Beliefs-driven Stock Market Entry and Exit *

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Abstract

We study how learning from experience affects stock market participation in overlapping generations. During episodes of market exuberance and low risk premiums, the unexperienced enter the market often with high leverage exposing them to the risk of abruptly losing wealth, with the consequence being exit. On the contrary, experienced cohorts typically remain in the market despite pessimism. These patterns generate procyclical participation and feedback effects between participation and learning, amplifying fluctuations in interest rates and market volatility and lead to large welfare losses. Importantly, the model reproduces empirical evidence on participation, entry and exit from multiple countries.

Keywords: Endogenous Stock Market Participation, Stock Market Entry and Exit, Learning from Experience, Endogenous Learning

JEL Classification: E2, G10, G11, G12

1 Introduction

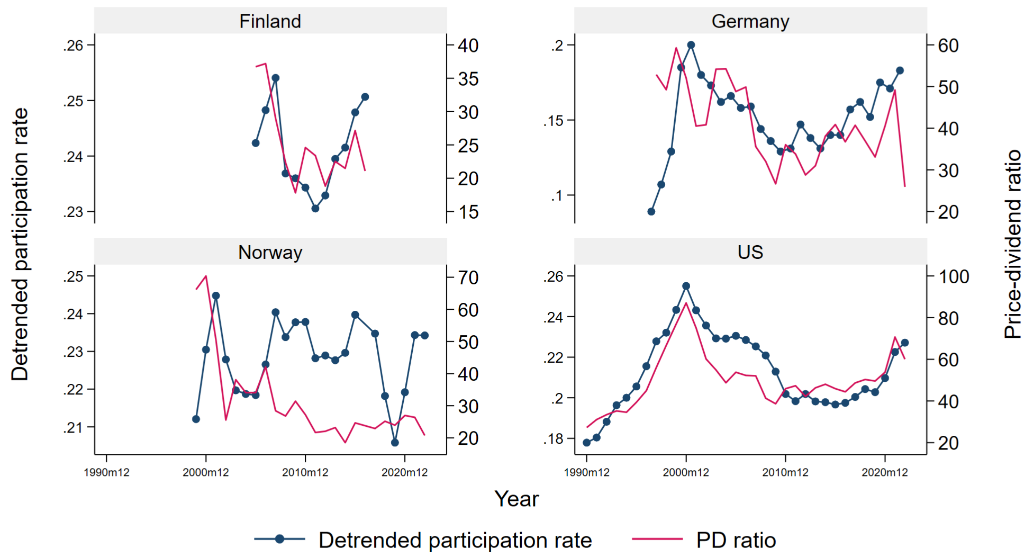
Our objective is to understand how beliefs about returns drive stock market entry, exit, and reentry decisions and how that affects asset prices and welfare. We study this question in a model with incomplete information in which a large number of constrained rational cohorts learn from data observed during their lifetime. This experience based bias and the fact that more than one source of information (a fundamental and a signal) is available for the learning problem, imply a large and time-varying cross-section of beliefs about growth. In equilibrium, the fluctuations of perceived expected returns drive the trading strategies of agents, which endogenously determine fluctuations in the real short rate of interest, market price of risk, stock market volatility, who participate in the stock market and who do not.

To understand the consumption and wealth fluctuations and entry and exit, it is important to look at who is in the market and how it depends on average optimism and pessimism. When the unexperienced are the optimists, they take levered long positions in the stock market. Further, when the unexperienced are the pessimists they usually exit the market. This is what one would expect. However, for the experienced we see a different pattern. When the experienced are the pessimists they often remain in the market because the endogenous cutoff is not just driven by estimation errors but significantly driven by the consumption distribution, and this is often gravitated towards the experienced. This asymmetric response to waves of pessimism leads to a time-varying wedge in consumption growth between experienced and unexperienced. Specifically, exuberance-driven unexperienced tend to enter the stock market with high leverage when the market price of risk is low. These are times when participants have lower consumption growth than nonparticipants. Subsequently, agents may exit the stock market in disappointment.

In this regard, Figure 1 suggests that market participation in Finland, Germany, Norway and the USA tend to rise during times of elevated valuation ratios.[1] As alluded above, such

[1] The correlation between the participation rate and lagged PD-ratio in Finland, Germany, Norway and the USA are 0.63, 0.10, 0.22, and 0.78 respectively.

Figure 1: **The Stock Market and Participation.** The plots show the time series of the price-dividend ratio. Price-dividend ratios are calculated using $\frac{P_t}{D_t} = \frac{R_t^{price}}{R_t^S - R_t^{price}}$, where R_t^{price} is the price return, and R_t^S is the total return including dividends. The data are from Amit Goyal's website for the US, and from Ken French's website for the other countries.



behavior may prove costly, since high past returns or elevated valuation ratios often signal lower, rather than higher, future risk premia. This suggests that investors may inadvertently adopt a procyclical entry-exit strategy that undermines long-term performance.

What is the cost of entry and exit in terms of utility? Interpreted through the lens of our model, life-time utility in a frictionless benchmark economy with complete information, where all cohorts participate, equates with the life-time utility in our economy when consumption growth is at most 1.3% compared to the actual 2%. In our view, the obvious policy response to such large costs from poorly timed sequences of entry and exit must be a substantially increased effort to improve the financial literacy of households. To succeed with such an effort may require, among other actions, including financial literacy in high school and university curriculum, as well as mandatory information through financial institutions.

Reproducing the time-series of stock market participation, and entry and exit reflects the essence of our analysis. To tease out realistic beliefs-driven effects, we feed —in the spirit of

Lettau, Ludvigson, and Wachter (2007)— our model with shocks extracted from the stock market and from various macroeconomic data standing in for the signal. This exercise isolates our proposed mechanism and generates implied time-series of participation rates for Finland, Germany, Norway and the USA. Broadly speaking, the correlations between the actual time-series of stock market participation and the model counterparts are statistically significant. For Finland and Norway, we can increase the requirement on the model and compare the actual time-series of entry and exit to the counterparts of the model. Specifically, while the participation rate is tightly linked to entry and exit in the model, this does not directly imply that reproducing the overall dynamics of the participation rate in a country also reproduces its entry and exit dynamics. Hence it is reassuring that this exercise produces statistically significant entry and exit correlations between the model and the data counterparts for Finland. Overall, each crucial bit of empirical evidence, which for the USA is based on three data sources, and the model-based evidence, which stands on data from four countries, support the beliefs-driven mechanism driving participation, entry, exit and reentry.

What are the basic ingredients of the model? The models has three key components - 1) overlapping generations, 2) experience based learning, and 3) a friction that prevents short selling. We have a broad interpretation of nonparticipation in mind that includes various behavioral biases or institutional restrictions, all of which effectively induce investors to leave the stock market when returns disappoint. Technically we impose short selling constraints on the agents in the economy. We do not try to further explain these underlying causes, but instead want to understand the consequences on the equilibrium.

Agents in our overlapping generations (OLG) economy use Bayes' formula to update their beliefs, but deviate from full rationality in two ways - 1) an agent uses only the data observed during her lifetime (experience based learning) and 2) nonparticipating agents pay less attention to news than participating agents. We model the experience based learning by assuming that agents start with a diffuse prior, but learn for a certain number of years before entering the market. Agents learn about expected dividend growth from the realization of dividends and a signal. When agents become nonparticipants, we assume that they stop learning from the signal, representing limited attention to the stock market. Due to limited attention by nonparticipants, the speed at which the agents learn is determined by the endogenous asset prices. This sets our model apart from previous models of learning with agents that agree to disagree, where the updating occurs independently of equilibrium prices. As a consequence, when stock market participation is low, agents in the economy learn slower on average.

For transparency and tractability, we use log utility. Since log utility produces constant stock market volatility even with heterogeneous beliefs, we assume that agents differ in their time preferences within each cohort. This breaks the constant consumption to wealth ratio at the aggregate level by allowing for differing consumption and wealth shares despite the log utility. Hence, there are four endogenous state variables: 1) the consumption share of the participating agents, 2) the wealth share of the participating agents, 3) participants consumption weighted estimation error, and 4) participants wealth weighted estimation error.

The closed-form expressions for the real short rate of interest and market price of risk, which derive from consumption market clearing, depend on the consumption share and the consumption weighted estimation error of the participants. We term this result the *beliefs-driven participation effect*. The volatility of the stock market, which derives from clearing of the stock market, depends on all four state variables. In essence, the stock volatility varies over time as the gap between the consumption- and the wealth-weighted belief widens and shrinks, and with the dynamics of the ratio between the aggregate wealth to consumption share of the participants.

Guided by the model, we relate differences in model participation rates to differences in experiences in an empirical fashion to conclude that the model reproduces the empirics based on survey data (Survey of Consumer Finance) in Malmendier and Nagel (2011). To strengthen the evidence, we employ the Michigan survey to show that experiencing particularly high returns leads one to enter the stock market and that experiencing particularly low returns leads one to exit. In addition, the model reproduces the re-entry frequencies conditional on experienced returns data from Finland. Further, using data from Finland, Germany, Norway and the USA, we provide suggestive evidence for the participation rate to predict returns.

Our main contribution is theoretical and relates closely to the literate that explains stock market nonparticipation through one-time entry costs or per period monitoring costs in an equilibrium OLG setting such as the classical work of Gomes and Michaelides (2007),[2] with focus on the one-time decision to either enter or not to enter the stock market. Technically, our continuous-time general equilibrium model with overlapping generations of agents builds on Gârleanu and Panageas (2015), that focuses on asset pricing, and Ehling, Graniero, and Heyerdahl-Larsen (2018b), that focuses on how lifetime experiences affect expectations[3] in an otherwise frictionless market.[4] Our contribution vis-à-vis the literature on endogenous nonparticipation is a dynamic return-driven mechanism explaining entry, exit, and reentry, which then simultaneously determines movements in asset prices. For instance, the time-varying participation rate —an observable proxy for the *beliefs-driven participation effect*— predicts stock market returns in the model and the data.

Close to our work in its focus on entry and exit is Bonaparte, Korniotis, Kumar, Michaelides, and Zhang (2025). In their life cycle portfolio choice model with constant expectations a rare disaster or an increased labor income uncertainty along with various participation costs produce more realistic frequencies for entry and exit, which is a mechanism that is complementary to ours. Another recent related paper also using a life-cycle model with constant expectations is Gomes and Smirnova (2023); through empirically motivated participation costs, decreasing relative risk aversion, and human capital the model generates stock market

[2]See also the seminal paper on nonparticipation by Basak and Cuoco (1998); Guvenen (2009) focuses on matching asset prices with a participant and a nonparticipant; and Favilukis (2013) focuses on inequality driven by nonparticipation.

[3]For empirical evidence on lifetime experiences affecting expectations, preferences, and choices see Malmendier and Nagel (2011), Malmendier and Nagel (2016), Knüpfer, Rantapuska, and Sarvimäki (2017) and more recently Cocco, Gomes, and Lopes (2025).

[4]For a quantitative equilibrium model with an experience effect see Collin-Dufresne, Johannes, and Lochstoer (2017); see also Malmendier, Pouzo, and Vanasco (2020) and the references therein.

participation that is a hump-shaped function of age. Further, Galaasen and Raja (2024) present a portfolio choice problem with experience effects to explain stock market participation dynamics without requiring high per-period participation costs. They document exit and reentry margins of stock market participation showing among other things that the longer Norwegian households participate, the less likely they are to exit. What sets our model apart from these works is not just its asset pricing implications, but also the insight that participation drives overall learning and the welfare consequences of nonparticipation or more specifically the welfare costs emerging from exit and reentry.

