Optimal Monetary and Prudential Policies in the Presence of Interest-Rate and Run Risk

Ilhyock Shim, Frank Smets, and Tack Yun

Bank of International Settlements and Seoul National University

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Abstract

We study the optimal monetary and prudential policies in a modified Diamond-Dybvig model with interest-rate and run risk to address the following four issues. The first issue is which regime between price-level targeting and inflation targeting regimes is more desirable one in the presence of bank's exposure to interest-rate risk. The second issue is how bank's exposure to interest-rate risk affects the interest-rate channel as a transmission channel of monetary policy. The third issue is how to coordinate monetary and prudential policies optimally in response to interest and run risk when the financial regulator is supposed to adopt a set of balance-sheet regulation such as Value-at-Risk management, capital ratio requirement and Volcker rule. The fourth issue is whether and how the public market information about short-term and long-term bonds affects depositor's run decision in the context of the Morris-Shin model.

JEL classification: E31, E43, E44, G21, G28

Keywords: Monetary Policy; Inflation Targeting; Prudential Policy; Long-Term Bonds; Interest-Rate Risk; Bank Run Risk

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1 Introduction

Implementing a desirable balance between monetary policy and financial stability is a long-standing issue in policy circles and academia. A well-known example is the criticism about the desirability of inflation targeting as a monetary policy framework because its focus on price stability force central banks to neglect implications of their policy actions for financial stability, which was culminated in the wake of the global financial crisis. A complex nature of this issue is that the first-line of defence to secure the stable functioning of financial market is the prudential balance-sheet management of financial intermediaries. Combining these perspectives leads to a warning principle that it is crucial to prevent a joint occurrence of central bank's negligence of monetary policy's implications for financial stability and loose balance-sheet management of financial intermediaries.

In fact, the predictability of such a warning principle is likely to be high in the actual world as can be seen from recent US bank failures. For example, Silicon Valley Bank and Signature Bank were bankrupt with unrealized capital losses in their asset-holdings during the monetary tightening period of 2022-2023 reflecting that market values of long-term bonds fell as the yield curve inverted with rapid rises in short-term interest rates. A distinguishing feature of the recent US bank failures is bank's inefficient exposure to interest-rate risk. But there has been little discussion of how central bank and financial regulator carry out their policy tasks optimally in the presence of bank's inefficient exposure to interest-rate risk.

We incorporate Diamond-Dybvig's financial frictions into an otherwise prototypical New Kenyesian model and study the optimal monetary and prudential policies in such a modified Diamond-Dybvig model with interest-rate and run risk to address the following four issues. The first issue is which regime between price-level targeting and inflation targeting regimes is more desirable one in the presence of bank's exposure to interest-rate risk. The second issue is how bank's exposure to interest-rate risk affects the interest-rate channel as a transmission channel of monetary policy. The third issue is how to coordinate monetary and prudential policies optimally in response to interest and run risk when the financial regulator is supposed to adopt a set of balance-sheet regulation such as Value-at-Risk management, capital ratio requirement and Volcker rule. The fourth issue is whether and how the public market information about short-term and long-term bonds affects depositor's run decision in the context of the Morris-Shin model.

It is well-known in the literature on the optimal monetary policy that when the social planner is supposed to maximize the social welfare subject to only the trade-off between inflation and output gap implied by a prototypical New Keynesian Phillips curve, the optimal monetary policy regime is the price-level targeting together with the optimality of a zero inflation as can be confirmed in Clarida, Gali, and Gertler (1999) and Woodford (2003). But this result does not necessarily hold when the social planner is supposed to maximize the social welfare subject to both the trade-off between inflation and output gap and the trade-off between current-period and next-period consumptions due to the financial frictions of Diamond-Dybvig model. The reason for this argument is that the trade-off between current-period and next-period consumptions is determined by the nominal interest rate and the nominal interest rate can be expressed as the sum of real interest rate and expected inflation. It means that the social planner can adjust the expected inflation of households and thus the nominal interest rate in the presence of a commitment mechanism about future inflation.

If the social planner chooses to use such a commitment mechanism about future inflation, the price-level targeting should be no longer the optimal monetary policy regime together with the sub-optimality of a zero inflation. In particular, since the introduction of the Diamond-Dybvig's financial frictions into an otherwise prototypical New Kenyesian model lowers the maximum attainable level of the social welfare, the social planner has incentive to increase the current output by its commitment for a positive future inflation.

Turning to the second issue, two additional features augment prototypical New Keynesian models that comprise of three equations: IS curve, Phillips curve and central bank's interest-rate rule. The first additional feature is the distinction between the central bank's policy rate and the short-term nominal interest rate. The addition of this feature to the model reflects the distinction between the cost of bank's reserves to meet early withdrawals and the opportunity cost associated with the trade-off between early and late withdrawals.

The second additional feature is the wealth effect of monetary policy on household's labor supply and consumption, which arises because commercial banks hold long-term bonds. The IS and Phillips curves therefore contain new additional variables that reflect the wealth effect of changes in commercial bank's portfolios, while the wealth effect acts as an endogenous supply shock in the Phillips curve. The wealth effect also can be regarded as a channel through which a severe disruption of financial market can affect inflation and output gap. As a result, the equilibrium determination of inflation and output gap is affected by monetary and prudential polices, which in turn motivates the analysis of optimal monetary and prudential policies in this paper.

The focus of the third issue is how to implement the optimal allocation at decentralized markets while the social planner's optimal solution contains the prescription for commercial bank's holdings of long-term bonds. The resolution of this issue is associated with how to model commercial banks. In this regard, we model commercial banks as arbitrageurs subject to Value-at-Risk(hereafter VaR) constraint can be compared with that of Adrian and Shin (2014) to model banks as active investors subject to VaR constraint for an appropriate level of capital cushion.

In the presence of the VaR constraint, commercial banks face a trade-off between meeting the

VaR constraint and buying more long-term bonds to exploit a higher net interest margin during a high interest period. In this regard, commercial banks can be regarded as both arbitrageurs and preferred-habitat investors in long-term bonds markets. Hence balance-sheets of commercial banks are affected by monetary and prudential polices, which in turn leads to the issue of how to coordinate monetary and prudential policies optimally in response to interest and run risk.

The final issue is the identification of conditions under which a lot of depositors choose to withdraw at the same time and thus bank runs occur. In this part, it would be worthwhile to mention the recent empirical result of Correia, Luck, and Verner (2025) in favor of the solvency view of bank failures that banks are more likely to fail when poor fundamentals such as realized credit risk, interest-rate risk or fraud trigger insolvency. The solvency view is in contrast with the bank run view that depositors collectively withdraw from otherwise either solvent banks or troubled but solvent banks, while the Diamond-Dybvig model has been used to explain the bank run view. The solvency view is reflected in the model of Gertler and Kiyotaki (2015) where the occurrence of bank runs depends solely on the insolvency of banks.

The model of this paper is in line with the solvency view in the sense that depositors determine whether to choose bank run depending on their private noisy signals about bank's net-worth. In addition, depositors are assumed to have public information about market and liquidation prices of long-term bonds on the ground that the information about the impact of the central bank's policy rate on market prices of long-term bonds is publicly available. Given such an information structure, depositors choose bank runs only when the message of the public information is mutually consistent with their private signals.

The rest of this paper is organized as follows. The next section reviews the related literature. Section 3 presents the benchmark model without bank run where banks are subject to interest-rate risk. Section 4 studies the optimal monetary and prudential policies for the benchmark model of section 3. Section 5 extends the benchmark model to allow for bank run and studies the optimal monetary and prudential policies with expected bank run. Section 6 concludes.

2 Related Literature

The first issue in the literature review is the discussion of whether the paper's focus on interest-rate risk and market-risk of commercial banks is an important one. The reason behind this statement is the traditional view of banks that depicts banks as deposit-taking institutions to perform the predominant role of lending their funds to households and firms. In this view, interest-rate risk or market risk can be regarded as a secondary risk, not a primary risk. But while credit risk can be the primary risk for commercial banks, it should also be mentioned that the business model of banks has changed substantially. For example, while loan provision is still the most important role of commercial banks, banks themselves sell over 50% of their loans instead of holding them on their balance sheet by using a variety of securitization techniques as discussed in Buchak, Matvos, Piskorski, and Seru (2024). While loan sale reflects the advance of financial engineering, another one comes with the digitization of bank services associated with deposits and loans. The impact of digitization on banks can be reflected in the following three aspects: lower access costs of bank services, facilitation of relocation between usual demand deposit and interest-bearing funds such as MMF, and reduction of marginal production cost of deposit services. Koont, Santos, and Zingales (2025) show that digitization increases the outflows or walk of deposits when the federal funds rate increases and also raises the sensitivity of bank deposit rates to changes in the federal funds rate.

The recent episodes of U.S. bank runs also point to the importance of interest-rate risk as follows. Rajan and Acharya (2023) identify four causes of 2023 bankruptcies of Silicon Valley Bank and Signature Bank as can be seen below. First, over 90 percent of their deposit liabilities were uninsured. Second, the two banks held significant amounts of long-term bonds. Third, the Fed's rapid swing from a ZLB with QE to a high interest-rate stance decreased market prices of long-term bonds. Four, many supervisors were not aware of the rising interest-rate exposure of banks or unable to force banks to reduce it. Seru (2023) explains bankruptcies of Silicon Valley Bank and Signature Bank by using a model of solvency runs where interest-rate increases initially lead to runs on banks whose assets are fully liquid and then become widespread self-fulfilling solvency runs as uninsured depositors pull their funds in the belief that others will do the same, generating strong incentives for runs among depositors. He also argues that banks are fragile when they have high ratios of uninsured deposits to assets and market values of their assets are far lower than corresponding book values.

The second issue in this literature review is the nature of bank run. Table 2.1 contains a set of selected papers that can be compared with the model of this paper as can be seen below. First, bank run can occur as a result of self-fulfilling expectations about non-fundamentals as in the original Diamond-Dybvig model. Second, bank run can occur if and only if fundamentals of the economy are below some threshold level as can be seen in Goldstein and Pauzner (2005, GP), Gertler and Kiyotaki (2015, GK), and Kashyap, Tsomocos, and Vardoulakis (2024, KTV). In the model of Gertler and Kiyotaki (2015), a bank run equilibrium exists if the realized rate of return on bank assets is sufficiently low relative to the gross interest rate on deposits and the leverage multiple is sufficiently high. In the model of Kashyap, Tsomocos, and Vardoulakis (2024), a bank run occurs when patient savers receive the private signals about liquidation value of bank assets whose values are lower than a unique threshold value. In the model of Goldstein and Pauzner (2005), each agent receives a private signal regarding fundamentals of the economy. This information is regarded as

Table 2.1: Information and Bank Run

	GP (2005)	GK (2015)	KTV (2024)	SSY (2025)
Signal Variable	Long-Term Return of Bank Assets (One Variable)	Bank Net Worth (One Variable)	Liquidation Value of Bank Assets (One Variable)	Bank Net Worth and Bond Price (Two Variables)
Model Set-Up	Finite Period and Risky Long-Term Public Investment	Infinite Period and Asymmetric Information between Band E	Finite Period and Asymmetric Information between Band E	Infinite Period and Interest Rate Risk of Long-Term Bonds
Run Strategy	Threshold Value of Economic State	Recovery Rate Less than One	Threshold Value of Bank Assets at Liquidation	Threshold Values of Net Worth and Bond Price
Unanticipated Bank Run	Unanticipated Adverse Economic State	Adverse Shock to Productivity of Banks	Adverse Shock to Liquidation Value	Large Rapid Hikes of Policy Rate

Note: The recovery rate in the GK is defined as the ratio of bank's asset value to its deposit repayment evaluated at the state of liquidation. B means bank and E means entrepreneur.

the agent's private information about long-term return on the investment project. In this model, a unique equilibrium exists where patient agents run if their private signals are below a unique threshold value and do not run otherwise.

