Value of Information in Social Learning*

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Abstract

This study extends Blackwell's (1953) comparison of information to a sequential social learning model in which agents make decisions sequentially based on both private signals and observed actions of others. In this context, we introduce a new binary relation over information structures: an information structure is *more socially valuable* than another if it yields higher expected payoffs for *all* agents, regardless of their preferences. First, we establish that this binary relation is strictly stronger than the Blackwell order. Next, we provide a necessary and sufficient condition for our binary relation and propose a simpler sufficient condition that is easier to verify. We further explore comparisons of information structures in terms of long-run and limit welfare.

Keywords: Comparison of experiments; Social learning; Herding.
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1 Introduction

In classical decision theory, an information source is considered more valuable than another if it enables an individual decision-maker to make better choices under uncertainty. This has been established by Blackwell's (1953) comparison of information structures, which evaluates information structures based on whether a single agent would always prefer one over another, regardless of preferences. Thus, the Blackwell order provides a preference-independent criterion for comparing the value of information sources for a single decision-maker.

However, in many real-world settings, decision-makers do not rely solely on their private signals; they also acquire information from the observed actions of others. This creates an information externality: an individual's decision not only influences their own outcome but also transmits information to future decision-makers. Because of this externality, simply comparing information structures based on their value for individual decision-making is no longer sufficient to evaluate the value of information in the society.

A motivating example arises in online shopping.[1] Buyers can obtain information about product quality from advertisements or product descriptions on each product page. In addition to these signals, buyers often have access to information about the volume of past purchases. When deciding whether to purchase a product, buyers use both their private signal and the information based on the popularity of previous buyers. As a result, when assessing and comparing advertisements or product pages for their informational efficiency, it is essential to consider the information externalities created by the observability of past purchasing behavior.

To address the value of information in the society, we extend Blackwell's (1953) comparison of information structures to the classical sequential social learning model (Banerjee, 1992; Bikhchandani et al., 1992; Smith and Sørensen, 2000). In this model, homogeneous agents make decisions sequentially based on the past actions of others (referred to as history) and their own private signals. These private signals are independently drawn from an identical information structure. Within this framework, we introduce a binary relation over information structures: an information structure is *more socially valuable* than another if it yields higher expected payoffs for *all* agents, regardless of their preferences and equilibrium selections, in the presence of social learning.

[1] A similar example is provided in Arieli et al. (2023).

Within this model, we pose a benchmark question: *Does a Blackwell more informative source always make all agents better off in a social learning environment?* The answer is no. We observe that our binary relation is strictly stronger than the Blackwell order (Proposition 1). This follows intuitively, because the history garbles signal realizations depending on the underlying decision problem. Consequently, our binary relation requires a sufficiently informative signal to ensure that the joint value of history and the private signal increases. This highlights an essential feature of the observability assumption. If agents could observe past signal realizations instead of actions, then a Blackwell more informative signal would always be more socially valuable. Thus, this strict gap between our binary relation and the Blackwell order ultimately arises from whether agents can observe past signal realizations or only past actions, and consequently, from the resulting information externalities.

We then ask the main question: *When is one information structure more socially valuable than another?* Theorem 1 provides a necessary and sufficient condition for our binary relation. Specifically, one information structure is more socially valuable than another if and only if it yields higher expected payoffs for all agents across all decision problems and equilibria, even in settings where past signals (rather than actions) are observable under the alternative information structure. The necessary condition, combined with classical results, indicates that an information structure is more socially valuable than another only if it induces unbounded (private) beliefs. Thus, if an information structure induces an information cascade, then it is no longer more socially valuable than any other information structure. Equivalently, the existence of conclusive signals, or asymptotic learning, is necessary for all agents to be better off.

Given the strong necessary condition, it is natural to ask: *Which pairs of information structures can be compared within our binary relation?* This question naturally directs our focus to the sufficiency part of Theorem 1, but verifying this condition is challenging, as it depends on the underlying decision problem. To address this, we provide a clear and simple sufficient condition. Specifically, Theorem 2 states that an information structure is more socially valuable than another if there exists a mixture of full and no information between them in the Blackwell order. To verify the existence of such a mixture, Proposition 2 provides an equivalent condition. By combining these results, we show that an information structure is more socially valuable than another if it assigns a sufficiently high probability of disclosing conclusive signals about each state. Therefore, a relatively higher degree of conclusiveness is sufficient for all agents to be better off.

This sufficient condition follows from the intrinsic properties of mixtures of two extreme information structures. Under any such mixture, the expected payoffs for all agents match those in a setting in which agents observe past signals rather than actions for any equilibrium and any decision problem. Moreover, any such mixture respects the Blackwell order: If the mixture is Blackwell more informative than another information structure, then this mixture is more socially valuable. Conversely, if an information structure is Blackwell more informative than the mixture, it is also more socially valuable. Thus, if a mixture of full and no information exists between two information structures in the Blackwell order, they are also comparable in our binary relation. Therefore, our proof finds and utilizes the unique property of the mixtures of full and no information. This avoids the complicated formula of expected payoffs for all agents, which is the main technical challenge of our analysis.

In Section 4, we discuss several extensions of our analysis. First, Section 4.1 introduces a binary relation based only on sufficiently later agents. Formally, we say that one information structure is *eventually more socially valuable* than another if it yields weakly higher expected payoffs than another for all agents after some cutoff for all decision problems. Proposition 3 provides a characterization and highlights the distinction between our binary relations and the eventual Blackwell order by Azrieli (2014) and Mu et al. (2021), as shown in Proposition 4. Moreover, by utilizing the eventual Blackwell order, Proposition 5 provides a sufficient condition for this long-run comparison, which is weaker than the condition in Theorem 2.

We further introduce a binary relation comparing only the limit welfare, which is one of the central objectives in the social learning literature. Specifically, we say that one information structure is *eventually more socially valuable in the limit* than another if it yields weakly higher expected payoffs than another in the limit for all decision problems. Theorem 3 shows that one information structure is eventually more socially valuable in the limit than another if and only if the former induces unbounded beliefs or the latter is no information. Thus, the necessary condition for the more socially valuable order becomes sufficient in this limit welfare comparison.

Next, we extend our results to more general settings. Section 4.2 demonstrates that our main results naturally extend to dynamic information structures, allowing for correlation or heterogeneity across agents. Proposition 6 provides a characterization, and Proposition 7 derives a sufficient condition by building on the characterizations developed by Renou and Venel (2024) and Whitmeyer and Williams (2024a). Section 4.3 then considers models in which agents may face heterogeneous decision problems. Proposition 8 shows that our characterization continues to hold, while Example 3 illustrates that our sufficient condition may fail in such settings.

Finally, our original binary relation relies on a strong equilibrium selection rule and therefore, is not a partial order. To relax this assumption, Section 4.4 introduces a weaker binary relation over equilibrium selections. We say that one information structure is *weakly more socially valuable* than another if it yields weakly higher payoffs for all agents under some equilibrium (rather than any equilibrium) than another. This weaker version is a partial order. Example 1 also shows that this weaker relation is strictly stronger than the Blackwell order. Furthermore, in Example 4, we show that it is strictly stronger than our original binary relation by providing a specific sufficient condition (Proposition 10) that is independent of Theorem 2. Relatedly, in Section 4.5, we further relax the requirement of a universal domain of decision problems and restrict our attention to canonical and generic binary environments.

1.1 Related Literature

Pioneered by Blackwell (1951, 1953), numerous studies have extended Blackwell's comparison of experiments.[2] Our study investigates comparisons in a game-theoretic setting, similar to Lehrer et al. (2010), Lehrer et al. (2013), Gossner (2000), Pęski (2008), Cherry and Smith (2012), Bergemann and Morris (2016), and de Oliveira (2018, Section 6), but we focus specifically on the social learning model, where strategic interaction arises from information externalities rather than payoff externalities.

Beyond the game-theoretic setting, our study is closely related to two strands of literature on comparisons of experiments. The first strand examines comparisons involving repeated samples (Stein, 1951; Torgersen, 1970; Moscarini and Smith, 2002; Azrieli, 2014; Mu et al., 2021). Although each agent in our model receives a private signal independently drawn from the identical information structure, they cannot observe past signal realizations. Thus, our analysis departs from the standard learning model, and our binary relation becomes strictly

[2]Some studies examine comparisons of experiments within a restricted domain of decision problems or a limited class of experiments (Lehmann, 1988; Persico, 2000; Athey and Levin, 2018; Ben-Shahar and Sulganik, 2024).

stronger than the Blackwell order. This aspect highlights the observability assumption inherent in the social learning model. We discuss the relation between (a weaker version of) our binary relation and the eventual Blackwell order in Azrieli (2014) and Mu et al. (2021) in Section 4.1.

The second strand explore comparisons of dynamic information structures in sequential decision problems, as studied by Greenshtein (1996), de Oliveira (2018, Section 5), Renou and Venel (2024), and Whitmeyer and Williams (2024a).[3] Similar to these studies, the information observed by agents is correlated across periods, but in our model this correlation arises from the correlation of past actions. However, unlike in previous studies, the information they observe crucially depends on past actions and the underlying decision problem.

Broadly, this study contributes to the literature on social learning. Since, for example, Banerjee (1992), Bikhchandani et al. (1992), and Smith and Sørensen (2000),[4] a fundamental question has been whether agents can eventually learn the true state in various settings.[5] Some recent studies, such as Arieli et al. (2023) and Arieli et al. (2024), examine how information structures can be optimally designed or regulated to influence herding or asymptotic behavior (see also Lorecchio (2022) and Parakhonyak and Vikander (2023)).[6] To the best of our knowledge, the comparison of experiments, which is the focus of this study, remains largely unexplored in the literature. The primary technical challenge arises from the complexity of expected payoffs when analyzing all agents, which is even more difficult than focusing solely on asymptotic agents. Our approach addresses this issue by leveraging the properties of mixtures of full and no information.

[3]Whitmeyer and Williams (2024b) also analyze comparisons in dynamic decision problems in the presence of additional information, following Brooks et al. (2024).

[4]For a recent comprehensive survey, see Bikhchandani et al. (2024).

[5]Examples include cases with limited observations of past actions (Çelen and Kariv, 2004; Acemoglu et al., 2011; Lobel and Sadler, 2015; Arieli and Mueller-Frank, 2019, 2021; Kartik et al., 2024; Xu, 2025) (see also Banerjee and Fudenberg (2004); Gale and Kariv (2003); Callander and Hörner (2009); Smith and Sorensen (2013)), as well as cases where observing past actions is costly (Kultti and Miettinen, 2006, 2007; Song, 2016), and cases involving the costly acquisition of private signals (Mueller-Frank and Pai, 2016; Ali, 2018).

[6]Other important questions include social learning with correlated signals (Liang and Mu, 2020; Awaya and Krishna, 2025), the speed and efficiency of learning (Hann-Caruthers et al., 2018; Rosenberg and Vieille, 2019), and learning about the informativeness (Huang, 2024).

2 Model

There is an infinite sequence of ordered, homogeneous agents indexed by $i = 1, 2, \dots,$ who make decisions sequentially. The state space is binary, $\Omega = \{L, H\}$ with a common prior.[7] Let $\mu_0 \in (0, 1)$ be the prior of $\omega = H$. The periods are discrete ($t = 0, 1, \dots$), and each agent i takes an action at period i from a finite action set A. A common payoff function $u : A \times \Omega \to \mathbb{R}$ determines each agent's payoff.[8] The payoff of agent i depends solely on their own action and the state, independent of actions taken by other agents.

The timing of this game is as follows: At period 0, nature first determines the true state, which remains unchanged throughout the game. In each period i, agent i first observes the entire history, which consists of the actions of all preceding agents $(1, 2, \dots, i-1)$. Additionally, agent i receives a private signal $s \in S$, drawn independently from an identical information structure $\pi : \Omega \to \Delta(S)$.[9] For simplicity, we assume that S is finite.[10] Following these observations, agent i selects an action from the action set A.

Given the decision problem $\mathcal{D} = (A, u)$ and the information structure $\pi : \Omega \to \Delta(S)$, the strategy of agent i is denoted by $\sigma_i : A^{i-1} \times S \to \Delta(A)$. Given $\mathcal{D} = (A, u)$, π, and the strategy profile $\boldsymbol{\sigma} = (\sigma_i)_{i \in \mathbb{N}}$, let $\alpha^{\omega}_{\le i}(\pi, \boldsymbol{\sigma}) \in \Delta(A^i)$ denote the distribution of actions taken by agents $1, 2, \dots, i$ when the state is ω, that is,

$$\alpha^{\omega}_{\le i}(\boldsymbol{a}|\pi, \boldsymbol{\sigma}) = \sum_{(s_1, \dots, s_i) \in S^i} \prod_{k=1}^{i} \sigma_k(a_k | a_1, \dots, a_{k-1}, s_k) \pi(s_k|\omega).$$

Similarly, let $\alpha^{\omega}_i(\pi, \boldsymbol{\sigma}) \in \Delta(A)$ be the distribution of actions taken by agent i when the state is ω, that is,

$$\alpha^{\omega}_i(a|\pi, \boldsymbol{\sigma}) = \sum_{(a'_1, \dots, a'_{i-1}) \in A^{i-1}} \alpha^{\omega}_{\le i}(a'_1, \dots, a'_{i-1}, a|\pi, \boldsymbol{\sigma}).$$

Note that $\alpha^{\omega}_i(\pi, \boldsymbol{\sigma})$ does not depend on the strategies of agents after i. Let

[7] For simplicity, we assume a binary state space, but our results can be extended to a finite state space.

[8] We discuss extensions to a heterogeneous-agent model in Section 4.3.

[9] We can relax the assumptions of independent and identically distributed private signals, as discussed in Section 4.2.

[10] Although the proof holds even when both A and S are countable, we impose this assumption to simplify notation.

$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma})$ be the ex-ante expected payoff for agent i. Precisely,

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}) = \mathbb{E}_\omega \left[\sum_{a \in A} \alpha_i^\omega(a|\pi, \boldsymbol{\sigma}) u(a, \omega) \right].$$

We say that the strategy profile $\boldsymbol{\sigma}^*$ is a Bayes-Nash equilibrium (hereafter referred to simply as an equilibrium) under $(\mathcal{D}, \pi)$ if

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq V_i^{\mathcal{D}}(\pi, (\sigma_i, \boldsymbol{\sigma}_{-i}^*))$$

for all σ_i and i.

For two information structures $\pi : \Omega \to \Delta(S)$ and $\pi' : \Omega \to \Delta(S')$, define their product $\pi \otimes \pi' : \Omega \to \Delta(S \times S')$ as

$$(\pi \otimes \pi')((s, s')|\omega) = \pi(s|\omega)\pi'(s'|\omega)$$

for all $s \in S$, $s' \in S'$, and $\omega \in \Omega$. We denote

$$\pi^{\otimes i} = \pi \otimes \cdots \otimes \pi$$

as the information structure generated by i conditionally independent observations from π. Define $\overline{V_i}^{\mathcal{D}}(\pi)$ as

$$\overline{V_i}^{\mathcal{D}}(\pi) = \max_{\sigma_i : S^i \to \Delta(A)} \mathbb{E}_\omega \left[\sum_{a \in A} \sum_{\boldsymbol{s} \in S^i} \sigma_i(a|\boldsymbol{s}) \pi^{\otimes i}(\boldsymbol{s}|\omega) u(a, \omega) \right].$$

In other words, this represents the maximized expected payoff when agent i independently observes the signal drawn from π for i times.

Given the information structure $\pi : \Omega \to \Delta(S)$, define $\mu \in \Delta[0, 1]$ as the *private belief distribution* induced by π, which represents the distribution over private beliefs about the state being H after observing private signals from π. Specifically, for $x \in [0, 1]$,

$$\mu(x) = \mathbb{E}_\omega \left[\sum_{s \in S(x)} \pi(s|\omega) \right],$$

where $S(x) = \{s \in S \mid \frac{\mu_0 \pi(s|H)}{\mu_0 \pi(s|H) + (1-\mu_0)\pi(s|L)} = x\}$. With a slight abuse of notation, we define $\pi(\mu = x|\omega) = \sum_{s \in S(x)} \pi(s|\omega)$ for each $\omega \in \Omega$. We say that a signal s is a *conclusive signal about* $\omega = H$ (resp. $\omega = L$) if $s \in S(1)$ (resp. $s \in S(0)$). Additionally, we say that an information structure π is *no information* if $\mathrm{supp}(\mu) = \{\mu_0\}$, and that π is *full information* if $\mathrm{supp}(\mu) = \{0, 1\}$. Slightly abusing the terminology, π is a *mixture of full and no information* if $\mathrm{supp}(\mu) = \{0, \mu_0, 1\}$.[11]

[11] This definition differs slightly from the usual definition of a mixture of information structures. Formally, we say that π is a *mixture* of π' and π'' if $\mu = \lambda\mu' + (1-\lambda)\mu''$ for some $\lambda \in (0, 1)$.

