FINANCIAL SANCTIONS AND CURRENCY HEGEMONY: A SEARCH-THEORETIC MODEL

ABSTRACT. This paper examines financial sanctions and currency hegemony by constructing a search-theoretic model involving two large countries, A and C, and a continuum of small countries. These small countries differ in their probabilities of matching with the large countries. Only the currencies of the large countries can circulate as international currencies. While agents in the large countries are committed to using their respective currencies, agents in the small countries independently decide whether to accept or reject the two currencies. The model yields a unique equilibrium in which small countries are categorized into three groups: those that accept only currency A, those that accept only currency C, and those that accept both currencies. The paper also conducts comparative statics and welfare analyses, including the impact of sanctions. It is shown, among others, that under any condition, there are some countries that are better off by being sanctioned provided that there is a competitor for currency hegemony. Furthermore, when country A imposes sanctions on small countries, they expand trade with country C, thereby increasing country C's welfare. These results highlight the limitations of financial sanctions in the presence of competition for currency hegemony.

Keywords: international settlement, financial sanction, currency competition, currency hegemony, search theory

1. INTRODUCTION

In the modern world, economic activities often rely on the use of money, and international commerce is no exception. Most trade payments are processed through international settlement systems. Among these, the Society for Worldwide Interbank Financial Telecommunication (SWIFT) is the dominant system, with participation from over 10,000 financial institutions.

Within SWIFT, the US dollar has maintained its status as the dominant currency. According to SWIFT data as of May 2024, the US dollar (USD) accounts for 59% of global settlements, followed by the euro (EUR), Japanese yen (JPY), British pound (GBP), and Chinese yuan (CNY).[1]

Leveraging this dominant position, the United States has used the payment system as a tool for financial sanctions by freezing assets and blocking payment channels. A prominent

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[1]https://www.statista.com/statistics/1189498/share-of-global-payments-by-currency/

historical example prior to SWIFT is the asset freeze that caused significant harm to Japan before and during World War II (Miller, 2007). More recently, sanctions have been imposed through SWIFT on various countries, including North Korea, Iran, and Russia (Dowlah, 2024; Cipriani, Goldberg, & La Spada, 2023).

The use of payment systems as instruments of sanctions has encouraged large economies to develop alternative infrastructures. Among them, the Cross-border Interbank Payment System (CIPS), launched by China in 2015, has emerged as a potential competitor to SWIFT and facilitated the international use of the Chinese yuan. Some researchers interpret this development as part of a transition toward a new monetary order (Goh, 2019). As Klein (2024) argues, the more frequently dominant countries employ financial sanctions, the stronger the incentives become for others to establish alternative payment systems.

With the coexistence of multiple payment systems and currencies, countries that do not issue an international currency may find themselves participating in one or several payment and currency networks simultaneously. In such an environment, the choice of which currency to use in international trade and settlement becomes nontrivial, and the strategic consequences of financial sanctions are no longer confined to a single dominant currency area. This raises several questions: What determines the optimal choice of international currency in decentralized trade? Under what conditions are financial sanctions effective? And can sanctions imposed by one currency area unintentionally benefit another?

To answer these questions, we need to analyze how trade is redirected across trading partners when access to a payment network is restricted. A key feature of these environments is that international transactions occur in a decentralized manner and trading opportunities arrive sequentially. Economic agents must decide whether to transact using a currently available currency without knowing when a more favorable opportunity will arise. Restricting access to a currency therefore affects the timing and availability of future transactions—considerations that cannot be captured by frictionless models in which agents can freely reallocate trade across currencies.

For this reason, we employ a search-theoretic framework. In decentralized exchange environments, international trade takes place bilaterally, and agents meet trading partners sequentially. In such environments, agents face intertemporal constraints on how many transactions they can complete over time. Because production and delivery take time and cannot be adjusted instantaneously, supplying a good today may preclude an agent from engaging in a more valuable transaction tomorrow. This intertemporal trade-off between present and future exchange opportunities shapes currency acceptance decisions and the welfare effects of redirecting trade across partners. A search model therefore provides a natural framework to study currency competition and financial sanctions under decentralized trade.

To analyze these issues, this paper develops a search-theoretic model with two large countries, A and C, and a continuum of small countries. The framework builds on Matsuyama, Kiyotaki, and Matsui (1993), extending their analysis from a single international currency to competition between multiple currencies. Small countries differ in their probabilities of matching with the large countries. Agents in large countries use their own currencies, while agents in small countries decide whether to accept either or both currencies. The model yields a unique equilibrium in which small countries are endogenously divided into three groups: those accepting only currency A, only currency C, or both.

The model allows us to study the welfare consequences of sanctions. We show that, under general conditions, some countries may become better off even when sanctioned in the presence of currency competition. Moreover, sanctions imposed by country A reallocate trade toward country C, increasing country C's welfare. These results highlight a limitation of financial sanctions when multiple international currencies coexist and suggest the possibility of unintended cross-country welfare effects.

The findings also have practical implications for Asian economies, including Japan, which do not have an international settlement system of its own, but are integrated with multiple competing currency areas. This paper studies the incentives of countries positioned between rival major currency blocs, focusing on how their trade patterns and currency usage adjust when competition between dominant currencies intensifies. The framework provides a structured way to analyze the strategic trade-offs faced by such countries as geopolitical and economic rivalries reshape the international monetary landscape.

1.1. **Literature review.** This paper is closely related to the search-theoretic literature on international currency and currency competition. A foundational contribution is Matsuyama et al. (1993), who developed a two-country, two-currency search model to study the emergence of an international currency. Their framework demonstrates how differences in trading opportunities and acceptance patterns can lead to the dominance of a particular currency in decentralized trade. Subsequent studies extended this approach in various directions. For example, Zhou (1997) and Wright and Trejos (2001) analyzed currency competition in open-economy search models, while Li and Matsui (2009) analyzed international trade between two countries and showed how trading frictions shape international currency usage, introducing endogenous matching probabilities as a modeling device. Building on this literature but departing from existing frameworks, the present paper introduces a continuum of heterogeneous small countries indexed by their economic distance to the major currency-issuing economies, providing a novel framework for studying financial sanctions and currency hegemony.

The present paper contributes to this literature in two main dimensions. First, as mentioned above, rather than focusing on a small number of symmetric or representative countries, the model introduces a continuum of heterogeneous small countries that differ in their trading intensities with the two major economies. This structure allows us to study the endogenous formation of currency blocs: some countries optimally use only one dominant currency, others use both currencies, and yet others gravitate toward the rival currency. By explicitly modeling heterogeneity in international trading relationships, the paper provides a tractable framework for understanding how patterns of international currency usage vary systematically across countries depending on their economic proximity to major currency issuers.

Second, the paper explicitly incorporates financial sanctions into a multi-currency search environment. While the existing search-theoretic literature has primarily focused on the endogenous emergence and competition of international currencies, it has largely abstracted from policy interventions that restrict access to dominant payment systems. In contrast, this paper analyzes how sanctions imposed by one major country affect trade patterns, currency acceptance, and welfare when an alternative international currency is available. By combining heterogeneous small countries with competing international currencies, the model captures how sanctions can induce substitution across currencies and trade partners, potentially undermining the effectiveness of sanctions.

A related and complementary strand of the literature examines financial sanctions and international payment systems from an institutional and empirical perspective. Cipriani et al. (2023) document the increasing use of financial sanctions and emphasize the growing importance of payment systems and currency infrastructure in the transmission of such policies. While their analysis is largely descriptive and empirical, the present paper contributes to this literature by providing a formal theoretical framework that explains how the presence of competing international currencies alters the welfare and trade effects of financial sanctions.

Another related line of research studies conditions under which a dominant international currency emerges even among otherwise symmetric economies. Fukuda and Tanaka (2022) analyze trade between two regions with different time zones and show that the U.S. dollar can uniquely emerge as the dominant international currency due to temporal trading advantages. While their focus is on time-zone frictions rather than sanctions, both their analysis and ours highlight how asymmetries in trading opportunities can generate persistent currency dominance.

Finally, a separate but influential literature incorporates centralized markets into search-theoretic monetary models. Lagos and Wright (2005) propose a framework in which agents trade in decentralized markets and subsequently participate in a Walrasian market, allowing for tractable analysis of monetary policy. Building on this framework, Kannan (2009) develop a multi-country, multi-currency open-economy model to study welfare and exchange-rate determination. Unlike these models, the present paper abstracts from centralized markets in order to isolate the role of decentralized trade, currency acceptance, and sanctions in shaping international payment patterns. Incorporating centralized markets into the analysis of sanctions and competing payment systems remains an important avenue for future research.

Overall, the existing literature has primarily examined the emergence and competition of international currencies and their implications for monetary policy. The contribution of this paper is to extend this line of research by providing a theoretical analysis of financial sanctions in an environment with heterogeneous countries and multiple competing payment systems, highlighting how currency competition fundamentally alters the trade and welfare consequences of sanctions. In doing so, the present paper offers a unified perspective on the global economy through the lens of international currency competition.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 defines strategy and equilibrium. Section 4 begins analysis, presenting some general properties of the model. Section 5 conducts comparative statics. Section 6 analyzes the impact of sanctions. Section 7 concludes the paper.

2. Model

Time is discrete and from zero to infinity. There are two major countries and a continuum of infinitesimal countries. The two major powers are denoted by A and C, and each infinitesimal country is denoted by $i \in B \equiv [0, n_B]$ where n_B is the total population of B. Let n_A and n_C denote the population size of countries A and C, respectively. We assume $n_A, n_B, n_C \in (0,1)$ and $n_A + n_B + n_C < 1$. A country i's agent ($i \in B$) is called agent i for short.[2]

In each country, $K (K \geq 3)$ types of indivisible commodities and agents exist. The proportions of K types of agents are equal in each country. We denote by k a generic type ($k = 1, \dots K$). A type k agent gains utility from the consumption of commodity k. After the consumption of commodity k, one and only one unit of commodity $k+1 \pmod{K}$ is produced with no cost. There is no storage cost up to one unit. The type k agent can neither produce nor store other types of goods. Given K that is greater than two, this situation

[2]Each agent $i \in B$ is infinitesimal in each country i, *i.e.*, technically speaking, we consider a continuum of continua as in Kaneda (1995).

implies no "double coincidence of wants" in this economy. We interpret K as the degree of heterogeneity or specialization in the economy.

Let $u > 0$ be the instantaneous utility from consuming the agent's consumption good, and $r > 0$ be the interest rate. Both of them are independent of the type and the nationality. The expected discounted utility of an agent as of time t is given by

$$V_t = E\left[\sum_{s=0}^{\infty} \frac{u}{(1+r)^s} I_{t+s} \middle| \Omega_t\right]$$

where I_{t+s} is a random indicator function that takes one if the agent consumes one's consumption good at period $t+s$ and zero otherwise; Ω_t is the information available at period t. When a type k agent obtains consumption good k, the agent consumes it immediately and produces one unit of one's production good. Note that $V_t \geq 0$ always holds.

In addition to the commodities, there are two distinguishable *fiat* monies with zero intrinsic utility. They are currency A and currency C. It is assumed that each currency is indivisible, and can be stored with no cost up to one unit by every agent if the agent does not carry the production good or the other currency. This implies that, at any period, the inventory of each agent contains either one unit of currency A, one unit of currency C, or one unit of one's production good. We assume that agents never dispose of their inventories. This is not restrictive because they do not gain by doing so.

Furthermore, we assume that agents in country A always agree to exchange currency A and a commodity good but never trade either currency A or a commodity good for currency C. Similarly, we assume that agents in country C always agree to exchange currency C and a commodity good but never trade either currency C or a commodity good for currency A. As we shall see in the sequel, this assumption enables us to focus on the analysis of the behavior of the agents in B.

We use the following notations for inventory and money holdings. Let m_A (resp. m_C) be the fraction of country A's agents holding currency A (resp. C). Let m_C be the fraction of country C agents holding currency C. We assume that m_A and m_C are given exogenously and that $m_A, m_C \in (0, 1)$ holds.

The inventory distribution of a country is represented by a three-element vector consisting of the fractions of agents who hold goods, currency A, and currency C. Since agents in A (resp. C) never accept currency C (resp. A), the inventory distributions of A and C are given by $X^A = (1 - m_A, m_A, 0)$ $X^C = (1 - m_C, 0, m_C)$, respectively.

Let m^i_A (resp. m^i_C) be the fraction of country i's agents holding currency A (resp. C). The fraction of country i's agents holding production goods is then $1 - m^i_A - m^i_C$, so that the inventory distribution for agent $i \in B$ can be summarized by a row vector

$$X^i = (1 - m_A^i - m_C^i, m_A^i, m_C^i).$$

We have assumed that the money supplies in the large countries, m_A and m_C, are exogenously fixed. This assumption implies that the global supply of each currency may vary in different situations. When agents from other countries accept and hold currency A, this should be interpreted as an expansion of the global circulation of currency A, rather than as a reduction in the quantity of currency circulating within country A: domestic money holdings in the issuing country remain unchanged, while foreign usage reflects additional issuance or circulation at the global level. For example, if the amount of currency A circulating in region B increases, the quantity issued by country A expands correspondingly so that the fraction m_A remains at its prescribed level.

There is no centralized market in which all agents can meet and exchange commodities multilaterally. Agents are matched randomly in pairs. When agents are matched, they must decide whether or not to trade bilaterally without any outside authority to impose any arrangement. Trade entails a one-for-one swap of inventories and takes place if and only if they both agree to trade. We assume the trade happens between two agents only when both sides gain strictly positive utility.

