Shopping Versus Shipping: The Strategic Choice of Retail Formats*

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October 28, 2025

Abstract

We develop a parsimonious model that incorporates shopping and shipping costs, a distaste cost, consumer taste heterogeneity, and the pricing strategies of online and offline retailers. We compare offline, mixed and online duopolies and show that the retailing industry equilibrium depends on the degree of taste heterogeneity, but also on the value of shopping and shipping costs. Consumers' most-preferred retail format depends on their location relative to the retailers. In the aggregate, consumers are better-off under online retailing. However, firms are worse-off because they get trapped in a prisoner's dilemma.

Keywords — Offline retailing, online retailing, spatial price policies

JEL codes —L13; L81; R10.

*We are grateful to Fukunari Kimura, Ian Coxhead, and seminar audience at University of Tokyo for helpful comments.

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1 Introduction

E-commerce has become a defining feature of modern economies and is likely to capture a dominant share of retailing in the near future. In the United States, the share of e-commerce in total retail sales increased from 6.5% in 2014 to 15.9% in 2023. In China, it rose from 10.6% to 27.6% over the same period. In the United Kingdom, e-commerce accounted for 11.3% of total retail sales in 2014 and reached 26.7% in 2023.

Our objective is to develop a parsimonious model that incorporates shopping and shipping transportation costs, consumer taste heterogeneity, and the pricing strategies of online and offline retailers. We analyze how these factors interact to determine the equilibrium structure of the retail industry. As reflected in our list of references, this topic has attracted considerable attention from scholars across various disciplines. The contribution of our paper lies in combining spatial pricing, as studied in industrial organization, with discrete choice theory within a unified game-theoretic framework that also integrates insights from marketing. Assuming that firms behave strategically is particularly relevant in an industry characterized by high concentration: among the top 100 online firms, the five largest typically account for about half of total sales. At the local level, even when retailers are small, spatial proximity is well known to foster strategic behavior regardless of the total number of firms operating in a city.

Even though it is common to say that location no longer matters in e-commerce, we hope to show that the choice of a particular retail format is by nature a spatial problem. As retailers compete for spatially dispersed consumers through different spatial price policies, it seems natural to nest our problem within the framework of spatial competition theory (Forman *et al.*, 2009; Bonfrer *et al.*, 2022).

An offline retailer charges the same *mill price* to all its customers while a buyer faces a full price that reflects exactly her travel cost to the retailer. Firms that offer home delivery of products face a wide range of possible pricing strategies. Basically, they are related to the way firms charge shipping costs to customers from local, to catalog, to Internet. Although the jury is still out to say what the best pricing strategy is, it is well documented that free shipping and fast delivery increase consumer satisfaction and positively influence purchase intention, but free shipping can also lead to an increase in returns (Lederer, 2012; Cavallo, 2017). A reasonable approach is thus to assume that e-retailers adopt a simple policy known as *uniform delivered pricing* in which a firm charges the same delivered price, inclusive of shipping and handling costs, regardless of its customers' locations.

To underscore the empirical relevance of this assumption, note first that Amazon implements a uniform pricing strategy, ensuring that each product variety is available across all counties at identical prices. Analogous pricing practices have been documented for other major retailers, such as Target and Walmart. Consistent with these observations, Cavallo (2017) reports that, between 2014 and 2016, online prices in the United States exhibited minimal regional variation. Moreoever, we will discuss in the concluding section how our model can be used to tackle alternative pricing strategies. Because e-retailers must ship commodities from a depot to spatially dispersed consumers, *shipping vastly differs from shopping*. Somewhat surprisingly, the existing literature often overlooks the distribution costs incurred by e-retailers, which play a central role in our paper.

Discrete choice models provide powerful tools for analyzing consumer demand, as they capture heterogeneity in consumer behavior in a tractable and realistic way (Chintagunta and Nair, 2011; Dubé, 2019). To the best of our knowledge, these models have been less extensively applied to the study of competition between retail formats (see below for a brief discussion of relevant contributions). Nevertheless, as we will show, *consumer heterogeneity plays a* *central role in the emergence of different retail formats*, even under otherwise identical market conditions. Equally important, heterogeneity is essential for establishing the existence of equilibrium.

Taste heterogeneity implies that consumers exhibit lower price sensitivity. This finding is consistent with recent empirical evidence by Döpper *et al.* (2025), who report that product-specific demand elasticities in the United States declined by an average of 30% between 2006 and 2019. Moreover, as market size expands, consumer heterogeneity tends to increase, reflecting a broader dispersion of individual preferences. This diversification of tastes is a key driver of e-commerce growth. It is thus unsurprising that China currently accounts for the largest share of global e-commerce, while India is projected to record the highest compound annual growth rate in e-commerce between 2024 and 2029 (22.1%).

We will see that firms' choices of a retail format and consumers' decision to buy from an offline or online firm often depend on firms' and consumers' locations. By integrating spatial pricing with discrete choice theory, we can analyze how transportation costs and pricing strategies interact with consumer tastes and locations in shaping the emergence of one of three equilibrium retail structures: an offline duopoly, an online duopoly, or a mixed duopoly involving both formats. Our framework enables us to address several key questions: (i) Does it pay to go online? (ii) What is the equilibrium structure of the retail industry? (iii) Is one retail format inherently more competitive than the other? (iv) What are the implications of the online business model for consumer and social welfare?

By drawing on insights from multiple disciplines, our study contributes to the development of a unified framework for analyzing competition between retail formats.

Our main findings may be summarized as follows. First, consumers who purchase online are deprived from the advantages associated with buying from a physical store. This gives rise to what is called a distaste cost (Balasubramanian, 1998; Loginova, 2009). We show that *a conventional retailer finds it profitable to become an e-retailer when the distaste cost is sufficiently low.* This result may explain why expensive goods for which consumers have strong idiosyncratic preferences are still provided by conventional shops. By contrast, more standardized goods, such as books, CDs or inexpensive clothes, involve low distaste costs (Smith and Brynjolfsson, 2001). Furthermore, both firms choose to become e-retailers when consumer tastes are sufficiently heterogeneous. In this case, *both firms make lower profits in an online duopoly than in an offline duopoly.*

Second, one of the main issues discussed in the literature is the expected procompetitive nature of online retailing generated by lower distribution and search costs (Brown and Goolsbee, 2002; Brynjolfsson *et al.*, 2003; Lieber and Syverson, 2012). The tenets of e-commerce hold that it reduces market friction, unleashes competition and makes consumers better-off. Here too, our analysis is more nuance in that *consumers' best retail format depends on where consumers are located relative to retailers.* As a result, it seems futile to look for general answers regarding the desirability of the online channel. Nevertheless, comparing consumer surpluses and profits shows that *the online channel benefits consumers but harms firms in aggregate when consumers are sufficiently heterogeneous.* However, this ceases to hold when heterogeneity is weak. Consequently, by affecting consumers' attitudes toward product differentiation, the nature of the goods supplied by retailers is likely to affect the desirable retail system.

Last, a growing number of conventional retailers have reacted to the entry of online retailers by offering delivery as an additional option to their customers (Bernstein *et al.*, 2008; Hübner *et al.*, 2022). In the context of this paper, a conventional retailer that adopts a multi-channel strategy combines *pick-up* and *delivery.* Its set of customers is then split between those located in its vicinity (they visit the store) and the remote consumers (they order online). The location of the consumer indifferent between the two channels also depends on the price chosen by the competing firm. We show that a conventional retailer becomes a multi-channel retailer when heterogeneity is weak enough for price competition to be fierce.

Related literature. Gauri *et al.* (2021) and Ratchford *et al.* (2022) are two detailed overviews of the now vast literature on competition between different retail formats. In what follows, we only discuss papers closely related to ours.

The idea that consumers are heterogeneous along several dimensions is not new, but the literature typically focuses on conventional retailers (Lauga and Ofek, 2011). Balasubramanian (1998) and Bouckaert (2000) are pioneering contributions that develop spatial competition models with offline and online retailers. Yoo and Lee (2011) assume that spatially dispersed consumers are heterogeneous in the disutility they incur for using an Internet channel.

Furthermore, by facilitating consumers' search, e-commerce was expected to strongly reduce price dispersion. Brynjolfsson and Smith (2000) observe that price dispersion still prevails among e-retailers. Smith and Brynjolfsson (2001) then use the multinomial logit to rationalize customers' behavior who do not always buy the cheapest homogeneous good, such as books or CDs. Bernstein *et al.* (2008) also use the multinomial logit and consider different markets involving only brick-and-mortar retailers, physical retailers and retailers which combine off- and online sales, and retailers selling through both channels. They find that combining the two channels is a dominant strategy for all retailers. This is reminiscent of our result that suggest that the industry equilibrium involves only online-firms when consumers are sufficiently heterogeneous. However, their setting includes no transportation costs.

More closely related to our work, Rhodes and Zhou (2024) use a general discrete choice model to assess how varying degrees of market coverage affect consumer surplus and firm profits when n firms can or cannot engage in price discrimination across heterogeneous consumers. However, Rhodes and Zhou do not consider spatial differentiation.

A key distinction between our study and the existing literature lies in our treatment of competition between retail formats. We compare different retail configurations for a given number of firms, whereas much of the prior literature examines the entry of online firms into markets already populated by conventional retailers. Because market entry typically intensifies competition, it becomes difficult to disentangle the effects of competition between retail formats from those stemming from increased market rivalry. By contrast, in our framework, changes in market outcomes arise solely from firms' endogenous choices of retail format.

The remainder of the paper is structured as follows. Section 2 presents the model and illustrates its mechanics by considering the simple case of two conventional retailers. Two cases may arise. In the first one, there is *full market coverage* when all consumers have a positive probability of purchasing from each firm; otherwise, *partial market coverage* arises. Our analysis focuses primarily on full market coverage, while a discussion of partial coverage is provided in the Appendix. Section 3 analyzes a mixed duopoly consisting of one offline and one online firm; we first derive the equilibrium prices and then identify the conditions under which a conventional retailer switches to an online format. Section 4 determines the conditions under which both firms become e-retailers. Equipped with these results, Section 5 challenges the common view that online firms are necessarily more competitive than offline firms, while Section 6 compares consumer surplus across retail formats. Section 7 examines whether a conventional retailer competing with an e-retailer finds it profitable to introduce a second (online) channel, thereby allowing consumers to choose between in-store pickup and home delivery. We show that this occurs when consumers are not too homogeneous but also not excessively heterogeneous. Interestingly, the e-retailer prefers its rival to remain a brick-and-mortar store. Section 8 discusses possible extensions.