Given π, $\boldsymbol{\sigma}$, and $i \geq 2$, define $\rho_i \in \Delta[0,1]$ as the *public belief distribution*, which is the distribution over public beliefs regarding the state being H induced by observing actions taken by agents $1, \ldots, i-1$. Formally, for each $x \in [0,1]$,

$$\rho_i(x) = \mathbb{E}_\omega \left[\sum_{\boldsymbol{a} \in A^{i-1}(x)} \alpha^\omega_{\leq i-1}(\boldsymbol{a}|\pi, \boldsymbol{\sigma}) \right],$$

where $A^{i-1}(x) = \{\boldsymbol{a} \in A^{i-1} \mid \frac{\mu_0 \alpha^H_{\leq i-1}(\boldsymbol{a}|\pi,\boldsymbol{\sigma})}{\mu_0 \alpha^H_{\leq i-1}(\boldsymbol{a}|\pi,\boldsymbol{\sigma}) + (1-\mu_0)\alpha^L_{\leq i-1}(\boldsymbol{a}|\pi,\boldsymbol{\sigma})} = x\}$. Then, based on their private and public beliefs, agents update their *posterior beliefs* about $\omega = H$ according to Bayes' rule.[12]

3 Results

For comparison, we introduce the standard Blackwell order, denoted $\pi \succsim_B \pi'$, when π is Blackwell more informative than π', that is, when π' is a garbling of π.[13] By the Blackwell's classical theorem, $\pi \succsim_B \pi'$ if and only if the single decision maker (agent 1 in our model) prefers π over π' for all decision problems.

By contrast, our primary focus is on the following binary relation:

Definition 1. π is *more socially valuable* than π', denoted $\pi \succsim_S \pi'$, if for any decision problem $\mathcal{D} = (A, u)$, $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ for any agent i, any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$, and any equilibrium $\boldsymbol{\sigma}^{**}$ under $(\mathcal{D}, \pi')$.

Thus, $\succsim_S$ requires that *all* agents weakly prefer π over π' in terms of expected payoff, across *all* decision problems and *all* equilibria.[14] This Pareto-type order is natural from a societal perspective and provides insights for the social learning literature. Specifically, this strong requirement of $\succsim_S$ implies that if π is more socially valuable than π', then π achieves higher welfare, defined as the discounted sum of payoffs, than π' for all discount factors, decision problems, and equilibrium selections.[15] Additionally, $\pi \succsim_S \pi'$ indicates that learning occurs faster under π than under π' for all decision problems, connecting our

When $\text{supp}(\pi') \cap \text{supp}(\pi'') = \emptyset$, this definition coincides with the standard one.

[12]Given a public belief x and a private belief y, the posterior belief is $\frac{xy}{xy + \frac{\mu_0}{1-\mu_0}(1-x)(1-y)}$.

[13]Formally, π' is a garbling of π if there exists a Markov kernel $\gamma : S \to \Delta(S')$ such that $\pi'(s'|\omega) = \sum_{s \in S} \gamma(s'|s)\pi(s|\omega)$.

[14]In Section 4.1, we introduce a weaker binary relation that requires all asymptotic agents to prefer one information structure to another for every decision problem.

[15]We discuss welfare comparisons under discount factors sufficiently close to 1 in Section 4.1.

binary relation $\succsim_S$ to the literature on the speed of learning in social learning (Hann-Caruthers et al., 2018; Rosenberg and Vieille, 2019).

Definition 1 also requires that all equilibrium under π is better than all equilibrium under π', and thus, we impose the strong equilibrium selection rules. In this sense, $\succsim_S$ provides a robust welfare-based criterion for comparing information structures in the social learning.[16] Consider the online shopping setting in the Introduction wherein the outside observer evaluates the informational efficiency of advertisements or product descriptions. Then, they usually cannot predict either users' decision problems or the realizing equilibrium. In this setting, our definition of $\succsim_S$ captures the most pessimistic and robust evaluation on the value of information from π.

Our first observation establishes that our binary relation is strictly stronger than the Blackwell order. In other words, if one information structure is more socially valuable than another, then it is also Blackwell more informative, but not vice versa.

Proposition 1. *$\succsim_S$ is a strictly stronger binary relation than $\succsim_B$.*

Note that $\pi \succsim_S \pi'$ implies that $\pi \succsim_B \pi'$ as agent 1 prefers π over π' for all decision problems. Thus, $\succsim_S$ is weakly stronger than $\succsim_B$. To complete the proof, we show the following example in which $\pi \succsim_B \pi'$ holds, but $\pi \succsim_S \pi'$ does not.

Example 1. Consider the information structure $\pi : \Omega \to \Delta(\{s_0, s_1, s_2\})$ defined by $\pi(s_1|H) = 1 - \varepsilon$, $\pi(s_2|H) = \varepsilon$, $\pi(s_0|L) = 1 - \delta$, and $\pi(s_2|L) = \delta$. Suppose $\varepsilon > \delta > 0$. Now, take $\varepsilon' \in (\varepsilon, 1)$ and define $\pi' : \Omega \to \Delta(\{s_0, s_1, s_2\})$ as $\pi'(s_1|H) = 1 - \varepsilon'$, $\pi'(s_2|H) = \varepsilon'$, $\pi'(s_0|L) = 1 - \delta$, and $\pi'(s_2|L) = \delta$. Then, we have $\pi \succsim_B \pi'$. Now, consider the following decision problem $\mathcal{D} = (A, u)$: $A = \{a_0, a_1\}$, $u(a_0, H) = u(a_0, L) = 0$, $u(a_1, H) = 1 - r$, and $u(a_1, L) = -r$, where $r \in \left(\frac{\mu_0\varepsilon}{\mu_0\varepsilon+(1-\mu_0)\delta}, \min\left\{\frac{\mu_0\varepsilon'}{\mu_0\varepsilon'+(1-\mu_0)\delta}, \frac{\mu_0\varepsilon^2}{\mu_0\varepsilon^2+(1-\mu_0)\delta^2}\right\}\right)$.[17]

Take any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$. First, agent 1 chooses action a_1 if and only if she receives $s = s_1$ as $r > \frac{\mu_0\varepsilon}{\mu_0\varepsilon+(1-\mu_0)\delta}$. Then, agent 2 can perfectly infer that the true state is H when either $s = s_1$ or agent 1 has chosen action a_1. Otherwise, agent 2's posterior belief is below the cutoff r. Thus, agent 2 chooses action a_1 if and only if she perfectly knows that the true state is H. Similarly,

[16] Section 4.4 examines a weaker binary relation that relaxes the equilibrium selection rule. Relatedly, Section 4.5 focuses on a restricted domain of decision problems, particularly those involving binary actions.

[17] Note that $\frac{\mu_0\varepsilon^2}{\mu_0\varepsilon^2+(1-\mu_0)\delta^2} > \frac{\mu_0\varepsilon}{\mu_0\varepsilon+(1-\mu_0)\delta}$ since $\varepsilon > \delta > 0$.

agent i chooses action a_1 if and only if (i) $s = s_1$ or (ii) at least one agent before i has chosen action a_1, as agent i can perfectly infer that the true state is H under these cases, while the posterior beliefs would otherwise be below r. Thus, agent i's expected payoff is $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \mu_0(1-\varepsilon^i)(1-r)$.[18]

Under $(\mathcal{D}, \pi')$, there is an equilibrium in which agent i takes a_0 if and only if he receives s_0 or at least one agent before i has taken a_0 since $\frac{\mu_0\varepsilon'}{\mu_0\varepsilon'+(1-\mu_0)\delta} > r$. Let $\boldsymbol{\sigma}^{**}$ denote this equilibrium strategy profile. In this equilibrium, the ex-ante expected payoff of agent i is $V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**}) = \mu_0(1-r) - (1-\mu_0)\delta^i r$.

Thus, the difference in payoffs of agent i (≥ 2) is

$$\begin{aligned} V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**}) - V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) =& \mu_0\varepsilon^i(1-r) - (1-\mu_0)\delta^i r \\ =& \mu_0\varepsilon^i\left(1 - \frac{\mu_0\varepsilon^i + (1-\mu_0)\delta^i}{\mu_0\varepsilon^i} r\right) \\ \geq& \mu_0\varepsilon^i\left(1 - \frac{\mu_0\varepsilon^2 + (1-\mu_0)\delta^2}{\mu_0\varepsilon^2} r\right) \\ >& 0. \end{aligned}$$

Therefore, $\pi \succsim_B \pi'$ but not $\pi \succsim_S \pi'$. □

Proposition 1 intuitively follows because past actions provide coarser information than past signal realizations. Consequently, our binary relation requires the information structure to be sufficiently informative to ensure that the joint value of history and private signals increases. By contrast, if agents could observe past signal realizations instead of actions, then a Blackwell more informative signal would always be more socially valuable. Essentially, the gap between $\succsim_S$ and $\succsim_B$ is driven by information externalities arising from whether agents can observe past signal realizations or only past actions.

In the setting described in Example 1, when signals are observable, the expected payoffs of agent $i \geq 2$ under π and π' are identical in this example. If past signals were observable, agent i receiving $s = s_2$ would choose a_1 whenever all preceding agents also received $s = s_2$. However, in the observable action setting, agent i with $s = s_2$ would choose a_0 if all predecessors had selected a_0 under π, even when all preceding agents receive $s = s_2$. In contrast, under π', agents can behave as if they can perfectly observe the past signal realizations. Therefore, all agents except for agent 1 strictly prefer π over π' in this decision problem.

How strong is our binary relation relative to the Blackwell order? To answer

[18] The formal proof is provided in Lemma 3 in the Appendix.

this, we provide a characterization as follows:

Theorem 1 (Characterization). *$\pi \succsim_S \pi'$ holds if and only if*

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq \overline{V}_i^{\mathcal{D}}(\pi')$$

for any decision problem $\mathcal{D}$, any agent i, and any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D},\pi)$.

Thus, by Theorem 1, one information structure is more socially valuable than another if and only if it yields higher expected payoffs for all agents, decision problems, and equilibria, even when past signals are observable under the alternative information structure.

The sufficiency part of Theorem 1 is immediate, as the observable signal setting provides an upper bound on any equilibrium payoffs (Lemma 5). For the necessity part of Theorem 1, we construct a decision problem in which agents can perfectly infer past signal realizations under π'. Specifically, take any decision problem $\mathcal{D} = (A, u)$, an equilibrium $\boldsymbol{\sigma}^*$ under π, and an equilibrium $\boldsymbol{\sigma}^{**}$ under π'. Then, we construct an auxiliary decision problem $\overline{\mathcal{D}} = (\overline{A}, \overline{u})$ where $\overline{A} = \{(a, k) \mid a \in A, k \in S'\}$ and $\overline{u}((a, k), \omega) = u(a, \omega)$ for all $a \in A$, $\omega \in \Omega$, and $k \in S'$. Next, we construct an equilibrium $\boldsymbol{\sigma}$ under $(\overline{\mathcal{D}}, \pi)$ in which agents choose optimal actions from among the replicated actions, independently of their private signals $s \in S$. By contrast, under $(\overline{\mathcal{D}}, \pi')$, we consider an equilibrium $\boldsymbol{\sigma}'$ in which each agent selects an optimal action that perfectly reflects their private signal realization $s' \in S'$. Thus, the expected payoff under π with $\boldsymbol{\sigma}$ is the same as in the original equilibrium $\boldsymbol{\sigma}^*$. However, under π' with $\boldsymbol{\sigma}'$, the expected payoff corresponds to that in the observable signal setting.

Intuitively, by sufficiently enlarging the action space, we can reinterpret agents' actions as messages that convey private signals in a cheap-talk environment. In this setting, the equilibrium selection effectively determines how informative these messages become. Under π', we can select a truth-telling equilibrium in which each agent breaks ties in a manner that perfectly reveals their private signal realization. By contrast, under π, we construct a babbling equilibrium in which agents' actions convey no additional information relative to the original setting. This comparison illustrates that when the action space is rich enough, the observable signal setting under π' can be induced as an equilibrium outcome that achieves maximal information revelation, whereas the equilibrium under π corresponds to the original observable action setting. Hence, by leveraging the strong equilibrium selection rule, this construction highlights the essential role of information externalities in determining how effectively private information is aggregated and transmitted in social learning, in its most extreme form.

By combining the classical result of Smith and Sørensen (2000), we derive a simple necessary condition from Theorem 1. We say that an information structure π induces *unbounded beliefs* if $\mathrm{co}(\mathrm{supp}(\mu)) = [0, 1]$. Since agents can eventually learn the true state in an observable signal setting, we obtain the following necessary condition:

Corollary 1 (Necessary condition)**.** *Suppose that π' is not no information. If $\pi \succsim_S \pi'$, then π induces unbounded beliefs.*

Corollary 1 states that, except in the trivial case, an information structure must induce unbounded beliefs to be more socially valuable than another. Thus, if an information cascade occurs under a given information structure, it is no longer more socially valuable than any other information structure except in certain trivial cases.

For the proof sketch, we construct a decision problem in which only almost conclusive signals yield a positive payoff. If π does not generate unbounded beliefs, the equilibrium payoff becomes zero. However, by Theorem 1, we can construct an auxiliary problem in which the equilibrium payoff under π remains unchanged, while π' yields the payoff corresponding to the observable signal setting. In this auxiliary problem, whenever π' is not no information, repeated observations of past signal realizations induce asymptotic learning and thus strictly positive expected payoffs. Hence, inducing unbounded beliefs under π is a necessary condition for $\succsim_S$.[19]

Theorem 1 and Corollary 1 underscore the strong requirements inherent in our binary relation. This naturally gives rise to the question: Which pairs of information structures can be compared within our binary relation? Accordingly, we shift our focus to the sufficiency part of Theorem 1. However, verifying this condition is challenging, as it depends on the underlying decision problem. Moreover, we can see that the necessary condition in Corollary 1 is not a sufficient condition by Example 1. To address this, we provide a sufficient condition that can be verified directly from the information structures.

[19]The construction of the original decision problem relies on "test" problems, which correspond to the extreme points of convex functions in the binary state space. We leave open the question of whether considering only test problems is sufficient for the original problem (rather than the auxiliary one) when verifying $\succsim_S$.

Theorem 2 (Sufficient condition)**.** *If there exists π'' such that supp$(\mu'') = \{0, \mu_0, 1\}$ and $\pi \succsim_B \pi'' \succsim_B \pi'$, then $\pi \succsim_S \pi'$.*

Thus, Theorem 2 indicates that π is more socially valuable than π' if there exists a mixture of full and no information π'' such that $\pi \succsim_B \pi'' \succsim_B \pi'$. To verify the existence of such a mixture, we provide an equivalent condition.

Proposition 2. *There exists π'' such that supp$(\mu'') = \{0, \mu_0, 1\}$ and $\pi \succsim_B \pi'' \succsim_B \pi'$ if and only if π and π' satisfy*

$$1 - \sum_{s \in supp(\pi')} \min\{\pi'(s|L), \pi'(s|H)\} \leq \min\{\pi(\mu = 0|L), \pi(\mu = 1|H)\}.$$

Recall that the necessary condition in Theorem 1 requires that π induces unbounded beliefs if π is more socially valuable than π' and π' is not no information. Then, by Proposition 2, the sufficient condition in Theorem 2 indicates that π is more socially valuable than π' if π assigns a sufficiently high probability to disclosing conclusive signals about each state.

