Partially Endogenous State Space Expansion Model of Growing Awareness

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Abstract

We propose a non-probabilistic model of state space expansion by integrating unawareness-structure and growing-awareness approaches. Subjective state spaces are interpreted as framework-based projections of an objective state. Unlike models assuming "Reverse Bayesianism"—the preservation of likelihood ratios—our framework captures the qualitative rejection of prior beliefs triggered by the recognition of new logical constraints. We endogenously formalize the expansion process as a response to cognitive dissonance arising from the divergence between predicted and realized outcomes. Under finiteness, this expansion eventually ceases. Our model explains institutional rigidity and non-linear transitions, providing a behavioral foundation for innovation as a backward-looking resolution.

Keywords: Unawareness, Growing Awareness, Cognitive Dissonance, State-Space Expansion, Framework Revision.

JEL Classification: D80, D81, D83.

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1. Introduction

Since Milgrom and Stokey's (1982) no-trade theorem, a large body of research has questioned the assumption of common knowledge in economic decision-making. One important branch of this literature concerns unawareness. Following Schipper (2014), unawareness is understood here as the absence of concepts rather than merely the absence of information. For example, cholera bacillus existed long before Robert Koch identified it in 1883, yet prior to this discovery the concept itself was unavailable. In this sense, nineteenth-century agents could not meaningfully describe states involving bacterial contamination because the relevant conceptual framework did not yet exist.

Early studies of unawareness in economics developed along two major lines: the modal logic approach of Fagin and Halpern (1988) and the non-partitional state-space approach initiated by Geanakoplos (2021, originally circulated as a 1989 working paper). Subsequently, Modica and Rustichini (1994) and Dekel, Lipman, and Rustichini (1998) showed that standard state-space models cannot provide a meaningful representation of unawareness despite allowing non-partitional information structures. In response to this difficulty, Heifetz, Meier, and Schipper (2006) introduced a lattice structure over state spaces in order to represent different levels of expressiveness in the description of the world. Their framework made it possible to formalize awareness differences arising from conceptual limitations. Building on this approach, Heifetz, Meier, and Schipper (2013a), Galanis (2018), and others studied economic environments under unawareness, including no-trade results and game-theoretic situations.

At the same time, another strand of the literature examined how agents revise beliefs when unawareness is resolved. Karni and Vierø (2013, 2017) proposed a growing-awareness framework in which awareness expansion changes the underlying state space, action set, or outcome set. Their approach relies on "Reverse

Bayesianism," according to which the relative likelihood ratios assigned to previously conceivable events are preserved after awareness expands. Vierø (2021) later extended this framework dynamically through event-tree analysis. Although these studies substantially advanced the analysis of growing awareness, they retain two important limitations. First, awareness expansion is treated exogenously. Second, the framework focuses primarily on probabilistic consistency rather than on the qualitative rejection of prior interpretive structures.

These limitations become particularly important when new awareness falsifies an existing framework itself. In the reverse-Bayesian framework of Karni and Vierø (2013), awareness expansion preserves the relative likelihood ratios assigned to previously conceivable events. However, when a newly discovered relationship reveals that some previously conceivable states are fundamentally impossible, such preservation may no longer be coherent. In such cases, certain states must instead be assigned probability zero. The issue is therefore not merely how probabilities should be redistributed after awareness expands, but whether the agent's prior interpretive framework itself must be rejected. As Galanis (2013) emphasizes in the context of "unawareness of theorems," the discovery of a previously unnoticed logical implication may invalidate earlier beliefs rather than merely refine them probabilistically. For example, if an agent newly realizes that the presence of an intruder necessarily implies a dog's barking, then states previously regarded as feasible may become impossible and must receive probability zero. In such situations, preserving prior likelihood ratios is no longer possible. More fundamentally, the issue is not merely probabilistic updating but framework-dependent falsification. Existing growing-awareness models do not formally capture how discrepancies between expected and realized outcomes can trigger the rejection of an old interpretive framework and induce the emergence of a new one.

Motivated by these observations, this paper develops a non-probabilistic model of generalized growing awareness that integrates the unawareness-structure framework of Heifetz, Meier, and Schipper (2006) with the growing-awareness approach of Karni and Vierø (2013). In the proposed framework, subjective state spaces are organized through a lattice structure and are interpreted as projections of a richer ideal environment. The central idea is that an agent's subjective state space and feasible action set are determined by socio-technological frameworks, namely interpretive and practical structures that govern how the agent understands the world and which actions are regarded as feasible. These frameworks include not only physical technologies but also norms, institutions, and value systems.

Within this framework, agents form expectations based on projected subjective states rather than directly observing the ideal environment. Consequently, predicted outcomes may differ from realized outcomes. We interpret such discrepancies as cognitive dissonance. For example, prior to the acceptance of germ theory, cholera was widely believed to spread through air. If this interpretation were correct, workers in heavily infected areas should also have contracted cholera. However, John Snow's (1855) observations contradicted this prediction. Such inconsistencies between expected and realized outcomes may induce dissatisfaction with the existing framework and motivate the adoption of a new one.

Based on this idea, the paper develops a partially endogenous model of state-space expansion. When cognitive dissonance arises, agents may revise their socio-technological framework, thereby expanding the subjective state space or feasible action set. Unlike the smooth probabilistic revisions emphasized in the existing literature, framework revision in our model is reactive and non-linear. Awareness expansion occurs not through forward-looking optimization over fully specified future contingencies, but through attempts to resolve accumulated dissonance generated by observed mismatches between predictions and outcomes. We show that, under finiteness assumptions, framework revision eventually ceases once cognitive dissonance arising along the realized history has been resolved within a finite number of steps.

This paper makes three main contributions. First, it integrates the unawareness-structure approach of Heifetz, Meier, and Schipper (2006) with the growing-awareness framework of Karni and Vierø (2013) within a unified lattice-based structure. Second, it develops a partially endogenous model of awareness expansion in which cognitive dissonance drives framework revision and the enlargement of subjective state spaces and feasible actions. Third, it establishes a stabilization result showing that, under finiteness assumptions, cognitive dissonance arising along the realized history is eventually resolved and no further framework-induced expansion occurs after a finite stage.