2 The model and preliminary results

Consider a market comprising a unit-mass of consumers and two firms, denoted by 1 and 2, which supply a horizontally differentiated good; both firms have access to similar production and distribution technologies. Each consumer consumes a fixed amount of the differentiated good, which is normalized to one. The firms are assumed to have identical and constant marginal costs, normalized to zero. Prior research in the field suggests that a duopoly often provides a sufficient framework for examining several key issues associated with the emergence of diverse retail formats.

2.1 Consumer heterogeneity

Following the literature on product differentiation theory, we assume that the indirect utility of a consumer at x who chooses variety i is given by

$$V_i = Y - p_i(x) + \varepsilon_i,$$

where Y is her income, $p_i(x)$ the price of variety i, and ε_i a random variable whose realization measures the quality of the match between individual preferences and variety i. Since we consider a duopoly, we adopt the canonical discrete choice model in which the random variable $\varepsilon = \varepsilon_2 - \varepsilon_1$ is uniformly distributed over the compact interval $[-L, L]$ with density $1/2L$. Thus, consumer heterogeneity is represented through the *linear probability model* (Amemiya, 1981; Anderson *et al.*, 1992). The parameter L serves as an index of heterogeneity: when $L = 0$, consumers have identical tastes, whereas higher values of L correspond to greater dispersion in individual preferences. Alternatively, the parameter L can measure consumers' attitude toward diversity: as L becomes larger, consumers care more about diversity in consumption and less about prices (Sajeesh and Raju, 2010).[1]

We extend the model by embedding consumers and firms in a linear spatial environment. Specifically, in line with the literature on spatial competition, firms 1 and 2 are located at the endpoints $x = 0$ and $x = 1$ of the unit interval $[0, 1]$, respectively. Consumers are uniformly distributed along this interval with unit density. Each consumer is therefore characterized by her geographical location $x \in [0, 1]$ and her preference address $z \in [-L/2, L/2]$. Our emphasis on spatial pricing policies leads us to distinguish between two prices. The *producer price* of a conventional retailer is its mill price while the corresponding *consumer price* is the full price paid by consumers, which increases with the distance to the retailer. On the other hand, when the firm is an e-retailer, the producer and consumer prices are constant across locations and equal up to the distaste cost.

Let $t > 0$ denote the per-unit shopping cost. Assuming that both firms operate as conventional retailers, the

[1] Since L is proportional to the standard-deviation of the variable ε, it plays the same role as $1/\sigma$, where σ is the elasticity of substitution, in CES models of monopolistic competition (Anderson *et al.*, 1992, Proposition 3.8).

consumer price at x for purchasing from firm i is

$$p_i(x) = p_i + t\,|x - x_i|\,,$$

where p_i is the mill price set by firm i, and $t\,|x - x_i|$ represents the cost of traveling to the firm's location. Accordingly, the indirect utility of a consumer located at x who chooses variety i can be expressed as

$$V_i = Y - p_i(x) + \varepsilon_i.$$

In sum, we work with the simplest model that combines the main ideas of spatial competition and discrete choice theories. We acknowledge that the above assumptions, which are made for analytical tractability, are "unrealistic". On the positive side, they allow us to uncover results that can be tested by means of quantitative spatial models.

Competition between firms is modeled as a two-stage game. In the first stage, firms choose a retail format, either offline or online. In the second stage, they compete in prices, contingent on their chosen price policies. The market outcome is given by a subgame perfect Nash equilibrium. As usual, the game is solved by backward induction.

2.2 Competition between two brick-and-mortar stores

Assume that the two firms are conventional retailers. In this case, consumers travel to the physical stores and consumer prices are given by firms' mill prices plus the corresponding travel costs to the shops: $p_1(x) = p_1 + tx$ and $p_2(x) = p_2 + t(1-x)$. The probabilities/quantities that a consumer located at x buys from firms 1 and 2 are, respectively, given by

$$\begin{aligned} q_1(p_1,p_2;x) &= \begin{cases} 0 & \text{if } x \geq x_{\max} \\ \frac{L-p_1-tx+p_2+t(1-x)}{2L} & \text{if } x_{\min} < x < x_{\max} \\ 1 & \text{if } \ x \leq x_{\min} \end{cases}, \\ q_2(p_1,p_2;x) &= \begin{cases} 1 & \text{if } x \geq x_{\max} \\ \frac{L+p_1+tx-p_2-t(1-x)}{2L} & \text{if } x_{\min} < x < x_{\max} \\ 0 & \text{if } \ x \leq x_{\min} \end{cases}, \end{aligned} \tag{1}$$

where

$$x_{\max} \equiv \min\left\{\frac{-p_1+p_2+t+L}{2t}, 1\right\}, \qquad x_{\min} \equiv \max\left\{0, \frac{-p_1+p_2+t-L}{2t}\right\} \tag{2}$$

are the locations of consumers indifferent between buying both varieties or only one of them ($q_1(p_1,p_2;x_{\max}) = 0$ and $q_2(p_1,p_2;x_{\min}) = 0$). Note that $q_1(p_1,p_2;x) + q_2(p_1,p_2;x) = 1$.

There is full market coverage when $x_{\min} = 0$ and $x_{\max} = 1$, and partial market coverage when $x_{\min} > 0$ and $x_{\max} < 1$. Using (2) shows that full market coverage arises when $L \geq t$. When $L < t$, there is partial market coverage: consumers in $[0, x_{\min}]$ and $[x_{\max}, 1]$ are loyal to one firm, whereas those in $[x_{\min}, x_{\max}]$ buy from both firms.

Using (1), it is easy to show that firms' demands are linear and given by

$$\begin{aligned} q_1(p_1,p_2) &= \int_0^1 \frac{L-p_1-tx+p_2+t(1-x)}{2L}dx = \frac{L-p_1+p_2}{2L}, \\ q_2(p_1,p_2) &= \int_0^1 \frac{L+p_1+tx-p_2-t(1-x)}{2L}dx = \frac{L+p_1-p_2}{2L}, \end{aligned} \tag{3}$$

which implies that market demands are less price-sensitive when L is larger.

Since firms' profits $\Pi_{1m} = p_1 q_1(p_1, p_2)$ and $\Pi_{2m} = p_2 q_2(p_1, p_2)$ are concave in their respective price, applying the first-order conditions yields the equilibrium prices:[2]

$$p_1^* = p_2^* = L, \tag{4}$$

which increase with the degree of heterogeneity. This is because firms' demands (3) become less elastic when L rises, thus allowing firms to charge higher prices.

Using (3) shows that firms' outputs are equal to 1/2, while the equilibrium profits are given by

$$\Pi_{1m}^*(L, L) = \Pi_{2m}^*(L, L) = \frac{L}{2}. \tag{5}$$

Summarizing, we have:

Proposition 1. *Assume a market with two brick-and-mortar stores. If $L \geq t$, there exists a unique price equilibrium with full market coverage, which is given by $p_1^* = p_2^* = L$.*

Thus, consumers must be sufficiently heterogeneous in tastes for full market coverage to arise. Furthermore, when $L \geq t$, consumers are willing to pay a higher price when L increases because varieties are perceived by consumers as being more differentiated.

The equilibrium spatial distribution of consumption is given by

$$q_1^*(x) \equiv q_1(p_1^*, p_2^*; x) = \frac{p_2^* + t(1 - 2x)}{2L},$$

which decreases as the distance to firm 1 rises. Similarly, $q_2^*(x) = 1 - q_1^*(x)$ increases with x. Since $q_1^*(1) = q_2^*(0) = 1/2$, there is full market coverage at the equilibrium prices.

3 Competition between offline and online firms

Because e-retailers must ship commodities from a depot to spatially dispersed consumers, shipping vastly differs from shopping. In this section, we consider a *mixed* duopoly in which firm 1 is offline while firm 2 is online and charges a uniform delivered (UD) price P. In other words, consumers bear location-specific shopping costs to firm 1 whereas firm 2 incurs location-specific delivery costs to consumers. Consumers reveal their true locations to the e-retailer under UD pricing as arbitrage between consumers is not desirable because they pay the same consumer price. Note that UD pricing implies price discrimination because the implicit mill price of an e-retailer, which is equal to $P - T(1 - x)$, increases with the distance to firm 2.

A mill pricing firm encounters a strategic trade-off: it is incentivized to lower its mill price to attract distant consumers, yet it has a preference for charging higher prices to the local customers. Conversely, a firm utilizing UD pricing faces the opposite trade-off. To cover the transportation costs associated with serving remote consumers, it is inclined to set a higher uniform price; however, doing so limit its ability to capture a higher surplus from nearby customers.

[2] When its competitor sells its variety at price L, a firm must charge a price equal to 0 to drive its competitor out of business. Furthermore, we show in the Online Appendix A that firms have no incentives to shift to partial market coverage by raising prices above $L + t$.

3.1 Shipping and distaste costs

As argued by Smith and Brynjolfsson (2001), consumers are sensitive to shipping fees and delivery times. For online firms, the choice of a delivery strategy is therefore a key issue. This has led these firms to implement sophisticated logistic strategies that allow them to serve at low cost geographically dispersed customers by using a fleet of trucks that each delivers the good to several customers. But doing this requires solving an NP-hard optimization problem which aims to determine the least-cost delivery routes that start and end at the same location. This problem, called the vehicle routing problem, has been extensively studied in operations research and management science (Laporte, 2009; Koç *et al.*, 2020). On the other hand, due to traffic congestion and a high opportunity cost of time, people's travel costs within cities remain substantial (Redding and Turner, 2015).

Since the vehicle routing problem is NP-hard, it seems hopeless to consider a game-theoretic model which would account for the strategic determination of a solution to this problem within each e-retailer's profit-maximization problem (Ahmadi-Javi *et al.*, 2018). To deal with this issue in an analytically tractable way, we assume that the e-retailer undertakes one return trip by customer and bears a shipping cost T that is significantly lower than the shopping cost, $T << t$. As the e-retailer pays the shipping cost, it never chooses a delivered price P smaller than T. Parameters t and T measure the cost of geographical differentiation in different retail contexts. Lower case letters, (p, t) are used for the conventional retailer and capital letters (P, T) for the e-retailer.