Our paper builds on a vast literature of heterogeneous beliefs models.[5] Further, we build on models in this extended literature that incorporate an additional signal as in Detemple and Murthy (1997) or the sentiment index in Scheinkman and Xiong (2003) and Dumas, Kurshev, and Uppal (2009), where the signal or index may improve or inhibit learning about growth.[6] On a technical note, we solve the incomplete market with participants and nonparticipants as a complete market with fictitious state prices as in He and Pearson (1991) and Karatzas, Lehoczky, Shreve, and Xu (1991).[7]

Importantly, in the heterogeneous beliefs literature it is common to assume two type of agents. Hence, optimism and pessimism are basically predetermined. Therefore, nonparticipation is also predetermined as in Gallmeyer and Hollifield (2008).[8] This is true even when agents update their beliefs through learning as the pessimist in Detemple and Murthy (1997) or Gallmeyer and Hollifield (2008) cannot transition to be the optimist. With a constant growth rate, as the agents learn the true parameter over time, the disagreement declines and the nonparticipant turns into a participant as well. In contrast, in our framework agents or

[5] From this body of literature see, for example, Harris and Raviv (1993), Detemple and Murthy (1994), Jouini and Napp (2007), Cvitanić, Jouini, Malamud, and Napp (2012), Bhamra and Uppal (2014), and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018a).

[6] See also Xiong and Yan (2010).

[7] Papers that also use fictitious state prices to solve models with incomplete markets include Cuoco (1997), Basak and Croitoru (2000), Gallmeyer and Hollifield (2008) and Dieckmann (2011).

[8] We mention Gârleanu, Panageas, and Zheng (2023), a recent complementary study that employs, just as we do, heterogeneous beliefs along with log utility in a continuous-time OLG environment, where their focus is on the performance of a short seller facing a shorting fee.

cohorts smoothly transition from being relatively optimistic to relatively pessimistic. Thus, the agents in our economy endogenously enter, exit and reenter with realistic frequencies.

2 The Model

In our continuous-time overlapping generations (OLG) exchange economy, agents die at rate $\nu > 0$ to be replaced by newborns at the same rate. Hence, the total population size remains constant and can be normalized to equal 1, where the time-t size of the cohort born at time $s < t$ is $\nu e^{-\nu(t-s)} ds$.[9]

Output Y_t evolves as follows

$$dY_t/Y_t = \mu_Y dt + \sigma_Y dz_t^Y, \tag{1}$$

with z_t^Y representing a standard Brownian motion.

Further, agents observe an additional signal as in Dumas, Kurshev, and Uppal (2009). Here, only agents who actively trade the stock market observe the signal SI_t, capturing the notion that participants pay additional attention. It evolves as

$$dSI_t = \phi dz_t^Y + \sqrt{1-\phi^2} dz_t^{SI}, \tag{2}$$

where ϕ is the correlation between shocks to output and the signal, with z_t^{SI} representing a standard Brownian motion, uncorrelated with z_t^Y.

For experience to matter, we assume that agents observe the output and may observe the signal but do not know the value of expected growth μ_Y. At time s newborn agents start with a prior belief about expected output growth $\hat{\mu}_{s,s}$, which is dependent on the time of birth, and variance $\hat{V} > 0$, which is independent of it. Based on the perceived expected output growth $\hat{\mu}_{s,t}$ agents who do not participate in the stock market (we refer to them as

[9] See Blanchard (1985) or Gârleanu and Panageas (2015).

nonparticipants; N) observe the dynamics of output in the following way:

$$dY_t/Y_t = \hat{\mu}_{s,t}dt + \sigma_Y dz^Y_{s,t}, \tag{3}$$

where $z^Y_{s,t}$ denotes a Brownian motion under the belief of an agent born at time s. In similar fashion, the stock market participants (P) perceive the following pair of stochastic di￿erential equations:

$$dY_t/Y_t = \hat{\mu}_{s,t}dt + \sigma_Y dz^Y_{s,t}, \quad dSI_t = \phi dz^Y_{s,t} + \sqrt{1-\phi^2}dz^{SI}_{s,t}, \tag{4}$$

where $z^Y_{s,t}$ and $z^{SI}_{s,t}$ denote two uncorrelated Brownian motions under the belief of an agent born at time s. Here, all agents in the cohort s participate or do not participate, and the decision to become a participant or nonparticipant will be endogenously determined.

Independent of whether the agents follow the additional signal or not, they update their beliefs about expected output growth through the Bayes' rule. Specifically, by standard filtering theory, at time t a cohort born at time s perceives the dynamics of the expected output growth and its posterior variance in the following way:

$$d\hat{\mu}_{s,t} = \begin{cases} \frac{\hat{V}_{s,t}}{\sigma_Y} dz^Y_{s,t}, & \text{if } N\text{, or} \\ \frac{\hat{V}_{s,t}}{\sigma_Y}\left(dz^Y_{s,t} - \frac{\phi}{\sqrt{1-\phi^2}}dz^{SI}_{s,t}\right), & \text{if } P\text{, where} \end{cases} \tag{5}$$

$$\hat{V}_{s,t} = \begin{cases} \frac{\sigma_Y^2 \hat{V}'_s}{\sigma_Y^2 + \hat{V}'_s\left(t - t'_s\right)}, & \text{if } N\text{, or} \\ \frac{\sigma_Y^2(1-\phi^2)\hat{V}'_s}{\sigma_Y^2(1-\phi^2) + \hat{V}'_s\left(t-t'_s\right)}, & \text{if } P\text{,} \end{cases} \tag{6}$$

where t'_s denotes the last time when all agents from a cohort s switched from participant to nonparticipant or vice versa. Else, $t'_s = s$. Similarly, $\hat{V}'_s$ stands for the variance of the perceived growth of output at t'_s. From Equations (5 - 6), we see that as cohorts learn about the true growth of output over time, their posterior variance decreases.

Given that agents know σ_Y and ϕ, the perceived and true shocks are linked with each other in the following manner:

$$dz_{s,t}^{Y} = dz_t^Y - \Delta_{s,t}dt, \quad \text{and} \tag{7}$$

$$dz_{s,t}^{SI} = dz_t^{SI} + \frac{\phi}{\sqrt{1-\phi^2}}\Delta_{s,t}dt, \tag{8}$$

where $\Delta_{s,t} = \frac{\hat{\mu}_{s,t}-\mu_Y}{\sigma_Y}$ is the standardized estimation error. According to Equations (7 - 8) the dynamics of the expected output growth of a cohort born at time s, under the true probability measure, are

$$d\hat{\mu}_{s,t} = \begin{cases} -\frac{\hat{V}_{s,t}}{\sigma_Y}\Delta_{s,t}dt + \frac{\hat{V}_{s,t}}{\sigma_Y}dz_t^Y, & \text{if } N, \text{ or} \\ -\frac{\hat{V}_{s,t}}{\sigma_Y}\frac{1}{1-\phi^2}\Delta_{s,t}dt + \frac{\hat{V}_{s,t}}{\sigma_Y}dz_t^Y - \frac{\hat{V}_{s,t}}{\sigma_Y}\frac{\phi}{\sqrt{1-\phi^2}}dz_t^{SI}, & \text{if } P. \end{cases} \tag{9}$$

Solving the stochastic differential equations in (9) leads to the following proposition.

Proposition 1. *The estimation error at time t of the cohort born at time s is*

$$\Delta_{s,t} = \begin{cases} \frac{\sigma_Y^2}{\sigma_Y^2+\hat{V}_s'\left(t-t_s'\right)}\Delta_{s,t_s'} + \frac{\hat{V}_s'}{\sigma_Y^2+\hat{V}_s'\left(t-t_s'\right)}\left(z_t^Y - z_{t_s'}^Y\right), & \text{if } N, \text{ or} \\ \frac{\sigma_Y^2(1-\phi^2)}{\sigma_Y^2(1-\phi^2)+\hat{V}_s'\left(t-t_s'\right)}\Delta_{s,t_s'} + \frac{\hat{V}_s'(1-\phi^2)}{\sigma_Y^2(1-\phi^2)+\hat{V}_s'\left(t-t_s'\right)}\left\{\left(z_t^Y - z_{t_s'}^Y\right) - \frac{\phi}{\sqrt{1-\phi^2}}\left(z_t^{SI} - z_{t_s'}^{SI}\right)\right\}, & \text{if } P, \end{cases} \tag{10}$$

where $t_s^{'} > s$ denotes switching from participant to nonparticipant, or vice versa, and where $\Delta_{s,t_s'} = \frac{\hat{\mu}_{s,t_s'}-\mu_Y}{\sigma_Y}$, $\lim_{t-t_s'\to\infty}\Delta_{s,t} = 0$ a.s.

We interpret the magnitude of ϕ as information quality. A higher magnitude of ϕ entails acquiring more precise information from observing the signal. Moreover, the negative coefficient in front of $\frac{\phi}{\sqrt{1-\phi^2}}$ implies that with a positive ϕ value, shocks to output dz_t^Y and those to the signal dz_t^{SI} affect updates to beliefs in different directions. For intuition, when dz_t^{SI} is large, an agent who observes the signal as in Equation (2) tends to overestimate dz_t^Y. She thus forms lower expectations about the output growth rate, compared to if she does not observe the signal.[10]

2.1 Security Markets and Prices

Cohorts trade in an instantaneously risk-free bond, shares of the stock market, and annuities. The instantaneously risk-free bond is in zero net supply and evolves according to

$$dB_t/B_t = r_t dt, \tag{11}$$

where r_t denotes the real short rate of interest determined in equilibrium.

Shares in the stock market are normalized to one with price S_t. All output is paid out as dividend, where stock returns R_t have dynamics

$$dR_t = \left(dS_t + D_t dt\right)/S_t = \left(dS_t + Y_t dt\right)/S_t = \mu_t^S dt + \sigma_t^S dz_t^Y = \mu_{s,t}^S dt + \sigma_t^S dz_{s,t}^Y, \tag{12}$$

where μ_t^S, $\mu_{s,t}^S$ and σ_t^S are determined in equilibrium. Cohorts agree on current prices, but disagree on their probability distribution in the future. From the relation between the perceived and actual shocks in (7), we have $\mu_{s,t}^S = \mu_t^S + \sigma_t^S \Delta_{s,t}$.

There is a competitive insurance industry that offers annuity contracts as in Yaari (1965) at the actuarily fair rate ν per unit of wealth, such that agents with positive financial wealth $W_{s,t}$ find it optimal to annuitize all wealth. This is because cohorts face mortality risk and do not derive utility from leaving financial wealth behind. The contract stipulates that the insurance industry receives the financial wealth of the dead and in return pays $\nu W_{s,t}$ to agents currently alive.

[10] The impact of ϕ therefore differs from Dumas, Kurshev, and Uppal (2009), as in their setting output growth follows a mean-reverting process, where the signal contains information about shocks to output growth instead of output.