It should be noted that a bank's long-term investment return is a random variable in GP model and that the short-term liquidation value of a bank asset is a random variable in KTV model. The common feature of these two models is that agents receive private signals about fundamentals and thus their behaviors depend on the values of their private signals. Specifically, there exists a threshold value of private signals that splits out between "run on bank deposits" and "no run on bank deposits".

In this paper, depositors have noisy signals about the future return on a bank's asset (hence bank's future net-worth) and update their forecast about it. The difference of this paper from the two papers (GP and KTV) is that agents have public signal about bank's future net-worth. Given this information, they compute the threshold value of the bank's future net-worth for bank run. The threshold value of net worth is used for the run strategy of depositors, which is different from the GK model whose threshold variable is the bank's recovery rate as specified in the table below

The third issue in this literature review is the market structure of long-term bond markets. In the model of this paper, commercial banks issue demand deposits to invest in long-term government bonds. In this regard, commercial banks can be regarded as arbitrageurs in long-term bond markets, while they are supposed to hold long-term bonds with specific maturity segments in their portfolios. But their demands for zero-coupon long-term bonds are restricted by their market-risk management on the basis of Value-at-Risk techniques. Banks holding a large amount of long-term bonds will

Table 2.2: Bond Market Specification

DS (2002)	VV (2021)	SSY (2025)
No Preferred-Habit Investors (affine model)	Preferred-Habitat Investors and Arbitrageurs (affine model)	Bank as both Arbitrageur and Preferred-Habitat Investor (non-linear model)
No Arbitrage Profit with Stochastic Discount Factor and Market Price of Risk	No Arbitrage Profit with Market-Clearing Condition	No Arbitrage Profit with Market-Clearing Condition
		Banks Issue Short-Term Debt and Hold Long-Term Bonds with VaR Constraint for Interest-Rate Risk

need to reduce their holdings of such bonds when they face a sharp increase in long-term bond yields to avoid hitting the VaR constraint. That is, there is a trade-off between meeting the VaR constraint and pursuing a higher net interest margin from buying long-term bond yields during a high interest period, which may explain why banks do not increase their holdings of long-term bonds quickly in a high interest rate environment.[1] Hence, commercial banks can be regarded as preferred-habitat investors in long-term bond markets.

The first question is whether commercial banks are important participants in long-term bond markets. Unless their trades have significant impacts on market prices of long-term bonds, their trades can be ignored in the determination of equilibrium prices of long-term bonds by omitting them in the market clearing conditions of long-term bonds. In addition, usual examples of preferred-habitat investors in long-term bonds are pension funds and insurance companies as discussed in Vayanos and Vila (2021). Pension funds prefer long-term bonds to match durations of their assets and liabilities, while insurance companies demand intermediate and long-term bonds to match durations of their liabilities and insurance products. In addition, Domanski, Shin, and Sushko (2017) point out that Solvency II classifies European government bonds in domestic currency as risk-free, creating an incentive for insurance firms to overweigh them in their portfolios. But one might wonder if there are such simple scenarios for commercial banks to hold long-term bonds. In particular, this skepticism might reflect the fact that it is difficult to reconcile the main business model of commercial banks with their incentives to hold long-term bonds. For example, before its collapse, Silicon Valley Bank held government debt to invest its surge of customer deposits from the tech boom in seemingly safe, long-term assets, seeking a higher yield than shorter-term bonds offered. By investing in US Treasuries and agency mortgage-backed securities, the bank aimed

[1] In reality, banks are subject to IRRBB (interest rate risk on banking book) standards which are similar to the VaR constraint.

Table 2.3: DSGE Model with Explicit Bond Markets

RS (2008,2012)	MSS (2023)	SSY (2025)
No Arbitrage Condition for Bond Market	No Arbitrage Condition for Bond Market	No Arbitrage Profit with Market-Clearing Condition
Geometric Structure for Maturities of Long-Term Bonds	Geometric Structure for Maturities of Long-Term Bonds	No Geometric Structure for Maturities of Long-Term Bonds

to generate earnings in a period of low interest rates. This behavior of banks can be viewed as the usual practice of arbitrageurs in bond markets. In addition, banks have incentives to purchase long-term government bonds for the diversification of their portfolios of long-term bonds when their loans can be regarded as corporate long-term bonds. For this reason, banks are arbitrageurs and preferred-habitat investors in bond markets as can be seen in Table 2.2.

The second question is how to model demands of commercial banks for long-term government bonds. Eren, Schrimpf and Xia (2023) report that demands of commercial banks, foreign private investors, pension funds, investment funds, and insurance companies for long-term bonds are very sensitive to changes in long-term yields, but to varying degrees in advanced countries including US, EU, UK, and Japan. This empirical result is consistent with the model of Vayanos and Vila (2021) where demands of preferred-habitat investors for long-term bonds are price-elastic. But this empirical result does not mean that commercial banks can be regarded as preferred-habitat investors. In the real world, commercial banks can be in between arbitrageurs and preferred-habitat investors.

The third question is whether the demand of preferred-habitat investors for long-term bonds increases or decreases in response to an increase in the short-term rate.[2] In particular, this one is associated with whether the demand of preferred-habitat commercial banks for long-term bonds increases with an increase in the central bank's policy rate. If commercial banks are regarded as preferred-habitat investors, commercial banks usually reduce their holdings of long-term bonds when the central bank raises its policy rate. For this reason, demands of commercial banks for long-term bonds decrease with increases in the policy rate.

[2] For banks holding long-term bonds, the most important interest rate is the long-term bond yield. If an increase in the short-term (policy) rate leads to an increase in the long-term bond yield, then we can consider the impact of monetary policy. By contrast, if an increase in the short-term rate does not entail an increase in the long-term bond yield for some reason, then there is no concern for market risk associated with long-term bonds. This point is related to the conflicting views of Hanson and Stein (2015) and Vayanos and Vila (2021) about the relationship between short-term interest rates and preferred habitat investors' demand for long-term bonds as discussed in Carboni and Ellison (2022). The positive correlation between the short-term interest rate and the demand for long-term bonds is line with the one of preferred-habitat investors assumed in Vayanos and Vila (2021). But Carboni and Ellison (2022) and Domanski, Shin and Sushko (2017) point out that the correlation would work in the opposite direction if insurance companies and pension funds adopt immunization strategies of interest-rate risks.

Table 2.4: Impact of Financial Regulation on Bank's Bond Holdings in the Model

	IRRBB	CAR	Volcker Rule	LCR
Weight of Long-Term Bonds	Decrease	Decrease	Decrease	Increase
Regulation Constraints	$\bar{v}_t \leq \bar{v}_t^*$ $\bar{c}_t \geq \bar{c}_t^*$	$\omega_t \leq \omega_t^*$	$\omega_t \leq \omega_t^*$	$\omega_t \geq \omega_t^*$

Note: ω_t is the ratio of the value of long-term government bonds held by banks to the amount of household's deposits net of reserves for early withdrawals, $\bar{v}_t$ is the potential maximum loss from interest-rate shock of holding long-term government bonds for one period, $\bar{c}_t$ is the confidence interval. In addition, $\bar{v}_t^*$, $\bar{c}_t^*$, and $\bar{\omega}_t^*$ represent target levels of the financial regulator. IRRBB is the interest rate risk in the banking book, CAR is the capital adequacy ratio, and LCR is the liquidity coverage ratio.

Table 2.3 summarizes a pair of recent works that analyze DSGE models with explicit bond markets. Miao, Shen and Su (2023, forthcoming in AEJ: Macro) extends the GK model into a New Keynesian DSGE model with banking sector for the analysis of the following four issues. What is the role of bank's holdings of long-term securities in banking crises? Can interest rate hikes cause a banking crisis? What are the underlying economic mechanisms? What policies can prevent or mitigate banking crises? In this regard, our research idea is very close to their research idea. The difference of ours from MSS (2023) lies in the specification of bond market and the relation between bank's demand for bond-holdings and risk-management especially for market risk as can be seen above.

The bond premium is determined by the covariance between the stochastic discount factor and future maturity yield (or future bond price). In order to allow for time-varying bond premium in numerical solutions of DGSE models, Rudebusch and Swanson (2008, 2012) adopt third-order nonlinear approximation to equilibrium conditions of their models with Epstein-Zin preferences. Miao, Shen and Su (2023, forthcoming in AEJ: Macro) also adopt non-linear approximation to equilibrium conditions of their models. The common issue facing RS (2008, 2021) and MSS (2023) is how to deal with the potential impact of changes in the maturity structure on the aggregate economy due to the failure of the Ricardian equivalence in the presence of economic frictions when the government issues a portfolio of nominal zero-coupon bonds of different maturities. In order to get around this problem, they adopt the specification of Cochrane (2001) and Woodford (2001) for the maturity structure of government bonds: maturities of government bonds follow a geometric structure.[3]

Table 2.4 shows how different types of financial regulations such as VaR regulation for interest

[3] In their models, one unit of government bond's portfolio at period t is assumed to pay υ^{k-1} in nominal unit of account at period $(t+k)$ with $0 < \upsilon < 1$.

rate risk (IRRBB), captal adequacy ratio regulation (CAR), Volcker rule, and liquidity ratio regulation (LCR) can affect bank's holdings of long-term government bonds in the model of this paper. Hence an advantage of the VaR specification facilitates the analysis of impacts of various financial regulations on balance sheets of commercial banks and thus the financial stability. In addition, the set of prudential policies in Table 2.4 is also different from that of Miao, Shen and Su (2023) whose focus is how the imposition of a permanent tax (and its relaxation) on bank's investment return of long-term bonds affects bank run.

3 Model

A representative household consists of a continuum of ex-ante homogeneous consumers whose measure is equal to one. A liquidity shock hits a fraction of consumers, θ, in the first-half of each period, which in turn lead them to make immediate consumption expenditures in the similar way as is done for early consumers in the Diamond-Dybvig model. The other fraction of consumers, $1-\theta$, are supposed to consume in the second-half of each period in the similar way as is done for late consumers in the Diamond-Dybvig model. The division of a prototypical one-period of discrete-time models into two subperiods is motivated by the incorporation of the Diamond-Dybvig's demand-deposit contract into the usual set-up of DSGE models with infinitely-lived households as shown below.

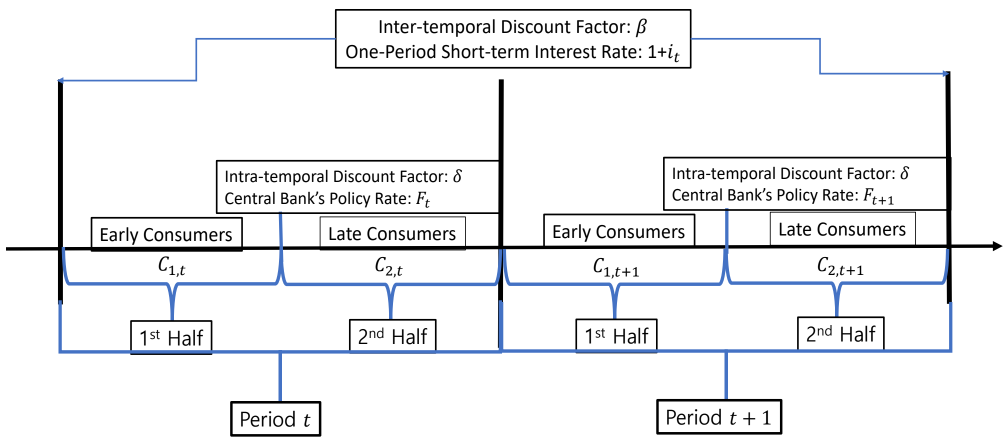
An important feature of the model of this paper is that a partial segmentation of bonds markets exists. While households can trade equities of firms, short-term government bonds, and long-term government bonds, there is a subset of long-term bonds where households do not participate directly but banks trade as their delegates. Banks trade federal funds with the central bank and one-period government bonds. In addition, bank have incentives to hold long-term bonds for the following reasons.

By investing in US Treasuries and agency mortgage-backed securities, banks generate earnings in a period of low interest rates as can be seen in the recent bank runs of Silicon Valley Bank and Signature Bank. Banks hold long-term government bonds for the diversification of their bond portfolios when their loans are regarded as corporate long-term bonds. For this reason, banks can be in between arbitrageurs and preferred-habitat investors in a subset of long-term bonds.