The remainder of the paper is organized as follows. Section 2 introduces the generalized growing-awareness model as a static framework and presents its main theorem. Section 3 develops the partially endogenous state-space expansion model and proves the eventual stabilization of framework change. Section 4 discusses the conceptual implications of the model, including framework rigidity and socio-technological transitions. Section 5 concludes.

2. Static Model: Generalized Growing Awareness

We define a generalized growing-awareness model by enriching the growing-awareness framework of Karni and Vierø (2013) with the lattice-theoretic structure of Heifetz, Meier, and Schipper (2006).

Definition 1 $\mathcal{M} = \langle \Omega, \bar{A}, O, \mathcal{S}, H, B \rangle$ is a generalized growing-awareness model, where

Ω is the finite ideal state space,

$\bar{A}$ is the set of ideal actions,

O is the set of outcomes,

$\mathcal{S} = \{S_\lambda\}_{\lambda \in \Lambda}$ is the family of state spaces (or signal spaces), where Λ is the finite index set,

H is the set of socio-technological frameworks, and

B is a belief function.

The first three components—the ideal state space Ω, ideal action set $\bar{A}$, and outcome set O —are commonly found in decision-making models under uncertainty; however, the interpretations might differ. $\mathcal{S}$ is defined as an unaware structure. In this paper, the interpretation of each state space $S \in \mathcal{S}$ is distinct from the interpretation of the ideal state space. The set of socio-technological frameworks H and belief function B proposed in our model are designed to integrate unawareness structures and growing awareness. Explanations for each symbol are provided in the following subsections.

2.1. Ideal State Space

The ideal state space Ω is the objective state space that comprises the finite set of complete statements about the world. Each state $\omega \in \Omega$ describes everything, ranging from matters directly relevant to those completely unrelated to the agent's decision-making.

For instance, consider a physician, Alice, residing in the United States. In her clinical practice, the relevant states of the world concern the potential symptoms of the patient before her, such as whether the patient has a cold, is infected with a contagious disease, or what their dietary habits are. Naturally, these are encapsulated within ω. However, ω is not limited to such clinical data; it also encompasses descriptions extraneous to the medical examination, such as the current state of the U.S. economy or weather conditions in foreign countries.

Furthermore, it includes correct proofs of unsolved mathematical problems, the origin of the Universe, and the existence of extraterrestrial life or the multiverse.

The ideal state space is not restricted to what agents can observe or verify. Rather, it is introduced as a theoretical benchmark representing a complete specification of the world, independent of the agent's current awareness, concepts, or available frameworks.

2.2. Ideal Action Set

The ideal action set $\bar{A}$ differs from the standard action set used in the decision model. Within this framework, certain actions are available depending on the era, whereas others are not. For instance, in the pre-common era, it was impossible for humans to fly; thus, "flying" was infeasible. In the modern era, however, the availability of aircraft makes "flying" a feasible action. In this sense, the set includes actions that might be currently unattainable but could potentially become available in the future. Furthermore, the ideal action set includes not only actions currently infeasible but potentially realizable through future technological development, such as interstellar travel, but also actions that may remain permanently infeasible under the current scientific framework.

We define $\mathcal{A} = 2^{\bar{A}} \setminus \{\emptyset\}$.

2.3. Outcome Set

For the finite outcome set O, we denote $o: \Omega \times \bar{A} \to O$ to be a true outcome function, where $o = o(\omega, a)$ indicates that $o \in O$ is induced from the true state $\omega \in \Omega$ and an agent's decision $a \in \bar{A}$. For example, if the true state is that well water is contaminated with cholera bacteria and villagers drink the water, then the resulting outcome is that they contract cholera.

2.4. Family of State Spaces

Because we cannot perceive the ideal state space directly, we observe only a coarse representation, for example, "whilst I don't have a detailed understanding of the Road Traffic Act, I do at least know that you must stop at a red light." To represent this situation, we use the unawareness structures proposed by Heifetz, Meier, and Schipper (2006), in which each state among the agents' subjective state spaces has different statements regarding whether they are unaware of an attribute. The family of a state space is defined by $\mathcal{S} = \{S_\lambda\}_{\lambda \in \Lambda}$, where Λ is the finite index set. The set $(\mathcal{S}, \succcurlyeq)$ is a complete lattice of disjoint state spaces (or signal spaces) with partial order $\succcurlyeq$. Given two state spaces $S, S' \in \mathcal{S}$, we define $S \vee S'$ to be join and $S \wedge S'$ to be meet. Because $(\mathcal{S}, \succcurlyeq)$ is a complete lattice and Λ is finite, there exist a maximum element $\bigvee_{S \in \mathcal{S}} S \in \mathcal{S}$ and a minimum element $\bigwedge_{S \in \mathcal{S}} S \in \mathcal{S}$. $S \vee S' = S$ and $S \wedge S' = S'$ if and only if $S \succcurlyeq S'$, which implies that S is at least as expressive/fine as S'. Therefore, there exists a surjective projection $r_{S'}^{S}: S \to S'$, where for any $s \in S$, there exists $s' \in S'$ such that $r_{S'}^{S}(s) = s'$. For example, our knowledge of cholera is more precise than that of the people in 19th century. If $S \succcurlyeq S' \succcurlyeq S''$, then $r_{S''}^{S} = r_{S''}^{S'} \circ r_{S'}^{S}$. Finally, let $\Sigma = \bigcup_{\lambda \in \Lambda} S_\lambda$.

2.5. Socio-Technological Framework Set

The state space perceived by an agent and the set of available actions are not arbitrarily determined; instead, they are conditioned by the technological and socio-institutional foundations prevalent in a given era. For example, the invention of the microscope was necessary to recognize the existence of pathogens, just as the development of aviation technology was required to include flight in the reachable action set. Beyond these physical technologies, the framework also encompasses "soft" structures, such as legal systems, social norms, and ethical values, which define the boundaries of admissible actions and outcomes. In this study, we formalize these combined underpinnings as "interpretive and practical frameworks," representing the technical and structural contexts that project the objective environment into the agent's subjective representation.[1]

Within H, the set of frameworks, each element $h \in H$ contains not only scientific technologies such as aircraft engines, shipbuilding technology, and microscopes but also social institutions such as law, government, and education, and values such as morality, ideology, and social norms. H is countable and finite.