2.2 The Disagreement Process

In our framework, nonparticipants in the stock market are simply agents who cannot or do not want to sell short. Since shares in the stock market are the only source of consumption risk in the economy, agents become nonparticipants only when their perceived instantaneous price of risk $\theta_{s,t}$ is negative. Importantly, effectively binding or perceived constraints do not affect relatively optimistic cohorts who have a positive demand for the stock. Therefore, when a cohort perceives $\theta_{s,t} \geq 0$, all agents in that cohort participate in the stock market. Coherently, when a cohort perceives $\theta_{s,t} < 0$, all wealth is invested in the bond market. To solve the model, we make use of the constrained problem with fictitious state prices as in He and Pearson (1991) and Karatzas, Lehoczky, Shreve, and Xu (1991), which is equivalent to a complete market setting. Hence, in our fictitious economy with participants and nonparticipants, the stochastic discount factor perceived by a cohort born at time s follows the process

$$d\xi_{s,t}/\xi_{s,t} = -r_t dt - \theta^{+}_{s,t} dz^{Y}_{s,t}, \tag{13}$$

where $\theta^{+}_{s,t} = \max(\theta_{s,t}, 0)$. On the true probability measure, ξ_t follows

$$d\xi_t/\xi_t = -r_t dt - \theta_t dz^{Y}_t, \tag{14}$$

where from Equation (12), we have that $\theta_{s,t} = \theta_t + \Delta_{s,t}$, implying that nonparticipation occurs when $\Delta_{s,t} < -\theta_t$.

Next, we define the disagreement process $\eta_{s,t}$ through the relation between objective and perceived stochastic discount factors $\xi_t = \eta_{s,t}\xi_{s,t}$, where the disagreement follows

$$d\eta_{s,t}/\eta_{s,t} = \left(\theta^{+}_{s,t} - \theta_t\right) dz^{Y}_t. \tag{15}$$

2.3 Preferences and Individual Optimization

For the sake of variation in stock market volatility, we assume that agents differ in their time preferences within each cohort, where the distribution of types remains invariant over time. We denote the time preference of type i agents as ρ_i, and the density of type i agents as α_i, with $\int_i \alpha_i di = 1$. Since all agents of the same cohort learn from the same experience, the subjective stochastic discount factor does not differ across different agent types within the same cohort.[11]

To finance the endowment of the newly born, we impose a tax on the consumption of all agents currently alive.[12] Specifically, a cohort born at time s with a cohort size of νdt share the total consumption tax revenue of $\tau C_s dt$, where τ is the tax rate, and C_s denotes the aggregate level of consumption in the economy at time s. Thus, each agent in the cohort s has an initial wealth of $W_{i,s,s} = W_{s,s} = \frac{\tau}{\nu} C_s$.

Agents face constant mortality risk and since the random time of death is independent of aggregate output, we integrate it out from the expected lifetime utility, which then is given by

$$E_{s,s}\left[\int_s^{\infty} e^{-(\rho_i+v)(t-s)} log(c_{i,s,t})dt\right]. \tag{16}$$

The dynamics of wealth $W_{i,s,t}$ follows

$$dW_{i,s,t} = \begin{cases} \left(r_t W_{i,s,t} + \nu W_{i,s,t} - \tau c_{i,s,t} - c_{i,s,t}\right) dt, & \text{if } N, \quad \text{or} \\ \left[r_t W_{i,s,t} + \pi_{i,s,t}\left(\mu^S_{s,t} - r_t\right) + \nu W_{i,s,t} - \tau c_{i,s,t} - c_{i,s,t}\right] dt + \pi_{i,s,t}\sigma^S_t dz^Y_{s,t}, & \text{if } P, \end{cases} \tag{17}$$

where $\pi_{i,s,t}$ denotes the dollar investment in the stock market.

[11] We thus omit i in the subscript for $dz^Y_{s,t}$, $\xi_{s,t}$, $\theta^+_{s,t}$, and $\eta_{s,t}$.

[12] The equilibrium with a wealth tax, which has similar structure, is summarized in Subsection C.1 in the Online Appendix.

2.4 Equilibrium

The fictitious unconstrained consumption portfolio choice problem, corresponding to the static problem of Cox and Huang (1989), is given by

$$\max_{c_{i,s}} E_{s,s}\left[\int_s^{\infty} e^{-(\rho_i+v)(t-s)} log(c_{i,s,t})dt\right]$$

s.t.

$$E_{s,s}\left[\int_s^{\infty} e^{-v(t-s)}\xi_{s,t}(c_{i,s,t}+\tau c_{i,s,t})dt\right] = W_{i,s,s}. \tag{18}$$

From the first-order conditions (FOCs), we have

$$\frac{e^{-(\rho_i+v)(t-u)}}{c_{i,s,t}} = \kappa_{i,s}\left(1+\tau\right)e^{-v(t-u)}\xi_{s,t}, \tag{19}$$

where $\kappa_{i,s}$ is the Lagrange multiplier associated with the static budget constraint.

Proposition 2. *At t, the optimal consumption for a living type i agent of cohort s is*

$$c_{i,s,t} = c_{i,s,s}e^{-\rho_i(t-s)}\frac{\eta_{s,t}}{\eta_{s,s}}\frac{\xi_s}{\xi_t}. \tag{20}$$

We conjecture and verify that $\frac{c_{i,s,t}}{W_{i,s,t}} = \beta_i$, which is consistent with the standard constant consumption-to-wealth ratio for log utility. We define the wealth share of type i agents born at time s as $f^W_{i,s,t} = \frac{\alpha_i v e^{-v(t-s)}W_{i,s,t}}{W_t}$, and the consumption share as $f^c_{i,s,t} = \frac{\alpha_i v e^{-v(t-s)}c_{i,s,t}}{Y_t}$.

Proposition 3. *In equilibrium, the real short rate of interest is*

$$r_t = \overbrace{\nu - \tau\beta}^{\text{OLG}} + \overbrace{\bar{\rho}_t + \mu_Y - \sigma_Y(}^{\text{Log utility}}\sigma_Y \underbrace{\tfrac{1}{\bar{\Phi}_t} - \bar{\Delta}_t}_{\text{Beliefs-driven participation}}) = \nu - \tau\beta + \bar{\rho}_t - \sigma_Y^2\frac{1}{\bar{\Phi}_t} + \bar{\mu}_t, \tag{21}$$

and the market price of risk is

$$\theta_t = \overbrace{\sigma_Y}^{\text{Log utility}} \underbrace{\frac{1}{\bar{\Phi}_t} \quad - \quad \bar{\Delta}_t}_{\text{Beliefs-driven participation}} = \sigma_Y \frac{1}{\bar{\Phi}_t} - \frac{\bar{\mu}_t - \mu_Y}{\sigma_Y}, \tag{22}$$

where $\beta = \int_i \alpha_i \beta_i di$ *is the consumption-to-wealth ratio of the newborn cohort,* $\bar{\rho}_t = \int_i \rho_i \int_{-\infty}^t f_{i,s,t}^c ds di$ *is the consumption weighted average time preference,* $\bar{\Phi}_t = \int_i \int_{-\infty}^t f_{i,s,t}^c \mid (\Delta_{s,t} \geq -\theta_t)\, ds di$ *denotes the aggregate consumption share of the agents who invest in the stock market,* $\bar{\Delta}_t = \frac{\int_i \int_{-\infty}^t f_{i,s,t}^c \Delta_{s,t} | (\Delta_{s,t} \geq -\theta_t) ds di}{\int_i \int_{-\infty}^t f_{i,s,t}^c | (\Delta_{s,t} \geq -\theta_t) ds di} = \frac{\int_i \int_{-\infty}^t f_{i,s,t}^c \Delta_{s,t} | (\Delta_{s,t} \geq -\theta_t) ds di}{\bar{\Phi}_t}$ *is the consumption weighted average standardized estimation error in the economy conditional on stock market participation, and* $\bar{\mu}_t = \frac{\int_i \int_{-\infty}^t f_{i,s,t}^c \hat{\mu}_{s,t} | \left(\frac{\hat{\mu}_{s,t} - \mu_Y}{\sigma_Y} \geq -\theta_t\right) ds di}{\bar{\Phi}_t}$ *denotes the consumption weighted average expected growth rate conditional on stock market participation. We label it the market view of the participants.*

In Equation (21), we see that the real short rate of interest is given by an OLG term, a log utility term and a *beliefs-driven participation e ect*. In addition, in Equation (22), the market price of risk is given by the standard log utility term and the *beliefs-driven participation e ect*. Here, the experience term di ers from the complete market case since it includes only investors holding the stock market.

We now inspect the numerator and denominator of the market price of risk in the proposition below.

Proposition 4. *The expected excess return on the stock market is given by*

$$\mu_t^S - r_t = \sigma_t^S \theta_t, \tag{23}$$

and the volatility of the stock market is

$$\sigma_t^S = \tilde{\Phi}_t \left(\theta_t + \tilde{\Delta}_t\right), \tag{24}$$

where $\tilde{\Phi}_t = \int_i \int_{-\infty}^t f^W_{i,s,t} \mid (\Delta_{s,t} \geq -\theta_t)\, dsdi$ *denotes the aggregate wealth share of the stock market participants, and* $\tilde{\Delta}_t = \frac{\int_i \int_{-\infty}^t f^W_{i,s,t} \Delta_{s,t} | (\Delta_{s,t} \geq -\theta_t) dsdi}{\int_i \int_{-\infty}^t f^W_{i,s,t} | (\Delta_{s,t} \geq -\theta_t) dsdi}$ *is the wealth weighted average belief conditional on stock market participation.*

As for the real short rate of interest and the market price of risk, the expected excess return on the stock market also depends on the *beliefs-driven participation e ect*, $\frac{1}{\Phi_t} - \bar{\Delta}_t$ embedded in θ_t. Similarly, the volatility of the stock varies over time as the gap between the consumption- and the wealth-weighted belief widens and shrinks, $-\bar{\Delta}_t + \tilde{\Delta}_t$, and with the dynamics of the ratio between the aggregate wealth to consumption share of the participants, $\frac{\tilde{\Phi}_t}{\Phi_t}$. What sets Proposition 4 apart from Proposition 3, and more generally from the literature, is that despite log utility, consumption- and wealth-weighted equilibrium quantities di er. When they do not di er, because the time preference is homogeneous, the volatility of the stock market collapses to σ_Y. What is the intuition for why wealth-weighted quantities drive the stock market volatility but not the real short rate of interest? This dichotomy comes from two market clearing conditions: The real short rate of interest and the market price of risk derive from consumption market clearing. It does not invoke wealth. Clearing of the stock market, however, does rely on the wealth of the agents.

The following proposition shows the investment in the stock market.

Proposition 5. *For participants, the optimal dollar investment in the stock market,* $\pi_{i,s,t}$, *is*

$$\pi_{i,s,t} = \frac{\Delta_{s,t} + \theta_t}{\sigma_t^S} W_{i,s,t}. \tag{25}$$

The portfolio policy in Proposition 5 has two components: $\frac{\Delta_{s,t} - \bar{\Delta}_t}{\sigma_t^S} = \frac{\hat{\mu}_{s,t} - \bar{\mu}_t}{\sigma_Y \sigma_t^S}$, which depends on the market view, and $\frac{\theta_t + \bar{\Delta}_t}{\sigma_t^S} = \frac{\sigma_Y}{\sigma_t^S} \frac{1}{\Phi_t}$, which depends on the ratio of exogenous to endogenous volatility times the inverse of the consumption share of the participants. Instead, in the complete financial market benchmark with log utility, variations in portfolio composition arise only through the belief component. Further, we learn that leverage and therefore also whether an agent participates in the stock market does not directly depend on her time preference.