3.1 Households

Each period, transactions between households and firms for consumption goods proceed as follows. Firms set nominal product prices in advance before transactions take place with the restriction of nominal price of $C_{1,t}$ = nominal price of $C_{2,t}$ in each period where $C_{1,t}$ represents early consumer's consumption and $C_{2,t}$ is late consumer's consumption. In other words, firms set nominal product

Figure 3.1: Household's Consumption and Discount Factor



prices in advance before transactions take place without price differentiation between early and late consumers. Hence firms set the same price for $C_{1,t}$ and $C_{2,t}$. In addition, early and late consumers can use the same payment system but are subject to different timings of spending consumption expenditures. For example, consumption expenditures of early consumers $C_{1,t}$ should be paid immediately on the occurrence of a liquidity shock. But consumption expenditures of late consumers $C_{2,t}$ are settled at the beginning of period $t+1$ by transferring money from their bank accounts to sellers' bank accounts.[4] Figure 3.1 also demonstrates that early consumers consume in the first-half of each period and late consumers consume in the second-half of each period.

Early consumers and late consumers have the same utility function. Specifically, $u(C_{1,t})$ represents the concave and differentiable utility function of early consumers and $u(C_{2,t})$ corresponds to the concave and differentiable utility function of late consumers. In addition, $v(H_t)$ is the household' dis-utility function of labor that is convex and differentiable in its argument. The intra-period discount factor $(= \delta)$ is also used to compute the beginning-of-period utility value of late consumers who consume in the second-half period. Each period, therefore, the beginning-of-period expected utility function for ex-ante homogeneous households can be written as follows: $(\theta u(C_{1,t}) + (1-\theta)\delta u(C_{2,t}) - v(H_t))$ where H_t is the number of hours worked at period t. As a

[4] If early consumers are required to use cash for their payments, $C_{1,t}$ is regarded as cash goods and $C_{2,t}$ is regarded as credit goods following the distinction of Lucas and Stokey (1987) because payment for $C_{2,t}$ is settled at the beginning of period t+1 by bank's transferring money from household's bank account to firm's bank account.

result, the preferences at period 0 of the representative household can be expressed as follows.

$$\sum_{t=0}^{\infty} \beta^t E_0[Z_t(\theta u(C_{1,t}) + (1-\theta)\delta u(C_{2,t}) - v(H_t))] \tag{3.1}$$

where Z_t represents an exogenous preference shock, β denotes the inter-period discount factor and δ is the intra-period discount factor. Figure 3.1 illustrates different roles of these two discount factors.

Households can trade a complete set of state-contingent claims. But the existence of a complete set of state-contingent claims is not helpful for individual consumers to attain the equality between $C_{1,t}$ and $C_{2,t}$. The reason for this feature reflects the fact that the occurrence of liquidity shock cannot be publicly verified in the same way as is done in the Diamond-Dybvig Model (1983). Each member of the representative household receives the same wage rate ($= w_t$) in return for its labor services to firms ($= H_t$) and dividend incomes for individual firms giving their profits as dividends for stock owners ($= \pi_{f,t}$), while ex-ante identical consumers have the same level of demand deposits in their bank accounts ($= D_t$). The representative household's consolidated budget constraint is therefore given by the following equation.

$$D_t + E_t[Q_{t,t+1}A_{t+1}] = w_t H_t + A_t + \pi_{f,t} + G_{H,t} \tag{3.2}$$

where A_{t+1} represents their demands at period t for contingent claims that give one unit of consumption goods conditional on states at period $t+1$ and $Q_{t,t+1}$ is the stochastic discount factor that is used to compute the value at period t of one unit of consumption goods at period $t+1$ and $G_{H,t}$ is the real value of the household's government account that includes its holdings of government bonds and net subsidies (taxes minus subsidies).

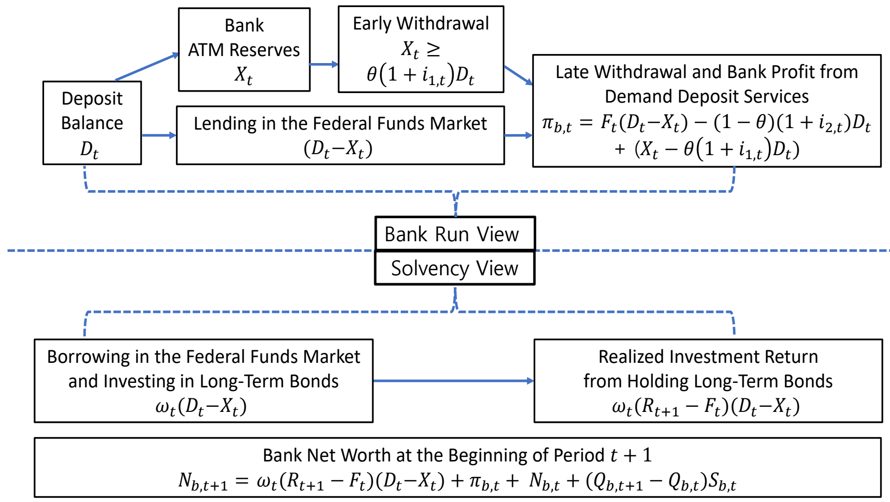
3.2 Banks

Figure 3.2 shows how individual banks determine their portfolios after households make deposits ($= D_t$). Banks divide deposits into two parts. One is reserves for early withdrawals ($= X_t$) and the other is the bank's lending in the federal funds market ($= D_t$ - X_t) whose gross nominal return is F_t. The total withdrawals from early consumers is $\theta(1+i_{1,t})D_t$ in the first-half of each period and total withdrawals of late consumers is $(1-\theta)(1+i_{2,t})D_t$ in the second-half of each period. In sum, the bank's profit flow from its deposit services is

$$\pi_{b,t} = F_t(D_t - X_t) - (1-\theta)(1+i_{2,t}) + (X_t - \theta(1+i_{1,t})D_t) \tag{3.3}$$

where $\pi_{b,t}$ is the bank's profit flow from its deposit services at period t. The cash flows specified in equation (3.3) can be interpreted as profit flows of bank's deposit services. The upper part of the

Figure 3.2: Bank's Portfolio Decision and Net Worth



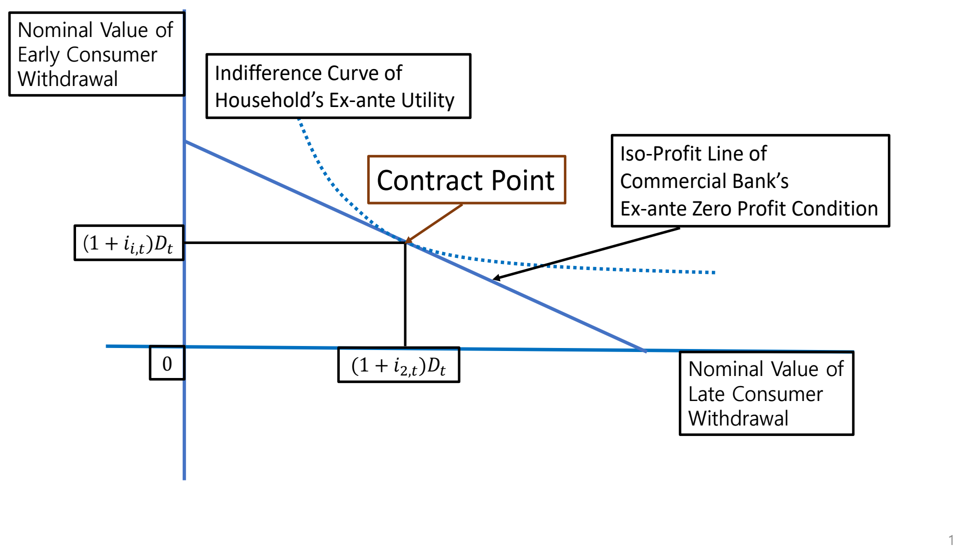
blue dotted line of Figure 3.2 can be described by the word of "Bank Run View" because equation (3.3) mimics the profit flows of the Diamond-Dybvig model. In this case, depositors can collectively withdraw from otherwise solvent banks even without having any stochastic returns from bank's assets.

The lower part of the blue dotted line of Figure 3.2 can be described by the word of "Solvency View" because it allows for the possibility that banks can suffer severe investment losses enough to completely deplete their net worths. In order to incorporate the solvency view into the model, banks are supposed to borrow funds from the federal funds market to invest in long-term bonds, so that banks act as arbitrageurs in markets of long-term bonds. Hence the bank's net worth evolves over time according to the following equation

$$N_{b,t+1} = \omega_t(R_{t+1} - F_t)(D_t - X_t) + \pi_{b,t} + (N_{b,t} - (Q_{b,t+1} - Q_{b,t})S_{b,t}) \tag{3.4}$$

where $N_{b,t+1}$ is the bank's net worth at the beginning of period $t+1$, R_{t+1} is the realized gross return from one-period holdings of long-term bonds, the amount of bank's borrowing in the federal funds market is $\omega_t(D_t - X_t)$, $Q_{b,t}$ is the nominal price at period t of bank's share and R_{t+1} is the realized gross return from one-period holdings of long-term government bonds. The parenthesis at the end of the right-hand side of equation (3.4) reflects changes in the market value of bank's net-worth between periods t and $t+1$. In addition, changes in the value of ω_t lead to changes in the amount of bank's borrowing in the federal funds market. The inclusion of ω_t in equation (3.4) is motivated by the fact that the bank's risk-management tends to adjust the size of potential

Figure 3.3: Optimal Demand Deposit Contract



investment loss relative to that of deposit (net of reserves) as will seen later.

3.3 Demand Deposit Contract and Household's Utility Maximization

Each period, banks and households are supposed to make demand deposit contracts before liquidity shocks take place in the same way as is done in the Diamond-Dybvig model. The ex-ante demand deposit contract between banks and households is determined as the result of the maximization of household's ex-ate expected utility subject to a zero expected profit condition of banks for their deposit services. The zero expected profit condition of banks can be regarded as a participation constraint of perfectly competitive markets of deposit services.[5]

Figure 3.3 demonstrates that the demand deposit contract is determined at the point which the zero-profit line of banks is tangent to the household's ex-ate indifference curve. The blue dotted curve represents the household's ex-ante indifference curve. The blue solid line is the zero ex-ante profit line of banks. The brown box of contract point marks the tangent point of the bank's zero-profit line to the household's ex-ate indifference curve. The ex-ante instantaneous utility can be regarded as the instantaneous social welfare function in the absence of bank runs. Hence the contract point of Figure 3.2 can be regarded as the optimal contract between banks and households

[5]A consequence of this condition is to make interest rates of demand deposits such as checking accounts lower than the federal funds rate. In actual data, the national rate of checking accounts in the US tends to be lower the federal funds rate. For example, the national deposit rate of checking account is 0.07% (as of September 15, 2025 at the FDIC home-page, https://www.fdic.gov/national-rates-and-rate-caps), while the target of the federal funds rate is a range of 4.00-4.25%.

that maximizes the social welfare in the absence of bank runs. The optimal demand deposit contract can be written as follows.

$$\begin{array}{ll} \max_{\{i_{1,t}, i_{2,t}\}} & \theta u((1+i_{1,t})D_t) + (1-\theta)\delta u((1+i_{2,t})D_t) \\ \text{subject to} & \pi_{b,t} = 0 \ \rightarrow \theta(1+i_{1,t}) + (1-\theta)(\frac{1+i_{2,t}}{F_t}) = 1 \end{array} \tag{3.5}$$

The optimal condition of the demand deposit contract can be derived as follows. First, the ex-ante marginal rate of substitution between early consumption and late consumption is

$$\frac{\Delta C_{2,t}}{\Delta C_{1,t}} = -\frac{u'(C_{1,t})}{\delta u'(C_{2,t})}$$