Agent i meets an agent from A or C but not another country $j \in B$. The probability that agent i meets an agent in country A (resp. country C) is $\alpha(i)n_A$ (resp. $\gamma(i)n_C$) where $\alpha(i)$, $\gamma(i) \in [0, 1]$ hold. We assume that the functions $\alpha(\cdot)$ and $\gamma(\cdot)$ are continuously differentiable. We also assume that $\alpha'(i) < 0$ and $\gamma'(i) > 0$ hold for all $i \in (0, n_B)$, and that $\alpha(n_B) = \gamma(0) = 0$ and $\alpha(0) = \gamma(n_B) = 1$ hold. Let μ be the Lebesgue measure on $\mathbb{R}$.

Take an arbitrary measurable subset J of B as given, *i.e.*, J is an arbitrary measurable subset of small countries that belong to B. We write $\alpha^J = \int_{i\in J} \alpha(i)d\mu$ and $\gamma^J = \int_{i\in J} \gamma(i)d\mu$: α^J (resp. γ^J) is the probability of an agent in country A (resp. C) meeting an agent in J.

Figure 1 shows a typical $\alpha(\cdot)$ and $\gamma(\cdot)$ as a function of the index i. We may interpret the shape of $\alpha(\cdot)$ and $\gamma(\cdot)$ in a geographical manner. The smaller the index i is, the closer it is to country A, and the farther to country C. The geographical distance is correlated with the probability of matching: the closer a country in region B is to country A (resp. C), the likelier its agents meet with agents in country A (resp. C). Reflecting this structure, we have assumed that $\alpha(i)$ is decreasing, and $\gamma(i)$ is increasing in i.

We also interpret that for each $i \in B$, $\alpha(i)$ and $\gamma(i)$ are the degrees of economic integration of country i with A and C, respectively. Note that an increase in the degree of integration

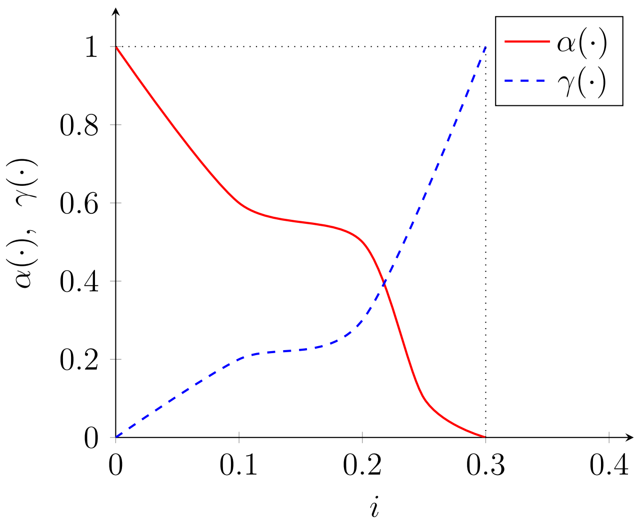

FIGURE 1. $\alpha(\cdot)$ and $\gamma(\cdot)$: $n_B = 0.3$

with country, say, A does not reduce the chance of meeting with agents from country C; rather, it results in a higher frequency of trading opportunities.

An agent in country A (resp. C) meets an agent from the same country with probability n_A (resp. n_C). We assume, for the sake of simplicity, that any agent from country A (resp. C) meets no agent from country C (resp. A).[3] As we have mentioned, for a given arbitrary subset J of B, each agent in A (resp. C) meets an agent in $J \subset B$ with probability α^J (resp. γ^J). The matching technology is given in Table 1.

	A	C	$J \subset B$	nobody
A	n_A	0	α^J	$1 - n_A - \alpha^B$
C	0	n_C	γ^J	$1 - n_C - \gamma^B$
i	$\alpha(i)n_A$	$\gamma(i)n_C$	0	$1 - \alpha(i)n_A - \gamma(i)n_C$

TABLE 1. The matching technology

One story behind this matching technology may be described as follows (see Figure 2). Consider a small country $i \in B$. Agents from country A arrive in country i with probability $\alpha(i)n_A$, while agents from country C arrive with probability $\gamma(i)n_C$. Visiting agents from either country A or C are assigned to one side of a trading table. While they stay in country i, local agents—who never travel abroad—visit these tables for the purpose of trade and are assigned to the other side. Once both sides are filled with visiting agents and local agents, bilateral trade negotiations take place. Thus, matching is two-sided between visiting agents

[3]Even if agents from A and C meet with each other, the incentives of agents in B do not change, while we have additional incentive constraints for A and C agents.

and local agents in B. For a trade between goods and currency A to occur, an agent in B holding currency A must be matched with an agent from A who holds a good. Conditional on such a meeting, the probability that the good matches the agent's consumption preference is $1/K$.

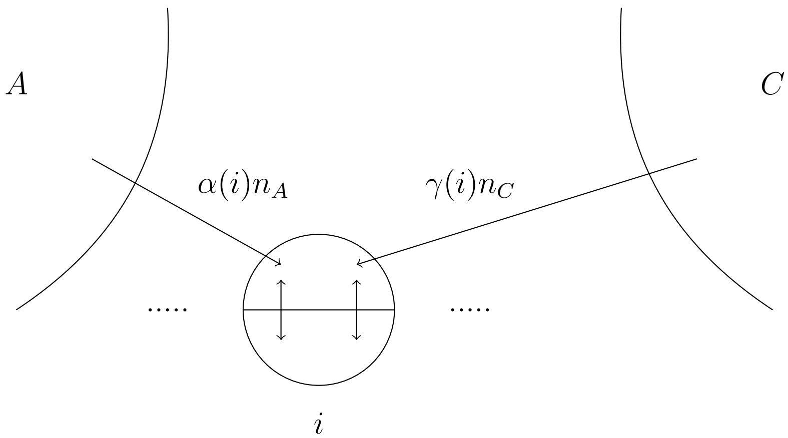

FIGURE 2. A story behind the matching technology

In this process, some agents may not meet any trading partner in a given period. A period in the model can be interpreted as a short time interval, such as a day or a week. Accordingly, the possibility that an agent does not meet anybody in one period reflects the fact that an agent may fail to meet a trading partner on a given day (for example, on a Friday), while still meeting someone in the future with probability one. Thus, this feature should not be interpreted as implying permanent exclusion from trade. Rather, it generates variation in agents' waiting time until a successful match occurs.

3. Strategy and equilibrium

We restrict our attention to pure strategies which only depend on an agent's nationality and the objects that this agent and the opponent have in inventory. The strategy of an agent in country $i \in B$ can be described as

$$\tau^i_{ab} = \begin{cases} 1 & \text{if the agent agrees to trade object } a \text{ and } b, \\ 0 & \text{otherwise,} \end{cases}$$

where $a, b = A, C$, or g, and $a \neq b$. Due to the present setup, an agent in country i never wants to exchange one currency with the other one. We simply assume that $\tau^i_{AC} = \tau^i_{CA} = 0$ holds for all $i \in B$.

The strategies of agents in A and C are $\tau^A_{Ag} = \tau^A_{gA} = 1$ and $\tau^C_{Cg} = \tau^C_{gC} = 1$ by assumptions. Since agents in A and C have only one strategy, we focus on the behavior of agents in B hereafter.

Let τ^i be a strategy profile of agent $i \in B$, *i.e.*, $\tau^i = (\tau^i_{gA}, \tau^i_{gC}, \tau^i_{Ag}, \tau^i_{Cg})$. We also denote $\tau = (\tau^i)_{i\in B}$.

For all $i \in B$, trade strategies τ^i's, inventory distributions X^i's, and the matching technology together generate the Markov process that each agent's inventory follows. This can be summarized by the following transition matrix:

$$\Pi^i = \begin{bmatrix} 1 - P^i_{gA} - P^i_{gC} & P^i_{gA} & P^i_{gC} \\ P^i_{Ag} & 1 - P^i_{Ag} & 0 \\ P^i_{Cg} & 0 & 1 - P^i_{Cg} \end{bmatrix}.$$

where P^i_{ab} is the transition probability with which an agent i switches one's inventory from object a to object b.

We say that $X = (X^i)_{i\in B}$ and $\Pi = (\Pi^i)_{i\in B}$ are a steady-state inventory profile and a steady-state transition matrix profile if $X^i\Pi^i = X^i$ holds.

This assumption is equivalent to the following:

$$P^i_{gA}(1 - m^i_A - m^i_C) = P^i_{Ag} m^i_A, \text{ and} \tag{1}$$

$$P^i_{gC}(1 - m^i_A - m^i_C) = P^i_{Cg} m^i_C. \tag{2}$$

We consider a steady-state, symmetric, pure-strategy Markov perfect equilibrium (a steady-state equilibrium for short) of this economy, which is a set of strategies $\tau = (\tau^i)_{i\in B}$, a steady-state inventory profile $X = (X^i)_{i\in B}$ and a steady-state transition matrix profile $\Pi = (\Pi^i)_{i\in B}$, that satisfies:

i. given the strategies of other agents, X and Π, each agent chooses a trading strategy to maximize one's conditional expected utility;
ii. X and Π are consistent with τ.

4. Analysis

In a steady-state equilibrium, each agent faces a stationary environment, which allows us to formulate each agent's decision problem in a dynamic programming framework. Let V^i_g, V^i_A, and V^i_C be the value functions of agent i in a particular equilibrium. Then, the Bellman equations are

$$V^i_g = [(1 - P^i_{gA} - P^i_{gC})V^i_g + P^i_{gA}V^i_A + P^i_{gC}V^i_C]/(1 + r), \tag{3}$$

$$V^i_A = [P^i_{Ag}(u + V^i_g) + (1 - P^i_{Ag})V^i_A]/(1 + r), \tag{4}$$

$$V^i_C = [P^i_{Cg}(u + V^i_g) + (1 - P^i_{Cg})V^i_C]/(1 + r). \tag{5}$$

In Equations (4) and (5), acquiring the consumption good yields utility u from consumption and the value V_g^i of holding the production good, as consumption enables the agent to produce the good immediately.

The value functions and equilibrium strategies satisfy the following incentive compatibility constraints:

$$\begin{aligned}\tau_{gb}^i &= 1 \quad \text{iff } V_g^i < V_b^i \quad (b = A, C), \text{ and} \\ \tau_{ag}^i &= 1 \quad \text{iff } V_a^i < u + V_g^i \quad (a = A, C),\end{aligned}$$

where we adopt a tie-breaking rule according to which agents do not trade if they are indifferent between trading and not trading.

In any steady-state equilibrium, for all $i \in B$, we have

$$P_{gA}^i = \frac{\alpha(i) n_A m_A}{K} \quad \text{if } \tau_{gA}^i = 1, \tag{6}$$

$$P_{gC}^i = \frac{\gamma(i) n_C m_C}{K} \quad \text{if } \tau_{gC}^i = 1, \tag{7}$$

$$P_{Ag}^i = \frac{\alpha(i) n_A (1 - m_A)}{K} \quad \text{if } \tau_{Ag}^i = 1, \tag{8}$$

$$P_{Cg}^i = \frac{\gamma(i) n_C (1 - m_C)}{K} \quad \text{if } \tau_{Cg}^i = 1. \tag{9}$$

Let us explain how we derive Equation (6). (7)-(9) are similar in derivation. Consider an agent $i \in B$. If the agent is willing to obtain currency A, *i.e.*, $\tau_{gA}^i = 1$, then he must meet a country A's agent who owns A and desires the commodity the agent has. The probability of i's meeting with an agent in country A is $\alpha(i) n_A$. This person must hold the currency, and the fraction is m_A. Also, this person must like what i has, the fraction of which is $1/K$. Multiplying them all, we obtain (6).

Next, solving (3)-(5), we have, after some calculations,

$$V_g^i = \bar{P} u, \tag{10}$$

$$V_A^i = \frac{P_{Ag}^i}{r + P_{Ag}^i}(1 + \bar{P})u, \text{ and} \tag{11}$$

$$V_C^i = \frac{P_{Cg}^i}{r + P_{Cg}^i}(1 + \bar{P})u, \tag{12}$$

where

$$\bar{P} = \frac{P_{Ag}^i P_{gA}^i (r + P_{Cg}^i) + P_{Cg}^i P_{gC}^i (r + P_{Ag}^i)}{r\{(r + P_{Ag}^i)(r + P_{Cg}^i) + P_{gA}^i (r + P_{Cg}^i) + P_{gC}^i (r + P_{Ag}^i)\}}$$

holds.

Also, for all $i \in B$, $\tau_{Ag}^i = 1$ and $\tau_{Cg}^i = 1$ hold. This is because $\tau_{Ag}^i = 0$ implies a payoff of 0 while $\tau_{Ag}^i = 1$ implies a positive payoff at least u.

The following lemma is essentially due to Matsuyama et al. (1993).

Lemma 4.1. *In any steady-state equilibrium, for all $i \in B$*

$$u + V_g^i > Max\{V_A^i, V_C^i\} > V_g^i > 0. \tag{13}$$

$$V_A^i \gtreqless V_C^i \text{ iff } P_{Ag}^i \gtreqless P_{Cg}^i. \tag{14}$$

$$V_A^i \gtreqless V_g^i \text{ iff } P_{Ag}^i(r + P_{gC}^i + P_{Cg}^i) \gtreqless P_{gC}^i P_{Cg}^i. \tag{15}$$

$$V_C^i \gtreqless V_g^i \text{ iff } P_{Cg}^i(r + P_{gA}^i + P_{Ag}^i) \gtreqless P_{gA}^i P_{Ag}^i. \tag{16}$$

$$r[(1 - m_A^i - m_C^i)V_g^i + m_A^i V_A^i + m_C^i V_C^i] = [m_A^i P_{Ag}^i + m_C^i P_{Cg}^i]u. \tag{17}$$

The proof of this lemma is relegated to Appendix A.1.