A partially ordered set $(H, \geq)$ is a complete lattice, with a maximum and minimum element. For any two frameworks $h, h' \in H$, $h \vee h' = h$ and $h \wedge h' = h'$ if and only if $h \geq h'$. We can then interpret h as equivalent to or even more advanced than h'. If there exist three different frameworks $h, h', h'' \in H$ such that $h = h' \vee h''$, we interpret that h is a framework that combines h' and h''.

A framework $h \in H$ defines a state space that an agent is aware of and a feasible action set. We define $S: H \to \mathcal{S}$ as a state space function where, given h, $S(h) = S$, which is interpreted as an agent being aware of her subjective state space S under the framework h. We denote $S(h) = S_h$ for any $h \in H$. Consider a state function $s: \Omega \times H \to \Sigma$ where, for a given ω and h, $s(\omega, h) = s$; this implies that an agent observes a state or signal s under h at ω. We denote $s(\omega, h) = s_h(\omega)$, and we define $A: H \to \mathcal{A}$ as an action set function, where $A(h) = A$ means that an agent has a feasible action set A under the technology h. We further denote $A(h) = A_h$.

Based on the above, this paper supposes the following assumptions:

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[1] While the primary focus of this paper is on technological and socio-institutional frameworks, the mathematical structure of h is sufficiently general to encompass broader cognitive frameworks, including imaginative or speculative paradigms that may influence agents' subjective representations.

Assumption 1 For any $S \in \mathcal{S}$, there exists $h \in H$ such that $S = S(h)$.

Assumption 2 For any $S, S' \in \mathcal{S}$ and $h, h' \in H$, suppose $S = S(h)$ and $S' = S(h')$. Then, $S \succcurlyeq S'$ if and only if $h \geq h'$.

Assumption 3 $h \geq h'$ implies $A_h \supseteq A_{h'}$.

Assumption 1 implies that each state in the family of state spaces is associated with a specific framework $h \in H$. Assumption 2 implies that the partial order on the family of state spaces is linked to the partial order on the set of frameworks. Assumption 3 implies that, if a framework is incremental, the set of technologically feasible actions expands.

Remark 1 Suppose H is finite. Then, there exists a maximal element $\bigvee_{h \in H} h$ such that $\bigvee_{S \in \mathcal{S}} S = S(\bigvee_{h \in H} h)$ and there exists a minimal element $\bigwedge_{h \in H} h$ such that $\bigwedge_{S \in \mathcal{S}} S = S(\bigwedge_{h \in H} h)$. If $(H, \geq)$ is a complete lattice, then $\bigvee_{h \in H} h$ is the maximum element and $\bigwedge_{h \in H} h$ is the minimum element.

This study does not assume the completeness of $(H, \geq)$ because we wish to allow for the coexistence of conflicting technologies and value systems as well as the presence of imaginary or fictional technologies. For example, the technologies of wooden and concrete construction are fundamentally different, yet each has developed along its own trajectory and they coexist. Similarly, value systems that differ across countries or regions do not need to be ranked in terms of superiority or inferiority, even if they are mutually incompatible. Moreover, some conceivable technologies may remain unrealizable under existing scientific constraints, yet agents may still reason about them hypothetically or imaginatively.

History shows that technologies and value systems with distinct underlying foundations can be combined to generate new forms. For instance, sailing technology and steam engines originated from different technological lineages, yet their combination gave rise to steamships. Similarly, the fusion of Indian Buddhism and Roman sculptural techniques produced Gandharan art, while alchemical traditions contributed to the development of modern chemistry. Thus, not all technologies and value systems lie on a single continuum, nor do they necessarily converge to a single ultimate form. Accordingly, refraining from assuming the completeness of $(H, \geq)$ amounts to recognizing the possibility of diverse paths of technological progress and the coexistence of plural social values.

Furthermore, the state function $s: \Omega \times H \to \Sigma$ in this paper implicitly distinguishes between the "state" itself and the agent's "observation" or "experience" of the true state. For example, whether an agent can recognize that abdominal pain is caused by cholera bacillus depends not only on whether the agent possesses the concept of cholera bacillus but also on whether the agent has the technology required to observe its presence. Although this distinction might initially appear to be of limited significance, it is important from the perspective that the state space is reconstructed and re-identified through observation and experience. For instance, 19th-century British physician John Snow was unaware of the existence of cholera bacillus. Nevertheless, as documented by Snow (1855), he identified contaminated well water as a cause of cholera. This is a paradigmatic example of a state-space constructed through observation and experience.

The present framework also captures the practical attitude that agents do not deliberate over aspects of the world that are irrelevant to their current decision-making problems. For example, although dark matter is an important scientific topic, most individuals do not form everyday decisions based on detailed beliefs about its nature. This illustrates that subjective state spaces are shaped not only by objective reality but also by practical relevance.

The framework also captures situations in which awareness and knowledge expand gradually through experience. A newborn infant is initially ignorant of nearly everything, yet gradually learns through repeated interaction with the environment that certain behaviors systematically generate particular outcomes. In this sense, subjective state spaces may evolve incrementally through accumulated experience even when initial awareness is extremely limited.

2.6. Belief Function

The outcome of an agent's decision is determined not solely by the action they have chosen but also the combination of that action with the underlying ideal state. For example, drinking well water per se does not cause cholera; rather, illness arises only when well water is contaminated with cholera bacilli. However, the agent does not have full awareness of the ideal state space. The agent can access only a state space S represented through a projection r_S^Ω. Consequently, the agent cannot accurately predict the outcomes generated by the combinations of states and actions. For instance, without the concept of cholera bacillus, the agent cannot even conceive of the possibility that a well is contaminated and therefore cannot anticipate the possibility of becoming ill. In other words, the agent must form expectations about outcomes based on combinations of actions and the state space accessible to them while remaining unaware of the true underlying ideal states.

We define $B: \Sigma \times H \times \bar{A} \to O$ as an agent (non-probabilistic) belief function representing an agent prediction about outcomes, where $B(s, h, a) = o$ means that, if an agent observes a state s under the framework h and chooses an action a, she predicts that the outcome is o. Because s and a are correlated, we denote that $B_h: S_h \times A_h \to O$ under $h \in H$. Then, $s = s(\omega, h)$ implies $B(s, h, a) = B_h(s(\omega,\ h),\ a)$.