This equation can be interpreted as the social planner's valuation of the opportunity cost of withdrawals of early consumers compared with those of late consumers. Second, the slope of the iso-profit curve obtained from the ex-ante zero profit condition is

$$\frac{\Delta C_{2,t}}{\Delta C_{1,t}} = -F_t$$

This equation can be interpreted as the market evaluation of the opportunity cost of withdrawals of early consumers compared with those of late consumers. Hence the equality between subjective and objective evaluations of the opportunity cost of withdrawals of early consumers compared with those of late consumers leads to the following optimal condition for the demand deposit contract specified in equation (3.5).

$$\frac{\delta u'(C_{2,t})}{u'(C_{1,t})} F_t = 1 \tag{3.6}$$

Having described the demand deposit contract between banks and households, the next discussion turns to the optimization conditions of households. The substitution of $\pi_{b,t} = 0$ and $X_t = \theta C_{1,t}$ into the household's budget constraint leads to the following equation:

$$\theta C_{1,t} + (1-\theta)\frac{C_{2,t}}{F_t} + E_t[Q_{t,t+1}A_{t+1}] = w_t H_t + A_t + \pi_{f,t} + G_{H,t} \tag{3.7}$$

Hence the representative household maximizes the life-time utility function (3.1) subject to a series of the following period-by-period budget constraint (3.7) taking $\{G_{H,t}\}_{t=0}^{\infty}$ as given. The first-order conditions of the representative household's utility maximization problem can be summarized as follows. The first-order condition for early consumer's consumption is

$$Z_t u'(C_{1,t}) = \Lambda_t \tag{3.8}$$

The first-order condition for late consumer's consumption is

$$Z_t \delta F_t u'(C_{2,t}) = \Lambda_t \tag{3.9}$$

The first-order condition for labor supply condition equates the marginal rate of substitution between consumption and leisure to the real wage as follows.

$$Z_t v'(H_t) = \Lambda_t w_t \tag{3.10}$$

The first-order condition for A_{t+1} gives the following relation between the stochastic discount factor and the inter-temporal marginal rate of substitution.

$$Q_{t,t+1} = \beta \Lambda_{t+1}/\Lambda_t \tag{3.11}$$

It should be noted that dividing both sides of equation (3.9) by corresponding sides of (3.8) leads to the optimal condition for demand deposit contract (3.6). The rest of optimization conditions are binding budget constraints for consumption expenditures as can be seen below.

$$\begin{array}{rcl} C_{1,t} & = & (1+i_{1,t})D_t \\ C_{2,t} & = & (1+i_{2,t})D_t \\ D_t & = & w_t H_t + \pi_{f,t} + G_{H,t} \\ D_t & = & \theta C_{1,t} + (1-\theta)(C_{2,t}/F_t) \end{array} \tag{3.12}$$

The next discussion is the derivation of the equilibrium relation between intra-period risk-free (gross) interest rate ($= F_t$) and inter-period risk-free (gross) interest rate ($= 1 + i_t$). The first point is that since the inter-period risk-free gross interest rate is the inverse of the price of one-period government bond whose face value is one unit of account, the absence of arbitrage profit leads to the following equation.

$$1 = \beta E_t[\frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}}](1 + i_t). \tag{3.13}$$

where P_t is the aggregate price index at period t. The second point is that even if households cannot participate in the federal funds market, commercial banks can sell risk-free short-term bonds whose payoffs mimic payoffs of securities traded in the intra-asset market. In particular, when such assets are available for late consumers in the second-half subperiod, the absence of arbitrage profit leads to the following condition.

$$\frac{Z_t \delta u'(C_{2,t})}{P_t} = \beta E_t[\frac{\Lambda_{t+1}}{P_{t+1}}]F_t$$

where both sides of this equation are evaluated in terms of utilities of early consumers. The left-hand side of this condition is the utility cost of one unit of nominal account that is invested at period t. The right-hand side is the expected discounted utility benefit of holding the risk-free short-term bonds whose payoffs mimic payoffs of securities traded in the intra-asset market. Dividing both sides of this equation by Λ_t leads to the following representation.

$$\frac{Z_t \delta u'(C_{2,t})}{\Lambda_t} = \beta E_t[\frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}}]F_t \quad \rightarrow \quad \frac{Z_t \delta u'(C_{2,t})}{\Lambda_t} = \frac{F_t}{1 + i_t}$$

The right equation of the arrow shown above is derived by substituting equation (3.13) into left equation of the arrow. The substitution of equation (3.6) into the equation of the arrow shown above the equilibrium relation between intra-period risk-free (gross) interest rate ($= F_t$) and inter-period risk-free (gross) interest rate ($= 1 + i_t$).

$$1 + i_t = F_t^2 \tag{3.14}$$

The final discussion of this subsection is the determination of equilibrium prices of government bonds. Since households do not hold a subset of government bonds, a partial segmentation exists for government bonds. The equilibrium price of long-term bonds whose market is not available for households are determined by equating demands of commercial banks to corresponding market supplies as will discussed later. But the equilibrium price of long-term bonds whose market is available for households is determined by the following condition.

$$P_t^{(k)} = \beta E_t[\frac{\Lambda_{t+1} P_{t+1}^{(k-1)}}{\Lambda_t \Pi_{t+1}}] \tag{3.15}$$

where $P_t^{(k)}$ is the nominal price at period t of government bonds whose remaining maturity is k and $P_{t+1}^{(0)} = 1$.

3.4 VaR and Bank's Demands for Long-Term Government Bonds

The aim of this subsection is the derivation of bank's demand function for long-term bonds under the assumption that banks adopt VaR to control exposures of their portfolios to interest-rate risk. The important assumption of this subsection is that banks have a target maturity of long-term bonds as preferred habitat investors where τ is the target maturity at period t of long-term bonds held by banks. Recall from equation (3.4) that bank's demands for long-term bonds can be written as follows.

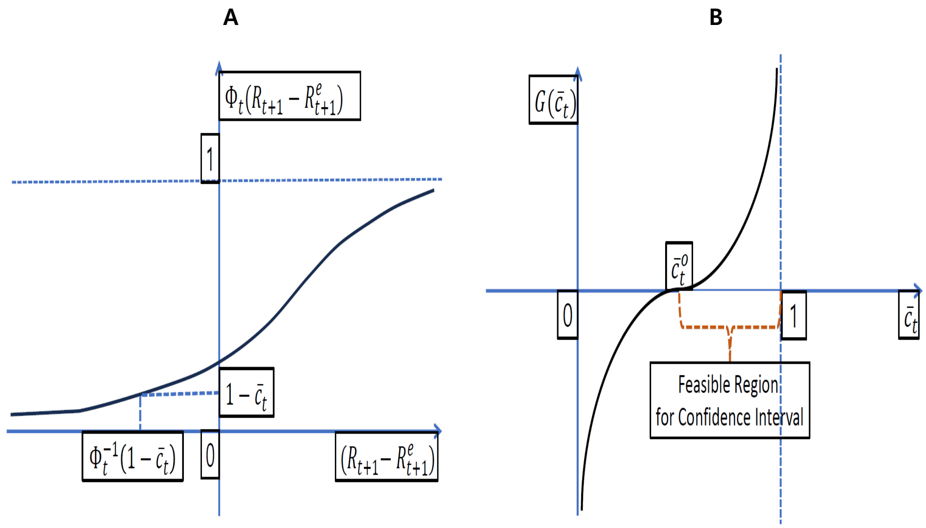
$$D_t^{(\tau)} = \omega_t (D_t - X_t) \tag{3.16}$$

where $D_t^{(\tau)}$ is the nominal value at period t of long-term bonds held by banks.

Now the derivation of bank's demand function for long-term bonds proceeds with the analysis of relation between VaR and ω_t with the emphasis of the following three points. First, $P_t^{(\tau)}$ is not determined by the equilibrium condition for bonds prices specified in equation (3.15). Second, $P_t^{(\tau)}$ is determined by the market clearing condition of long-term bonds whose remaining maturity is τ at period t as will be confirmed below. Third, bank's adoption of VaR is associated with financial regulation, while it also can be rationalized by the contract-based approach of Adrian and Shin (2008, 2014).[6]

[6]It would be helpful to quote the following statement of the BIS's IRRBB document, "Interest rate risk in the

Figure 3.4: VaR Confidence Interval



In order to incorporate VaR into bank's investment decision for long-term bonds, banks are assumed to know the probability distribution of their expectation errors about one-period holding returns of long-term bonds. The confidence interval can be then written in terms of the expected excess premium of one-period holdings of long-term bonds. The portfolio ratio for long-term bonds ($= \omega_t$) becomes a function of VaR parameters such as the maximum potential loss and confidence interval.

The determination of portfolio ratio for long-term bonds ($= \omega_t$) goes through the following three steps. The first step is the specification of the VaR constraint facing individual banks:

$$(F_t - R_{t+1})\omega_t \leq \bar{v}_t \quad \rightarrow \quad (R_{t+1} - R^e_{t+1}) \geq -(\xi_t + \omega_t^{-1}\bar{v}_t)$$

where $\xi_t = R^e_{t+1} - F_t$ and bank's investment return is measured in terms of nominal unit of account: $R_{t+1} = P^{(\tau-1)}_{t+1}/P^{(\tau)}_t$. The right side of the arrow shown above means that VaR generates a lower bound on bank's expectation error about its investment return on long-term bonds. The second step is the determination of the associated confidence interval:

$$\bar{c}_t = 1 - \Phi(-(\xi_t + \omega_t^{-1}\bar{v}_t)) \tag{3.17}$$

The third step is to get the inverse function of the cumulative distribution function Φ to express the

banking book (IRRBB) is part of the Basel capital framework's Pillar 2 (Supervisory Review Process) and subject to the Committee's guidance set out in the 2004 Principles for the management and supervision of interest rate risk (henceforth, the IRR Principles)" In addition, this document gives an explicit account of VaR as follows. "Economic value at risk (EVaR) measures the expected maximum reduction of market value that can be incurred under normal market circumstances over a given time horizon or holding period and subject to a given confidence level."

bond weight in terms of both the expected excess return of bond investment and VaR parameters. As a result, the portfolio ratio of long-term bonds becomes a function of the two parameters of VaR such as maximum potential loss and confidence interval as well as expected excess return as can be seen below.

$$\omega_t = \frac{\bar{v}_t}{G(\bar{c}_t) - \xi_t}; \quad G(\bar{c}_t) = -\Phi^{-1}(1 - \bar{c}_t) \tag{3.18}$$

The important features of equation (3.18) can be summarized as follows. First, portfolio ratio of long-term bonds is an increasing function of the expected excess investment return of long-term bonds. Second, portfolio ratio of long-term bonds is an increasing function of the maximum potential loss. Third, portfolio ratio of long-term bonds is a decreasing function of confidence interval. In relation to the third feature, Figure 3.4 demonstrates that $G(\bar{c}_t)$ is an increasing function of $\bar{c}_t$. Panel A of Figure 3.4 shows how the inverse of cumulative distribution function of expectation errors is affect by confidence interval. Panel A corresponds to the graph of $\Phi^{-1}(1 - \bar{c}_t)$. Panel B of Figure 3.4 illustrates the graph of $G(\bar{c}_t)$ that is obtained by using the graph of $\Phi^{-1}(1 - \bar{c}_t)$.

In sum, it follows from the substitution of equation (3.18) into equation (3.16) that individual banks have a downward-sloping demand curve for long-term bonds.

$$P_t^{(\tau)} = \frac{E_t[P_{t+1}^{(\tau-1)}]}{F_t + G(\bar{c}_t) - \bar{v}_t(D_t - X_t)(D_t^{(\tau)})^{-1}} \tag{3.19}$$

The bank's demand curve for long-term bonds shifts upward as the expected value of next-period's bond price rises, while it shifts downward as the policy rate decreases. In addition, bank's tough risk-management also shifts downward its demand curve for long-term bonds.