Inequality (13) states that trade is beneficial for an agent, in the sense that engaging in trade strictly increases the agent's value relative to holding goods. Condition (14) implies that holding currency A is more valuable than holding currency C if and only if the probability of obtaining goods through currency A is higher than through currency C. Condition (15) characterizes the condition under which an agent prefers to accept currency A rather than to keep goods. To understand the basic intuition behind this condition, divide both sides of the condition by $P_{Ag}^i P_{gC}^i P_{Cg}^i$ and let r go to zero:

$$\frac{1}{P_{Cg}^i} + \frac{1}{P_{gC}^i} \gtreqless \frac{1}{P_{Ag}^i}. \tag{18}$$

In (18), $\frac{1}{P_{ab}^i}$ $(a, b = g, A, C)$ is the expected waiting time to exchange a for b. The left hand side (LHS) of (18) is the expected time to obtain her consumption good through currency C, while the right hand side (RHS) is that through currency A provided that she faces an opportunity to trade good for currency A. She accepts currency A if and only if LHS is greater than RHS. Condition (16) is the mirror image of (15). Finally, Equation (17) shows that an agent's equilibrium utility is proportional to the frequency with which the agent consumes goods.

Although the model in Matsuyama et al. (1993) differs from the present one, the equilibrium incentive structure for agents in B who have access to two currencies for consumption has the same functional relationship between value functions and transition probabilities.

Using this relationship, we consider the threshold agents i and j for whom $V_A^i = V_g^i$ and $V_C^j = V_g^j$ hold, respectively. By Lemma 4.1, for such i and j, we have $P_{Ag}^i(r + P_{gC}^i + P_{Cg}^i) = P_{gC}^i P_{Cg}^i$ and $P_{Cg}^j(r + P_{gA}^j + P_{Ag}^j) = P_{gA}^j P_{Ag}^j$, respectively.

Suppose that $\tau_{gC}^i = 1$ holds. Then, after some calculations, we have

$$\alpha(i) \gtreqless f_A(i) \quad \text{iff} \quad P_{Ag}^i(r + P_{gC}^i + P_{Cg}^i) \gtreqless P_{gC}^i P_{Cg}^i. \tag{19}$$

where

$$f_A(i) = \frac{(\gamma(i)n_C)^2 m_C(1-m_C)}{(Kr+\gamma(i)n_C)n_A(1-m_A)}.$$

Also, suppose that $\tau^i_{gA}=1$. Then, we have

$$\gamma(i) \gtreqless f_C(i) \quad \text{iff} \quad P^i_{Cg}(r+P^i_{gA}+P^i_{Ag}) \gtreqless P^i_{gA}P^i_{Ag}, \tag{20}$$

where

$$f_C(i) = \frac{(\alpha(i)n_A)^2 m_A(1-m_A)}{(Kr+\alpha(i)n_A)n_C(1-m_C)}.$$

It is verified that function $f_A(i)$ is strictly increasing in i if $i\neq 0$, and function $f_C(i)$ is strictly decreasing in i if $i \neq n_B$ (see Appendix A.1).

Define j_A and j_C as the solutions of the following equations:

$$\alpha(j_A) = f_A(j_A); \tag{21}$$

$$\gamma(j_C) = f_C(j_C). \tag{22}$$

Due to Conditions (15) and (19), j_A satisfies $V^{j_A}_A = V^{j_A}_g$. Similarly, j_C satisfies $V^{j_C}_C = V^{j_C}_g$.

It can be shown that the solution j_A to equation (21) exists uniquely in the interval $(0,n_B)$. By analogous arguments, the solution j_C to equation (22) also exists uniquely in $(0,n_B)$. A detailed discussion is provided in Appendix A.1.

An agent located at j_A who holds the good is indifferent between accepting currency A and remaining with g. By Inequality (13), such an agent is therefore strictly better off accepting currency C. Consequently, the agent located at j_C, who is indifferent between accepting currency C and g, must be farther from country C than the agent located at j_A.

Therefore, we have

$$j_C < j_A.$$

Let $J_C=[0,j_C]$ and $J_A=[j_A,n_B]$. Next, define $\bar\tau$ as follows:
for all $i\in J_C$, $\bar\tau^i_{gA}=1$ and $\bar\tau^i_{gC}=0$ hold;
for all $i\in J_A$, $\bar\tau^i_{gC}=1$ and $\bar\tau^i_{gA}=0$ hold;
for all $i\in B\setminus\{J_A\cup J_C\}$, $\bar\tau^i_{gC}=1$ and $\bar\tau^i_{gA}=1$ hold.

Following the strategy profile $\bar\tau$, agents in J_C never accept currency C in exchange for goods, and agents in J_A never accept currency A in exchange for goods. Agents who belong to neither J_C nor J_A accept both currencies.

Given $\bar\tau$, let $X_{\bar\tau}$ and $\Pi_{\bar\tau}$ be an inventory profile and a transition probability matrix profile induced by $\bar\tau$, respectively. We then have the following lemma.

Lemma 4.2. *$X_{\bar{\tau}}$ and $\Pi_{\bar{\tau}}$ are a steady-state inventory profile and a transition probability matrix profile, and satisfy the following:*

for all $i \in J_C$,

$$\begin{aligned} P^i_{gA} &= \frac{\alpha(i) n_A m_A}{K}, \\ P^i_{gC} &= 0, \\ P^i_{Ag} &= \frac{\alpha(i) n_A (1-m_A)}{K}, \\ P^i_{Cg} &= \frac{\gamma(i) n_C (1-m_C)}{K}, \\ m^i_A &= m_A, \text{ and} \\ m^i_C &= 0; \end{aligned}$$

for all $i \in J_A$,

$$\begin{aligned} P^i_{gA} &= 0, \\ P^i_{gC} &= \frac{\gamma(i) n_C m_C}{K}, \\ P^i_{Ag} &= \frac{\alpha(i) n_A (1-m_A)}{K}, \\ P^i_{Cg} &= \frac{\gamma(i) n_C (1-m_C)}{K}, \\ m^i_A &= 0, \text{ and} \\ m^i_C &= m_C; \end{aligned}$$

for all $i \in B \setminus (J_A \cup J_C)$,

$$\begin{aligned} P^i_{gA} &= \frac{\alpha(i) n_A m_A}{K}, \\ P^i_{gC} &= \frac{\gamma(i) n_C m_C}{K}, \\ P^i_{Ag} &= \frac{\alpha(i) n_A (1-m_A)}{K}, \\ P^i_{Cg} &= \frac{\gamma(i) n_C (1-m_C)}{K}, \\ m^i_A &= \frac{m_A(1-m_C)}{1-m_A m_C}, \\ m^i_C &= \frac{m_C(1-m_A)}{1-m_A m_C}, \text{ and} \\ 1 - m^i_A - m^i_C &= \frac{(1-m_A)(1-m_C)}{1-m_A m_C} \end{aligned}$$

The proof of this lemma is relegated to Appendix A.3.

By definition, agents in J_C do not exchange goods for currency C when they hold goods. Hence, the transition probability from goods to currency C is zero, and the fraction of agents holding currency C is also zero. An analogous argument applies to agents in J_A with respect to currency A. Agents who belong to neither J_C nor J_A use both currencies. Whenever an exchange between goods and a currency occurs, the corresponding transition probabilities are given by Equations (6)–(9).

Since j_A and j_C are uniquely determined, the steady-state equilibrium is uniquely determined as well.

Theorem 4.3. *The profile $(\bar{\tau}, X_{\bar{\tau}}, \Pi_{\bar{\tau}})$ is a unique steady-state equilibrium.*

The proof of this theorem is relegated to Appendix A.4.

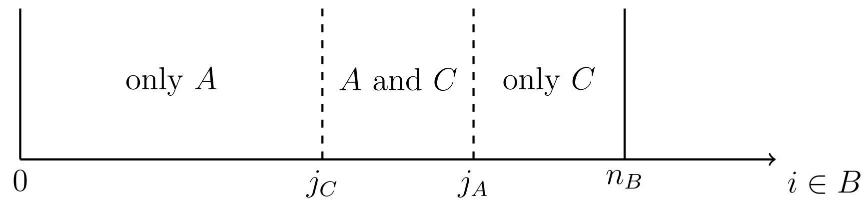

FIGURE 3. Currencies accepted by $i \in B$

Figure 3 illustrates how countries are endogenously classified into three regions depending on their relative economic proximity to the two major countries, A (the United States) and C (China). Countries in the region "only A" optimally use only currency A, those in "A and C" use both currencies, and those in "only C" rely exclusively on currency C. The classification reflects differences in trade intensity and liquidity conditions rather than political alignment.

This partition into three regions captures systematic cross-country patterns of international currency use observed in the real world. Region "only A" refers to the set of countries that predominantly use the currency of Country A for international transactions. Countries in North America fall naturally into this category. These countries are tightly integrated with the U.S. economy through trade, supply chains, and financial markets, and the U.S. dollar serves as the dominant invoicing and settlement currency in their cross-border transactions. According to Boz et al. (2025) (see p.15), North American countries rarely use Renminbi for invoicing compared with Asian countries.[4] In terms of the model, their economic distance to Country A is sufficiently small that adopting only currency A is optimal.

Region "A and C" consists of countries that use both currencies in international trade. A prominent example is found in Asia (see, *e.g.*, Boz et al. (2025)). These economies maintain substantial trade relationships with both the United States and China. In practice, the U.S. dollar remains the dominant invoicing currency, while trade with China increasingly involves China-related settlement arrangements, including limited use of the renminbi. This mixed currency usage corresponds to an intermediate position in the model, where neither currency fully dominates due to comparable economic proximity to both major countries.

Region "only C" corresponds to economic environments in which, as an equilibrium outcome, agents predominantly conduct transactions using currency C, capturing limited effective use of currency A along particular dimensions of trade or finance. North Korea may be viewed as an illustrative example, as international transactions involving the country

[4]After Russian invasion to Ukraine, European countries' share of Renminbi sharply increases.

rely heavily on arrangements linked to Country C, reflecting restricted access to the global dollar-based financial system.

5. Comparative Statics

This section does some comparative statics on the unique steady-state equilibrium. First of all, we have the following statement.

Theorem 5.1. *We have the following relationship:*

(i) j_A *is decreasing in* m_A*;*
(ii) j_A *is increasing in* n_A *and decreasing in* n_C*;*
(iii) j_A *is decreasing in* m_C *if* $m_C \leq 0.5$*, and it is increasing in* m_C *if* $m_C > 0.5$*.*

We have the mirror image for j_C*.*

The proof of this theorem is relegated to Appendix A.5.

Theorem 5.1 characterizes how the acceptability thresholds j_A and j_C respond to changes in the distribution of money holdings and population size. The key mechanism underlying these comparative statics is expected waiting time to complete a full trade cycle:

$$\text{good} \to \text{money} \to \text{good}.$$

Consider (i) of Theorem 5.1 first. An increase in m_A raises the fraction of agents holding currency A and therefore reduces the fraction holding goods, $1 - m_A$. While a seller is more likely to meet a currency-A holder, once the currency is accepted it becomes more difficult to exchange it back for goods. Consequently, the expected waiting time to obtain the consumption good increases, lowering the continuation value of currency A and thus reducing the acceptability threshold j_A.

This waiting-time channel is driven by a supply constraint rather than by the restriction on portfolio holdings (i.e., that agents can hold at most one unit of goods or currency at a time). Once production good is sold out, the producer cannot supply the next one until production occurs again. When m_A increases, demand for goods rises and stockouts become more frequent, which prolongs the time required to obtain goods, – *i.e.*, complete the cycle by "currency $A \to$ good"– relative to the alternative cycle "good $\to$ currency $C \to$ good".

The key economic force is, therefore, that production is lumpy and time-consuming: higher demand exhausts current productive capacity and delays future trading opportunities. This supply constraint can be interpreted as limited production capacity, inventory shortages, or delivery lags that prevent agents from responding to all the demands.

Next, consider (ii) of Theorem 5.1. An increase in the economic size of country A, captured by a larger population measure n_A, increases j_A. A higher n_A implies an expansion in goods production, increasing the availability of goods, and therefore, reducing the expected waiting time. As a result, the supply constraint is relaxed, the real value of currency A rises, and the acceptability threshold j_A increases.

Third, consider (iii) of Theorem 5.1, the effect of m_C on j_A. For an agent who currently holds currency A but intends to trade it for currency C in order to complete the cycle "good $\to$ currency $C \to$ good", the value of currency C depends on how frequently agents holding C are met. When m_C is low, currency C is scarce, and increasing m_C improves its liquidity by reducing the expected waiting time to encounter a currency-C holder. However, when m_C exceeds a certain threshold, congestion among currency-C holders dominates, increasing the waiting time to convert currency C into goods. As a result, the value of currency C—and hence the incentive to wait for trades via C—is maximized at $m_C = 0.5$. Accordingly, as m_C approaches 0.5, the cutoff j_A decreases.

This non-monotonicity implies that $m_C = 0.5$ plays the role of an "optimal money supply" for currency C in this environment. While the exact value 0.5 is specific to the structure of the model, the underlying logic is more general: an intermediate level of money supply balances liquidity provision against congestion effects. This balance determines the relative attractiveness of trading via different currencies and underlies the comparative statics in Theorem 5.1 (iii).

