of unfunded fiscal shocks: it is the only setting in which such shocks generate a determinate equilibrium.

3 Empirical analysis

The stylzed model predicts that funded fiscal shocks, both lump-sum tax shocks and government spending shocks, generate divergent inflation responses across currency union members, while unfunded shocks cause inflation to co-move. We now test these predictions empirically. In this section, we study how fiscal shocks originating in Germany, France, and Italy—the three largest euro area economies—transmit to the domestic economy and to the rest of the union. Using local projections à la Jordà (2005) and fiscal shocks obtained from forecast errors, we establish three facts. First, fiscal expansions in Germany and Italy are inflationary both domestically and in other euro area countries, whereas fiscal expansions in France are not. Second, monetary policy responses differ by source country: the central bank reacts in a contractionary way to French fiscal expansions, but less so to German and Italian expansions. Third, consistent with this heterogeneous policy response, real rate dynamics also differ by source country. Taken together, these results suggest that the central bank accommodates inflation caused by fiscal expansions in Germany and Italy (*fiscal inflation*), but not fiscal expansions in France.

3.1 Methodology and data

Fiscal shocks. We construct fiscal shocks using the OECD Economic Outlook (OECD, 2025). The OECD publishes the Economic Outlook twice a year, and it includes forecasts of macroeconomic variables as point estimates, including government spending, for the next few quarters. As in Auerbach and Gorodnichenko (2012, 2017) and Boehm (2020), we use those forecasts to construct forecast errors of government spending as a share of gross domestic product (GDP) for each country. We have quarterly data starting from *Economic Outlook No 74* (December 2003), so our sample spans 2004Q1–2025Q4.

We construct forecast errors of government spending using data on government consumption in volume (constant-price) terms. We extract forecasts of government consumption for Q1 and Q2 of each year from the second *Economic Outlook* of the previous year, and forecasts for Q3 and Q4 from the first *Economic Outlook* in the same year. For example, for the forecast of government spending in 2016Q1 and 2016Q2, we consult the second *Economic Outlook* of 2015. We then scale the forecasted growth rate of government spending by GDP to obtain a forecast for the government spending growth rate as a percentage of GDP. The forecast error $FE_{t|t-1}$ is the difference between the realized government-spending growth rate (as a share of GDP) and the forecast made in the previous period. We use realized government-spending growth rates from the most recent *Economic Outlook*.

Even though the forecast error $FE_{t|t-1}$ is already the unpredictable innovation in government-spending growth (as in Auerbach and Gorodnichenko (2017)), we further purge predictable components by regressing $FE_{t|t-1}$ on lags of macroeconomic variables. We use the residuals from this panel regression as the fiscal shock. Appendix A shows all forecast errors in Figure A.1 and their correlations in Table A.1. Purged government spending shocks are weakly positively correlated with next-quarter debt/GDP growth (r = 0.10-–0.21 across countries) as shown in A.2, consistent with spending expansions being partially debt-financed, though the relationship is unsurprisingly noisy given the many other drivers of debt dynamics.

Other data. All other data that we use as controls in the local projections are also publicly available at the OECD. We construct quarterly government spending growth and GDP growth from the government spending and GDP measures above. The harmonized inflation rate and the short-term nominal interest rate are expressed in annualized percentage points. The real rate is the difference between the nominal interest rate and inflation. We focus on the original euro area countries to include countries that have shared a central bank for the longest period.[4]

Local projection specification. We study three types of local projections: (i) country-specific local projections for the effect of a fiscal shock on domestic inflation and real rate, (ii) spillover local projections for the effect of a fiscal shock on inflation and real rate in other countries, and (iii) short rate local projections for the effect of a fiscal shock on the short-term nominal interest rate.

The specification of the country-specific local projections, used for y as inflation and real rate, are as follows:

$$y_{c,t+h} - y_{c,t} = \beta_{c,h}\text{shock}_{c,t} + \sum_{j=0}^{2} \gamma_{c,h,j} X_{c,t-j} + \varepsilon_{c,t+h}, \tag{43}$$

where $c \in \{\text{Germany, France, Italy}\}$. $y_{c,t+h}$ is the inflation or real rate of country c at time $t+h$, and $\text{shock}_{c,t}$ is the fiscal shock, as explained above. $X_{c,t-j}$ includes the control variables: government spending growth, GDP growth, inflation, and their lags.[5] We estimate the response of inflation and the real rate up to $h = 1, 2, \ldots, 8$ quarters ahead with heteroskedastic standard errors. We estimate equation (43) twice for each country, for the inflation and real rate, to obtain the effect of a fiscal shock on domestic inflation and real rate.

The spillover panel local projections estimate the effect of the fiscal shock in the source country on the inflation and real rate in the other countries in the euro area:

$$y_{i\neq c,t+h} - y_{i\neq c,t} = \beta_{c,h}\text{shock}_{c,t} + \sum_{j=0}^{2} \gamma_{i,h,j} X_{i,t-j} + \alpha_{i\neq c} + \varepsilon_{i\neq c,t+h}, \tag{44}$$

where i are all countries in the sample and countries $c \in \{\text{Germany, France, Italy}\}$ are

4. Austria, Belgium, Germany, Spain, France, Ireland, Italy, Netherlands, Portugal. We exclude Luxembourg and Finland for data availability reasons.

5. For the real rate local projections, we add the lags of the change of the real rate as controls. Moreover, we include euro area wide GDP and inflation, since those are naturally confounded with the euro area wide short-term interest rate, included in constructing the dependent variable.

the source country of the fiscal shock. Inflation and the real rate (the dependent variable) and the country fixed effects are of countries other than the source country. The controls include lagged variables from both the receiver and source country and are the same as the country-specific local projections, plus contemporaneous and lagged euro area wide controls for inflation and GDP growth. We cannot add quarterly time fixed effects since the shock is identical across countries and therefore collinear with quarterly time-fixed effects. The spillover panel local projections use Driscoll-Kraay standard errors with GDP weights. With equation (44), we estimate $\beta_{c,h}$ with panel local projections for each source country, which we interpret as the effect of a fiscal shock in the source country on inflation and real rate in other euro area countries.

The short-rate local projections are similar to the country-specific local projections, but with the euro area wide short-term interest rate, i_t, as the dependent variable:

$$i_{t+h} - i_t = \beta_{c,h}\text{shock}_{c,t} + \sum_{j=0}^{2} \gamma_{c,h,j} X_{c,t-j} + \varepsilon_{c,t+h}. \tag{45}$$

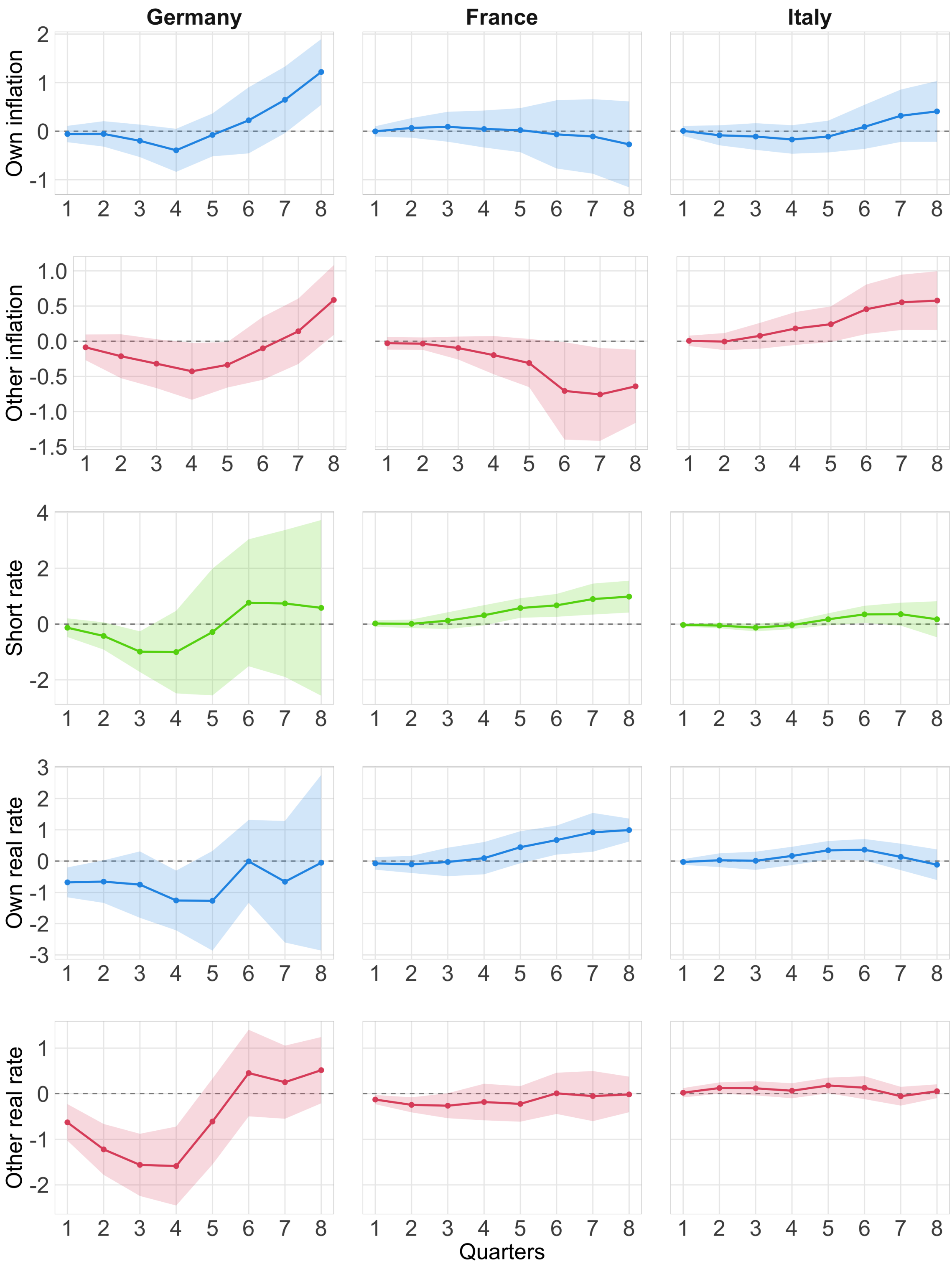
For this specification, the controls also include contemporaneous euro area wide inflation and GDP growth rate, and their lags, as well as the lags of the change in the short-term interest rate. We estimate $\beta_{c,h}$ with this equation, which we interpret as the effect of a fiscal shock on the short-term nominal interest rate. As in the country-specific local projections, we use heteroskedastic standard errors. Since the short-term interest rate in the euro area drops below zero in 2014Q2, we consider this as the start of the effective lower bound. We run all short-rate and real-rate local projections with data up to 2014Q1.

3.2 Results

Figure 3 presents the results from estimating five local projections, on the effect of a fiscal shock on (i) domestic inflation (eq. (43)), (ii) other countries' inflation (eq. (44)), (iii) the short-term interest rate (eq.(45)), (iv) domestic real rate (eq.(43)), and (v) other countries' real rate (eq.(44)). The column labels indicate the source country of the fiscal shock.

The first and the second row in Figure 3 show the effects of a fiscal shock in Germany, France, and Italy on domestic inflation and inflation in other countries in the euro area. Our findings are as follows: fiscal expansions in Germany are inflationary for both the domestic economy and the rest of the euro area. There is a similar pattern for fiscal expansions in Italy, but not significant domestically. In France, fiscal expansions do not have a significant effect on domestic inflation, but they decrease inflation in other countries. These results suggest that the monetary policy stance after fiscal ex-

Figure 3: Impulse responses to a 1 pp shock in govt spending (% of GDP)



Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

pansions in France is more contractionary than after expansions in Germany or Italy. The responses in the third row of Figure 3 confirm this conjecture: the short-term interest rate increases after a fiscal expansion in France, but not after expansions in Germany or Italy.

The responses of the real rate, as shown in the fourth and fifth row in Figure 3, show that fiscal expansions in Germany decrease the real rate domestically and in other countries. Fiscal expansions in France, however, increase the real rate domestically. For Italy, the results are insignificant. The results are in line with the responses for inflation and the short-term interest rate. On one hand, since German fiscal expansions do not move the short-term interest rate, they increase inflation and decrease real rate domestically. Moreover, these effects spill over to other countries in the euro area. On the other hand, French fiscal expansions elicit a contractionary response from the central bank, and hence decrease inflation in other countries sharing the central bank.

In summary, we find that fiscal expansions in Germany, France and Italy have heterogeneous effects on inflation, the short-term interest rate, and the real rate. While fiscal expansions in France elicit an interest hike as a contractionary response from the central bank, expansions in Germany and Italy do not. Hence, expansions in Germany and Italy are inflationary, both domestically and for other countries. In France, domestic inflation remains constant after fiscal expansions, likely because the fiscal boost and the interest hike counteract each other. However, in other countries, inflation decreases due to the interest rate increase. The real rate responses reinforce this narrative: the inflationary effect from the German fiscal expansions and an absent monetary policy response lead to a decline in real rates domestically and in other euro area countries. The real rate in France increases after a fiscal expansion through the contractionary response from the central bank.

The empirical results establish that fiscal shocks in Germany and Italy have an inflationary effect on the rest of the euro area, suggesting that the central bank accommodates inflation. This result alludes to *fiscal inflation* coming from *unfunded* fiscal shocks, as we established in the stylized model. By contrast, fiscal shocks in France have a deflationary effect on the rest of the euro area, implying that the central bank reacts to fiscal expansions and that these shocks may be *funded*.

As robustness checks, Appendix A includes bilateral spillover responses for Germany, France, and Italy for inflation in Figure A.3 and the real rate in Figure A.4. Moreover, we do the entire exercise with unpurged shocks, so the pure forecast errors, in Figure A.5 and with different standard error specifications in Figure A.6. Figure A.7 shows the results when all local projections include lags of fiscal shocks as controls and Figure A.8 include contemporaneous weighted fiscal shocks of other countries and its lags as controls. Figure A.9 shows the results when we include one more period of lags in all the controls. We also run a timing-robust specification in Figure A.10 that removes within-semester reallocation variation in forecast errors before estimating the same local projections.[6] All results are in line with the baseline results. We also conduct local projections on the sovereign spread in Figure A.11.

4 Quantitative model

In this section, we build a medium-scale currency union model with partially unfunded debt. The model will feature habit formation by households, production of tradable and non-tradable goods with nominal rigidities, capital formation, and long-term bonds. The aim of the model is to successfully capture the business cycles in the euro area with macroeconomic shocks, such that we can quantify the effect of funded and unfunded fiscal shocks on the economy. The model builds on the stylized model, so we only explain the changed equations.

4.1 Households

Households derive utility from consumption C_t and disutility from providing labor L_t. There are external habits in consumption, which means that utility from consumption relates to the previous value of aggregate consumption. The per-period utility function of the household is as follows:

$$\ln\left(C_t - hC_{t-1}\right) - \frac{1}{1+\chi} L_t^{1+\chi} \exp(\varepsilon_t^h), \tag{46}$$

where h is the habit parameter, χ the inverse-Frisch elasticity of labor supply, and ε_t^h is an IID labor supply shock. As in Ferroni, Fisher, and Melosi (2024), we use this shock to accommodate the COVID-19 pandemic period (2020Q1–2021Q2), during which lockdown policies caused unprecedented and exogenous labor market disruptions. The shock variance is restricted to zero outside the pandemic period using heteroskedastic shock variances, so that ε_t^h does not affect the model's dynamics outside of the COVID quarters.

Households derive income from renting out capital K_t at rate R_{Kt} to tradable goods producers and exerting labor for wages W_t. The price of a unit of capital is Q_t and the

6. For each country and semester (S1: Q1–Q2, S2: Q3–Q4), we compute the quarterly forecast error as before, but replace each quarter's error with the semester average of the two quarterly forecast errors. This keeps the semester-level surprise in government spending but removes within-semester timing reallocation (e.g., if spending was expected in Q3 but realized in Q4). We then apply the same purging step and re-estimate all local projections with the same controls and horizons.

depreciation rate is δ. Households can invest in capital through perfectly competitive capital producers as explained below.

Households can save and smooth consumption through one-period government bonds B_t which are in zero net supply, and through long-term government bonds from both country A, B_{At}, and country B, B_{Bt}, in nonzero net supply. The one-period bonds pay interest rate R_t which is set by the central bank. We follow Woodford (2001) and model long-term bonds as a perpetuity with geometrically declining coupons with average maturity of $(1-\beta\rho)^{-1}$ where $\rho \in [0,1]$ is a constant rate of decay. Households can purchase the long-term bonds from country A and B at price P_t^L and P_t^{L*} respectively. Arbitrage conditions determine the price of these bonds, together with country specific risk premium shocks ξ_t^{rp} and ξ_t^{rp*}:

$$R_t = \mathbb{E}_t R_{t+1}^L e^{\xi_t^{rp}} \equiv \mathbb{E}_t \left[\frac{1+\rho P_{t+1}^L}{P_t^L}\right] e^{\xi_t^{rp}}, \tag{47}$$

$$R_t = \mathbb{E}_t R_{t+1}^{L*} e^{\xi_t^{rp*}} \equiv \mathbb{E}_t \left[\frac{1+\rho P_{t+1}^{L*}}{P_t^{L*}}\right] e^{\xi_t^{rp*}}. \tag{48}$$

We can then write the per-period budget constraint of the household as:

$$\begin{aligned} &P_t(1+\tau_{Ct})C_t + Q_t K_t + P_t^L B_{At} + P_t^{L*} B_{Bt} + B_t \\ &= W_t(1-\tau_{Lt})L_t + P_t Z_t + \left[R_{Kt}(1-\tau_{Kt})\mu_{Kt} - a(\mu_{Kt}) + Q_t(1-\delta)\right] K_{t-1} + R_{t-1}B_{t-1} \\ &+ \left(1+\rho P_t^L\right) B_{A,t-1} + \left(1+\rho P_t^{L*}\right) B_{B,t-1} - \frac{\psi_b P_t \left(\bar{b}_A + \bar{b}_B\right)}{2} \left(\delta_b \frac{B_{Bt}}{B_{At}} - 1\right)^2, \end{aligned} \tag{49}$$

where C_t is composed of non-tradable and tradable goods and Z_t are lump-sum transfers from the government. τ_{Ct}, τ_{Lt}, and τ_{Kt} are consumption tax, labor income tax, and capital income tax, respectively. The notation is consistent with the stylized model presented in Section 2. μ_{Kt} is the capital utilization rate and $a(\mu_{Kt})$ the resource cost, as explained below.

The household maximizes its utility choosing consumption, labor, capital, and the three types of bonds, subject to the budget constraint. The optimization problem gives rise to the labor supply equation:

$$C_t^{\text{eff}} L_t^{\chi} \exp(\varepsilon_t^h) = MRS_t \tag{50}$$

where $C_t^{\text{eff}} = C_t - hC_{t-1}$ is effective consumption and MRS_t denotes the marginal rate of substitution between consumption and labor. At the optimum, $MRS_t = (1-\tau_{Lt})W_t/P_t$. Moreover, households also choose the amount of capital to rent out and the bonds to hold following the Euler equations:

$$Q_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[R_{K,t+1}(1-\tau_{K,t+1})\mu_{K,t+1} - a(\mu_{K,t+1}) + (1-\delta)Q_{t+1} \right], \tag{51}$$

$$1 = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} R_t, \tag{52}$$

$$\lambda_t P_t^L = \beta \mathbb{E}_t \lambda_{t+1} \left(1 + \rho P_{t+1}^L\right) - \psi_b(\overline{b}_A + \overline{b}_B) \left(\delta_b \frac{b_{Bt}}{b_{At}} - 1\right) \delta_b \frac{b_{Bt}}{b_{At}^2}, \tag{53}$$

$$\lambda_t P_t^{L*} = \beta \mathbb{E}_t \lambda_{t+1} \left(1 + \rho P_{t+1}^{L*}\right) + \psi_b \left(\overline{b}_A + \overline{b}_B\right) \left(\delta_b \frac{b_{Bt}}{b_{At}} - 1\right) \delta_b \frac{1}{b_{At}}, \tag{54}$$

where $\lambda_t = \frac{1}{C_t^{\text{eff}}} - \beta h \mathbb{E}_t \frac{1}{C_{t+1}^{\text{eff}}}$ is the marginal utility of consumption. These Euler equations give rise to no-arbitrage conditions across the different asset holdings.

Households also choose the capital utilization rate μ_{Kt}, which determines the intensity at which the capital stock is used in production. Following Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), varying utilization incurs a resource cost $a(\mu_{Kt})$ per unit of capital, where $a(\cdot)$ satisfies $a(1) = 0$, $a'(1) = \overline{R}_K$, and $a''(1)/a'(1) = \psi_u/(1-\psi_u)$. The parameter $\psi_u \in (0,1)$ governs the curvature of the cost function: as $\psi_u \to 1$, utilization becomes prohibitively costly and effectively fixed at its steady-state value. The first-order condition for utilization equates the rental rate of capital to the marginal cost of utilization:

$$R_{Kt} = a'(\mu_{Kt}) = \alpha \frac{Y_{Tt}}{\mu_{Kt} K_{t-1}}. \tag{55}$$

C_t is a constant elasticity of substitution (CES) aggregate of tradable goods C_{Tt} and non-tradable goods C_{Nt}, and P_t is the price of the aggregate bundle (with Cobb-Douglas aggregation in the limit as $\mu_N \to 1$):

$$C_t = \left[(1-\alpha_N)^{1/\mu_N} (C_{Tt})^{(\mu_N - 1)/\mu_N} + \alpha_N^{1/\mu_N} (C_{Nt})^{(\mu_N - 1)/\mu_N} \right]^{\mu_N/(\mu_N - 1)}, \tag{56}$$

$$P_t = \left[(1-\alpha_N) P_{Tt}^{1-\mu_N} + \alpha_N P_{Nt}^{1-\mu_N} \right]^{1/(1-\mu_N)}, \tag{57}$$

where C_{Tt} and C_{Nt} are consumption of tradable and non-tradable goods, respectively, and P_{Tt} and P_{Nt} their prices. α_N is the share of non-tradable goods in the consumption basket and μ_N the elasticity of substitution between tradable and non-tradable goods. The bundle of tradable goods C_{Tt} is equivalent to the bundle of consumption goods in the stylized model. Similarly, the price of the tradable goods P_{Tt} is equivalent to the price of the consumption good in the stylized model. The cost-minimization problem provides the following relative demand between tradable and non-tradable goods:[7]

$$\frac{C_{Tt}}{C_{Nt}} = \frac{1-\alpha_N}{\alpha_N}\left(\frac{P_{Tt}}{P_{Nt}}\right)^{-\mu_N}. \tag{58}$$

4.2 Wage setting

We introduce differentiated labor and wage rigidities as in Christiano, Eichenbaum, and Evans (2005). We assume that each household supplies a unique labor type $L_t(l)$ at nominal wage $W_t(l)$, where $l \in [0,1]$. A labor union then transforms the differentiated labor outputs into aggregate labor L_t using a CES aggregator:

$$L_t = \left(\int_0^1 L_t(l)^{\frac{\epsilon_w-1}{\epsilon_w}} dl\right)^{\frac{\epsilon_w}{\epsilon_w-1}}, \tag{59}$$

where ϵ_w is the elasticity of substitution across labor types. The labor union minimizes the cost $\int_0^1 W_t(l)L_t(l)dl$ subject to the aggregation constraint, which yields the labor demand for each type and the aggregate wage index:

$$L_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\epsilon_w} L_t, \tag{60}$$

$$W_t = \left(\int_0^1 W_t(l)^{1-\epsilon_w} dl\right)^{\frac{1}{1-\epsilon_w}}. \tag{61}$$

With probability $(1-\theta_w)$, each labor type can reset its wage each period. When the labor type is not re-optimizing its wage, the wage evolves via the following indexation:

$$W_t(l) = W_{t-1}(l)\Pi_{t-1}^{\chi_w}, \tag{62}$$

where Π_t is gross price inflation and $\chi_w \in [0,1]$ the degree of wage indexation. When the labor type is re-optimizing its wage, it solves the following problem:

$$\max_{W_t^{\#}(l)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_w)^s \left[\ln C_{t+s} - \frac{1}{1+\chi} L_{t+s}^{1+\chi}\right], \tag{63}$$

taking into account the labor type demand (61), the wage indexation (62), and the household budget constraint, so that the relevant trade-off is between labor disutility and the after-tax real wage. The first-order condition, after applying the symmetry condition $W_t(l) = W_t$ and log-linearizing, gives the standard wage Phillips Curve:

$$\hat{\pi}_t^w = \beta\mathbb{E}_t\hat{\pi}_{t+1}^w + \chi_w\hat{\pi}_{t-1} + \kappa_w\left(\widehat{mrs}_t - \hat{w}_t^{real}\right) - \kappa_w\frac{\overline{\tau}_L}{1-\overline{\tau}_L}\hat{\tau}_{Lt} + u_t^w, \tag{64}$$

7. Consequently, the tradable consumption bundle C_{Tt} retains the CES structure from the stylized model, such that $C_{Tt} = \left[(1-\alpha_I)^{1/\mu}(C_{At})^{(\mu-1)/\mu} + \alpha_I^{1/\mu}(C_{Bt})^{(\mu-1)/\mu}\right]^{\mu/(\mu-1)}$, where C_{At} and C_{Bt} denote consumption of goods produced in countries A and B, respectively. The associated demand equations are $C_{At} = (1-\alpha_I)\left(\frac{P_{At}}{P_{Tt}}\right)^{-\mu} C_{Tt}$ and $C_{Bt} = \alpha_I\left(\frac{P_{Bt}}{P_{Tt}}\right)^{-\mu} C_{Tt}$, with analogous expressions for country B households.

where $\kappa_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w}\frac{1}{1+\chi}$ and u_t^w is a wage markup shock.