We denote the outcome set $O_h = \{B_h(s,\ a) \in O | s \in S_h,\ a \in A_h\}$, which implies that an agent considers the feasible outcome set under framework h. However, she is aware of some $o \in O \setminus O_h$. Importantly, the agent may be unable to infer certain consequences from combinations of actions and states of which they are aware; however, this does not imply that the agent is unaware of the possibility of such consequences. In other words, even if a consequence cannot be derived from the agent's current awareness or inferential resources, the agent may still be aware that it could become derivable through an expansion of awareness or the acquisition of new inferential methods. For example, the content of a correct proof of $P \neq NP$ may currently lie beyond the agent's awareness, and it may be the case that no existing mathematical proof technique is sufficient to establish $P \neq NP$. However, this does not mean that the possibility of the conclusion $P \neq NP$ itself lies beyond awareness. Rather, one may well be aware that, if new proof methods beyond those currently available were to be discovered, it might become possible to establish $P \neq NP$ in the future.

Note that an agent's outcome prediction does not always require correction. Hence, the outcome anticipated by an agent may differ from the actual outcome. In this study, we say that cognitive dissonance arises when such a discrepancy emerges between anticipated and actual outcomes. More precisely, for some $(\omega, h, a) \in \Omega \times H \times A_h$, if $B_h(s(\omega,\ h),\ a) \neq o(\omega, a)$, then we say that the belief B_h is potentially cognitively dissonant.

We make the following assumptions:

Assumption 4 For any $h \in H$, $\omega \in \Omega$, and $a \in A_h$, if $B_h(s(\omega,\ h),\ a) \neq o(\omega, a)$, then there exists $h' \geq h$ such that $B_{h'}(s(\omega,\ h'),\ a) = o(\omega, a)$.

Assumption 5 For any $h \in H$, $\omega \in \Omega$, and $a \in A_h$, if $B_h(s(\omega,\ h),\ a) \neq o(\omega, a)$, then $B_{h'}(s(\omega,\ h'),\ a) = o(\omega, a)$ for any $h' \geq h$.

Assumption 4 states that, if a decision a made under a framework h in the true ideal state ω gives rise to cognitive dissonance, then there exists a more advanced framework h' than h under which the cognitive dissonance generated by a in ω can be resolved. Assumption 5 states that, if a decision a made under a framework h in the true ideal state ω gives rise to cognitive dissonance, then the cognitive dissonance generated by a in ω can be resolved under all frameworks that are more advanced than h.

Assumption 4 immediately yields the following observation:

Remark 2 Suppose that $h \in H$ is maximal. Then, for any $\omega \in \Omega$ and any $a \in A_h$, $B_h(s(\omega, h), a) = o(\omega, a)$. Otherwise, if $B_h(s(\omega, h), a) \neq o(\omega, a)$ for some $\omega \in \Omega$ and $a \in A_h$, then by Assumption 4 there would exist $h' \geq h$ such that $B_{h'}(s(\omega, h'), a) = o(\omega, a)$. As h is maximal, this is impossible unless $h' = h$, contradicting $B_h(s(\omega, h), a) \neq o(\omega, a)$.

2.7. Main Result in Static Models

The main theorem derived in this section is as follows:

Theorem 1 Suppose $\mathcal{S}$, $\bar{A}$, O, and H are finite. Then, there exists $h^* \in H$ such that for any $\omega \in \Omega$ and $a \in A(h^*)$, $B_{h^*}(s(\omega,\ h^*),\ a) = o(\omega,\ a)$.

Proof. Let $\{h_k\}_{k \geq 1}$ be a sequence of frameworks constructed recursively as follows. Let $h_1 = h \in H$ be arbitrary. If for every $\omega \in \Omega$ and every $a \in A_{h_1}$, $B_{h_1}(s(\omega, h_1), a) = o(\omega, a)$, then the claim holds with $h = h^*$. Otherwise, there exists a pair $(\omega_1, a_1) \in \Omega \times A_{h_1}$ satisfying $B_{h_1}(s(\omega_1,\ h_1),\ a_1) \neq o(\omega_1, a_1)$. Then, by Assumption 4, we can choose $h_2 \geq h_1$ such that $B_{h_2}(s(\omega_1,\ h_2),\ a_1) = o(\omega_1, a_1)$. Moreover, by Assumption 5, for any $h \geq h_1$, $B_h(s(\omega_1,\ h),\ a_1) = o(\omega_1, a_1)$. Therefore, for any $m \geq 2$, $B_{h_m}(s(\omega_1,\ h_m),\ a_1) = o(\omega_1, a_1)$. If, for any