For example, for any information structure π', we can construct an information structure π by taking a mixture of π' and π^{full} where π^{full} is full information. Then, there exists $\lambda \in (0, 1)$ such that $\pi(\mu = x|\omega) = (1-\lambda)\pi'(\mu' = x|\omega)$ and $\pi(\mu = y|\omega) = (1 - \lambda)\pi'(\mu = y|\omega) + \lambda\pi^{full}(\mu = y|\omega)$ for all $x \neq 0, 1$, $y \in \{0, 1\}$ and ω. We can see that if λ is above the threshold, $\pi \succsim_S \pi'$ holds.[20] One of the most popular information structures in the social learning literature is the symmetric binary information structures. Suppose that $\mu_0 = 1/2$. A symmetric binary signal π' consists of the binary signals $S' = \{s_l, s_h\}$ with disclosure rule $\pi'(s_l|L) = \pi'(s_h|H) = 1 - p$ where $p \in [0, 1/2]$. Then, π' is parameterized by the single parameter p, where lower p implies a more informative signal in the Blackwell order. Now, we define π as the mixture of π^{full} and π' with probability λ and $1 - \lambda$. By the previous discussion, π is more socially valuable than π' if $\lambda \geq 1 - 2p$. Thus, π is more socially valuable than π' if π assigns enough high probability to disclose full information relative to the informativeness of π'.

The formal proof of Theorem 2 is complex and provided in the Appendix. The key step focuses on the intrinsic properties of mixtures of full and no information. Specifically, if such a mixture is Blackwell more informative than

[20] Note that $\min\{\pi(\mu = 0|L), \pi(\mu = 1|H)\} = \lambda + (1 - \lambda)\min\{\pi'(\mu' = 0|L), \pi'(\mu' = 1|H)\}$. Thus, the inequality in Proposition 2 holds if

$$\lambda \geq 1 - \frac{\sum_{s \in \text{supp}(\pi')} \min\{\pi'(s|L), \pi'(s|H)\}}{1 - \min\{\pi'(\mu' = 0|L), \pi'(\mu' = 1|H)\}}.$$

another information structure, it is also more socially valuable (Lemma 7). Moreover, if an information structure is Blackwell more informative than the mixture, it is also more socially valuable (Lemma 8). Therefore, whenever a mixture of full and no information exists between two information structures in the Blackwell order, they remain comparable in our binary relation.

We now briefly explain why the Blackwell order with a mixture of full and no information implies our binary relation. The proof of Lemma 7 proceeds as follows. First, as shown in Lemma 6, under any mixture of full and no information, all agents can achieve the same expected payoff as if they had observed past signal realizations, even though they cannot directly infer their predecessors' private signals. Intuitively, this follows for the following reasons. First, under any mixture of full and no information, it is optimal for all agents in any decision problem to choose the optimal action when receiving a conclusive signal and to mimic the immediately preceding agent when receiving an uninformative signal. Second, this strategy profile ensures that agents act as if they could observe past signals, both when (i) some previous agent or the agent herself has received a conclusive signal, and when (ii) all agents have received uninformative signals.[21] Given the above discussion, if π'' is Blackwell more informative than π' and π'' is a mixture of full and no information, then the expected payoff of agent i under π'' is weakly higher than that under i conditionally independent observations of π' (that is, $\pi'^{\otimes i}$). Since past signals are always Blackwell more informative than history (Lemma 5), this expected payoff remains higher than that in any equilibrium under π'.

For the second step, Lemma 8 constructs a strategy profile under π that achieves a lower bound on any equilibrium payoff under π. Additionally, this strategy profile induces the same expected payoff as that under π'' for any equilibrium when π'' is a mixture of full and no information. Intuitively, the construction follows this logic: Consider any equilibrium strategy under π. First, any other strategy weakly decreases the agent's payoff due to the equilibrium condition. In particular, take a strategy in which agent i behaves as if she observes π'' rather than π. Since π'' is a mixture of full and no information, such a strategy involves choosing the optimal actions upon receiving conclusive signals about each state and mimicking agent $i-1$'s action otherwise. Given this, we further modify agent $i-1$'s strategy to follow the same one. This

[21] This feature is nontrivial because even a slight deviation in the support of the private belief distribution from that of the mixture can result in decision problems and equilibria that violate this property, as can be inferred from the proof of Proposition 10 in Section 4.4.

change decreases agent $i-1$'s expected payoff, which, in turn, reduces agent i's (conditional) payoff from mimicking agent $i-1$ as the private belief coincides with the prior. Repeating this process yields a strategy profile that induces the lower bound of any equilibrium payoff under π. Moreover, this lower bound coincides with the expected payoff under π'' for any equilibrium by construction and Lemma 6.

Note that, under the single decision problem, it is sufficient to construct a strategy in which an agent can behave as if she observes π'' rather than π to prove that π is Blackwell more informative than π'' because information is free disposal as in the logic behind garbling. By contrast, this is not sufficient in our social learning setting because of the information externalities. In the proof of Lemma 8, we further utilize the property of the equilibrium strategy under the mixtures of full and no information π''. Then, behaving as if an agent observes π'' rather than π negatively affects the payoffs of future agents, and thus, π is more socially valuable than π''.

As a remark, we do not know whether the converse of Theorem 2 holds, or how close this sufficient condition is to being necessary in general. The main difficulty, as discussed in the Conclusion, lies in conducting a general analysis of learning speeds in each period. By circumventing this difficulty, one can derive a necessary condition using techniques from large deviation theory, when focusing only on sufficiently late agents and specific decision problems. Although the resulting necessary condition might not be tight, the following Example 2 indicates that the sufficient condition in Theorem 2 is close to being necessary to some extent.

Example 2. Suppose that $\mu_0 = 1/2$. We now focus on two mixtures of full information and symmetric binary information structures. Formally, π consists of $S = \{s_0, s_l, s_h, s_1\}$, $\pi(s_1|L) = \pi(s_0|H) = 0$, $\pi(s_0|L) = \pi(s_1|H) = \lambda$, $\pi(s_l|L) = \pi(s_h|H) = (1-\lambda)(1-p)$, and $\pi(s_h|L) = \pi(s_l|H) = (1-\lambda)p$, where $\lambda \in [0,1]$ and $p \in (0, 1/2)$. Thus, π discloses conclusive signals s_0 or s_1 with probability λ and symmetric binary signals s_l or s_h with the remaining probability $1-\lambda$. Similarly, π' consists of $S = \{s_0, s_l, s_h, s_1\}$, $\pi'(s_1|L) = \pi'(s_0|H) = 0$, $\pi'(s_0|L) = \pi'(s_1|H) = \lambda'$, $\pi'(s_l|L) = \pi'(s_h|H) = (1-\lambda')(1-p')$, and $\pi'(s_h|L) = \pi'(s_l|H) = (1-\lambda')p'$, where $\lambda' \in [0,1]$ and $p' \in (0,1)$. We assume that $\lambda > \lambda'$ and $p < p'$, and thus $\pi \succsim_B \pi'$.

Consider the following (original) decision problem $\mathcal{D} = (A, u)$. There are two actions, $A = \{a_0, a_1\}$, with payoffs given by $u(a_0, L) = u(a_0, H) = 0$, $u(a_1, H) = 1 - r$, and $u(a_1, L) = -r$, where $r \in (1-p, 1)$. Then, by Theorem 1, we can construct an auxiliary problem $\overline{\mathcal{D}}$ such that the expected payoff under π is the same as in the original problem, while the expected payoff under π' coincides with that in the observable-signal setting.

Under the assumption that $r > 1 - p$, the payoffs under π can be computed straightforwardly. Specifically, there exists a unique equilibrium $\boldsymbol{\sigma}$ under π, which yields a payoff of $V_i^{\overline{\mathcal{D}}}(\pi, \boldsymbol{\sigma}) = \frac{1}{2}[1 - (1 - \lambda)^i](1 - r)$ for each i. In the Appendix, we derive $\overline{V}_i^{\overline{\mathcal{D}}}(\pi')$ by focusing on sufficiently large i and applying large deviation theory. The derivation implies that $V_i^{\overline{\mathcal{D}}}(\pi, \boldsymbol{\sigma}) \geq \overline{V}_i^{\overline{\mathcal{D}}}(\pi')$ for all $r \in (1 - p, 1)$ and sufficiently large i if and only if

$$\lambda \geq 1 - 2(1 - \lambda')\sqrt{p'(1 - p')},$$

where this is a necessary condition for $\pi \succsim_S \pi'$.

Note that Theorem 2 and Proposition 2 together imply that our sufficient condition is equivalent to $\lambda \geq 1 - 2(1 - \lambda')p'$. Thus, by comparing with the above necessary condition, this condition is, to some extent, close to being necessary. In particular, our sufficient condition becomes tight when p' is large.[22] □

In Example 2, $\pi \succsim_S \pi'$ does not hold when π and π' have the same probability of disclosing the conclusive signals. This is true in general and derives a specific condition under which the converse of Theorem 2 holds, as shown below.

Corollary 2. *Suppose that* $\pi(\mu = 1|H) = \pi'(\mu' = 1|H) > 0$ *and* $\pi(\mu = 0|L) = \pi'(\mu' = 0|L) > 0$. *Then,* $\pi \succsim_S \pi'$ *(if and) only if there exists* π'' *such that supp*$(\mu'') = \{0, \mu_0, 1\}$ *and* $\pi \succsim_B \pi'' \succsim_B \pi'$.

Proof. By Theorem 2, it is enough to show the necessity part. Take any π and π' such that $\pi(\mu = 1|H) = \pi'(\mu' = 1|H) > 0$ and $\pi(\mu = 0|L) = \pi'(\mu' = 0|L) > 0$. As a contraposition, suppose that there is no π'' such that $\text{supp}(\mu'') = \{0, \mu_0, 1\}$ and $\pi \succsim_B \pi'' \succsim_B \pi'$. Then, under π', there exists a signal s' inducing a private belief in $(0, 1) \setminus \{\mu_0\}$.[23] As in the construction behind the proof of Corollary 1, there exists some decision problem and its auxiliary problem, in which the observable signal setting of π' yields strictly higher payoff than π for some sufficiently later agent i because observing signal s' i-times induce the strictly better payoff by $\pi(\mu = 1|H) = \pi'(\mu' = 1|H)$ and $\pi(\mu = 0|L) = \pi'(\mu' = 0|L)$. □

[22] As $p' \to \frac{1}{2}$, the inequality converges to $\lambda \geq \lambda'$. This is consistent with Theorem 2, since π' approaches a mixture of full and no information.

[23] Note that π' is not full or no information.

4 Discussion

4.1 Long-Run Comparison

Our original binary relation appears strong, as it requires that all agents prefer one information structure to another. A plausible alternative definition would require only that all sufficiently late agents prefer one information structure over another. We focus on this weaker version and show that our original characterization still provides insights into it.

Definition 2. π is *eventually more socially valuable* than π', denoted $\pi \succsim_{ES} \pi'$, if there is one threshold $N \in \mathbb{N}$ such that for any decision problem $\mathcal{D} = (A, u)$, $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ for all $i \geq N$, any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$, and any equilibrium $\boldsymbol{\sigma}^{**}$ under $(\mathcal{D}, \pi')$.

By utilizing the proof of Theorem 1, we have the following characterization:

Proposition 3. *$\pi \succsim_{ES} \pi'$ if and only if there exists $N \in \mathbb{N}$ such that $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq \overline{V}_i^{\mathcal{D}}(\pi')$ for any decision problem $\mathcal{D}$, all $i \geq N$, and any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D},\pi)$.*

The proof is almost the same as the one in Theorem 1, and thus it is omitted. It turns out that the necessary condition in Corollary 1 is also a necessary condition in this setting. Thus, it is necessary that π induces unbounded beliefs if $\pi \succsim_{ES} \pi'$ for some π' except for the trivial case.

Corollary 3. *Suppose that π' is not no information. If $\pi \succsim_{ES} \pi'$, then π induces unbounded beliefs.*

In Example 1, $V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ is strictly higher than $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*)$ for all $i \geq 2$, and thus π is not eventually more socially valuable than π'. This means that inducing unbounded beliefs is not a sufficient condition even in this case.

By Proposition 3, we can highlight the difference between $\succsim_{ES}$ and the eventual Blackwell order/the large sample order in Azrieli (2014) and Mu et al. (2021).

Definition 3 (Mu et al. (2021))**.** π is *eventually Blackwell more informative than* π', denoted $\pi \succsim_{EB} \pi'$, if there exists $N \in \mathbb{N}$ such that $\pi^{\otimes i} \succsim_B \pi'^{\otimes i}$ for all $i \geq N$.

By the same logic behind Example 1, the following Proposition 4 shows that $\succsim_{ES}$ is strictly stronger than the eventual Blackwell order $\succsim_{EB}$, mirroring the relationship between $\succsim_S$ and the Blackwell order $\succsim_B$.

Proposition 4. *$\succsim_{ES}$ is a strictly stronger binary relation than $\succsim_{EB}$.*

As $\succsim_S$ is stronger than $\succsim_{ES}$, Theorem 2 still provides a sufficient condition for $\succsim_{ES}$. However, we can provide a weaker sufficient condition for $\succsim_{ES}$ by leveraging the eventual Blackwell order $\succsim_{EB}$.

Proposition 5. *If there exists π'' such that supp$(\mu'') = \{0, \mu_0, 1\}$ and $\pi \succsim_{EB} \pi'' \succsim_{EB} \pi'$, then $\pi \succsim_{ES} \pi'$.*

In other words, if there exists a mixture of full and no information between π and π' in the eventual Blackwell order, then π is eventually more socially valuable than π'.

Note that, as shown in Azrieli (2014) and Mu et al. (2021), there exists a mixture of full and no information π'' and a symmetric binary information structure π' such that $\pi'' \succsim_{EB} \pi'$ holds, while $\pi'' \succsim_B \pi'$ does not. Our proof demonstrates that, under the condition $\text{supp}(\mu'') = \{0, \mu_0, 1\}$, $\pi'' \succsim_{EB} \pi'$ implies $\pi'' \succsim_{ES} \pi'$. Therefore, combining these observations shows that there exists an example in which $\pi'' \succsim_{ES} \pi'$ holds even though $\pi'' \succsim_B \pi'$ does not.[24] Furthermore, together with Example 1, this implies that $\succsim_{ES}$ is independent of the Blackwell order $\succsim_B$.

As noted above, $\succsim_{ES}$ compares two information structures based on the expected payoffs of agents who arrive sufficiently late. To further weaken this requirement, we now introduce a coarser binary relation that focuses solely on the expected payoffs in the limit.

Definition 4. π is *eventually more socially valuable in the limit* than π', denoted $\pi \succsim_{LES} \pi'$, if $\lim_{i\to\infty} V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq \lim_{i\to\infty} V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ for any decision problem $\mathcal{D} = (A, u)$, any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$, and any equilibrium $\boldsymbol{\sigma}^{**}$ under $(\mathcal{D}, \pi')$.

Note that $\succsim_{LES}$ is equivalent to the welfare comparison at the limit. Formally, $\pi \succsim_{LES} \pi'$ if and only if the discounted sum of payoffs is higher under π rather than π' for all discount factors sufficiently close to 1, any decision problems, and any equilibrium selections.

Under this weaker notion, we have a tight characterization.

Theorem 3. *Suppose that π' is not no information. $\pi \succsim_{LES} \pi'$ if and only if π induces unbounded beliefs.*

[24] This implies that $\succsim_S$ is strictly stronger than $\succsim_{ES}$, since $\succsim_S$ is strictly stronger than $\succsim_B$ by Proposition 1.

Thus, if we are concerned only with limit welfare, it is sufficient to check whether one information structure induces unbounded beliefs or discloses conclusive signals about each state, which is a necessary condition for $\succsim_S$ as in Corollary 1. Moreover, Theorem 3, combined with Example 1, indicates that $\succsim_{LES}$ is strictly stronger than $\succsim_{ES}$. The relationships among the binary relations introduced so far are summarized in Figure 1.

$$\begin{tikzcd}
\succsim_S \arrow[r] \arrow[d] & \succsim_{ES} \arrow[r] \arrow[d] & \succsim_{LES} \\
\succsim_B \arrow[r] & \succsim_{EB} &
\end{tikzcd}$$

Figure 1: Relationships among Binary Relations

Notes: $\succsim_B$ (resp. $\succsim_{EB}$) represent the (resp. eventual) Blackwell order. $\succsim_S$ denotes the socially valuable order defined in Definition 1. $\succsim_{ES}$ represents the eventual socially valuable order introduced in Definition 2. $\succsim_{LES}$ corresponds to the limit eventual socially valuable order defined in Definition 4. Arrows indicate the strength of binary relations: an arrow from relation P to relation Q means that P is (strictly) stronger than Q.

4.2 Dynamic Information Structures

Throughout Section 3, we have assumed that agents receive private signals independently from an identical information structure. In this section, we extend our analysis to dynamic information structures, following the dynamic extensions of the Blackwell comparison developed by Renou and Venel (2024) and Whitmeyer and Williams (2024a) under adaptive decision problems. We show that our main results continue to hold in this more general setting.