Consumers who buy from an e-retailer save travel costs but bear a *distaste cost*. This cost includes the deprivation of the various advantages provided by the physical inspection of goods, the delay in receiving the product, the difficulty in returning products, and the after-sales services supplied by a conventional store. All of this generate a utility loss, which is expressed as a monetary cost. Smith and Brynjolfsson (2001) find that this cost is positive even for homogeneous goods like books. We capture the distaste cost by assuming that consumers at x make purchase decisions based on the consumer prices $p_1(x) = p + tx$ and $p_2 = \gamma P$ where p and P are the producer prices and $\gamma > 1$ the distaste cost. The finding that, among multichannel retailers, online and offline prices are identical approximately 72% of the time suggests that consumers' distaste costs are not negligible (Cavallo, 2017). More generally, γ can be viewed as a statistic that subsumes the various costs and benefits of purchasing from e-retailing.

In the literature, this cost is supposed to be additive, $P + \gamma$ (Balasubramanian, 1998; Cao *et al.*, 2016; Guo and Lai, 2017). We choose instead a multiplicative cost because the distaste cost is likely to be higher when consumers buy a luxury and expensive product, such as cloths produced by prestigious fashion houses, leather goods, jewelry, and watches, than when they acquire more standardized products like books or CDs. We expect the distaste cost to be negligible if the product bought from an online seller has previously been purchased at a brick-and-mortar store.

Our setting involves four parameters: L, t, T, and γ. We reduce the number of cases by making two assumptions that seem empirically plausible. First, we assume that $t > \gamma T$. This inequality seems empirically more relevant because (i) online firms have access to efficient logistic technologies that allow them to supply customers at lower costs ($T \downarrow$), (ii) shopping costs did not decrease as much as shipping costs because the opportunity cost of time grew with income ($t \uparrow$), and (iii) advanced information technologies allow consumers to better assess the quality of online products ($\gamma \downarrow$). Chintagunta *et al.* (2012) find that buying online is cheaper than travelling to an offline retailer. Second, as one of the main purpose of the paper is to investigate the impact of consumer heterogeneity and transportation costs on the equilibrium outcome, we also assume $L > t$, which implies full market coverage. In sum, throughout the remaining of the paper, we will assume the following inequalities:

$$L > t > \gamma T. \tag{6}$$

It is worth stressing that *Propositions 2-9 discussed below hold under* (6), though several results hold under weaker conditions that are necessary and sufficient.

3.2 Equilibrium in a mixed retailing market

The demands of a consumer at x for varieties 1 and 2 are, respectively, given by

$$q(p,P;x) = \begin{cases} 0 & \text{if } x > 1 \\ \frac{L-p-tx+\gamma P}{2L}, & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } \ x < 0 \end{cases},$$

$$Q(p,P;x) = \begin{cases} 1 & \text{if } x > 1 \\ \frac{L+p+tx-\gamma P}{2L}, & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } \ x < 0 \end{cases}.$$

The condition

$$t - L < \gamma P^* - p^* < L$$

must hold for all consumers to be supplied by both firms. This interval is not empty if and only if $L > t/2$.

The demands of consumers at x are as follows:

$$q(p,P;x) = \frac{L-p-tx+\gamma P}{2L}, \qquad Q(p,P;x) = \frac{L+p+tx-\gamma P}{2L}. \tag{7}$$

Profits are thus defined as follows:

$$\begin{aligned} \Pi_m(p,P) &= p\int_0^1 q(p,P;x)dx = p\left(\frac{1}{2} + \frac{\gamma P - p}{2L} - \frac{t}{4L}\right), \\ \Pi_U(p,P) &= \int_0^1 Q(p,P;x)(P - T(1-x))dx = \left(P - \frac{T}{2}\right)\left(\frac{1}{2} + \frac{p-\gamma P}{2L} + \frac{t}{4L}\right) + \frac{tT}{24L}. \end{aligned}$$

Note that our definition of Π_U supposes that each customer is supplied by firm 2 through a specific shipment. By assuming that the shipping rate T can be arbitrarily low, and is always smaller than the shopping rate t, we retain enough analytical flexibility while avoiding appealing to complicated logistic strategies. Recall that in the real-world e-retailers typically use sophisticated delivery strategies such as the vehicle routing problem discussed in the introduction.

Since both profit functions are concave in their own price, applying the first-order conditions yield the candidate equilibrium prices of the off- and online shops:

$$p^* = L - \frac{t-\gamma T}{6} > 0, \qquad P^* = \frac{L}{\gamma} + \frac{t+2\gamma T}{6\gamma}. \tag{8}$$

For the prices (8) to be a Nash equilibrium of the game between the offline and online retailers, the following two conditions must be satisfied:

$$p^* > 0 \Leftrightarrow 6L > t - \gamma T \Leftrightarrow \gamma T > 0 > t - 6L, \tag{9}$$

and

$$P^* > T \Leftrightarrow 6L > 4\gamma T - t \Leftrightarrow \gamma T < \frac{6L + t}{4}. \tag{10}$$

Furthermore, the condition for the two firms to serve the whole market at the prices (8) is given by

$$-L < -p^* - tx + \gamma P^* < L.$$

Substituting $x = 0$ and $x = 1$ in (7) leads to the inequalities

$$-p^* + \gamma P^* < L \Leftrightarrow 6L > 2t + \gamma T \Leftrightarrow \gamma T < 6L - 2t, \tag{11}$$

and

$$-L < -p^* - t + \gamma P^* \Leftrightarrow 6L > 4t - \gamma T \Leftrightarrow \gamma T > 0 > 4t - 6L \tag{12}$$

Among (9)-(12), the most demanding condition is given by

$$\gamma T < \frac{6L + t}{4}, \tag{13}$$

which means that a price equilibrium exists in a mixed duopoly if and only if the full shipping cost γT is not too high. Note that this condition is satisfied under (6).[3]

Furthermore, (13) becomes less stringent when consumers are more heterogeneous and/or when shopping costs are high because price competition is relaxed. Note also that (13) is equivalent to $L > (4\gamma T - t)/6$, that is, L must be large enough for an equilibrium to exist. In other words, an equilibrium (in pure strategies) does not exist when the population is homogeneous.

At the equilibrium prices (8), the quantities sold at x are as follows:

$$q^*(x) = \frac{6(L - tx) + \gamma T + 2t}{12L}, \qquad Q^*(x) = \frac{6(L + tx) - \gamma T - 2t}{12L}, \tag{14}$$

which shows that $q^*(x)$ decreases with x while $Q^*(x)$ increases with x. In other words, the consumption of a variety decreases as the distance to its supplier rises (Lieber and Syverson, 2012). Next, when t and T are negligible, $q^*(x) \approx Q^*(x)$ holds because the spatial differentiation effect is negligible too.

Dolfen *et al.* (2013) observe that the main share of the gains generated by e-commerce in the U.S. stems from substituting to retailers available online but not locally. This concurs with (14) which shows that a more efficient shipping strategy ($T \downarrow$) implies that consumers will buy more from the online channel. By contrast, when consumers have better access to a conventional retailer ($t \downarrow$), they substitute away from the e-retailer.

Furthermore, when $t > \gamma T$, *the online firm's sales are larger than those of the offline firm:*

$$0 < \int_0^1 q^*(x)dx = \frac{6L - t + \gamma T}{12L} < \frac{1}{2} < \int_0^1 Q^*(x)dx = \frac{6L + t - \gamma T}{12L}. \tag{15}$$

[3] We prove in Online Appendix A that firm 1 does not find it profitable to raise its price such that $x_{\max} < 1$.

As shown by (4) and (8), $p^* < p_1^*$ when $t > \gamma T$. In other words, firm 1 must decrease its mill price when firms 2 becomes an e-retailer. Nevertheless, its output also declines because its market share is smaller than one-half. Consequently, *firm 2's shift to e-retailing proves highly detrimental to firm* 1. In other words, by shifting from mill pricing to UD pricing, firm 2 adopts a more aggressive competitive stance. On the other hand, profits made by firm 2 when it becomes an e-retailer need not be larger than the profits earned when it is a conventional retailer. We will see below when firm 2 finds it profitable to adopt the online channel.

In addition, the price gap $p_1^* - p^* > 0$ widens as shopping costs increase or shipping costs decrease. For example, a higher density of population is likely to permit better access to conventional retailers in larger cities than in rural areas. This in turn should foster the gradual disappearance of conventional retailing in small cities and rural areas, which will boost the development of e-commerce in such regions. This concurs with Fan *et al.* (2018) who find that the average welfare gains from e-commerce are 1.11% for Chinese cities in the largest population quintile but 2.01% for those in the lowest quintile.

As the distance from $x = 0$ increases, the consumer price of the conventional retailer ($p^* + tx$) increases while that of the online retailer remains constant (γP^*). The former is smaller than the latter if and only if $x < \tilde{x} \equiv 1/3 + \gamma T/6t < 1/2$. Beyond this location, the e-retailer prices at a lower level than its competitor.

Summarizing yields the following proposition.

Proposition 2. *In a mixed duopoly, there exists a unique price equilibrium given by* (8) *if and only if shipping and distaste costs are not too high. Furthermore, the conventional retailer reduces its mill price and output when its rival becomes an e-retailer.*

As shown by (8), higher shopping costs ($t \uparrow$) lead the conventional retailer to decrease its price because its demand is reduced. By contrast, higher shipping costs ($T \uparrow$) lead the e-retailer to raise its delivered price because its transportation cost rises. Shopping and shipping costs thus have different impacts on market prices, confirming once more that the two retail formats differ in nature. However, both equilibrium prices increase when consumers have more heterogeneous tastes. Moreover, the impact of t and T on prices is similar under partial market coverage expect for p^* that increases with t.

Note also that a higher distaste cost γ allows the conventional retailer to raise its price because the e-retailer becomes less competitive. Consequently, the e-retailer must lower its price (P^* decreases with γ). Therefore, *the costs T and γ have different impacts on the online firm's price.* In addition, the pass-through is incomplete in both retail formats: the conventional retailer decreases its price by 1/6 of the shopping cost increase while the e-retailer absorbs 2/3 of its shipping costs.