4.3 Firms

4.3.1 Capital producers

Capital producers transform final goods into investment subject to adjustment costs as in Christiano, Eichenbaum, and Evans (2005). The optimization problem for the capital producers is as follows:

$$\max_{I_t} \mathbb{E}_t \sum_{t=0}^{\infty} \Lambda_{t,t+1} \left[Q_t - 1 - \frac{\psi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \right] I_t, \tag{65}$$

where $\Lambda_{t,t+1} = \beta\frac{C_t}{C_{t+1}}$ is the stochastic discount factor and ψ_I the investment adjustment cost. The first order condition of the capital producers determines the price of capital:

$$Q_t = 1 + \frac{\psi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 + \psi_I\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}} - \psi_I \mathbb{E}_t \Lambda_{t+1}\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2. \tag{66}$$

Investment turns into capital K_t, which evolves according to the following law of motion:

$$K_t = (1-\delta)K_{t-1} + I_t, \tag{67}$$

and we assume that capital is only used in the tradable sector.

4.3.2 Final goods producers

Final goods producers aggregate differentiated intermediate goods from monopolistically competitive firms in both the tradable and non-tradable goods sector, using a CES aggregator:

$$Y_{Tt} = \left(\int_0^1 Y_{Tt}(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{68}$$

$$Y_{Nt} = \left(\int_0^1 Y_{Nt}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{69}$$

where $i \in [0,1]$ represents the continuum of firms in the tradable goods sector, producing goods $Y_{Tt}(i)$, and $j \in [0,1]$ the continuum of firms in the non-tradable sector, producing goods $Y_{Nt}(i)$. ϵ is the elasticity of substitution between the differentiated goods. The cost-minimization problem of the final goods producers yields the following demand schedules for each differentiated good:

$$Y_{Tt}(i) = \left(\frac{P_{Tt}(i)}{P_{Tt}}\right)^{-\epsilon} Y_{Tt}, \tag{70}$$

$$Y_{Nt}(j) = \left(\frac{P_{Nt}(j)}{P_{Nt}}\right)^{-\epsilon} Y_{Nt}, \tag{71}$$

where $P_{Tt}(i)$ and $P_{Nt}(j)$ are the price of tradable and non-tradable from firm i and j respectively.

4.3.3 Tradable goods producers

There is a continuum of tradable goods firms $i \in [0,1]$ who produce differentiated intermediate inputs $Y_{Tt}(i)$ for the final goods producer using the following production technology:

$$Y_{Tt}(i) = A_{Tt}(\mu_{Kt}K_{t-1}(i))^{\alpha} L_{Tt}(i)^{1-\alpha}, \tag{72}$$

where α is the output elasticity of capital. A_{Tt} is the labor augmenting technology and follows an AR(1) process. The firms are under monopolistic competition and are subject to Calvo (1983) price rigidities. Hence, the optimization problem of the firm is:

$$\max_{P_{At}(i),L_{Tt}(i),K_t(i)} \mathbb{E}_0 \sum_{s=0}^{\infty} \theta_T^s \Lambda_{t,t+1} \left[\left(\frac{P_{At}(i)}{P_{A,t+s}} - MC_{t+s}(i)\right) Y_{T,t+s}(i)\right], \tag{73}$$

subject to the production function and the demand for differentiated tradable goods (70). $(1-\theta_T)$ is the fraction of firms that can set their price in each period and $MC_t(i)$ is the real marginal cost for firm i. After applying the symmetry conditions $P_{At}(i) = P_{At}$ and $MC_t(i) = MC_t$ and log-linearizing, the optimization problem gives rise to the standard New Keynesian Phillips Curve (NKPC):

$$\hat{\pi}_{Tt} = \kappa_T \widehat{mc}_{Tt} + \beta \mathbb{E}_t \hat{\pi}_{T,t+1} + u_t^T \tag{74}$$

where $\kappa_T \equiv \frac{(1-\theta_T\beta)(1-\theta_T)}{\theta_T}$, $\widehat{mc}_{Tt} = \hat{w}_t^{real} + \hat{l}_{Tt} - \hat{y}_{Tt}$ is the log-linear real marginal cost of the firm, and u_t^T is a tradable goods markup shock.

4.3.4 Non-tradable goods producers

There is a continuum of non-tradable firms $j \in [0,1]$ producing differentiated intermediate inputs $Y_{Nt}(j)$ for the final goods producer. In contrast to the tradable goods producers, they only use labor as a production input:

$$Y_{Nt}(j) = A_{Nt}L_{Nt}(j)^{1-\alpha}. \tag{75}$$

A_{Nt} is the labor augmenting technology and follows an AR(1) process. Similarly to the firms producing tradable goods, the non-tradable goods producers are under monopolistic competition subject to Calvo (1983) price rigidities. The demand equation for labor and the NKPC are analogous to those of the tradable goods firms.

4.3.5 Fiscal authority

Assuming a zero net supply of one-period government bonds, the nominal budget constraint of the government is as follows:

$$P_t^L B_{At} + \tau_{Kt} R_{Kt} K_t + \tau_{Lt} W_t L_t + \tau_{Ct} P_t C_t = \left(1 + \rho P_t^L\right) B_{A,t-1} + P_t G_t + P_t Z_t. \tag{76}$$

The government raises revenue through distortionary taxes on capital income at rate τ_{Kt}, labor income at rate $\tau_{L,t}$, and consumption at rate $\tau_{Ct} = \tau_C$ (held constant), together with lump-sum transfers T_t. The steady-state tax rates are calibrated to euro area averages: $\bar{\tau}_K = 0.25$, $\bar{\tau}_L = 0.38$, and $\bar{\tau}_C = 0.20$.

Transfer rule. The rule for lump-sum transfers extends the stylized model's fiscal rule with persistence and an output response, following Bianchi, Faccini, and Melosi (2023):

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + (1 - \rho_z) \left[\gamma \hat{s}_{b,t-1} + \phi_{zy} \hat{y}_t\right] + \frac{1}{\bar{z}} \left(\zeta_t + \zeta_{z,t}\right), \tag{77}$$

where we redefine $z_t = \frac{Z_t}{P_t Y_t}$ as the transfers-to-output ratio and $s_{bt} = \frac{P_t^L B_{At}}{P_t Y_t}$ as the debt-to-GDP ratio. ρ_z is the persistence of the transfer rule, ϕ_{zy} captures the response of transfers to output fluctuations, and $\zeta_{z,t}$ is a transitory transfer shock (following BFM Appendix D.20) that avoids triple-counting of transfer shocks across the three-economy decomposition. As in the stylized model, γ determines how strongly transfers adjust to debt.

Government spending rule. Government spending follows an AR(1) process with debt feedback:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} - (1 - \rho_g)\gamma_G \hat{s}_{b,t-1} + \varepsilon_t^g, \tag{78}$$

where γ_G governs the spending response to government debt.

Distortionary tax rules. Capital and labor income tax rates follow persistent rules with debt feedback:

$$\hat{\tau}_{Kt} = \rho_K \hat{\tau}_{K,t-1} + (1 - \rho_K)\gamma_K \hat{s}_{b,t-1}, \tag{79}$$

$$\hat{\tau}_{L,t} = \rho_L \hat{\tau}_{L,t-1} + (1 - \rho_L)\gamma_L \hat{s}_{b,t-1}. \tag{80}$$

Three-economy fiscal decomposition. As in the stylized model, we decompose each fiscal rule into monetary-led (ML) and fiscal-led (FLA for fiscal-led by A, FLB for fiscal-led by B) components. In the ML economy, both countries follow passive fiscal rules

($\gamma^M, \gamma^{M*} > 0$); in the FLA economy, country A follows an active fiscal rule ($\gamma_A^{FLA} \approx 0$) while country B remains passive; in the FLB economy the roles are reversed. The same decomposition applies to the government spending and distortionary tax rules, where the active economy's debt feedback coefficient is set to zero.

4.4 Equilibrium

The monetary policy rule is augmented relative to the stylized model with an AR(1) monetary policy shock ξ_t^m:

$$\hat{i}_t = \phi^M \hat{\pi}_t^{MU} + \xi_t^m, \qquad \xi_t^m = \rho_m \xi_{t-1}^m + \varepsilon_t^m, \tag{81}$$

where ρ_m is the persistence of the monetary policy shock. During estimation, the nominal interest rate is additionally subject to an effective lower bound (ELB), implemented as an occasionally binding constraint using the piecewise-linear method of Guerrieri and Iacoviello (2015). When the ELB binds, the interest rate is fixed at the lower bound and the monetary policy shock innovation is shut down.

The equilibrium of this economy is characterized by a sequence of prices

$$\left\{P_{At}, P_{Bt}, P_{Nt}, P_{Nt}^*, W_t, W_t^*, R_t, R_{Kt}, R_{Kt}^*\right\},$$

and allocations

$$\left\{Y_{Tt}, Y_{Tt}^*, Y_{Nt}, Y_{Nt}^*, K_t, K_t^*, \mu_{Kt}, \mu_{Kt}^*, I_t, I_t^*, G_t, G_t^*, C_{Tt}, C_{Tt}^*, C_{Nt}, C_{Nt}^*, L_{Tt}, L_{Tt}^*, L_{Nt}, L_{Nt}^*, B_{At}, B_{Bt}, B_{At}^*, B_{Bt}^*\right\},$$

such that the goods markets are cleared for both country A and B goods, the labor market is cleared, and the bonds market is cleared for bonds supplied by fiscal authorities in both countries. The goods market clearing conditions for country A and B are the following:

$$Y_{Tt} = C_{At} + C_{At}^* + I_t\left[1 + \frac{\psi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] + G_t + a(\mu_{Kt})K_{t-1}, \tag{82}$$

$$Y_{Tt}^* = C_{Bt} + C_{Bt}^* + I_t^*\left[1 + \frac{\psi_I}{2}\left(\frac{I_t^*}{I_{t-1}^*} - 1\right)^2\right] + G_t^* + a(\mu_{Kt}^*)K_{t-1}^*, \tag{83}$$

$$Y_{Nt} = C_{Nt}, \tag{84}$$

$$Y_{Nt}^* = C_{Nt}^*. \tag{85}$$

Labor market clearing conditions for country A and B are the following:

$$L_t = L_{Tt} + L_{Nt}, \tag{86}$$

$$L_t^* = L_{Tt}^* + L_{Nt}^*. \tag{87}$$

We express the dynamics of net foreign assets in country A with the current account definition:

$$
\begin{aligned}
CA_t \equiv & P_{At} Y_{Tt} - P_{Tt} C_{Tt} - P_{At} I_t \left[1 + \frac{\psi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] - P_{At} a(\mu_{Kt}) K_{t-1} \\
= & B_{Bt} - B^*_{At} - R_{t-1} \left(B_{B,t-1} - B^*_{A,t-1} \right).
\end{aligned}
\tag{88}
$$

In Appendix E we write down the additional equations associated with the quantitative model and derive the steady state.

4.5 Estimation

We discipline the structural parameters through Bayesian estimation of the two-country model. Following the literature, we obtain the posterior distribution by combining our priors with the model's likelihood function via the Kalman filter. The posterior results of the estimation with Germany as country A and the rest of the euro area as country B are reported in Table 5. We estimate the model separately for each country pair, with country A being either France, Germany, or Italy, and country B being the GDP-weighted aggregate of the remaining euro area countries. This allows us to recover country-specific structural parameters and fiscal shock processes, enabling us to identify the mix of funded versus unfunded fiscal shocks across countries.

We follow Guo, Han, and Smit (2026) and estimate the latent inflation and debt targets of each fiscal regime. As in Bianchi, Faccini, and Melosi (2023), the monetary and fiscal rules have time-varying targets equivalent to inflation and debt in the shadow economies. Guo, Han, and Smit (2026) show that these targets are functions of unfunded transfers, whose parameters we estimate.

Data. We use the publicly available large datasets for the euro area, collected by Barigozzi, Lissona, and Tonni (2025), for 2000Q1–2025Q3. For each country pair, we construct 19 observables: nine per region (real consumption, real government spending, real investment, hours, real GDP, and gross debt-to-GDP in growth rates, wage and CPI inflation, and the long-term interest rate) plus one shared short-term interest rate. We construct the data for transfers ourselves with data from Eurostat, similarly to Bianchi, Faccini, and Melosi (2023). See the Online Appendix for details. The shocks we introduce into our model for the estimation are: technology (tradable and non-tradable), monetary policy, funded and unfunded taxes, risk premium, preference, price markup (tradable and non-tradable), wage markup, investment, and government spending. We add measurement errors for investments, consumption, hours, wages, output, debt, government spending, and inflation in each region.

Priors. Table 4 shows the parameters that are fixed in the estimation. Those parameters either have a direct counterpart in the data or a standard value in the literature. The discount factor β is 0.99, consistent with a steady-state annualized real rate of 4%. The degree of trade openness, α_I, is 0.25 to match the share of imported goods in the euro area Eurostat (2023). The share of non-tradable goods, α_N, is 0.6 and the elasticity of substitution between tradable and non-tradables, μ_N, is 1 according to literature. The relative country size, Θ, is set to the GDP share of country A in the estimation sample. Capital depreciates at rate $\delta = 0.025$ following Christiano, Eichenbaum, and Evans (2005). As is standard, the output elasticity of capital, α, is 0.33.

The rate of decay of the long-term bond ρ^L is 0.975 to match European Monetary Union data, which implies that the average maturity of the long-term bond portfolio is 10 years. The debt adjustment costs ψ are 0.0074 following Schmitt-Grohé and Uribe (2003). We take the steady-state real market value of debt, $\overline{s}_b = 0.80$, and the steady-state ratio of domestic and foreign bonds, $\delta_b = 0.44/0.56$, from Eurostat Data (2025) and Bruegel (2025) respectively.

For the parameters in the model with partially unfunded debt, we adopt the parameters from BFM: $\phi^M = \phi^{M*} = 2$ and $\phi^F = \phi^{F*} = 0$ to ensure the central bank reacts more than one-to-one to monetary-led inflation and none to fiscal-led inflation. $\gamma^F = 0$ allows for unfunded debt in country A, whereas all other components of debt are funded: $\gamma^M = \gamma^{M*} = \gamma^{F*} = 20$.

Table 5 shows the prior distributions for the parameters we estimate. We estimate region-specific structural parameters for both countries A and B, including the Frisch labor elasticity χ, habit persistence h, Calvo price and wage stickiness (θ_T, θ_N, θ_w), wage indexation χ_w, and the trade elasticity μ. Investment adjustment cost ψ_I and capital utilization cost ψ_u are estimated as common parameters. We also estimate the fiscal rule debt-stabilization coefficients γ^M, γ^{M*}, γ^{F*}, and γ^{FB}, and the Taylor rule coefficient ϕ^M. The prior mean of the Frisch labor elasticity is 2 (Normal), Calvo probabilities are 0.75 (Beta), wage indexation is 0.25 (Beta), and investment adjustment cost is 2 (Gamma). The shock persistence parameters, ρ_X, and standard deviation parameters σ_X assume standard prior distributions.

Posteriors. Table 5 presents the posterior distributions of each parameter for the estimation with Germany as country A and the rest of the euro area as country B. In Table A.19 and A.20 in Appendix A we show the posterior distributions from an estimation with France and Italy as country A. We observe that the estimated values are all roughly in line with the standard literature.

Figure 4: Estimation results: fixed parameters

Parameter	Description	Value	Source
Households			
β	Discount factor	0.99	literature
α_I	Degree of trade openness	0.25	Eurostat (2023)
μ	Elasticity of subst. goods A vs. B	1	literature
α_N	Share of non-tradable goods	0.6	literature
μ_N	Elasticity of subst. trad. vs. non-trad. goods	1	literature
Θ	Size country A	0.5	–
Firms			
δ	Capital depreciation rate	0.025	Christiano, Eichenbaum, and Evans (2005)
α	Output elasticity of capital	0.33	literature
Monetary			
ϕ^M, ϕ^{M*}	Taylor-rule weight on monetary-led infl.	2	BFM
ϕ^F, ϕ^{F*}	Taylor-rule weight on fiscal-led infl.	0	BFM
Fiscal			
ρ^L	Rate of decay long-term bond	0.975	EMU Criterion
ψ	Debt adjustment cost	0.0074	Schmitt-Grohé and Uribe (2003)
$\bar{s}_b$	Steady-state real market value debt	0.80	Eurostat Data (2025)
δ_b	Steady-state ratio dom. vs. foreign bonds	0.44/0.56	Bruegel (2025)
$\gamma^M, \gamma^{M*}, \gamma^{F*}$	Fiscal rule weight on monetary-led debt	20	BFM
γ^F	Fiscal rule weight on fiscal-led debt	0	BFM

4.6 Results

The simulation results of the quantitative model are in line with the stylized model, but with more persistence as expected. The same set of impulse responses as for the stylized model are in Figure A.16a and A.16b in Appendix A, where we confirm that under a funded fiscal shock, inflation diverges within the union and under an unfunded fiscal shock inflation rates in both countries increase. Moreover, funded and unfunded government spending shocks (as % of GDP), to match the shock used in local projections, also generate similar propagations as the baseline funded and unfunded fiscal shocks through lump-sum taxes. The responses to funded and unfunded government spending shocks are in Figure A.17a and A.17b.

We perform Bayesian estimations of the quantitative currency union model with France, Germany, and Italy as country A to investigate how funded and unfunded shocks have contributed to inflation rates historically. Figure 6 presents the historical shock decomposition of inflation for Germany, France, and Italy. Those are results from three separate estimations of the quantitative currency union model, in which we treat each country as country A and the rest of the euro area as country B. The decomposition results for the rest of the euro area are in Figure A.21 in Appendix A. For each shock decomposition, we focus on how domestic fiscal-led, foreign fiscal-led shocks, and monetary policy shocks contribute to the fluctuations in inflation, and group all other shocks into one bar.

The historical shock decomposition of inflation in Figure 6 suggests that during the financial crisis (2009Q1–2009Q4), monetary easing pushed inflation up, while domestic and foreign unfunded fiscal contractions pushed it down. In this context, an un-

Figure 5: Estimation results: prior and posteriors of the two-country Bayesian estimation (Germany vs. rest of EA)

		Prior			Posterior		
Param.	Description	Type	Mean	Std.	Mode	Median	90% HPD CI
Region A structural							
χ	Inverse Frisch elasticity of labor supply	N	2.00	0.50	2.6130	2.8442	[2.6029, 3.2001]
χ_w	Wage indexation	B	0.25	0.10	0.0650	0.0394	[0.0274, 0.0526]
μ	Elasticity of substitution across tradable goods	N	1.00	0.25	0.0939	-0.2808	[-0.4518, -0.1159]
θ_T	Calvo probability tradable prices	B	0.75	0.10	0.3358	0.0905	[0.0596, 0.1241]
θ_N	Calvo probability non-tradable prices	B	0.75	0.10	0.4625	0.4774	[0.4415, 0.5127]
θ_w	Calvo probability wages	B	0.75	0.10	0.9910	0.9942	[0.9911, 0.9972]
Region B structural							
χ^*	Inverse Frisch elasticity B	N	2.00	0.50	2.5247	2.6857	[2.3245, 2.9601]
χ_w^*	Wage indexation B	B	0.25	0.10	0.0936	0.0991	[0.0407, 0.1599]
μ^*	Trade elasticity B	N	1.00	0.25	0.5281	0.1035	[-0.0611, 0.2589]
θ_T^*	Calvo probability tradable prices B	B	0.75	0.10	0.3297	0.1778	[0.1439, 0.2147]
θ_N^*	Calvo probability non-tradable prices B	B	0.75	0.10	0.3736	0.3347	[0.2991, 0.3690]
Common structural							
ψ_u	Capital utilization cost	B	0.50	0.15	0.0363	0.0567	[0.0339, 0.0799]
Monetary / target policy							
ϕ^{ML}	Taylor rule inflation coefficient (ML)	N	2.00	0.25	2.0406	1.8296	[1.4610, 2.0670]
ρ_r	Taylor rule interest rate smoothing (ML)	B	0.50	0.25	0.9567	0.9613	[0.9521, 0.9695]
ϕ_y	Taylor rule output response coefficient (union output, GDP-weighted)	N	0.125	0.05	0.0145	0.0325	[0.0216, 0.0409]
ϕ_F	Inflation target persistence (Singh et al. eq. 26)	B	0.95	0.03	0.6909	0.5922	[0.5691, 0.6146]
Shock persistence							
ρ_a	Technology shock persistence	B	0.80	0.10	0.8060	0.8537	[0.8238, 0.8835]
ρ_{rp}	Risk premium shock persistence	B	0.80	0.10	0.9751	0.9743	[0.9642, 0.9835]
ρ_d	Preference shock persistence	B	0.50	0.15	0.1724	0.1328	[0.0834, 0.2159]
ρ_u	Tradable markup shock persistence	B	0.80	0.10	0.9552	0.8785	[0.8163, 0.9353]
ρ_{uN}	NT markup shock persistence	B	0.80	0.10	0.8517	0.7614	[0.7091, 0.8069]
ρ_w	Wage markup shock persistence	B	0.80	0.10	0.6660	0.6916	[0.6343, 0.7371]
ρ_k	MEI shock persistence	B	0.50	0.15	0.3718	0.5809	[0.4902, 0.6646]
ρ_ξ	Monetary policy shock persistence	B	0.50	0.15	0.6273	0.4996	[0.4414, 0.5343]
Shock volatilities							
σ_a	St.dev. technology shock	IG	0.50	inf	2.1956	1.1841	[1.0248, 1.3408]
σ_{aN}	St.dev. NT technology shock	IG	0.50	inf	1.8641	1.7631	[1.6061, 1.9241]
σ_{rp}	St.dev. risk premium shock	IG	0.25	inf	0.0529	0.0361	[0.0304, 0.0418]
σ_d	St.dev. preference shock	IG	0.50	inf	13.2794	13.5594	[11.5831, 15.5487]
σ_u	St.dev. tradable markup shock	IG	0.50	inf	0.8883	1.6191	[0.5783, 3.0090]
σ_{uN}	St.dev. NT markup shock	IG	0.50	inf	3.5953	4.4274	[3.9831, 4.9240]
σ_w	St.dev. wage markup shock	IG	0.50	inf	0.4495	0.4179	[0.3568, 0.4911]
σ_k	St.dev. MEI shock	IG	0.50	inf	12.6270	8.0891	[6.5184, 9.7400]
σ_ξ	St.dev. monetary policy shock	IG	0.25	inf	0.2134	0.1768	[0.1500, 0.2033]
Unfunded floating-target							
$\sigma_{U,A}$	St.dev. unfunded shock A	IG	0.50	inf	20.1293	20.3581	[19.0126, 22.1875]
$\sigma_{U,B}$	St.dev. unfunded shock B	IG	0.50	inf	19.1496	19.2047	[17.8241, 20.4626]

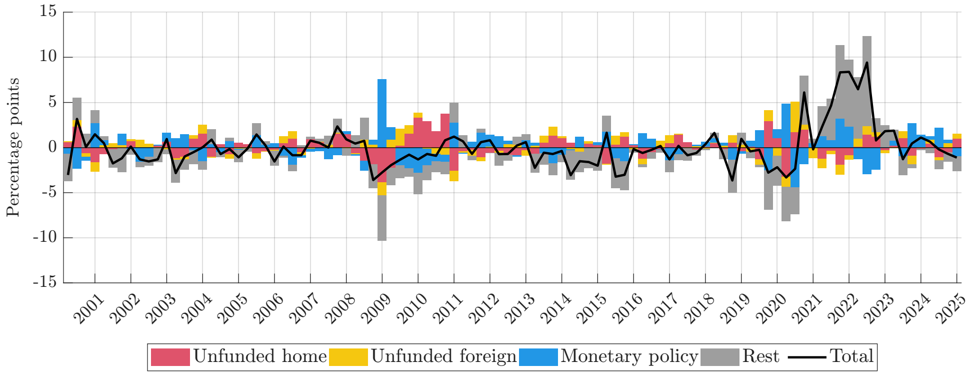
Notes: Results of the estimation of the structural parameters. N = Normal, B = Beta, G = Gamma, IG = Inverse-Gamma, AR coeff. = autoregressive coefficient, St.dev. = standard deviation, HPD = highest posterior density.

funded fiscal contraction can be interpreted as an increase in the primary surplus that is not offset by expected future tax reductions, consistent with the fiscal adjustments observed in several European economies during the austerity phase of the crisis. During the pandemic quarters, 2020Q2–2020Q4, unfunded fiscal expansions pushed both inflation up, whereas in the quarters that followed, monetary easing was the main inflationary force. During the energy crises, 2022Q1–2023Q1, we see an increase in fiscal inflation again.