Let $S = \prod_{i=1}^{\infty} S_i$, and let $\pi : \Omega \to \Delta(S)$ denote the dynamic information structure. For each i, let π_i be the marginal distribution of π over S_i, and let μ_i denote the induced distribution of private beliefs. For a decision problem $\mathcal{D}$ and a strategy profile $\boldsymbol{\sigma}$, define $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma})$ as the expected payoff of agent i when she observes both the past action history and her private signal. Let $\overline{V}_i^{\mathcal{D}}(\pi)$ denote the maximized expected payoff under the imaginary setting wherein agent i can observe signals drawn from $\pi_{\leq i}$, the marginal distribution over $S_1 \times \cdots \times S_i$ induced by π.

Note that private signals s_i and s_j may be correlated, so the public belief can depend on the realization of each agent's private signal. Moreover, the marginal distributions π_i may differ across agents, allowing our framework to naturally encompass settings with heterogeneous private signals.

Then, Theorem 1 is naturally extended as follows:

Proposition 6. *$\pi \succsim_S \pi'$ holds if and only if $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq \overline{V}_i^{\mathcal{D}}(\pi')$ for any decision problem $\mathcal{D}$, any agent i, and any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D},\pi)$.*

Proof. We only show that $\overline{V}_i^{\mathcal{D}}(\pi) \geq V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma})$ holds for all $i, \mathcal{D}, \pi$, and $\boldsymbol{\sigma}$ because the remaining part of the proof is identical to that of Theorem 1. Take any $\mathcal{D}, \pi$, and $\boldsymbol{\sigma}$. Note that $\overline{V}_1^{\mathcal{D}}(\pi) = V_1^{\mathcal{D}}(\pi, \boldsymbol{\sigma})$. Fix $i \geq 2$. For each $\boldsymbol{s} \in S_1 \times \cdots \times S_{i-1}$, define $f_{i-1}(\boldsymbol{s}) \in \Delta(A^{i-1})$ as

$$f_{i-1}(\boldsymbol{a}|\boldsymbol{s}) = \prod_{k=1}^{i-1} \sigma_k(a_k|a_1, \ldots, a_{k-1}, s_k).$$

Thus, $f_{i-1}(\boldsymbol{a}|\boldsymbol{s})$ is the probability that agents $1, \ldots, i-1$ take action $\boldsymbol{a} = (a_1, \ldots, a_{i-1})$ when they receive private signals $\boldsymbol{s} = (s_1, \ldots, s_{i-1})$. Let $g_i^{\omega}(\boldsymbol{a}, s_i'|\pi, \boldsymbol{\sigma})$ denote the probability that agent i observes $(\boldsymbol{a}, s_i')$ when the state is ω and agents take strategy profile $\boldsymbol{\sigma}$. Then, $g_i(\cdot|\pi, \boldsymbol{\sigma})$ is a garbling of $\pi_{\leq i}$, where $g_i(\cdot|\pi, \boldsymbol{\sigma})$ is the ex-ante distribution induced by $g_i(\cdot|\pi, \boldsymbol{\sigma})$.[25] $\square$

It is natural to extend Theorem 2 as follows: If there exists π'' such that $\mathrm{supp}(\mu_i'') = \{0, \mu_0, 1\}$ and $\pi_i \succsim_B \pi_i'' \succsim_B \pi_i'$ for all i, then $\pi \succsim_S \pi'$. However, this claim does not hold in general because the agents' private signals may be correlated. The issue arises in Lemmas 6 and 7: when signals are correlated, the Blackwell order may not be respected even when past signals are observable.

To address this issue, we utilize the characterizations developed by Renou and Venel (2024) and Whitmeyer and Williams (2024a). They show that, in additively separable dynamic decision problems, dominance in expected payoff is equivalent to dominance in the Blackwell order of the distribution over posterior beliefs induced by all past signal realizations. Building on this insight, we derive the following sufficient condition, analogous to Theorem 2.

Proposition 7. *If there exists π'' such that for all i, (i) $\mathrm{supp}(\mu_i'') = \{0, \mu_0, 1\}$, (ii) $\pi_i \succsim_B \pi_i''$, (iii) $\pi_{\leq i}'' \succsim_B \pi_{\leq i}'$, and (iv) π'' is independent across agents, then $\pi \succsim_S \pi'$.*

[25] Note that

$$\begin{aligned} g_i^{\omega}(\boldsymbol{a}, s_i'|\pi, \boldsymbol{\sigma}) &= \sum_{\boldsymbol{s} \in S_1 \times \cdots \times S_{i-1}} \prod_{k=1}^{i-1} \sigma_k(a_k|a_1, \ldots, a_{k-1}, s_k) \pi_{\leq i}(\boldsymbol{s}, s_i'|\omega) \\ &= \sum_{\boldsymbol{s} \in S_1 \times \cdots \times S_{i-1}} f_{i-1}(\boldsymbol{a}|\boldsymbol{s}) \pi_{\leq i}(\boldsymbol{s}, s_i'|\omega) \\ &= \sum_{\boldsymbol{s} \in S_1 \times \cdots \times S_i} f_{i-1}(\boldsymbol{a}|\boldsymbol{s}) \pi_{\leq i}(\boldsymbol{s}|\omega) \mathbb{1}_{s_i = s_i'}. \end{aligned}$$

The proof is omitted, as it is essentially the same as that of Theorem 2. Instead, we provide a sketch of the main argument. The existence of a mixture of full and no information, denoted by π'', still provides a sufficient condition for $\succsim_S$. Because π'' is independent across agents by condition (iv), Lemma 6 continues to hold when extended to heterogeneous precisions. Conditions (i), (ii), and (iv) together with Lemma 8, ensure $\pi \succsim_S \pi''$ even when π exhibits serial correlation. The key insight from Renou and Venel (2024) and Whitmeyer and Williams (2024a) is reflected in the step establishing $\pi'' \succsim_S \pi'$. In particular, condition (iii), along with Lemma 6, guarantees that $\overline{V}_i^{\mathcal{D}}(\pi'') \geq \overline{V}_i^{\mathcal{D}}(\pi')$ for all agents and decision problems, thereby yielding $\pi'' \succsim_S \pi'$.

4.3 Heterogeneous Decision Problems

We assume that all agents are homogeneous, that is, each agent faces the same decision problem. In this section, we discuss how our result persist when we relax this assumption.

Let $\mathcal{D}_i = (A_i, u_i)$ be the agent i's decision problem.

Proposition 8. *$\pi \succsim_S \pi'$ holds if and only if $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq \overline{V}_i^{\mathcal{D}}(\pi')$ for any decision problem $\mathcal{D} = (\mathcal{D}_i)_{i\in\mathbb{N}}$, any agent i, and any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$.*

Thus, the same consequence from Theorem 1 holds, and hence, Corollary 1 is still a necessary condition.

However, the following Example 3 shows that our sufficient condition in Theorem 2 does not hold under the heterogeneous decision problems.

Example 3. Suppose that an information structure π consists of $S = \{s_0, s_{\mu_0}, s_1\}$, $\pi(s_1|H) = \pi(s_0|L) = 1-\varepsilon$, and $\pi(s_{\mu_0}|H) = \pi(s_{\mu_0}|L) = \varepsilon$, which induces $\mathrm{supp}(\mu) = \{0, \mu_0, 1\}$. In addition, we consider another information structure π' such that $S' = \{s'_l, s'_1\}$, $\pi'(s'_1|H) = 1-\varepsilon$, $\pi'(s'_1|L) = \varepsilon$, and $\pi'(s'_l|L) = 1$, which induces $\mathrm{supp}(\mu') = \{l, 1\}$ where $l = \frac{\mu_0\varepsilon}{\mu_0\varepsilon + 1 - \mu_0}$. Then, by the construction, we have $\pi \succsim_B \pi'$ and $\pi(\mu = 1) = \pi'(\mu = 1)$.

We now consider the following decision problem $(\mathcal{D}_1, \mathcal{D}_2)$: Let $\mathcal{D}_1 = (A_1, u_1)$, where $A_1 = \{a_0, a_1\}$, $u_1(a_0, \omega) = 0$, $u_1(a_1, H) = 1 - r_1$, $u_1(a_1, L) = r_1$, and $r_1 \in (l, \mu_0)$. Additionally, let $\mathcal{D}_2 = (A_2, u_2)$, where $A_2 = \{b_0, b_1\}$, $u_2(b_0, \omega) = 0$, $u_2(b_1, H) = 1 - r_2$, $u_2(b_1, L) = 1 - r_2$, and $r_2 \in (\mu_0, 1)$. Agent 1 faces on $\mathcal{D}_1$ and agent 2 faces on $\mathcal{D}_2$.

Take any equilibrium $\boldsymbol{\sigma}$ under π. Then, agent 1 chooses action a_1 if and only if she receives signals either s_1 or s_{μ_0}. Now, agent 2's posterior belief after observing a_1 and s_{μ_0} is below the cutoff r_2, and thus agent 2 chooses action b_1 if and only if he observes a conclusive signal s_1. Thus, the expected payoffs are $V_1(\pi, \boldsymbol{\sigma}) = \mu_0(1 - r_1) - (1 - \mu_0)\varepsilon r_1$ and $V_2(\pi, \boldsymbol{\sigma}) = \mu_0(1 - \varepsilon)(1 - r_2)$.

By contrast, take any equilibrium $\boldsymbol{\sigma}'$ under π'. Then, agent 1 chooses action a_1 if and only if she observes a conclusive signal s'_1. Given this, agent 2 can inference the agent 1's private signal by her action. Thus, agent 2 chooses action b_1 if and only if agent 1 chose action a_1 or private signal is s'_1. The expected payoffs are given by $V_1(\pi', \boldsymbol{\sigma}') = \mu_0\varepsilon(1 - r_1)$ and $V_2(\pi', \boldsymbol{\sigma}') = \mu_0(1 - \varepsilon^2)(1 - r_2)$.

Therefore, $V_1(\pi, \boldsymbol{\sigma}) > V_1(\pi', \boldsymbol{\sigma}')$ but $V_2(\pi, \boldsymbol{\sigma}) < V_2(\pi', \boldsymbol{\sigma}')$ even though $\text{supp}(\mu) = \{0, \mu_0, 1\}$ and $\pi \succsim_B \pi'$. This indicates that Theorem 2 does not hold under the heterogeneous decision problems. $\square$

4.4 Equilibrium Selection

Our definitions so far are too strong, particularly in relation to the equilibrium selection rule. As a result, $\succsim_S$ is not a partial order for example.

Proposition 9. *$\pi \succsim_S \pi$ if and only if $\text{supp}(\mu) \subseteq \{0, \mu_0, 1\}$.*

Proof. Suppose $\text{supp}(\mu) \subseteq \{0, \mu_0, 1\}$. Then, $1 - \sum_{s \in \text{supp}(\pi')} \min\{\pi'(s|L), \pi'(s|H)\} = 1 - \pi(\mu = \mu_0|L) = \pi(\mu = 0|L) = \min\{\pi(\mu = 0|L), \pi(\mu = 1|H)\}$. Therefore, by Theorem 2 and Proposition 2, we have $\pi \succsim_S \pi$.

Now, we show that $\pi \succsim_s \pi$ does not hold if $\text{supp}(\mu) \not\subseteq \{0, \mu_0, 1\}$. It is sufficient to show for the case where there exists some $x \in (\mu_0, 1)$ such that $x \in \text{supp}(\mu)$. Take $r \in [0, 1]$ that satisfies $x < r < \frac{x^2}{x^2 + \frac{\mu_0}{1-\mu_0}(1-x)^2}$. Consider the decision problem $\mathcal{D} = (A, u)$: $A = \{a_0, a_1\}$ and the payoff function is defined as $u(a_0, H) = u(a_0, L) = 0$, $u(a_1, H) = 1 - r$, and $u(a_1, L) = -r$. Take any equilibrium $\boldsymbol{\sigma}^* = (\sigma_i^*)_{i \in \mathbb{N}}$ and $s_1, s_2 \in S(x)$. Then, it follows that $\sigma_1^*(a_0|s_1) = 1$ and $\sigma_2^*(a_0|a_0, s_2) = 1$. Thus, $V_2^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*|s_1, s_2) = 0$. Additionally, we have $\overline{V}_2^{\mathcal{D}}(\pi|s_1, s_2) > 0$ since $r < \frac{x^2}{x^2 + \frac{\mu_0}{1-\mu_0}(1-x)^2}$. Note that for all $s'_1, s'_2 \in S$, $\overline{V}_2^{\mathcal{D}}(\pi|(s'_1, s'_2)) \geq V_2^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*|(s'_1, s'_2))$.[26] Therefore, $\overline{V}_2^{\mathcal{D}}(\pi) > V_2^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*)$. By Theorem 1, it follows that $\pi \succsim_S \pi$ does not hold. $\square$

An alternative binary relation considers a weaker notion of comparison. This definition relaxes the pessimistic and robust attitude toward equilibrium selection rules in Definition 1.

[26]This statement follows from the same argument as Lemma 5 in the Appendix.

Definition 5. π is *weakly more socially valuable* than π', denoted $\pi \succsim_W \pi'$, if for any decision problem $\mathcal{D} = (A, u)$ and any equilibrium $\boldsymbol{\sigma}^{**}$ under $(\mathcal{D}, \pi')$, there exists an equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$ such that $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ for any agent i.

Under this definition, it is straightforward to see that $\pi \succsim_W \pi$ holds for any π. Note that Example 1 does not use equilibrium selection under π, and therefore, the same result holds even if we consider this weak order. This means that $\succsim_W$ is also a strictly stronger order than $\succsim_B$.

In the following example, we highlight the difference between $\succsim_S$ and $\succsim_W$.

Example 4. Suppose that $\mu_0 = 1/2$. Let $\pi : \Omega \to \Delta(\{s_0, s_1, s_2\})$ as $\pi(s_1|H) = 1 - \varepsilon$, $\pi(s_2|H) = \varepsilon$, $\pi(s_0|L) = 1 - \delta$, and $\pi(s_2|L) = \delta$. Additionally, let $\pi' : \Omega \to \Delta(\{s_0', s_1', s_2'\})$ as $\pi(s_1'|H) = 1 - \varepsilon'$, $\pi'(s_2'|H) = \varepsilon'$, $\pi'(s_0'|L) = 1 - \delta'$, and $\pi'(s_2'|L) = \delta'$. Assume that $\delta < \delta' < \varepsilon < \varepsilon'$.[27] Thus, $\pi \succsim_B \pi'$ holds, as this condition is equivalent to $\varepsilon \leq \varepsilon'$ and $\delta \leq \delta'$. Moreover, we assume that $\frac{\varepsilon'}{\varepsilon'+\delta'} < \frac{\varepsilon}{\varepsilon+\delta} < \frac{\varepsilon'^2}{\varepsilon'^2+\delta'^2}$.

We now construct a decision problem in which the necessary condition of Theorem 1 is violated, implying that $\pi \succsim_S \pi'$ does not hold. Consider decision problem $\mathcal{D}$ defined as follows: Let $x = \frac{\varepsilon}{\varepsilon+\delta}$. The action set is given by $A = \{a_0, a_1, a_2\}$, with payoffs specified as follows: $u(a_0, L) = u(a_0, H) = u(a_2, L) = u(a_2, H) = 0$ and $u(a_1, H) = 1 - x$, $u(a_1, L) = -x$.

Now consider equilibrium strategy $\boldsymbol{\sigma}^*$ under π such that agent 1 chooses action a_0 if $s = s_0$ or s_2 and a_1 if $s = s_1$. Given this strategy, the posterior belief of agent 2 when agent 1's action is a_0 and $s = s_2$ is $\frac{\varepsilon^2}{\varepsilon^2+\delta}$, which is lower than x. Given this, agent 2 optimally chooses action a_1 if and only if (i) $s = s_1$ or (ii) $s = s_2$ and agent 1 chooses action a_1. Thus, the expected payoff for agent 2 under this equilibrium is given by $V_2^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = (1 - \varepsilon^2)(1 - x)/2$.