Substituting the equilibrium prices (8) into firms' profits yields

$$\Pi_m^* = \frac{1}{72L}\left(6L + \gamma T - t\right)^2, \qquad \Pi_U^* = \frac{1}{72\gamma L}\left(6L + t - \gamma T\right)^2 + \frac{tT}{24L}. \tag{16}$$

Observe that the conventional retailer's profits decrease with shopping costs ($t \uparrow$), but increase with shipping costs ($T \uparrow$), whereas the opposite holds for the e-retailer. Next, a higher distaste cost ($\gamma \uparrow$) is always detrimental to the online retailer but is beneficial to the offline retailer. In sum, by relaxing competition, a retailer's higher transaction cost is always detrimental to this retailer, which agrees with Chintagunta *et al.* (2012). By contrast, its competitor always earns larger profits. Last, *when consumers are more heterogeneous, both types of firms charge higher producer prices*, which agrees with well-established results in industrial organization.

Though e-commerce favors trade toward remote locations, the e-retailer does not drive the conventional retailer out of business. Indeed, $q^*(0) > 0$ while Π_m^* is always positive (see (16) and (A.5)). Recall here our assumption that both firms operate under the same marginal cost. If it operates under a higher marginal cost, the offline firm could be forced to exit the market because its mill price cannot fall below its marginal cost.

So far, we have assumed that all consumers face the same expenditure when they order from an online firm. Yet, in remote, rural regions, consumers may face logistical and cultural obstacles to the adoption of e-commerce, which allows conventional retailers to survive (Couture *et al.*, 2021). Formally, this idea may be captured by assuming that consumers in, say, region $[0, a]$ bear a higher cost γ than those in $(a, 1]$. Because the full price of the online firm becomes higher in $[0, a]$, the conventional retailer may be able to retain more customers.

3.3 Is it profitable for an offline retailer to become an online retailer?

In the foregoing, we assumed that firm 2 was an e-retailer while firm 1 remained a physical retailer. It is, however, legitimate to ask whether firm 2 finds it profitable to become an e-retailer? In other words, does profit maximization incentivizes firm 2 to shift from the conventional channel to the online channel, and if so under which conditions?

Using (5) and (16), we have:

$$\Pi_{2m}^* - \Pi_U^* = H(L, \gamma) \equiv \frac{L}{2} - \frac{tT}{24L} - \frac{1}{72\gamma L}\left(6L + t - \gamma T\right)^2 .$$

Observe first that $\Pi_U^* > \Pi_{2m}^*$ holds when $t = T$ and $\gamma = 1$. By continuity, the same inequality holds true if γ is slightly higher than 1 or t slightly bigger than T. More specifically, it is readily verified that $H(L, 1) < 0$, $H(L, \gamma)$ increases with γ over the interval $(0, \gamma_{\max})$ where $\gamma_{\max} = (6L + t)/T > t/T > 1$ is the maximizer of $H(L, \gamma)$. Since $H(L, \gamma_{\max}) > 0$ when $L > t$, there exists a unique value $\bar{\gamma} \in (1, \gamma_{\max})$ such that $H(L, \bar{\gamma}) = 0$ and $H(L, \gamma) < 0$ for $\gamma \in (1, \bar{\gamma})$.[4] Otherwise, firm 2 remains a conventional retailer because the full shipping cost γT is too large.

To sum up, we have:

Proposition 3. *Assume that firm* 1 *is a conventional retailer. If* (*13*) *holds, then there exists a unique value* $\bar{\gamma}$ *such that firm* 2 *chooses to become an e-retailer when* $\gamma < \bar{\gamma}$. *If* $\gamma > \bar{\gamma}$ *or if* (*13*) *does not hold, firm* 2 *remains a conventional retailer.*

Thus, firm 2 shifts to the e-channel when the distaste cost and the shipping cost are sufficiently low. This may explain why luxury and expensive goods, such as fashion clothes, jewelry, and watches, for which consumers have strong idiosyncratic preferences are still provided by conventional shops. Such goods By contrast, more standardized goods, such as books, CDs, appliances or inexpensive clothes, involve low distaste costs. It is no surprise, therefore, that Amazon started by selling such products. Furthermore, the adoption of new information technologies that allow for a better perception of goods by consumers are likely to favor e-retailing at the expense of brick-and-mortar stores.

Proposition 3 has several noteworthy implications. First, firm 2 may choose to become an e-retailer even when shopping costs are low. In this case, $\gamma T < t$ implies that shipping and distaste costs must be even lower. This concurs with Datta and Sudhir (2013) who argue that *retailers choose different formats when spatial differentiation is weak.*

[4] To be precise, $\bar{\gamma}(L)$ is the smallest root of the equation $H(L, \gamma) = 0$, that is, $\bar{\gamma}(L)T^2 = 18L^2 + 6LT - (tT)/2 - \sqrt{3/4}\sqrt{(12L^2 - Tt)(24LT + Tt + 36L^2)}$

Second, the function $H(L,\gamma)$ is shifted upward when L increases. As a result, the cut-off $\bar{\gamma}$ decreases with the degree of consumer heterogeneity. Proposition 3 then implies firm 2 is less inclined to become an e-retailer. This is because consumers are less sensitive to the full price they pay. In sum, in a market involving several firms, *a weak spatial differentiation effect and/or a strong consumer heterogeneity are likely to induce at least one firm to shift to the e-channel.*

Last, when $\gamma T < t$, the threshold $\bar{\gamma}$ must belong to $(1, t/T]$, which holds if and only if $\bar{\gamma} - t/T > 0$,that is, $-12L^2(t-T) + Tt^2 < 0$. Since this expression decreases with L, it is equal to $12t - 13T < 0$ when L takes its minimum value t. This inequality amounts to $t < 13T/12$, which means that the distaste cost must account for less than 8.33% of the uniform price for a conventional retail industry to shift to a mixed format.

4 Competition between e-retailers

In this section, we consider the case in which both firms are e-retailers. Let P_i be the delivered price chosen by firm i and γP_i the corresponding full price paid by consumers. Since γP_i is independent of x, there is always full market coverage. Hence, the profit functions are given by

$$\begin{aligned} \Pi_{1U}(P_1,P_2) &= \int_0^1 (P_1 - Tx)\frac{L-\gamma P_1+\gamma P_2}{2L}dx = \left(P_1 - \frac{T}{2}\right)\frac{L+\gamma P_2-\gamma P_1}{2L}, \\ \Pi_{2U}(P_1,P_2) &= \left(P_2 - \frac{T}{2}\right)\frac{L+\gamma P_1-\gamma P_2}{2L}, \end{aligned}$$

where each profit function is concave in its own price. Applying the first-order conditions yields the common candidate equilibrium prices:

$$P_1^* = P_2^* = \frac{L}{\gamma} + \frac{T}{2}. \tag{17}$$

Since Π_{iU} is concave in P_i, the following condition is necessary and sufficient for these prices to be an equilibrium: P_i^* is larger than T, which holds if and only if

$$\gamma T < 2L, \tag{18}$$

which holds under the condition (13). Under (18), there is full market coverage because the demand is evenly split between the two firms at each location x.[5]

Prices (17) imply that the pass-through of shipping cost is positive but smaller than 1. The equilibrium uniform price increases with shipping costs, but decreases with the distaste cost. This confirms the idea that shipping and distaste costs differ in nature.

Furthermore, like in Section 4, the e-retailer, here firm 2, must decrease its delivered price, $P_2^* < P^*$, when firm 1 shifts to the e-channel. The equilibrium profits are then given by

$$\Pi_{1U}^* = \Pi_{2U}^* = \frac{1}{2}\int_0^1 (P_1^* - Tx)dx = \frac{L}{2\gamma}, \tag{19}$$

which decrease with the distaste cost but increase with consumer heterogeneity. Thus, distaste costs play in the online duopoly the same role as shopping costs in the conventional duopoly. Note that profits are higher when both firms are offline retailers.

[5] Under (18), an e-retailer never finds it profitable to supply a submarket by setting a price lower than T (see Online Appendix A).

Proposition 4. *Assume two online firms. There exists a unique price equilibrium given by* (17) *if and only if* (18) *holds. Furthermore, when the conventional retailer shifts to the e-format, the pre-existing online store must lower its delivered price.*

This proposition shows that L must be large enough for a price equilibrium to exist in an online duopoly. This should not come as a surprise as uniform pricing has the nature of an aggressive pricing strategy that leads to the non-existence of an equilibrium in pure strategies when consumers are differentiated along the sole spatial dimension, that is, $L = 0$ (Kats and Thisse, 1993).

Can both firms choose to become e-retailers? Since the online retailer supplies the whole market, firm 1 chooses to become an e-retailer if

$$\Pi_{1m}^{*} = \frac{1}{72L}\left(6L + \gamma T - t\right)^2 < \Pi_{1U}^{*} = \frac{L}{2\gamma}, \tag{20}$$

where profits are defined as above for $L > t/2$. It is readily verified that (20) holds if and only if

$$t - \gamma T > 6L(1 - 1/\sqrt{\gamma}). \tag{21}$$

Given (18), we have $\gamma \in (1, 2L/T)$. Observe that (21) holds for $\gamma = 1$ because the left-hand side (resp., right-hand side) of (21) is equal to $t - T > 0$ (resp., 0). When γ increases, the condition (21) becomes more stringent because the left-hand side (resp., right-hand side) of (20) decreases (resp., increases) with γ. At $\gamma = 2L/T$, (21) does not hold. Therefore, there exists a unique value $\hat{\gamma} > 1$ such that (21) holds if and only if $\gamma < \hat{\gamma}$. Similarly, (21) is more likely to hold when the distribution cost T decreases.

The following proposition comprises a summary.

Proposition 5. *Assume that firm* 2 *is an e-retailer. If* (21) *holds, then there exists a unique value* $\hat{\gamma} > 1$ *such that firm* 1 *chooses to become an e-retailer if and only if* $\gamma < \hat{\gamma}$. *Otherwise, firm* 1 *remains a conventional retailer.*

We are now equipped to determine the equilibrium structure of the industry. First, Proposition 3 shows that firm 2 adopts an online channel if $\gamma < \bar{\gamma}$. In the resulting mixed duopoly, firm 1 chooses in turn to become an online retailer when $\gamma < \hat{\gamma}$. Thus, the equilibrium structure of the industry involves two online firms if $1 < \gamma < \min\{\bar{\gamma}, \hat{\gamma}\}$. When $\gamma > \min\{\bar{\gamma}, \hat{\gamma}\}$, at least one firm does not shift to the online channel.