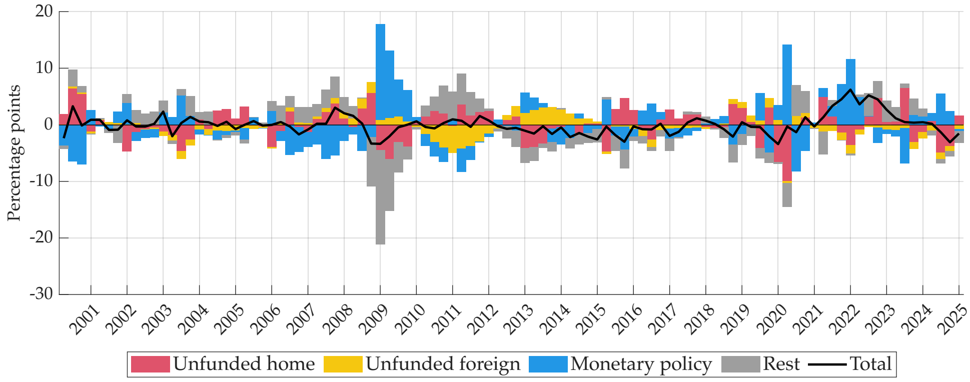
5 Conclusion

This paper studies how funded and unfunded fiscal expansions propagate in a currency union with one central bank and multiple fiscal authorities. We combine evi-

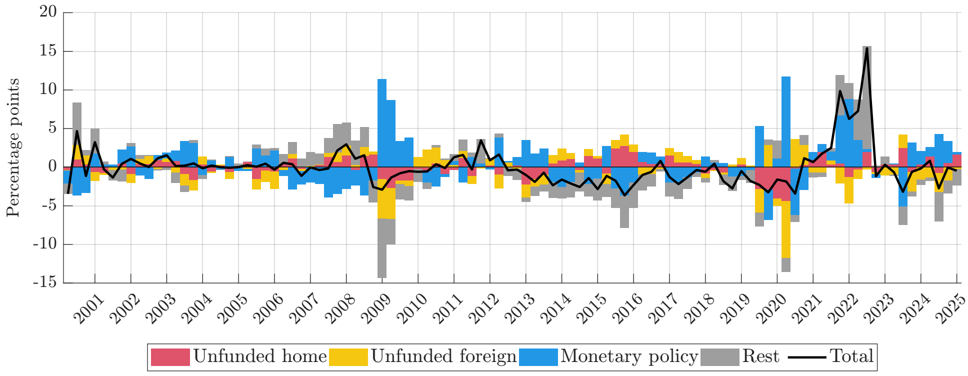
Figure 6: Shock decomposition of inflation in Germany, France, and Italy



(a) Inflation in Germany



(b) Inflation in France



(c) Inflation in Italy

Notes: Historical shock decomposition of the inflation in Germany, France, and Italy in percentage point deviations from the sample mean. We estimate the quantitative two-country model three times, with each of those countries as country A and the rest of the euro area as country B. Monetary-led fiscal shocks are ζ_t^{ML} and g_t^{ML}, and fiscal-led fiscal shocks are ζ_t^{FL} and g_t^{FL}. The x-axis is in quarters.

dence from local projections for euro area countries with a two-country New Keynesian model that allows for partially unfunded debt. The central question is whether differences in inflation spillovers across source countries of fiscal shocks can be interpreted through the lens of fiscal backing.

First, we investigate the price determinacy conditions of a currency union and the propagation mechanism of funded and unfunded shocks in a stylized model. We show that at most one fiscal authority can issue unfunded debt when countries share a common central bank. Moreover, we find that funded fiscal expansions increase inflation at home while decreasing inflation abroad because the common monetary policy rate is too expansionary for the country of the shock and too contractionary for others. Unfunded fiscal expansions increase inflation in both countries because the central bank accommodates *fiscal inflation*.

Our empirical analysis documents substantial heterogeneity in spillovers which we interpret through the lens of the model. Fiscal expansions in Germany and Italy are associated with inflationary effects domestically and in other countries, while fiscal expansions in France do not generate the same inflationary spillovers and are accompanied by a more contractionary monetary policy response. These patterns suggest that the central bank accommodates fiscal inflation originating in Germany and Italy, implying fiscal expansions in those countries are less funded, or backed by future tax adjustments, than fiscal expansions in France.

The quantitative model extends the stylized model to a richer environment with habit formation, nominal rigidities, non-tradables, capital accumulation, and long-term bonds. Preliminary simulations are consistent with the stylized model and produce more persistent responses. We estimate the full currency-union model across alternative country groupings to identify episodes of fiscal inflation. We observe fiscal inflation in the pandemic and energy crises episodes, but also find that monetary easing during those times was a large driver of inflation as well. Our estimation is still a work in progress. We are currently working on constructing a measure for real government transfers to improve the identification of the funded and unfunded shocks.

References

Ascari, Guido, Dennis Bonam, Lorenzo Mori, and Andra Smadu. 2024. *Fiscal Policy and Inflation in the Euro Area.* Technical report. De Nederlandsche Bank Working Paper No. 820.

Auerbach, Alan J, and Yuriy Gorodnichenko. 2017. *Fiscal Stimulus and Fiscal Sustainability.* Working Paper, Working Paper Series 23789. National Bureau of Economic Research, September. https://doi.org/10.3386/w23789. http://www.nber.org/papers/w23789.

Auerbach, Alan J., and Yuriy Gorodnichenko. 2012. "Measuring the Output Responses to Fiscal Policy." *American Economic Journal: Economic Policy* 4, no. 2 (May): 1–27. https://doi.org/10.1257/pol.4.2.1. https://www.aeaweb.org/articles?id=10.1257/pol.4.2.1.

Barbier-Gauchard, Amélie, and Thierry Betti. 2021. "Spillover effects of fiscal policy in a monetary union: Why do fiscal instruments matter?" *Bulletin of Economic Research* 73 (1): 1–33. https://doi.org/https://doi.org/10.1111/boer.12231. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/boer.12231. https://onlinelibrary.wiley.com/doi/abs/10.1111/boer.12231.

Barigozzi, Matteo, Claudio Lissona, and Lorenzo Tonni. 2025. *Large datasets for the Euro Area and its member countries and the dynamic effects of the common monetary policy.* arXiv: 2410.05082 [econ.EM]. https://arxiv.org/abs/2410.05082.

Beetsma, Roel M.W.J., and Henrik Jensen. 2005. "Monetary and Fiscal Interactions in a Micro-Founded Model of a Monetary Union." *Journal of International Economics* 67 (2): 320–352.

Bergin, Paul R. 2000. "Fiscal solvency and price level determination in a monetary union." *Journal of Monetary Economics* 45 (1): 37–53. ISSN: 0304-3932. https://doi.org/https://doi.org/10.1016/S0304-3932(99)00041-0. https://www.sciencedirect.com/science/article/pii/S0304393299000410.

Bianchi, Francesco, Renato Faccini, and Leonardo Melosi. 2023. "A Fiscal Theory of Persistent Inflation*." *The Quarterly Journal of Economics* 138, no. 4 (November): 2127–2179. ISSN: 0033-5533, accessed April 30, 2025. https://doi.org/10.1093/qje/qjad027. https://doi.org/10.1093/qje/qjad027.

Boehm, Christoph E. 2020. "Government consumption and investment: Does the composition of purchases affect the multiplier?" *Journal of Monetary Economics* 115:80–93. ISSN: 0304-3932. https://doi.org/https://doi.org/10.1016/j.jmoneco.2019.05.003. https://www.sciencedirect.com/science/article/pii/S0304393219300935.

Born, Benjamin, Gernot J. Müller, and Johannes Pfeifer. 2020. "Does Austerity Pay Off?" *The Review of Economics and Statistics* 102, no. 2 (May): 323–338. https://doi.org/None. https://ideas.repec.org/a/tpr/restat/v102y2020i2p323-338.html.

Bruegel. 2025. *Bruegel database of sovereign bond holdings developed in Merler and Pisani-Ferry (2012).* https://www.bruegel.org/dataset/sovereign-bond-holdings?utm_source=chatgpt.com.

Calvo, Guillermo A. 1983. "Staggered Prices in a Utility-Maximising Framework." *Journal of Monetary Economics* 12 (3): 383–398.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113 (1): 1–45. ISSN: 00223808, 1537534X, accessed January 12, 2026. http://www.jstor.org/stable/10.1086/426038.

Cochrane, John H. 2021. *The Fiscal Theory of the Price Level.* Princeton University Press.

Debrun, Xavier, Klaus Masuch, Giuseppe Ferrero, Isabel Vansteenkiste, Marien Ferdinandusse, Leopold von Thadden, Sebastian Hauptmeier, Mario Alloza, ChloÃ© Derouen, and Krzyszto BaÅ„kowski. 2021. *Monetary-fiscal policy interactions in the euro area.* Occasional Paper Series 273. European Central Bank, September. https://ideas.repec.org/p/ecb/ecbops/2021273.html.

Dupor, Bill. 2000. "Exchange rates and the fiscal theory of the price level." *Journal of Monetary Economics* 45 (3): 613–630. ISSN: 0304-3932. https://doi.org/https://doi.org/10.1016/S0304-3932(00)00006-4. https://www.sciencedirect.com/science/article/pii/S0304393200000064.

Eurostat. 2023. *World Trade in Goods and Services - An Overview.* https://ec.europa.eu/eurostat/statistics-explained/index.php?title=World_trade_in_goods_and_services_-_an_overview.

Eurostat Data. 2025. https://ec.europa.eu/eurostat/data/database.

Ferrero, Andrea. 2009. "Fiscal and Monetary Rules for a Currency Union." *Journal of International Economics* 77 (1): 1–10.

Ferroni, Filippo, Jonas D.M. Fisher, and Leonardo Melosi. 2024. "Unusual shocks in our usual models." *Journal of Monetary Economics* 147:103598. ISSN: 0304-3932. https://doi.org/https://doi.org/10.1016/j.jmoneco.2024.103598. https://www.sciencedirect.com/science/article/pii/S0304393224000515.

Galí, Jordi, and Tommaso Monacelli. 2008. "Optimal Monetary and Fiscal Policy in a Currency Union." *Journal of International Economics* 76:116–132.

Goujard, Antoine. 2017. "Cross-Country Spillovers from Fiscal Consolidations." *Fiscal Studies* 38 (2): pp. 219–267. ISSN: 01435671, 14755890, accessed November 28, 2025. https://www.jstor.org/stable/26605619.

Guerrieri, Luca, and Matteo Iacoviello. 2015. "OccBin: A Toolkit for Solving Dynamic Models with Occasionally Binding Constraints Easily." *Journal of Monetary Economics* 70 (C): 22–38.

Guo, Junjie, Zhao Han, and Abhiprerna Smit. 2026. "Fiscal Inflation with Incomplete Information." *SSRN working paper,* https://papers.ssrn.com/sol3/papers.cfm?abstract_id=5555120.

Hjortsoe, Ida. 2016. "Imbalances and fiscal policy in a monetary union." *Journal of International Economics* 102:225–241. ISSN: 0022-1996. https://doi.org/https://doi.org/10.1016/j.jinteco.2016.07.002. https://www.sciencedirect.com/science/article/pii/S0022199616300800.

Jordà, Òscar. 2005. "Estimation and Inference of Impulse Responses by Local Projections." *American Economic Review* 95, no. 1 (March): 161–182. https://doi.org/10.1257/0002828053828518. https://www.aeaweb.org/articles?id=10.1257/0002828053828518.

Leeper, Eric M. 1991. "Equilibria under 'active' and 'passive' monetary and fiscal policies." *Journal of Monetary Economics* 27, no. 1 (February): 129–147. ISSN: 0304-3932. https://doi.org/10.1016/0304-3932(91)90007-b. http://dx.doi.org/10.1016/0304-3932(91)90007-B.

OECD. 2025. *OECD Economic Outlook.*

Sargent, Thomas J., and Neil Wallace. 1981. "Some Unpleasant Monetarist Arithmetic." *Quarterly Review 5, no. 3 of Federal Reserve Bank of Minneapolis,* https://doi.org/https://doi.org/10.21034/qr.531.

Schmitt-Grohé, Stephanie, and Martín Uribe. 2003. "Closing Small Open Economy Models." *Journal of International Economics* 61:163–185.

Sims, Christopher A. 1994. "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy." *Economic Theory* 4 (3): 381–399. ISSN: 09382259, 14320479, accessed November 24, 2025. http://www.jstor.org/stable/25054771.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97 (3): 586–606.

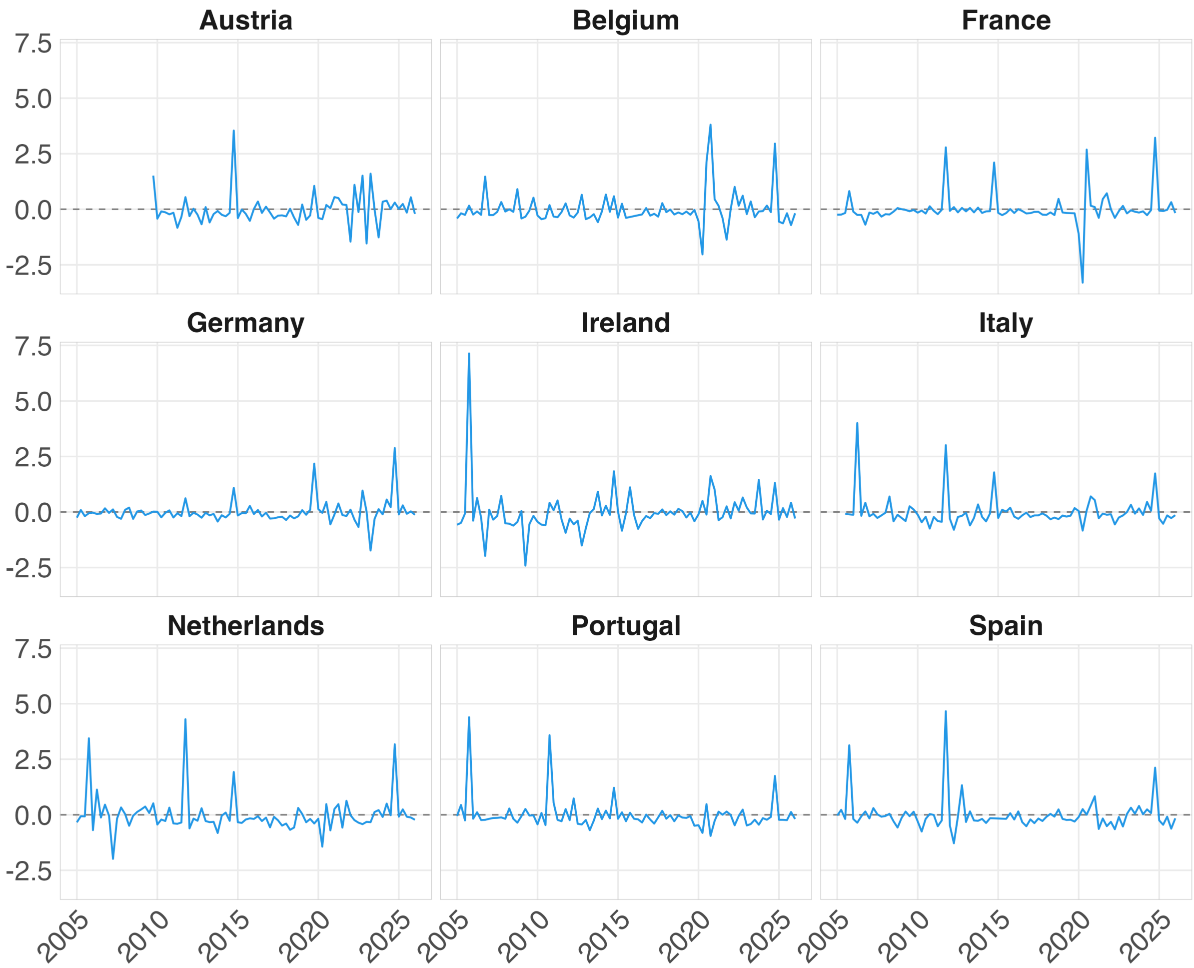
Woodford, Michael. 1994. "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy." *Economic Theory* 4 (3): 345–380. ISSN: 09382259, 14320479, accessed November 24, 2025. http://www.jstor.org/stable/25054770.

———. 2001. “Fiscal Requirements for Price Stability.” *Journal of Money, Credit and Banking* 33 (3): 669–728. https://doi.org/10.2307/2673890.

Appendix

A Additional figures

Figure A.1: Forecast errors of government spending (percentage point of GDP)



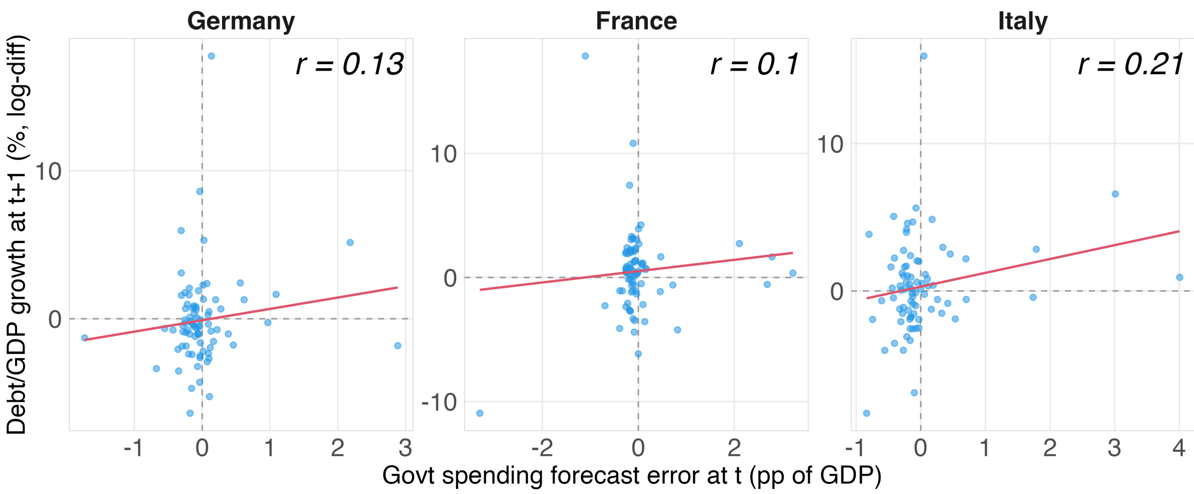
Notes: Purged government spending forecast errors for each country in the sample, 2004Q1-–2025Q2. Each series is the residual from a within-country regression of raw forecast errors on lagged macroeconomic controls (output growth, inflation, interest rates), removing predictable variation. Units are percentage points of lagged GDP.

Table A.1: Correlation of forecast errors

	Austria	Belgium	France	Germany	Ireland	Italy	Netherlands	Portugal	Spain
Austria	1.00	0.24	0.33	0.26	0.22	0.41	0.33	0.22	0.07
Belgium	0.24	1.00	0.51	0.29	0.16	0.33	0.32	0.15	0.28
France	0.33	0.51	1.00	0.45	0.20	0.47	0.69	0.42	0.51
Germany	0.26	0.29	0.45	1.00	0.10	0.33	0.43	0.25	0.25
Ireland	0.22	0.16	0.20	0.10	1.00	0.13	0.39	0.59	0.35
Italy	0.41	0.33	0.47	0.33	0.13	1.00	0.59	0.11	0.50
Netherlands	0.33	0.32	0.69	0.43	0.39	0.59	1.00	0.56	0.76
Portugal	0.22	0.15	0.42	0.25	0.59	0.11	0.56	1.00	0.40
Spain	0.07	0.28	0.51	0.25	0.35	0.50	0.76	0.40	1.00

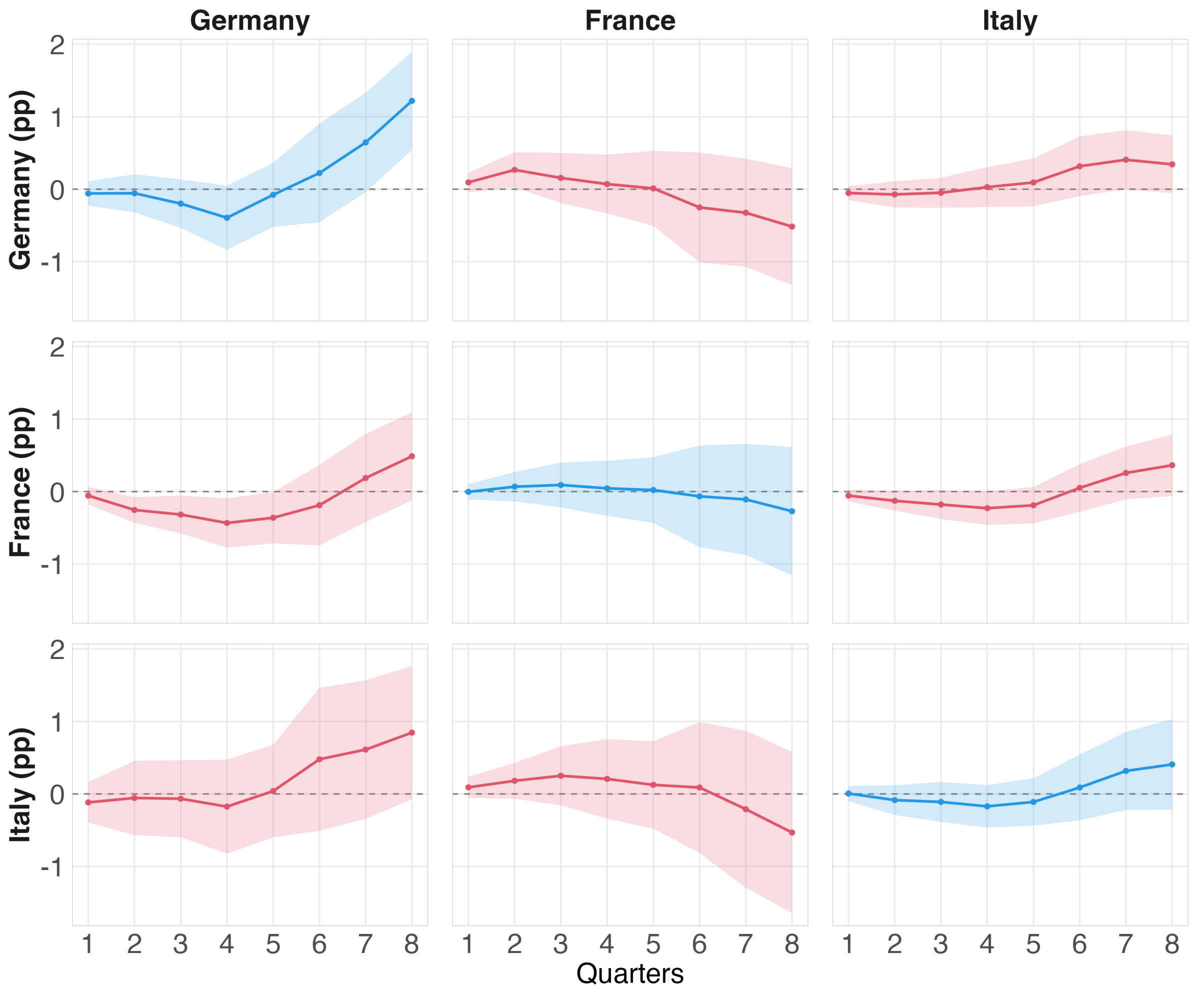
Notes: Pairwise Pearson correlation coefficients of purged government spending forecast errors across countries. Computed over the common sample using pairwise-complete observations.