However, under π', when agent 1 chooses action a_0 if $s' = s_0'$, a_2 if $s' = s_2'$, and a_1 if $s' = s_1'$, agent 2 can perfectly infer agent 1's private signal.[28] Given the assumption $x = \frac{\varepsilon}{\varepsilon+\delta} < \frac{\varepsilon'^2}{\varepsilon'^2+\delta'^2}$, when agent 2 observes that agent 1 chooses action a_2 and receives the private signal $s' = s_2'$, the optimal action is a_1. Thus, agent

[27] Note that this violates the sufficient condition of Theorem 2 as

$$1 - \sum_{s \in \mathrm{supp}(\pi')} \min\{\pi'(s|L), \pi'(s|H)\} = 1 - \delta' \quad \text{and} \quad \min\{\pi(\mu = 0|L), \pi(\mu = 1|H)\} = 1 - \varepsilon.$$

[28] Recall that a_2 always induces the same payoffs as a_0. Thus, this strategy is also optimal for agent 1.

2 optimally chooses action a_1 if and only if either (i) $s' = s'_1$ or (ii) $s' = s'_2$ and agent 1 chooses either action a_1 or a_2. Let $\boldsymbol{\sigma}^{**}$ denote the equilibrium strategy profile following this tie-breaking rule. Then, the expected payoff of agent 2 is

$$\begin{aligned}
V_2^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**}) = \overline{V}_2^{\mathcal{D}}(\pi') \\
&= \frac{1}{2}(1-\varepsilon')(1-x) + \frac{1}{2}\varepsilon'(1-\varepsilon')(1-x) \\
&\quad + \frac{1}{2}(\varepsilon'^2 + \delta'^2)\left(\frac{\varepsilon'^2}{\varepsilon'^2+\delta'^2}(1-x) + \frac{\delta'^2}{\varepsilon'^2+\delta'^2}(-x)\right) \\
&= \frac{1}{2}(1-x) - \frac{1}{2}\delta'^2 x.
\end{aligned}$$

Since $V_2^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) < V_2^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ is equivalent to $\frac{\varepsilon}{\varepsilon+\delta} < \frac{\varepsilon'^2}{\varepsilon'^2+\delta'^2}$, it follows that $\pi \succsim_S \pi'$ does not hold.

Next, we establish that $\pi \succsim_W \pi'$. By directly constructing the equilibrium, we have a slightly more general observation:

Proposition 10. *Suppose $\pi \succsim_B \pi'$ and supp$(\mu) = \{0, x, 1\}$ such that $|\mu_0 - x| \geq |\mu_0 - y|$ for all $y \in$ supp$(\mu') \cap (0,1)$, where $x, y \in [0,1]$.*[29] *Then, $\pi \succsim_W \pi'$.*

By applying Proposition 10, we confirm that in this example, $\pi \succsim_W \pi'$ holds. The key feature is that under π, if agent 1 chooses action a_1 or a_2 when $s = s_2$, then agent 2 can obtain the expected payoff as if she were able to observe the past signal realization. □

Beyond Example 4, we cannot obtain a general characterization or a simple sufficient condition for the weaker order $\succsim_W$.

4.5 Restricted Decision Problems

Up to this point, we have assumed that the domain of decision problems is unusually rich, following the original work of Blackwell (1953). This strong assumption simplifies our analysis in Theorem 1. In this subsection, we instead focus on canonical (and generic) binary decision problems and assume that $\mu_0 = 1/2$, which are widely studied in the social learning literature. Let $A = \{a_0, a_1\}$ and $r \in (0,1)$ be the cutoff belief such that agents' optimal action is a_1 if posterior beliefs exceed r and a_0 otherwise.

Moreover, as in the classical works of Banerjee (1992) and Bikhchandani et al. (1992), we consider symmetric binary information structures parameterized by $p \in (0, 1/2)$, where $S = \{s_l, s_h\}$, $\pi(s_l|L) = \pi(s_h|H) = 1 - p$, and $\pi(s_h|L) =$

[29] If $x = 0$ or $x = 1$, then supp$(\mu) = \{0, 1\}$.

$\pi(s_l|H) = p$. Given any two symmetric binary information structures π and π' parameterized by p and p' with $p < p'$, we focus on the generic binary decision problems to avoid issues related to equilibrium selection. More formally, we assume that $r \notin \{p', 1/2, 1 - p'\}$.[30]

Proposition 11. *Suppose that $\mu_0 = 1/2$ and that π and π' are symmetric binary information structures. If $\pi \succsim_B \pi'$, then the expected payoff under π is weakly higher than that under π' for all agents and all generic binary decision problems.*

This observation highlights the strong assumptions inherent in $\succsim_S$ because any binary information structures cannot be compared when we consider the universal domain of the decision problems by Corollary 1. Note that Example 1 shows that π may yield lower payoffs for some agents than π', even when π is Blackwell more informative than π' under canonical and generic binary decision problems. Thus, focusing only on binary decision problems significantly weakens our requirements, but the relation remains strictly stronger than the Blackwell order. Beyond these examples, it remains unclear how our results extend to the restricted domain of simple decision problems, due to the complexity of the dynamics of public beliefs and agents' optimal strategies in general even under the binary decision problems.

5 Conclusion

In this study, we analyzed the comparison of information structures in the classical social learning model. Our main binary relation, $\succsim_S$, defines one information structure as more socially valuable than another if it yields weakly higher expected payoffs for all agents, across all decision problems and equilibrium selections. Proposition 1 shows that a Blackwell more informative information structure may be worse for some agents. Theorem 1 provides a characterization and establishes a necessary condition, demonstrating that inducing unbounded beliefs is required for all agents to be better off. Building on this, Theorem 2

[30] What is essential here is that there is no equilibrium selection problem. Thus, if we assume that the agent always takes action a_0 in case of a tie, the following proposition holds for any $r \in (0, 1)$. However, even under binary decision problems and symmetric binary information structures, the equilibrium selection rules are relevant for the comparison in general. For example, if indifference occurs at the prior, then a tie-breaking rule that favors mimicking the previous agent's action under one Blackwell-more-informative information structure may lead to worse outcomes than a rule that favors taking one's own private signal under another.

derives a simple sufficient condition by exploiting the properties of mixtures of full and no information. Furthermore, our framework naturally extends to long-run and limit comparisons, and we discuss other extensions in Section 4.

We leave open a number of questions. Our limitations stem primarily from two main sources of analytical difficulty: information externalities and learning speeds. First, public beliefs depend heavily on the underlying decision problem, and there is generally no Blackwell order among the induced public beliefs, even when one information structure yields higher expected payoffs than another for a particular decision problem. As a result, explicitly characterizing or comparing the expected payoffs or posterior beliefs becomes highly complex. This complexity is the technical reason we rely on the strong equilibrium selection rule in Theorem 1, and why we cannot provide a more general characterization of $\succsim_W$ and $\succsim_S$ in terms of the underlying information structures (even under the restricted domain of decision problems).

Second, even when we employ Theorem 1, a general analysis of learning speeds over all periods is still required, as this is closely related to the distribution induced by repeated sampling from the information structure. This necessity prevents us from establishing or negating the converse of Theorem 2, and more broadly, from determining whether the expected payoffs under one information structure are always higher than those under the observable signal setting of another in cases in which the payoffs and beliefs can be explicitly derived.

Therefore, deriving the payoffs or beliefs is difficult, and moreover, making comparisons remains challenging even when these can be computed exactly. A natural direction for future research is to pursue a more complete characterization of the social value of information by deepening its connection with and further developing the analysis of repeated Blackwell comparisons.

Appendix A: Omitted Proofs

A.1 Preliminaries

In this subsection, we present some preliminary results that will be used in subsequent proofs.

For each $a^* \in A$ and $z \in [0, 1]$, define

$$B(a^*) = \left\{ z \in [0,1] \mid a^* \in \arg\max_{a \in A} \left[zu(a, H) + (1-z)u(a, L) \right] \right\},$$

and

$$B^{-1}(z) = \{a \in A \mid z \in B(a)\} = \arg\max_{a \in A} [zu(a,H) + (1-z)u(a,L)].$$

Lemma 1. *Fix any $\mathcal{D} = (A, u)$. For each $a^* \in A$, $B(a^*)$ is a closed interval.*

Proof. Since $zu(a,H) + (1-z)u(a,L)$ is continuous with respect to z, $B(a^*)$ is a closed set. Suppose $z_1 \in B(a^*)$ and $z_2 \in B(a^*)$. It follows that $z_1 u(a^*,H) + (1-z_1)u(a^*,L) \geq z_1 u(a,H) + (1-z_1)u(a,L)$ and $z_2 u(a^*,H) + (1-z_2)u(a^*,L) \geq z_2 u(a,H) + (1-z_2)u(a,L)$ for any $a \in A$. Take any $t \in [0,1]$, then we have

$$\begin{aligned}
&[tz_1 + (1-t)z_2]u(a^*,H) + [1 - tz_1 - (1-t)z_2]u(a^*,L) \\
&= t[z_1 u(a^*,H) + (1-z_1)u(a^*,L)] + (1-t)[z_2 u(a^*,H) + (1-z_2)u(a^*,L)] \\
&\geq t[z_1 u(a,H) + (1-z_1)u(a,L)] + (1-t)[z_2 u(a,H) + (1-z_2)u(a,L)] \\
&= [tz_1 + (1-t)z_2]u(a,H) + [1 - tz_1 - (1-t)z_2]u(a,L).
\end{aligned}$$

Hence, $tz_1 + (1-t)z_2 \in B(a^*)$. □

Lemma 2. *Fix any $\mathcal{D} = (A, u)$. Suppose $B^{-1}(z_1) \cap B^{-1}(z_2) \neq \emptyset$ for some $0 \leq z_1 < z_2 \leq 1$. Then, $B^{-1}(w) = B^{-1}(z_1) \cap B^{-1}(z_2)$ for all $w \in (z_1, z_2)$.*

Proof. Take any $a_0 \in B^{-1}(z_1) \cap B^{-1}(z_2)$. Then, $z_1 u(a_0,H) + (1-z_1)u(a_0,L) \geq z_1 u(a,H) + (1-z_1)u(a,L)$ and $z_2 u(a_0,H) + (1-z_2)u(a_0,L) \geq z_2 u(a,H) + (1-z_2)u(a,L)$ for all $a \in A$. Note that at least one inequality holds strictly if $a \notin B^{-1}(z_1) \cap B^{-1}(z_2)$. Hence, for any $w \in (z_1, z_2)$,

$$\begin{aligned}
&wu(a_0,H) + (1-w)u(a_0,L) \\
&= \frac{w-z_2}{z_1-z_2}[z_1 u(a_0,H) + (1-z_1)u(a_0,L)] \\
&\qquad\qquad + \left(1 - \frac{w-z_2}{z_1-z_2}\right)[z_2 u(a_0,H) + (1-z_2)u(a_0,L)] \\
&\geq \frac{w-z_2}{z_1-z_2}[z_1 u(a,H) + (1-z_1)u(a,L)] \\
&\qquad\qquad + \left(1 - \frac{w-z_2}{z_1-z_2}\right)[z_2 u(a,H) + (1-z_2)u(a,L)] \\
&= wu(a,H) + (1-w)u(a,L)
\end{aligned}$$

for all $a \in A$ and strict inequality holds for all $a \notin B^{-1}(z_1) \cap B^{-1}(z_2)$. Thus, $B^{-1}(w) = B^{-1}(z_1) \cap B^{-1}(z_2)$. □

Lemma 3. *Suppose $(\mathcal{D}, \pi)$ satisfies $supp(\mu) \cap (x, 1) = \emptyset$ and $B^{-1}(0) = B^{-1}(x) = \{a_0\}$ for some $x \geq \mu_0$ and $a_0 \in A$. Take arbitrary equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$. Then,*

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \mu_0[(1-p^i)u(a_1,H) + p^i u(a_0,H)] + (1-\mu_0)u(a_0,L),$$

where $p = 1 - \pi(\mu = 1|H)$ *and* $a_1 \in B^{-1}(1)$.

Proof. If $a_0 \in B^{-1}(1)$, the statement holds because

$$\begin{aligned} V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) &= \mu_0 u(a_0, H) + (1-\mu_0)u(a_0, L) \\ &= \mu_0[(1-p^i)u(a_0, H) + p^i u(a_0, H)] + (1-\mu_0)u(a_0, L) \\ &= \mu_0[(1-p^i)u(a_1, H) + p^i u(a_0, H)] + (1-\mu_0)u(a_0, L). \end{aligned}$$

Suppose $a_0 \notin B^{-1}(1)$. Take any equilibrium under π. Then, by Lemma 2, agent 1 chooses a_0 if and only if he receives $s \notin S(1)$. Agent 2 chooses an action from $B^{-1}(1)$ if she receives $s \in S(1)$ or agent 1 takes an action other than a_0 because she knows that the state is H. Notably, the public belief after observing a_0 is less than μ_0 and Lemma 2 implies that $B^{-1}(z) = \{a_0\}$ for all $z \in [0, x]$. Hence, agent 2 must choose a_0 if she receives $s \notin S(1)$ and agent 1 chooses a_0. Analogously, agent i takes action from $B^{-1}(1)$ if and only if he receives $s \in S(1)$ or at least one previous agent chooses an action other than a_0. Otherwise, agent i takes a_0. Therefore, we have

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \mu_0[(1-p^i)u(a_1, H) + p^i u(a_0, H)] + (1-\mu_0)u(a_0, L).$$

□

A.2 Proofs of Theorem 1 and Corollary 1

We first provide a self-contained proof of the following lemma.

Lemma 4. *If* $\pi \succsim_B \pi'$ *and* $\rho \succsim_B \rho'$, *then* $\pi \otimes \rho \succsim_B \pi' \otimes \rho'$.

Proof. Suppose $\pi \succsim_B \pi'$ and $\rho \succsim_B \rho'$. Then, there exist Markov kernel γ_1 and γ_2 such that

$$\pi'(s'|\omega) = \sum_{s \in \mathrm{supp}(\pi)} \gamma_1(s'|s)\pi(s|\omega) \quad \text{and} \quad \rho'(t'|\omega) = \sum_{t \in \mathrm{supp}(\rho)} \gamma_2(t'|t)\rho(t|\omega)$$

for all $s' \in \mathrm{supp}(\pi')$ and $t' \in \mathrm{supp}(\rho')$. Then, we have

$$\begin{aligned} (\pi' \otimes \rho')((s', t')|\omega) &= \pi'(s'|\omega)\rho'(t'|\omega) \\ &= \sum_{s \in \mathrm{supp}(\pi)} \gamma_1(s'|s)\pi(s|\omega) \sum_{t \in \mathrm{supp}(\rho)} \gamma_2(t'|t)\rho(t|\omega) \\ &= \sum_{(s,t) \in \mathrm{supp}(\pi \otimes \rho)} \gamma_1(s'|s)\gamma_2(t'|t)\pi(s|\omega)\rho(t|\omega) \\ &= \sum_{(s,t) \in \mathrm{supp}(\pi \otimes \rho)} \gamma((s', t')|(s, t))(\pi \otimes \rho)((s, t)|\omega), \end{aligned}$$

where $\gamma((s',t')|(s,t)) = \gamma_1(s'|s)\gamma_2(t'|t)$. Since γ is a Markov kernel, $\pi' \otimes \rho'$ is a garbling of $\pi \otimes \rho$. $\square$

Then, the next lemma establishes that the expected payoff under the observable signal setting provides an upper bound for each agent, any decision problem, and any equilibrium.

Lemma 5. $\overline{V}_i^{\mathcal{D}}(\pi) \geq V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma})$ *for all* $i, \mathcal{D}, \pi$, *and* $\boldsymbol{\sigma}$.