5.1. **Countries A and C.** Let P^A_{ab} be the transition probability with which an agent i in country A switches one's inventory from object a to object b $(a, b = A, g)$. Similarly define P^C_{ab} $(a, b = C, g)$. Then, we define V^A_g, V^A_A,V^C_g, and V^C_C as follows.

$$\begin{aligned} V^A_g &= [(1 - P^A_{gA})V^A_g + P^A_{gA}V^A_A]/(1 + r), \\ V^A_A &= [P^A_{Ag}(u + V^A_g) + (1 - P^A_{Ag})V^A_A]/(1 + r), \\ V^C_g &= [(1 - P^C_{gC})V^C_g + P^C_{gC}V^C_C]/(1 + r), \text{ and} \\ V^C_C &= [P^C_{Cg}(u + V^C_g) + (1 - P^C_{Cg})V^C_C]/(1 + r). \end{aligned}$$

Given equilibrium $(\bar{\tau}, X_{\bar{\tau}}, \Pi_{\bar{\tau}})$, let the welfare W^A of country A be the expected payoff of the agents therein, *i.e.*,

$$\begin{aligned} W^A &= r[(1 - m_A)V^A_g + m_A V^A_A] \\ &= m_A P^A_{Ag} u. \end{aligned} \tag{23}$$

Similarly, the welfare W^C of country C be the expected payoff of the agents therein, *i.e.*,

$$\begin{aligned} W^C &= r[(1-m_C)V_g^C + m_C V_C^C] \\ &= m_C P_{Cg}^C u. \end{aligned} \tag{24}$$

Given equilibrium $(\bar{\tau}, X_{\bar{\tau}}, \Pi_{\bar{\tau}})$, we have

$$\begin{aligned} P_{Ag}^A &= \frac{1}{K}[(1-m_A)n_A + \int_0^{j_C} \alpha(i)(1-m_A^i)d\mu + \int_{j_C}^{j_A} \alpha(i)(1-m_A^i - m_C^i)d\mu] \\ &= \frac{1-m_A}{K(1-m_A m_C)}\left[(1-m_A m_C)n_A + (1-m_A m_C)\int_0^{j_C} \alpha(i)d\mu + (1-m_C)\int_{j_C}^{j_A} \alpha(i)d\mu\right]. \end{aligned}$$

Similarly, we have

$$\begin{aligned} P_{Cg}^C &= \frac{1}{K}[(1-m_C)n_C + \int_{j_C}^{j_A} \gamma(i)(1-m_A^i - m_C^i)d\mu + \int_{j_A}^{n_B} \gamma(i)(1-m_C^i)d\mu] \\ &= \frac{1-m_C}{K(1-m_A m_C)}\left[(1-m_A m_C)n_C + (1-m_A)\int_{j_C}^{j_A} \gamma(i)d\mu + (1-m_A m_C)\int_{j_A}^{n_B} \gamma(i)d\mu\right]. \end{aligned}$$

Lemma 5.2. *W^A is increasing in n_A. Similarly, W^C is increasing in n_C.*

The proof of this lemma is relegated to Appendix A.6.

An increase in a country's population raises its welfare through two effects. Focusing on country A, the first is a direct effect: population growth increases the volume of agents holding goods. The second is an indirect effect: as shown in Theorem 5.1, both j_C and j_A increase with population in country A. An increase in j_C corresponds to an expansion of the area in which currency A is used exclusively, while an increase in j_A leads to the broader circulation area of currency A. Trading opportunities expand through these two effects, which leads to higher welfare.

To analyze the effect of matching probability with agents in B on W^A, we introduce a non-negative continuously differentiable function $h : [0, n_B] \to \mathbb{R}$ such that $h(i) > 0$ for all $i \in (0, n_B)$, and $h(0) = h(n_B) = 0$. Now consider $\alpha(i) + \varepsilon h(i)$ for $\varepsilon \geq 0$. Note that $\alpha(i) + \varepsilon h(i) > \alpha(i)$ for all $i \in (0, n_B)$ if $\varepsilon > 0$.

By construction of the function h, the altered matching probability $\alpha(\cdot) + \varepsilon h(\cdot)$ lies weakly above the original $\alpha(\cdot)$ while preserving the same boundary values. Figure 4 illustrates such an alteration of $\alpha(\cdot)$.

Because it is difficult to characterize how welfare responds to an arbitrary direct change in $\alpha(\cdot)$, we instead analyze comparative statics with respect to ε. That is, we study how country A's welfare changes as ε increases, i.e., as the matching probability between country i and country A rises.

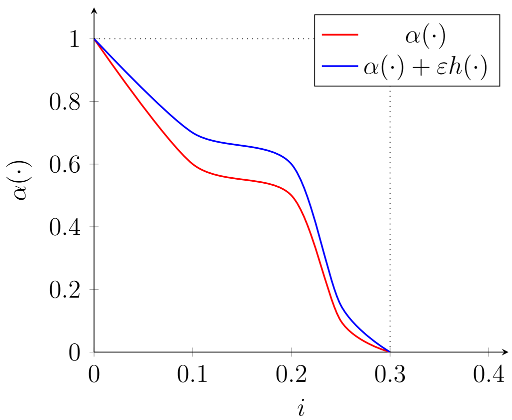

FIGURE 4. $\alpha(\cdot)$ and $h(\cdot)$: $n_B = 0.3$

Given other parameters, $W^A(\varepsilon; h)$ denotes the welfare of country A when $\alpha + \varepsilon h$ is used in place of α.

Similarly, $W^C(\varepsilon; h)$ is defined when $\gamma + \varepsilon h$ is used in place of γ. We then have the following statement.

Lemma 5.3. *Suppose that h is a function as defined above. Then $W^A(\varepsilon; h)$ is increasing in ε. Also, $W^C(\varepsilon; h)$ is increasing in ε.*

The proof of this lemma is relegated to Appendix A.7.

The welfare of country A increases as the probability of meeting agents in region B rises, reflecting an expansion in trading opportunities. The same result holds for country C. To see this, consider country A. An increase in the probability of meeting agents in region B expands trading opportunities through two distinct channels.

First, it raises the likelihood that agents from country A meet agents in countries where currency A is already in circulation. Second, and more importantly, the equilibrium thresholds j_C and j_A shift to the right, expanding the circulation area of currency A, including the region in which it is used exclusively. Together, these two effects increase trading opportunities and raise welfare.

6. Sanctions

This section examines the effect of financial sanctions by country A on a small country. Sanctions by country C can be analyzed in a symmetric manner (see Appendix A.9).

A *financial sanction* by country A on country $j \in B$ is defined by setting $\alpha(j) = 0$, which is treated as an exogenous constraint imposed on country j. Note that the parameter $\alpha(j)$ governs the matching technology between agents in countries A and j. This definition implies that agents from country A do not enter the matching pool for country j during the sanction period, so that agents in j are never matched with agents from A.

In the present paper, sanctions are not modeled as a strategic choice by country A. Instead, they are treated as an exogenous policy. In this sense, the analysis of the present section is comparative statics in order to study their equilibrium effects on currency acceptability and welfare in decentralized trade.

In the real world, a financial sanction typically takes the form of a prohibition on the use of currency A by agents in country j. This implies that country j is excluded from the currency-A system, either voluntarily or as a result of external enforcement. We incorporate this restriction into the model in reduced form through the matching parameter $\alpha(j)$ in order to analyze the redistribution of currency use following the sanction.

This modeling choice is motivated by the fact that access to currency A is governed by financial and payment infrastructures controlled by country A, so that a sanction represents a unilateral restriction on access to this infrastructure.

6.1. **Welfare.** First, we consider country j's welfare in the absence of any sanction.

The welfare W^j of country $j \in B$ is given by

$$W^j = \left[m_A^j P_{Ag}^j + m_C^j P_{Cg}^j\right] u. \tag{25}$$

Suppose that this country trades with both A and C.

Due to Lemma 4.2, we have

$$\begin{aligned} m_A^j &= \frac{m_A(1-m_C)}{1-m_A m_C}, \\ m_C^j &= \frac{m_C(1-m_A)}{1-m_A m_C}. \end{aligned}$$

Substituting these equations into (25), we obtain

$$W^j = \frac{(1-m_A)(1-m_C)}{1-m_A m_C}\left[\alpha(j) n_A m_A + \gamma(j) n_C m_C\right] \frac{u}{K}. \tag{26}$$

If, on the other hand, j uses only currency A without sanction, *i.e.*, $j \leq j_C$ holds, then $m_A^j = m_A$ holds, and the welfare W^j is given by

$$W^j = m_A(1-m_A)\alpha(j) n_A \frac{u}{K}. \tag{27}$$

Next, we consider a sanction by country A on country j. Suppose that if sanctioned, j can no longer trade through currency A.

Let $\hat{\alpha}(j)$ and $\hat{\gamma}(j)$ denote the post-sanction matching probabilities of an agent in country j with agents from countries A and C, respectively. By definition of the sanction, we have $\hat{\alpha}(j) = 0$. For $\hat{\gamma}(j)$, we specify

$$\hat{\gamma}(j) = \gamma(j) + \lambda\alpha(j),$$

where $\lambda \in [0, 1]$. This formulation allows the matching probability with agents from country C to remain unchanged or to increase following the sanction.

The economic interpretation is as follows. As a result of the sanction, agents in country j who would otherwise have been matched with agents from country A may instead be matched with agents from country C. In the baseline specification, the arrival probability of agents from country C to country j is assumed to remain unchanged, corresponding to $\lambda = 0$. In reality, sanctions may induce agents from country C to visit country j more frequently, for example because trading opportunities with country A become unavailable. The parameter λ captures this possibility by allowing the matching probability between agents from countries C and j to increase after the sanction. We conduct the welfare analysis under this modified matching structure.

The welfare $\hat{W}^j$ of country $j \in B$ after the sanction is given by

$$\hat{W}^j = \left[\hat{m}_C^j \hat{P}_{Cg}^j\right] u.$$

Since $\hat{m}_C^j = m_C$ and $\hat{P}_{Cg}^j = \hat{\gamma}(j) n_C (1 - m_C)$ hold, we obtain

$$\hat{W}^j = [m_C \hat{\gamma}(j) n_C (1 - m_C)] \frac{u}{K}. \tag{28}$$

We divide the analysis into two cases.

6.2. **Case 1:** $j_C < j < j_A$**.** First, consider the case of $j_C < j < j_A$, *i.e.*, country j accepts both currencies. Subtracting (26) from (28), we obtain

$$\hat{W}^j - W^j \propto m_A m_C (1 - m_C) \gamma(j) n_C + (1 - m_A m_C) m_C \lambda \alpha(j) n_C - (1 - m_A) m_A \alpha(j) n_A. \tag{29}$$

Note that the last expression is increasing in λ.

We first evaluate (29) at $j = j_A$. Substituting (21) into (29), given $\lambda = 0$, we obtain

$$\hat{W}^{j_A} - W^{j_A} \propto \gamma(j_A) n_C r > 0, \tag{30}$$

where W^{j_A} is calculated as the welfare when j_A accepts currency A.

Equation (30) indicates that the welfare after the sanction is greater than before, even though each agent is indifferent between accepting and rejecting currency A. The intuition is as follows. The welfare before the sanction is given by

$$W^{j_A} = r\left[m_A^{j_A} V_A^{j_A} + m_C^{J_A} V_C^{j_A} + (1 - m_A^{j_A} - m_C^{j_A}) V_g^{j_A}\right].$$

In this expression, we have $V_C^{j_A} > V_g^{j_A} = V_A^{j_A}$, where the last equality comes from the fact that the agent is indifferent between accepting and rejecting currency A. If individual agents are banned from using currency A, they simply shift to using currency C more often than before, which leads to a higher welfare after the sanction.

Since $\hat{W}^j - W^j$ is coninuous in j, we have the following result.

Theorem 6.1. *There exists a neighborhood U of j_A for which for all $j \in U$ with $j \leq j_A$, $\hat{W}^j > W^j$ holds.*

Theorem 6.1 is a striking result as it states that under any condition, there are some countries that are better off by being sanctioned provided that there is a competitor for currency hegemony. These are the countries in which agents are almost indifferent between accepting and rejecting currency A.

The intuition behind this result is as follows. Consider a country whose agents use both currencies A and C, and who trade more frequently with Country C than with Country A. For such an agent, exchanging her production good for currency C yields a relatively large surplus, whereas an exchange for currency A generates a smaller, *albeit* still positive, surplus. If the agent is initially matched with a trader offering currency A while she holds her production good, it is optimal for her to accept the trade.

While this acceptance decision is individually optimal, it has implications for aggregate trading efficiency. To evaluate these implications, we measure welfare by the expected waiting time to complete a trade cycle of the form:

$$\text{good} \to \text{money} \to \text{good}.$$

In this cycle, trading via A may crowd out trading via C, and the presence of A worsens the welfare.

Even in such a case, each agent may accept A because what he compares is the expected *discounted* waiting time between

$$\text{currency } A \to \text{good}$$

and

$$\text{good} \to \text{currency } C \to \text{good}.$$

That is, the agent accepts currency A if the expected discounted time required to complete a full trade cycle through currency A may be shorter than that through currency C provided that the agent faces the trade opportunity to obtain currency A right away. As a result, accepting currency A can be optimal whenever the expected discounted time to convert currency A back into goods is shorter than that required to complete a trade cycle via currency C.

This discrepancy between agents' private incentives to accept currency A and the social welfare evaluation generates an inefficiency: the presence of currency A crowds out trades via the more valuable currency C. Consequently, shutting down the trading channel via currency A may improve aggregate welfare.

As in Theorem 5.1 (i), the mechanism behind Theorem 6.1 is not the restriction on portfolio holdings but the presence of a supply constraint. Once production goods are sold out, the producer cannot supply another unit until production occurs again. Consequently, accepting a low-surplus currency today may preclude the agent from participating in a higher-surplus exchange later, even when each trade is individually profitable.

In this sense, the mechanism does not rely on restrictions on simultaneous currency holdings *per se*. Even in an environment where agents are allowed to hold one unit of production goods and one unit of each currency simultaneously, the same logic would apply as long as supplying a good exhausts the agent's current productive capacity. What matters is that production is a lumpy and time-consuming process, so that agents face a trade-off between current and future exchange opportunities.

From an economic perspective, this supply constraint can be interpreted as capturing limited production capacity, inventory constraints, or delivery lags that prevent agents from instantaneously responding to multiple demands. These features are natural in environments where access to trading opportunities is uncertain and politically or institutionally mediated, as in the context of competing currency systems.

An illustrative example of forced currency substitution under sanctions is provided by Iran. Following the tightening of U.S. financial sanctions, Iran's access to dollar-denominated payment systems was severely restricted. As a result, Iranian firms and trading partners were largely unable to settle international transactions using currency A.