An important step in identifying equilibrium is that participation and θ_t are determined jointly. Specifically, cohorts with cutoff estimation error $\Delta^*_{s,t} = \frac{\hat{\mu}^*_{s,t} - \mu_Y}{\sigma_Y} = -\theta_t$ have exactly zero demand for the stock. Agents who are more optimistic are participants, while agents who are more pessimistic are nonparticipants. A consequence of the binding constraint is the differing (log) consumption dynamics of participants and nonparticipants:

Proposition 6. *The log consumption at time t for an agent of type i born at time s has the dynamics*

$$d\log(c_{i,s,t}) = \begin{cases} (-\rho_i + r_t)\, dt, & \text{if } N\text{, or} \\ \left(-\rho_i + r_t + \frac{1}{2}\theta_t^2 - \frac{1}{2}\Delta_{s,t}^2\right) dt + (\theta_t + \Delta_{s,t})\, dz_t^Y, & \text{if } P. \end{cases} \tag{26}$$

Proposition 6 captures when an agent is expected to lose out in terms of lower expected log consumption growth. Nonparticipant lose simply because they do not earn the risk premium. Participants lose whenever the expected value of the squared estimation error is larger than that of the market price of risk. The last proposition summarizes the dynamics of the consumption share of a cohort:

Proposition 7. *The consumption share at time t of type i agents born at s follows the process*

$$\begin{aligned} df^c_{i,s,t}/f^c_{i,s,t} &= (-\tau\beta + \bar{\rho}_t - \rho_i)\, dt + (\theta_t - \sigma_Y)\left(\theta^+_{s,t} - \sigma_Y\right) dt + \left(\theta^+_{s,t} - \sigma_Y\right) dz_t^Y \\ &= \begin{cases} \left[-\tau\beta + \bar{\rho}_t - \rho_i - \sigma_Y(\theta_t - \sigma_Y)\right] dt - \sigma_Y dz_t^Y, & \text{if } N\text{, or} \\ \left[-\tau\beta + \bar{\rho}_t - \rho_i - \sigma_Y(\theta_t - \sigma_Y) + (\theta_t + \Delta_{s,t})(\theta_t - \sigma_Y)\right] dt & \\ \qquad + (\theta_t + \Delta_{s,t} - \sigma_Y)\, dz_t^Y, & \text{if } P, \end{cases} \end{aligned} \tag{27}$$

and the wealth share at time t of type i agents born at s follows the process

$$df^W_{i,s,t}/f^W_{i,s,t} = \begin{cases} \left[-\nu + \tilde{\beta}_t - \rho_i - \sigma^S_t\left(\theta_t - \sigma^S_t\right)\right]dt - \sigma^S_t dz^Y_t, & \text{if } N \text{, or} \\ \left[-\nu + \tilde{\beta}_t - \rho_i - \sigma^S_t\left(\theta_t - \sigma^S_t\right) + \left(\theta_t + \Delta_{s,t}\right)\left(\theta_t - \sigma^S_t\right)\right]dt & \\ \quad + \left(\theta_t + \Delta_{s,t} - \sigma^S_t\right)dz^Y_t, & \text{if } P. \end{cases} , \quad (28)$$

where $\tilde{\beta}_t \equiv \frac{C_t}{W_t}$ is the aggregate consumption-to-wealth ratio.

The significance of Proposition 7 is that although our framework contains two Brownian shocks, of which only the fundamental shock to output is priced, financial markets are complete; and that although we obtain closed-form solution throughout this entire section, we still have to follow all consumption and wealth shares to simulate the economy forward.

3 Model Dynamics

We now examine the equilibrium dynamics of the model presented in Section 2 with focus on stock market participation, entry, and exit.

3.1 Parameters

The model has nine parameters $\left(\rho_a, \rho_b, \nu, \mu_Y, \sigma_Y, \hat{\mu}_{s,s}, \hat{V}, \phi, \tau\right)$. We follow Gârleanu and Panageas (2015) and set the birth and death rate ν at 2%, and the drift and volatility of aggregate output μ_Y and σ_Y at 2% and 3.3%, respectively. Ehling, Graniero, and Heyerdahl-Larsen (2018b) connect the prior belief $\hat{\mu}_{s,s}$ and the prior variance $\hat{V}$ through an initial window with n years during which an agent builds formative experience with stock returns before entering the economy, and set $\hat{\mu}_{s,s} = \mu_Y + \sigma_Y \frac{z^Y_s - z^Y_{s-n}}{n}$, and $\hat{V} = \frac{\sigma^2_Y}{n}$. Following this approach, we focus on $n = 5$, and we regard agents as 20 years old when entering the economy. For the time discount factor, we set ρ_a at 0.1% for half of the agents,[13] and 0.5%

[13] Gârleanu and Panageas (2015) uses $\rho = 0.1\%$ for all agents.

for the other half. For the correlation between the fundamental and the signal ϕ, we focus on $\phi = 0.5$.[14] We set τ at 0.35, which implies that a-types consume $\frac{c_{a,t,t}}{C_t} = 27.2\%$ and b-types consume $\frac{c_{b,t,t}}{C_t} = 32.4\%$.[15] Therefore, an average newborn consumes 29.8% of the average consumption in the economy.[16]

3.2 Stock Market Participation Data

The data on stock market participation rates come from the following countries: Finland,[17] Germany, Norway,[18] and the United States of America. A summary of the data is shown in Table 1. We define a participant in the stock market as an individual or household that has direct or indirect holdings of stocks in a given year. Using income data, a participant in the stock market is defined as an individual or household that receives dividends in a given year. The participation rates for the USA are from the Internal Revenue Services, rates from Finland and Norway are from the tax authorities covering all the residents with a tax obligation, and are recorded at the year end. We also supplement with survey-based data for the USA and use survey-based data from Germany.[19]

We detrend participation rates using country-specific time trends, to remove any potential e￿ects due to changes in participation costs, tax treatment, and demographic compositions. This removes any significant linear time trend, while keeping the time-series mean participa-

[14]See for example Chen, Roll, and Ross (1986), which is a classical paper on the relation between fluctuations in macroeconomic state variables and the stock market; it presents absolute correlations ranging from 0 to at most 0.5.

[15]The consumption tax rate τ directly affects the consumption share of a new born agent, $\frac{c_{i,t,t}}{C_t} = \frac{\tau}{\nu}\beta_i = \frac{\tau}{\nu}\frac{\nu+\rho_i}{1+\tau} = \frac{\tau}{1+\tau}(1 + \frac{\rho_i}{\nu})$.

[16]Asset pricing moments are delegated to Table 4 in Subsection C.5 of the Online Appendix.

[17]Data for Finland are compiled by Breitkopf, Knüpfer, and Rantapuska (2021) using data from Statistics Finland.

[18]We commissioned Statistics Norway for annual participation rates. In the Norwegian data, we define a participant in the stock market as an individual who reports holdings in shares in equity mutual funds in a given year, to isolate the exposure to the stock market. The Norwegian tax authorities impose a wealth tax and require reporting any taxable value of the following four categories: (1) listed and unlisted Norwegian shares, bonds, equity certificate and options registered in Central Securities Depository (VPS); (2) those not registered in VPS; (3) shares in equity mutual funds; and (4) bonds and money market funds. (The first two accounts are related to direct holding of securities. However, we cannot isolate equity exposure from them.)

[19]The data for Germany are annual aggregates of 12 monthly waves, and thus we shift dating to mid-year.

Country	Measure	Time-series	Frequency	Entry and exit	Data Source
Finland	Portfolio	2004 - 2016	Annual	Yes	Statistics Finland
Germany	Survey	1997 - 2022	Annual	No	Deutsches Aktieninstitut
Norway	Portfolio	1999 - 2022	Annual	Yes	Statistics Norway
USA	Dividend	1990 - 2022	Annual	No	Internal Revenue Services
	Survey	1989 - 2022	Triannual	No	Survey of Consumer Finances
	Survey	1998 - 2023	Monthly	Yes	Michigan Surveys of Consumers

Table 1: **Participation data.** Participation rates are based on registry data, dividends reported on tax returns, and surveys. Entry and exit rates are available through registry data and the Surveys of Consumers of the University of Michigan.

tion rate unchanged. Specifically, we remove time-trends in Finland, Norway and the USA, while there appears to be no significant trend in Germany.[20]

For Finland and Norway,[21] we also have data on entry and exit from the stock market. Entry and exit are constructed from stock or portfolio holdings at the year-end. Investors are regarded as entering if they hold stocks or mutual funds at the end of the current year but did not hold any stocks or mutual funds at the end of the previous year. Conversely, investors are regarded as exiting if they owned stocks or mutual funds at the end of the previous year but do not own stocks or mutual funds at the end of the current year.[22]

For the USA, we focus on the dividend income tax measure of stock market participation as it covers the general population. We supplement the data with Survey of Consumer Finances and the Michigan Survey. Survey of Consumer Finances is conducted every three years, and we use aggregate participation rate time-series by age groups from 1989 to 2022. The Michigan Surveys of Consumers select a sample of 300 to 400 respondents each month. From August 1998, the survey contains question on investment portfolio. A subsample of respondents are interviewed a second time after 6 months. We utilize this panel structure to also study entry and exit dynamics in the USA.

[20] The original and the detrended time-series of participation rates for Finland, Germany, Norway and the USA are Table 10 in Subsection C.4 in the Online Appendix.

[21] For Norway, entry and exit data for the year 2016 are missing due to change of tax treatment on equity and bond mutual funds.

[22] The original and the detrended time-series of entry and exit rates for Finland and Norway are Table 11 in Subsection C.4 in the Online Appendix.

3.3 Shocks to Stock Market and Signal

For Finland, Germany and Norway, we use the monthly international country portfolio data from Kenneth French's website, starting from year 1975. Specifically, we use the value weighted average of the total returns in the local currency. Prior to 1975 for Finland, we use market returns constructed by Nyberg and Vaihekoski (2014), which are available from the year 1912 onward. For the USA, the stock returns we use are CRSP value weighted total returns after 1926, and index returns between 1871 and 1926 from Amit Goyal's website.

The macroeconomic indicators we use for Finland, Germany and Norway include the exchange rate against the USD, industrial production, inflation, and unemployment rate. These data are seasonally adjusted monthly time-series from the OECD database. For the USA, the macroeconomic indicators include changes in 12-month earnings, industrial production, Michigan consumer sentiment, and macroeconomic factors from Ludvigson and Ng (2009).

To simulate the time-series of model implied participation rates, we feed in shocks to the fundamental dz_t^Y and shocks to the signal dz_t^{SI} constructed using the historical data on the stock returns and a pool of macroeconomic indicators. For this exercise, we standardize (with mean zero and standard deviation $\sqrt{dt}$) the monthly stock returns from the four countries to obtain the fundamental shocks dz_t^Y. For the shocks to the signal dz_t^{SI}, we first regress dz_t^Y on the pool of macroeconomic indicators to obtain the fitted values, which we regard as dSI_t. We then regress dSI_t on dz_t^Y, take the residual, and use the standardized residual as dz_t^{SI}.

3.4 Forward Simulations

The forward simulations of the economy use the final values from a burn-in simulation. In the burn-in as in the forward simulation, a period equals one month and in every period one additional cohort is born. After 6000 periods in the burn-in, there are 6000 cohorts, which produce the starting values and from then on we keep the number of cohorts constant at 6000. For computational convenience in the first 240 periods of the burn-in, we do not impose any constraint and use $\hat{\mu}_{s,s} = \mu_Y$, and $\hat{V} = \frac{\sigma_Y^2}{n}$.

To simulate the time-series of model implied quantities, we use 6000 periods (500 years) for each path. For the simulated participation, entry and exit rates, we use random shocks whenever data on implied shocks are not available as additional burn-in, and use the average from 50 burn-in paths.