Proof of Lemma 5. Take any $\mathcal{D}, \pi$, and $\boldsymbol{\sigma}$. Note that $\overline{V}_1^{\mathcal{D}}(\pi) = V_1^{\mathcal{D}}(\pi, \boldsymbol{\sigma})$. Fix $i \geq 2$. For each $s \in S^{i-1}$, define $f_{i-1}(s) \in \Delta(A^{i-1})$ as

$$f_{i-1}(\boldsymbol{a}|s) = \prod_{k=1}^{i-1} \sigma_k(a_k|a_1, \dots, a_{k-1}, s_k).$$

Hence, $f_{i-1}(a|s)$ is the probability that agent 1 to agent $i-1$ takes action $\boldsymbol{a} = (a_1, \dots, a_{i-1})$ when agent 1 to agent $i-1$ receives private signal $\boldsymbol{s} = (s_1, \dots, s_{i-1})$. Then,

$$\begin{aligned} \alpha^{\omega}_{\leq i-1}(\boldsymbol{a}|\pi, \boldsymbol{\sigma}) &= \sum_{s \in S^{i-1}} \prod_{k=1}^{i-1} \sigma_k(a_k|a_1, \dots, a_{k-1}, s_k)\pi(s_k|\omega) \\ &= \sum_{s \in S^{i-1}} f_{i-1}(\boldsymbol{a}|\boldsymbol{s})\pi^{\otimes i-1}(\boldsymbol{s}|\omega). \end{aligned}$$

Thus, $\alpha_{\leq i-1}(\cdot|\pi, \boldsymbol{\sigma})$ is a garbling of $\pi^{\otimes i-1}$, where $\alpha_{\leq i-1}$ is an ex-ante action distribution induced by $\alpha^{\omega}_{\leq i-1}$.[31] By Lemma 4, we have $\pi^{\otimes i-1} \otimes \pi \succsim_B \alpha_{\leq i-1}(\cdot|\pi, \boldsymbol{\sigma}) \otimes \pi$. Hence, $\overline{V}_i^{\mathcal{D}}(\pi) \geq V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma})$ holds for all i, $\mathcal{D}$, π, and $\boldsymbol{\sigma}$. $\square$

Proof of Theorem 1. Since $\overline{V}_i^{\mathcal{D}}(\pi') \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}')$ holds for all strategy profile $\boldsymbol{\sigma}'$ by Lemma 5, $\pi \succsim_S \pi'$ holds if $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq \overline{V}_i^{\mathcal{D}}(\pi')$.

Conversely, suppose $\pi \succsim_S \pi'$. Take any $\mathcal{D} = (A, u)$, equilibrium $\boldsymbol{\sigma}^*$ under $\pi : \Omega \to \Delta(S)$, and equilibrium $\boldsymbol{\sigma}^{**}$ under $\pi' : \Omega \to \Delta(S')$. Then, $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ by $\pi \succsim_S \pi'$. Consider the decision problem $\overline{\mathcal{D}} = (\overline{A}, \overline{u})$, where $\overline{A} = \{(a,k) \mid a \in A, k \in S'\}$ and $\overline{u}((a,k),\omega) = u(a,\omega)$ for all $a \in A, \omega \in \Omega$. Fix $s_1 \in S'$ and define strategy profile $\boldsymbol{\sigma} = (\sigma_i)_{i \in \mathbb{N}}$ under $(\overline{\mathcal{D}}, \pi)$ as following:

$$\begin{cases} \sigma_i((a,s_1)|(a_1,k_1),(a_2,k_2),\dots,(a_{i-1},k_{i-1}),s) = \sigma_i^*(a|a_1,a_2,\dots,a_{i-1},s) \\ \sigma_i((a,k)|(a_1,k_1),(a_2,k_2),\dots,(a_{i-1},k_{i-1}),s) = 0, \end{cases}$$

for all $a \in A$, $s \in S$, $(a_1, \dots, a_{i-1}) \in A^{i-1}$, $k_1, k_2, \dots, k_{i-1} \in S'$, and $k \in S' \setminus \{s_1\}$. Note that $\boldsymbol{\sigma}$ is an equilibrium under $(\overline{\mathcal{D}}, \pi)$. Moreover, it follows that

[31] Precisely, $\alpha_{\leq i-1}(\boldsymbol{a}|\pi, \boldsymbol{\sigma}) = (1-\mu_0)\alpha^L_{\leq i-1}(\boldsymbol{a}|\pi, \boldsymbol{\sigma}) + \mu_0 \alpha^H_{\leq i-1}(\boldsymbol{a}|\pi, \boldsymbol{\sigma})$.

$V_i^{\overline{\mathcal{D}}}(\pi, \boldsymbol{\sigma}) = V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*)$. Under $(\overline{\mathcal{D}}, \pi')$, if we consider the following equilibrium $\boldsymbol{\sigma}'$, the expected playoff of agent i at equilibrium $(V_i^{\overline{\mathcal{D}}}(\pi', \boldsymbol{\sigma}'))$ coincides with $\overline{V}_i^{\mathcal{D}}(\pi')$. Specifically, each agent i chooses an action that maximizes his expected payoff on the equilibrium path, but always chooses an action of the form (a, k) $(a \in A)$ when the received signal is $k \in S'$. Since each agent can observe signals received by their predecessor on the equilibrium path, it follows that $V_i^{\overline{\mathcal{D}}}(\pi', \boldsymbol{\sigma}') = \overline{V}_i^{\overline{\mathcal{D}}}(\pi') = \overline{V}_i^{\mathcal{D}}(\pi')$. Therefore,

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = V_i^{\overline{\mathcal{D}}}(\pi, \boldsymbol{\sigma}) \geq V_i^{\overline{\mathcal{D}}}(\pi', \boldsymbol{\sigma}') = \overline{V}_i^{\mathcal{D}}(\pi').$$

□

Proof of Corollary 1. Prove by contradiction. Suppose $\mathrm{co}(\mathrm{supp}(\mu)) \neq [0, 1]$. Then, either $1 \notin \mathrm{supp}(\mu)$ or $0 \notin \mathrm{supp}(\mu)$. By symmetry, it suffices to consider the case where $1 \notin \mathrm{supp}(\mu)$. Since $\mathrm{supp}(\mu)$ is a closed set, there exists $r \in [\mu_0, 1)$ such that $\mathrm{supp}(\mu) \subseteq [0, r]$. Consider the following decision problem $\mathcal{D} = (A, u)$: $A = \{a_0, a_1\}$, $u(a_0, L) = u(a_0, H) = 0$, $u(a_1, H) = 1 - r$ and $u(a_1, L) = -r$. Then, the strategy profile $\boldsymbol{\sigma}^*$ that all agents always choose a_0 is an equilibrium under $(\mathcal{D}, \pi)$. It follows that $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = 0$. Since π' is not no information, repeated observations of π' allow agents to learn the state in the limit. Hence, $\overline{V}_i^{\mathcal{D}}(\pi') > 0$ holds for sufficiently large i. By Theorem 1, π is not more socially valuable than π'. □

A.3 Proof of Theorem 2

The following lemma shows that the expected payoff under the mixture of full and no information is the same as that under observable signal setting for any decision problem and equilibrium.

Lemma 6. *Suppose $\mathrm{supp}(\mu) = \{0, \mu_0, 1\}$. Fix the decision problem $\mathcal{D} = (A, u)$. Take arbitrary equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$. Then,*

$$\begin{aligned} V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) &= \overline{V}_i^{\mathcal{D}}(\pi) \\ &= \mu_0[(1 - p^i)U_1] + (1 - \mu_0)[(1 - p^i)U_0] + p^i U_{\mu_0}, \end{aligned}$$

where $U_1 = \max_a u(a, H)$, $U_0 = \max_a u(a, L)$, $U_{\mu_0} = \max_a[\mu_0 u(a, H) + (1 - \mu_0)u(a, L)]$, and $p = \pi(\mu = \mu_0|H) = \pi(\mu = \mu_0|L)$.

Proof. First, it is easily calculated that

$$\overline{V}_i^{\mathcal{D}}(\pi) = \mu_0[(1 - p^i)U_1] + (1 - \mu_0)[(1 - p^i)U_0] + p^i U_{\mu_0}.$$

We now show that $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \overline{V}_i^{\mathcal{D}}(\pi)$. First, this obviously holds for agent 1:

$$V_1^{\mathcal{D}}(\pi, \sigma^*) = \overline{V}_1^{\mathcal{D}}(\pi) = \mu_0(1-p)U_1 + (1-\mu_0)(1-p)U_0 + pU_{\mu_0}.$$

Then, for each $i \geq 2$, we consider a strategy in which agent i chooses the optimal actions upon receiving conclusive signals about each state and mimics agent $i-1$'s action otherwise. By the optimality of the equilibrium strategy, for each $i \geq 2$, we have

$$\begin{aligned}
V_i^{\mathcal{D}}(\pi, \sigma^*) \geq\ & \mu_0(1-p)U_1 + (1-\mu_0)(1-p)U_0 \\
& + p\left[\mu_0 \sum_a \alpha_{i-1}^H(a|\pi, \sigma^*)u(a, H) + (1-\mu_0)\sum_a \alpha_{i-1}^L(a|\pi, \sigma^*)u(a, L)\right] \\
=\ & \mu_0(1-p)U_1 + (1-\mu_0)(1-p)U_0 + pV_{i-1}^{\mathcal{D}}(\pi, \sigma^*).
\end{aligned}$$

Conversely, from Lemma 5, for each i

$$V_i^{\mathcal{D}}(\pi, \sigma^*) \leq \overline{V}_i^{\mathcal{D}}(\pi) = \mu_0(1-p^i)U_1 + (1-\mu_0)(1-p^i)U_0 + p^iU_{\mu_0}.$$

Fix some $j \geq 1$ and suppose $V_{j-1}^{\mathcal{D}}(\pi, \sigma^*) = \overline{V}_{j-1}^{\mathcal{D}}(\pi)$. Then,

$$\begin{aligned}
V_j^{\mathcal{D}}(\pi, \sigma^*) &\geq \mu_0(1-p)U_1 + (1-\mu_0)(1-p)U_0 + pV_{j-1}^{\mathcal{D}}(\pi, \sigma^*) \\
&= \mu_0(1-p)U_1 + (1-\mu_0)(1-p)U_0 \\
&\qquad + p\left[\mu_0(1-p^{j-1})U_1 + (1-\mu_0)(1-p^{j-1})U_0 + p^{j-1}U_{\mu_0}\right] \\
&= \mu_0(1-p^j)U_1 + (1-\mu_0)(1-p^j)U_0 + p^jU_{\mu_0} \\
&= \overline{V}_j^{\mathcal{D}}(\pi).
\end{aligned}$$

Hence, we have $V_j^{\mathcal{D}}(\pi, \sigma^*) = \overline{V}_j^{\mathcal{D}}(\pi)$. By mathematical induction, it follows that $V_i^{\mathcal{D}}(\pi, \sigma^*) = \overline{V}_i^{\mathcal{D}}(\pi)$ for all i. □

Utilizing Lemma 6 and Blackwell's theorem (Lemma 4), we can show that the expected payoff under π is weakly higher than the upper bound under π' for any decision problems if π is a mixture of full and no information.

Lemma 7. *Suppose $\pi \succsim_B \pi'$ and supp$(\mu) = \{0, \mu_0, 1\}$. Then, $\pi \succsim_S \pi'$.*

Proof. Take any $\mathcal{D} = (A, u)$. Take arbitrary equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$. From Lemma 6, we have $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \overline{V}_i^{\mathcal{D}}(\pi)$. Hence, the expected payoff of agent i in any equilibrium is the same as the expected payoff of agent i when agent i can observe not the actions taken by past agents but the signals received by past agents.

Next, take any equilibrium $\boldsymbol{\sigma}^{**}$ under $(\mathcal{D}, \pi')$. Note that $\overline{V}_i^{\mathcal{D}}(\pi') \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$ holds by Lemma 5. Since $\pi^{\otimes i} \succsim_B \pi'^{\otimes i}$ by Lemma 4, we have $\overline{V}_i^{\mathcal{D}}(\pi) \geq \overline{V}_i^{\mathcal{D}}(\pi')$. Hence, it follows that

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \overline{V}_i^{\mathcal{D}}(\pi) \geq \overline{V}_i^{\mathcal{D}}(\pi') \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**}).$$

Therefore, we obtain $\pi \succsim_S \pi'$. □

We then construct a strategy profile under π that achieves the same equilibrium expected payoff under π' when π' is a mixture of full and no information. Additionally, we show that this strategy profile provides a lower bound for the payoffs of all agents under π.

Lemma 8. *Suppose that π and π' satisfy supp$(\mu') = \{0, \mu_0, 1\}$ and $\min\{\pi(\mu = 0|L), \pi(\mu = 1|H)\} \geq 1 - p$, where $p = \pi'(\mu' = \mu_0|L) = \pi'(\mu' = \mu_0|H)$. Then, $\pi \succsim_S \pi'$.*

Proof. Let $q_L = \frac{\pi'(\mu'=0|L)}{\pi(\mu=0|L)}$ and $q_H = \frac{\pi'(\mu'=1|H)}{\pi(\mu=1|H)}$. Take any $\mathcal{D}$ and define $\boldsymbol{\sigma}^{**} = (\sigma_i^{**})_{i\in\mathbb{N}}$ as the following strategy under $(\mathcal{D}, \pi)$. Agent 1 chooses $a_0 \in B^{-1}(0)$ with probability q_L and chooses $a_2 \in B^{-1}(\mu_0)$ with probability $1 - q_L$ if he receives conclusive signal about $\omega = L$. Agent 1 chooses $a_1 \in B^{-1}(1)$ with probability q_H and chooses a_2 with probability $1 - q_H$ if he receives conclusive signal about $\omega = H$. Otherwise, agent 1 chooses a_2. For $i \geq 2$, agent i chooses a_0 with probability q_L and chooses the same action as agent $i - 1$ with probability $1 - q_L$ if he receives a conclusive signal about $\omega = L$. Agent i chooses a_1 with probability q_H and chooses the same action as agent $i - 1$ with probability $1 - q_H$ if he receives a conclusive signal about $\omega = H$. Otherwise, agent i chooses the same action as agent $i - 1$. First, note that

$$\begin{aligned} V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^{**}) &= \mu_0[(1 - p^i)U_1] + (1 - \mu_0)[(1 - p^i)U_0] + p^i U_{\mu_0} \\ &= \overline{V}_i^{\mathcal{D}}(\pi'), \end{aligned}$$

where the last equality comes from Lemma 6.

Fix any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$ and define $\boldsymbol{\sigma}(k)$ as

$$\boldsymbol{\sigma}(k) = (\sigma_1^*, \sigma_2^*, \ldots, \sigma_k^*, \sigma_{k+1}^{**}, \sigma_{k+2}^{**}, \ldots).$$

Show that if $i \geq k + 1$,

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}(k)) = \mu_0(1 - p)U_1 + (1 - \mu_0)(1 - p)U_0 + pV_{i-1}^{\mathcal{D}}(\pi, \boldsymbol{\sigma}(k)).$$

Note that

$$V_{i-1}^{\mathcal{D}}(\pi, \boldsymbol{\sigma}(k)) = \mu_0 \sum_{a\in A} \alpha_{i-1}^H(a|\pi, \boldsymbol{\sigma}(k))u(a, H)$$

$$+ (1-\mu_0) \sum_{a \in A} \alpha^L_{i-1}(a|\pi, \boldsymbol{\sigma}(k)) u(a, L).$$

$i \geq k+1$ implies that $\boldsymbol{\sigma}(k)_i = \sigma^{**}_i$. Hence,

$$\begin{aligned}
V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(k)) &= \mu_0 \left[\pi(\mu = 1|H) q_H U_1 + (1 - \pi(\mu = 1|H) q_H) \sum_{a \in A} \alpha^H_{i-1}(a|\pi, \boldsymbol{\sigma}(k)) u(a, H) \right] \\
&\quad + (1-\mu_0) \left[\pi(\mu = 0|L) q_L U_0 + (1 - \pi(\mu = 0|L) q_L) \sum_{a \in A} \alpha^L_{i-1}(a|\pi, \boldsymbol{\sigma}(k)) u(a, L) \right] \\
&= \mu_0 \left[(1-p) U_1 + p \sum_{a \in A} \alpha^H_{i-1}(a|\pi, \boldsymbol{\sigma}(k)) u(a, H) \right] \\
&\qquad + (1-\mu_0) \left[(1-p) U_0 + p \sum_{a \in A} \alpha^L_{i-1}(a|\pi, \boldsymbol{\sigma}(k)) u(a, L) \right] \\
&= \mu_0 (1-p) U_1 + (1-\mu_0)(1-p) U_0 + p V^{\mathcal{D}}_{i-1}(\pi, \boldsymbol{\sigma}(k)).
\end{aligned}$$

By the definition of $\boldsymbol{\sigma}(k)$,

$$\begin{cases} V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}^*) = V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(k)) & \text{if } i < k+1, \\ V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}^*) \geq V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(k)) & \text{if } i = k+1. \end{cases}$$

The second inequality is held by the optimality of σ^*_i. We now show that

$$V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(k+1)) \geq V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(k))$$

for all i and k. First, if $k \geq i-1$, we have $V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(k+1)) = V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}^*) \geq V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(k))$. Next, we have $V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(i-1)) \geq V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(i-2))$ for $i \geq 2$ since

$$\begin{aligned}
V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(i-2)) &= \mu_0(1-p)U_1 + (1-\mu_0))1-p)U_0 + p V^{\mathcal{D}}_{i-1}(\pi, \boldsymbol{\sigma}(i-2)) \\
&\leq \mu_0(1-p)U_1 + (1-\mu_0)(1-p)U_0 + p V^{\mathcal{D}}_{i-1}(\pi, \boldsymbol{\sigma}(i-1)) \\
&= V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(i-1)).
\end{aligned}$$