Figure A.2: Correlation of government spending shocks and debt-to-GDP growth



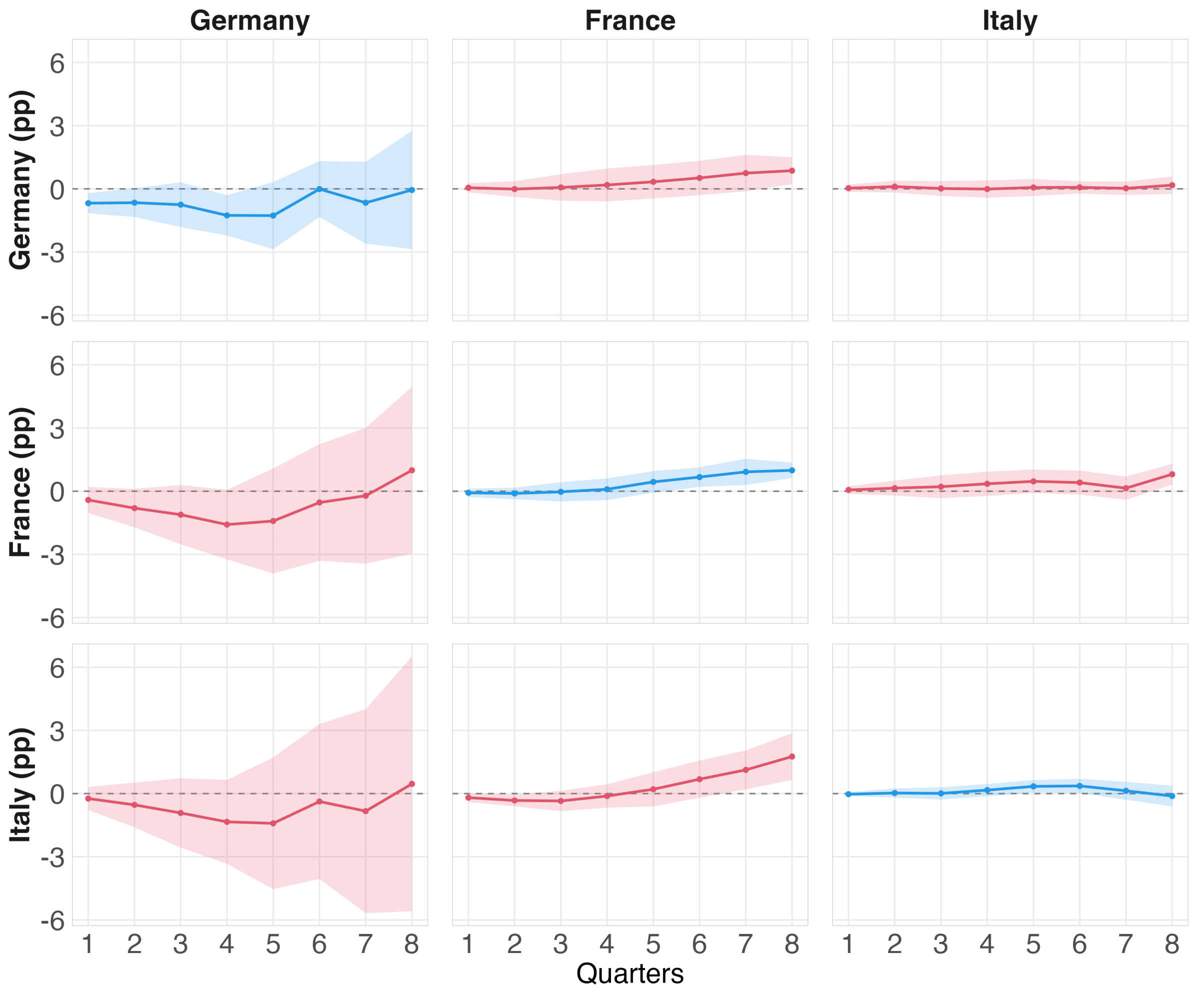
Notes: Scatter plots of purged government spending forecast errors at time t against the log-change in gross government debt-to-GDP at $t+1$, for Germany, France, and Italy (2004Q1–2025Q2). The debt-to-GDP ratio is constructed from the EA-MD-QD dataset by Barigozzi, Lissona, and Tonni (2025) following the same deflation procedure used in estimation. The correlation coefficient r is reported in each panel. The fitted line is estimated by OLS.

Figure A.3: Impulse responses of inflation to a 1pp shock in govt spending (% of GDP)



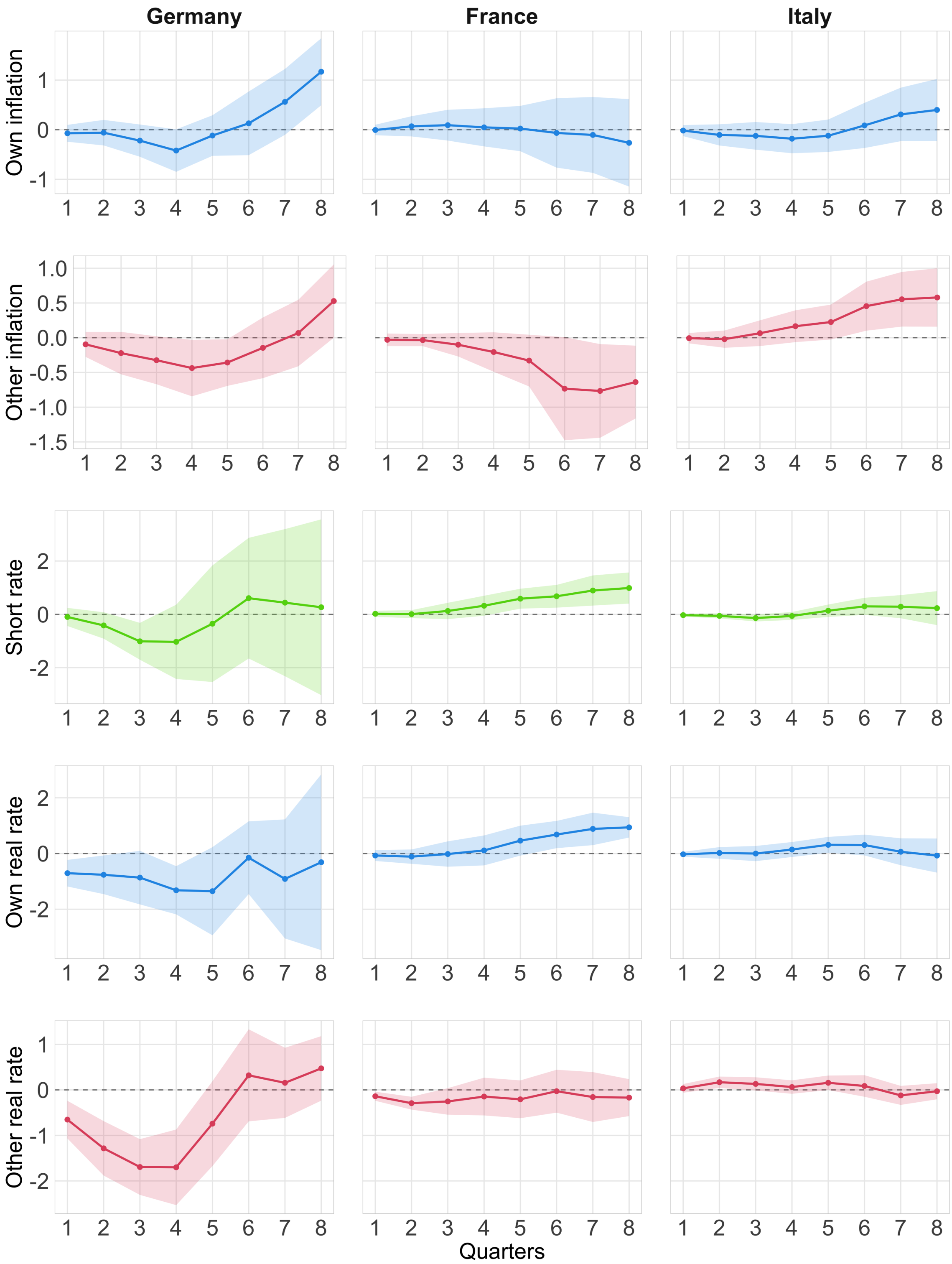
Notes: Pooled estimates from panel local projections on inflation in other countries ($i \neq c$) as response to fiscal shock in country c. Countries are weighted by GDP. Shaded area shows 90% confidence interval. Driscoll-Kraay standard errors.

Figure A.4: Impulse responses of real rate to a 1pp shock in govt spending (% of GDP)



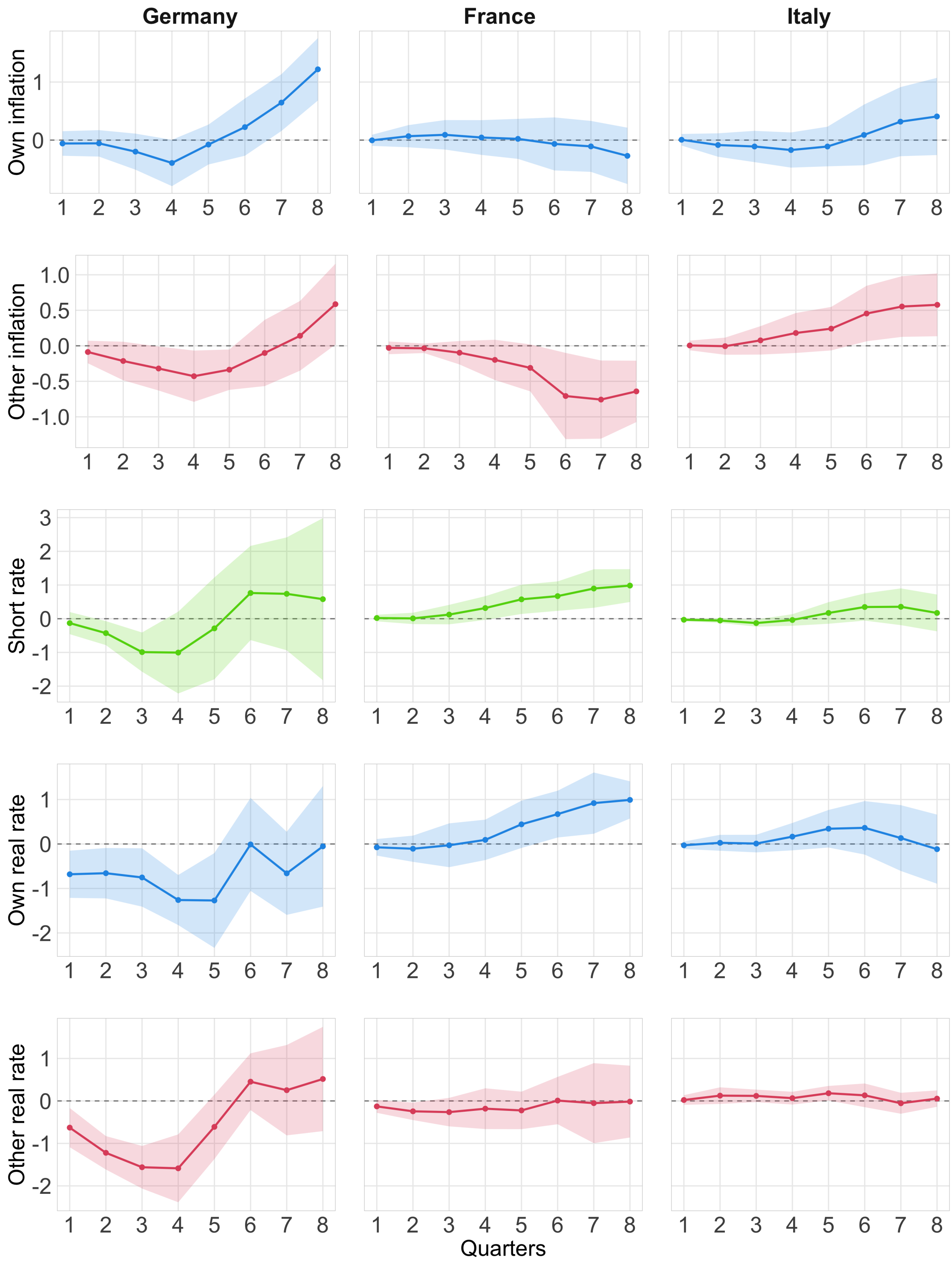
Notes: Pooled estimates from panel local projections on real rate in other countries ($i \neq c$) as response to fiscal shock in country c. Countries are weighted by GDP. Shaded area shows 90% confidence interval. Driscoll-Kraay standard errors.

Figure A.5: Impulse responses to a 1 pp shock in govt spending (% of GDP) with unpurged shocks



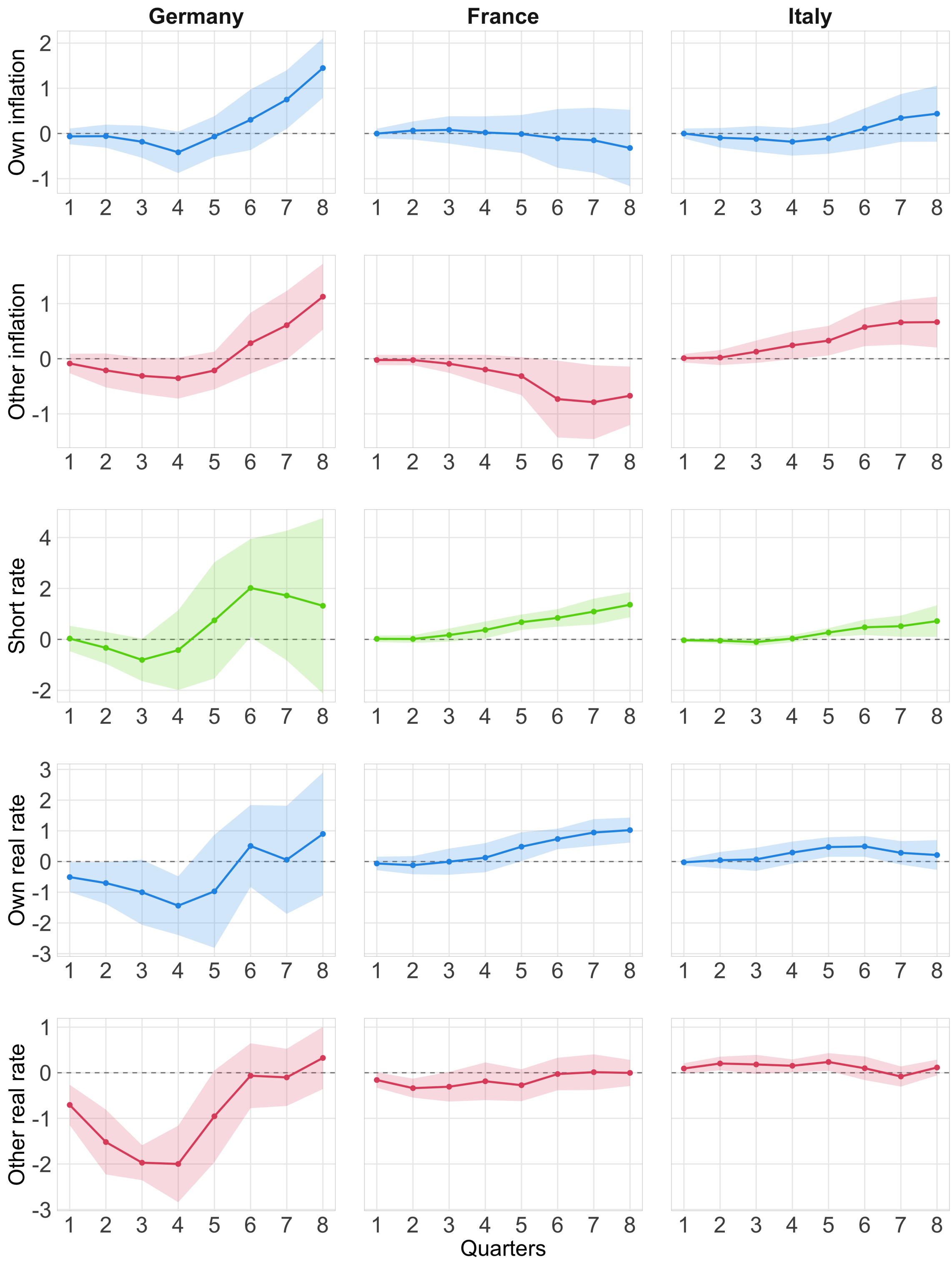
Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure A.6: Impulse responses to a 1 pp shock in govt spending (% of GDP) with different standard errors



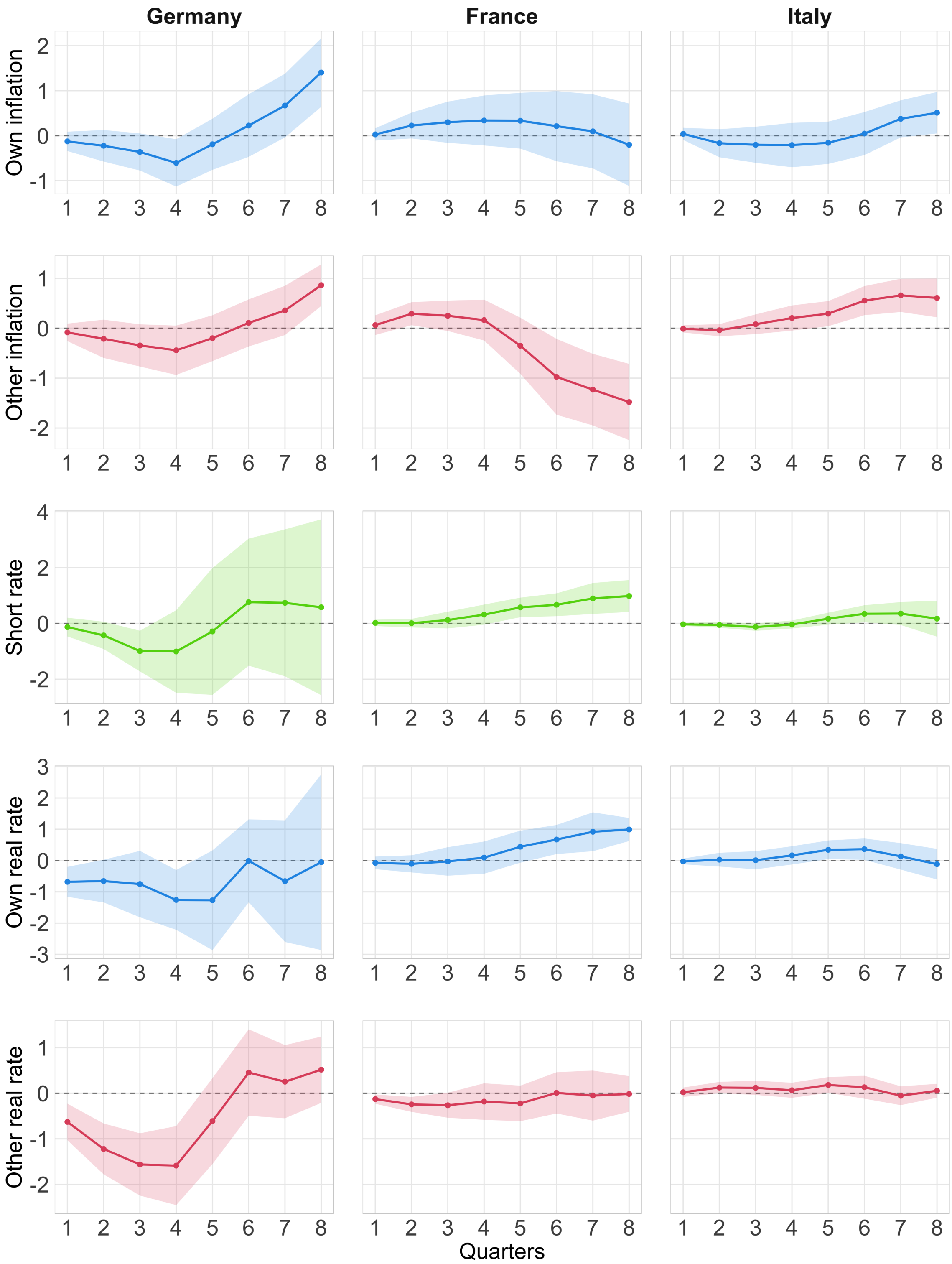
Notes: The own country and short rate local projections use Newey-West HAC standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with two-way standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure A.7: Impulse responses to a 1 pp shock in govt spending (% of GDP) with lags of past shocks as extra controls



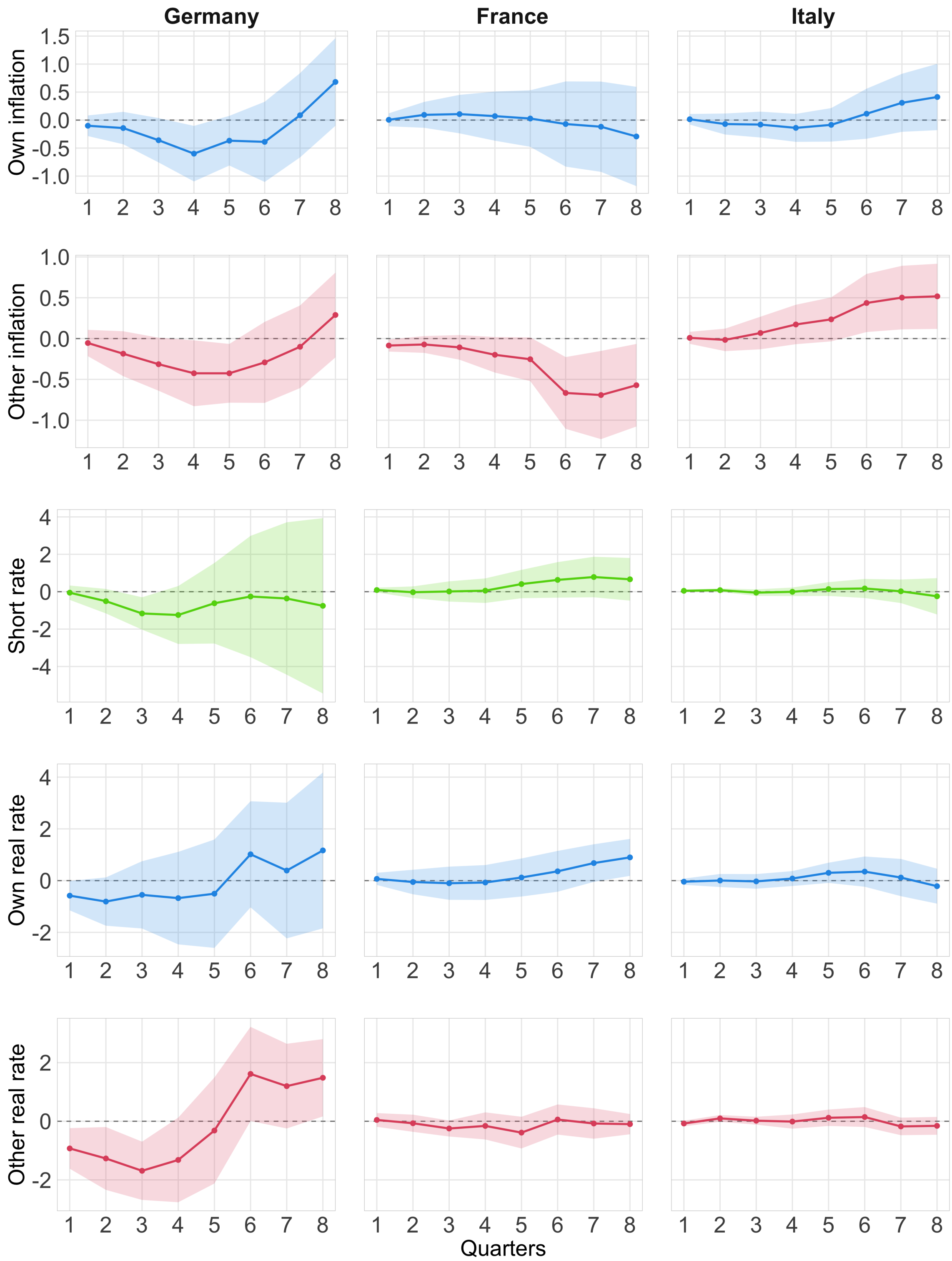
Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure A.8: Impulse responses to a 1 pp shock in govt spending (% of GDP) with contemporaneous weighted other-country fiscal shocks and its lags as extra controls



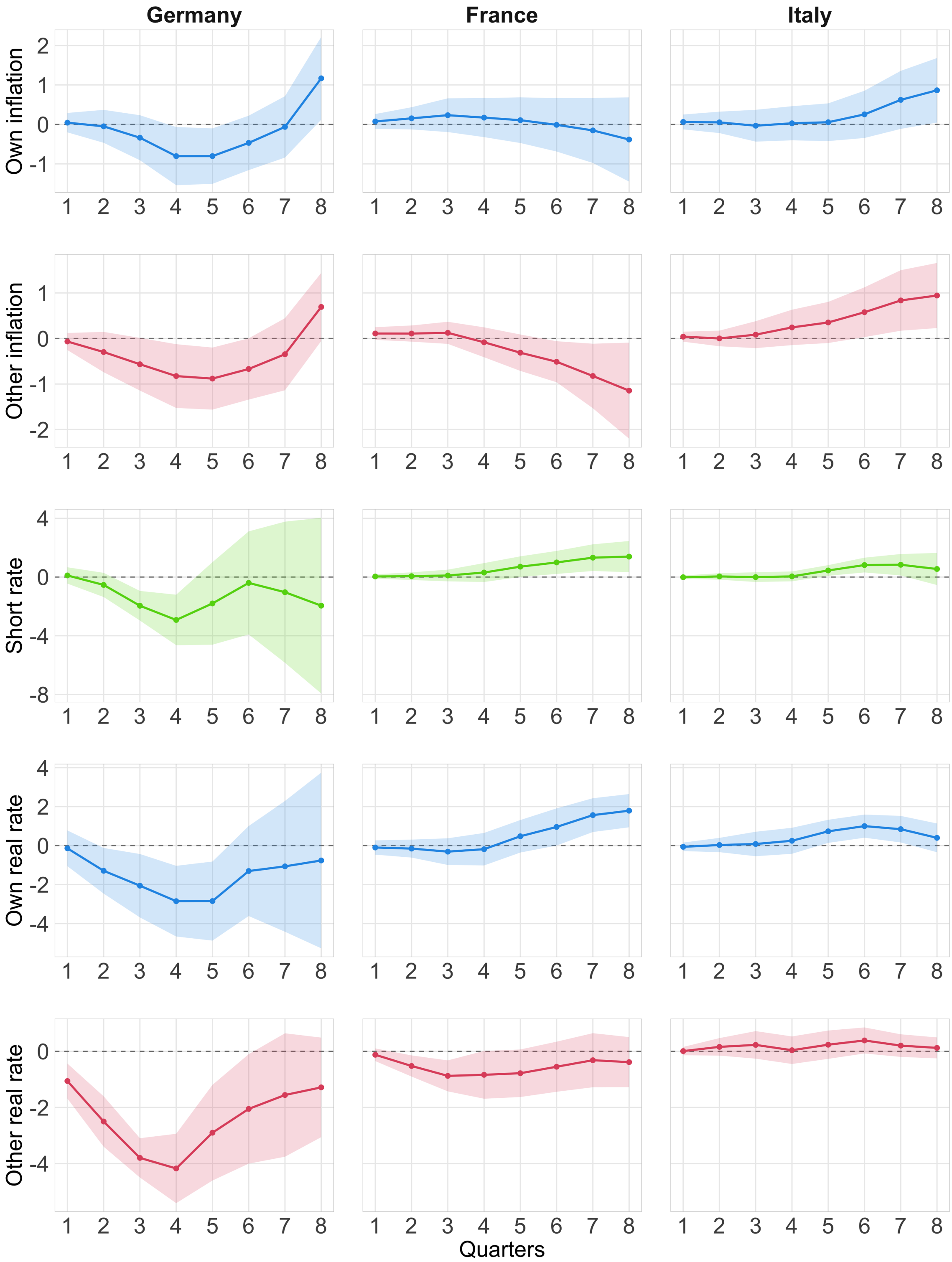
Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure A.9: Impulse responses to a 1 pp shock in govt spending (% of GDP) with lags of three periods