3.5 Scenarios

We consider two scenarios:

Definition 1. *In the reentry scenario a cohort leaves the stock market upon the event of the perceived instantaneous price of risk being negative, that is $\theta_{s,t} < 0$, and returns to the stock market upon the event of the perceived instantaneous price of risk being positive, that is $\theta_{s,t} > 0$. This corresponds to the equilibrium in Section 2.*

We use the *reentry* scenario mainly to present unconditional averages.

Definition 2. *In the mix scenario, the economy is populated by four groups of agents: reentry type, disappointment type,*[23] *designated participants, and designated nonparticipants. Here, designated participants are unconstrained and designated nonparticipants never hold the stock market.*

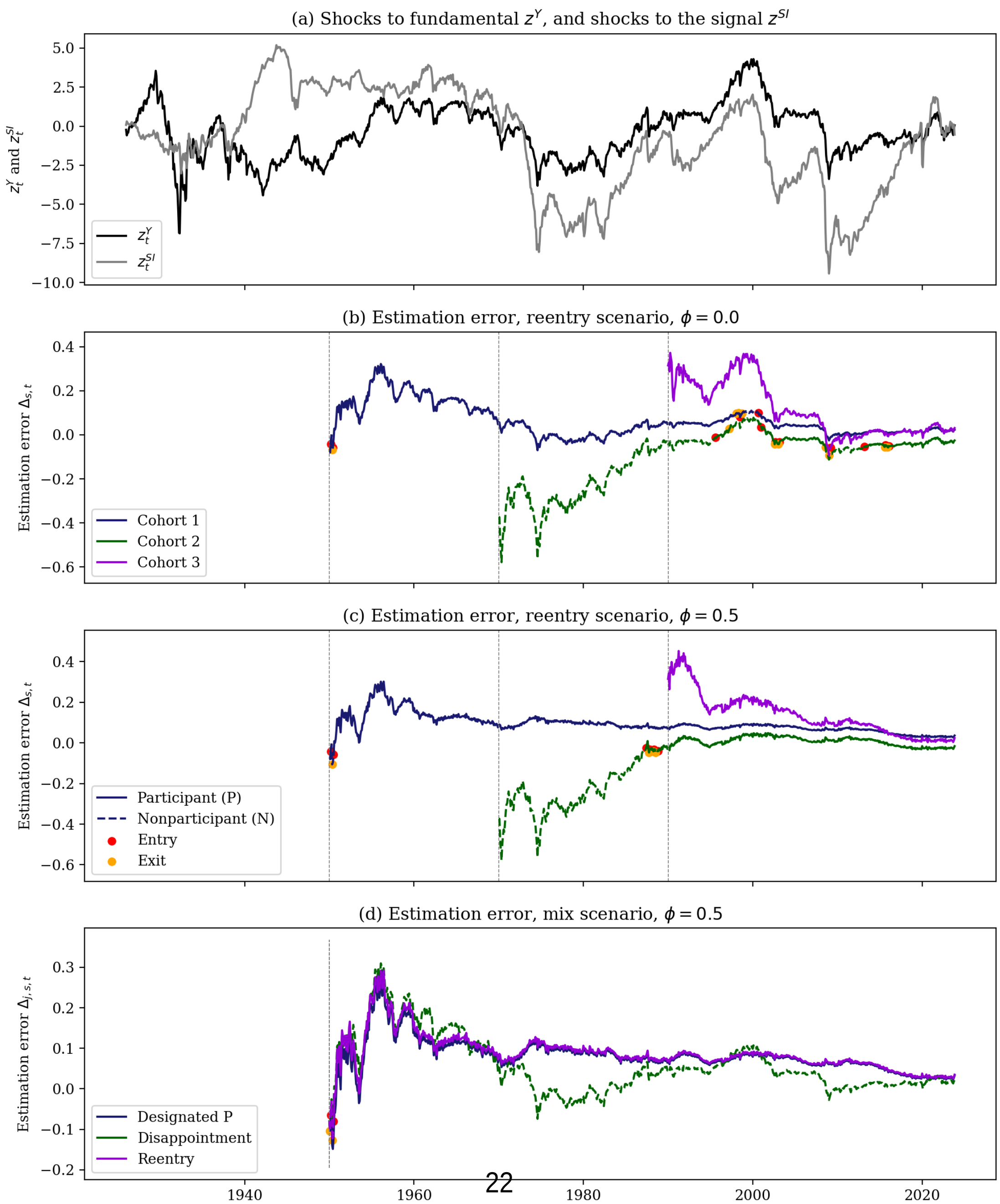
We use the *mix* scenario mainly to present time-series results from the implied model.

3.6 Cohort Estimation Error

We start discussing results by inspecting how different cohorts of agents update beliefs in response to the two shocks.

[23] The *disappointment* type leave the stock market for good upon the event of the perceived instantaneous price of risk being negative, that is $\theta_{j,s,t} < 0$. Proofs for an equilibrium with *disappointment* type are in subsection C.2 in the Online Appendix.

Figure 2: **Estimation error across joint path of output and signal.** The top plot show the implied joint path of shocks to the fundamental and to the signal for the US economy. The middle plots show the standardized estimation error $\Delta_{s,t}$ of three cohorts in the *reentry* scenario, with $\phi = 0.0$ and $\phi = 0.5$, respectively. The cohorts are born in the years 1950, 1970, and 1990, marked by dotted vertical lines. The bottom plot shows $\Delta_{j,s,t}$ for designated participants, disappointment type, and reentry type born in the year 1950 in the *mix* scenario, with $\phi = 0.5$ and equal density on types. Solid time-series lines denote times when a cohort participates in the stock market, whereas the dotted lines represent times when a cohort does not participate, where red dots indicate the timing of a switch from nonparticipant to participant, and orange dots indicate the timing of a switch from participant to nonparticipant. For parameters see Subsection 3.1. For implied shocks see Subsection 3.3.



In this regard, the top and bottom middle plots in Figure 2 show the standardized estimation error $\Delta_{s,t}$ of three cohorts, born in the years 1950, 1970 and 1990 respectively, where red solid dots represent transitions from nonparticipants to participants, and orange dots represent transitions from participants to nonparticipants. These estimation errors produced in the *reentry* scenario are driven by the joint path for the shocks to output dz_t^Y and the signal dz_t^{SI} (top plot of Figure 2) that are implied by the US economy. In the top middle plot the signal is uncorrelated with dz_t^Y and hence participants and nonparticipants have identical information, while in the bottom middle plot the correlation equals the baseline of 0.5. A solid line for the estimation error indicates that the cohort participates in the stock market, whereas a dashed line indicates nonparticipation.

From the plots, we see that each cohort updates aggressively when young, responding to shocks with large changes in $\Delta_{s,t}$, and gradually learn from experience and through that the precision of estimates about output growth increases and updates to $\Delta_{s,t}$ decline in magnitude. The plots also show that even after collecting long time-series of data, the estimation error across the three cohorts di er considerably. Further, from the plot it is evident that cohorts switch from participation to nonparticipation at di ering frequency or just stay in the market all the time such as the cohort born in 1990. The intricate influence of the signal on the estimation error is most evident from the di ering participation of the two older cohorts with and without the macroeconomic signal.

The bottom plot contains the estimation error of designated participants, disappointment type and reentry type of agents belonging to the cohort born in the year 1950 for the *mix* scenario. Here, the di erences in the estimation error across types within the same cohort arise only through entry and exit, as all agents in the same cohort share the same experience, while only those who participate observe the signal. Such di erences in estimation error, independently of whether they arise within or across cohorts, are at the core of our model and thus drive fluctuations in the financial market, which then drive economy-wide learning.

The main takeaways from these plots, such as aggressive updating by the young or entry and exit driving persistent wedges between the estimation errors of differing cohorts or differing agent types, are typical in that they are not confined to the selected cohorts or even the shocks implied by US data.

3.7 Endogenous Learning

One unique feature of our model is that the speed of learning directly depends on equilibrium outcomes, i.e., it is endogenous. This is the case although the updating equations per se depend only on the exogenous shocks to output and the signal as is standard in the literature building on Detemple and Murthy (1997). The reason for why our learning scheme depends on equilibrium itself is that a cohort of agents switches from participants to nonparticipants on the event $\Delta_{s,t} < -\theta_t$. Since nonparticipants do not pay attention to the signal, there is a learning-driven wedge between participants and nonparticipants. As the market price of risk θ_t itself depends on the participants wealth shares, the model then produces endogenously a learning from experience that differs across participants and nonparticipants.

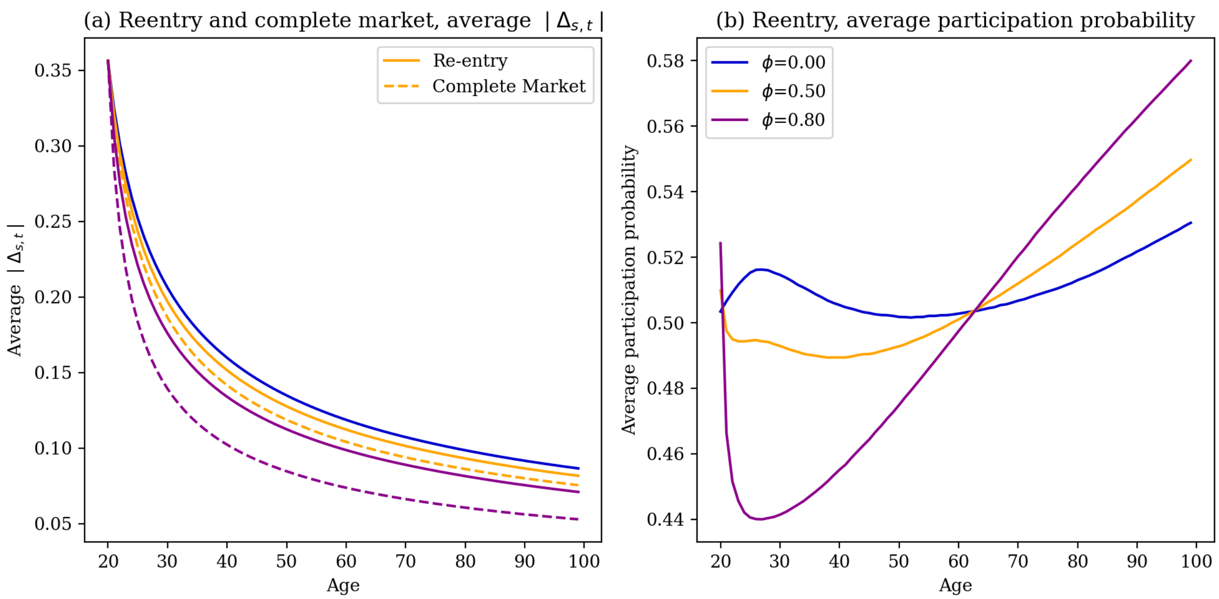
To show the impact of the endogenous learning mechanism, we compute the average absolute values of the estimation error over age for different ϕ values shown in the left plot of Figure 3. The average estimation error decreases in age and in ϕ. Clearly, agents learn slower in the *reentry* scenario compared to the complete market benchmark (shown as dashed line), as only the participants observe the signal. Specifically, ϕ has a convex impact over the learning gap between the two scenarios.