$\omega \in \Omega$ and $a \in A_{h_2}$, $B_{h_2}(s(\omega,\ h_2),\ a) = o(\omega, a)$, the proof is complete. Otherwise, there exists a pair $(\omega_2, a_2) \in \Omega \times A_{h_2} \setminus \{(\omega_1, a_1)\}$ satisfying $B_{h_2}(s(\omega_2,\ h_2),\ a_2) \neq o(\omega_2, a_2)$. Then, by Assumption 4, we can choose $h_3 \geq h_2$ such that $B_{h_3}(s(\omega_2,\ h_3),\ a_2) = o(\omega_2, a_2)$. Moreover, by Assumption 5, for any $h \geq h_2$, $B_h(s(\omega_2,\ h),\ a_2) = o(\omega_2, a_2)$. Therefore, for any $m \geq 3$, $B_{h_m}(s(\omega_2,\ h_m),\ a_2) = o(\omega_2, a_2)$. If, for any $\omega \in \Omega$ and $a \in A_{h_3}$, $B_{h_3}(s(\omega,\ h_3),\ a) = o(\omega, a)$, the proof is complete. Otherwise, there exists a pair $(\omega_3, a_3) \in \Omega \times A_{h_3} \setminus \{(\omega_1, a_1), (\omega_2, a_2)\}$ satisfying $B_{h_3}(s(\omega_3,\ h_3),\ a_3) \neq o(\omega_3, a_3)$. Proceeding inductively, we suppose that the claim has not yet been established at stage $k-1$. Then, there exists a pair $(\omega_k, a_k) \in \Omega \times A_{h_k} \setminus \{(\omega_1, a_1), \dots, (\omega_{k-1}, a_{k-1})\}$ satisfying $B_{h_k}(s(\omega_k,\ h_k),\ a_k) \neq o(\omega_k, a_k)$. Then, by Assumption 4, we can choose $h_{k+1} \geq h_k$ such that $B_{h_{k+1}}(s(\omega_k,\ h_{k+1}),\ a_k) = o(\omega_k, a_k)$. Moreover, by Assumption 5, for any $h \geq h_k$, $B_h(s(\omega_k,\ h),\ a_k) = o(\omega_k, a_k)$. Hence, for any $m \geq k+1$, $B_{h_m}(s(\omega_k,\ h_m),\ a_k) = o(\omega_k, a_k)$. Because the previously resolved pairs remain resolved by Assumption 5, no pair is selected more than once. Thus, each step eliminates at least one previously unresolved pair from $\Omega \times \bar{A}$. By repeating this procedure, because Ω is finite by Definition 1 and $\bar{A}$ is finite, there are only finitely many possible pairs in $\Omega \times \bar{A}$. Moreover, at each step, at least one new pair is permanently resolved. Therefore, the procedure is terminated after a finite number of steps. Hence, there exists k^* such that for any $\omega \in \Omega$ and $a \in A_{h_{k^*}}$, $B_{h_{k^*}}(s(\omega,\ h_{k^*}),\ a) = o(\omega, a)$. This completes the Proof. ■

Because the initial framework $h_1 = h$ was arbitrary, the same construction applies from any starting point in H, yielding Corollary 1:

Corollary 1 Suppose $\mathcal{S}$, $\bar{A}$, O, and H are finite. Then, for any initial framework $h \in H$, there exists $h^* \in H$ with $h^* \geq h$ such that for any $\omega \in \Omega$ and $a \in A(h^*)$, $B_{h^*}(s(\omega,\ h^*),\ a) = o(\omega,\ a)$.

In the main theorem, what governs Theorem 1 is not only the structure of $(H, \geq)$ and finiteness of the ideal state space Ω, the family of state spaces $\mathcal{S}$, the outcome set O, and the ideal action set $\bar{A}$ but also Assumption 5.

The theorem implies that there always exists a framework $h \in H$ that does not give rise to cognitive dissonance. Moreover, as is clear from the proof, the initial framework can be chosen arbitrarily. In other words, regardless of which technology, norms, and value systems are taken as the starting point, there always exists, among the technologies reachable under the partial order $\geq$, a framework that resolves cognitive dissonance.

3. Dynamic Model: Partially Endogenous State Space Expansion Model

3.1. Definition of Dynamic Models

Section 2 established a framework in which cognitive dissonance is eliminated. The remaining issue is how to achieve such a framework over time. To address this question, we develop a dynamic model in which growing awareness is represented as a historical framework for the revision process. The central idea is that cognitive dissonance creates pressure for change; when the outcome anticipated under the current framework differs from the realized outcome, the agent may respond by adopting a new framework, which may, in turn, expand the subjective state space or the feasible action set. Thus, awareness expansion is only partially endogenous. The available family of state spaces and frameworks is taken as given but movement across frameworks is driven by the agent's mismatch between belief and reality. This dynamic formulation makes it possible to study not only the eventual elimination of cognitive dissonance but also historical contingency, path dependence, and the stabilization of frameworks over time.

Definition 2 $\mathcal{P} = \langle \mathcal{M}, T, D \rangle$ is a partially endogenous state space expansion model where

- $T = \{0, 1, 2, \cdots\}$ is time;
- $\omega_t \in \Omega$ is an ideal state at t;
- $h_t \in H$ is a framework at t. At any t and $\omega \in \Omega$,

  $S_t = S_{h_t}$ is the state space at t;

  $A_t = A_{h_t} = \bigcup_{h \in H_t} A_h$ is the action set at t;

  $O_t = O_{h_t}$ is the set of outcomes at t representable under h_t;

  $B_t = B_{h_t}$ is a period-t belief function.

- $D: \Omega \times H \times O \to H$ is a framework development function, where $h_{t+1} = D(\omega_t, h_t, o_t)$.

The essential new ingredient is the framework development function D, which determines how the current framework changes in response to the experienced outcomes. The partially endogenous state space expansion model $\mathcal{P}$ is an infinite-horizon model of non-probabilistic belief revision constructed based on a model of generalized growing awareness $\mathcal{M}$. The state space, action set, and outcome set of which the agent is aware in period t, denoted by S_t, A_t, and O_t, respectively, are defined by the framework h_t prevailing in period t. Also, B_t is the period-t belief function in period *t*.

We denote by D the framework's development function, which derives the framework h_{t+1} in period $t+1$ from the true ideal state ω_t, technology h_t, and true outcome o_t in the previous period t. This function represents the process through which the agent searches for a framework that restores consistency with that belief when an outcome is discovered that renders the agent's belief cognitively dissonant. For example, after Koch discovered the cholera bacillus, various responses followed, including the establishment of medical technologies premised on the existence of the cholera bacillus, the development of water supply and sewage systems, and the diffusion of the concept of public health. The framework development function captures the technological innovation processes and institutional or social system improvements.

Crucially, our model treats the falsification of a prior theory—such as the rejection of the miasma theory after the discovery of the cholera bacillus—as an expansion rather than a contraction of the state space. While Karni and Vierø (2013) typically represent the dismissal of a hypothesis by nullifying or removing states, this framework incorporates the "realization of a prior error" as higher-order information that enriches the agent's new framework. By doing so, the transition $h_t \to h_{t+1}$ consistently results in a monotonic expansion of the subjective state space, ensuring a unified treatment of discovery and refutation within the lattice structure.

We impose the following assumptions on the framework development function D, where $(H, \geq)$ is a finite complete lattice:

Assumption 6 For any $h \in H$, $D(\omega,\ h,\ o) \geq h$.