Furthermore, comparing (5) and (19) shows that conventional stores always make higher profits ($L/2$) than as e-retailers ($L/2\gamma$).

Consequently, we have the following proposition.

Proposition 6. *Assume* (21)*. Firms* 1 *and* 2 *choose to be e-retailers if and only if the distaste cost is sufficiently low. Furthermore, firms get trapped in a prisoner's dilemma when they choose strategically their retail format.*

The second result is in line with Thisse and Vives (1988) and Bernstein *et al.* (2008). The former study spatial price discrimination and mill pricing when consumers are homogeneous, whereas the latter consider heterogeneous consumers but do not include transportation costs in their setting.

The consumer price in an offline duopoly is lower than the consumer price in an online duopoly when

$$x < \bar{x} = \frac{1}{t}\left(\frac{T}{2} - L\frac{\gamma - 1}{\gamma}\right) < \frac{1}{2}.$$

Note that $\bar{x} > 0$ when $\gamma < 2$, while $\bar{x} < 1$ because $t > T$. Therefore, consumers are divided between those who prefer mill pricing and those who prefer UD pricing.

Thus, *compared with mill pricing, UD pricing hurts firms while benefiting a relatively large share of consumers.*

5 Is e-commerce more competitive than conventional retailing?

We have seen that a retailer must decrease its mill or delivered price when its rival chooses to become an e-retailer. We now study the impact of e-retailing shift on consumer prices, that is, the full prices paid by consumers.

(i) We first compare prices in the offline and mixed duopolies when firm 1 is a conventional retailer. It follows from (8) that $p^* + tx < p_1^* + tx = L + tx$. Hence, the full prices paid to the offline retailer are lower in the mixed duopoly than in the conventional duopoly.

As for firm 2, we must compare $p_2^* + t(1-x)$ and γP^*. The marginal consumer is at the location where the two full prices are equal:

$$p_2^* + t(1-\bar{x}) = \gamma P^* \Leftrightarrow \bar{x} = \frac{5}{6} - \frac{\gamma T}{3t}.$$

Thus, we have $\gamma P^* < p_2^* + t(1-x)$ for $x < \bar{x} \in (0,1)$. Put differently, when firm 2 is an e-retailer, consumers located in $[0, \bar{x})$ pay a lower full price than when firm 2 is a conventional retailer, which impels firm 1 to reduce its own price from p_1^* to p^*. By contrast, consumers located in $(\bar{x}, 1]$ pay a higher full price when firm 2 is an e-retailer. This is because an e-retailer balances the profits made by selling to distant customers and those made from selling to the close customers when it chooses its delivered price. As $\bar{x}$ is larger than 1/2, more than half consumers are better-off when firm 2 is an e-retailer. Nevertheless, the remaining consumers are worse-off. Therefore, firm 2's decision to become an e-retailer has opposite impacts on consumers, so that *we cannot conclude that one format is globally more competitive than the other from the consumers' point of view.* Furthermore, it follows from (13) that the online firm cannot drive the offline firm out of business, even when $\gamma \approx 1$ or $T \approx 0$. In other words, product differentiation protects the conventional retailer.

(ii) We now turn our attention to the offline and online duopolies. Considering firm 1, the marginal consumer is located at $\bar{x}_1$, which solves

$$p_1^* + t\bar{x}_1 = \gamma P_1^* \Leftrightarrow \bar{x}_1 = \frac{\gamma T}{2t} < \frac{1}{2}.$$

Hence, consumers located to the left of $\bar{x}_1$ pay a lower full price when firm 1 is an offline retailer whereas those located to the right of $\bar{x}_1$ bear a lower price when firm 1 is an e-retailer.

As for firm 2, we have

$$p_2^* + t(1-\bar{x}_2) = \gamma P_2^* \Leftrightarrow \bar{x}_2 = \frac{2t - \gamma T}{2t} > \frac{1}{2},$$

where $\bar{x}_1 < \bar{x}_2$. Hence, consumers located to the left of $\bar{x}_2$ pay a higher full price when firm 2 is a conventional retailer whereas those located to the right of $\bar{x}_2$ bear a higher price when firm 2 is an e-retailer.

Therefore, three cases may arise. Consumers locate in $[0, \bar{x}_1)$ pay a lower full price when firm 1 is a conventional retailer, but a higher full price when this firm is an e-retailer. The same holds for consumers located in $(\bar{x}_2, 1]$ when they buy from firm 2. By contrast, consumers located in $[\bar{x}_1, \bar{x}_2]$ always bear a lower full price when both firms are e-retailers.

(iii) It remains to discuss the case of mixed and online duopolies when firm 2 is an e-retailer. Starting with firm 1, we find

$$p^* + t\hat{x} = \gamma P_1^* \Leftrightarrow \hat{x} = \frac{t + 2\gamma T}{6t} < \frac{1}{2},$$

which means that consumers located in $(\hat{x}, 1]$ prefer firm 1 to be an e-retailer. Regarding firm 2, we must compare γP^* and γP_2^*:

$$\gamma P^* - \gamma P_2^* = \frac{t - \gamma T}{6} > 0.$$

In other words, all consumers pay a lower price to firm 2 in an online duopoly. Therefore, consumers located to the right of $\hat{x}$ always pay lower prices in an online duopoly. However, consumer in $[0, \hat{x})$ are better-off when firm 1 is a conventional retailer. Once more, consumers are split between those who prefer a mixed duopoly and those who prefer an online duopoly.

Straightforward calculations show that the different marginal consumers may be ranked as follows:

$$0 < \bar{x}_1 < \hat{x} < 1/2 < \bar{x} < \bar{x}_2 < 1.$$

Using the above price comparisons for the different retail formats, we may conclude as follows:

$0 < x < \bar{x}_1$	Mixed $\succ$ Offline $\succ$ Online
$\bar{x}_1 < x < \hat{x}$	Mixed $\succ$ Online $\succ$ Offline
$\hat{x} < x < \bar{x}$	Online $\succ$ Mixed $\succ$ Offline
$\bar{x} < x < \bar{x}_2$	Online $\succ$ Offline $\succ$ Mixed
$\bar{x}_2 < x < 1$	Offline $\succ$ Online $\succ$ Mixed

By lowering the cost of distribution and by making search easier for consumers, online retailing is expected to intensify price competition. Our findings cast doubt on this claim and show that *the most competitive retail format depends on the location of consumers relative to firms.* There is always a market segment over which each retail format yields the lowest consumer prices, thus confirming the idea that spatial proximity still matters in consumers' choices. In other words, *no single retail format is universally preferred across the entire population.* This divergence is intuitive—consumers' preferred format largely depends on their geographic proximity to retailers, consistent with the empirical findings of Forman *et al.* (2009).

Furthermore, which retail format is the most competitive also depends on the parameters t and γ that characterize consumers, as well as the parameter T that describes the delivery strategy of e-retailers. In addition, the conventional duopoly is the most competitive format for a minority of consumers located in the area $(\bar{x}_2, 1]$, which implies that the presence of an online firm allows more than half consumers to benefit from lower prices.

We may thus conclude as follows.

Proposition 7. *The presence of an online retailer leads to lower consumer prices for a majority of consumers, but a positive share of consumers face higher prices.*

The above results are in line with the conventional wisdom that the entry of online firms fosters competition, although the empirical evidence is somewhat inconclusive (Yoo and Lee, 2011; Goldfarb and Tucker, 2019).

6 What is the most desirable retail format?

Are consumers better-off when firms shift to e-retailing, or do they prefer to keep brick-and-mortar stores? Using Theorems 3.3 and 3.4 of Anderson *et al.* (1992) implies that the linear probability model describes a population of heterogeneous consumers who can be represented by a single consumer whose indirect utility is given by

$$V(Y,\mathbf{p}(x)) = Y + H(\mathbf{p}(x)) \equiv \begin{cases} Y + \frac{-2L(p_1(x)+p_2(x))+(p_1(x)-p_2(x))^2}{4L} & \text{if } q_1(\mathbf{p}(x)) + q_2(\mathbf{p}(x)) = 1 \\ -\infty & \text{otherwise} \end{cases}, \quad (22)$$

where $p_i(x)$ is the consumer price at x.

Since individual preferences are quasi-linear, the expression

$$S = \int_0^1 V(Y,\mathbf{p}(x))dx$$

measures the consumer surplus.

6.1 Consumer surplus

Consider an offline duopoly. Plugging the equilibrium prices (4) in (22), we obtain the indirect utility of a consumer at x in a offline duopoly:

$$V_m(x) = Y - L - \frac{2Lt - t^2(1-2x)^2}{4L},$$

so that the consumer surplus is equal to

$$S_m = \int_0^1 V_m(x)dx = Y - L - \frac{t}{2} + \frac{t^2}{12L}. \quad (23)$$

Consider now the online duopoly. Since both firms charge the same delivered price, the indirect utility is given by

$$V_U = Y - \gamma P_i^* = Y - L - \frac{\gamma T}{2},$$

which is constant across locations. Since $V(Y, \gamma P_1^*, \gamma P_2^*)$ is independent of x, we have:

$$S_U = Y - L - \frac{\gamma T}{2}. \quad (24)$$

The utility differential is defined as follows:

$$f(x) \equiv V_m(x) - V_U = \frac{t^2(2x-1)^2 - 2L(t-\gamma T)}{4L},$$

which is convex, symmetric, minimized at $x = 1/2$ and maximized at $x = 0$ and $x = 1$. Determining the signs of $f(0)$ and $f(1/2)$ is thus sufficient to find the sign of $f(x)$.

Evaluating $f(x)$ at $x = 0$ and $x = 1/2$ yields the following expressions:

$$f(0) = \frac{t^2 - 2L(t-\gamma T)}{4L}, \qquad f(1/2) = -\frac{t-\gamma T}{2} < f(0).$$

Therefore, we have: (i) if $f(0) < 0$, then all consumers prefer an online duopoly and (ii) if $f(0) > 0$, then $f(x) = 0$ has one root between 0 and 1/2 and a second root between 1/2 and 1. In other words, if $t^2 - 2L(t - \gamma T) > 0$, consumers around the market center prefer an online duopoly because the consumer price is flat, whereas consumers close to the market endpoints prefer an offline duopoly because they are close to a conventional retailer.