3.5 Household's Holdings of Government Bonds and Government Budget Constraint

It would be worthwhile to discuss some issues of $G_{H,t}$ in the household's budget constraint where $G_{H,t}$ is called the household's government account. The real value of household's government account is defined as after-tax net-revenue of its bond portfolio as follows.

$$G_{H,t} = -T_{H,t} + \frac{B_{H,t}^{(1)} + \sum_{k=2}^{K} P_t^{(k)} B_{H,t}^{(k-1)}}{P_t} - \frac{\sum_{k=1}^{K} P_t^{(k)} B_{H,t+1}^{(k)}}{P_t}$$

where $B_{H,t+1}^{(k)}$ is the number of bonds with maturity k held by the household, $P_t^{(k)}$ is the corresponding nominal bond price at period t, P_t is the aggregate price index and $T_{H,t}$ is the net lump-sum tax for households. In this representation, it should be noted that household's participation restriction for bonds whose maturity is τ leads to $B_{H,t+1}^{(\tau)} = 0$.

In particular, a non-zero value of $G_{H,t}$ is associated with the failure of Ricardian equivalence. Specifically, unless households hold all government bonds, a change in the maturity structure of government debt can affect the household's flow budget constraint through its impact on the value of $G_{H,t}$. For this reason, a non-zero value of $G_{H,t}$ (i.e. $G_{H,t} \neq 0$) raises the possibility that household's consumption demand can be affected by government's choice between lump-sum tax and debt for a given level of its spending. In relation with this issue, Cochrane (2001), Woodford (2001), Rudebusch and Swanson (2008, 2012), and Miao, Shen and SU (2023) adopt the assumption that maturities of government bonds follow a geometric structure with no-arbitrage pricing for bonds. The model of this paper abstracts from this assumption and proceeds with the imposition of a lump-sum transfer to set $G_{H,t} = 0$.

Government issues only zero coupon bonds whose face values are one nominal unit of account with the maximum maturity K (> 1). Given a set of new issues (measured as real values at period t) of government bonds $\{N_t^{(k)}\}_{k=1}^{K}$, the government flow budget constraint at period t is

$$\sum_{k=1}^{K} \frac{P_t^{(k)} N_t^{(k)}}{P_t} + T_t = \frac{B_t^{(1)}}{P_t} + G_t$$

where T_t is the aggregate tax revenue and G_t is the aggregate government expenditures. The outstanding stocks of government bonds evolve over time as follows.

$$B_{t+1}^{(k)} = N_t^{(k)} + B_t^{(k+1)}$$

for $k = 1, \cdots, K-1$ and $B_{t+1}^{(K)} = N_t^{(K)}$.

Turning to the market supplies of long-term government bonds, the following equation describes market supplies of government long-term bonds that the central bank and households do not hold in their balance sheets.

$$S_t^{(\tau)} = B_t^{(\tau+1)} + N_t^{(\tau)}$$

where τ is a positive integer greater than one but less than K with $S_t^{(K)} = N_t^{(K)}$. The central bank's quantitative easing can affect market supplies of government bonds as can be seen below.

$$S_t^{(\tau)} = (B_t^{(\tau+1)} - B_{F,t}^{(\tau+1)}) + (N_t^{(\tau)} - B_{F,t+1}^{(\tau)})$$

where $B_{F,t}^{(k)}$ represents the central bank's demands for long-term bonds whose maturity is k.

3.6 Firms

A continuum of monopolistically competitive firms are in charge of the production of differentiated goods using a linear production function:

$$Y_t(i) = A_t H_t(i)$$

where $Y_t(i)$ is the output of firm i, $H_t(i)$ is the amount of labor input, and A_t is the productivity level at period t. Following the Dixit-Stiglitz model, the demand curve of firm i whose nominal product price is $P_t(i)$ is given by

$$Y_t(i) = (\frac{P_t(i)}{P_t})^{-\epsilon} Y_t$$

where Y_t is the aggregate output, P_t is the aggregate price index, and ϵ is a positive constant. The aggregation condition of ($P_t Y_t = \int_{i=0}^{i=1} P_t(i) Y_t(i) di$) implies that the aggregate price level ($= P_t$) is defined as a non-linear sum of individual nominal prices ($= P_t(i)$).

$$P_t = (\int_{i=0}^{i=1} P_t(i)^{1-\epsilon} di)^{\frac{1}{1-\epsilon}}$$

Each period, a fraction of firms $1-\alpha$ set a new nominal product price P_t^* but the other fraction α do not change prices following the Calvo model. In this case, the aggregate price index ($= P_t$) is determined as follows.

$$P_t^* = (1-\alpha)(P_t^*)^{1-\epsilon} + \alpha P_{t-1}^{1-\epsilon}$$

The profit maximization problem of firms that reset their prices can be written as follows.

$$\max_{P_t^*} \sum_{k=0}^{\infty} (\alpha\beta)^k E_t[\frac{\Lambda_{t+k+1}}{\Lambda_{t+k}}((\frac{P_t^*}{P_{t+k}})^{1-\epsilon} - mc_{t+k}(\frac{P_t^*}{P_{t+k}})^{-\epsilon}) Y_{t+k}]$$

where mc_t is the real marginal cost of production. The corresponding optimization condition is also summarized as follows.

$$\begin{array}{l} K_t = Y_t + \alpha\beta E_t[\frac{\Lambda_{t+1}}{\Lambda_t}\Pi_{t+1}^{\epsilon-1} K_{t+1}] \\ J_t = \frac{\epsilon mc_t Y_t}{(\epsilon-1)} + \alpha\beta E_t[\frac{\Lambda_{t+1}}{\Lambda_t}\Pi_{t+1}^{\epsilon} J_{t+1}] \\ 1 = (1-\alpha)(\frac{J_t}{K_t})^{1-\epsilon} + \alpha\Pi_t^{\epsilon-1} \end{array} \tag{3.20}$$

The first and second lines of equation (3.20) are derived from the profit maximization conditions of firms. The third line of equation (3.20) reflects the definition of the aggregate price index. The tree lines of equation (3.20) can be solved for three variables such as K_t, J_t, and Π_t given a set of values Λ_t and mc_t. In addition, a long-linear approximation of equation (3.20) leads to a linearized Phillips curve equation as will be seen later.

3.7 Collection of Equilibrium Conditions

Before proceeding to the collection of the equilibrium conditions, it would be helpful to discuss macroeconomic impacts of prudential policy measures. In order to do so, it would be necessary to discuss what prudential policy measures are included in the model of this paper. In this light, it should be mentioned that the two parameters of the VaR can be regarded as prudential policy measures on the basis of the view that financial regulation is responsible for the bank's adoption of

Table 3.1: Impact of Financial Regulation on Γ_t

	IRRBB	CAR	Volcker Rule	LCR
Response of ω_t	Decrease	Decrease	Decrease	Increase
Response of Γ_t	Increase	Increase	Increase	Decrease
Regulation Constraints	$\bar{v}_t \leq \bar{v}_t^*$ $\bar{c}_t \geq \bar{c}_t^*$	$\omega_t \leq \bar{\omega}_t$	$\omega_t \leq \bar{\omega}_t$	$\omega_t \geq \bar{\omega}_t$

Note: $\bar{c}_t$, $\bar{v}_t^*$, and $\bar{\omega}_t$ represent target levels of the financial regulator. The impact of each financial regulation on the value of Γ_t occurs when the corresponding regulation constraint is binding. IRRBB is the interest rate risk in the banking book, CAR is the capital adequacy ratio, and LCR is the liquidity coverage ratio.

the VaR as a tool of its risk management as can be seen in the discussion of the IRRBB. Specifically, the financial regulator can require that banks should not deviate from specific ranges for values of the two VaR parameters in an effort to control their holdings of long-term bonds. The target variable of this kind of financial regulation is the fraction of long-term bonds in the bank's balance sheet. On top of this one, the financial regulator can require that the amount of risky assets held by banks should not exceed a certain fraction of their deposits in an effort to secure repayments of deposits in line with the well-known BIS requirement of the capital adequacy ratio for commercial banks.

In order to see how these policy measures of the financial regulator affect bank's investment in long-term bonds in the model of this paper, it should be noted that the IRRB regulation affects ω_t and the CAR or CARR regulation affects the ratio of bank's investment to its deposits is $D_t^{(\tau)}/D_t$. Moreover, Γ_t contains impacts of these two financial regulatory measures in the model of this paper reflecting its definition of $\Gamma_t = D_t^{(\tau)}/(\omega_t D_t)$. It should be also noted that Γ_t summarizes impacts of financial regulatory measures in the aggregate equilibrium conditions because it is the only one variable that shows up in the aggregate equilibrium conditions.

In order to facilitate the analysis of equilibrium solution to the model described above, this section is focused on a self-sufficient set of equilibrium conditions. In this subsection, 10 equilibrium conditions are collected for 10 endogenous variables of K_t, J_t, Π_t, Y_t, Δ_t, F_t, $(1+i_t)$, Λ_t, mc_t, Γ_t given two exogenous fundamental shocks such as productivity and preference shocks A_t and Z_t as can be seen below.

$$
\begin{array}{l}
K_t = Y_t + \alpha\beta E_t[\frac{\Lambda_{t+1}}{\Lambda_t}\Pi_{t+1}^{\epsilon-1}K_{t+1}] \\
J_t = \frac{\epsilon mc_t Y_t}{(\epsilon-1)} + \alpha\beta E_t[\frac{\Lambda_{t+1}}{\Lambda_t}\Pi_{t+1}^{\epsilon}J_{t+1}] \\
1 = (1-\alpha)(\frac{J_t}{K_t})^{1-\epsilon} + \alpha\Pi_t^{\epsilon-1} \\
mc_t = \frac{v'(Y_t\Delta_t/A_t)}{A_t u'(\theta^{-1}(1-\Gamma_t)Y_t)} \\
1+i_t = F_t^2 \\
\Lambda_t = Z_t u'(\theta^{-1}(1-\Gamma_t)Y_t) \\
\beta E_t[\frac{\Lambda_{t+1}}{\Lambda_t\Pi_{t+1}}](1+i_t) = 1 \\
\Delta_t = (1-\alpha)(\frac{1-\alpha\Pi_t^{\epsilon-1}}{1-\alpha})^{\frac{\epsilon}{\epsilon-1}} + \alpha\Pi_t^{\epsilon}\Delta_{t-1} \\
\text{Determination of } \Gamma_t\text{: Prudential Policy Variable} \\
\text{Determination of } F_t\text{: Monetary Policy Variable}
\end{array}
\tag{3.21}
$$

In the last two lines of equation (3.21), we do not present concrete specifications of relevant equations. It would be thus worthwhile to discuss how these two equations are determined. First, the central bank controls the value of F_t on the basis of an interest rate rule that connects the central bank's policy rate to deviations of inflation and output from their targets. Second, the equilibrium value of Γ_t is set equal to $\Gamma_t = S_t^{(\tau)}/(\bar{\omega}_t Y_t)$ with two equilibrium conditions of $D_t = Y_t$ and $D_t^{(\tau)} = S_t^{(\tau)}$ when financial regulation constraints in Table 3.1 are binding. In this case, Γ_t is directly affected by changes of prudential policies given $S_t^{(\tau)}$ and Y_t. For this reason, Γ_t is regarded as a policy tool of financial regulation in this section as will be seen below.

3.8 Inflation and Output Effects of Financial Regulation

The aim of this subsection is to compare a log-linearized small-scale model of this paper with the prototypical New Keynesian model of three equations such as the Phillips curve, the IS curve and the interest rate rule. The difference between these two models is two-fold. One is the addition of Γ_t to the set of equilibrium conditions. The other is the change in the indeterminacy region for numerical values of feedback parameters of inflation and output gaps that is associated with the Taylor principle.