Assumption 7 For any t, an agent chooses $a_t \in A_t$ at $\omega_t \in \Omega$ and $h_t \in H$. Then, $h_{t+1} \in H$ satisfies the following:

(i) If $b_t(s(\omega_t,\ h_t),\ a_t) = o(\omega_t, a_t)$, then $h_{t+1} = D(\omega_t,\ h_t,\ o_t) = h_t$;

(ii) If $b_t(s(\omega_t,\ h_t),\ a_t) \neq o(\omega_t, a_t)$, then $h_{t+1} = D(\omega_t,\ h_t,\ o_t) \neq h_t$. In this case, the agent adopts a framework that resolves the experienced cognitive dissonance in the sense of Assumption 4.

Assumption 6 requires that, for any given framework, the framework development function yields a framework that is either equivalent to or more advanced than the current one. Assumption 7 specifies that the framework adopted in the current period is retained in the next period. Specifically, if the agent's belief does not generate cognitive dissonance in the current period, the agent adopts the same framework in the next period. By contrast, if cognitive dissonance arises in the current period, the agent searches for an alternative framework in the next period to resolve it.

Remark 3 By Assumptions 5 and 6, once a given pair $(\omega, a) \in \Omega \times \bar{A}$ ceases to generate cognitive dissonance, it does not generate cognitive dissonance again along any subsequent framework path.

Remark 3 states that if a given ideal state and action induce cognitive dissonance, any framework adopted in subsequent periods must resolve the cognitive dissonance for the same ideal state and action. This assumption rules out the possibility that cognitive dissonance reemerges for the same ideal state and decision over time. In other words, socio-technological framework evolution have eliminated previously experienced cognitive dissonance.

This consistency is aligned with our interpretation that the refutation of a theory is a step toward a more comprehensive state space, rather than a mere contraction of awareness.

3.2. Main Result in Dynamic Models

The following is the main theorem in our dynamic model:

Theorem 2 Suppose $\mathcal{S}$, $\bar{A}$, O, and H are finite. Then, in a partially endogenous state space expansion model based on a generalized growing-awareness model $\mathcal{M}$, there exists t^* such that (i) $B_{t^*}(s(\omega, h_{t^*}), a) = o(\omega, a)$ for any $(\omega, a) \in \{(\omega_t, a_t) | t \leq t^*\}$, and (ii) in every $t \geq t^*$, $h_{t+1} = h_t$.

Proof. Consider the realized sequence of ideal states and actions $\{(\omega_t, a_t)\}_{t\geq 0}$ and the corresponding sequence of frameworks $\{h_t\}_{t\geq 0}$ generated by the framework development function. By Assumption 6, the sequence of frameworks is weakly increasing; that is, $h_t \leq h_{t+1}$ for every t.

Suppose that cognitive dissonance arises at period t. By definition, this means that $B_t(s(\omega_t, h_t), a_t) \neq o(\omega_t, a_t)$. By Assumption 4, there exists a more advanced framework that can resolve the cognitive dissonance generated by the realized pair (\omega_t,a_t). By Assumption 7 (ii), when such cognitive dissonance arises, the agent adopts in the next period a framework that resolves the experienced cognitive dissonance in the sense of Assumption 4. Hence, $B_{t+1}(s(\omega_t, h_{t+1}), a_t) = o(\omega_t, a_t)$. Moreover, by Assumption 5, once the cognitive dissonance generated by a given pair (ω_t, a_t) is resolved under a framework, it remains resolved under any more advanced framework. Since the framework sequence is weakly increasing by Assumption 6, the same realized pair cannot generate cognitive dissonance again in any later period.

Now consider the set of state-action pairs encountered along the realized history. Since S and $\bar{A}$ are finite, the set of possible realized state-action pairs is finite. Each time cognitive dissonance occurs, at least one previously unresolved realized pair is resolved. By the preceding paragraph, no resolved pair becomes unresolved again. Therefore, cognitive dissonance can occur only finitely many times along the realized dynamic path. Thus, there exists a finite period t^* such that no further cognitive dissonance occurs after t^*. Consequently, for every realized pair $(\omega, a) \in \{(\omega_t, a_t) \mid t \leq t^*\}$, the stabilized framework h_{t^*} resolves the cognitive dissonance associated with that pair. Therefore, $B_{t^*}(s(\omega, h_{t^*}), a) = o(\omega, a)$. This proves part (i).

For part (ii), since no cognitive dissonance occurs at any period $t \geq t^*$, Assumption 7 (i) implies that the current framework is retained in the next period. Hence, $h_{t+1} = h_t$ for every $t \geq t^*$. This proves part (ii). ■

Remark 4 This theorem is a pathwise convergence result. It does not imply that every logically possible state-action pair in $\Omega \times \bar{A}$ is eventually examined or resolved. State-action pairs that are not encountered along the realized history may remain unresolved indefinitely, provided that they do not generate cognitive dissonance along that history.

Remark 5 (Theoretical Benchmark and Satisficing) While Theorem 2 establishes the eventual resolution of cognitive dissonance and the cessation of framework updating, this result should be interpreted as a theoretical benchmark for the system's long-run convergence. In practical economic environments, the updating process may terminate before reaching the global maximum h^* for two primary reasons. First, if agents face a "satisficing" threshold (Simon 1955), they may stop updating once the dissonance falls below a subjective level of tolerance. Second, if the marginal cost of adopting a more complex framework outweighs the perceived benefits of error reduction, the agent may remain at a "locally stable" framework. In such cases, technological progress "ceases" not because the truth is fully uncovered but because the current framework provides a sufficient subjective representation of the reachable environment.

Part (i) in Theorem 2 states that all realized state-action pairs encountered before stabilization are retrospectively interpreted consistently under the stabilized framework h_{t^*}. Theorem 2 implies that, regardless of the framework from which one starts, as long as S, $\bar{A}$, O, and H are finite, cognitive dissonance arising along the realized history can eventually be resolved through socio-technological framework evolution. Simultaneously, Assumption 7 implies that technological progress ceases when cognitive dissonance is eliminated. In other words, a framework change must eventually end. Equivalently, a state in which no cognitive dissonance arises may be interpreted as one in which the agent is satisfied with the status quo.