Finally, it is readily verified that $S_m - S_U$ and $t^2 - 6L(t - \gamma T)$ have the same sign. Consequently, we have the following proposition.

Proposition 8. *Consumers' preferred retail format depends on their location relative to the retailers. Furthermore, the consumer surplus is higher in an online duopoly than in an offline duopoly if and only if* $L > t^2/(6(t-\gamma T))$.

6.2 Social welfare

Using (5) and (23), we obtain total welfare in the offline duopoly:

$$W_m = S_m + \Pi_{1m}^* + \Pi_{2m}^* = Y - \frac{t}{2} + \frac{t^2}{12L}.$$

Similarly, it follows from (19) and (24) that total welfare in the online duopoly is given by

$$W_U = S_U + \Pi_{1U}^* + \Pi_{2U}^* = Y - \frac{\gamma - 1}{\gamma}L - \frac{\gamma T}{2}.$$

Clearly, W_i decreases with L for $i = m, U$. Social welfare also decreases in both retailing formats when transportation costs, t and T, or the distaste cost increase. Hence, raising t or T or γ leads to a lower welfare level.

It is straightforward that

$$W_m - W_U = \frac{\gamma - 1}{\gamma}L + \frac{t^2 - 6(t - \gamma T)L}{12L}. \tag{25}$$

If $t^2 - 6(t - \gamma T)L > 0$, then $W_m > W_U$. Because a high distaste cost is damaging to e-retailers, $W_m - W_U$ increases with γ. Hence, the right-hand sign of (25) is minimized at $\gamma = 1$. In this case, if $t^2 - 6(t - \gamma T)L < 0$, we have $W_U > W_m$, which still holds for small values of γ. Otherwise, we have $W_U < W_m$.

Put differently, when consumers are not too heterogeneous while $t > 1.2\gamma T$, both the consumer surplus and the social welfare are higher in an offline duopoly. By contrast, if consumers are sufficiently heterogeneous ($L > \max\{t, t^2/6(t - \gamma T)\}$), the online duopoly generates a higher social welfare provided that the distaste cost is sufficiently small.

To sum up, we have the following proposition.

Proposition 9. *When consumers are not very heterogeneous, mill pricing benefits firms and consumers. When consumers are sufficiently heterogeneous and the distaste cost is low, UD pricing benefits consumers but harms firms in aggregate.*

We show in the Appendix what this proposition becomes under partial market coverage.

7 Pick-up or delivery?

Consider a mixed duopoly where firm 1 considers offering their customers two purchasing options instead of one: either consumers pick-up variety 1 at the physical store or have the same variety delivered to their homes; firm 2 is an online retailer whose UD price is P_{2O} (where O stands for "omni-channel"). When firm 1 becomes a multi-channel retailer, it has to choose a mill price p_{1O} and a delivered price P_{1O}.

Since firm 1 supplies the same variety through both channels, customers care only about the consumer price they pay. Consequently, they are split in two potential groups: those at x where $p_{1O} + tx < \gamma P_{1O}$ choose in-store pick-up, whereas those at x where $p_{1O} + tx > \gamma P_{1O}$ opt for home delivery. If consumers are split in two groups, then $y = (\gamma P_{1O} - p_{1O})/t \in (0, 1)$ is the boundary between firm 1's offline and online market areas. In particular, firm 1 may choose to remain a conventional retailer by setting a high value of P_{1O} for y to be equal to 1 or to become an e-retailer by setting a low value of P_{1O} for y to be equal to 0.

Using the same notation as in the previous sections, firms' profit functions are defined as follows:

$$\begin{aligned} \Pi_1(p_{1O}, P_{1O}, P_{2O}) &= \int_0^y p_{1O} q_1(p_{1O}, P_{2O}; x)dx + \int_y^1 (P_{1O} - Tx)Q_1(P_{1O}, P_{2O})dx, \\ \Pi_2(p_{1O}, P_{1O}, P_{2O}) &= \int_0^y (P_{2O} - T(1-x))Q_2(p_{1O}, P_{2O}; x)dx + \int_y^1 (P_{2O} - T(1-x))Q_2(P_{1O}, P_{2O})dx, \end{aligned} \tag{26}$$

where demand functions are as follows:

$$\begin{aligned} q_1(p_{1O}, P_{2O}; x) &= \frac{L - p_{1O} - tx + \gamma P_{2O}}{2L}, \qquad Q_1(P_{1O}, P_{2O}) = \frac{L - \gamma P_{1O} + \gamma P_{2O}}{2L}, \\ Q_2(p_{1O}, P_{2O}; x) &= \frac{L + p_{1O} + tx - \gamma P_{2O}}{2L}, \qquad Q_2(P_{1O}, P_{2O}) = \frac{L + \gamma P_{1O} - \gamma P_{2O}}{2L}. \end{aligned}$$

Consider a game in which firm 1 chooses a mill price p_{1O} and a delivered price P_{1O}, firm 2 selects a delivered P_{2O}, and consumers choose between pick-up and delivery, which yields the marginal consumer y indifferent between the two channels.

Conditional on $y \in [0, 1]$, equilibrium prices are given by

$$\begin{aligned} p^*_{1O}(y) &= L + \frac{1}{4}\gamma T - \frac{1}{4}ty + \frac{(t - \gamma T)y^2}{12}, \\ P^*_{1O}(y) &= \frac{L}{\gamma} + \frac{1}{2}T + \frac{1}{4}Ty + \frac{(t - \gamma T)y^2}{12\gamma}, \\ P^*_{2O}(y) &= \frac{L}{\gamma} + \frac{1}{2}T + \frac{(t - \gamma T)y^2}{6\gamma}. \end{aligned} \tag{27}$$

Prices p^*_{1O} and P^*_{2O} boil down to (8) when $y = 1$, that is, the equilibrium prices in the mixed duopoly. Likewise, P^*_{1O} and P^*_{2O} are equal to (17) when $y = 0$, that is, when both firms are e-retailers.

Given prices (27), the equilibrium value y^* must solve the indifference condition $p^*_{1O}(y) + ty = \gamma P^*_{1O}(y)$. Substituting $p^*_{1O}(y)$ and $P^*_{1O}(y)$ into this condition and solving for y lead to the unique solution:

$$y^* = \frac{\gamma T}{3t - \gamma T}, \tag{28}$$

which is positive and smaller than 1/2.[6] Indeed, consumers at $x = 1$ always prefer delivery to pick-up because $p^*_{1O}(1) + t > \gamma P^*_{1O}(1)$, which implies $y^* < 1$.

Substituting y^* into prices (27) yields

[6] Alternatively, firm 1 could choose the value of y that solves the unconstrained profit-maximization problem. Plugging prices (27) in Π_1 and differentiating with respect to y yields a 5-order equation in y, which we were unable to solve.

$$
\begin{aligned}
p_{1O}^* &= L + \frac{T\gamma\left(\gamma^2T^2 + 9t^2 - 7\gamma tT\right)}{6\left(3t - T\gamma\right)^2}, \\
P_{1O}^* &= \frac{L}{\gamma} + \frac{T\left(\gamma^2T^2 + 27t^2 - 13\gamma tT\right)}{6\left(3t - \gamma T\right)^2}, \\
P_{2O}^* &= \frac{L}{\gamma} + \frac{T\left(2\gamma^2T^2 + 27t^2 - 17\gamma tT\right)}{6\left(3t - \gamma T\right)^2},
\end{aligned} \tag{29}
$$

Note that firm 1's price in a mixed duopoly (see (8)) is lower than p_{1O}^* because firm 1 must attract distant consumers in a mixed duopoly whereas it supplies only nearby consumers in a multi-channel format. Similarly, the online retailer sets a delivered price P^* higher than P_{2O}^*. Note that $P_{1O}^* > P_{2O}^*$ because firm 1 competes for nearby consumers by using a mill price strategy, which allows this retailer to charge a higher delivered price to the remote consumers.

Plugging prices (29) into demand functions yields the following equilibrium quantities:

$$
\begin{aligned}
q_1^*(x) &= \frac{6\left(3t - T\gamma\right)^2\left(L - tx\right) + 2\gamma tT(9t - 5T\gamma) + \gamma^3T^3}{12L\left(3t - \gamma T\right)^2}, \qquad x \in [0, y^*) \\
Q_1^* &= \frac{6L\left(3t - \gamma T\right)^2 - \gamma^2T^2(4t - \gamma T)}{12L\left(3t - \gamma T\right)^2}, \qquad x \in (y^*, 1] \\
Q_2^*(x) &= \frac{6(3t - \gamma T)^2(L + tx) - 2\gamma tT(9t - 5\gamma T) - T^3\gamma^3}{12L\left(3t - \gamma T\right)^2}, \qquad x \in [0, y^*) \\
Q_2^* &= \frac{6L\left(3t - \gamma T\right)^2 + \gamma^2T^2(4t - \gamma T)}{12L\left(3t - \gamma T\right)^2}, \qquad x \in (y^*, 1].
\end{aligned}
$$

All these quantities are positive under (6). Furthermore, it also follows from (6) that $P_{1O}^* - T > 0$ and $P_{2O}^* - T > 0$. As a result, both $p_{1O}^* q_1^*(x)$ and $(P_{1O}^* - T)Q_1^*$ are positive for all x. Note that $Q_1^* < Q_2^*$ because $P_{1O}^* > P_{2O}^*$.

It remains to check that the full market coverage conditions are satisfied at prices (29) where

$$
x_{\max} \equiv \min\left\{\frac{-\gamma P_{1O}^* + \gamma P_{2O}^* + L}{t}, 1\right\}, \qquad x_{\min} \equiv \max\left\{0, \frac{-p_{1O}^* + \gamma P_{2O}^* - L}{t}\right\}.
$$

We show in Online Appendix B that, at prices (29), $x_{\min} = 0$ while $x_{\max} = 1$ if $L > 1.125t$, which is slightly more stringent than our condition $L > t$ in (6).