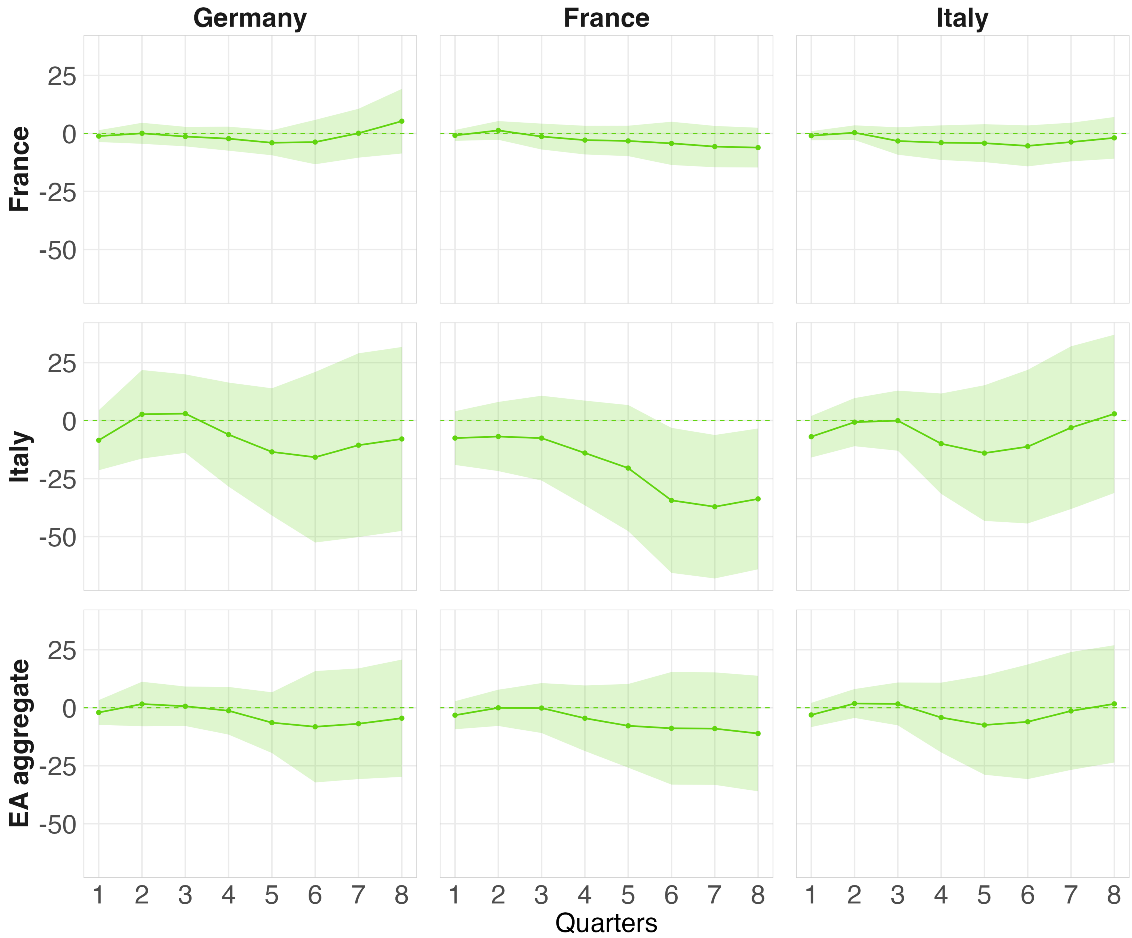
Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure A.10: Impulse responses to a 1 pp shock in govt spending (% of GDP) with timing fix



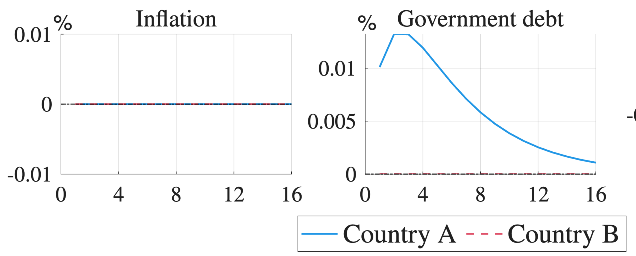
Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure A.11: Impulse responses of sovereign spreads to a 1 pp shock in govt spending (% of GDP)



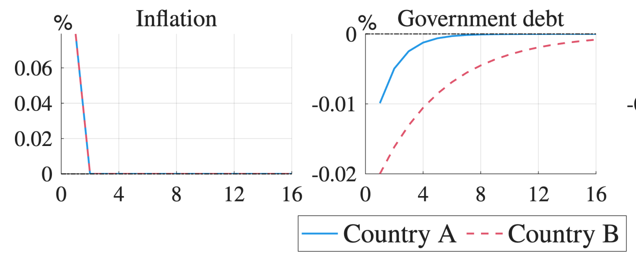
Notes: The dependent variable is the 10-year sovereign bond spread vis-à-vis Germany in basis points. Local projections use heteroskedastic standard errors. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4. Controls include source country controls, receiving country controls (if not the same as source country), German controls, and euro area aggregate controls.

Figure A.12: Impulse responses to funded and unfunded fiscal expansions in country A when $NFA = 0$



(a) Funded fiscal expansion in country A

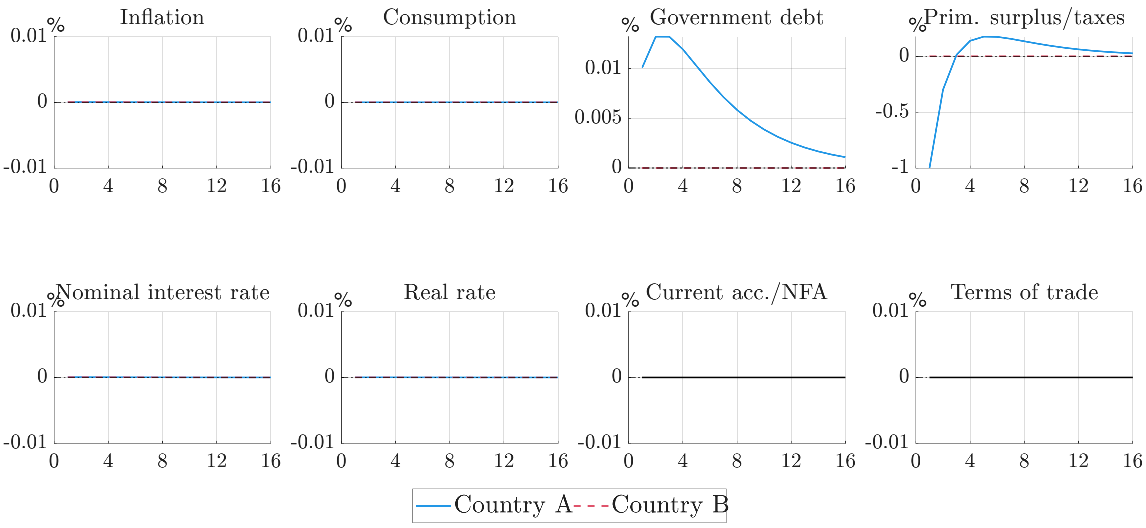
Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The fiscal expansion is funded, i.e. monetary-led, so a shock to ζ_t^M. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.



(b) Unfunded fiscal expansion in country A

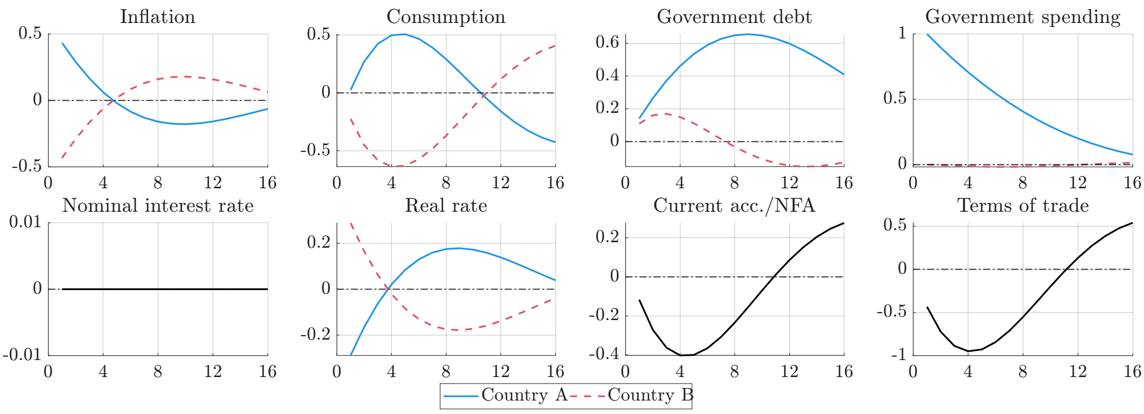
Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The fiscal expansion is unfunded, i.e. fiscal-led, so a shock to ζ_t^F. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

Figure A.13: Impulse responses to a funded fiscal expansion in country *A* under flexible exchange rates



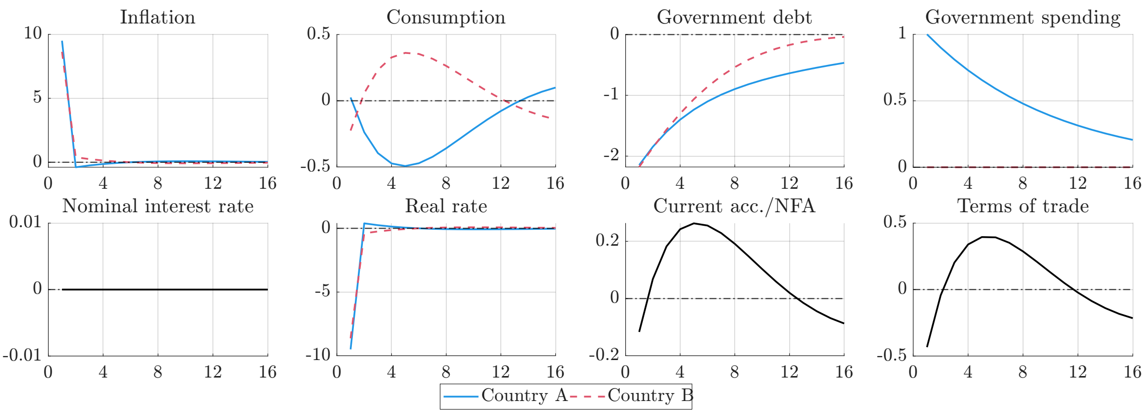
Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The fiscal expansion is funded, i.e. monetary-led, so a shock to ζ_t^M. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

Figure A.14: Impulse responses to funded and unfunded fiscal government spending shocks in country A in the stylized model



(a) Funded government spending shock in country A

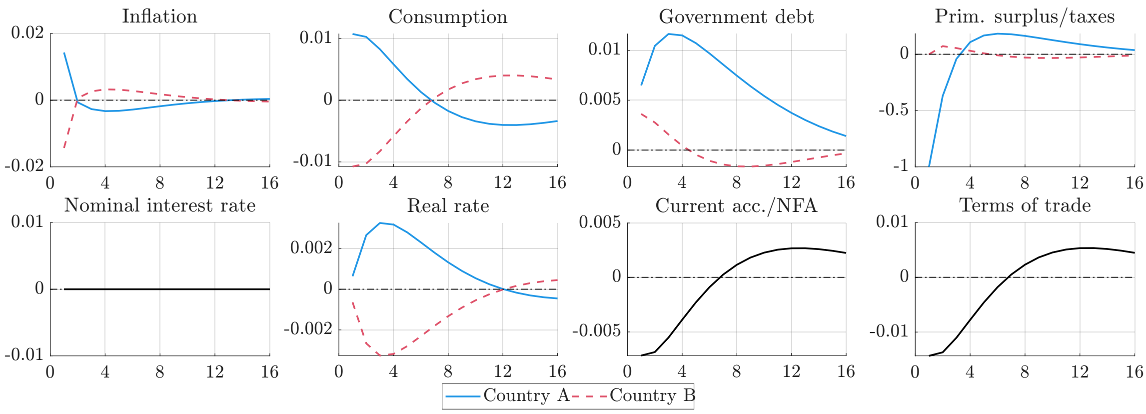
Notes: Impulse responses to a 1% increase in government spending in country A. The fiscal expansion is funded, i.e. monetary-led, so a shock to g_t^M. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.



(b) Unfunded government spending shock in country A

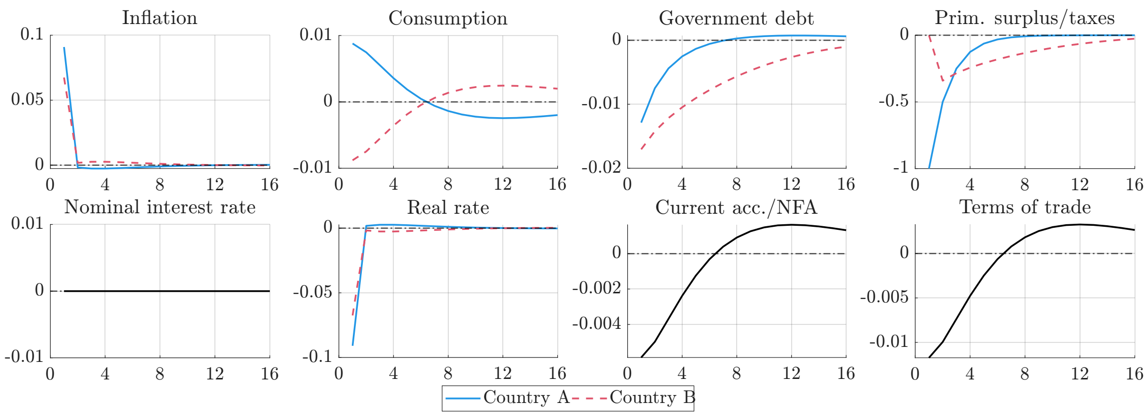
Notes: Impulse responses to a 1% increase in government spending in country A. The fiscal expansion is unfunded, i.e. fiscal-led, so a shock to ζ_t^F. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

Figure A.15: Impulse responses to funded and unfunded fiscal expansions in country A in the stylized model with CES aggregator for bonds (see section G)



(a) Funded fiscal expansion in country A

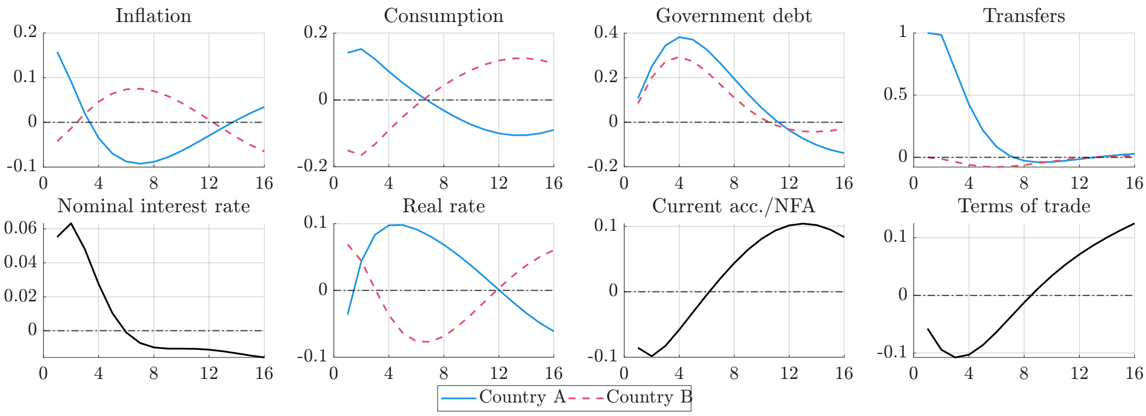
Notes: Impulse responses to a 1% increase in transfers in country A. The fiscal expansion is funded, i.e. monetary-led, so a shock to ζ_t^M. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.



(b) Unfunded fiscal expansion in country A

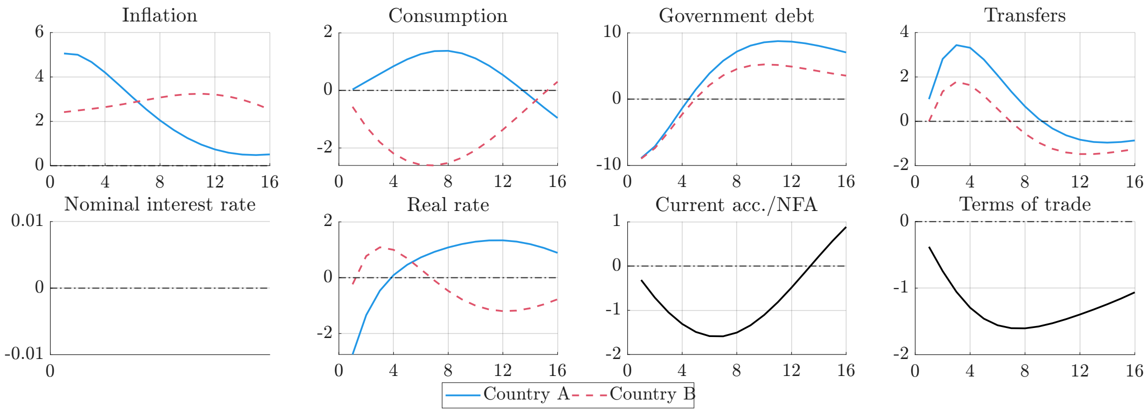
Notes: Impulse responses to a 1% increase in transfer in country A. The fiscal expansion is unfunded, i.e. fiscal-led, so a shock to ζ_t^F. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

Figure A.16: Impulse responses to funded and unfunded fiscal expansions in country A in the quantitative model



(a) Funded fiscal expansion in country A

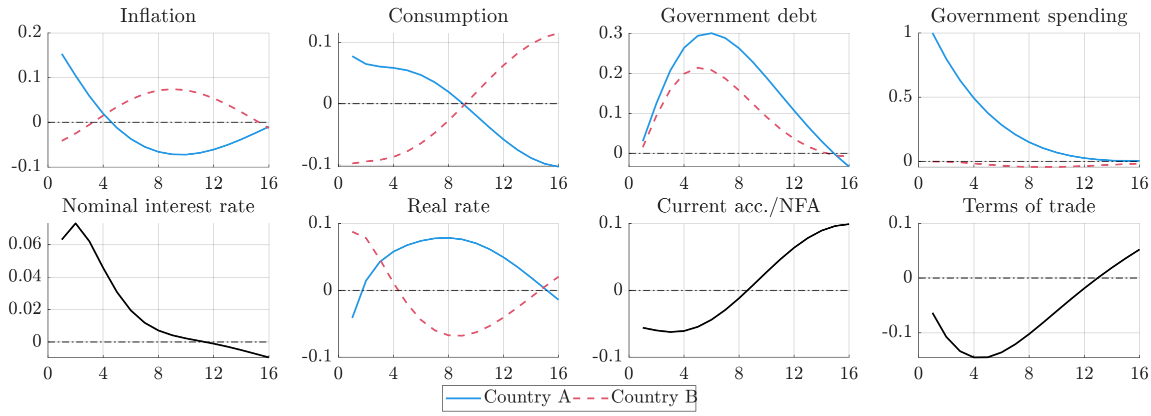
Notes: Impulse responses to a 1% increase in transfers in country A. The fiscal expansion is funded, i.e. monetary-led, so a shock to ζ_t^M. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.



(b) Unfunded fiscal expansion in country A

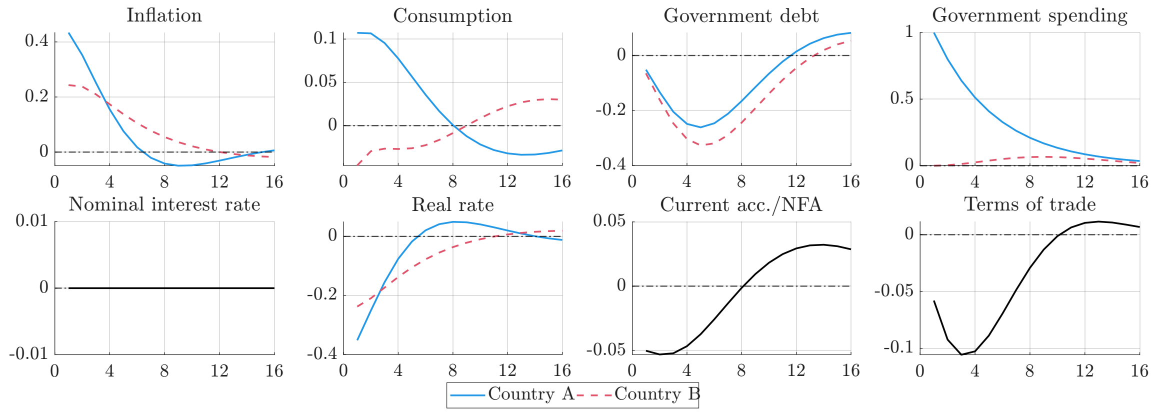
Notes: Impulse responses to a 1% increase in transfer in country A. The fiscal expansion is unfunded, i.e. fiscal-led, so a shock to ζ_t^F. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

Figure A.17: Impulse responses to funded and unfunded government spending shocks in country A



(a) Funded government spending shock in country A

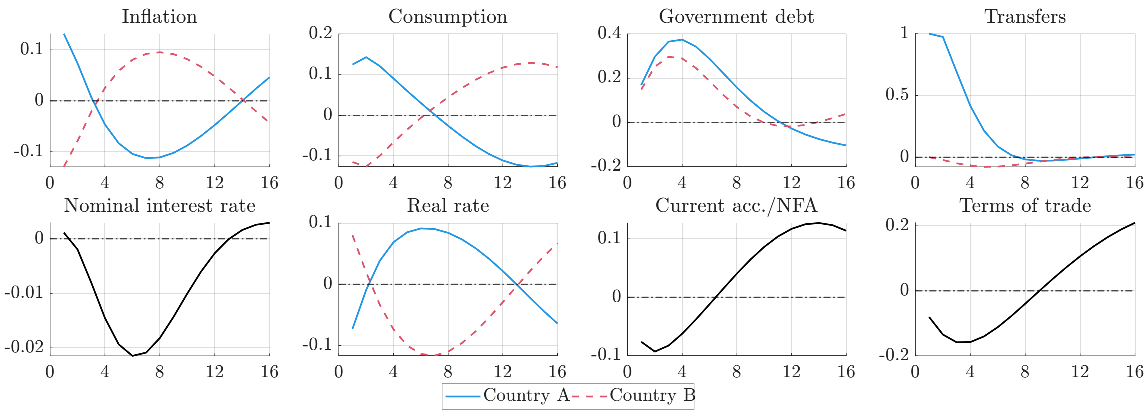
Notes: Impulse responses to a 1% increase in government spending (as % of GDP) in country A. The government spending is funded, i.e. monetary-led. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.



(b) Unfunded government spending shock in country A

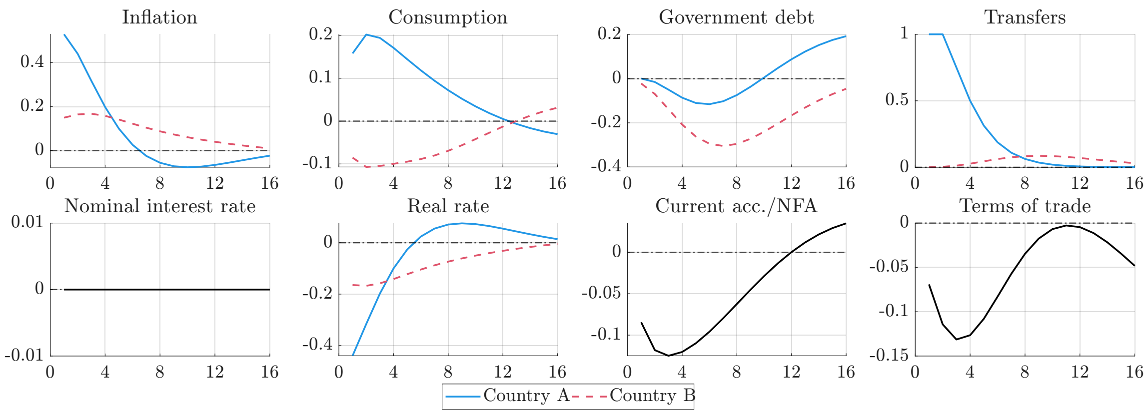
Notes: Impulse responses to a 1% increase in government spending (as % of GDP) in country A. The government spending is unfunded, i.e. fiscal-led. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

Figure A.18: Impulse responses to funded and unfunded fiscal expansions in country A in the quantitative model with CES aggregator for bonds (see section G)



(a) Funded fiscal expansion in country A

Notes: Impulse responses to a 1% increase in transfers in country A. The fiscal expansion is funded, i.e. monetary-led, so a shock to ζ_t^M. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.