Despite these restrictions, Iran continued to engage in international trade, particularly with China, by relying on alternative settlement mechanisms that did not involve the dollar-based financial system. In practice, this involved trade arrangements denominated in non-dollar currencies and bilateral settlement mechanisms that bypassed dollar clearing.

This episode suggests that when trade channels via a dominant currency are exogenously shut down by sanctions, countries may resort to alternative currencies even if those currencies would not be chosen under normal market conditions. Importantly, this substitution was not driven by superior liquidity or efficiency of the alternative currency, but rather by the elimination of the dollar-based trading channel itself.

In the context of the present model, sanctions can be interpreted as an exogenous removal of the trading channel via currency A. Consistent with our theoretical results, such a restriction may increase the relative importance of alternative currencies in affected trade relationships, even though the dominant currency remains welfare-superior in the absence of sanctions.

One might ask whether an economy that stands to gain from sanctions could improve welfare by committing in advance—through legislation or other institutional mechanisms—to refrain from using currency A. The model suggests that such a commitment is difficult to sustain because of a misalignment between aggregate welfare and individual incentives.

While shutting down the trading channel via currency A can increase aggregate welfare, some individual agents incur immediate losses by forgoing profitable trading opportunities involving currency A. As a result, abandoning currency A is not incentive compatible at the individual level. Because these losses are borne by a non-negligible fraction of agents, a unilateral and permanent restriction on the use of a widely accepted currency would generate substantial domestic resistance.

In this sense, the difficulty of commitment arises endogenously from the economic incentives emphasized by the model, rather than from exogenous political or institutional frictions. This helps explain why economies that may benefit *ex post* from sanctions do not necessarily choose to restrict currency usage *ex ante*.

Theorem 6.1 also suggests that welfare may increase for countries positioned at intermediate distances between major currency areas. For such economies, reliance on a single dominant currency is not optimal in equilibrium. Implementing a voluntary and permanent restriction on a widely used currency would require overcoming substantial domestic resistance from agents who benefit from its liquidity, as well as political and institutional constraints. As a result, while sanctions can exogenously eliminate a trading channel, a similar outcome is unlikely to arise endogenously through policy commitment in the real world.

A companion result states that for any country $j \in B$, if country C is sufficiently large compared with country A, then the sanction by A results in the improvement of the sanctioned country.

Theorem 6.2. *For any country $j \in (j_C, j_A)$, $\hat{W}^j > W^j$ holds if and only if*

$$\frac{n_C}{n_A} > \frac{m_A(1-m_A)\alpha(j)}{m_A m_C(1-m_C)\gamma(j) + (1-m_A m_C)m_C\lambda\alpha(j)}$$

holds.

6.3. **Case 2: $j \leq j_C$.** Next, we consider the case of $j \leq j_C$, *i.e.*, country j accepts only currency A before sanction. In this case, the welfare before sanction is given by (27). Subtracting it from (28), we obtain

$$\hat{W}^j - W^j \propto m_C(1-m_C)(\gamma(j) + \lambda\alpha(j))n_C - (1-m_A)m_A\alpha(j)n_A.$$

We state this as a theorem.

Theorem 6.3. *For any country $j < j_C$, $\hat{W}^j > W^j$ holds if and only*

$$\frac{n_C}{n_A} > \frac{m_A(1-m_A)\alpha(j)}{m_C(1-m_C)(\gamma(j)+\lambda\alpha(j))}$$

holds.

6.4. **A sanction on multiple countries.** Given equilibrium $(\bar{\tau}, X_{\bar{\tau}}, \Pi_{\bar{\tau}})$, Suppose country A imposes a sanction on $\tilde{J} \subset [0, j_A]$ with $\mu(\tilde{J}) > 0$.

Let J_1 be a subset of $[0, j_C]$ satisfying

$$J_1 = [0, j_C] \setminus \tilde{J}.$$

Similarly, define J_2 be a subset of $[j_C, j_A]$ satisfying

$$J_2 = (j_C, j_A) \setminus \tilde{J}.$$

Note that, due to Equation (23), the welfare of country A is written as $m_A P^A_{Ag} u$.

Let $\tilde{W}^A$ be the welfare of country A after sanctioning countries $\tilde{J}$, *i.e.*, $\tilde{W}^A = m_A \tilde{P}^A_{Ag} u$ where $\tilde{P}^A_{Ag}$ is

$$\begin{aligned}
\tilde{P}^A_{Ag} &= \frac{1}{K}[(1-m_A)n_A + \int_{j\in J_1} \alpha(i)(1-m^i_A)d\mu + \int_{j\in J_2} \alpha(i)(1-m^i_A-m^i_C)d\mu] \\
&= \frac{1}{K}[(1-m_A)n_A + \int_{j\in J_1} \alpha(i)(1-m_A)d\mu + \int_{j\in J_2} \alpha(i)\frac{(1-m_A)(1-m_C)}{1-m_A m_C}d\mu] \\
&< \frac{1}{K}\left[(1-m_A)n_A + \int_0^{j_C} \alpha(i)(1-m_A)d\mu + \int_{j_C}^{j_A} \alpha(i)\frac{(1-m_A)(1-m_C)}{1-m_A m_C}d\mu\right] \\
&= P^A_{Ag},
\end{aligned}$$

where the strict inequality comes from $J_1 \subsetneq [0, j_C]$ and/or $J_2 \subsetneq (j_c, j_A)$. Thus, we have

$$\tilde{W}^A < W^A. \tag{31}$$

Also, due to Equation (24), the welfare of country C is

$$W^C = m_C P^C_{Cg} u.$$

Define the welfare of country C after country A's sanction on $\tilde{J}$ as $\tilde{W}^C = m_C \tilde{P}^C_{Cg} u$ with

$$\begin{aligned}
\tilde{P}^C_{Cg} &= \frac{1}{K}[(1-m_C)n_C + \int_{j_C}^{j_A} \gamma(i)(1-m^i_A - m^i_C)d\mu + \int_{j_A}^{n_B} \gamma(i)(1-m^i_C)d\mu \\
&\quad + \int_{j\in\tilde{J}} (\gamma(i)+\lambda\alpha(i))(1-m^i_C)d\mu] \\
&= \frac{1}{K}[(1-m_C)n_C + \int_{j_C}^{j_A} \gamma(i)\frac{(1-m_A)(1-m_C)}{1-m_Am_C}d\mu + \int_{j_A}^{n_B} \gamma(i)(1-m_C)d\mu \\
&\quad + \int_{j\in\tilde{J}} (\gamma(i)+\lambda\alpha(i))(1-m_C)d\mu] \\
&> \frac{1}{K}\left[(1-m_C)n_C + \int_{j_C}^{j_A} \gamma(i)\frac{(1-m_A)(1-m_C)}{1-m_Am_C}d\mu + \int_{j_A}^{n_B} \gamma(i)(1-m_C)d\mu\right] \\
&= P^C_{Cg},
\end{aligned}$$

where the strict inequality comes from the additional positive term. Thus, we have

$$\tilde{W}^C > W^C. \tag{32}$$

Due to Equations (31) and (32), we have the following theorem.

Theorem 6.4. *Sanction by country A on a set of small countries with a positive measure decreases A's welfare, but increases C's welfare.*

Theorem 6.4 states that sanctions imposed by country A reduce A's own welfare while increasing the welfare of its rival currency issuer C.

This welfare effect is not a mechanical trade-diversion result in which a country with more trading opportunities gains automatically. Rather, sanctions alter equilibrium currency acceptability in decentralized trade, changing the composition of trading relationships across currencies. Consequently, the welfare reversal arises from a general-equilibrium shift in the currency regime rather than a partial-equilibrium reallocation of trade volumes. Importantly, this effect can occur even when aggregate trading opportunities remain unchanged (e.g., when $\lambda = 0$), as the reorganization of currency usage alone modifies liquidity and welfare.

Sanctions are often viewed as instruments intended to weaken targeted economies while strengthening the sanctioning country's geopolitical or economic position. Existing analyses have therefore focused primarily on the effects of sanctions on trade volumes, political outcomes, and the welfare of sanctioning and sanctioned countries themselves (see, *e.g.*, a literature review in Pala (2021)). From the perspective of competition between currency systems, however, the welfare implications for third countries—especially rival currency-issuing countries—have received far less attention. The present analysis highlights an additional and previously unexplored channel through which financial sanctions can reshape the international currency system and thereby affect welfare beyond the directly involved parties.

In this paper, the welfare effect arises endogenously from the optimal reallocation of trade and currency use by agents in sanctioned countries. When country A imposes sanctions on a subset of small countries, it restricts their access to trade transactions involving currency A, thereby altering agents' currency acceptance decisions. As a result, these countries optimally reallocate trade toward country C, increasing the circulation and acceptability of currency C in international transactions.

Importantly, this outcome does not rely on *ad hoc* assumptions about political alignment, exogenous trade diversion, or direct expansion of C's trading opportunities, especially when $\lambda = 0$. Instead, the expansion of trade via country C arises endogenously from changes in currency acceptability and liquidity in a decentralized trading environment. Taken together, these results identify an endogenous channel through which financial sanctions may generate unintended welfare losses in the presence of competition between currency systems.

7. Conclusion

This paper studies the interaction between financial sanctions and currency hegemony in a decentralized trading environment. We develop a search-theoretic model featuring two large countries, A and C, and a continuum of small countries that differ in their probabilities of trading with the two large economies. Only the currencies issued by the large countries can circulate internationally, while agents in small countries choose endogenously whether to accept each currency. In equilibrium, small countries are endogenously sorted into three groups: those that accept only currency A, those that accept only currency C, and those that accept both currencies.

The model delivers several implications for the effectiveness of financial sanctions. When there exists competition for currency hegemony, sanctions imposed by country A do not necessarily reduce the welfare of targeted countries. On the contrary, some sanctioned countries may benefit from sanctions by reallocating trade toward country C. Moreover, sanctions imposed by country A can increase country C's welfare by expanding its role as a trading partner and strengthening the circulation of its currency. These results highlight a limitation of financial sanctions in an international environment where alternative currency systems coexist.

From a broader perspective, the analysis suggests that the effectiveness of financial sanctions depends critically on the structure of international currency competition. When access to a dominant currency is restricted, targeted countries may adjust by relying more heavily on alternative currencies and trading partners, thereby weakening the intended impact of sanctions. This mechanism is consistent with recent real-world observations in which sanctioned economies have increasingly sought trade and settlement arrangements outside U.S. dollar–based financial infrastructure.

Beyond these implications for the effectiveness of financial sanctions, this paper also provides a framework for analyzing the incentives of countries that do not issue a major international currency but are economically integrated with multiple competing currency areas. By focusing on small or intermediate economies positioned between rival currency blocs, the model clarifies how shifts in geopolitical and economic rivalry affect trade patterns, currency usage, and welfare.

This perspective is particularly relevant for Asian countries, including Japan, which do not have a settlement system of its own. Although many Asian countries have long been economically and financially tied to the United States, they are geographically close to China, and their trade and financial linkages with China have expanded substantially in recent years, alongside China's economic growth. As rivalry between major currency areas intensifies, Asian countries face strategic choices regarding the extent to which it engages with each currency bloc. The analysis in this paper highlights that such choices are not neutral: changes in trade integration and currency usage alter the incentives faced by domestic agents and can affect the relative value and liquidity of currencies used in international transactions.

In light of these considerations, the model points to important directions for future research. While we allow for heterogeneity in the degree of economic integration between small countries and major currency issuers, this integration is captured by an exogenous matching structure. In reality, however, the extent of economic integration between a small country and competing major powers is itself an endogenous policy and strategic outcome. Incorporating endogenous integration decisions into the model would be a natural and important extension. In this sense, the present framework offers a useful starting point for thinking systematically about the policy choices faced by countries like Japan as they navigate an increasingly fragmented international monetary system.

Several other extensions remain for future research. First, this paper abstracts from trade among small countries in order to maintain a tractable equilibrium structure. Allowing trade among small countries could introduce additional channels for currency exchange and substitution, potentially amplifying or dampening the effects identified here. Second, incorporating a centralized Walrasian market, following the approach of Lagos and Wright (2005) or Kannan (2009), would allow the analysis of open-economy monetary policies alongside sanctions within a unified framework. Exploring these extensions would deepen our understanding of how monetary arrangements and policy interventions interact in an increasingly fragmented international financial system.

Appendix A. Proofs

A.1. **Proof of Lemma 4.1.**

Proof. For any $i \in B$, either $\tau^i_{gA}\tau^i_{Ag} = 1$ or $\tau^i_{gC}\tau^i_{Cg} = 1$ holds (better than no-trading). Thus, in any steady-state equilibrium, for all $i \in B$, $V^i_g > 0$ holds.

Substituting (10) into (11) and (12), we obtain

$$V^i_A = \frac{P^i_{Ag}}{r + P^i_{Ag}}(u + V^i_g), \text{ and} \tag{33}$$

$$V^i_C = \frac{P^i_{Cg}}{r + P^i_{Cg}}(u + V^i_g), \tag{34}$$

respectively.

Thus, we have

$$\begin{aligned} & u + V^i_g - V^i_A \\ = \; & u + V^i_g - \frac{P^i_{Ag}}{r + P^i_{Ag}}(u + V^i_g) = \frac{r}{r + P^i_{Ag}}(u + V^i_g) > 0, \text{ and} \\ & u + V^i_g - V^i_C \\ = \; & u + V^i_g - \frac{P^i_{Cg}}{r + P^i_{Cg}}(u + V^i_g) = \frac{r}{r + P^i_{Cg}}(u + V^i_g) > 0. \end{aligned}$$

Therefore, $u + V^i_g > \text{Max}\{V^i_A, V^i_C\}$ holds.