At first glance, this conclusion appears counter-intuitive. However, this naturally follows if the incentive for technological development or the creation of new value systems is assumed to depend solely on the resolution of cognitive dissonance. One may argue that this claim is implausible if the incentives for technological development are understood in monetary terms. However, even monetary incentives can be reinterpreted within the same framework as a form of cognitive dissonance between the belief "it is more profitable to maintain the current technology" and the true outcome "it is not profitable to maintain the current technology." This reinterpretation preserves the present reading. For example, suppose that the development of environmentally friendly technologies does not advance because firms believe that retaining the current technology is more profitable. Furthermore, suppose that actual observations do not contradict this belief and therefore do not generate cognitive dissonance. In this case, firms have no incentive to develop environmentally friendly technologies. Accordingly, if one wishes to promote the development of such technologies, it is necessary to introduce some factor that generates cognitive dissonance with respect to the belief "it is more profitable to maintain the current technology." Examples include government subsidies and consumer boycott.

Moreover, the partially endogenous state-space expansion model $\mathcal{P}$, together with Theorem 2, demonstrates that the framework incorporates both historical contingency and historical necessity. For example, Robert Koch discovered the cholerae Bacillus in 1883, but there was nothing essential about Koch himself as a discoverer. John Snow, for instance, had already recognized at an early stage that cholera was caused by oral infection through contaminated water and, had he pursued his research further, it is possible that he would have discovered the cholera bacillus himself. By the same logic, the discovery of the cholera bacillus itself may be regarded as historically necessary; it was merely contingent that Koch happened to make the discovery. Given that there already existed figures such as Snow who had come close to the core of cholera research, it is plausible that someone else would eventually have made the same discovery, even if Koch had not.

The model also contains the idea of path dependence in the sense that past historical contingencies constrain the subsequent formation and development of social customs and institutions.

Remark 6 (Path dependence): Because the framework development function D depends on the realized state and outcome, the evolution of frameworks need not be uniquely determined by the current framework alone. In particular, for a given $h \in H$, there may exist different ideal states $\omega, \omega' \in \Omega$ and different actions $a, a' \in \bar{A}$ such that $D(\omega, h, o(\omega, a)) \neq h$, $D(\omega', h, o(\omega', a')) \neq h$, and $D(\omega, h, o(\omega, a)) \neq D(\omega', h, o(\omega', a'))$,' with either $\omega \neq \omega'$ or $a \neq a'$. Thus, even when agents start from the same current framework, different realized histories may lead to different future frameworks.

Although the framework development function D yields a unique framework at each point in time, the framework obtained in the next period may still differ, depending on the true ideal state and the action realized in the previous period, even when the current framework is the same. An illustrative historical example is provided by the German optical lens company Carl Zeiss. Although Germany was a single country originally, its post-1945 division led its eastern and western branches to develop distinct technological trajectories. These branches were eventually reunified; however, there have been many cases in which countries or regions followed different paths of technological development and historical evolution. This remark formalizes the possibility of such divergence.

4. Discussion

4.1. Framework Rigidity and Organizational Failure

Our model provides a cognitive foundation for why successful incumbent firms often fail to adapt to disruptive innovations. A classic real-world example of this mechanism is the failure of Eastman Kodak. Although Kodak invented the first digital camera in 1975, it failed to transition effectively to the digital era, leading to its eventual bankruptcy in 2012.

From the perspective of the growing-awareness models of Karni and Vierø (2013, 2017), which are built upon expected utility theory, Kodak's failure would typically be interpreted through the lens of probabilistic belief revision. In their framework, once an agent becomes aware of a new technology (a new state), they are assumed to assign a probability to it and optimize their actions to maximize expected utility. Consequently, Kodak's failure must be treated either as a miscalculation of future payoffs or as an irrational bias in updating their "reverse Bayesian" probabilities.

However, our model offers a more nuanced explanation based on the rigidity of the framework h. In our setting, the outcome an agent expects is determined by the belief function $B_h(s, a)$, where s is the subjective state projected from the true ideal state ω through the framework h (i.e., $s = s(\omega, h)$).

For decades, Kodak's framework h was optimized for a silver-halide film business. When the digital technology—the true state ω—emerged, Kodak's rigid framework h projected this ω into a coarse subjective state s. This s represented digital technology merely as a low-quality niche product, failing to capture the full expressive richness of the true disruptive state. Consequently, their belief function $B_h(s, a)$ predicted that maintaining the status quo (a) would still result in a satisfactory outcome o' (continued dominance in the film market), even though the true outcome function $o(\omega, a)$ was already shifting toward a catastrophic loss.

Crucially, because the perceived outcome o' based on their current h did not initially contradict their observations, the firm experienced no immediate cognitive dissonance. It was only when the actual market outcomes o began to diverge drastically from the framework's predictions—generating intolerable cognitive dissonance—that the firm was forced to "reject" its obsolete framework h. By this time, however, the state space expansion had occurred too late. This case illustrates that organizational failure is not necessarily a lack of information but a failure of the socio-technological framework to correctly project the true state space into a meaningful subjective representation.

4.2. Institutional Evolution and Socio-technical Transitions

Beyond individual firm behavior, our model provides a formal cognitive foundation for large-scale institutional changes. This perspective resonates with the Multi-Level Perspective (MLP) proposed by Geels (2002), which describes how "socio-technical regimes"—a configuration of technologies, rules, and beliefs—undergo fundamental transitions.

In our framework, a socio-technical regime can be defined as a dominant framework h. This h functions as a cognitive filter that projects complex landscape pressures (the true state ω, such as accelerating climate change) into a simplified subjective state s. Because this entrenched h is optimized for existing economic activities, the agent's belief function $B_h(s, a)$ initially predicts that the status quo remains optimal, effectively "ignoring" the long-term risks inherent in ω.

While the expected utility-based model of Karni and Vierø (2013) would suggest that agents incrementally update their beliefs as new information arrives, our model explains the "rigidity" and "non-linear jump" of institutional change. According to Geels (2002), a regime remains stable until landscape pressures create a significant "mismatch" or tension. In our terms, this mismatch is the accumulation of cognitive dissonance between predicted outcomes and reality.