(b) Unfunded fiscal expansion in country A

Notes: Impulse responses to a 1% increase in transfer in country A. The fiscal expansion is unfunded, i.e. fiscal-led, so a shock to ζ_t^F. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

Figure A.19: Estimation results: prior and posteriors of the two-country Bayesian estimation (France vs. rest of EA)

		Prior			Posterior		
Param.	Description	Type	Mean	Std.	Mode	Median	90% HPD CI
Region A structural							
χ	Inverse Frisch elasticity of labor supply	N	2.00	0.50	1.3207	1.1063	[0.4696, 1.5857]
χ_w	Wage indexation	B	0.25	0.10	0.2465	0.2305	[0.1437, 0.3242]
μ	Elasticity of substitution across tradable goods	N	1.00	0.25	1.9821	1.8994	[1.7428, 2.0624]
θ_T	Calvo probability tradable prices	B	0.75	0.10	0.6527	0.6125	[0.5497, 0.6771]
θ_N	Calvo probability non-tradable prices	B	0.75	0.10	0.9641	0.9635	[0.9486, 0.9771]
θ_w	Calvo probability wages	B	0.75	0.10	0.8361	0.8356	[0.8152, 0.8554]
Region B structural							
χ^*	Inverse Frisch elasticity B	N	2.00	0.50	2.5339	2.7966	[2.3045, 3.2650]
χ^*_w	Wage indexation B	B	0.25	0.10	0.2197	0.1988	[0.1191, 0.2875]
μ^*	Trade elasticity B	N	1.00	0.25	2.1345	2.1162	[1.7935, 2.3983]
θ^*_T	Calvo probability tradable prices B	B	0.75	0.10	0.4039	0.4310	[0.3640, 0.4945]
θ^*_N	Calvo probability non-tradable prices B	B	0.75	0.10	0.7717	0.7525	[0.7046, 0.7999]
Common structural							
ψ_u	Capital utilization cost	B	0.50	0.15	0.0439	0.0488	[0.0233, 0.0767]
Monetary / target policy							
ϕ^{ML}	Taylor rule inflation coefficient (ML)	N	2.00	0.25	1.7316	1.5901	[1.3031, 1.9024]
ρ_r	Taylor rule interest rate smoothing (ML)	B	0.50	0.25	0.9209	0.9206	[0.9033, 0.9380]
ϕ_y	Taylor rule output response coefficient (union output, GDP-weighted)	N	0.125	0.05	0.0281	0.0247	[0.0103, 0.0443]
ϕ_F	Inflation target persistence (Singh et al. eq. 26)	B	0.95	0.03	0.6657	0.6267	[0.5914, 0.6610]
Shock persistence							
ρ_a	Technology shock persistence	B	0.80	0.10	0.9061	0.8991	[0.8721, 0.9249]
ρ_{rp}	Risk premium shock persistence	B	0.80	0.10	0.9688	0.9673	[0.9560, 0.9783]
ρ_d	Preference shock persistence	B	0.50	0.15	0.1316	0.1514	[0.0817, 0.2261]
ρ_u	Tradable markup shock persistence	B	0.80	0.10	0.7048	0.7188	[0.5091, 0.8560]
ρ_{uN}	NT markup shock persistence	B	0.80	0.10	0.6222	0.5967	[0.5426, 0.6512]
ρ_w	Wage markup shock persistence	B	0.80	0.10	0.7043	0.7054	[0.6397, 0.7663]
ρ_k	MEI shock persistence	B	0.50	0.15	0.5351	0.5347	[0.4823, 0.5890]
ρ_ξ	Monetary policy shock persistence	B	0.50	0.15	0.6713	0.6256	[0.5756, 0.6670]
Shock volatilities							
σ_a	St.dev. technology shock	IG	0.50	inf	1.4137	1.6308	[1.2692, 1.9777]
σ_{aN}	St.dev. NT technology shock	IG	0.50	inf	1.6952	1.6814	[1.5148, 1.8480]
σ_{rp}	St.dev. risk premium shock	IG	0.25	inf	0.0378	0.0391	[0.0330, 0.0455]
σ_d	St.dev. preference shock	IG	0.50	inf	8.3534	8.6309	[7.5835, 9.7630]
σ_u	St.dev. tradable markup shock	IG	0.50	inf	0.8933	0.8266	[0.3864, 1.2515]
σ_{uN}	St.dev. NT markup shock	IG	0.50	inf	0.9035	0.9564	[0.8192, 1.0992]
σ_w	St.dev. wage markup shock	IG	0.50	inf	0.3908	0.3904	[0.3384, 0.4463]
σ_k	St.dev. MEI shock	IG	0.50	inf	5.9331	5.9469	[5.2500, 6.6484]
σ_ξ	St.dev. monetary policy shock	IG	0.25	inf	0.3308	0.3087	[0.2578, 0.3612]
Unfunded floating-target							
$\sigma_{U,A}$	St.dev. unfunded shock A	IG	0.50	inf	18.1238	17.7758	[16.1152, 19.5814]
$\sigma_{U,B}$	St.dev. unfunded shock B	IG	0.50	inf	22.2268	22.8853	[20.5085, 25.7281]

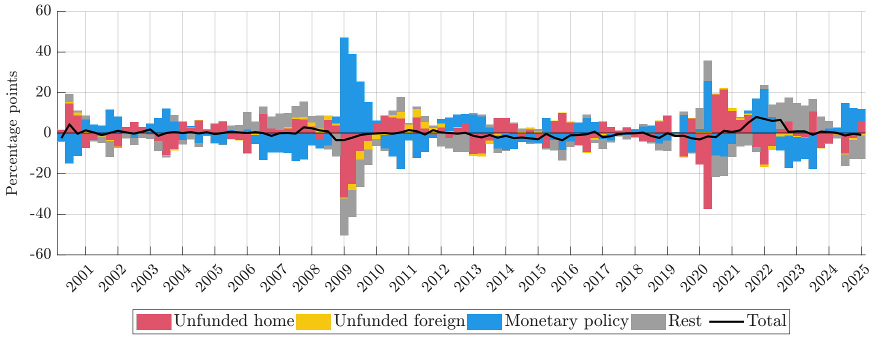
Notes: Results of the estimation of the structural parameters. N = Normal, B = Beta, G = Gamma, IG = Inverse-Gamma, AR coeff. = autoregressive coefficient, St.dev. = standard deviation, HPD = highest posterior density.

Figure A.20: Estimation results: prior and posteriors of the two-country Bayesian estimation (Italy vs. rest of EA)

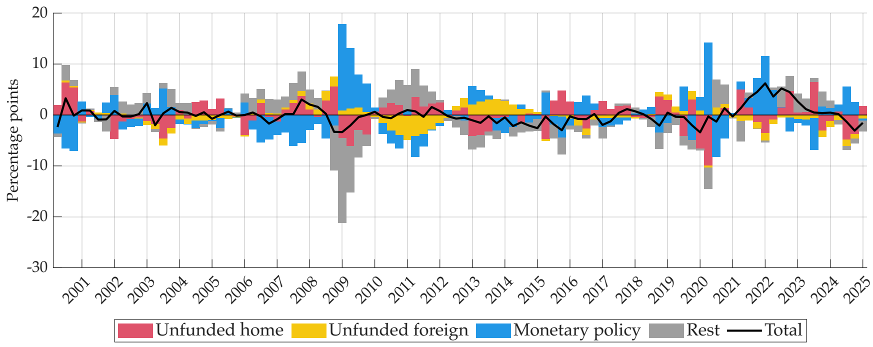
		Prior			Posterior		
Param.	Description	Type	Mean	Std.	Mode	Median	90% HPD CI
Region A structural							
χ	Inverse Frisch elasticity of labor supply	N	2.00	0.50	1.7350	1.5661	[1.0919, 2.0297]
χ_w	Wage indexation	B	0.25	0.10	0.1207	0.1235	[0.0424, 0.2184]
μ	Elasticity of substitution across tradable goods	N	1.00	0.25	2.1163	2.1228	[1.9818, 2.3245]
θ_T	Calvo probability tradable prices	B	0.75	0.10	0.8494	0.8318	[0.7905, 0.8705]
θ_N	Calvo probability non-tradable prices	B	0.75	0.10	0.9843	0.9827	[0.9753, 0.9894]
θ_w	Calvo probability wages	B	0.75	0.10	0.8627	0.8610	[0.8460, 0.8746]
Region B structural							
χ^*	Inverse Frisch elasticity B	N	2.00	0.50	3.1293	3.0426	[2.5751, 3.5074]
χ_w^*	Wage indexation B	B	0.25	0.10	0.1030	0.0972	[0.0432, 0.1523]
μ^*	Trade elasticity B	N	1.00	0.25	1.7391	1.7372	[1.4902, 1.9989]
θ_T^*	Calvo probability tradable prices B	B	0.75	0.10	0.5287	0.4930	[0.4250, 0.5637]
θ_N^*	Calvo probability non-tradable prices B	B	0.75	0.10	0.7803	0.7849	[0.7484, 0.8183]
Common structural							
ψ_u	Capital utilization cost	B	0.50	0.15	0.0339	0.0374	[0.0185, 0.0581]
Monetary / target policy							
ϕ^{ML}	Taylor rule inflation coefficient (ML)	N	2.00	0.25	2.3697	2.2939	[2.1391, 2.4881]
ρ_r	Taylor rule interest rate smoothing (ML)	B	0.50	0.25	0.9409	0.9389	[0.9295, 0.9482]
ϕ_y	Taylor rule output response coefficient (union output, GDP-weighted)	N	0.125	0.05	0.0407	0.0403	[0.0219, 0.0587]
ϕ_F	Inflation target persistence (Singh et al. eq. 26)	B	0.95	0.03	0.7007	0.7053	[0.6810, 0.7283]
Shock persistence							
ρ_a	Technology shock persistence	B	0.80	0.10	0.9320	0.9234	[0.9007, 0.9462]
ρ_{rp}	Risk premium shock persistence	B	0.80	0.10	0.9625	0.9626	[0.9497, 0.9745]
ρ_d	Preference shock persistence	B	0.50	0.15	0.3283	0.2656	[0.1995, 0.3351]
ρ_u	Tradable markup shock persistence	B	0.80	0.10	0.5675	0.5798	[0.5153, 0.6449]
ρ_{uN}	NT markup shock persistence	B	0.80	0.10	0.4667	0.4752	[0.4228, 0.5456]
ρ_w	Wage markup shock persistence	B	0.80	0.10	0.7295	0.7006	[0.6319, 0.7674]
ρ_k	MEI shock persistence	B	0.50	0.15	0.4046	0.4220	[0.3493, 0.5055]
ρ_ξ	Monetary policy shock persistence	B	0.50	0.15	0.7129	0.7184	[0.6928, 0.7436]
Shock volatilities							
σ_a	St.dev. technology shock	IG	0.50	inf	2.0036	1.8102	[1.3941, 2.2075]
σ_{aN}	St.dev. NT technology shock	IG	0.50	inf	1.7350	1.7850	[1.5833, 1.9818]
σ_{rp}	St.dev. risk premium shock	IG	0.25	inf	0.0406	0.0401	[0.0338, 0.0473]
σ_d	St.dev. preference shock	IG	0.50	inf	9.2329	8.9428	[7.9234, 10.0232]
σ_u	St.dev. tradable markup shock	IG	0.50	inf	0.8915	0.8886	[0.7235, 1.0522]
σ_{uN}	St.dev. NT markup shock	IG	0.50	inf	0.9094	0.9214	[0.7914, 1.0558]
σ_w	St.dev. wage markup shock	IG	0.50	inf	0.5112	0.5462	[0.4709, 0.6221]
σ_k	St.dev. MEI shock	IG	0.50	inf	8.0980	8.0435	[6.7512, 9.3153]
σ_ξ	St.dev. monetary policy shock	IG	0.25	inf	0.3586	0.3646	[0.2991, 0.4331]
Unfunded floating-target							
$\sigma_{U,A}$	St.dev. unfunded shock A	IG	0.50	inf	18.7742	19.8435	[17.9180, 22.0399]
$\sigma_{U,B}$	St.dev. unfunded shock B	IG	0.50	inf	24.4645	25.4367	[23.5577, 27.3931]

Notes: Results of the estimation of the structural parameters. N = Normal, B = Beta, G = Gamma, IG = Inverse-Gamma, AR coeff. = autoregressive coefficient, St.dev. = standard deviation, HPD = highest posterior density.

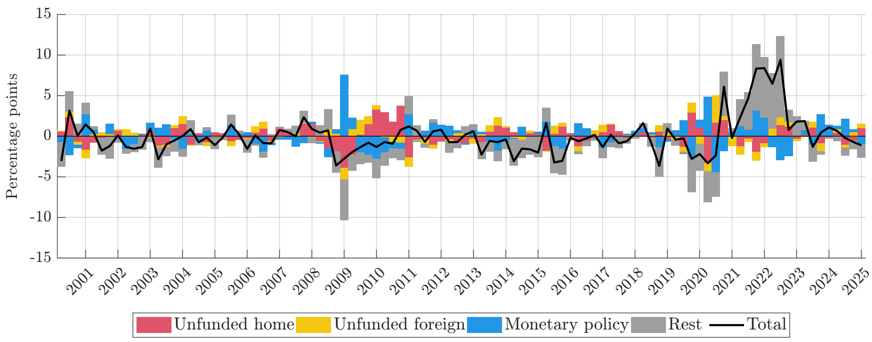
Figure A.21: Shock decomposition of inflation in rest of euro area



(a) Inflation in the rest of the euro area (vs. Germany)



(b) Inflation in the rest of the euro area (vs. France)



(c) Inflation in the rest of the euro area (vs. Italy)

Notes: Historical shock decomposition of the inflation in the rest of the euro area (vs. Germany, France, and Italy) in percentage point deviations from the sample mean. We estimate the quantitative two-country model three times, with each of those countries as country A and the rest of the euro area as country B. Monetary-led fiscal shocks are ζ_t^{ML} and g_t^{ML}, and fiscal-led fiscal shocks are ζ_t^{FL} and g_t^{FL}. The x-axis is in quarters.

B Equations stylized model

We summarize the stylized model with the following equations, linearized around the steady state as $\hat{x}_t = \ln(X_t) - \ln(\overline{X}) \approx \frac{X_t - \overline{X}}{\overline{X}}$.

$$\text{Agg. Euler equation} \quad \hat{c}_t^{MU} = \mathbb{E}_t \hat{c}_{t+1}^{MU} - \left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}^{MU}\right) - \Theta \psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A} \left(\hat{b}_{Bt} - \hat{b}_{At}\right) - (1-\Theta)\psi_b \frac{\overline{b}_A^* + \overline{b}_B^*}{\overline{b}_A^*} \left(\hat{b}_{Bt}^* - \hat{b}_{At}^*\right) \tag{B.1}$$

$$\text{Govt. budget constraint A} \quad \hat{s}_{bt} = \frac{1}{\beta}\left[\hat{s}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_t - (1-\beta)\hat{\tau}_t\right] \tag{B.2}$$

$$\text{Govt. budget constraint B} \quad \hat{s}_{bt}^* = \frac{1}{\beta}\left[\hat{s}_{b,t-1}^* + \hat{i}_{t-1} - \hat{\pi}_t^* - (1-\beta)\hat{\tau}_t^*\right] \tag{B.3}$$

$$\text{Fiscal rule A} \quad \hat{\tau}_t = \gamma \hat{s}_{b,t-1} + \zeta_t \tag{B.4}$$

$$\text{Fiscal rule B} \quad \hat{\tau}_t^* = \gamma^* \hat{s}_{b,t-1}^* + \zeta_t^* \tag{B.5}$$

$$\text{Monetary rule} \quad \hat{i}_t = \phi \hat{\pi}_t^{MU} \tag{B.6}$$

$$\text{Market clearing A} \quad 0 = (1-\alpha_I)\hat{c}_t + \alpha_I \hat{c}_t^* + 2\mu\alpha_I(1-\alpha_I)\hat{s}_t \tag{B.7}$$

$$\text{Agg. goods market clearing} \quad 0 = \Theta \hat{c}_t + (1-\Theta)\hat{c}_t^* \tag{B.8}$$

$$\text{Bonds market clearing A} \quad \overline{s}_b \hat{s}_{bt} = \overline{b}_A \hat{b}_{At} + \overline{b}_A^* \hat{b}_{At}^* \tag{B.9}$$

$$\text{Bonds market clearing B} \quad \overline{s}_b^* \hat{s}_{bt}^* = \overline{b}_B \hat{b}_{Bt} + \overline{b}_B^* \hat{b}_{Bt}^* \tag{B.10}$$

$$\text{No arbitrage A} \quad \hat{b}_{At} = \hat{b}_{Bt} \tag{B.11}$$

$$\text{No arbitrage B} \quad \hat{b}_{At}^* = \hat{b}_{Bt}^* \tag{B.12}$$

$$\text{Terms of trade} \quad \alpha_I\left(\hat{s}_t - \hat{s}_{t-1}\right) = \hat{\pi}_t - \hat{\pi}_{Tt} \tag{B.13}$$

$$\text{Current account} \quad -\hat{c}_t - \alpha_I \hat{s}_t = \hat{ca}_t \tag{B.14}$$

$$\text{Dynamics of NFA} \quad \hat{ca}_t = \overline{b}_B \hat{b}_{Bt} - \overline{b}_A^* \hat{b}_{At}^* - \frac{1}{\beta}\left(\overline{b}_B \hat{b}_{B,t-1} - \overline{b}_A^* \hat{b}_{A,t-1}^*\right) \tag{B.15}$$

$$\text{Agg. inflation} \quad \hat{\pi}_t^{MU} = \Theta \hat{\pi}_t + (1-\Theta)\hat{\pi}_t^* \tag{B.16}$$

$$\text{Inflation A} \quad \hat{\pi}_t = (1-\alpha_I)\hat{\pi}_{Tt} + \alpha_I \hat{\pi}_{Tt}^* \tag{B.17}$$

$$\text{Inflation B} \quad \hat{\pi}_t^* = \alpha_I \hat{\pi}_{Tt} + (1-\alpha_I)\hat{\pi}_{Tt}^* \tag{B.18}$$

where $\hat{x}_t^{MU} \equiv \Theta \hat{x}_t + (1-\Theta)\hat{x}_t^*$ and bond holdings are in real terms (as shares of output), e.g. $\hat{b}_{At} = \frac{\frac{B_{At}}{P_t Y} - \frac{\overline{B}_A}{\overline{P}Y}}{\frac{\overline{B}_A}{\overline{P}Y}}$.

Steady state. From the Euler equations, we obtain the steady-state nominal interest rate:

$$\overline{R} = \frac{1}{\beta} \tag{B.19}$$

For simplicity, we impose that in steady state the prices of the goods are unitary:

$$\overline{P}_A = \overline{P}_B = 1 \tag{B.20}$$

From the relative demand equations and the goods market clearing conditions, we get the steady-state consumption values, assuming that the endowments $Y = Y^* = 1$:

$$\overline{C}_A = \overline{C}^*_B = 1 - \alpha_I \tag{B.21}$$

$$\overline{C}_B = \overline{C}^*_A = \alpha_I \tag{B.22}$$

$$\overline{C} = \overline{C}^* = 1 \tag{B.23}$$

With these steady-state values, we can get the government budget constraint in steady state:

$$\overline{B} = \frac{\beta}{1-\beta}\overline{T} \tag{B.24}$$

We calibrate the debt-to-GDP ratio $\frac{\overline{B}}{\overline{Y}} = \overline{B} = 0.80$. See the Calibration section in the main text for more details.

C Solving the stylized model with partially unfunded debt

In the model with partially unfunded debt in country A, we can keep all equations the same as the model without, except the fiscal rules and the monetary rule which we replace with equations (30), (31), and (32). Moreover, for every equation we add the equivalent equation for the shadow economies.

To prove that the equations for the model with partially unfunded debt are correct, we need to prove that the following is true: first, that $\hat{s}^M_{bt} = \hat{s}_{bt} - \hat{s}^F_{bt}$ and $\hat{s}^{M*}_{bt} = \hat{s}^*_{bt} - \hat{s}^{F*}_{bt}$, i.e. the difference of the overall stock of debt and fiscal-led stock of debt is the monetary-led stock of debt. Second, that $\hat{\pi}^M_t = \hat{\pi}_t - \hat{\pi}^F_t$ and $\hat{\pi}^{M*}_t = \hat{\pi}^*_t - \hat{\pi}^{F*}_t$, i.e. the inflation rate the central bank tries to stabilize in each economy is the overall inflation net of the fiscal-led part of inflation.

To prove those claims, we follow BFM and construct another parallel economy to characterize the part of inflation the central bank has control over, $\hat{\pi}^M_t$ and $\hat{\pi}^{M*}_t$:

$$\Theta \mathbb{E}_t \hat{\pi}^M_{t+1} + (1-\Theta)\mathbb{E}_t \hat{\pi}^{M*}_{t+1} = \hat{i}^M_t - \Theta\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) - (1-\Theta)\psi_b \frac{\overline{b}^*_A + \overline{b}^*_B}{\overline{b}^*_A}\left(\hat{b}^*_{Bt} - \hat{b}^*_{At}\right) \tag{C.1}$$

$$\hat{s}^M_{bt} = \frac{1}{\beta}\left[\hat{s}^M_{b,t-1} + \hat{i}^M_{t-1} - \hat{\pi}^M_t - (1-\beta)\hat{\tau}^M_t\right] \tag{C.2}$$

$$\hat{s}^{M*}_{bt} = \frac{1}{\beta}\left[\hat{s}^{M*}_{b,t-1} + \hat{i}^M_{t-1} - \hat{\pi}^{M*}_t - (1-\beta)\hat{\tau}^{M*}_t\right] \tag{C.3}$$

$$\hat{\tau}^M_t = \gamma^M \hat{s}^M_{b,t-1} + \zeta^M_t \tag{C.4}$$

$$\hat{\tau}^{M*}_t = \gamma^{M*} \hat{s}^{M*}_{b,t-1} + \zeta^{M*}_t \tag{C.5}$$

$$\hat{i}^M_t = \Theta\phi^M \hat{\pi}^M_t + (1-\Theta)\phi^{M*}\hat{\pi}^{M*}_t \tag{C.6}$$

We obtain the monetary and fiscal blocks of this parallel economy as with other economies in the main text:

$$\Theta \mathbb{E}_t \hat{\pi}^M_{t+1} + (1-\Theta)\mathbb{E}_t \hat{\pi}^{M*}_{t+1} = \Theta\phi^M \hat{\pi}^M_t + (1-\Theta)\phi^{M*}\hat{\pi}^{M*}_t - \Theta\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) - (1-\Theta)\psi_b \frac{\overline{b}^*_A + \overline{b}^*_B}{\overline{b}^*_A}\left(\hat{b}^*_{Bt} - \hat{b}^*_{At}\right) \tag{C.7}$$

$$\hat{s}^M_{bt} = \frac{1}{\beta}\left[(1-(1-\beta)\gamma^M)\hat{s}^M_{b,t-1} + \hat{i}^M_{t-1} - \hat{\pi}^M_t - (1-\beta)\zeta^M_t\right] \tag{C.8}$$

$$\hat{s}^{M*}_{bt} = \frac{1}{\beta}\left[(1-(1-\beta)\gamma^{M*})\hat{s}^{M*}_{b,t-1} + \hat{i}^M_{t-1} - \hat{\pi}^{M*}_t - (1-\beta)\zeta^{M*}_t\right] \tag{C.9}$$

We prove $\hat{\pi}^M_t = \hat{\pi}_t - \hat{\pi}^F_t$ by subtracting the equations for the monetary block of the monetary-led economy (C.7) and fiscal-led economy (37) and (38) from the monetary block equation of the overall economy (33):[8]

$$\Theta\mathbb{E}_t\left[\hat{\pi}_{t+1}-\hat{\pi}^M_{t+1}-\hat{\pi}^F_{t+1}\right]+(1-\Theta)\mathbb{E}_t\left[\hat{\pi}^*_{t+1}-\hat{\pi}^{M*}_{t+1}-\hat{\pi}^{F*}_{t+1}\right]$$
$$=\Theta\phi^M\left(\hat{\pi}_t-\hat{\pi}^M_t-\hat{\pi}^F_t\right)+(1-\Theta)\phi^{M*}\left(\hat{\pi}^*_t-\hat{\pi}^{M*}_t-\hat{\pi}^{F*}_t\right)\quad \text{(C.10)}$$

where we multiply the fiscal-led economy equations with the respective country weights before subtracting. Since $\phi^M=\phi^{M*}>1$, the above equation implies $\hat{\pi}_t-\hat{\pi}^M_t-\hat{\pi}^F_t=0$ and $\hat{\pi}^*_t-\hat{\pi}^{M*}_t-\hat{\pi}^{F*}_t=0$, thus $\hat{\pi}^M_t=\hat{\pi}_t-\hat{\pi}^F_t$ and $\hat{\pi}^{M*}_t=\hat{\pi}^*_t-\hat{\pi}^{F*}_t$.