The log-linear approximation to the profit-maximization condition of price-adjusting firms (first tree equations of equation (3.21) around the deterministic steady state with a zero inflation together with the definition of the aggregate price index) leads to the following Phillips curve equation.[7]

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa x_t - \kappa_\Gamma \gamma_t \tag{3.22}$$

where the slope of Phillips curve is $\kappa = (\sigma+\chi)(1-\alpha)(1-\alpha\beta)/\alpha$ and $\kappa_\Gamma = \Gamma\kappa_x/(1-\Gamma)$ with $0 < \Gamma < 1$. The log-linear approximation to the Euler equation (sixth line of equation (3.21) around the

[7] The log-linearized variables are represented by small characters, whereas non-linear original variables are represented by large characters. In addition, the household's utility function for consumption is $u(c) = (c^{1-\sigma}-1)/(1-\sigma)$ and the dis-utility function for hours is $v(H) = H^{1+\chi}/(1+\chi)$ where both σ and χ are positive.

deterministic steady state with a zero inflation together with the definition of the aggregate price index) leads to the following IS curve equation.

$$x_t = E_t[x_{t+1}] - \sigma^{-1}(i_t - r_t^* - E_t[\pi_{t+1}]) + \sigma_\Gamma(\gamma_t - E_t[\gamma_{t+1}]) \tag{3.23}$$

where i_t^* is the natural rate of interest, σ is the inverse of the inter-temporal elasticity of substitution and $\sigma_\Gamma = \Gamma/(\sigma(1-\Gamma))$. The central bank's interest rate rule leads to the following specification

$$i_t = r_t^* + \phi_\pi \pi_t + \phi_x x_t + e_t \tag{3.24}$$

where coefficients ϕ_π and ϕ_x all positive and e_t is an exogenous interest-rate shock.

The apparent difference of this paper from the prototypical New Keynesian model is that equations (3.22) and (3.23) include the log-linearized variable of Γ_t $(= \gamma_t)$ as mentioned before. Specifically, the Phillips curve shifts downward as the value of γ_t decreases. A loose financial regulation leads to increase in ω_t and thus a decrease in the value of Γ_t as can be seen in Table 3.1. Hence a decrease in γ_t can be regarded as a result of a loose financial regulation. Hence equation (3.22) indicates that a loose financial regulation acts as an endogenous supply shock to raise the aggregate inflation rate.

Smets (2014) also emphasizes that this feature can play an important role in the analysis of the optimal financial regulation with a different channel through which financial regulation affects the Phillips curve. The mechanism behind this result in the model of this paper is income effect channel of financial regulation. The financial regulation generates an income effect on the household's labor supply, which shifts the labor supply curve in the labor market. Specifically, being other things equal, the real wage decreases when Γ_t increases reflecting its impact on labor supply curve. Hence the equilibrium real wage rises with a loose financial regulation.

Turning to the second topic of this subsection, it should be noted that the log-linear relation of $f_t = (1/2)\, i_t$ leads to a change in the indeterminacy region for numerical values of feedback parameters of inflation and output gaps that is associated with the Taylor principle when the central bank is supposed to adjust its policy rate according to the following equation.[8]

$$f_t = f_t^* + \hat{\phi}_\pi \pi_t + \hat{\phi}_x x_t + \hat{e}_t \tag{3.25}$$

where f_t^* is the neutral level of federal funds rate, coefficients $\hat{\phi}_\pi$ and $\hat{\phi}_x$ all positive and $\hat{e}_t$ is an exogenous shock to the federal funds rate. Comparing equation (3.24) with equation (3.25),

$$i_t^* = 2f_t^*, \quad \phi_\pi = 2\hat{\phi}_\pi \quad \phi_x = 2\hat{\phi}_x.$$

[8]The equation of $f_t = (1/2)\, i_t$ can be obtained by taking log-differences between current-period and steady-state values to both sides of equation (3.14).

The Taylor principle should be applied to values of ϕ_π and ϕ_x in the model of this paper to secure the determinacy of equilibrium solutions, whereas the Taylor principle is applied to values of $\hat{\phi}_\pi$ and $\hat{\phi}_x$ in the prototypical New Keynesian model where $i_t = f_t$. Specifically, the model of this paper does not need the principle of $\hat{\phi}_\pi > 1$ to secure the determinacy of equilibrium solutions.

4 Optimal Monetary and Prudential Policies

In this section, we begin with the characterization of an instantaneous social welfare function. The first step is the use of the ex-ante zero profit condition of banks $\pi_{b,t} = 0$ to derive the household's ex-ante present-value budget constraint of the following form:

$$\theta C_{1,t} + (1-\theta)\frac{C_{2,t}}{F_t} = D_t.$$

The second step is to express consumptions of early and late consumers as functions of D_t, Γ_t and F_t as follows.

$$\Gamma_t = 1 - \frac{X_t}{D_t} \;\rightarrow\; X_t = (1-\Gamma_t)D_t \;\rightarrow\; C_{1,t} = \frac{(1-\Gamma_t)D_t}{\theta} \;\rightarrow\; C_{2,t} = \frac{\Gamma_t D_t F_t}{1-\theta}$$

The third step is to derive the equality of $D_t = Y_t$ from the representative household's budget constraint evaluated at the equilibrium. Hence the substitution of this relation into this equation implies that consumptions of early and late consumers become functions of Y_t, Γ_t and $(1+i_t)$ as follows.

$$\begin{aligned} C_{1,t} &= \tfrac{(1-\Gamma_t)D_t}{\theta} \\ C_{2,t} &= \tfrac{\Gamma_t D_t (1+i_t)^{1/2}}{1-\theta} \end{aligned}$$

As a result, the instantaneous social welfare function can be expressed in terms of the aggregate output, prudential policy measure ($= \Gamma_t$), relative price distortion ($= \Delta_t$), and short-term (gross) nominal interest rate as follows.

$$\mathcal{U}(Y_t, \Gamma_t, \Delta_t, 1+i_t) = \theta u(\frac{(1-\Gamma_t)Y_t}{\theta}) + (1-\theta)u(\frac{\Gamma_t Y_t \mathcal{Q}(1+i_t)}{1-\theta}) - v(\frac{Y_t\Delta_t}{A_t})$$

where $\mathcal{Q}(1+i_t) = (1+i_t)^{1/2}$ is a monotonically increasing and differentiable function of $1+i_t$.

Having derived the social planner's objective function, the next topic is the characterization of constraints facing the social planner. In fact, there are two constraints for the social planner's optimization problem. The first one is the evolution equation of relative price distortion. The second one is the Euler equation that corresponds to the household's holdings of short-term government bonds. The reason why the second constraint is included is that the social planner's objective function is directly affected by the short-term nominal interest rate. In fact, this result reflects the important feature of the Diamond-Dybvig model where there are distinction between early and late consumers.

In sum, the social planner's optimization problem can be formulated as follows.

$$\max \sum_{t=0}^{\infty} \beta^t E_0[\mathcal{U}(Y_t, \Gamma_t, \Delta_t, 1+i_t) - \Lambda(Y_t, \Gamma_t)(\frac{\Xi_t}{1+i_t} - \frac{\Xi_{t-1}}{\Pi_t}) + \Psi_t(\Delta_t - \Delta(\Pi_t, \Delta_{t-1}))] \tag{4.1}$$

where Ξ_t is the Lagrange multiplier of the Euler equation and Ψ_t is the Lagrange multiplier of relative price distortion. The social planner's optimization problem (4.1) deserves a couple of points. The first point is that the choice variables of the social planner's problem are five variables such as (Y_t, Π_t, Γ_t, Δ_t, $1+i_t$). In this light, this optimal policy problem can be regarded an extended nonlinear version of New Keynesian optimal policy problem that minimizes an weighted sum of output-gap variability and inflation variability given the Phillips curve equation. The second point is that this optimal policy problem allows for the timeless perspective of Benigno and Woodford (2012).

The social planner's optimization conditions can be written as follows.

$$\begin{array}{lcl} \mathcal{U}_Y(Y_t, \Gamma_t, \Delta_t, 1+i_t) & = & \Lambda_Y(Y_t, \Gamma_t)(\frac{\Xi_t}{1+i_t} - \frac{\Xi_{t-1}}{\Pi_t}) \\ \mathcal{U}_\Gamma(Y_t, \Gamma_t, \Delta_t, 1+i_t) & = & \Lambda_\Gamma(Y_t, \Gamma_t)(\frac{\Xi_t}{1+i_t} - \frac{\Xi_{t-1}}{\Pi_t}) \\ \mathcal{U}_{(1+i)}(Y_t, \Gamma_t, \Delta_t, 1+i_t) & = & -\frac{\Xi_t \Lambda(Y_t, \Gamma_t, 1+i_t)}{(1+i_t)^2} \\ \mathcal{U}_\Delta(Y_t, \Gamma_t, \Delta_t, 1+i_t) & = & -(\Psi_t - \alpha\beta E_t[\Pi_{t+1}^\epsilon \Psi_{t+1}]) \\ \Delta_\Pi(\Pi_t, \Delta_{t-1})\Psi_t & = & -\Lambda(Y_t, \Gamma_t)\frac{\Xi_{t-1}}{\Pi_t^2} \end{array} \tag{4.2}$$

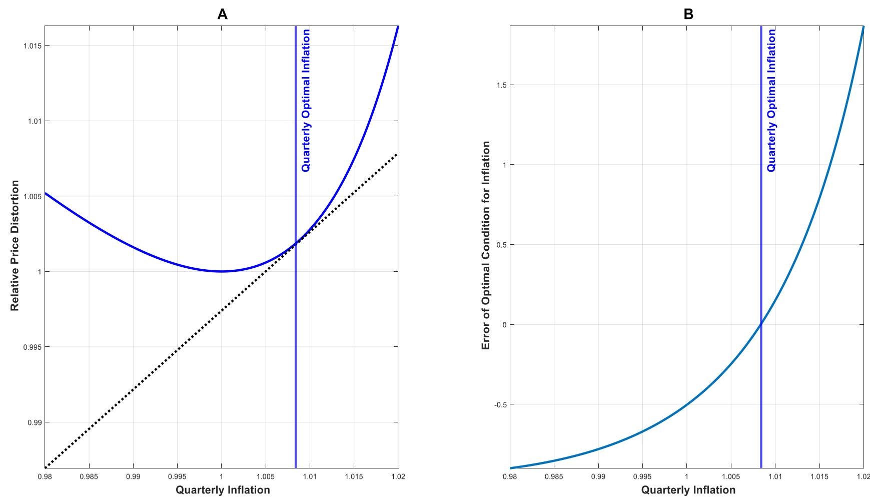
where each function's subscripts represent partial derivatives of corresponding variables. The first line is the first-order condition for output. The second line is the first order condition for Γ_t. The third line is the first-order condition for nominal interest rate. The fourth line is the first order condition for relative price distortion. The fifth equation is the first order condition for inflation.

It should be noted that the social planner's optimization problem of this paper from that of prototypical New Kenyesian model is the inclusion of the Euler equation into the set of constraints of the social planner's problem, which in turn alters the prescription of the social planner's optimization problem for both the optimal inflation target and the optimal monetary policy regime. The main reason for such a difference is that the inclusion of the Euler equation into the set of constraints of the social planner's problem creates a commitment value for future inflation that can affect the real value of future payoff of one-period holding of government bonds.

More precisely, the social planner can face a trade-off between the minimization of relative price distortion and the maximization of the commitment value for future inflation that helps to increase the social welfare. For this reason, the social planner can deviate from a zero inflation (that minimizes relative price distortion) in order to obtain the welfare benefit that is associated with the commitment value of future inflation discussed above.

We now turn to the implication for optimal inflation under discretion can be summarized as follows. In the absence of commitment, the inclusion of the Euler equation into the set of constraints

Figure 4.1: Optimal Inflation: Numerical Result



of the social planner's problem does not create a commitment value for future inflation that can affect the real value of future payoff of one-period holding of government bonds. The reason for this statement is that the social planner lose the power to control the public's expectation about future inflation. Under discretion, the social planner does not deviate from a zero inflation that minimizes relative price distortion.

Turning to the optimization condition for the prudential policy measure Γ_t, the two optimization conditions for the aggregate output and the prudential policy measure can be used to produce a tangent condition for two curves.

$$\frac{\mathcal{U}_Y(Y_t, \Gamma_t, \Delta_t, 1+i_t)}{\mathcal{U}_\Gamma(Y_t, \Gamma_t, \Delta_t, 1+i_t)} = \frac{\Lambda_Y(Y_t, \Gamma_t, 1+i_t)}{\Lambda_\Gamma(Y_t, \Gamma_t, 1+i_t)}$$

The left-hand side of this equation corresponds to the marginal rate of substitution between output and prudential measure from the social planner's indifference curve. The right-hand side of this equation corresponds to the marginal rate of substitution between output and prudential measure from the marginal utility of consumption that is used for the stochastic discount factor in asset markets. This condition demonstrates the trade-off between the aggregate output and prudential policy measure facing the benevolent social planner.

