“Necessary and sufficient conditions for convergence in distribution of quantile and P-P processes in $L^1(0,1)$”

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Abstract: We establish a necessary and sufficient condition for the quantile process based on iid sampling to converge in distribution in $L^1(0,1)$. The condition is that the quantile function is locally absolutely continuous on the open unit interval and satisfies a slight strengthening of square integrability. We further establish a necessary and sufficient condition for the P-P process based on iid sampling from two populations to converge in distribution in $L^1(0,1)$. The condition is that the P-P curve is locally absolutely continuous on the open unit interval. If either process converges in distribution then it may be approximated using the bootstrap.

“The Hellinger Bounds on the Kullback–Leibler Divergence and Bernstein Norm”

Tetsuya Kaji

Abstract: The Kullback–Leibler divergence, the Kullback–Leibler variation, and the Bernstein “norm” are used to quantify discrepancies among probability measures in likelihood models such as nonparametric maximum likelihood and nonparametric Bayes. They are closely related to the Hellinger distance, which is often easier to work with. Consequently, it is of interest to characterize conditions under which the Hellinger distance serves as an upper bound for these discrepancy measures. This article develops a necessary and sufficient condition for each of the discrepancy measures to be bounded by the Hellinger distance. They accommodate unbounded likelihood ratios and generalize all previously known results. We then apply it to relax an existing regularity condition for the sieve maximum likelihood estimator.