Now, we prove that $\hat{s}^M_{bt}=\hat{s}_{bt}-\hat{s}^F_{bt}$. Substituting the fiscal rule of country A and monetary rule that allow for partially unfunded debt, equations (30) and (32) respectively, into the country A government budget constraint (20), we obtain:

$$\beta\hat{s}_{bt}=\left(1-(1-\beta)\gamma^M\right)\hat{s}_{b,t-1}+(1-\beta)\gamma^M\hat{s}^F_{b,t-1}-(1-\beta)\gamma^F\hat{s}^F_{b,t-1}-\hat{\pi}_t-(1-\beta)\left(\zeta^M_t+\zeta^F_t\right)$$
$$+\Theta\left[\phi^M\left(\hat{\pi}_{t-1}-\hat{\pi}^F_{t-1}\right)+\phi^F\hat{\pi}^F_{t-1}\right]+(1-\Theta)\left[\phi^{M*}\left(\hat{\pi}^*_{t-1}-\hat{\pi}^{F*}_{t-1}\right)+\phi^{F*}\hat{\pi}^{F*}_{t-1}\right].\quad \text{(C.11)}$$

We obtain the equivalent equations for the monetary-led and fiscal-led parallel economies:

$$\beta\hat{s}^M_{bt}=(1-(1-\beta)\gamma^M)\hat{s}^M_{b,t-1}+\Theta\phi^M\hat{\pi}^M_{t-1}+(1-\Theta)\phi^{M*}\hat{\pi}^{M*}_{t-1}-\hat{\pi}^M_t-(1-\beta)\zeta^M_t\quad \text{(C.12)}$$
$$\beta\hat{s}^F_{bt}=(1-(1-\beta)\gamma^F)\hat{s}^F_{b,t-1}+\Theta\phi^F\hat{\pi}^F_{t-1}+(1-\Theta)\phi^{F*}\hat{\pi}^{F*}_{t-1}-\hat{\pi}^F_t-(1-\beta)\zeta^F_t\quad \text{(C.13)}$$

Subtract $\beta\hat{s}^M_{bt}$ and $\beta\hat{s}^F_{bt}$ from $\beta\hat{s}_{bt}$ to obtain:

$$\beta\left(\hat{s}_{bt}-\hat{s}^M_{bt}-\hat{s}^F_{bt}\right)=\left(1-(1-\beta)\gamma^M\right)\left(\hat{s}_{b,t-1}-\hat{s}^M_{b,t-1}-\hat{s}^F_{b,t-1}\right)-\left(\hat{\pi}_t-\hat{\pi}^M_t-\hat{\pi}^F_t\right)$$
$$+\Theta\phi^M\left(\hat{\pi}_{t-1}-\hat{\pi}^M_{t-1}-\hat{\pi}^F_{t-1}\right)+(1-\Theta)\phi^{M*}\left(\hat{\pi}^*_{t-1}-\hat{\pi}^{M*}_{t-1}-\hat{\pi}^{F*}_{t-1}\right)\quad \text{(C.14)}$$

Using $\hat{\pi}^M_t=\hat{\pi}_t-\hat{\pi}^F_t$ and $\hat{\pi}^{M*}_t=\hat{\pi}^*_t-\hat{\pi}^{F*}_t$, it must be that $\hat{s}_{bt}-\hat{s}^M_{bt}-\hat{s}^F_{bt}=0$ and hence $\hat{s}^M_{bt}=\hat{s}_{bt}-\hat{s}^F_{bt}$ for all periods. The proof for $\hat{s}^{M*}_{bt}=\hat{s}^*_{bt}-\hat{s}^{F*}_{bt}$ is analogous.

8. Impose the no-arbitrage conditions to simplify the adjustment cost term.

D Proof for signs of spillovers from funded and unfunded shocks

Proposition 2 (Inflation divergence under funded shocks, co-movement under unfunded shocks)**.** *Consider a currency union with two endowment economies and cross-border bond holdings subject to portfolio adjustment costs, characterized by the equations in Appendix B. A decline in primary surplus ($\zeta_t < 0$) creates the following inflation dynamics:*

(*i*) ***Inflation divergence*** *in a* ***monetary-led*** *economy, i.e. $\zeta_t < 0$ is a* ***funded*** *shock: on impact, $\hat{\pi}_t > 0, \hat{\pi}_t^* < 0$.*

(*ii*) ***Inflation co-movement*** *in a* ***fiscal-led*** *economy, i.e. $\zeta_t < 0$ is an* ***unfunded*** *shock: on impact, $\hat{\pi}_t > 0, \hat{\pi}_t^* > 0$.*

Proof. **(i) Funded shock (monetary-led economy)**

Step 1: Aggregate inflation response is zero. In the monetary-led economy, the monetary block, after applying the no-arbitrage conditions $\hat{b}_{At}^M = \hat{b}_{Bt}$ and $\hat{b}_{At}^* = \hat{b}_{Bt}^*$, reduces to:

$$\mathbb{E}_t \hat{\pi}_{t+1}^{MU} = \phi \hat{\pi}_t^{MU}. \tag{D.1}$$

Since $\phi > 1$ when the economy is monetary-led, the unique bounded (non-explosive) solution is:

$$\hat{\pi}_t^{MU} = 0 \quad \forall t. \tag{D.2}$$

Step 2: Inflation divergence. With $0 < \Theta < 1$, the condition $\hat{\pi}_t^{MU} = 0$ yields:

$$\Theta\hat{\pi}_t + (1-\Theta)\hat{\pi}_t^* = 0 \quad \Longrightarrow \quad \hat{\pi}_t^* = -\frac{\Theta}{1-\Theta}\hat{\pi}_t \quad \Longrightarrow \quad \frac{\partial \hat{\pi}_t^*}{\partial \hat{\pi}_t} = -\frac{\Theta}{1-\Theta} < 0. \tag{D.3}$$

Step 3: $\hat{\pi}_t > 0, \hat{\pi}_t^* < 0$ on impact. By definition, a funded decline in primary surplus in country A raises $\hat{s}_{bt}$, government debt in country A. With zero bond positions in $t-1$ and no shock in country B, so $\hat{s}_{bt}^* = 0$ at the time of the shock, the two bond market clearing conditions give:

$$\bar{s}_b \hat{s}_{bt} = \bar{b}_A \hat{b}_{At} + \bar{b}_A^* \hat{b}_{At}^* \tag{D.4}$$

$$0 = \bar{b}_B \hat{b}_{Bt} + \bar{b}_B^* \hat{b}_{Bt}^*. \tag{D.5}$$

Solving the system for $\hat{b}_{At}^*$ using the no-arbitrage conditions and, for expositional clarity, also using symmetric steady-state bond holdings $\bar{b}_A = \bar{b}_B^*$ and $\bar{b}_B = \bar{b}_A^*$, we get:

$$\hat{b}_{At}^* = \frac{\bar{s}_b \bar{b}_B}{\bar{b}_B^2 - \bar{b}_A^2}\hat{s}_{bt} \quad \Longrightarrow \quad \frac{\partial \hat{b}_{At}^*}{\partial \hat{s}_{bt}} = \frac{\bar{s}_b \bar{b}_B}{\bar{b}_B^2 - \bar{b}_A^2} > 0, \quad \text{if } \bar{b}_B > \bar{b}_A. \tag{D.6}$$

So, as long as cross-border holdings exceed home holdings, which is what we observe in the data, when the government debt in country A, $\hat{s}_{bt}$, increases, households in country B absorb part of that, with an increase in $\hat{b}^*_{At}$. Given zero bond positions in $t-1$, the net foreign assets equation gives $\hat{ca}_t < 0$ when $\hat{b}^*_{At} > 0$: country A runs a current account deficit whereas country B becomes a net saver. From rearranging the two goods market conditions, we solve for the terms of trade:

$$\hat{s}_t = -\frac{1-2\alpha_I}{2\mu\alpha_I(1-\alpha_I)}\hat{c}_t. \tag{D.7}$$

Then, substituting the above equation into the current account equation gives us:

$$\hat{c}_t = \frac{2\mu(1-\alpha_I)}{1-2\alpha_I-2\mu(1-\alpha_I)}\hat{ca}_t$$
$$\implies \quad \frac{\partial \hat{c}_t}{\partial \hat{ca}_t} = \frac{2\mu(1-\alpha_I)}{1-2\alpha_I-2\mu(1-\alpha_I)} < 0 \quad \text{if } \alpha_I < 0.5 \text{ and } \mu > \frac{1-2\alpha_I}{2(1-\alpha_I)}. \tag{D.8}$$

So, with the baseline home bias parameter at $\alpha_I = 0.25$, the threshold for the elasticity of substitution between goods is $\mu > 1/3$, which holds easily, and a decrease in the current account increases consumption in country A. Since the supply is fixed at Y and $\hat{c}_t > 0$, excess demand for A goods requires $\hat{\pi}_{At} > 0$ to clear the market. Combining the definitions for CPI and applying the inflation divergence result from step 2, we get:[9]

$$\hat{\pi}_t = (1-2\alpha_I)\hat{\pi}_{At} \quad \implies \quad \frac{\partial \pi_t}{\partial \hat{\pi}_{At}} = 1-2\alpha_I > 0 \quad \text{if } \alpha_I < 0.5. \tag{D.9}$$

So, with home bias, we get that CPI inflation in country A increases after a funded fiscal shock: $\hat{\pi}_t > 0$. Step 2 provides the inflation divergence result: $\hat{\pi}^*_t < 0$.

(ii) Unfunded shock (fiscal-led economy)

Step 1: Fiscal-led interest rate does not respond. From the setup of the fiscal-led economy, with $\phi = \phi^* = 0$, the monetary rule in the fiscal-led economy gives $\hat{i}_t = 0 \; \forall t$.

Step 2: Country A's inflation is positive. Since $\gamma = 0$, country A's fiscal-led debt has an eigenvalue of $1/\beta > 1$ in the government budget constraint. With $\hat{i}_t = 0$, the constraint becomes:

$$\hat{s}_{bt} = \frac{1}{\beta}\left[\hat{s}_{b,t-1} - \hat{\pi}_t - (1-\beta)\zeta_t\right]. \tag{D.10}$$

The explosive root $1/\beta$ implies that for a bounded equilibrium, $\hat{\pi}_t$ must jump to satisfy the intertemporal government budget constraint. In particular, $\zeta_t < 0$, a decrease in primary surplus, must be met by an increase in $\hat{\pi}_t > 0$ to prevent the real value of debt from exploding. This is the standard FTPL mechanism: unfunded deficits are resolved through inflation. An increase in inflation, combined with the absence of monetary

9. Here, we assume symmetric country sizes $\Theta = 0.5$ without loss of generality.

policy tightening, decreases the real rate in country A and increases demand: $\hat{c}_t > 0$. Given the home bias and the fixed supply, the price of A goods increases: $\hat{\pi}_{At} > 0$.

Step 3: Country B's inflation is also positive. Since there is no interest rate response ($\hat{i}_t = 0$), the increase in A-goods prices, $\hat{\pi}_{At} > 0$ feeds directly into country B's CPI through the import share $\alpha_I > 0$:

$$\hat{\pi}_t^* = \alpha_I \hat{\pi}_{At} + (1 - \alpha_I)\hat{\pi}_{Bt}. \tag{D.11}$$

The first term, $\alpha_I \hat{\pi}_{At} > 0$, is the *direct import price channel*: country B imports goods from A, whose prices have risen.

Furthermore, the absence of monetary tightening together with positive expected inflation $\mathbb{E}_t \hat{\pi}_t^* > 0$, from the direct import price channel and a persistent shock, decreases the real interest rate in country B. A decline in the real interest rate stimulates demand in country B: $\hat{c}_t^* > 0$.

With a fixed endowment, the excess demand also drives up prices of country B goods: $\hat{\pi}_{Bt} \geq 0$. Hence:

$$\hat{\pi}_t^* = \underbrace{\alpha_I \hat{\pi}_{At}}_{>0} + \underbrace{(1 - \alpha_I)\hat{\pi}_{Bt}}_{\geq 0} > 0. \tag{D.12}$$

Country B's inflation is positive. Both countries experience inflation increases, so inflation rates *co-move*.

□

E Log-linearized equations (quantitative model)

We summarize the quantitative model with the following equations, linearized around the steady state as in Appendix B.

Demand

Effective consumption A
$$\hat{c}_t^{\text{eff}} = \frac{1}{1-h}\hat{c}_t - \frac{h}{1-h}\hat{c}_{t-1} \tag{E.1}$$

Effective consumption B
$$\hat{c}_t^{\text{eff}*} = \frac{1}{1-h}\hat{c}_t^* - \frac{h}{1-h}\hat{c}_{t-1}^* \tag{E.2}$$

Marginal utility A
$$\hat{\lambda}_t = -\frac{1}{1-\beta h}\left(\hat{c}_t^{\text{eff}} - \beta h \mathbb{E}_t \hat{c}_{t+1}^{\text{eff}}\right) + \frac{1}{1-\beta h}\left(\xi_t^d - \beta h \mathbb{E}_t \xi_{t+1}^d\right) \tag{E.3}$$

Marginal utility B
$$\hat{\lambda}_t^* = -\frac{1}{1-\beta h}\left(\hat{c}_t^{\text{eff}*} - \beta h \mathbb{E}_t \hat{c}_{t+1}^{\text{eff}*}\right) + \frac{1}{1-\beta h}\left(\xi_t^{d*} - \beta h \mathbb{E}_t \xi_{t+1}^{d*}\right) \tag{E.4}$$

Labor supply A
$$\hat{c}_t^{\text{eff}} + \chi \hat{l}_t + \varepsilon_t^h = \widehat{mrs}_t \tag{E.5}$$

Labor supply B
$$\hat{c}_t^{\text{eff}*} + \chi \hat{l}_t^* + \varepsilon_t^{h*} = \widehat{mrs}_t^* \tag{E.6}$$

Agg. Euler equation
$$\mathbb{E}_t \hat{\lambda}_{t+1}^{MU} = \hat{\lambda}_t^{MU} - \left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}^{MU}\right) \tag{E.7}$$

Agg. capital Euler equation
$$\begin{aligned}\hat{q}_t^{MU} &= \mathbb{E}_t \hat{\lambda}_{t+1}^{MU} - \hat{\lambda}_t^{MU} + \beta(1-\delta)\mathbb{E}_t \hat{q}_{t+1}^{MU} \\ &+ [1-\beta(1-\delta)]\mathbb{E}_t\left[\hat{r}_{K,t+1}^{MU} - \frac{\overline{\tau}_K}{1-\overline{\tau}_K}\left(\Theta\hat{\tau}_{K,t+1} + (1-\Theta)\hat{\tau}_{K,t+1}^*\right)\right]\end{aligned} \tag{E.8}$$

No arbitrage capital and ST bonds A
$$\begin{aligned}-\left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) &= \hat{q}_t - \beta(1-\delta)\mathbb{E}_t \hat{q}_{t+1} \\ &- [1-\beta(1-\delta)]\mathbb{E}_t\left[\hat{r}_{K,t+1} - \frac{\overline{\tau}_K}{1-\overline{\tau}_K}\hat{\tau}_{K,t+1}\right]\end{aligned} \tag{E.9}$$

No arbitrage across LT bonds A
$$\begin{aligned}\hat{p}_t^L - \hat{p}_t^{L*} &= \beta\rho^L \mathbb{E}_t\left(\hat{p}_{t+1}^L - \hat{p}_{t+1}^{L*}\right) + \left(\xi_t^{rp} - \xi_t^{rp*}\right) \\ &- \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) + \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_B}\left(\hat{b}_{Bt} - \hat{b}_{At}\right)\end{aligned} \tag{E.10}$$

No arbitrage across LT bonds B
$$\begin{aligned}\hat{p}_t^L - \hat{p}_t^{L*} &= \beta\rho^L \mathbb{E}_t\left(\hat{p}_{t+1}^L - \hat{p}_{t+1}^{L*}\right) + \left(\xi_t^{rp} - \xi_t^{rp*}\right) \\ &- \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}_A^* + \overline{b}_B^*}{\overline{b}_A^*}\left(\hat{b}_{Bt}^* - \hat{b}_{At}^*\right) + \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}_A^* + \overline{b}_B^*}{\overline{b}_B^*}\left(\hat{b}_{Bt}^* - \hat{b}_{At}^*\right)\end{aligned} \tag{E.11}$$

No arbitrage LT and ST bonds A $$\hat{i}_t = \mathbb{E}_t \hat{r}^L_{t+1} + \xi^{rp}_t - \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) \tag{E.12}$$

No arbitrage LT and ST bonds B $$\hat{i}_t = \mathbb{E}_t \hat{r}^{L*}_{t+1} + \xi^{rp*}_t - \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}^*_A + \overline{b}^*_B}{\overline{b}^*_B}\left(\hat{b}^*_{Bt} - \hat{b}^*_{At}\right) \tag{E.13}$$

Return on LT bonds A $$\hat{r}^L_t = \rho^L \beta \hat{p}^L_t - \hat{p}^L_{t-1} \tag{E.14}$$

Return on LT bonds B $$\hat{r}^{L*}_t = \rho^L \beta \hat{p}^{L*}_t - \hat{p}^{L*}_{t-1} \tag{E.15}$$

Demand trad./non-trad. A $$\hat{c}_{Tt} - \hat{c}_{Nt} = -\mu_N\left(\hat{p}_{Tt} - \hat{p}_{Nt}\right) \tag{E.16}$$

Demand trad./non-trad. B $$\hat{c}^*_{Tt} - \hat{c}^*_{Nt} = -\mu_N\left(\hat{p}^*_{Tt} - \hat{p}^*_{Nt}\right) \tag{E.17}$$

Wages

Wage Phillips curve A $$\hat{\pi}^w_t = \beta \mathbb{E}_t \hat{\pi}^w_{t+1} + \chi_w \hat{\pi}_{t-1} + \kappa_w\left(\widehat{mrs}_t - \hat{w}^{real}_t\right) - \kappa_w \frac{\overline{\tau}_L}{1-\overline{\tau}_L}\hat{\tau}_{Lt} + u^w_t \tag{E.18}$$

Wage Phillips curve B $$\hat{\pi}^{w*}_t = \beta \mathbb{E}_t \hat{\pi}^{w*}_{t+1} + \chi_w \hat{\pi}^*_{t-1} + \kappa^*_w\left(\widehat{mrs}^*_t - \hat{w}^{*,real}_t\right) - \kappa^*_w \frac{\overline{\tau}_L}{1-\overline{\tau}_L}\hat{\tau}^*_{Lt} + u^{w*}_t \tag{E.19}$$

Wage inflation indexation A $$\hat{\pi}^w_t = \hat{w}^{real}_t - \hat{w}^{real}_{t-1} + \hat{\pi}_t \tag{E.20}$$

Wage inflation indexation B $$\hat{\pi}^{w*}_t = \hat{w}^{*,real}_t - \hat{w}^{*,real}_{t-1} + \hat{\pi}^*_t \tag{E.21}$$

Supply

Tobin's Q A $$\hat{q}_t = \psi_I\left[\widehat{inv}_t - \widehat{inv}_{t-1} - \beta\left(\mathbb{E}_t \widehat{inv}_{t+1} - \widehat{inv}_t\right)\right] - \xi^k_t \tag{E.22}$$

Tobin's Q B $$\hat{q}^*_t = \psi_I\left[\widehat{inv}^*_t - \widehat{inv}^*_{t-1} - \beta\left(\mathbb{E}_t \widehat{inv}^*_{t+1} - \widehat{inv}^*_t\right)\right] - \xi^{k*}_t \tag{E.23}$$

Capital accumulation A $$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\left(\widehat{inv}_t + \xi^k_t\right) \tag{E.24}$$

Capital accumulation B $$\hat{k}^*_t = (1-\delta)\hat{k}^*_{t-1} + \delta\left(\widehat{inv}^*_t + \xi^{k*}_t\right) \tag{E.25}$$

Capital services A $$\hat{k}^s_t = \hat{k}_{t-1} + \hat{\mu}_{K,t} \tag{E.26}$$

Capital services B $$\hat{k}^{s*}_t = \hat{k}^*_{t-1} + \hat{\mu}^*_{K,t} \tag{E.27}$$

Production function trad. A	$\hat{y}_{Tt} = \hat{a}_{Tt} + \alpha\hat{k}^s_t + (1-\alpha)\hat{l}_{Tt}$	(E.28)
Production function trad. B	$\hat{y}^*_{Tt} = \hat{a}^*_{Tt} + \alpha\hat{k}^{s*}_t + (1-\alpha)\hat{l}^*_{Tt}$	(E.29)
Production function non-trad. A	$\hat{y}_{Nt} = \hat{a}_{Nt} + (1-\alpha)\hat{l}_{Nt}$	(E.30)
Production function non-trad. B	$\hat{y}^*_{Nt} = \hat{a}^*_{Nt} + (1-\alpha)\hat{l}^*_{Nt}$	(E.31)
Capital demand A	$\hat{y}_{Tt} - \hat{k}^s_t = \hat{r}_{Kt}$	(E.32)
Capital demand B	$\hat{y}^*_{Tt} - \hat{k}^{s*}_t = \hat{r}^*_{Kt}$	(E.33)
FOC utilization A	$\hat{\mu}_{K,t} = \frac{1-\psi_u}{\psi_u}\hat{r}_{Kt}$	(E.34)
FOC utilization B	$\hat{\mu}^*_{K,t} = \frac{1-\psi_u}{\psi_u}\hat{r}^*_{Kt}$	(E.35)
Marginal cost trad. A	$\widehat{mc}_{Tt} = \hat{w}^{real}_t + \hat{l}_{Tt} - \hat{y}_{Tt}$	(E.36)
Marginal cost trad. B	$\widehat{mc}^*_{Tt} = \hat{w}^{*,real}_t + \hat{l}^*_{Tt} - \hat{y}^*_{Tt}$	(E.37)
NKPC trad. A	$\hat{\pi}_{At} = \beta\mathbb{E}_t\hat{\pi}_{A,t+1} + \kappa_T\widehat{mc}_{Tt} + u^T_t$	(E.38)
NKPC trad. B	$\hat{\pi}_{Bt} = \beta\mathbb{E}_t\hat{\pi}_{B,t+1} + \kappa_T\widehat{mc}^*_{Tt} + u^{T*}_t$	(E.39)
NKPC non-trad. A	$\hat{\pi}_{Nt} = \beta\mathbb{E}_t\hat{\pi}_{N,t+1} + \kappa_N\widehat{mc}_{Nt} + u^N_t$	(E.40)
NKPC non-trad. B	$\hat{\pi}^*_{Nt} = \beta\mathbb{E}_t\hat{\pi}^*_{N,t+1} + \kappa_N\widehat{mc}^*_{Nt} + u^{N*}_t$	(E.41)
Marginal cost non-trad. A	$\widehat{mc}_{Nt} = \hat{w}^{real}_t + \hat{l}_{Nt} - \hat{y}_{Nt}$	(E.42)
Marginal cost non-trad. B	$\widehat{mc}^*_{Nt} = \hat{w}^{*,real}_t + \hat{l}^*_{Nt} - \hat{y}^*_{Nt}$	(E.43)

Fiscal

Govt. budget constraint A

$$\hat{s}_{bt} = \frac{1}{\beta}\left[\hat{s}_{b,t-1} + \hat{i}^L_t - \hat{\pi}_t\right] - \overline{\tau}_K\left(\hat{\tau}_{Kt} + \hat{r}_{Kt} + \hat{k}_{t-1} + \hat{\mu}_{Kt}\right) - \overline{\tau}_L\left(\hat{\tau}_{Lt} + \hat{w}^{real}_t + \hat{l}_t\right) - \overline{\tau}_C\hat{c}_t + \frac{\overline{G}/\overline{Y}}{\overline{s}_b}\hat{g}_t + \frac{\overline{Z}/\overline{Y}}{\overline{s}_b}\hat{z}_t \quad (E.44)$$

Govt. budget constraint B

$$\hat{s}^*_{bt} = \frac{1}{\beta}\left[\hat{s}^*_{b,t-1} + \hat{i}^{L*}_t - \hat{\pi}^*_t\right] - \overline{\tau}_K\left(\hat{\tau}^*_{Kt} + \hat{r}^*_{Kt} + \hat{k}^*_{t-1} + \hat{\mu}^*_{Kt}\right) - \overline{\tau}_L\left(\hat{\tau}^*_{Lt} + \hat{w}^{*,real}_t + \hat{l}^*_t\right) - \overline{\tau}_C\hat{c}^*_t + \frac{\overline{G}/\overline{Y}}{\overline{s}_b}\hat{g}^*_t + \frac{\overline{Z}/\overline{Y}}{\overline{s}_b}\hat{z}^*_t \quad (E.45)$$

Transfer rule A

$$\hat{z}_t = \rho_z\hat{z}_{t-1} + (1-\rho_z)\left[\gamma\hat{s}_{b,t-1} + \phi_{zy}\hat{y}_t\right] + \frac{1}{\overline{Z}/\overline{Y}}(\zeta_t + \zeta_{z,t}) \quad (E.46)$$