Plugging the equilibrium prices and quantities into the profit function (26) yields the equilibrium profit of firm 1 when it is a multi-channel retailer:

$$
\begin{aligned}
\Pi_{1O}(p_{1O}^*, P_{1O}^*, P_{2O}^*) \;=\; & \frac{\gamma T}{72\left(3t - \gamma T\right)^5 L}\left(\gamma^3T^3 + 54Lt^2 + 6L\gamma^2T^2 - 7t\gamma^2T^2 + 9t^2\gamma T - 36Lt\gamma T\right)^2 \\
& + \frac{3t - 2\gamma T}{72\gamma\left(3t - \gamma T\right)^5 L}\left(\gamma^3T^3 + 54Lt^2 + 6L\gamma^2T^2 - 4t\gamma^2T^2 - 36Lt\gamma T\right)^2.
\end{aligned}
$$

Thus, the profit difference between being a multi-channel retailer and being a conventional retailer in the mixed duopoly (see Π_m^* given by (16)) is given by

$$
\Delta\Pi_1 \equiv \Pi_1(p_{1O}^*, P_{1O}^*, P_{2O}^*) - \Pi_m^* = \frac{3t - 2\gamma T}{72\gamma L(3t - \gamma T)^5}(aL^2 + bL + c), \tag{30}
$$

whose sign is the same as $aL^2 + bL + c$ where

$$\begin{aligned}
a &\equiv -36(\gamma-1)(3t-\gamma T)^4 < 0, \\
b &\equiv 12\gamma(3t-\gamma T)^2[3t(t-\gamma T)(3t-\gamma T) + T^2\gamma(\gamma-1)(4t-\gamma T)] > 0, \\
c &\equiv -3\gamma t^3\left[27(t-3\gamma T)t^2 + 31(3t-2\gamma T)\gamma^2 T^2\right] - (73\gamma-16)t^2\gamma^4 T^4 + \gamma^5 T^5\left[2t(7\gamma-4) - (\gamma-1)\gamma T\right].
\end{aligned}$$

When $\gamma = 1$, $a = 0$ and $\Delta\Pi_1$ is a linear function of L. In this case, $\Delta\Pi_1 > 0$ when $L > \max\{-c/b, 0\}$. By continuity, $\Delta\Pi_1 > 0$ still holds when $\gamma > 1$ is sufficiently small.

Summarizing leads to the following proposition.

Proposition 10. *Assume that the distaste cost is small. Then, a conventional retailer competing with an e-retailer adopts a multi-channel format when the degree of heterogeneity is sufficiently large.*

While firm 1 may be better-off when it becomes multi-channel, what happens to firm 2? Using firm 2's equilibrium profit in the mixed duopoly of Section 3.1 (see Π_U^* given by (16)), some tedious calculations show that

$$\Pi_U^* - \Pi_2^*(p_{1O}^*, P_{1O}^*, P_{2O}^*) = \frac{3t-2\gamma T}{(3t-\gamma T)^2} F > 0,$$

where

$$F \equiv \frac{t(t-\gamma T)}{2\gamma} + \frac{t\gamma^3 T^4}{3L(3t-\gamma T)^2} + \frac{t(t-\gamma T)(9t^3 + 12t^2\gamma T + 17t\gamma^2 T^2 + 4\gamma^3 T^3)}{24\gamma L(3t-\gamma T)^2} > 0.$$

In other words, *the e-retailer* 2 *is better-off when firm* 1 *remains a brick-and-mortar store.*

Does choosing a multi-channel format generate higher or lower prices? To answer this question, we compare prices (29) to those charged by the duopolists when firm 1 remains offline. Using (8) and (29), we find that

$$\begin{aligned}
p^* - p_{1O}^* &< 0, \qquad P_{1U}^* - P_{1O}^* < 0, \\
P^* - P_{2O}^* &> 0, \qquad P_{2U}^* - P_{2O}^* < 0.
\end{aligned}$$

In other words, *a multichannel firm charges a higher mill price than a conventional retailer and a higher delivered price than an e-retailer*, which concurs with the result obtained by Furlong and Slotsve (1983) in the case of a monopoly firm. In other words, firm 1 adopts a less aggressive pricing strategy when it chooses to be a multichannel retailer rather than a single channel retailer. The intuition is easy to grasp: the multi-channel firm chooses its mill price to attract only nearby consumers whereas it sets a higher delivered price charge only to remote consumers. Regarding firm 2, it charges a lower delivered price when firm 1 is multi-channel, but a higher price than in an online duopoly in which competition is fierce over a wider range of locations.

8 Concluding remarks

We develop a spatial discrete-choice model to examine competition between offline and online firms. The analysis demonstrates that the relative performance of these retail formats is critically contingent upon the spatial distribution of consumers and retailers. At first glance, this finding may appear counterintuitive, as e-commerce is often portrayed as a retail format in which spatial frictions are negligible. We argue, however, that this interpretation is overly reductive for at least two reasons.

First, the geographical location of brick-and-mortar retailers continues to exert a substantial influence on consumers' shopping behavior. Second, the spatial distribution of consumers necessitates the design of transportation and logistics strategies whose efficiency directly depends on the underlying geography of demand. Together, these factors underscore the enduring relevance of spatial considerations in shaping competitive dynamics within the digital economy.

Our framework builds on standard assumptions in industrial organization and marketing. Accordingly, it suffices to demonstrate that general efficiency results are unlikely to extend to more complex or heterogeneous market settings. We focus on a deliberately simple pricing policy for e-retailers; however, alternative mechanisms—such as partitioned pricing or subscription-based pricing—are equally significant and can be readily incorporated into the model.

Specifically, the unit interval may be divided into a finite number of zones, within which the e-retailer specifies a base price π_i and a shipping fee ϕ_i for each zone i (Burman and Biswas, 2007). Since $\pi_i + \varphi_i = \gamma P_i$, this formulation is equivalent to selecting a UD price P_i for each zone. As the number of zones increases, this policy converges to spatial discriminatory pricing (Lu and Matsushima, 2024; Rhodes and Zhou, 2024). Similarly, an e-retailer could implement a multizone pricing strategy in which prices vary with zonal shipping and handling costs (Adams and Williams, 2019). Moreover, an e-retailer offering multiple goods may adopt a subscription model by charging a fixed membership fee while providing free shipping (Balakrishnan *et al.*, 2024).

For simplicity, we have assumed that the two retailers face the same marginal cost. Here too, the model can be extended to deal with heterogeneous firms operating under different costs that may vary with their format. For example, if firm $i = 1, 2$ faces a marginal cost $c_i > 0$ in the setting considered in Section 3, the equilibrium prices become $p_1^* = t + 2c_1/3 + c_2/3$ and $p_2^* = t + c_1/3 + 2c_2/3$, which remain simple to handle.

This paper highlights the central role of consumer heterogeneity in determining market outcomes. While we have relied on a linear probability model—consistent with linear demand in a Hotelling-type duopoly—future research could explore more general frameworks. In particular, multinomial logit models represent a natural alternative, though their analytical intractability poses challenges. Recent advances in discrete choice modeling (Chambers *et al.*, 2025) may help address these difficulties and open promising avenues for further work. Moreover, our analysis has taken firms' locations as exogenous; understanding how firms operating under different formats endogenously choose their locations remains an important topic for future research.

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Appendix

Our purpose is to determine the equilibria of offline and online duopolies under partial market coverage and compare their social desirability. We also compare the equilibrium outcomes under full and partial market coverage.

1. Offline duopoly. Market demands are given by

$$q_1(p_1,p_2) \equiv \int_0^{x_{\min}} dx + \int_{x_{\min}}^{x_{\max}} \frac{-p_1 - tx + p_2 + t(1-x) + L}{2L} dx = \frac{t - p_1 + p_2}{2t}, \frac{L - p_1 + p_2}{2L} \tag{A.1}$$

$$q_2(p_1,p_2) \equiv \int_{x_{\min}}^{x_{\max}} \frac{L + p_1 + tx - p_2 - t(1-x)}{2L} dx + \int_{x_{\max}}^{1} dx = \frac{t + p_1 - p_2}{2t}. \tag{A.2}$$

Firms' profits are given by $\Pi_{1m} = p_1 q_1(p_1,p_2)$ and $\Pi_{2m} = p_2 q_2(p_1,p_2)$, which are both concave in their respective price. Applying the first-order conditions yields the equilibrium producer prices:

$$p_{1m}^* = p_{2m}^* = t > L. \tag{A.3}$$

It is straightforward to check that $x_{\min} = (t-L)/2t > 0$ and $x_{\max} = (t+L)/2t < 1$. Hence, consumers located in $[0,(t-L)/2t)$ patronize firm 1 only while those located in $((t+L)/2t, 1]$ buy only from firm 2. Put differently, firm 1 (resp., firm 2) is the sole firm that supplies the market segment $[0,(t-L)/2t)$ (resp., $((t+L)/2t,1])$. The two firms compete over the area of contention given by $[(t-L)/2t,(t+L)/2t]$. This area expands as the degree of consumer heterogeneity increases ($L\uparrow$), whereas it shrinks as the shopping cost rises ($t\uparrow$). Furthermore, over the interval $[(t-L)/2t,(t+L)/2t]$ the probability of purchasing from firm 1 decreases as the distance x increases. In the limit, when $L\to 0$, $x_{\min}=x_{\max}=1/2$ and each consumer buys from a single firm because the two goods are homogeneous.

When $L=t$, the equilibrium prices (4) and (A.3) are equal, which implies that p_i^* is a continuous function of L. However, p_i^* has an inward kink at $L=t$.

Using the market demands (A.1) and (A.2), the equilibrium profits are equal to $\Pi_{1m}^*=\Pi_{2m}^*=t/2$.

2. Online duopoly. Firms' profits are

$$\begin{aligned}\Pi_{1U}(P_1,P_2) &= \int_0^{1-P_2/T}(P_1-Tx)dx+\int_{1-P_2/T}^{P_1/T}(P_1-Tx)\frac{L-\gamma P_1+\gamma P_2}{2L}dx,\\ \Pi_{2U}(P_1,P_2) &= \int_{1-P_2/T}^{P_1/T}(P_2-T(1-x))\frac{L-\gamma P_2+\gamma P_1}{2L}dx+\int_{P_1/T}^{1}(P_2-T(1-x))dx.\end{aligned}$$

Applying the FOC yields the candidate equilibrium prices:

$$P_1^*=P_2^*=\frac{T}{2}+\sqrt{\frac{TL}{2\gamma}}. \tag{A.4}$$

Checking the condition $P_1^*/T<1$, we obtain:

$$\gamma T>2L,$$

which is the counterpart of (18). In other words, there is partial market coverage because the shipping cost is too high for the consumers located near the endpoints of the linear market to buy from the more distant firm.