With nonparticipation, ϕ affects the speed of learning through modifying not only the informativeness of the exogenous signals, but also the endogenous probability of observing the signals, as shown in the right plot of Figure 3.[24] The key insight here is that the average participation rate in the *reentry* scenario is relatively high for the young and then first it decreases and then it increases in age.[25] These variations of participation over age are the

[24] It is apparent that ϕ affects participation at birth. Since initial belief are not affected by ϕ as newborns train only on the realizations of dividends, a higher probability of entry at birth can only result from lower average real short rate of interest and higher market price of risk.

[25] The increase in participation rate of the old is significantly stronger with longer pre-entry learning.

Figure 3: **Updating and participation.** The figure shows the average estimation error in the complete market (dashed line) and the *reentry* scenario (solid line) in the left plot, and the average participation probability in the *reentry* scenario over age in the right plot, where colors represent different ϕ's. To compute the averages, we simulate the economies to generate data based on 10,000 simulations, each with 6000 periods or 500 years. For parameters see Subsection 3.1.



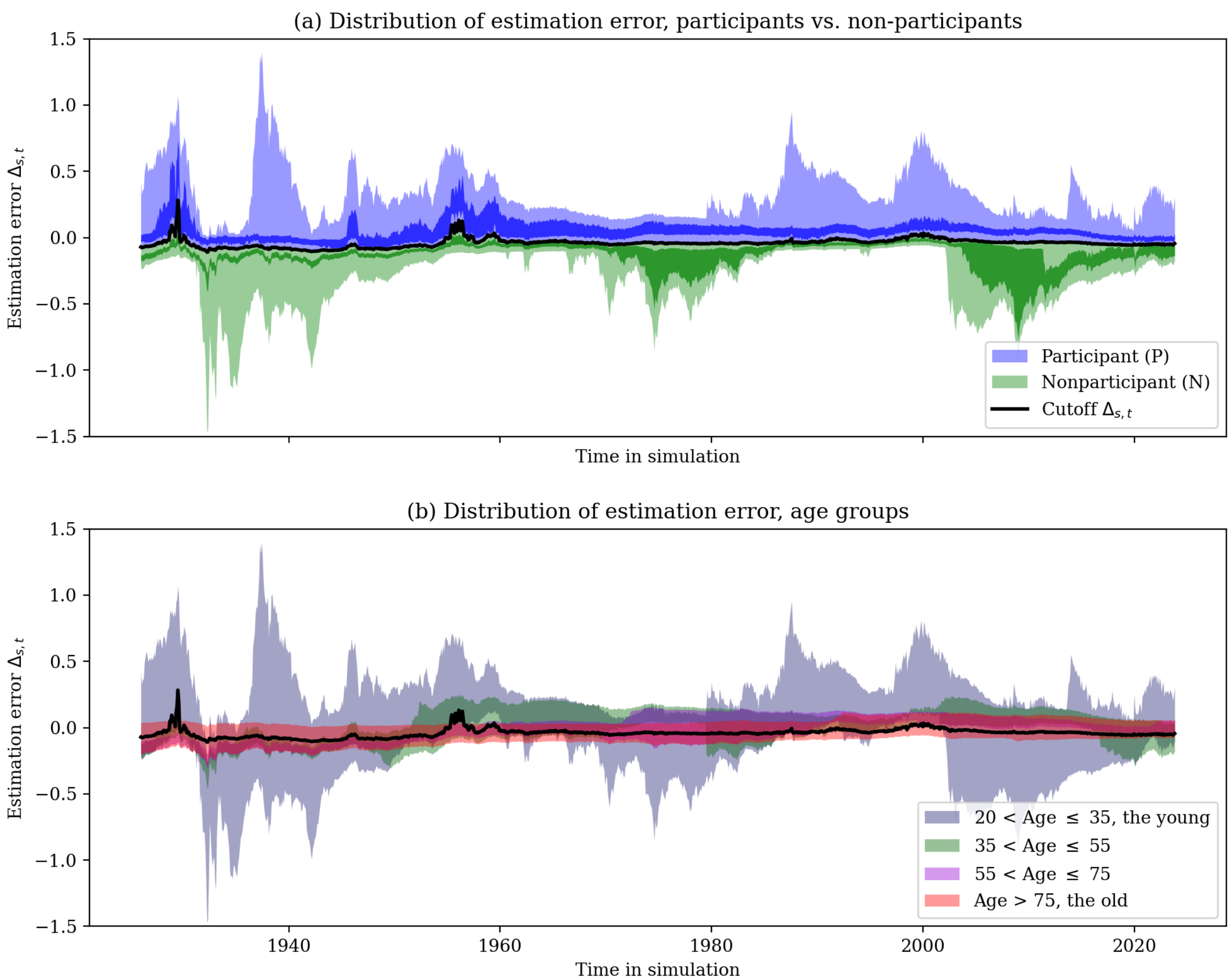
larger the more informative is the signal. The increase of participation rate among the old is mainly because of an asymmetry between how relatively optimistic young agents and old agents affect the market view, which is the consumption-weighted belief of the participants. Specifically, the young drive the market view through their exuberance, while the old drive it through their wealth and experience.

We now present the interplay between beliefs and the equilibrium in more detail: the top plot in Figure 4 shows the cross-sectional distribution of estimation errors over time for participants and nonparticipants using the implied shocks of the US economy. The blue area represents the distribution of estimation errors of the participants, the green area for that of the nonparticipants, and the dark areas represent the range between the 25^{th} and 75^{th} percentiles. The total width of the estimation error distribution maps the distance between the most optimistic and pessimistic agents. This range varies remarkably over time. Further, the relative division between P and N is largely independent of the total width, and each shows large time-series variation.

We next look at the distribution of estimation error for age groups in the bottom plot of Figure 4. The youngest group experience large swings in their estimation error, while the oldest group enjoy significantly more stable beliefs that, in addition, are closer to the true parameter. Thus, the youngest group become the most optimistic agents in the economy following a series of positive shocks (mainly shocks to output), and the most pessimistic following a series of negative shocks.

We see that the distribution of the estimation error of the oldest group of agents is sometimes entirely above the cutoff belief or at least a large part of the distribution is above the cutoff. In contrast, the distribution of the estimation error of the youngest group of agents shows large swings and is therefore equally likely to be mainly above or mainly below the cutoff belief. The cutoff belief gravitates around the beliefs of the older cohorts, as they have large wealth shares. As a result, only over three short episodes where the young show large exuberance in beliefs, we see the entire group of old leave the stock market.

Figure 4: **Distribution of estimation error.** The figure plots the time-series distribution of the implied standardized estimation errors of the the *re-entry* for the USA. The top plot shows the cross-sectional distribution of the estimation error over time for participants and nonparticipants, with the blue area representing the estimation error of the participants, the green area for that of the nonparticipants, where dark areas are in-between the 25^{th} and 75^{th} percentile by population. The cuto estimation error separating the participants from the nonparticipants corresponds to $\Delta_{s,t} < -\theta_t$. The bottom plot shows the distribution of the estimation error for age groups. For parameters see Subsection 3.1. For implied shocks see Subsection 3.3.



4 Stock Market Participation, Entry and Exit

Turning to equilibrium effects, we stress that the consumption and wealth weighted quantities in the real short rate of interest, the market price of risk and the volatility of the stock market are unobservable. We, therefore, focus on the participation rate and entry to and exit from the stock market as these quantities are observable.

4.1 Stock Market Participation

Figure 5 shows the detrended stock market participation rates in Finland, Germany, Norway and the US. Comparing these participation rates to rates implied by the model based on the *mix* scenario using $\phi = 0.0$ (shown as black solid lines) and $\phi = 0.5$ (shown as gray dotted lines), we see that although the level of the model implied participation rates are higher than in the data the dynamics are similar.[26] Out of these eight correlations only one is statistically insignificant, which is remarkable, especially to the extent that the model-implied participation rate is only driven by constrained learning from experience.[27]

Next, and inspired by the works of Malmendier and Nagel (2011), Malmendier and Nagel (2016) or Knüpfer, Rantapuska, and Sarvimäki (2017), we turn to differences in participation rates and relate them to differences in experiences. We stress that these empirical contributions have inspired several theoretical works such as Collin-Dufresne, Johannes, and Lochstoer (2017) or Ehling, Graniero, and Heyerdahl-Larsen (2018b); yet, in these models agents always participate in the stock market. To this end, we compute model driven differences in beliefs based on differences in experienced returns from the US stock market. Specifically, we compute the beliefs between agents that are 35 and younger (agents in

[26] For the US, where the signal has a richer structure the difference in the model based participation rates can be as high as 8%.

[27] We view the correlation between the data and the model counterpart as a relevant measure of fit because the literature has offered other important explanations for nonparticipation, entry and exit such as participation costs (see Gomes and Michaelides (2007)) or labor income uncertainty (Bonaparte, Korniotis, Kumar, Michaelides, and Zhang (2025)). Since we built the model to isolate the *beliefs-driven participation effect* it cannot account for these other important contributing factors. Hence, we do not test the hypothesis of a perfect fit.

Figure 5: **Participation rate: data versus model.** The figure contains the time-series of detrended participation rate in Finland, Germany, Norway and the USA (blue solid lines with dots), juxtaposed with simulated participation rates of the model based on the *mix* scenario using $\phi = 0.0$ (solid lines) and $\phi = 0.5$ (dashed lines), where n is the length of the initial window for the prior belief and T_ϕ is the t-statistics on the actual participation rate from regressing the model participation rate on a constant and the actual participation rate. For parameters see Subsection 3.1. For implied shocks see Subsection 3.3. Agent types (reentry, disappointment, designated participants, and designated nonparticipants) have 25% weight at birth.