Transfer rule B

$$\hat{z}^*_t = \rho_z\hat{z}^*_{t-1} + (1-\rho_z)\left[\gamma^*\hat{s}^*_{b,t-1} + \phi^*_{zy}\hat{y}^*_t\right] + \frac{1}{\overline{Z}/\overline{Y}}(\zeta^*_t + \zeta^*_{z,t}) \quad (E.47)$$

$$\text{Capital tax rule } A \qquad \hat{\tau}_{K,t} = \rho_K \hat{\tau}_{K,t-1} + (1-\rho_K)\gamma_K \hat{s}_{b,t-1} \tag{E.48}$$

$$\text{Capital tax rule } B \qquad \hat{\tau}^*_{K,t} = \rho_K \hat{\tau}^*_{K,t-1} + (1-\rho_K)\gamma^*_K \hat{s}^*_{b,t-1} \tag{E.49}$$

$$\text{Labor tax rule } A \qquad \hat{\tau}_{L,t} = \rho_L \hat{\tau}_{L,t-1} + (1-\rho_L)\gamma_L \hat{s}_{b,t-1} \tag{E.50}$$

$$\text{Labor tax rule } B \qquad \hat{\tau}^*_{L,t} = \rho_L \hat{\tau}^*_{L,t-1} + (1-\rho_L)\gamma^*_L \hat{s}^*_{b,t-1} \tag{E.51}$$

Monetary

$$\text{Taylor rule} \qquad \hat{i}_t = \rho_i \hat{i}_{t-1} + (1-\rho_i)\phi \hat{\pi}^{MU}_t + \xi^m_t \tag{E.52}$$

Equilibrium

$$\text{Market clearing trad. } A \qquad \hat{y}_{Tt} = (1-\alpha_I)\hat{c}_{Tt} + \alpha_I \frac{1-\Theta}{\Theta}\hat{c}^*_{Tt} + 2\mu\alpha_I(1-\alpha_I)\hat{s}_t + \frac{\overline{I}}{\overline{Y}}\widehat{inv}_t + \frac{\overline{G}}{\overline{Y}}\hat{g}_t + \alpha\hat{\mu}_{K,t} \tag{E.53}$$

$$\text{Market clearing trad. } B \qquad \hat{y}^*_{Tt} = (1-\alpha_I)\hat{c}^*_{Tt} + \alpha_I \frac{\Theta}{1-\Theta}\hat{c}_{Tt} - 2\mu\alpha_I(1-\alpha_I)\hat{s}_t + \frac{\overline{I}}{\overline{Y}}\widehat{inv}^*_t + \frac{\overline{G}}{\overline{Y}}\hat{g}^*_t + \alpha\hat{\mu}^*_{K,t} \tag{E.54}$$

$$\text{Current account} \qquad \hat{y}_{Tt} - \hat{c}_{Tt} - \alpha_I\hat{s}_t - \frac{\overline{I}}{\overline{Y}}\widehat{inv}_t - \frac{\overline{G}}{\overline{Y}}\hat{g}_t - \alpha\hat{\mu}_{K,t} = \widehat{ca}_t \tag{E.55}$$

$$\text{Market clearing non-trad. } A \qquad \hat{y}_{Nt} = \hat{c}_{Nt} \tag{E.56}$$

$$\text{Market clearing non-trad. } B \qquad \hat{y}^*_{Nt} = \hat{c}^*_{Nt} \tag{E.57}$$

$$\text{Labor market clearing } A \qquad \hat{l}_t = (1-\alpha_N)\hat{l}_{Tt} + \alpha_N\hat{l}_{Nt} \tag{E.58}$$

$$\text{Labor market clearing } B \qquad \hat{l}^*_t = (1-\alpha_N)\hat{l}^*_{Tt} + \alpha_N\hat{l}^*_{Nt} \tag{E.59}$$

$$\text{Dynamics of NFA} \qquad \widehat{ca}_t = \overline{b}_B\hat{b}_{Bt} - \overline{b}^*_A\hat{b}^*_{At} - \frac{1}{\beta}\left(\overline{b}_B\hat{b}_{B,t-1} - \overline{b}^*_A\hat{b}^*_{A,t-1}\right) \tag{E.60}$$

$$\text{Bonds market clearing A} \qquad \overline{s}_b\hat{s}_{bt} = \overline{b}_A\hat{b}_{At} + \overline{b}^*_A\hat{b}^*_{At} \tag{E.61}$$

$$\text{Bonds market clearing B} \qquad \overline{s}^*_b\hat{s}^*_{bt} = \overline{b}_B\hat{b}_{Bt} + \overline{b}^*_B\hat{b}^*_{Bt} \tag{E.62}$$

Definitions

Agg. output A	$\hat{y}_t = (1-\alpha_N)\hat{y}_{Tt} + \alpha_N \hat{y}_{Nt}$	(E.63)
Agg. output B	$\hat{y}^*_t = (1-\alpha_N)\hat{y}^*_{Tt} + \alpha_N \hat{y}^*_{Nt}$	(E.64)
Terms of trade	$\alpha_I (\hat{s}_t - \hat{s}_{t-1}) = \hat{\pi}_{Tt} - \hat{\pi}_{At}$	(E.65)
Agg. consumption A	$\hat{c}_t = (1-\alpha_N)\hat{c}_{Tt} + \alpha_N \hat{c}_{Nt}$	(E.66)
Agg. consumption B	$\hat{c}^*_t = (1-\alpha_N)\hat{c}^*_{Tt} + \alpha_N \hat{c}^*_{Nt}$	(E.67)
Tradable inflation A	$\hat{\pi}_{Tt} = (1-\alpha_I)\hat{\pi}_{At} + \alpha_I \hat{\pi}_{Bt}$	(E.68)
Tradable inflation B	$\hat{\pi}^*_{Tt} = \alpha_I \hat{\pi}_{At} + (1-\alpha_I)\hat{\pi}_{Bt}$	(E.69)
CPI inflation A	$\hat{\pi}_t = (1-\alpha_N)\hat{\pi}_{Tt} + \alpha_N \hat{\pi}_{Nt}$	(E.70)
CPI inflation B	$\hat{\pi}^*_t = (1-\alpha_N)\hat{\pi}^*_{Tt} + \alpha_N \hat{\pi}^*_{Nt}$	(E.71)
Real interest rate A	$\hat{r}^{real}_t = \hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}$	(E.72)
Real interest rate B	$\hat{r}^{*,real}_t = \hat{i}_t - \mathbb{E}_t \hat{\pi}^*_{t+1}$	(E.73)

Structural shock AR(1) processes

Technology trad. A	$\hat{a}_{Tt} = \rho_a \hat{a}_{T,t-1} + \varepsilon^T_t$	(E.74)
Technology trad. B	$\hat{a}^*_{Tt} = \rho_a \hat{a}^*_{T,t-1} + \varepsilon^{T*}_t$	(E.75)
Technology non-trad. A	$\hat{a}_{Nt} = \rho_a \hat{a}_{N,t-1} + \varepsilon^N_t$	(E.76)
Technology non-trad. B	$\hat{a}^*_{Nt} = \rho_a \hat{a}^*_{N,t-1} + \varepsilon^{N*}_t$	(E.77)
Risk premium shock A	$\xi^{rp}_t = \rho_{rp} \xi^{rp}_{t-1} + \varepsilon^{rp}_t$	(E.78)
Risk premium shock B	$\xi^{rp*}_t = \rho_{rp} \xi^{rp*}_{t-1} + \varepsilon^{rp*}_t$	(E.79)
Preference shock A	$\xi^d_t = \rho_d \xi^d_{t-1} + \varepsilon^d_t$	(E.80)
Preference shock B	$\xi^{d*}_t = \rho_d \xi^{d*}_{t-1} + \varepsilon^{d*}_t$	(E.81)
Tradable goods markup shock A	$u^T_t = \rho_T u^T_{t-1} + \varepsilon^{uT}_t$	(E.82)
Tradable goods markup shock B	$u^{T*}_t = \rho_T u^{T*}_{t-1} + \varepsilon^{uT*}_t$	(E.83)
Non-tradable goods markup shock A	$u^N_t = \rho_N u^N_{t-1} + \varepsilon^{uN}_t$	(E.84)
Non-tradable goods markup shock B	$u^{N*}_t = \rho_N u^{N*}_{t-1} + \varepsilon^{uN*}_t$	(E.85)
Wage markup shock A	$u^w_t = \rho_w u^w_{t-1} + \varepsilon^w_t$	(E.86)
Wage markup shock B	$u^{w*}_t = \rho_w u^{w*}_{t-1} + \varepsilon^{w*}_t$	(E.87)
MEI shock A	$\xi^k_t = \rho_k \xi^k_{t-1} + \varepsilon^k_t$	(E.88)
MEI shock B	$\xi^{k*}_t = \rho_k \xi^{k*}_{t-1} + \varepsilon^{k*}_t$	(E.89)
Monetary policy shock	$\xi^m_t = \rho_m \xi^m_{t-1} + \varepsilon^m_t$	(E.90)

COVID labor supply A	$\varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2 \mathbb{1}_{t \in \text{COVID}})$	(E.91)
COVID labor supply B	$\varepsilon_t^{h*} \sim \mathcal{N}(0, \sigma_{h*}^2 \mathbb{1}_{t \in \text{COVID}})$	(E.92)

Fiscal policy shock AR(1) processes

Primary surplus A	$\zeta_t = \rho_\zeta \zeta_{t-1} - \varepsilon_t^\zeta$	(E.93)
Primary surplus B	$\zeta_t^* = \rho_\zeta^* \zeta_{t-1}^* - \varepsilon_t^{\zeta*}$	(E.94)
Transitory transfer A	$\zeta_{z,t} = \rho_z \zeta_{z,t-1} - \varepsilon_t^z$	(E.95)
Transitory transfer B	$\zeta_{z,t}^* = \rho_z \zeta_{z,t-1}^* - \varepsilon_t^{z*}$	(E.96)
Government spending A	$\hat{g}_t = \rho_g \hat{g}_{t-1} - (1-\rho_g)\gamma_G \hat{s}_{b,t-1} + \varepsilon_t^g$	(E.97)
Government spending B	$\hat{g}_t^* = \rho_g \hat{g}_{t-1}^* - (1-\rho_g)\gamma_G^* \hat{s}_{b,t-1}^* + \varepsilon_t^{g*}$	(E.98)

Steady state. We characterize the steady state around which we log-linearize. We impose zero steady-state inflation ($\overline{\Pi} = 1$) and normalize the steady-state price level as $\overline{P} = 1$. The steady-state gross nominal interest rate is $\overline{R} = \beta^{-1}$.

Long-term bond price. With geometrically decaying coupons with rate ρ^L, the steady-state long-term bond price satisfies

$$\overline{P}^L = \beta\left(1 + \rho^L \overline{P}^L\right) \implies \overline{P}^L = \frac{\beta}{1-\beta\rho^L}. \tag{E.99}$$

Defining the real market value of government debt as $\overline{s}_b \equiv \frac{\overline{P}^L \overline{B}}{\overline{P}\,\overline{Y}}$, the steady-state government budget constraint implies

$$\overline{T} = \frac{1-\beta}{\beta}\overline{s}_b \iff \overline{s}_b = \frac{\beta}{1-\beta}\overline{T}. \tag{E.100}$$

Capital and investment shares. In steady state Tobin's Q is one, and the capital Euler equation implies

$$1 = \beta\left(\overline{R}_K + 1 - \delta\right) \implies \overline{R}_K = \frac{1}{\beta} - (1-\delta). \tag{E.101}$$

Using the tradable-sector capital demand condition $\overline{R}_K = \alpha \frac{\overline{Y}_A}{\overline{K}}$, we obtain the steady-state capital-output ratio in the tradable sector,

$$\frac{\overline{K}}{\overline{Y}_A} = \frac{\alpha}{\overline{R}_K}. \tag{E.102}$$

Finally, the capital law of motion implies $\overline{I} = \delta\overline{K}$, so the steady-state investment share (used in the log-linear tradable market clearing and current account equations) is

$$\frac{\overline{I}}{\overline{Y}_A} = \delta\frac{\overline{K}}{\overline{Y}_A} = \delta\frac{\alpha}{\frac{1}{\beta} - (1 - \delta)}. \tag{E.103}$$

Government spending share. The steady-state government spending-to-output ratio $\frac{\overline{G}}{\overline{Y}}$ is a calibrated parameter, set to match the average share of government final consumption expenditure in GDP. This ratio appears in the log-linearized tradable market clearing and current account equations.

F Two-country model with flexible nominal exchange rates

To consider the role of the common central bank in a currency union, we also build an equivalent two-country model with flexible nominal exchange rates. We define the nominal exchange rate $\mathcal{E}_t$ as the price of the country B currency in terms of the currency in country A and incur the following changes to the currency union model.

We change the nominal budget constraint of the households in country A to:

$$P_t C_t + B_{At} + \mathcal{E}_t B_{Bt} + P_t T_t$$
$$= P_t Y + R_{t-1} B_{A,t-1} + \mathcal{E}_t R^*_{t-1} B_{B,t-1} - \frac{\psi_b P_t \left(\overline{b}_A + \overline{b}_B\right)}{2} \left(\delta_b \frac{\mathcal{E}_t B_{Bt}}{B_{At}} - 1\right)^2 \quad \text{(F.1)}$$

Then, the two Euler equations become:

$$1 = \frac{\beta R_t}{1 - \psi_b(\overline{b}_A + \overline{b}_B)\left(\delta_b \frac{\mathcal{E}_t b_{Bt}}{b_{At}} - 1\right) \delta_b \frac{\mathcal{E}_t b_{Bt}}{b^2_{At}}} \mathbb{E}_t \left(\Pi^{-1}_{t+1} \frac{C_t}{C_{t+1}}\right) \quad \text{(F.2)}$$

$$1 = \frac{\beta R^*_t}{1 + \psi_b \left(\overline{b}_A + \overline{b}_B\right) \left(\delta_b \frac{\mathcal{E}_t b_{Bt}}{b_{At}} - 1\right) \delta_b \frac{1}{b_{At}}} \mathbb{E}_t \left(\Pi^{-1}_{t+1} \frac{C_t}{C_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) \quad \text{(F.3)}$$

where the first Euler equation, with respect to b_{At}, remains unchanged.

The definition of the terms of trade remains $S_t = \frac{P_{Bt}}{P_{At}}$ but the interpretation changes to the relative price of the good produced in country B to the good produced in country A, expressed in the country A currency. Assuming the law of one price holds, the goods prices in the country B currency are the following:

$$P_{At} = P^*_{At} \mathcal{E}_t \quad \text{(F.4)}$$

$$P_{Bt} = P^*_{Bt} \mathcal{E}_t \quad \text{(F.5)}$$

We define the real exchange rate as:

$$RER_t \equiv \frac{\mathcal{E}_t P^*_t}{P_t} \quad \text{(F.6)}$$

There are now two monetary authorities, one for each country, targeting the domestic gross inflation:

$$\frac{R_t}{\overline{R}} = \left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi} \quad \text{(F.7)}$$

$$\frac{R^*_t}{\overline{R}^*} = \left(\frac{\Pi^*_t}{\overline{\Pi}^*}\right)^{\phi^*} \quad \text{(F.8)}$$

Finally, we modify the current account equation as:

$$CA_t \equiv P_{At} Y - P_t C_t = \mathcal{E}_t B_{Bt} - B^*_{At} - \left(R^*_{t-1} \mathcal{E}_t B_{B,t-1} - R_{t-1} B^*_{A,t-1}\right) \quad \text{(F.9)}$$

With the updated Euler equations, the aggregate Euler equations and the no-arbitrage equations in the log-linear system become:

$$\hat{c}_t^{MU} = \mathbb{E}_t \hat{c}_{t+1}^{MU} - \left[\Theta \hat{i}_t + (1-\Theta)\hat{i}_t^* - \mathbb{E}_t \hat{\pi}_{t+1}^{MU}\right]$$
$$- \Theta \psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} + \hat{e}_t - \hat{b}_{At}\right) + (1-\Theta)\psi_b \frac{\overline{b}_A^* + \overline{b}_B^*}{\overline{b}_A^*}\left(\hat{b}_{Bt}^* - \hat{b}_{At}^* - \hat{e}_t\right) \quad \text{(F.10)}$$

$$-\psi_b \frac{\bar{b}_A + \bar{b}_B}{\bar{b}_A}\left(\hat{b}_{Bt} + \hat{e}_t - \hat{b}_{At}\right) - \hat{i}_t = \psi_b \frac{\bar{b}_A + \bar{b}_B}{\bar{b}_B}\left(\hat{b}_{Bt} + \hat{e}_t - \hat{b}_{At}\right) - \hat{i}_t^* - \Delta \hat{e}_{t+1} \quad \text{(F.11)}$$

$$-\psi_b \frac{\bar{b}_A^* + \bar{b}_B^*}{\bar{b}_A^*}\left(\hat{b}_{Bt}^* - \hat{b}_{At}^* - \hat{e}_t\right) - \hat{i}_t + \Delta \hat{e}_{t+1} = \psi_b \frac{\bar{b}_A^* + \bar{b}_B^*}{\bar{b}_B^*}\left(\hat{b}_{Bt}^* - \hat{b}_{At}^* - \hat{e}_t\right) - \hat{i}_t^* \quad \text{(F.12)}$$

For the aggregate Euler equation, we use the Euler equation in country A with respect to B_{At} and the Euler equation in country B with respect to B_{Bt}^* to avoid adding the nominal depreciation $\Delta\hat{e}_t = \hat{e}_t - \hat{e}_{t-1}$ in the Euler equation. The no-arbitrage equations are the uncovered interest parity (UIP) conditions.

The log-linear monetary rules for each country are:

$$\hat{i}_t = \phi \hat{\pi}_t \quad \text{(F.13)}$$

$$\hat{i}_t^* = \phi^* \hat{\pi}_t^* \quad \text{(F.14)}$$

Finally, the current account equation becomes:

$$-\hat{c}_t - \alpha_I \hat{s}_t = \overline{b}_B\left(\hat{b}_{Bt} + \hat{e}_t\right) - \overline{b}_A^* b_{At}^* - \frac{1}{\beta}\left[\overline{b}_B\left(\hat{b}_{B,t-1} + \hat{r}_{t-1}^* + \hat{e}_{t-1}\right) - \overline{b}_A^*\left(b_{A,t-1}^* + \hat{r}_{t-1}\right)\right] \quad \text{(F.15)}$$

G Model with CES aggregator for bonds

In this section, we explore the implications of a different structure regarding bond holdings. Households aggregate bonds with a Constant Elasticity of Substitution (CES) function, which generates endogenous sovereign spreads. We discuss the equations that this alternative assumption affect.

Stylized model. In the stylized model, the household budget constraint with a CES aggregator for bond holdings is:

$$P_t C_t + \mathcal{B}_t + P_t T_t = P_t Y + R_{t-1}^{CES} \mathcal{B}_{t-1}, \tag{G.1}$$

where $\mathcal{B}_t$ is the bundle of bond holdings, defined as a CES aggregate of nominal holdings in country A and B bonds:

$$\mathcal{B}_t = \left[\alpha_b^{1/\mu_b} B_{At}^{(\mu_b-1)/\mu_b} + (1-\alpha_b)^{1/\mu_b} B_{Bt}^{(\mu_b-1)/\mu_b} \right]^{\mu_b/(\mu_b-1)}, \tag{G.2}$$

where $\alpha_b \in (0,1)$ is the portfolio weight on country A bonds and $\mu_b > 0$ is the elasticity of substitution between the two bonds. From the cost-minimization problem, taking bond discount prices $1/R_{At}$ and $1/R_{Bt}$ as given, we derive the price index of the bond bundle:

$$\frac{1}{R_t^{CES}} = \left[\alpha_b \left(\frac{1}{R_{At}} \right)^{1-\mu_b} + (1-\alpha_b) \left(\frac{1}{R_{Bt}} \right)^{1-\mu_b} \right]^{\frac{1}{1-\mu_b}}, \tag{G.3}$$

where R_{At} and R_{Bt} are the gross returns on bonds issued by countries A and B respectively. Analogous to the goods market, the demand for each bond is:

$$B_{At} = \alpha_b \left(\frac{1/R_{At}}{1/R_t^{CES}} \right)^{-\mu_b} \mathcal{B}_t, \qquad B_{Bt} = (1-\alpha_b) \left(\frac{1/R_{Bt}}{1/R_t^{CES}} \right)^{-\mu_b} \mathcal{B}_t. \tag{G.4}$$

Households maximize their utility subject to the new budget constraint. The Euler equation is:

$$1 = \beta R_t^{CES} \mathbb{E}_t \left(\Pi_{t+1}^{-1} \frac{C_t}{C_{t+1}} \right) \tag{G.5}$$

We set $\alpha_b = 0.44$, driven by bond holdings data as in the baseline model, and $\mu_b = 0.1$ to generate similar magnitudes to the baseline model. Aggregating the Euler equation for country A and country B gives the same log-linear equation for the monetary block of the economy as in the baseline model, equation (19), such that the conditions for existence and determinacy remains the same as in the baseline stylized model. Moreover, the impulse responses to funded and unfunded fiscal expansions are quantitatively and qualitatively similar to the baseline, as shown in Figure A.15a and A.15b.

Quantitative model. In the quantitative model, the household budget constraint with a CES aggregator for long-term bond holdings is:

$$
\begin{aligned}
&P_t(1+\tau_{Ct})C_t + Q_t K_t + P_t^{L,CES}\mathcal{B}_t^L + B_t \\
&\quad = W_t(1-\tau_{Lt})L_t + P_t Z_t + \left[R_{Kt}(1-\tau_{Kt})\mu_{Kt} - a(\mu_{Kt}) + Q_t(1-\delta)\right] K_{t-1} + R_{t-1}B_{t-1} \\
&\qquad + \left(1+\rho^L P_t^L\right) B_{A,t-1} + \left(1+\rho^L P_t^{L*}\right) B_{B,t-1}, \quad \text{(G.6)}
\end{aligned}
$$

where $\mathcal{B}_t^L$ is the bundle of long-term bond holdings, defined as a CES aggregate of nominal holdings of long-term bonds B_{At} and B_{Bt}:

$$
\mathcal{B}_t^L = \left[\alpha_b^{1/\mu_b} B_{At}^{(\mu_b-1)/\mu_b} + (1-\alpha_b)^{1/\mu_b} B_{Bt}^{(\mu_b-1)/\mu_b}\right]^{\mu_b/(\mu_b-1)}, \tag{G.7}
$$

and the price index of $\mathcal{B}_t^L$ following the cost-minimization problem is:

$$
P_t^{L,CES} = \left[\alpha_b \left(P_t^L\right)^{1-\mu_b} + (1-\alpha_b)\left(P_t^{L*}\right)^{1-\mu_b}\right]^{\frac{1}{1-\mu_b}}. \tag{G.8}
$$

Analogous to the goods market, the demand for each bond is:

$$
B_{At} = \alpha_b \left(\frac{P_t^L}{P_t^{L,CES}}\right)^{-\mu_b} \mathcal{B}_t, \qquad B_{Bt} = (1-\alpha_b)\left(\frac{P_t^{L*}}{P_t^{L,CES}}\right)^{-\mu_b} \mathcal{B}_t. \tag{G.9}
$$

Households maxmimize their utility using three assets: capital, short-term bonds, and a bundle of long-term bonds. The first order conditions to those assets, respectively, are:[10]

$$
Q_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[R_{K,t+1}(1-\tau_{K,t+1})\mu_{K,t+1} - a(\mu_{K,t+1}) + (1-\delta)Q_{t+1}\right], \tag{G.11}
$$

$$
1 = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} R_t, \tag{G.12}
$$

$$
1 = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{P_t^{L,CES}}\left[\left(1+\rho^L P_{t+1}^L\right)\alpha_b\left(\frac{P_t^L}{P_t^{L,CES}}\right)^{-\mu_b} + \left(1+\rho^L P_{t+1}^{L*}\right)(1-\alpha_b)\left(\frac{P_t^{L*}}{P_t^{L,CES}}\right)^{-\mu_b}\right]. \tag{G.13}
$$

We loglinearize the first order condition with respect to the bundle of long-term bonds as:

$$
\hat{\lambda}_t = \mathbb{E}_t\hat{\lambda}_{t+1} + \rho^L \mathbb{E}_t \hat{p}_{t+1}^{L,CES} - \frac{1}{\overline{P}^L}\left(1+\rho^L\overline{P}^L\right)\hat{p}_t^{L,CES}, \tag{G.14}
$$

using the definition of the price index of the bond bundle. The impulse responses are almost identical to the baseline model with quadratic bond adjustment costs, as shown

10. For the long-term bond bundle, the Lagrangian first order condition is

$$
\lambda_t P_t^{L,CES} = \beta \mathbb{E}_t\left[\frac{\partial}{\partial \mathcal{B}_t^L}\left(\left(1+\rho^L P_{t+1}^L\right)B_{At} + \left(1+\rho^L P_{t+1}^{L*}\right)B_{Bt}\right)\right]. \tag{G.10}
$$

We obtain $\frac{\partial B_{At}}{\partial \mathcal{B}_t}$ and $\frac{\partial B_{Bt}}{\partial \mathcal{B}_t}$ from the demand equations.

in Figure A.18a and A.18b.