Then, we have $V_i(\pi, \boldsymbol{\sigma}(i-2)) \geq V_i(\pi, \boldsymbol{\sigma}(i-3))$ for $i \geq 3$ since

$$\begin{aligned}
V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(i-3)) &= \mu_0(1-p)U_1 + (1-\mu_0)(1-p)U_0 + p V^{\mathcal{D}}_{i-1}(\pi, \boldsymbol{\sigma}(i-3)) \\
&\leq \mu_0(1-p)U_1 + (1-\mu_0)(1-p)U_0 + p V^{\mathcal{D}}_{i-1}(\pi, \boldsymbol{\sigma}(i-2)) \\
&= V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(i-2)).
\end{aligned}$$

Analogously, it follows that $V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(i-m)) \geq V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(i-m-1))$ for all i, m that satisfies $i - m - 1 \geq 0$. Hence, $V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(k+1)) \geq V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(k))$ for all i, k. Therefore, we have

$$V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}^*) = V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(i)) \geq V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}(0)) = V^{\mathcal{D}}_i(\pi, \boldsymbol{\sigma}^{**}) = \overline{V_i}^{\mathcal{D}}(\pi').$$

□

Proof of Theorem 2. Suppose that $\pi \succsim_B \pi'' \succsim_B \pi'$ and $\text{supp}(\mu'') = \{0, \mu_0, 1\}$. From Lemma 7, we conclude that $\pi'' \succsim_S \pi'$ holds. Since $\pi \succsim_B \pi''$ and $\text{supp}(\mu'') = \{0, \mu_0, 1\}$, it follows that $\min\{\pi(\mu = 0|L), \pi(\mu = 0|H)\} \geq \pi''(\mu'' = \mu_0|L) = \pi''(\mu'' = \mu_0|H)$. Thus, from Lemma 8, we also conclude that $\pi \succsim_S \pi''$ holds. Therefore, we have $\pi \succsim_S \pi'$. □

A.4 Proof of Proposition 2

Let $S' = \text{supp}(\pi')$. Suppose $\text{supp}(\mu'') = \{0, \mu_0, 1\}$. We now show that $\pi \succsim_B \pi''$ is equivalent to

$$\pi(\mu = 0|L) \geq \pi''(\mu'' = 0|L) \quad \text{and} \quad \pi(\mu = 1|H) \geq \pi''(\mu'' = 1|H).$$

Note that

$$\pi''(\mu'' = 0|\omega) = \frac{\pi''(\mu'' = 0|L)}{\pi(\mu = 0|L)}\pi(\mu = 0|\omega)$$
$$\pi''(\mu'' = 1|\omega) = \frac{\pi''(\mu'' = 1|H)}{\pi(\mu = 1|H)}\pi(\mu = 1|\omega)$$
$$\pi''(\mu'' = \mu_0|\omega) = \left[1 - \frac{\pi''(\mu'' = 0|L)}{\pi(\mu = 0|L)}\right]\pi(\mu = 0|\omega) + \left[1 - \frac{\pi''(\mu'' = 1|H)}{\pi(\mu = 1|H)}\right]\pi(\mu = 1|\omega) + \sum_{x \in \text{supp}(\mu)\setminus\{0,1\}} \pi(\mu = x|\omega).$$

Thus, π'' is a garbling of π if $\pi(\mu = 0|L) \geq \pi''(\mu'' = 0|L)$ and $\pi(\mu = 1|H) \geq \pi''(\mu'' = 1|H)$. Conversely, suppose π'' is a garbling of π. Then, we have

$$\pi''(\mu'' = 0|\omega) = \sum_{x \in \text{supp}(\mu)} \gamma(x)\pi(\mu = x|\omega)$$

for some $\gamma \in \Delta(\text{supp}(\mu))$. Since $\pi''(\mu'' = 0|H) = 0$ and $\pi(\mu = x|H) > 0$ for all $x \neq 0$, it follows that $\gamma(x) = 0$ for all $x \neq 0$. Hence, $\pi(\mu = 0|L) \geq \pi''(\mu'' = 0|L)$. Similarly, we have $\pi(\mu = 1|H) \geq \pi''(\mu'' = 1|H)$.

Next, we show that $\pi'' \succsim_B \pi'$ is equivalent to

$$\pi''(\mu'' = \mu_0|L) = \pi''(\mu'' = \mu_0|H) \leq \sum_{s \in S'} \min\{\pi'(s|L), \pi'(s|H)\}.$$

Suppose $\pi''(\mu'' = \mu_0|L) = \pi''(\mu'' = \mu_0|H) \leq \sum_{s \in S'} \min\{\pi'(s|L), \pi'(s|H)\}$. Define $\rho : \Omega \to \Delta\{s_0, s_1, s_2\}$ that satisfies

$$\rho(s_1|L) = 0, \quad \rho(s_0|H) = 0, \quad \rho(s_2|H) = \rho(s_2|L) = \sum_{s \in S'} \min\{\pi'(s|L), \pi'(s|H)\}.$$

Then, we have $\pi'' \succsim_B \rho$ as $\text{supp}(\mu'') = \{0, \mu_0, 1\}$.

If $\rho(s_2|L) = \rho(s_2|H) = 1$, $\rho \succsim_B \pi'$ as π' is no information. If $\rho(s_2|L) = \rho(s_2|H) = 0$, $\rho \succsim_B \pi'$ as both ρ and π' are full information. Otherwise,

$$\begin{aligned}\pi'(s|\omega) =& \frac{\max\{\pi'(s|L) - \pi'(s|H), 0\}}{\rho(s_0|L)}\rho(s_0|\omega) + \frac{\max\{\pi'(s|H) - \pi'(s|L), 0\}}{\rho(s_1|H)}\rho(s_1|\omega) \\ &+ \frac{\min\{\pi'(s|L), \pi'(s|H)\}}{\rho(s_2|L)}\rho(s_2|\omega)\end{aligned}$$

and

$$\begin{aligned}\sum_{s\in S'} \frac{\max\{\pi'(s|L) - \pi'(s|H), 0\}}{\rho(s_0|L)} &= \frac{\sum_{s\in S'} \max\{\pi'(s|L) - \pi'(s|H), 0\}}{1 - \sum_{s\in S'} \min\{\pi'(s|L), \pi'(s|H)\}} = 1 \\ \sum_{s\in S'} \frac{\max\{\pi'(s|H) - \pi'(s|L), 0\}}{\rho(s_1|H)} &= \frac{\sum_{s\in S'} \max\{\pi'(s|H) - \pi'(s|L), 0\}}{1 - \sum_{s\in S'} \min\{\pi'(s|L), \pi'(s|H)\}} = 1 \\ \sum_{s\in S'} \frac{\min\{\pi'(s|L), \pi'(s|H)\}}{\rho(s_2|L)} &= 1.\end{aligned}$$

Hence, π' is a garbling of ρ and we have $\rho \succsim_B \pi'$. Note that $\pi'' \succsim_B \rho$ and $\rho \succsim_B \pi'$ implies $\pi'' \succsim_B \pi'$. Therefore, $\pi''(\mu'' = \mu_0|H) = \pi''(\mu'' = \mu_0|L) \leq \sum_{s\in S'} \min\{\pi'(s|L), \pi'(s|H)\}$ is a sufficient condition for $\pi'' \succsim_B \pi'$.

Conversely, suppose $\pi'' \succsim_B \pi'$. Then, there exists probability distribution γ_0, γ_1, and γ_{μ_0} over S' such that

$$\pi'(s|\omega) = \gamma_0(s)\pi''(\mu'' = 0|\omega) + \gamma_1(s)\pi''(\mu'' = 1|\omega) + \gamma_{\mu_0}(s)\pi''(\mu'' = \mu_0|\omega)$$

for all $s \in S'$ and $\omega \in \Omega$. Then, for each $\omega \in \Omega$,

$$\begin{aligned}&\sum_{s\in S'} \min\{\pi'(s|L), \pi'(s|H)\} \\ &= \sum_{s\in S'} \min\left\{\begin{array}{c}\gamma_0(s)\pi''(\mu'' = 0|L) + \gamma_{\mu_0}(s)\pi''(\mu'' = \mu_0|L), \\ \gamma_1(s)\pi''(\mu'' = 1|H) + \gamma_{\mu_0}(s)\pi''(\mu'' = \mu_0|H)\end{array}\right\} \\ &= \sum_{s\in S'} \left[\min\left\{\gamma_0(s)\pi''(\mu'' = 0|L), \gamma_1(s)\pi''(\mu'' = 1|H)\right\} + \gamma_{\mu_0}(s)\pi''(\mu'' = \mu_0|L)\right] \\ &\geq \sum_{s\in S'} \gamma_{\mu_0}(s)\pi''(\mu'' = \mu_0|L) \\ &= \pi''(\mu'' = \mu_0|\omega).\end{aligned}$$

Hence, $\pi''(\mu'' = \mu_0|L) = \pi''(\mu'' = \mu_0|H) \leq \sum_{s\in S'} \min\{\pi'(s|L), \pi'(s|H)\}$ is a necessary condition for $\pi'' \succsim_B \pi'$. Therefore, $\pi'' \succsim_B \pi'$ is equivalent to $\pi''(\mu'' = \mu_0|L) = \pi''(\mu'' = \mu_0|H) \leq \sum_{s\in S'} \min\{\pi'(s|L), \pi'(s|H)\}$, or $\pi''(\mu'' = 0|L) = \pi''(\mu'' = 1|H) \geq 1 - \sum_{s\in S'} \min\{\pi'(s|L), \pi'(s|H)\}$. By combining the first half and the second half, it can be seen that Proposition 2 holds.

A.5 Derivation of a Necessary Condition in Example 2

For the first step, we derive the expected payoff $\overline{V}_i^{\mathcal{D}}(\pi')$ for each sufficiently large i. We first decompose the expected payoff into two parts. First, if an agent observes a conclusive signal about H (resp. L), then she chooses the optimal action a_1 (resp. a_0). Hence, the expected payoff from the events in which at least one of the first i agents observes a conclusive signal is $\frac{1}{2} \cdot [1 - (1 - \lambda')^i](1 - r)$. We then focus on the event in which all agents from 1 to i observe either s_l or s_h and derive the corresponding interim payoff. Note that this interim payoff coincides with that obtained from i conditionally independent observations of the symmetric binary signal $\pi'_{p'}$, where $S_{p'} = \{s_l, s_h\}$, $\pi'_{p'}(s_l|L) = \pi'_{p'}(s_h|H) = 1 - p'$, and $\pi'_{p'}(s_h|L) = \pi'_{p'}(s_l|H) = p'$ with $p' \in (0, 1/2)$.

Let $l(s)$ denote the log-likelihood ratio after observing signal s from $\pi'_{p'}$, that is, $l(s) = \log \frac{\pi'_{p'}(s|H)}{\pi'_{p'}(s|L)}$. Then, we have $\mathbb{E}[l(s)|L] < 0 < \mathbb{E}[l(s)|H]$. Let $K^L : \mathbb{R} \to \mathbb{R}$ denote the *cumulant generating function* conditional on state L, defined as follows:

$$
\begin{aligned}
K^L(t) &= \log \sum_{s \in S_{p'}} \pi'_{p'}(s|L) \left(\frac{\pi'_{p'}(s|H)}{\pi'_{p'}(s|L)} \right)^t \\
&= \log \left((1 - p') \left(\frac{p'}{1 - p'} \right)^t + p' \left(\frac{1 - p'}{p'} \right)^t \right),
\end{aligned}
$$

for every $t \in \mathbb{R}$.

By Cramér's theorem (Cramér, 1938), for each $r \in (1 - p, 1)$ and $p \in (0, 1/2)$ (the parameter for π), we have

$$
\begin{aligned}
\pi'^{\otimes i}_{p'}(\mu' \geq r|L) &= \mathbb{P}\left[l(s_1) + \cdots + l(s_i) \geq \log \frac{r}{1 - r} \mid L \right] \\
&= e^{i \min_t K^L(t) + o(i)}.
\end{aligned}
$$

As $\min_t K^L(t) = \log(2\sqrt{p'(1 - p')})$ by the arithmetic–geometric mean inequality, we have $\pi'^{\otimes i}_{p'}(\mu' \geq r|L) = (2\sqrt{p'(1 - p')})^i e^{o(i)}$. By the symmetric argument, we obtain $\pi'^{\otimes i}_{p'}(\mu' \leq r|H) = (2\sqrt{p'(1 - p')})^i e^{o(i)}$.

Therefore, the interim payoff is

$$
\begin{aligned}
&\frac{1}{2} \cdot \left((1 - \pi'^{\otimes i}_{p'}(\mu' \leq r|H))(1 - r) + \pi'^{\otimes i}_{p'}(\mu' \geq r|L)(-r) \right) \\
&= \frac{1}{2} \cdot \left(1 - r - \left(2\sqrt{p'(1 - p')} \right)^i e^{o(i)} \right).
\end{aligned}
$$

Thus, for each sufficiently large i, the expected payoff is

$$\overline{V}_i^{\mathcal{D}}(\pi') = \frac{1}{2}\left[[1-(1-\lambda')^i](1-r) + (1-\lambda')^i\left(1-r-\left(2\sqrt{p'(1-p')}\right)^i e^{o(i)}\right)\right].$$

Then, a necessary condition for $\pi \succsim_S \pi'$ is $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}) \geq \overline{V}_i^{\mathcal{D}}(\pi')$ for all $r \in (1-p, 1)$ and sufficiently large i. By the above derivation, the necessary condition is

$$(1-\lambda')^i\left(2\sqrt{p'(1-p')}\right)^i e^{o(i)} \geq (1-\lambda)^i(1-r),$$

which is equivaent to

$$r \geq 1 - \left(\frac{1-\lambda'}{1-\lambda}\cdot 2\sqrt{p'(1-p')}\right)^i e^{o(i)}.$$

The left-hand-side is smallest when r is close to $1-p$, and $\pi \succsim_S \pi'$ does not hold if the inequality does not hold at $r = 1-p$. Thus, applying the $1/i$-th power to both sides at $r = 1-p$ yields the necessary condition as

$$\frac{1-\lambda'}{1-\lambda}\cdot 2\sqrt{p'(1-p')}e^{\frac{o(i)}{i}} \geq p^{\frac{1}{i}}.$$

Therefore, for sufficiently large i, we can derive a necessary condition as

$$\lambda \geq 1 - 2(1-\lambda')\sqrt{p'(1-p')}.$$

A.6 Proofs of Proposition 4, Proposition 5, and Theorem 3

Proof of Proposition 4. Suppose $\pi \succsim_{ES} \pi'$. Take $N \in \mathbb{N}$ such that $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq \overline{V}_i^{\mathcal{D}}(\pi')$ for all $\mathcal{D}$, $i \geq N$, and equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$. By Lemma 4, we have $\overline{V}_i^{\mathcal{D}}(\pi) \geq \overline{V}_i^{\mathcal{D}}(\pi')$ for all $\mathcal{D}$ and $i \geq N$. Thus, $\pi \succsim_{EB} \pi'$.

Consider the same example as Example 1. Then, we have $V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}') > V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma})$ for all $i \geq 2$. Hence, π is not eventually more socially valuable than π', although $\pi \succsim_{EB} \pi'$ holds as $\pi \succsim_B \pi'$ and Lemma 4. $\square$

Proof of Proposition 5. Since $\succsim_{ES}$ satisfies transitivity, showing $\pi \succsim_{ES} \pi''$ and $\pi'' \succsim_{ES} \pi'$ is sufficient. First, we have $\pi'' \succsim_{ES} \pi'$ because

$$V_i^{\mathcal{D}}(\pi'', \boldsymbol{\sigma}^*) = \overline{V}_i^{\mathcal{D}}(\pi'') \geq \overline{V}_i^{\mathcal{D}}(\pi') \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**})$$

for sufficiently large i, where $\boldsymbol{\sigma}^*$ is an arbitrary equilibrium under $(\mathcal{D}, \pi'')$ and $\boldsymbol{\sigma}^{**}$ is an arbitrary equilibrium under $(\mathcal{D}, \pi')$. Each equality or inequality follows from Lemma 6, $\pi'' \succsim_{EB} \pi'$, and Lemma 5 respectively. Thus, $\pi'' \succsim_{ES} \pi'$.