When $\gamma T=2L$, the equilibrium prices (17) and (A.4) are equal to T. Hence, the equilibrium price $p_i^*(L)$ is continuous in L and given by the upper envelop of the equilibrium outcomes obtained under full and partial market coverage.

3. Comparing mill and UD pricing. Plugging the equilibrium prices (A.3) in (22), we obtain the indirect utility of a consumer located at x:

$$V_m(x)=\begin{cases} Y-t-tx, & x\in[0,(t-L)/2t)\\ Y-\frac{3}{2}t+(1-2x)^2\frac{t^2}{4L}, & x\in[(t-L)/2t,(t+L)/2t]\\ Y-t-t(1-x), & x\in((t+L)/2t,1].\end{cases}$$

Hence, the consumer surplus is given by

$$S_m=\int_0^{\frac{t-L}{2t}}(Y-t-tx)dx+\int_{\frac{t-L}{2t}}^{\frac{t+L}{2t}}\left(Y-\frac{3}{2}t+\frac{(t-2tx)^2}{4L}\right)dx+\int_{\frac{t+L}{2t}}^{1}(Y-t-t(1-x))dx=Y-\frac{1}{12}\frac{2L^2+15t^2}{t}, \tag{A.5}$$

which is equal to (23) when $t=L$. However, the function S is not differentiable at $L=t$ because p_i^* has a kink at this point, which implies that the left-hand and right-hand side slopes of S_m are different.

Plugging the equilibrium consumer prices (A.4) in (22), we obtain the consumer surplus in an online duopoly:

$$S_U = \int_0^{1-P_2^*/T} (Y - \gamma P_1^*)dx + \int_{1-P_2^*/T}^{P_1^*/T} V(P_1^*, P_2^*)dx + \int_{P_1^*/T}^{1} (Y - \gamma P_2^*)dx = Y - \frac{1}{2}\gamma T - \sqrt{\frac{\gamma T}{2}} L.$$

Using (A.5), we obtain

$$S_m - S_U = -\frac{1}{12}\frac{2L^2 + 15t^2}{t} + \frac{1}{2}\gamma T + \sqrt{\frac{\gamma T}{2}} L,$$

which is a decreasing function of t. Since $t > \gamma T$, we have

$$S_m - S_U < -\frac{1}{12}\frac{2L^2 + 15(\gamma T)^2}{\gamma T} + \frac{1}{2}\gamma T + \sqrt{\frac{\gamma T}{2}} L.$$

Since the right-hand side of this inequality is a decreasing function of γT and $\gamma T > 2L$, we have

$$S_m - S_U < -\frac{1}{12}\frac{2L^2 + 15(2L)^2}{2L} + 2L = -\frac{7}{12}L < 0.$$

In other words, the consumer surplus in an online duopoly is always higher than in an offline duopoly.

Using (A.2) yields the equilibrium profits:

$$\Pi_{1U}^* = \Pi_{2U}^* = \frac{\gamma T - 2L + \sqrt{8\gamma T L}}{8\gamma}.$$

Since Π_U^* is a decreasing function of γ, evaluating Π_{1U}^* at the lower bound $\gamma = 2L/T$ yields $T/4$. As $T/4$ is smaller than $t/2$, firms earn higher profits in an offline duopoly than in an online duopoly.

To sum up, we have the following proposition .

Proposition A. *Under partial market coverage, UD pricing harms firms and benefits consumers in aggregate.*

This result contrasts with that obtained under full market coverage when consumers are weakly heterogeneous (see Proposition 9). The difference should not be surprising, as firms compete only along the intensive margin under full coverage, whereas they compete along both the extensive and intensive margins under partial coverage. Proposition A is consistent with Proposition 1 in Rhodes and Zhou (2024).

Online Appendix A

1.Offline duopoly. (i) Assume that $t > L$. If firm 1 decreases its price below t for $x_{\max} = 1$ to hold when $p_2^* = t$, firm 1's demand is equal to

$$q_1(p_1,t) = \int_0^{\frac{-p_1+2t-L}{2t}} dx + \int_{\frac{-p_1+2t-L}{2t}}^{1} \frac{-p_1 - tx + t + t(1-x) + L}{2L} dx = \frac{-2Lp_1 - p_1^2 + 8Lt - L^2}{8tL}.$$

Differentiating $p_1 q_1(p_1,t)$ w.r.t. p_1 and solving the FOC for p_1 yields $\bar{p}_1 = -(\sqrt{L^2+24tL} + 2L)/3 < 0$ and $\hat{p}_1 = (\sqrt{L^2+24tL} - 2L)/3 < t$. Plugging $\hat{p}_1$ in $x_{\max} = (-p_1 + 2t + L)/2t$ yields

$$x_{\max} = 1 - \frac{\sqrt{24Lt + L^2} - 5L}{6t} < 1,$$

which contradicts the definition of $q_1(p_1,t)$. The same argument applies to firm 2.

(ii) Assume that $L \geq t$. If firm 1 increases its price above L for $x_{\max} < 1$ to hold when $p_2 = L$, firm 1's demand is equal to

$$q_1(p_1,L) = \int_0^{\frac{2L+t-p_1}{2t}} \frac{-p_1 - tx + L + t(1-x) + L}{2L} dx = \frac{(2L+t-p_1)^2}{8tL}. \quad \text{(A.1)}$$

Differentiating $p_1 q_1(p_1,L)$ w.r.t. p_1 and solving the FOC for p_1 yields the following solutions: $\bar{p}_1 = (2L+t)/3 < L$ and $\hat{p}_1 = 2L + t > L$. Clearly, we have $q_1(\hat{p}_1,L) = 0$, and thus $\Pi_{1m}(\hat{p}_1,L) = 0 < \Pi_{1m}(L,L)$.

It remains to check that $x_{\min} = 0$ for (A.1) to be firm 1's demand at $\hat{p}_1$. Plugging $\hat{p}_1$ in $x_{\min} = (-p_1+t)/2t$ yields $-L/t < 0$. Therefore, $x_{\min} = \max\{-L/t, 0\} = 0$.

By symmetry, the same arguments apply to firm 2.

2.Mixed duopoly. Assume that firm 1 raises its price for $x_{\max} < 1$ to hold when firm 2 charges the UD price P^*. Then, we have:

$$q_1(p,P^*) = \int_0^{\frac{-p+\gamma P^*+L}{t}} \frac{L - p - tx + \gamma P^*}{2L} dx = \frac{(12L - 6p + t + 2\gamma T)^2}{4tL}.$$

Differentiating $pq_1(p,P^*)$ w.r.t. p and solving the FOC for p yields the following solutions: $\bar{p} = (12L+t+2\gamma T)/18$ and $\hat{p} = (12L + t + 2\gamma T)/6$. First, we have $\Pi_{1m}(\hat{p},P^*) = 0$ because of $x_{\max} = 0$ for $\hat{p}$. Second, plugging $\bar{p}$ in $x_{\max} = (-p + \gamma P^* + L)/t$ and using (13) yields $x_{\max} = 1$, which contradicts $q_1(p,P^*)$.

3. Online duopoly. Assume that firm 1 decreases its price P_1 below T. In this case, the e-retailer does supplies the market until $x = P_1/T$, so that its profits are given by

$$\Pi_{1U}(P_1,P_2^*) = \int_0^{P_1/T} (P_1 - Tx)\frac{L - \gamma P_1 + \gamma P_2^*}{2L} dx = P_1^2 \frac{L - \gamma P_1 + L + \gamma T/2}{4LT}.$$

Differentiating $\Pi_{1U}(P_1,P_2^*)$ w.r.t. P_1 yields $\bar{P}_1 = 0$ and $\hat{P}_1 = (4L + T\gamma)/3\gamma > T$, which contradicts $\Pi_{1U}(P_1,P_2^*)$.

Online Appendix B

Plugging (29) in

$$x_{\max} \equiv \min\left\{\frac{-\gamma P_{1O}^* + \gamma P_{2O}^* + L}{t}, 1\right\}, \qquad x_{\min} \equiv \max\left\{0, \frac{-p_{1O}^* + \gamma P_{2O}^* - L}{t}\right\}.$$

we find that $x_{\max} = 1$ if and only if

$$6L\,(3t - T\gamma)^2 - (54t^3 - 36Tt^2\gamma + 10T^2t\gamma^2 - T^3\gamma^3) \geq 0, \tag{B.1}$$

while $x_{\min} = 0$ if and only if

$$6L\,(3t - T\gamma)^2 - T\gamma\left(18t^2 - 10\gamma Tt + \gamma^2T^2\right) \geq 0. \tag{B.2}$$

Set $X \equiv \gamma T \in [0, t]$. Clearly, (B.2) holds if

$$\alpha(X) \equiv 6t\,(3t - X)^2 - X\left(18t^2 - 10Xt + X^2\right) \geq 0.$$

Since the equation $\alpha(X) = 0$ has a unique real solution larger than t, while $\alpha(0) > 0$ and $\alpha(t) > 0$, (B.2) always holds.

We now consider (B.1). Setting

$$\beta(X) \equiv 6L\,(3t - X)^2 - (54t^3 - 36Xt^2 + 10X^2t - X^3),$$

it is straightforward that $\beta(X) < 0$ when $L = t$. Since $\beta(X)$ is an increasing linear function of L, (B.1) is satisfied if and only if

$$L > L_0(X) \equiv \frac{54t^3 - 36Xt^2 + 10X^2t - X^3}{6\,(3t - X)^2}. \tag{B.3}$$

Since $L_0(0) = t$,

$$\frac{dL_0}{dX} = \frac{X}{6} \cdot \frac{-9Xt + X^2 + 24t^2}{(3t - X)^3} > 0,$$

and $L_0(t) = 1.125t$, the function $L_0(X)$ is increasing over $[0, t]$. Therefore, (B.3) always holds if and only if $L > 1.125t$.