The policy prescription of the social planner's optimization problem (4.1) can be summarized as follows. First, the optimal inflation rate is positive under commitment for the timeless perspective. Second, the price-level targeting is the solution to the social planner's problem under discretion, which is different from the result of existing works on the optimal monetary policy. The reason for this difference is that the social planer's problem of this paper includes two independent constraints such as the Phillips curve and the Euler equations, whereas the social planner's problem of typical New Keynesian models includes only one constraint namely the Phillips curve equation. If the Euler equation is included as the constraint for the social planner's problem, the social planner's has power to control the expected real interest rate by making commitment on future inflation, which is absent in the case of the social planner's problem with only one constraint of the Phillips curve equation.

Figure 4 shows a numerical solution to the social planner's optimization conditions (4.2) that can be obtained under a set of plausible parameter values. Panel A is the graph of relative price distortion where x-axis is quarterly gross inflation and y-axis is relative price distortion. The slope of relative price distortion corresponds to the left-hand side of the fifth line in equation (4.2). The straight line with a positive slope corresponds to the right-hand side of the fifth line in equation (4.2). Panel A thus illustrates that the optimal inflation is determined at the point where the straight line is tangent to the graph of relative price distortion. Panel B demonstrates how the difference between the right-hand and left-hand sides of the fifth line in equation (4.2) changes as steady-state gross inflation rises where x-axis is quarterly gross inflation and y-axis is the difference between between the right-hand and left-hand sides of the fifth line. Panel B thus shows that the optimal steady-state gross inflation is determined at the point where the difference becomes zero.

In each panel, the optimal inflation is marked by a vertical line. Panels A and B confirm that vertical lines mark the same optimal inflation. In sum, Figure 4 indicates that the optimal (yearly) steady-state inflation rate is 3.4% which is higher than the actual inflation target set by central banks of advanced countries, while it should be admitted that numerical results for the optimal inflation target are sensitive to changes in parameter values.[9]

5 Bank Run

In this section, we now turn to a bank-run model that reflects the solvency view of bank failures emphasized by a series of recent works including Gertler and Kiyotaki (2015) and Correia, Luck and Verner (2025) and e.t.c. An important feature of the model in this section is that households receive noisy signals about bank's net-worth at the end of each period. The motivation of this assumption is to make household's behavior in the model consistent with the observed behavior of depositors in the recent episode of Silicon Valley Bank and Signature Bank. The other important feature is that individual depositors refer to two information variables such as bank's net-worth and market bond price (or yield) at the same time when they choose to run on banks. The introduction

[9]The parameter values used for the numerical result in this section are $\sigma = 1$, $\chi = 0$, $\alpha = 0.60$, $\beta = 0.995$ and $\epsilon = 11$. The graph of relative price distortion is taken from its recursive specification presented in Yun (2005).

of this feature into the model is also motivated by the recent failures of Silicon Valley Bank and Signature Bank.

5.1 Bank Run and Liquidation of Bank's Assets

Each bank can retrieve cash from its asset investments when an emergency state breaks out. In fact, each bank owns its emergency fund at period t $(= E_{b,t})$ that is defined to be the sum of funds retrieved from lending in the federal funds market and net revenue from selling long-term bonds as can be seen below.

$$E_{b,t} = \underbrace{D_t - X_t}_{\text{Federl Funds Market}} + \underbrace{(P_t^{(\tau)} - 1)\omega_t(D_t - X_t)}_{\text{Net Revenue from Selling Bonds}} \tag{5.1}$$

Each bank uses its own emergency fund when ATM reserves is short of withdrawals of early consumers as can be seen below.

$$X_t < \phi_t(1 + i_{1,t})D_t \tag{5.2}$$

where $\phi_t(> \theta)$ is the measure of depositors who withdraw in the first-half subperiod of period t. It should be noted that equation (5.2) corresponds to the violation of sequential service constraint in the Diamond-Dybvig model. The difference of the model of this paper from the Diamond-Dybvig model is that even when the condition of equation (5.2) is satisfied, bank runs do not take places. The reason for this argument is that individual banks can quickly retrieve cash from selling their assets and thus prepare their own emergency funds as defined in equation (5.1). Instead of equation (5.2), bank runs take place when the following condition holds.

$$\begin{array}{rcl} X_t + E_{b,t} & < & \phi_t(1 + i_{1,t})D_t \\ N_{b,t} & < & (1 - \phi_t)(1 + i_{2,t})D_t \end{array} \tag{5.3}$$

The left-hand side of the second line of equation (5.3) includes bank's beginning-of-period net-worth $(= N_{b,t})$ in order to allow for the possibility that banks can sell their own shares in the stock market and thus raise funds to meet withdrawals of late consumers. It should be noted that when bank runs take place, the bank's net-worth can be written as

$$N_{b,t+1} = X_t + E_{b,t} + N_{b,t} - (\phi_t(1 + i_{1,t})D_t + (1 - \phi_t)(1 + i_{2,t})D_t) \tag{5.4}$$

It follows from equation (5.4) that banks are insolvent when the two inequalities of equation (5.3) are met. In this light, equation (5.2) corresponds to the bank run view, whereas equation (5.3) corresponds to the solvency view. The scenario of failing banks in the model of this paper requires both of the two equations (5.2) and (5.3) in the following sense. When the condition of (5.2) holds, depositors begin to line up at withdrawal services of banks. But it does not immediately lead to failures of banks until the condition of (5.3) is met.

The next discussion turns to the determination of the liquidation price of long-term bonds. The determination of the liquidation price of long-term bonds is associated with the liquidation condition of long-term bonds held by banks. The liquidation condition of long-term bonds is the first-line of equation (5.3). The liquidation price of long-term bonds is defined to be the market price of long-term bonds that satisfies the equality of $X_t + E_{b,t} = \phi_t(1 + i_{1,t})D_t$. Hence the liquidation price of long-term bonds ($= \underline{\mathrm{P}}_t^{(\tau)}$) satisfies the following condition.

$$1 - \omega_t(1 - \underline{\mathrm{P}}_t^{(\tau)})(1 - \frac{X_t}{D_t}) = \phi_t(1 + i_{1,t}) \ \rightarrow \ \underline{\mathrm{P}}_t^{(\tau)} = 1 - \frac{1 - (\Gamma_t^{-1} - 1)(\phi_t/\theta - 1)}{\omega_t} \tag{5.5}$$

given a positive value of ϕ_t less than one and where a sufficient condition for the liquidation price of long-term bonds to be less than one is $\phi_t < \theta/(1 - \Gamma_t)$. The liquidation price of long-term bonds is then used to determine if the aggregate economy falls onto the state of bank run. The aggregate economy falls onto the state of bank run depending on the following condition.

$$\begin{array}{lll} \text{Bank Run} & \text{if} & P_t^{(\tau)} < \underline{\mathrm{P}}_t^{(\tau)} \\ \text{No Bank Run} & \text{if} & P_t^{(\tau)} \geq \underline{\mathrm{P}}_t^{(\tau)} \end{array} \tag{5.6}$$

5.2 Household's Information about Bank's Net Worth

A noisy signal about bank's net worth hits the information set of households in the second-half of each period. The noisy signal ($= S_{b,t+1}$) is defined as the sum of the true value of next-period's bank net worth ($= N_{b,t+1}$) and a noise ($= \epsilon_t$).

$$S_{b,t+1} = N_{b,t+1} + \epsilon_t \tag{5.7}$$

The noisy signal about bank's net worth is not the same across households. The measure of households for each noise value ϵ_t is determined by a probability distribution whose cumulative distribution function is $\mathcal{F}(\epsilon_t)$ with probability density function $f(\epsilon_t)$.

The expected solvency constraint is $E(N_{b,t+1}|S_{b,t+1}) \geq 0$ for a household whose noisy signal about bank's net-worth is $S_{b,t+1}$. It means that there is a threshold value of the noisy signal about bank's net worth at which bank's expected solvency constraint is binding: $E[N_{b,t+1}|\underline{S}_{b,t+1}] = 0$. The threshold value of the noisy signal ($= \underline{S}_{b,t+1}$) about bank's net worth can be obtained as follows. The household's ex-post forecast of bank's net worth is

$$\begin{aligned} E[N_{b,t+1}|S_{b,t+1}] &= E[N_{b,t+1}|I_t] + \beta_{H,t}(S_{b,t+1} - E[N_{b,t+1}|I_t]) \\ \beta_{H,t} &= \tfrac{VAR(N_{b,t+1}|I_t)}{VAR(N_{b,t+1}|I_t) + VAR(\epsilon_t)} \end{aligned} \tag{5.8}$$

where $\beta_{H,t}$ is the household's adjustment coefficient to its expectation errors evaluated on the basis of its signal with $0 < \beta_{H,t} < 1$. It follows from the first line of equation (5.8) that $E[N_{b,t+1}|\underline{S}_{b,t+1}]$ $= 0$ leads to the following representation of the threshold value of the noisy signal about bank's net worth.

$$\underline{S}_{b,t+1} = -\frac{(1 - \beta_{H,t})E[N_{b,t+1}|I_t]}{\beta_{H,t}} \tag{5.9}$$

5.3 Equilibrium Aggregate Withdrawal Function

Each household determines whether to withdraw on the basis of his or her information about bank's net-worth given that the household has a noisy signal about bank's net worth discussed above. How many late consumers do forecast that banks are insolvent on the basis of their noisy signals about bank's net-worth? In order to answer this question, one should find the level of forecast error that corresponds to the threshold value of the noisy signal about bank's net-worth. It follows from equation (5.7) that $\underline{\epsilon}_t = \underline{S}_{b,t+1}$ - $N_{b,t+1}$. If $\epsilon_t \leq \underline{\epsilon}_t$, then households forecast that banks are insolvent. Hence the fraction of late consumers who forecast that banks are insolvent is $\mathcal{F}(\underline{S}_{b,t+1} - N_{b,t+1})$. In the meanwhile, it follows from equations (5.3) and (5.4) that the liquidation price of long-term bonds is the market price of long-term bonds when strict inequalities are replaced by equalities in equation (5.3). It means that $N_{b,t+1} = 0$ when bank runs take place, which is consistent with the solvency view. Hence the fraction of late consumers who forecast that banks are insolvent can be rewritten as $\mathcal{F}(\underline{S}_{b,t+1})$. In sum, the aggregate withdrawal function can be written as follows.

$$\phi_t = \theta + (1-\theta)\mathcal{F}(\underline{S}_{b,t+1})I_{[P_t^{(\tau)} < \underline{P}_t^{(\tau)}]} \tag{5.10}$$

where $I_{[P_t^{(\tau)} < \underline{P}_t^{(\tau)}]}$ is an indicator function whose value is equal to one only when $P_t^{(\tau)} < \underline{P}_t^{(\tau)}$ and zero otherwise.

It would be now helpful to explain the motivation of the inclusion of two information variables in equation (5.10) as the trigger of household's run on bank. More banks should sell long-term bonds as more households choose to run on banks. It means that an increase (a decrease) in the level of $\underline{S}_{b,t+1}$ leads to a decrease (an increase) in the level of $\underline{P}_t^{(\tau)}$, which in turn leads to a decrease (an increase) in the level of bank's fund available for its deposit repayment to households who choose to withdraw. Knowing the presence of such interaction described above, households at an equilibrium determine whether to run on banks on the basis of two information variables $\underline{S}_{b,t+1}$ and $\underline{P}_t^{(\tau)}$.