Next, we want to show $\text{Max}\{V^i_A, V^i_C\} > V^i_g$. Suppose the contrary, *i.e.*, $\text{Max}\{V^i_A, V^i_C\} \leq V^i_g$. Then, Equation (3) implies

$$(1 + r)V^i_g \leq V^i_g.$$

This is a contradiction because $V^i_g > 0$ holds, and because $(1 + r) > 1$ holds.

Equations (10), (11), and (12) imply the following:

$$V^i_A - V^i_C = \frac{r(P^i_{Ag} - P^i_{Cg})}{(r + P^i_{Ag})(r + P^i_{Cg})}(u + V^i_g), \tag{35}$$

$$V^i_A - V^i_g = \frac{P^i_{Ag}(r + P^i_{gC} + P^i_{Cg}) - P^i_{gC}P^i_{Cg}}{(r + P^i_{Ag})(r + P^i_{Cg}) + P^i_{gA}(r + P^i_{Cg}) + P^i_{gC}(r + P^i_{Ag})}u, \text{ and} \tag{36}$$

$$V^i_C - V^i_g = \frac{P^i_{Cg}(r + P^i_{gA} + P^i_{Ag}) - P^i_{gA}P^i_{Ag}}{(r + P^i_{Ag})(r + P^i_{Cg}) + P^i_{gA}(r + P^i_{Cg}) + P^i_{gC}(r + P^i_{Ag})}u. \tag{37}$$

In Equations (35), (36), and (37), the denominators, r, u, and V^i_g are positive. Therefore, they imply conditions (14), (15), and (16), respectively.

Finally, we show Equation (17). From Equations (1) and (2), we have

$$m_A^i = \frac{P_{gA}^i}{P_{Ag}^i}(1 - m_A^i - m_C^i), \text{ and} \tag{38}$$

$$m_C^i = \frac{P_{gC}^i}{P_{Cg}^i}(1 - m_A^i - m_C^i). \tag{39}$$

Substituting Equations (38) and (39) into the left hand side of Equation (17), we have

$$\begin{aligned} & r[(1 - m_A^i - m_C^i)V_g^i + m_A^i V_A^i + m_C^i V_C^i] \\ = \; & r(1 - m_A^i - m_C^i)\left(V_g^i + \frac{P_{gA}^i}{P_{Ag}^i}V_A^i + \frac{P_{gC}^i}{P_{Cg}^i}V_C^i\right). \end{aligned} \tag{40}$$

Substituting (33) and (34) into (40), we obtain

$$\begin{aligned} & r[(1 - m_A^i - m_C^i)V_g^i + m_A^i V_A^i + m_C^i V_C^i] \\ = \; & \frac{r(1 - m_A^i - m_C^i)}{(r + P_{Ag}^i)(r + P_{Cg}^i)}\Big\{ \left((r + P_{Ag}^i)(r + P_{Cg}^i) + P_{gA}^i(r + P_{Cg}^i) + P_{gC}^i(r + P_{Ag}^i)\right) V_g^i \\ & + \left(P_{gA}^i(r + P_{Cg}^i) + P_{gC}^i(r + P_{Ag}^i)\right) u \Big\} \end{aligned} \tag{41}$$

Substituting (10) into (41), we obtain, after some calculation,

$$\begin{aligned} & r[(1 - m_A^i - m_C^i)V_g^i + m_A^i V_A^i + m_C^i V_C^i] \\ = \; & \frac{(1 - m_A^i - m_C^i)}{(r + P_{Ag}^i)(r + P_{Cg}^i)}\Big\{ \left(P_{Ag}^i P_{gA}^i(r + P_{Cg}^i) + P_{Cg}^i P_{gC}^i(r + P_{Ag}^i)\right) \\ & + r\left(P_{gA}^i(r + P_{Cg}^i) + P_{gC}^i(r + P_{Ag}^i)\right) \Big\} u \\ = \; & \frac{(1 - m_A^i - m_C^i)}{(r + P_{Ag}^i)(r + P_{Cg}^i)}(r + P_{Ag}^i)(r + P_{Cg}^i)(P_{gA}^i + P_{gC}^i)u \\ = \; & (1 - m_A^i - m_C^i)(P_{gA}^i + P_{gC}^i)u. \end{aligned}$$

Using Equations (1) and (2) again, we have

$$(1 - m_A^i - m_C^i)P_{gA}^i = m_A^i P_{Ag}^i, \text{ and} \tag{42}$$

$$(1 - m_A^i - m_C^i)P_{gC}^i = m_C^i P_{Cg}^i. \tag{43}$$

Inserting these equations yield

$$(1 - m_A^i - m_C^i)(P_{gA}^i + P_{gC}^i)u = (m_A^i P_{Ag}^i + m_C^i P_{Cg}^i)u.$$

Thus, we have

$$r[(1 - m_A^i - m_C^i)V_g^i + m_A^i V_A^i + m_C^i V_C^i] = (m_A^i P_{Ag}^i + m_C^i P_{Cg}^i)u.$$

□

A.2. **Lemmata.**

Lemma A.1. *Function f_A is strictly increasing in i if $i \neq 0$, and function f_C is strictly decreasing in i if $i \neq n_B$.*

Proof. Since $\dfrac{(n_C)^2 m_C(1-m_C)}{n_A(1-m_A)}$ is a positive constant value, we have

$$\begin{aligned}\frac{df_A}{di} &\propto 2\gamma(i)\gamma'(i)(Kr+\gamma(i)n_C)-(\gamma(i))^2 n_C\gamma'(i)\\ &= \gamma(i)\gamma'(i)(2Kr+\gamma(i)n_C) > 0 \ \ \forall i \in (0, n_B),\end{aligned}$$

where the last inequality holds because $\gamma'(\cdot) > 0$ holds along with $\gamma(i)(2Kr + \gamma(i)n_C) > 0$ for all $i \in (0, n_B)$.

Similarly, since $\dfrac{(n_A)^2 m_A(1-m_A)}{n_C(1-m_C)}$ is a positive constant value we have

$$\begin{aligned}\frac{df_C}{di} &\propto 2\alpha(i)\alpha'(i)(Kr+\alpha(i)n_A)-(\alpha(i))^2 n_A\alpha'(i)\\ &= \alpha(i)\alpha'(i)(2Kr+\alpha(i)n_A) > 0 \ \ \forall i \in (0, n_B),\end{aligned}$$

where the last inequality holds because $\alpha'(\cdot) > 0$ holds along with $\alpha(i)(2Kr + \alpha(i)n_A) > 0$ for all $i \in (0, n_B)$.

□

Lemma A.2. *The solution j_A to Equation* (21) *uniquely exists in* $(0, n_B)$*. Similarly, the solution j_C to Equation* (22) *uniquely exists in* $(0, n_B)$*.*

Proof. Note that $f_A(0) = 0, \alpha(0) = 1, f_A(n_B) > 0$ and $\alpha(n_B) = 0$ hold and that α and f_A are continuous. Since Lemma A.1 holds, the solution j_A exists in $(0, n_B)$, and it is unique.

Similarly note that $f_C(n_B) = 0, \gamma(n_B) = 1, f_C(0) > 0$ and $\gamma(0) = 0$ hold and that γ and f_C are continuous. Since Lemma A.1 holds, the solution j_C exists in $(0, n_B)$, and it is unique.

□

Lemma A.3. *$j_C < j_A$ holds.*

Proof. Since α is strictly decreasing, and f_A is strictly increasing for $i \in (0, n_B)$ due to Lemma A.1, $j_C < j_A$ holds if and only if

$$\alpha(j_C) > f_A(j_C). \tag{44}$$

Inequality (44) is equivalent to

$$P_{Ag}^{j_C}(r + P_{gC}^{j_C} + P_{Cg}^{j_C}) > P_{gC}^{j_C} P_{Cg}^{j_C} \tag{45}$$

$$\Leftrightarrow \frac{r}{P_{gC}^{j_C} P_{Cg}^{j_C}} + \frac{1}{P_{Cg}^{j_C}} + \frac{1}{P_{gC}^{j_C}} - \frac{1}{P_{Ag}^{j_C}} > 0. \tag{46}$$

By definition, we have

$$\begin{aligned} P^{j_A}_{Ag}(r + P^{j_A}_{gC} + P^{j_A}_{Cg}) &= P^{j_A}_{gC}P^{j_A}_{Cg}, \text{ and} \\ P^{j_C}_{Cg}(r + P^{j_C}_{gA} + P^{j_C}_{Ag}) &= P^{j_C}_{gA}P^{j_C}_{Ag}, \end{aligned}$$

or

$$\frac{r}{P^{j_A}_{gC}P^{j_A}_{Cg}} + \frac{1}{P^{j_A}_{Cg}} + \frac{1}{P^{j_A}_{gC}} = \frac{1}{P^{j_A}_{Ag}}, \text{ and} \tag{47}$$

$$\frac{r}{P^{j_C}_{gA}P^{j_C}_{Ag}} + \frac{1}{P^{j_C}_{Ag}} + \frac{1}{P^{j_C}_{gA}} = \frac{1}{P^{j_C}_{Cg}}. \tag{48}$$

The left hand side of (46) is equal to

$$\begin{aligned} & \frac{r}{P^{j_C}_{gC}P^{j_C}_{Cg}} + \frac{1}{P^{j_C}_{Cg}} + \frac{1}{P^{j_C}_{gC}} - \frac{1}{P^{j_C}_{Ag}} \\ = \; & \frac{r}{P^{j_C}_{gC}P^{j_C}_{Cg}} + \{\frac{r}{P^{j_C}_{gA}P^{j_C}_{Ag}} + \frac{1}{P^{j_C}_{Ag}} + \frac{1}{P^{j_C}_{gA}}\} + \frac{1}{P^{j_C}_{gC}} - \frac{1}{P^{j_C}_{Ag}} \\ = \; & \frac{r}{P^{j_C}_{gC}P^{j_C}_{Cg}} + \{\frac{r}{P^{j_C}_{gA}P^{j_C}_{Ag}} + \frac{1}{P^{j_C}_{gA}}\} + \frac{1}{P^{j_C}_{gC}} \end{aligned}$$

where we make use of Equation (48).

Since all terms are positive, we have

$$\frac{r}{P^{j_C}_{gC}P^{j_C}_{Cg}} + \frac{1}{P^{j_C}_{Cg}} + \frac{1}{P^{j_C}_{gC}} - \frac{1}{P^{j_C}_{Ag}} > 0,$$

as desired.

□

A.3. **Proof of Lemma 4.2.**

Proof. Transition probabilities are obtained after some calculation. We check $X_{\bar{\tau}}$ and $\Pi_{\bar{\tau}}$ satisfy $X^i_{\bar{\tau}}\Pi^i_{\bar{\tau}} = X^i_{\bar{\tau}}$ for all $i \in B$.

For all $i \in J_C$, $\bar{\tau}^i_{gC} = 0$ implies that $m^i_C = 0$ holds. The steady-state implies

$$m^i_A P^i_{Ag} = (1 - m^i_A)P^i_{gA}.$$

Then we have

$$\begin{aligned} & m^i_A \frac{\alpha(i)n_A(1-m_A)}{K} = (1-m^i_A)\frac{\alpha(i)n_A m_A}{K} \\ \Leftrightarrow \; & \frac{m^i_A}{1-m^i_A} = \frac{m_A}{1-m_A}. \end{aligned}$$

Since for all $i \in J_C$, $m^i_A = m_A$ holds, this equation is satisfied.

A similar argument holds for $i \in J_A$.

For all $i \in B \setminus (J_A \cup J_C)$, the steady-state implies

$$\begin{aligned} m_A^i P_{Ag}^i &= (1 - m_A^i - m_C^i) P_{gA}^i \\ m_C^i P_{Cg}^i &= (1 - m_A^i - m_C^i) P_{gC}^i \end{aligned}$$

Then we have

$$\begin{aligned} m_A^i \frac{\alpha(i) n_A (1 - m_A)}{K} &= (1 - m_A^i - m_C^i) \frac{\alpha(i) n_A m_A}{K} \\ m_C^i \frac{\gamma(i) n_C (1 - m_C)}{K} &= (1 - m_A^i - m_C^i) \frac{\gamma(i) n_C m_C}{K}. \end{aligned}$$

By inserting $m_A^i = \dfrac{m_A(1 - m_C)}{1 - m_A m_C}$ and $m_C^i = \dfrac{m_C(1 - m_A)}{1 - m_A m_C}$, these two equations are satisfied.

□

A.4. **Proof of Theorem 4.3.**

Proof. First, we show the profile $(\bar{\tau}, X_{\bar{\tau}}, \Pi_{\bar{\tau}})$ is a steady-state equilibrium.

[Case 1: $i \in J_C$] It suffices to show that for all $i \in J_C$, $V_A^i > V_g^i \geq V_C^i$ holds.

Since $\tau_{gA}^i = 1$ and $\tau_{gC}^i = 0$ hold, we have

$$\begin{aligned} P_{gA}^i &= \frac{\alpha(i) n_A m_A}{K}, \\ P_{gC}^i &= 0, \\ P_{Ag}^i &= \frac{\alpha(i) n_A (1 - m_A)}{K}, \text{ and} \\ P_{Cg}^i &= \frac{\gamma(i) n_C (1 - m_C)}{K}. \end{aligned}$$

Then, we have

$$\begin{aligned} & P_{Ag}^i (r + P_{gC}^i + P_{Cg}^i) - P_{gC}^i P_{Cg}^i \\ = \; & \frac{\alpha(i) n_A (1 - m_A)}{K} \left(r + \frac{\gamma(i) n_C (1 - m_C)}{K}\right) > 0. \end{aligned}$$

Thus, due to Lemma 4.1, $V_A^i > V_g^i$ holds.