To show $\pi \succsim_{ES} \pi''$, note that if $\text{supp}(\mu'') = \{0, \mu_0, 1\}$, the support of private belief distribution induced by $\pi''^{\otimes i}$ is also $\{0, \mu_0, 1\}$ for all i. Then by the proof of Proposition 2, $\pi \succsim_{EB} \pi''$ if and only if there exists $N \in \mathbb{N}$ such that $\pi^{\otimes i}(\mu = 0|L) \geq \pi''^{\otimes i}(\mu'' = 0|L)$ and $\pi^{\otimes i}(\mu = 1|H) \geq \pi''^{\otimes i}(\mu'' = 1|H)$ for all $i \geq N$. By the nature of conclusive signals, agents form the extreme beliefs $\mu = 1$ or 0 if they observe the conclusive signal at least once. Thus, these conditions are equivalent to

$$1 - [1 - \pi(\mu = 0|L)]^i \geq 1 - [1 - \pi''(\mu'' = 0|L)]^i,$$

and

$$1 - [1 - \pi(\mu = 1|H)]^i \geq 1 - [1 - \pi''(\mu'' = 1|H)]^i,$$

for all sufficiently large i. Furthermore, these inequalities are equivalent to

$$\pi(\mu = 0|L) \geq \pi''(\mu'' = 0|L) \quad \text{and} \quad \pi(\mu = 1|H) \geq \pi''(\mu'' = 1|H).$$

Thus, under $\mathrm{supp}(\mu'') = \{0, \mu_0, 1\}$, $\pi \succsim_{EB} \pi''$ if and only if $\pi \succsim_B \pi''$. Therefore, we also have $\pi \succsim_S \pi''$ by Lemma 7. □

Proof of Theorem 3. Suppose that π does not induce unbounded beliefs. By symmetry, we can assume $1 \notin \mathrm{supp}(\mu)$ without loss of generality. Take $r \in (0,1)$ such that $\mathrm{supp}(\mu) \subset [0, r]$. Let $\mathcal{D} = (A, u)$ be the decision problem that is the same as in the proof of Corollary 1. Define $\overline{\mathcal{D}}$ as the replicated decision problem in the same way as in the proof of Theorem 1. Then, we can construct the equilibrium $\boldsymbol{\sigma}^{**}$ under $(\overline{\mathcal{D}}, \pi')$ whose history completely reveals the signals received by each agent. Fix $k_1 \in \mathrm{supp}(\pi')$. Note that it is always an equilibrium under $(\overline{\mathcal{D}}, \pi)$ for all agents to choose (a_1, k_1). Then, it follows that $V_i^{\overline{\mathcal{D}}}(\pi, \boldsymbol{\sigma}^*) = 0$ for all i. Since π' is not no information, for any $\varepsilon > 0$, $V_i^{\overline{\mathcal{D}}}(\pi', \boldsymbol{\sigma}^{**}) > \mu_0(1 - r) - \varepsilon$ for sufficiently large i. Thus, $\lim_{i\to\infty} V_i^{\overline{\mathcal{D}}}(\pi', \boldsymbol{\sigma}^{**}) > \lim_{i\to\infty} V_i^{\overline{\mathcal{D}}}(\pi, \boldsymbol{\sigma}^*)$, and hence $\pi \succsim_{LES} \pi'$ does not hold.

Conversely, suppose that π induces unbounded beliefs. Note that $V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**}) \leq \mu_0 U_1 + (1-\mu_0) U_0$, where $U_0 = \arg\max_{a\in A} u(a, L)$ and $U_1 = \arg\max_{a\in A} u(a, H)$, because the right-hand side is an expected utility under full information. Hence, it is enough to show that that, for any $\mathcal{D}$, $\lim_{i\to\infty} V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \mu_0 U_1 + (1 - \mu_0) U_0$ for any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$. Take any decision problem $\mathcal{D} = (A, u)$. Let $q_L = \frac{\min\{\pi(\mu=1|H), \pi(\mu=0|L)\}}{\pi(\mu=0|L)}$ and $q_H = \frac{\min\{\pi(\mu=1|H), \pi(\mu=0|L)\}}{\pi(\mu=1|H)}$. Note that $q_L, q_H > 0$ holds since π induces unbounded beliefs. Define $\boldsymbol{\sigma}^{**}$ in the same way as in the proof of Lemma 8. Then, by the same argument as the proof of Lemma 8, we have $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) \geq V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^{**})$ for all equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$. Since

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^{**}) = \mu_0[(1 - p^i)U_1] + (1 - \mu_0)[(1 - p^i)U_0] + p^i U_{\mu_0},$$

where $p = 1 - \min\{\pi(\mu = 1|H), \pi(\mu = 0|L)\}$ and $U_{\mu_0} = \arg\max_{a \in A}[\mu_0 u(a, H) + (1 - \mu_0)u(a, L)]$, it follows that

$$\mu_0 U_1 + (1 - \mu_0)U_0 \geq V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*)$$
$$\geq \mu_0[(1 - p^i)U_1] + (1 - \mu_0)[(1 - p^i)U_0] + p^i U_{\mu_0}.$$

Since $p < 1$, we have $\mu_0[(1-p^i)U_1]+(1-\mu_0)[(1-p^i)U_0]+p^i U_{\mu_0} \to \mu_0 U_1+(1-\mu_0)U_0$ as $i \to \infty$. Thus, by the squeeze theorem, it follows that $\lim_{i\to\infty} V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \mu_0 U_1 + (1 - \mu_0)U_0$ for any $\mathcal{D}$ and any equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$. Thus, $\pi \succsim_{LES} \pi'$ for any π'. □

A.7 Proof of Proposition 10

Without loss of generality, assume that $x > \mu_0$, $\text{supp}(\pi) = \{s_0, s_1, s_2\}$ and $\pi(s_0|H) = 0$, $\pi(s_1|H) = 1 - \varepsilon$, $\pi(s_2|H) = \varepsilon$, $\pi(s_0|L) = 1 - \delta$, $\pi(s_1|L) = 0$, and $\pi(s_2|L) = \delta$, where ε and δ satisfy the condition that $x = \frac{\mu_0\varepsilon}{\mu_0\varepsilon+(1-\mu_0)\delta}$. We divide decision problem $\mathcal{D}$ into three cases and construct the following equilibrium $\boldsymbol{\sigma}^*$ under $(\mathcal{D}, \pi)$.

Case (i): $B^{-1}(0) \cap B^{-1}(1) \neq \emptyset$. Fix $a^* \in B^{-1}(0) \cap B^{-1}(1)$. In this case, all agents choose a^* regardless of private signal and action histories.

Case (ii): $B^{-1}(1) \cap B^{-1}(x) = \emptyset$, $B^{-1}(0) = B^{-1}(x) = \{a_0\}$ for some $a_0 \in A$. Fix any $a_1 \in B^{-1}(1)$. Agent 1 chooses a_0 if he receives s_0 or s_2 and chooses a_1 otherwise. For $i \geq 2$, agent i chooses a_0 if she receives s_0, or receives s_2 and all previous agent takes a_0. Otherwise, i chooses a_1.

Case (iii): Otherwise. First, fix $a_0 \in B^{-1}(0)$ such that for all $z \in [x, 1]$, $B^{-1}(z) \neq \{a_0\}$. (Such a_0 must exists by Lemma 2.) In this case, agent 1 chooses action a_0 if he receives s_0, chooses action from $B^{-1}(1)$ if he receives s_1, and chooses action from $B^{-1}(x)$ if he receives s_2. For $i \geq 2$, agent i chooses action a_0 if she receives s_0 or at least one agent before i has taken a_0, chooses action from $B^{-1}(1)$ if she receives s_1, and chooses action from $B^{-1}(\frac{x^i}{x^i+(\frac{\mu_0}{1-\mu_0})^{i-1}(1-x)^i}) \setminus \{a_0\}$ if she receives s_2 and no one before i has taken action a_0 or action from $B^{-1}(1)$. Otherwise, she chooses the same action as agent $i - 1$.

In *Case (i)*, it is always optimal to take a^* regardless of the posterior belief. Hence, this strategy $\boldsymbol{\sigma}^*$ is an equilibrium and we have $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \overline{V}_i^{\mathcal{D}}(\pi)$. In *Case (iii)*, action a_0 is taken if someone has received the signal s_0 in the past, an action from $B^{-1}(1)$ is taken if someone has received the signal s_1 in the past, and an action from $B^{-1}(\frac{x^i}{x^i+(\frac{\mu_0}{1-\mu_0})^{i-1}(1-x)^i})$ or an action yielding the same expected payoff is taken when all past agents have received s_2. Therefore, we have $V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \overline{V}_i^{\mathcal{D}}(\pi)$. Hence, $\boldsymbol{\sigma}^*$ is an equilibrium. Then, in *Case (i)* and *Case (iii)*, by the same argument as Lemma 7,

$$V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*) = \overline{V}_i^{\mathcal{D}}(\pi) \geq \overline{V}_i^{\mathcal{D}}(\pi') \geq V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**}),$$

for any equilibrium $\boldsymbol{\sigma}^{**}$ under $(\mathcal{D}, \pi')$.

The only case left is *Case (ii)*. In *Case (ii)*, from Lemma 3,

$$\begin{aligned} V_i^{\mathcal{D}}(\pi', \boldsymbol{\sigma}^{**}) =& \mu_0[(1 - (1 - \pi'(s_1|H))^i)u(a_1, H) + \\ & (1 - \pi'(s_1|H))^i u(a_0, H)] + (1 - \mu_0)u(a_0, L), \end{aligned}$$

for any equilibrium $\boldsymbol{\sigma}^{**}$ under $(\mathcal{D}, \pi')$. Since $\pi'(s_1|H) \leq 1 - \varepsilon$ (by $\pi \succsim_B \pi'$) and $u(a_1, H) > u(a_0, H)$, it follows that

$$\begin{aligned} & \mu_0[(1 - (1 - \pi'(s_1|H))^i)u(a_1, H) + (1 - \pi'(s_1|H))^i u(a_0, H)] + (1 - \mu_0)u(a_0, L) \\ &= \mu_0 u(a_1, H) - \mu_0(1 - \pi'(s_1|H))^i[u(a_1, H) - u(a_0, H)] + (1 - \mu_0)u(a_0, L) \\ &\leq \mu_0 u(a_1, H) - \mu_0 \varepsilon^i[u(a_1, H) - u(a_0, H)] + (1 - \mu_0)u(a_0, L) \\ &= V_i^{\mathcal{D}}(\pi, \boldsymbol{\sigma}^*), \end{aligned}$$

where $\boldsymbol{\sigma}^*$ is an equilibrium described above. Therefore, $\pi \succsim_W \pi'$.

A.8 Proof of Proposition 11

Without loss of generality, we can normalize $u(a_0, L) = u(a_0, H) = 0$, $u(a_1, H) = 1 - r$, and $u(a_1, L) = -r$.

If $r \in (0, p')$ or $r \in (1 - p', 1)$, then, under π', all agents choose the same action regardless of their private signals. Thus, the Blackwell order between π and π' implies that π yields a weakly higher payoff than π' for all agents.

It therefore suffices to show that the expected payoff, given the parameter p, is decreasing in p. We now derive the expected payoff given p for two cases (I) $r \in (p, 1/2)$ and (II) $r \in (1/2, 1 - p)$. Agent 1 is obviously prefer smaller value of p by the Blackwell order, and thus we take $i \geq 2$ arbitrary.

Case (I): Agent i chooses action a_1 if and only if (i) agent i's public belief is $1 - p$ or (ii) agent i's public belief is p or $1/2$ and private signal is s_h. Thus, the conditional probability of choosing action a_1 at state H is $(1-p)\sum_{k=1}^{\frac{i+1}{2}}[(1-p)p]^{k-1}$ if i is odd, and $(1 - p)\sum_{k=1}^{\frac{i}{2}}[(1 - p)p]^{k-1} + [(1 - p)p]^{\frac{i}{2}}$ if i is even. Similarly, the conditional probability of choosing action a_1 at state L is $p\sum_{k=1}^{\frac{i+1}{2}}[(1 - p)p]^{k-1}$ if i is odd, and $p\sum_{k=1}^{\frac{i}{2}}[(1 - p)p]^{k-1} + [(1 - p)p]^{\frac{i}{2}}$ if i is even. Therefore, the expected payoff of agent i is

$$\begin{cases} \frac{1-p-r}{2} \cdot \sum_{k=1}^{\frac{i+1}{2}} [(1-p)p]^{k-1} & \text{if } i \text{ is odd} \\ \frac{1-p-r}{2} \cdot \sum_{k=1}^{\frac{i}{2}} [(1-p)p]^{k-1} + \frac{1-2r}{2} \cdot [(1-p)p]^{\frac{i}{2}} & \text{if } i \text{ is even.} \end{cases}$$

The calculation shows that this payoff is decreasing in $p \in (0, 1/2)$ under $r \in (p, 1/2)$.[32]

Case (II): Agent i chooses action a_1 if and only if (i) public belief is $\frac{(1-p)^2}{(1-p)^2+p^2}$ or (ii) public belief is $1/2$ or $1-p$ and private signal is s_h. Similarly to case (I), the expected payoff of agent i is

$$\begin{cases} \frac{(1-p)^2(1-r)-p^2 r}{2} \sum_{k=1}^{\frac{i-1}{2}} [(1-p)p]^{k-1} + \frac{1-p-r}{2} [(1-p)p]^{\frac{i-1}{2}} & \text{if } i \text{ is odd} \\ \frac{(1-p)^2(1-r)-p^2 r}{2} \sum_{k=1}^{\frac{i}{2}} [(1-p)p]^{k-1} & \text{if } i \text{ is even.} \end{cases}$$

The calculation shows that this payoff is decreasing in $p \in (0, 1/2)$ under $r \in (p, 1/2)$.[33]

---

[32]Let $f(p)$ be the payoff. Then, if i is odd, $f'(p) \le 0$ if and only if

$$\begin{aligned} &-(1-p-r)[1-(1-p)p]\left[\frac{i+1}{2} \cdot [(1-p)p]^{\frac{i-1}{2}}(1-2p)\right] \\ &\qquad + [1-[(1-p)p]^{\frac{i+1}{2}}][p(p-2)+r(2p-1)] \le 0, \end{aligned}$$

where the inequality holds as $p \in (0, 1/2)$. Also, if i is even, $f'(p) \le 0$ if and only if

$$\begin{aligned} &[1-[(1-p)p]^{\frac{i}{2}}][p(p-2)+r(2p-1)] \\ &\qquad + [1-(1-p)p]\left[\frac{i}{2} \cdot [(1-p)p]^{\frac{i}{2}-1}(1-2p)\right][(p-r)-(1-p)p(1-2r)] \le 0, \end{aligned}$$

where the second term is not positive by $r \in (p, 1/2)$. Therefore, $f(p)$ is decreasing in p.

[33]Let $f(p)$ be the expected payoff of agent i. If i is even, then $f'(p) \le 0$ if and only if

$$\begin{aligned} &[1-[(1-p)p]^{\frac{i}{2}}]\left(-2[1-(1-p)p][(1-p)(1-r)+pr] + [(1-p)^2(1-r)-p^2 r](1-2p)\right) \\ &\qquad - [1-(1-p)p][(1-p)^2(1-r)-p^2 r]\left[\frac{i}{2} \cdot [(1-p)p]^{\frac{i}{2}-1}(1-2p)\right] \le 0, \end{aligned}$$

where the inequality holds as $p \in (0, 1/2)$. If i is odd, then $f'(p) \le 0$ if and only if

$$\begin{aligned} &[1-[(1-p)p]^{\frac{i-1}{2}}]\left(-2[1-(1-p)p][(1-p)(1-r)+pr] + [(1-p)^2(1-r)-p^2 r](1-2p)\right) \\ &+\left[\frac{i-1}{2}[(1-p)p]^{\frac{i-1}{2}-1}(1-2p)\right][1-(1-p)p]\left(-[(1-p)^2(1-r)-p^2 r] + (1-p-r)[1-(1-p)p]\right) \\ &\qquad - [1-(1-p)p]^2[(1-p)p]^{\frac{i-1}{2}} \le 0 \end{aligned}$$

where the second term is weakly below 0 because $-[(1-p)^2(1-r)-p^2 r]+(1-p-r)[1-(1-p)p] \le 0 \iff p \le r$. Therefore, $f(p)$ is decreasing in p.

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