The aggregate withdrawal function specified in equation (5.10) implies that both of private and public signals are used for households to determine their behaviors in the model of this paper. Specifically, individual households observe $S_{b,t+1}$, $\underline{S}_{b,t+1}$, $P_t^{(\tau)}$, and $\underline{P}_t^{(\tau)}$ but not $N_{b,t+1}$ before they choose whether to run on bank. The market price of long-term bonds is determined as a market equilibrium outcome. In this context, $S_{b,t+1}$ is a private signal and ($P_t^{(\tau)}$, $\underline{P}_t^{(\tau)}$) is interpreted as a two-dimensional public signal for individual households. In the case of conflicting messages between private and public signals, households do not choose to run on bank. Households choose to run on bank only when both private and public signals produce consistent messages about the aggregate bank run.

5.4 Impact of Bank Run on the Aggregate Consumption Demand

The main topic of this subsection is the impact of bank runs on the aggregate consumption demand. The main result of this subsection is that $C_{N,t} > C_{R,t}$ where $C_{N,t}$ and $C_{R,t}$ denote the aggregate consumption demand without and with bank runs respectively. The reason for this result is that the aggregate consumption demand under bank run is affected by liquidation values of bank's assets that are lower than their normal market values.

In order to show this result, the starting point is the substitution of $N_{b,t+1} = 0$ as the condition for bank runs discussed above into equation (5.4) to obtain the following relation.

$$N_{b,t+1} = 0 \;\rightarrow\; X_t + \underline{E}_{b,t} + N_{b,t} = \phi_t(1 + i_{1,t})D_t + \hat{\phi}_t(1 + i_{2,t})D_t$$

where $\phi_t + \hat{\phi}_t \leq 1$. The left-hand side of the equation after the arrow is bank's liquidation value at the time of bank runs, while the right-hand side of the equation after the arrow is depositor's withdrawals from banks at the time of bank runs. In the case of $\hat{\phi}_t < 1 - \phi_t$, a fraction of depositors do not have paybacks of their deposits even with the liquidation of all bank's assets including bank's own shares.

The realized aggregate consumption under bank runs does not include the amount of bank's funds raised from the sale of bank's own shares because it takes time for a bank to sell all of its own shares in the stock market. For this reason, $C_{R,t} = X_t + \underline{E}_{b,t}$ reflecting that the realized aggregate consumption demand under bank runs is equal to the total amount of withdrawals from banks at the time of bank runs. Hence the aggregate consumption demand under bank run is affected by liquidation values of bank's assets.

$$\begin{array}{rcl} C_{R,t} & = & X_t + \underline{E}_{b,t} \;\rightarrow\; C_{R,t} = D_t(1 - \omega_t(1 - \underline{\mathrm{P}}_t^{(\tau)})\Gamma_t) \\ \frac{C_{R,t}}{C_{N,t}} & = & (1 - \omega_t(1 - \underline{\mathrm{P}}_t^{(\tau)})\Gamma_t) \end{array} \tag{5.11}$$

The second line of equation (5.11) reflects the fact that the aggregate consumption demand in normal times is equal to the amount of deposit ($C_{N,t} = D_t$). As a result, the ratio of the aggregate consumption demand ($= C_{R,t}$) under bank run to the aggregate consumption demand ($= C_{N,t}$) in normal times is less than one if and only if $0 < \omega_t < 1$, $0 < \underline{\mathrm{P}}_t^{(\tau)} < 1$ and $0 < \Gamma_t < 1$.

5.5 Expected Bank Run and Bond Prices

In each period t, all agents expect that the aggregate bank run breaks out in the next period with a probability of p_t. The household's anticipation about next-period's bank run should be consistent with the aggregate withdrawal function specified in equation (5.10). The aggregate withdrawal function implies that bank run breaks out when $\phi_t > \theta$. In addition, equation (5.10) implies that $\phi_t > \theta$ if and only if $\mathcal{F}(\underline{S}_{b,t+1}) > 0$. The household's anticipation at period t about the possibility of bank run at period $t+1$ should be consistent with the condition that $\phi_{t+1} > \theta$ if and only if $\mathcal{F}(\underline{S}_{b,t+2}) > 0$. For this reason, households at period t are supposed to believe that a bank run can happen at period $t+1$ with a probability of p_t where p_t is defined as follows.

$$p_t = E_t[\mathcal{F}(\underline{S}_{b,t+2}) I_{[P^{(\tau)}_{t+1} < \underline{P}^{(\tau)}_{t+1}]}] \tag{5.12}$$

The expected bank run can affect market prices of all assets. For example, the stochastic discount factor can be defined as a weighted average of the inter-temporal marginal rates of substitution under bank run and in normal times. The no-arbitrage conditions for equilibrium prices of bonds at normal times and under bank run are symmetric so that the following condition holds.

$$\begin{aligned} P^{(k)}_t &= \beta E_t[(p_t \tfrac{\Lambda_{R,t+1}}{\Lambda_{N,t}\Pi_{R,t+1}} + (1-p_t)\tfrac{\Lambda_{N,t+1} P^{(k-1)}_{N,t+1}}{\Lambda_{N,t}\Pi_{N,t+1}}) P^{(k-1)}_{t+1}] \\ P^{(\tau)}_t &= \frac{E_t[P^{(\tau-1)}_{t+1}]}{F_t + G(\overline{c}_t - \overline{v}_t(D_t - X_t)/S^{(\tau)}_t)} \end{aligned} \tag{5.13}$$

for $k = 1{,}2 \cdots$ but $k \neq \tau$.

5.6 Optimal Monetary and Prudential Policies with Expected Bank Runs

The analysis of this subsection adopts a dynamic programming approach for the analysis of the optimal policy from the timeless perspective in the spirit of Benigno and Woodford (2012). In this framework, the distinction between commitment and discretion determines whether the value function is affected by the lagged values of lagrange multipliers of constraints that includes expectations of private agents.

In the absence of bank runs, the social planner's problem can be rewritten as follows. The first step is to define the Lagrangian of the social planner's problem as follows.

$$\begin{aligned} \mathcal{L}(Y_t, \Gamma_t, \Delta_t, \Delta_{t-1}, 1+i_t, \Pi_t, \Psi_t, \Xi_t, \Xi_{t-1}) &= \mathcal{U}(Y_t, \Gamma_t, \Delta_t, 1+i_t) - \Lambda(Y_t, \Gamma_t)(\tfrac{\Xi_t}{1+i_t} - \tfrac{\Xi_{t-1}}{\Pi_t}) \\ &+ \Psi_t(\Delta_t - \Delta(\Pi_t, \Delta_{t-1})) \end{aligned}$$

In this Lagrangian of the social planner's problem, lagged values of the lagrange multiplier of the nonlinear IS curve equation and relative price distortion are included. The second step is to formulate a Bellman equation for the value function that is defined as the expected discounted sum of the current and future values of the Lagrangian defined above. The Bellman equation for the optimal policy without bank run can be written as follows.

$$v_t(\Delta_{t-1}, \Xi_{t-1}) = \max\{\mathcal{L}(Y_t, \Gamma_t, \Delta_t, \Delta_{t-1}, 1+i_t, \Pi_t, \Psi_t, \Xi_t, \Xi_{t-1}) + \beta E_t[v_{t+1}(\Delta_t, \Xi_t)]\}$$

where $v_t(\Delta_{t-1}, \Xi_{t-1})$ represents the value function at period t.

The next discussion moves onto the social planner's problem with expected bank run. In order to distinguish between states with and without bank run, s_t is used to mark the state without bank run as $s_t = 1$ and the state with bank run as $s_t = 2$. Given the definition of s_t, the Bellman equation for the optimal policy problem with bank run can be written as follows.

$$v_{k,t}(\Delta_{t-1}, \Xi_{t-1}) = \max\{\mathcal{L}_k(Y_t, \Gamma_t, \Delta_t, \Delta_{t-1}, 1+i_t, \Pi_t, \Psi_t, \Xi_t, \Xi_{t-1}) + \beta E_t[\sum_{k=1}^{2} p_{k,t} v_{k,t+1}(\Delta_t, \Xi_t)]\}$$

where $\mathcal{L}_k(Y_t, \Gamma_t, \Delta_t, \Delta_{t-1}, 1+i_t, \Pi_t, \Psi_t, \Xi_t, \Xi_{t-1}) = \mathcal{L}_k(Y_t, \Gamma_t, \Delta_t, \Delta_{t-1}, 1+i_t, \Pi_t, \Psi_t, \Xi_t, \Xi_{t-1}, s_t = k)$, $v_{k,t}(\Delta_{t-1}, \Xi_{t-1}) = v_t(\Delta_{t-1}, \Xi_{t-1}, s_t = k)$ and $p_{k,t} = p_t(s_{t+1} = k)$ for $k = 1$ and 2.

Comparing these the two Bellman equations with and without expected bank runs, the difference between them is whether to include $s_t = k$ for $k = 1$ and 2 in the Bellman equation of the social planner's optimization problem with expected bank runs. Specifically, the optimization conditions of the social planner's optimization problem with expected bank runs can be summarized as follows.

$$\begin{array}{lcl}
\mathcal{U}_Y(Y_t, \Gamma_t, \Delta_t, 1+i_t) & = & \Lambda_Y(Y_t, \Gamma_t)(\frac{\Xi_t}{1+i_t} - \frac{\Xi_{t-1}}{\Pi_t}) \\
\mathcal{U}_\Gamma(Y_t, \Gamma_t, \Delta_t, 1+i_t) & = & \Lambda_\Gamma(Y_t, \Gamma_t)(\frac{\Xi_t}{1+i_t} - \frac{\Xi_{t-1}}{\Pi_t}) \\
\frac{\mathcal{U}_{(1+i)}(Y_t, \Gamma_t, \Delta_t, 1+i_t)}{\Lambda(Y_t, \Gamma_t)} & = & -\frac{\Xi_t}{(1+i_t)^2} \\
\mathcal{U}_\Delta(Y_t, \Gamma_t, \Delta_t, 1+i_t) & = & -(\Psi_t - \alpha\beta E_t[\sum_{k=1}^{2} p(s_{t+1} = k)\Pi_{t+1}(s_{t+1} = k)^\epsilon \Psi_{t+1}(s_{t+1} = k)]) \\
\Delta_\Pi(\Pi_t, \Delta_{t-1})\Psi_t & = & \Lambda(Y_t, \Gamma_t)\frac{\Xi_{t-1}}{\Pi_t^2}
\end{array}$$

It follows from this equation that the social planner's commitment about future inflation is affected by the inclusion of expected bank runs. The set of optimization conditions summarized above also implies that the incorporation of expected bank runs into the model is not likely to change the optimality of a positive inflation target shown in the previous section.

6 Conclusion

It has been shown in this paper that the introduction of the Diamond-Dybvig's financial frictions into an otherwise prototypical New Keynesian model can break down the divine coincidence of pursuing a zero inflation because the social planner strikes the balance between two distinct trade-offs. One is the trade-off between inflation and output gap and the other is the trade-off between current-period and next-period consumptions.

The concluding part of this paper summarizes its implication for potential desirability of co-ordination between prudential and public policies in a broad sense. The analysis of this paper opens the possibility that if the government's fiscal policies and the central bank's unconventional monetary policies can affect market supplies of long-term government bonds held by commercial banks, such public policies can have unintended impacts on financial stability. It would be thus potentially desirable to incorporate this channel into forecasts about the aggregate consequences of public policies that are followed by changes in market supplies of long-term government bonds.

Recent episodes of US bank failures also indicate that this channel is likely to have more profound impacts especially when financial intermediaries heavily rely on interest earnings of long-term bonds and thus their balance sheets are relatively more sensitive to changes in the yield curve.

An important motivation of this paper is that balance sheets of financial intermediaries can deteriorate severely and quickly with their excessive exposures to the interest rate risk. On top of this issue, balance sheets of central banks in advanced countries are likely to possess similar perils of the interest rate risk because of large-scale holdings of long-term bonds. In this vein, it would be possible to argue that the interest rate risk can be a crucial ingredient of financial cycles in advanced countries through its impacts on balance sheets of financial intermediaries and even central banks. A related future research topic would be thus the analysis of welfare consequences of unconventional monetary policies to take into account both social costs of central bank balance sheet's exposure to the interest rate shock and social benefits of the stabilization of inflation and output.

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