Also, for all $i \in J_C$, due to $\tau_{gA}^i = 1$, we have

$$\gamma(i) \leq f_C(i).$$

This implies the following inequalities.

$$\begin{aligned}
&\gamma(i) \leq \frac{(\alpha(i)n_A)^2 m_A(1-m_A)}{(Kr+\alpha(i)n_A)n_C(1-m_C)} \\
\Leftrightarrow\ & \gamma(i)n_C(1-m_C)\{Kr+\alpha(i)n_A m_A+\alpha(i)n_A(1-m_A)\} \leq (\alpha(i)n_A m_A)\{\alpha(i)n_A(1-m_A)\} \\
\Leftrightarrow\ & \frac{\gamma(i)n_C(1-m_C)}{K}\{r+\frac{\alpha(i)n_A m_A}{K}+\frac{\alpha(i)n_A(1-m_A)}{K}\} \leq \frac{\alpha(i)n_A m_A}{K}\frac{\alpha(i)n_A(1-m_A)}{K} \\
\Leftrightarrow\ & P^i_{Cg}(r+P^i_{gA}+P^i_{Ag}) \leq P^i_{gA}P^i_{Ag}.
\end{aligned}$$

Thus, due to Lemma 4.1, $V^i_C \leq V^i_g$ holds. Then, for all $i \in J_C$, we have $V^i_A > V^i_g \geq V^i_C$.

[Case 2: $i \in J_A$]

The proof of case 2 is similar to that of case 1.

For all $i \in J_A$, $V^i_C > V^i_g \geq V^i_A$ holds.

[Case 3: $i \in B \setminus \{J_A \cup J_C\}$] It suffices to show that for all $i \in B \setminus \{J_A \cup J_C\}$, $V^i_A, V^i_C > V^i_g$ holds.

Since $\tau^i_{gC} = 1$ and $\tau^i_{gA} = 1$ hold, we have

$$\begin{aligned}
P^i_{gA} &= \frac{\alpha(i)n_A m_A}{K}, \\
P^i_{gC} &= \frac{\gamma(i)n_C m_C}{K}, \\
P^i_{Ag} &= \frac{\alpha(i)n_A(1-m_A)}{K}, \text{ and} \\
P^i_{Cg} &= \frac{\gamma(i)n_C(1-m_C)}{K}.
\end{aligned}$$

For all $i \in B \setminus \{J_A \cup J_C\}$, we have

$$\alpha(i) > f_A(i), \text{ and} \tag{49}$$

$$\gamma(i) > f_C(i). \tag{50}$$

Equation (49) implies the following inequalities.

$$\begin{aligned}
&\alpha(i) > \frac{(\gamma(i)n_C)^2 m_C(1-m_C)}{(Kr+\gamma(i)n_C)n_A(1-m_A)} \\
\Leftrightarrow\ & \alpha(i)n_A(1-m_A)\{Kr+\gamma(i)n_C m_C+\gamma(i)n_C(1-m_C)\} > (\gamma(i)n_C m_C)\{\gamma(i)n_C(1-m_C)\} \\
\Leftrightarrow\ & \frac{\alpha(i)n_A(1-m_A)}{K}\{r+\frac{\gamma(i)n_C m_C}{K}+\frac{\gamma(i)n_C(1-m_C)}{K}\} > \frac{\gamma(i)n_C m_C}{K}\frac{\gamma(i)n_C(1-m_C)}{K} \\
\Leftrightarrow\ & P^i_{Ag}(r+P^i_{gC}+P^i_{Cg}) > P^i_{gC}P^i_{Cg}.
\end{aligned}$$

Thus, due to Lemma 4.1, $V^i_A > V^i_g$ holds. Similarly, Equation (50) implies that $V^i_C > V^i_g$ holds. Then, for all $i \in B \setminus \{J_A \cup J_C\}$, we have $V^i_A, V^i_g > V^i_C$.

This ends the proof of showing that the profile $(\bar{\tau}, X_{\bar{\tau}}, \Pi_{\bar{\tau}})$ is a steady-state equilibrium.

Next, we show the uniqueness.

Suppose τ is another steady-state equilibrium. It suffices to show $\tau = \bar{\tau}$.

Suppose the contrary, *i.e.* there is $h \in B$ $\tau^h \neq \bar{\tau}^h$.

We divide the analysis into three cases.

[Case 1: $h \in J_C$.]

There are two possible subcases.

[Subcase 1: $\tau^h_{gA} = 0$ and $\tau^h_{gC} = 1$] In this subcase, we have

$$P^h_{gA} = 0, \quad (51)$$

$$P^h_{gC} = \frac{\gamma(h) n_C m_C}{K}, \quad (52)$$

$$P^h_{Ag} = \frac{\alpha(h) n_A (1 - m_A)}{K}, \quad (53)$$

$$P^h_{Cg} = \frac{\gamma(h) n_C (1 - m_C)}{K}, \text{ and} \quad (54)$$

$$V^h_C > V^h_g > V^h_A. \quad (55)$$

Then, due to Lemma 4.1, $V^h_C > V^h_A$ implies

$$P^h_{Cg} > P^h_{Ag},$$

which is equivalent to

$$\gamma(h) n_C (1 - m_C) > \alpha(h) n_A (1 - m_A). \quad (56)$$

Since $j \in J_C$, $\gamma(j) < f_C(j)$ holds. This implies

$$\gamma(h) n_C (1 - m_C) < \alpha(h) n_A (1 - m_A) \left(\frac{\alpha(h) n_A m_A}{Kr + \alpha(h) n_A} \right) \quad (57)$$

Since $\dfrac{\alpha(h) n_A m_A}{Kr + \alpha(h) n_A} < 1$ holds, Equations (56) and (57) contradict each other.

[Subcase 2: $\tau^h_{gA} = 1$ and $\tau^h_{gC} = 1$]

In this subcase, we must have

$$P^h_{gA} = \frac{\alpha(h) n_A m_A}{K},$$

$$P^h_{gC} = \frac{\gamma(h) n_C m_C}{K},$$

$$P^h_{Ag} = \frac{\alpha(h) n_A (1 - m_A)}{K},$$

$$P^h_{Cg} = \frac{\gamma(h) n_C (1 - m_C)}{K}, \text{ and}$$

$$V^h_C, V^h_A > V^h_g.$$

Then, due to Lemma 4.1, $V_C^h > V_g^h$ implies

$$\begin{aligned} & P_{Cg}^h(r + P_{gA}^h + P_{Ag}^h) > P_{gA}^h P_{Ag}^h \\ \Leftrightarrow \quad & \frac{\gamma(h) n_C(1-m_C)}{K}\left(r + \frac{\alpha(h) n_A m_A}{K} + \frac{\alpha(h) n_A(1-m_A)}{K}\right) > \frac{\alpha(h) n_A m_A}{K}\frac{\alpha(h) n_A(1-m_A)}{K}. \end{aligned}$$

Since $h \in J_C$, $\gamma(h) < f_C(h)$ holds. This implies that $\frac{\gamma(h) n_C(1-m_C)}{K}(r + \frac{\alpha(h) n_A m_A}{K} + \frac{\alpha(h) n_A(1-m_A)}{K}) < \frac{\alpha(h) n_A m_A}{K}\frac{\alpha(h) n_A(1-m_A)}{K}$.

This is a contradiction.

[Case 2: $h \in J_A$.]

The proof of this case is similar to that of Case 1.

[Case 3: $h \in B \setminus (J_A \cup J_C)$.]

There are two possible subcases.

[Subcase 1: $\tau_{gA}^h = 1$ and $\tau_{gC}^h = 0$] In this subcase, we have

$$\begin{aligned} P_{gA}^h &= \frac{\alpha(h) n_A m_A}{K}, \\ P_{gC}^h &= 0, \\ P_{Ag}^h &= \frac{\alpha(h) n_A(1-m_A)}{K}, \\ P_{Cg}^h &= \frac{\gamma(h) n_C(1-m_C)}{K}, \text{ and} \\ V_A^h &> V_g^h \geq V_C^h. \end{aligned}$$

Then, due to Lemma 4.1, $V_g^h \geq V_C^h$ implies

$$P_{Cg}^i(r + P_{gA}^i + P_{Ag}^i) \leq P_{gA}^i P_{Ag}^i. \tag{58}$$

Given the transition probabilities, Equation (58) implies

$$\gamma(i) \leq f_C(i). \tag{59}$$

However, $i \in B \setminus (J_A \cup J_C)$ implies $\gamma(i) > f_C(i)$. This contradicts Equation (59). This ends the proof of subcase 1.

[Subcase 2: $\tau_{gA}^h = 0$ and $\tau_{gC}^h = 1$] In this subcase, we have

$$\begin{aligned} P_{gA}^h &= 0, \\ P_{gC}^h &= \frac{\gamma(h) n_C m_C}{K}, \\ P_{Ag}^h &= \frac{\alpha(h) n_A(1-m_A)}{K}, \\ P_{Cg}^h &= \frac{\gamma(h) n_C(1-m_C)}{K}, \text{ and} \\ V_C^h &> V_g^h \geq V_A^h. \end{aligned}$$

Then, due to Lemma 4.1, $V_g^h \geq V_A^h$ implies

$$P_{Ag}^i(r + P_{gC}^i + P_{Cg}^i) \leq P_{gC}^i P_{Cg}^i. \tag{60}$$

Given the transition probabilities, Equation (60) implies

$$\alpha(i) \leq f_A(i). \tag{61}$$

However, $i \in B \setminus (J_A \cup J_C)$ implies $\alpha(i) > f_A(i)$. This contradicts Equation (61). This ends the proof of subcase 2.

Thus, the proof of this theorem is completed. □

A.5. **Proof of Theorem 5.1.**

Proof. Define $F_A(i, m_A, m_C, n_A, n_C)$ and $F_C(i, m_A, m_C, n_A, n_C)$ as follows.

$$\begin{aligned}
F_A(i, m_A, m_C, n_A, n_C) &= \frac{(\gamma(i)n_C)^2 m_C(1-m_C)}{(Kr + \gamma(i)n_C)n_A(1-m_A)} - \alpha(i) \\
F_C(i, m_A, m_C, n_A, n_C) &= \frac{(\alpha(i)n_A)^2 m_A(1-m_A)}{(Kr + \alpha(i)n_A)n_C(1-m_C)} - \gamma(i).
\end{aligned}$$

Note that $F_A(j_A, \cdot) = 0$ and $F_C(j_C, \cdot) = 0$

We have the following partial derivatives.

$$\begin{aligned}
\frac{\partial F_A}{\partial i} &= \frac{(n_C)^2 m_C(1-m_C)}{(1-m_A)n_A} \frac{\gamma(i)(2Kr + \gamma(i)n_C)}{(Kr + \gamma(i)n_C)^2} \times \gamma'(i) - \alpha'(i) \\
\frac{\partial F_A}{\partial m_A} &= \frac{(\gamma(i)n_C)^2 m_C(1-m_C)}{(Kr + \gamma(i)n_C)n_A} \times \frac{1}{(1-m_A)^2} \\
\frac{\partial F_A}{\partial m_C} &= \frac{(\gamma(i)n_C)^2}{(Kr + \gamma(i)n_C)n_A(1-m_A)} \times (1 - 2m_C) \\
\frac{\partial F_A}{\partial n_A} &= \frac{(\gamma(i)n_C)^2 m_C(1-m_C)}{(Kr + \gamma(i)n_C)(1-m_A)} \times \frac{-1}{(n_A)^2} \\
\frac{\partial F_A}{\partial n_C} &= \frac{(\gamma(i))^2 m_C(1-m_C)}{n_A(1-m_A)} \times \frac{n_C(2Kr + \gamma(i)n_C)}{(Kr + \gamma(i)n_C)^2}
\end{aligned}$$

Note that $\dfrac{\partial F_A}{\partial i} \neq 0$ for all $i \in (0, n_B)$.

Therefore, we have

$$\frac{dj_A}{dm_A} = -\frac{\partial F_A/\partial m_A}{\partial F_A/\partial i}\bigg|_{i=j_A} < 0$$

$$\frac{dj_A}{dm_C} = -\frac{\partial F_A/\partial m_C}{\partial F_A/\partial i}\bigg|_{i=j_A} \leq 0 \text{ if } m_C \leq 0.5$$

$$\frac{dj_A}{dm_C} = -\frac{\partial F_A/\partial m_C}{\partial F_A/\partial i}\bigg|_{i=j_A} > 0 \text{ if } m_C > 0.5$$

$$\frac{dj_A}{dn_A} = -\frac{\partial F_A/\partial n_A}{\partial F_A/\partial i}\bigg|_{i=j_A} > 0$$

$$\frac{dj_A}{dn_C} = -\frac{\partial F_A/\partial n_C}{\partial F_A/\partial i}\bigg|_{i=j_A} < 0$$

$$\frac{\partial F_C}{\partial i} = \frac{(n_A)^2 m_A(1-m_A)}{(1-m_C)n_C}\frac{\alpha(i)(2Kr+\alpha(i)n_A)}{(Kr+\alpha(i)n_A)^2} \times \alpha'(i) - \gamma'(i)$$

$$\frac{\partial F_C}{\partial m_A} = \frac{(\alpha(i)n_A)^2}{(Kr+\alpha(i)n_A)n_C(1-m_C)} \times (1-2m_A)$$

$$\frac{\partial F_C}{\partial m_C} = \frac{(\alpha(i)n_A)^2 m_A(1-m_A)}{(Kr+\alpha(i)n_A)n_C} \times \frac{1}{(1-m_C)^2}$$

$$\frac{\partial F_A}{\partial n_A} = \frac{(\alpha(i))^2 m_A(1-m_A)}{n_C(1-m_C)} \times \frac{n_A(2Kr+\alpha(i)n_A)}{(Kr+\alpha(i)n_A)^2}$$

$$\frac{\partial F_C}{\partial n_C} = \frac{(\alpha(i)n_A)^2 m_A(1-m_A)}{(Kr+\alpha(i)n_A)(1-m_C)} \times \frac{-1}{(n_C)^2}$$

Note that $\dfrac{\partial F_C}{\partial i} \neq 0$ for all $i \in (0, n_B)$.
Therefore, we have

$$\frac{dj_C}{dm_A} = -\frac{\partial F_C/\partial m_A}{\partial F_A/\partial i}\bigg|_{i=j_C} \geq 0 \text{ if } m_A \leq 0.5$$

$$\frac{dj_C}{dm_A} = -\frac{\partial F_C/\partial m_A}{\partial F_A/\partial i}\bigg|_{i=j_C} < 0 \text{ if } m_A > 0.5$$

$$\frac{dj_C}{dm_C} = -\frac{\partial F_C/\partial m_C}{\partial F_A/\partial i}\bigg|_{i=j_C} > 0$$

$$\frac{dj_C}{dn_A} = -\frac{\partial F_C/\partial n_A}{\partial F_A/\partial i}\bigg|_{i=j_C} > 0$$

$$\frac{dj_C}{dn_C} = -\frac{\partial F_C/\partial n_C}{\partial F_A/\partial i}\bigg|_{i=j_C} < 0$$

Thus, by the implicit function theorem, the statements hold.