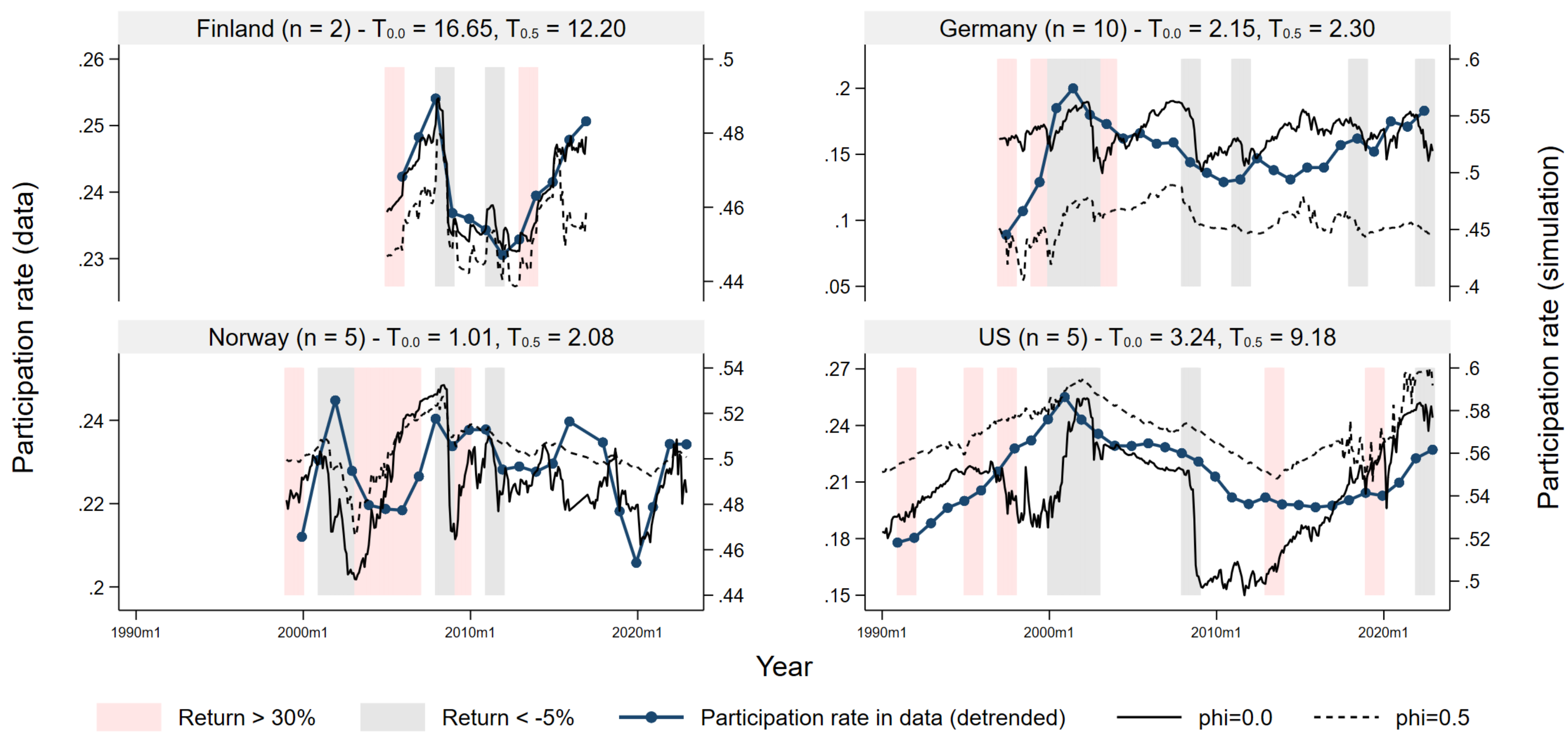
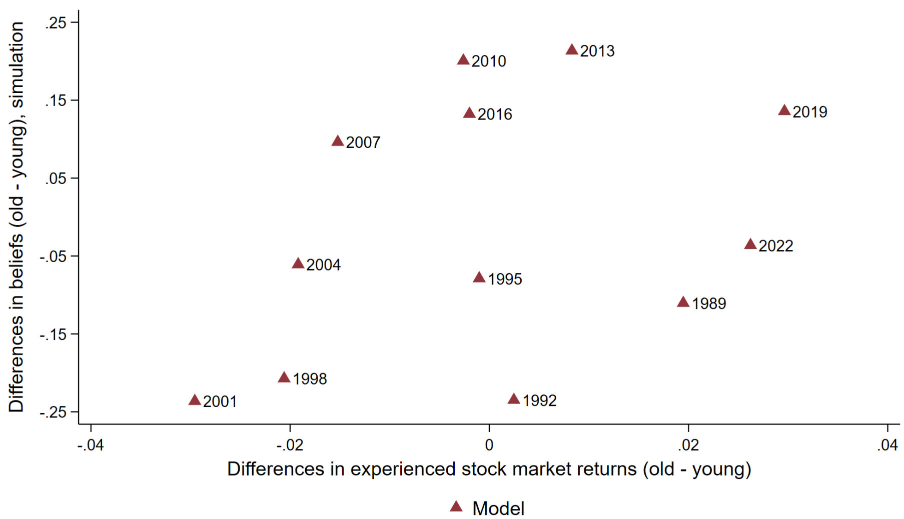
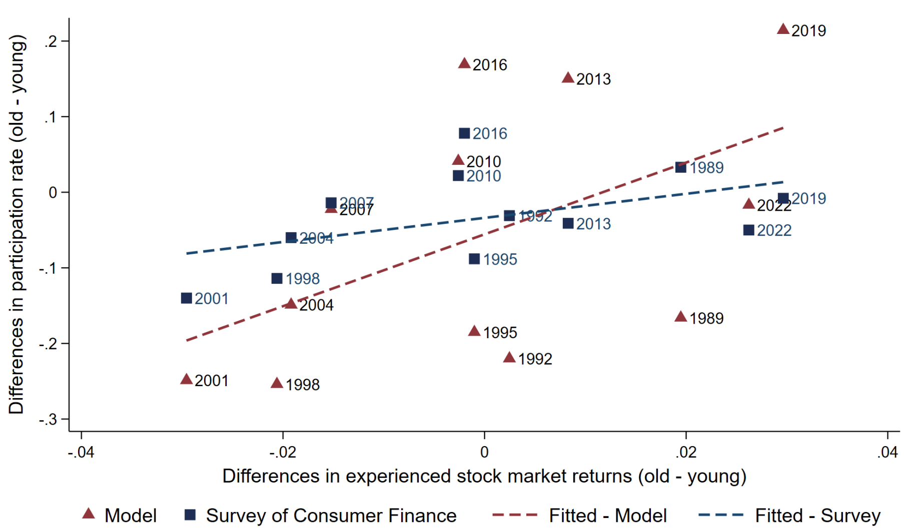


Figure 6: **Experience, beliefs, and participation: data versus model.** The top plot (a) shows the model implied difference in beliefs (in red) between the old and young based on experienced returns. The bottom plot (b) shows the model implied (in red) and the Survey of Consumer Finance (SCF, in blue) based difference in stock market participation rates between the old and young based on experienced returns. The bottom plot also shows the fitted relation between differences in experienced returns and differences in participation rates. We proxy for the difference in experienced stock market returns by taking the (time-series) average stock market return over the prior 50 years minus the return over the prior 20 years. For SCF survey, old refers to households over 75 years old, and young refers to those below 35. In the simulation, we use the same cutoffs, where agents that are 35 years old have 15 year of experience. The calendar years refer to the respective SCF survey waves. For parameters see Subsection 3.1. For implied shocks see Subsection 3.3. Agent types (reentry, disappointment, designated participants, and designated nonparticipants) have 25% weight at birth.

(a) **Experience and belief - model**



(b) **Experience and participation rate - data versus model**



the model start trading at age 20) and agents that are older than 75. These differences in implied beliefs are shown in Figure 6a for years with SCF (Survey of Consumer Finance) survey waves as red arrows. Based on these implied beliefs, we then compute model implied differences in participation rates between the old and young (red arrows) to juxtapose these with the differences in participation rates based on data from SCF (blue squares). To us, this evidence shown in Figure 6b, including their fitted relations (dashed lines), suggests that the return-driven or constrained experienced based differences in participation between the old and young in the model reproduce the survey data. Specifically, regressing the difference in participation rate between old and young from the simulation on that from the data produces a t-statistic of 3.84, and R-squared of 40%.

4.2 Entry and Exit

The dynamics of stock market participation can be broken down into dynamics of entry and exit. To study how experienced returns drive entry and exit respectively, we start with employing the Michigan survey. The survey selects around 400 respondents every month, and approximately 200 among them are interviewed a second time after 6 months.[28] We utilize this panel structure, and define entering households as those respondents who are nonparticipants in the first interview (the entry sample), but have become participants in the second interview. Similarly, we define exiting households as those respondents who are participants in the first interview (the exit sample), but have become nonparticipants in the second interview. To test whether investors are more likely to enter the stock market upon experiencing high returns and vice versa, we use 6-month returns to match the interval between the interview rounds.

As households in the entry and exit samples differ, we perform a Heckman correction. Specifically, since only the non-participants (participants) in the first interview may enter (exit), regression coefficients obtained from directly estimating how experienced returns drive

[28] Additional details about the Michigan Survey are delegated to the Online Appendix C.6.

Entry among non-participants	(1)	(2)	(3)
6m High Return	0.071**	0.090***	0.083***
	(0.028)	(0.029)	(0.028)
High Income	0.432***	0.466***	0.431***
	(0.048)	(0.051)	(0.048)
High Wealth	1.223***	1.224***	1.268***
	(0.073)	(0.073)	(0.077)
High Income × 6m High Return		-0.165*	
		(0.087)	
High Wealth × 6m High Return			-0.206*
			(0.119)
Controls	Y	Y	Y
N	53002	53002	53002

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Exit among participants	(4)	(5)	(6)
6m Low Return	0.087***	0.082***	0.086***
	(0.021)	(0.024)	(0.022)
High Income	-0.126***	-0.133***	-0.125***
	(0.026)	(0.029)	(0.026)
High Wealth	-0.823***	-0.823***	-0.829***
	(0.036)	(0.036)	(0.042)
High Income × 6m Low Return		0.027	
		(0.051)	
High Wealth × 6m Low Return			0.020
			(0.073)
Controls	Y	Y	Y
N	53002	53002	53002

Table 2: **Michigan data: high (low) stock returns and entry in (exit from) the stock market.** We use a probit model with sample selection (Heckman two-step estimator) to regress the probability of a household entering (exiting) the stock market on high (low) 6-month stock return, where we use the panel structure of the Michigan survey to define stock market entry (exit). We report second-step coefficients and standard errors. High (low) 6-month return is a dummy variable that equals to 1 if the return is within the top (bottom) 25% between 1998 and 2023. High income and high wealth are dummy variables, which equal to 1 if a household has income or wealth above the 75^{th} percentile in an interview wave. We use the following control variables in both first and second steps: high income, high wealth, age group which equal to 1 if an individual has age above 65, level of education (from lowest to highest grade completed), the housing situation of the household (own or rent), the region of residence of the household (North, South, Northeast, Central North), gender of respondent, and political affiliation (democrat, republican, non affiliated). We include a time trend in the first-step. Returns on the stock market are from Amit Goyal's website.

entry and exit may suffer from selection bias. Hence, in the first step, we estimate the probability of being selected in the entry sample and the exit sample using the characteristics in Table 5 of the Online Appendix. For wealth, income and age, we use dummy variables which indicate whether a household has income or wealth above the 75^{th} percentile in the interview wave, or is above 65 years old. We also include linear time trend in the first-step regression to account for long term changes in the selection to entry or exit sample.

In the second step, taking entry for example, we regress the probability of a household entering the stock market on high 6-month stock return, the same set of characteristics, and the probability of being selected in the entry sample given the characteristics. High 6-month return is defined as a dummy variable that equals to 1 if the return is within the top 25% between 1998 and 2023. To examine the heterogeneity of responses to high returns, we also include interaction terms between high return and high income, as well as high wealth. We perform analogous analysis for exit.

In Table 2, we report coefficient estimates from the second step for high (low) 6-month stock return, high income, high wealth, and the interaction terms. These regression results support the model's premise that experiencing particularly high returns leads one to enter the stock market in that the coefficient estimates for 6m High Return in the left panel of Table 2 are positive and significant at least at the 0.05-level. In the same vein, the regressions also support the model's premise that experiencing particularly low returns leads one to exit the stock market in that the coefficient estimates for 6m Low Return in the right panel of Table 2 are positive and significant at the 0.01-level. The other coefficients appear also as plausible: high income and wealth respondents are more likely to enter (not to exit) the stock market in general, where all these coefficients are significant at the 0.01-level. These latter effects are not particularly sensitive to returns as the interaction terms show low or no significance. This also appears to be in line with our model as in the language of our model high income or wealth likely corresponds to high experience, which implies that recent returns show low impact on beliefs for the experienced investors. Further, the results are robust to excluding survey respondents who report a change in the income quartile across the two waves of the survey.[29]

We now inspect the time series of entry and exit in the data and implied by our model by levering on the registry data from Finland and Norway. Just as for stock market participation rates, we compare the data to entry and exit rates implied by the model based on the *mix* scenario using $\phi = 0.0$ (shown as black solid lines) and $\phi = 0.5$ (shown as gray dotted lines). Specifically, Figure 7a shows the entry rates for Finland (left) and Norway (right), and we again see that the model implied entry rates largely mimic the dynamics of the data and even match the level of entry especially so for Finland. The picture for the exit rates are comparable as can be seen from Figure 7b. For Finland, three of four correlations come with t-statistic of at least 2.33, while for Norway only one out of four correlations shows a t-statistic of 1.97.