Institutional evolution occurs non-linearly: the society remains in a state of "satisficing" with the old framework h until the dissonance reaches a critical threshold. Once this threshold is crossed, the society "rejects" the obsolete h and undergoes a transition to a new framework h'—such as a shift toward sustainability norms. This transition expands the subjective state space, allowing the society to internalize the previously unobserved dimensions of ω. Thus, our model formalizes Geels' qualitative transition theory by defining institutional change as a reactive expansion of awareness triggered by intolerable dissonance.

4.3. Logical Discovery and Framework Revision

This point further highlights the necessity of adopting the lattice structure of Heifetz, Meier, and Schipper (2006) over the simple state-space expansion of Karni and Vierø (2013).

In the "Reverse Bayesianism" framework of Karni and Vierø (2013, 2017), growing awareness is modeled by adding new states while preserving the likelihood ratios of existing ones. However, as implied by the insights of Galanis (2013), the discovery of a new “theorem”—such as a logical link between an intruder and the dog’s reaction—fundamentally alters the internal logic of the existing state space.

Consider the classic example from Sherlock Holmes’ Silver Blaze, discussed by Galanis (2013). While Watson may initially consider the presence of an intruder without realizing its logical implications, Holmes recognizes a crucial “theorem”: if there is an intruder, the dog must bark. For someone unaware of this theorem, the observation that the dog remained silent provides no reason to reject the possibility of an intruder. However, once this logical link is recognized, the state where “an intruder was present but the dog did not bark” becomes logically impossible and must be assigned a probability of zero.

Such a qualitative shift—where the recognition of a new concept necessitates the absolute rejection of prior likelihoods—cannot be captured by merely appending states to a flat space. In Karni and Vierø’s (2013) model, adding states ex-post often fails to account for how new awareness “falsifies” the old framework’s internal structure. By contrast, our model, utilizing the lattice structure of Heifetz, Meier, and Schipper (2006), allows for a hierarchical representation. Here, the transition to a higher-order state space represents the acquisition of new logical constraints (theorems), explaining why an agent would endogenously reject a previously held theory. This provides a more robust foundation for understanding the non-linear, reactive transitions in awareness that characterize major socio-technological regime shifts.

4.4. Related Literature

This paper is also related to several strands of research that are not central to the formal construction of the model but help clarify its position in the broader literature. First, Galanis and Kotoronis (2021), Schipper (2021), and Tada (2022) study dynamic aspects of awareness and discovery in economic and game-theoretic environments. Galanis and Kotoronis (2021) analyze the resolution of asymmetric unawareness through communication, whereas Schipper (2021) and Tada (2022) examine discovery processes in games with unawareness. These studies show how agents may revise their awareness through interaction or observed behavior. By contrast, the present paper focuses on state-space expansion driven by discrepancies between predicted and realized outcomes, rather than on strategic communication or discovery of opponents' actions.

Second, this paper is related to recent work on growing awareness and decision theory beyond expected utility. Dominiak and Tserenjigmid (2022), for example, examine the relationship between growing awareness and ambiguity aversion. Their analysis shows that awareness expansion can interact with non-probabilistic attitudes toward uncertainty. The present paper is also non-probabilistic, but its emphasis is different. Rather than studying ambiguity attitudes over an expanding but given uncertainty structure, it analyzes how cognitive dissonance can induce a change in the interpretive framework that determines the agent's subjective state space itself.

Third, the present paper is connected to the literature on unforeseen evidence. Piermont (2021) studies how agents should update beliefs when evidence itself was previously inconceivable. His contribution provides a normative rule for belief revision under unforeseen evidence. The present model is complementary to this approach. It does not primarily ask how beliefs should be updated after unforeseen evidence arrives; instead, it asks how the mismatch between expected and realized outcomes can generate pressure to revise the agent's framework and expand the state space.

Finally, the behavioral interpretation of the model is related to Simon's (1955) notion of bounded rationality. In the standard growing-awareness literature, agents are often modeled as expected utility maximizers over the state space available at a given level of awareness. By contrast, the present model emphasizes backward-looking adjustment: agents do not optimize over fully specified future state spaces, but respond to cognitive dissonance generated by realized discrepancies. In this sense, the model provides a formal account of how progress may occur under genuine unawareness, where future states, actions, or outcomes are not merely unknown but not yet representable within the current framework.

5. Conclusion

This study developed a generalized growing-awareness model and its dynamic counterpart, the partially endogenous state-space expansion model. The findings show that as long as the family of state spaces, ideal action set, and outcome set is finite, at least one interpretive and practical framework exists in the generalized growing-awareness model that completely resolves cognitive dissonance. Furthermore, in the partially endogenous state space expansion model, both technological progress and the updating of value systems come to a halt once cognitive dissonance arising along the realized history is eventually resolved within a finite number of steps. In this sense, the model succeeds in integrating the unawareness structure and growing-awareness models, which have thus far remained mathematically disconnected, while also incorporating both historical necessity and contingency. By shifting the focus from forward-looking expected utility maximization to backward-looking dissonance resolution, our framework provides a novel behavioral foundation for understanding institutional rigidity and non-linear, reactive socio-technical transitions.

However, several issues remain to be resolved. First, although the present model partially endogenizes growing awareness, it can hardly be regarded as a fully endogenous model because both the family of state spaces and the framework set are taken as given. In particular, it does not explain why specific technologies or value systems are discovered or created through technological development or the search for new value. Therefore, endogenizing these creation processes remains an important direction for future research. Second, the present model is non-probabilistic. By contrast, Heifetz, Meier, and Schipper (2013b) and Karni and Vierø (2013) also consider probabilistic beliefs, and the growing-awareness framework may likewise be understood as a model of probabilistic belief revision. Therefore, it is necessary to extend the present framework to explicitly describe how an agent's probabilistic beliefs evolve over time. Third, although the unawareness structure model was developed using concepts such as possibility correspondences, knowledge operators, and unawareness operators, this study did not address these issues. How knowledge and unawareness operators evolve dynamically within the present framework remains to be explored. Another issue concerns the role of finiteness assumptions in Theorem 2. Although the theorem is stated under the finiteness of S, $\bar{A}$, O, and H, the proof itself relies primarily on the finiteness of realized state-action pairs together with the permanence property of resolved cognitive dissonance. This suggests that some finiteness assumptions, particularly those on O and H, may potentially be weakened in future refinements of the model. These issues remain as open questions for future research.

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