□

A.6. **Proof of Lemma 5.2.**

Proof. First, we have

$$\begin{aligned}\frac{\partial P^A_{Ag}}{\partial n_A} &\propto (1-m_Am_C)+(1-m_Am_C)\alpha(j_C)\frac{\partial j_C}{\partial n_A}+(\alpha(j_A)-\alpha(j_C))\frac{1-m_C}{1-m_Am_C}\frac{\partial j_C}{\partial n_A}\\ &= (1-m_Am_C)+m_C(1-m_A)\alpha(j_C)\frac{\partial j_C}{\partial n_A}+\alpha(j_A)(1-m_C)\frac{\partial j_C}{\partial n_A}>0.\end{aligned}$$

The last inequality is due to Lemma 5.1. Thus, we have $\frac{\partial W^A}{\partial n_A}>0$. Similarly, we have $\frac{\partial W^C}{\partial n_C}>0$. □

A.7. **Proof of Lemma 5.3.**

Proof. Suppose $\varepsilon\geq 0$. Using $\alpha+\varepsilon h$ in place of α for $\hat{P}^A_{Ag}$, we have

$$\begin{aligned}\hat{P}^A_{Ag} &= \frac{1-m_A}{K(1-m_Am_C)}\Bigg[(1-m_Am_C)n_A+(1-m_Am_C)\int_0^{j_C}(\alpha(i)+\varepsilon h(i))d\mu\\ &+ (1-m_C)\int_{j_C}^{j_A}(\alpha(i)+\varepsilon h(i))d\mu\Bigg].\end{aligned}$$

Differentiating it with respect to ε and evaluating it at $\varepsilon=0$, we obtain

$$\begin{aligned}\left.\frac{\partial \hat{P}^A_{Ag}}{\partial\varepsilon}\right|_{\varepsilon=0} &\propto (1-m_Am_C)\left[(\alpha(j_C))\left.\frac{\partial j_C}{\partial\varepsilon}\right|_{\varepsilon=0}+\int_0^{j_C}h(i)d\mu\right]\\ &+ (1-m_C)\left[(\alpha(j_A))\left.\frac{\partial j_A}{\partial\varepsilon}\right|_{\varepsilon=0}-(\alpha(j_C))\left.\frac{\partial j_C}{\partial\varepsilon}\right|_{\varepsilon=0}+\int_{j_C}^{j_A}h(i)d\mu\right]\\ &= m_C(1-m_A)\left[(\alpha(j_C))\left.\frac{\partial j_C}{\partial\varepsilon}\right|_{\varepsilon=0}\right]+(1-m_Am_C)\int_0^{j_C}h(i)d\mu\\ &+ (1-m_C)\left[(\alpha(j_A))\left.\frac{\partial j_A}{\partial\varepsilon}\right|_{\varepsilon=0}+\int_{j_C}^{j_A}h(i)d\mu\right].\end{aligned}$$

Due to Equation (21), using the implicit function theorem, we have

$$\left.\frac{\partial j_A}{\partial\varepsilon}\right|_{\varepsilon=0}=-\frac{h(j_A)}{\alpha'(j_A)-f'_A(j_A)}.$$

By the assumption on α and Lemma A.1, we have

$$\left.\frac{\partial j_A}{\partial\varepsilon}\right|_{\varepsilon=0}>0. \tag{62}$$

Similarly, due to Equation (22), define function G as follows:

$$G(\varepsilon,j_C)=\gamma(j_C)-\frac{(\alpha(j_C+\varepsilon h(j_C)))^2n_A^2m_A(1-m_A)}{\{Kr+(\alpha(j_C)+\varepsilon h(j_C)n_A)\}n_C(1-m_C)}$$

Then, after some calculation we have $\frac{\partial G}{\partial j_C}\Big|_{\varepsilon=0} = \gamma'(j_C) - f_C'(j_C)$. Also, we have

$$\begin{aligned}\frac{\partial G}{\partial \varepsilon}\Big|_{\varepsilon=0} &\propto -[2\alpha(j_C)h(j_C)(Kr+\alpha(j_C)n_A) - \alpha^2(j_C)h(j_C)n_A] \\ &= -[\alpha^2(j_C)h(j_C)n_A + 2\alpha(j_C)h(j_C)Kr] < 0.\end{aligned}$$

Thus, using the implicit function theorem, we obtain

$$\frac{\partial j_C}{\partial \varepsilon}\Big|_{\varepsilon=0} \propto -\frac{\frac{\partial G}{\partial \varepsilon}\Big|_{\varepsilon=0}}{\frac{\partial G}{\partial j_C}\Big|_{\varepsilon=0}} > 0. \tag{63}$$

Therefore, using Equations (62) and (63), we derive

$$\begin{aligned}\frac{\partial \hat{P}^A_{Ag}}{\partial \varepsilon}\Bigg|_{\varepsilon=0} &\propto m_C(1-m_A)\left[(\alpha(j_C))\frac{\partial j_C}{\partial \varepsilon}\Big|_{\varepsilon=0}\right] + (1-m_Am_C)\int_0^{j_C} h(i)d\mu \\ &+ (1-m_C)\left[(\alpha(j_A))\frac{\partial j_A}{\partial \varepsilon}\Big|_{\varepsilon=0} + \int_{j_C}^{j_A} h(i)d\mu\right] > 0\end{aligned}$$

Thus, we have $\frac{\partial W^A}{\partial \varepsilon} > 0$. We conclude that W_A is increasing in α. Similarly, W_C is increasing in γ. □

A.8. **Sanction by A.**

A.8.1. *Case 1:$j_C < j < j_A$.*

$$\begin{aligned}&\hat{W}^j - W^j \\ =\ & [m_C\hat{\gamma}(j)n_C(1-m_C)]\frac{u}{K} - \frac{(1-m_A)(1-m_C)}{1-m_Am_C}[\alpha(j)m_An_A + \gamma(j)m_Cn_C]\frac{u}{K} \\ =\ & \frac{(1-m_C)u}{(1-m_Am_C)K}[m_Am_C(1-m_C)\gamma(j)n_C + (1-m_Am_C)m_C\lambda\alpha(j)n_C - m_A(1-m_A)\alpha(j)n_A]\end{aligned}$$

If $\lambda = 0$ holds, then we have

$$\begin{aligned}\hat{W}^{j_A} - W^{j_A} &= \frac{(1-m_C)u}{(1-m_Am_C)K}[m_Am_C(1-m_C)\gamma(j_A)n_C - m_A(1-m_A)\alpha(j_A)n_A] \\ &= \frac{m_Am_C(1-m_C)^2u}{(Kr+\gamma(j_A)n_C)(1-m_Am_C)}\gamma(j_A)n_Cr\end{aligned}$$

A.8.2. *Case 2: $j \le j_C$.*

$$\begin{aligned}&\hat{W}^j - W^j \\ =\ & [m_C\hat{\gamma}(j)n_C(1-m_C)]\frac{u}{K} - m_A(1-m_A)\alpha(j)n_A\frac{u}{K} \\ =\ & \frac{u}{K}[m_C(1-m_C)(\gamma(j)+\lambda\alpha(j))n_C - m_A(1-m_A)\alpha(j)n_A]\end{aligned}$$

A.9. **Sanction by C.** If j uses only currency C without sanction, *i.e.*,$j_A < j$ holds, then $m_C^j = m_C$ holds, and the welfare W^j is given by

$$W^j = m_C(1-m_C)\gamma(j)n_C\frac{u}{K}. \tag{64}$$

Next, we consider a sanction by country C on country j. Suppose that if sanctioned, j can no longer trade through currency C. This corresponds to the situation in which $\gamma(j) = 0$. Part of those who cannot be matched with agents in C may meet agents in A instead. Let $\hat{\alpha}(j) = \alpha(j) + \lambda\gamma(j)$ with $\lambda \in [0, 1]$ and $\hat{\gamma}(j) = 0$, where the variables with "ˆ" are the ones after the sanction.

The welfare $\hat{W}^j$ of country $j \in B$ after the sanction is given by

$$\hat{W}^j = \left[\hat{m}_A^j \hat{P}_{Ag}^j\right] u.$$

Since $\hat{m}_A^j = m_A$ and $\hat{P}_{Ag}^j = \hat{\alpha}(j)n_A(1-m_A)$ hold, we obtain

$$\hat{W}^j = \left[m_A\hat{\alpha}(j)n_A(1-m_A)\right]\frac{u}{K}. \tag{65}$$

We now divide the analysis into two cases.

A.9.1. *Case 1: $j_C < j < j_A$.* First, consider the case of $j_C < j < j_A$, *i.e.*, country j accepts both currencies. Subtracting (26) from (65), we have

$$\begin{aligned}
&\hat{W}^j - W^j \\
= \;& \left[m_A\hat{\alpha}(j)n_A(1-m_A)\right]\frac{u}{K} - \frac{(1-m_A)(1-m_C)}{1-m_Am_C}\left[\alpha(j)m_An_A + \gamma(j)m_Cn_C\right]\frac{u}{K} \\
= \;& \frac{(1-m_A)u}{(1-m_Am_C)K}\left[(1-m_Am_C)m_An_A(\alpha(j)+\lambda\gamma(j)) - (1-m_C)\{\alpha(j)m_An_A + \gamma(j)m_Cn_C\}\right] \\
= \;& \frac{(1-m_A)u}{(1-m_Am_C)K}\left[m_A(1-m_A)m_C\alpha(j)n_A + (1-m_Am_C)m_A\lambda\gamma(j)n_A - m_C(1-m_C)\gamma(j)n_C\right].
\end{aligned}$$

Then, we obtain

$$\hat{W}^j - W^j \propto m_A(1-m_A)m_C\alpha(j)n_A + (1-m_Am_C)m_A\lambda\gamma(j)n_A - m_C(1-m_C)\gamma(j)n_C. \tag{66}$$

Note that the last expression is increasing in λ.

We first evaluate (66) at $j = j_C$. Substituting (22) into (66), given $\lambda = 0$, we have

$$\begin{aligned}
\hat{W}^{j_C} - W^{j_C} &= \frac{(1-m_A)u}{(1-m_Am_C)K}\left[m_A(1-m_A)m_C\alpha(j_C)n_A - m_C(1-m_C)\gamma(j_C)n_C\right] \\
&= \frac{m_A(1-m_A)^2m_Cu}{(Kr+\alpha(j_C)n_A)(1-m_Am_C)}\alpha(j_C)n_Ar.
\end{aligned}$$

Then, we obtain

$$\hat{W}^{j_C} - W^{j_C} \propto \alpha(j_C)n_Ar > 0. \tag{67}$$

Since $\hat{W}^j - W^j$ is continuous in j, we have the following result.

Theorem A.4. *There exists a neighborhood U of j_C for which for all $j \in U$ with $j \geq j_C$, $\hat{W}^j > W^j$ holds.*

Theorem A.4 is a striking result as it states that under any condition, there are some countries that are better off by being sanctioned provided that there is a competitor for currency hegemony. These are the countries in which agents are almost indifferent between accepting and rejecting currency C.

A companion result states that for any country $i \in B$, if country A is sufficiently large compared with country C, then the sanction by C results in the improvement of the sanctioned country.

Theorem A.5. *For any country $j \in (j_C, j_A)$, $\hat{W}^j > W^j$ holds if and only if*

$$\frac{n_A}{n_C} > \frac{m_C(1-m_C)\gamma(j)}{m_A(1-m_A)m_C\alpha(j) + (1-m_A m_C)m_A\lambda\gamma(j)}$$

holds.

A.9.2. *Case 2: $j \geq j_A$.* Next, we consider the case of $j < j_C$, *i.e.*, country j accepts only currency C before sanction. In this case, the welfare before sanction is given by (64). Subtracting it from (65), we have

$$\begin{aligned} & \hat{W}^j - W^j \\ = \; & \left[m_A\hat{\alpha}(j)n_A(1-m_A)\right]\frac{u}{K} - m_C(1-m_C)\gamma(j)n_C\frac{u}{K} \\ = \; & \frac{u}{K}\left[m_A(1-m_A)(\alpha(j)+\lambda\gamma(j))n_A - m_C(1-m_C)\gamma(j)n_C\right]. \end{aligned}$$

Then, we obtain

$$\hat{W}^j - W^j \propto m_A(1-m_A)(\alpha(j)+\lambda\gamma(j))n_A - m_C(1-m_C)\gamma(j)n_C. \tag{68}$$

We state this as a theorem.

Theorem A.6. *For any country $j > j_A$, $\hat{W}^j > W^j$ holds if and only*

$$\frac{n_A}{n_C} > \frac{m_C(1-m_C)\gamma(j)}{m_A(1-m_A)(\alpha(j)+\lambda\gamma(j))}$$

holds.

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