4.3 Re-Entry Frequencies

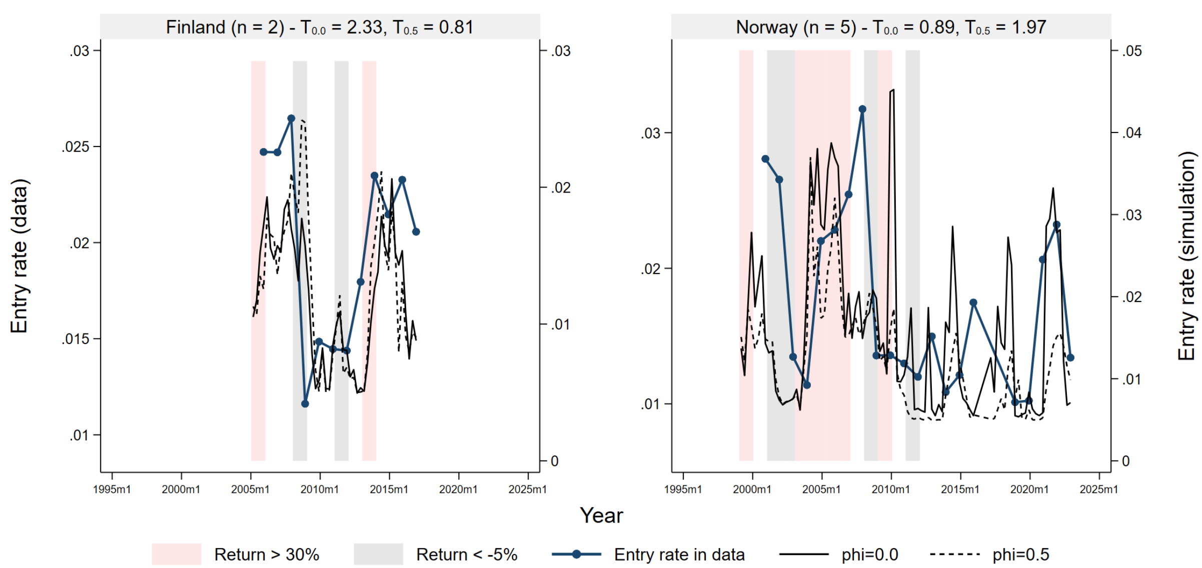
Next, we study re-entry frequencies conditional on experienced returns. In this regard, the top plot in Figure 8a shows the fraction of investors subsequently re-entering the stock market in Finland, from the year 2005 on through 2011. According to the data, approximately 30% of the investors who exit the stock market re-enter within 5 years. What stands out is that the fraction of re-entering investors among those who exited in year 2008, when the total stock return in Finland was -51%, is at the absolute bottom of re-entering investors. Such a pattern highlights the role of experience-based learning.

Our model replicates this pattern. Simulating our model forward, we follow the exiting cohorts for a non-overlapping twenty-year window and calculate the proportion of agents who re-enter the first time after a given number of years. This comparison to the data is shown in Figure 8b. In the plot, the blue solid lines show the unconditional average re-entry rate by time since exit. The red solid lines shows the re-entry rate conditional on the 1-year

[29] See Table 6 in the Online Appendix.

Figure 7: **Entry and exit: data versus model.** The figure shows the time-series of entry and exit from the stock market in Finland and Norway (the blue lines with dots), compared to the simulated entry and exit for the *mix* scenario using $\phi = 0.0$ (solid lines) and $\phi = 0.5$ (dashed lines), where n is the length of the initial window for the prior belief and T_ϕ is the t-statistics on the actual entry (exit) rate from regressing the model entry (exit) rate on a constant and the actual entry (exit) rate. The number of individuals entering and exiting from the stock market in Finland are from Breitkopf, Knüpfer, and Rantapuska (2021), and the total population from Statistics Finland. The percentage of individuals entering and exiting from the stock market in Norway are from Statistics Norway. For parameters see Subsection 3.1. For implied shocks see Subsection 3.3. Agent types (reentry, disappointment, designated participants, and designated nonparticipants) have 25% weight at birth.

(a) **Entry in the stock market - Finland and Norway**



(b) **Exit from the stock market - Finland and Norway**

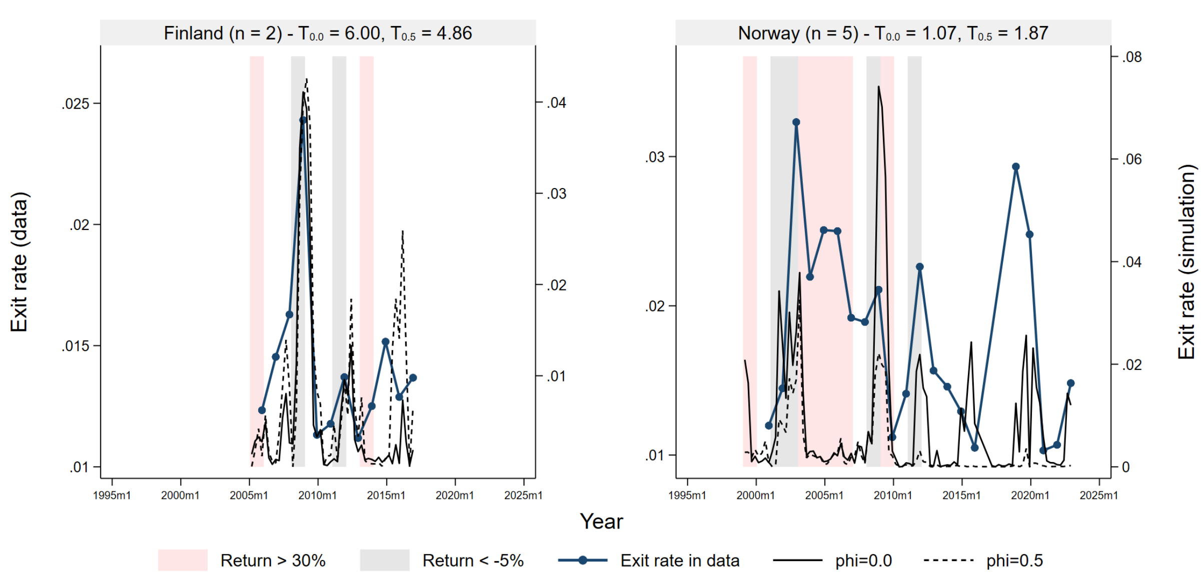
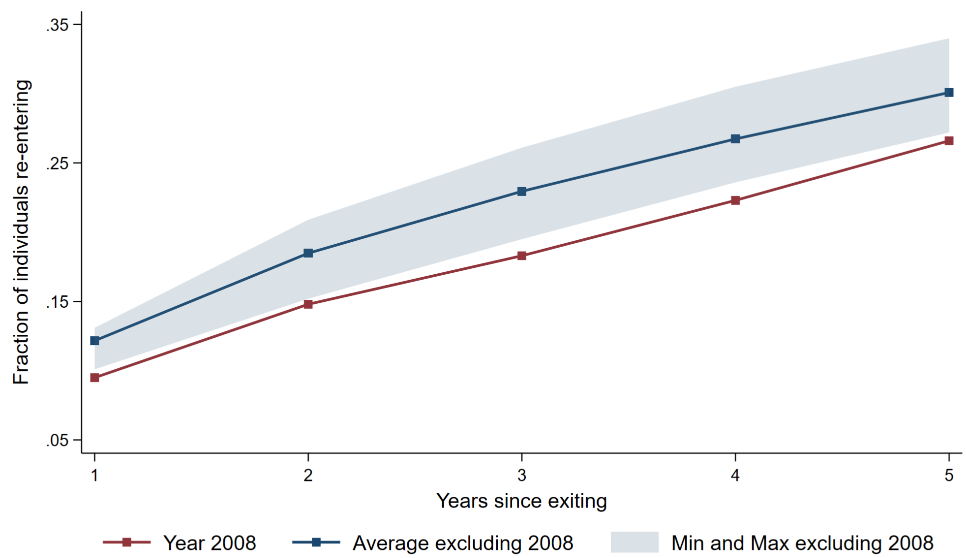
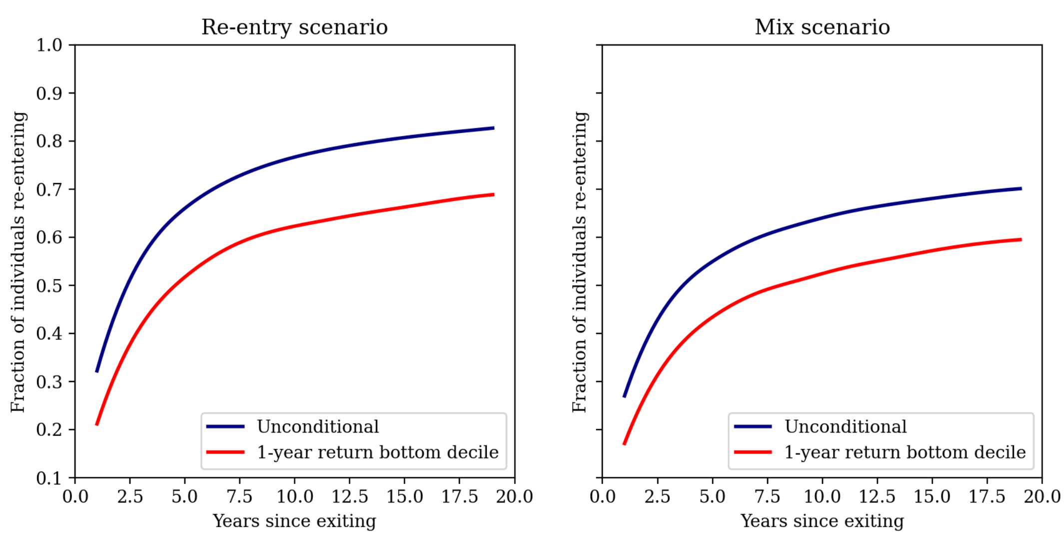


Figure 8: **Fraction of investors re-entering the stock market - data versus model.** The top plot shows the fraction of investors re-entering the stock market within certain years since exit, using Finnish data. The red line is the fraction of re-entering investors among those exit in 2008, and the blue line is the average among those exit in year 2005 to 2011, excluding 2008. The bottom plots show the fraction of investors returning to the stock market since exit, for the reentry scenario (left) and the mix scenario (right), conditional on realized 1-year stock returns at the time of exit. The blue lines are unconditional average, and the red lines are when the 1-year stock return are within the bottom decile at the time of exit. We set $\phi = 0.5$, $\tau = 0.35$, and use a 5-year time-series for pre-entry learning.

(a) **Fraction of investors re-entering within n years after exiting - Finnish data**



(b) **Fraction of investors re-entering within n years after exiting - simulation**



stock return at the time of exit being below the 10^{th} percentile in the time-series. The take away from this exercise is that, as in the data, we see a large gap in the fraction of re-entering investors in the *reentry* scenario when investors leave upon experiencing particularly negative returns. In the *mix* scenario, where 25% of agents are of the disappointment type who leave the stock market permanently upon perceiving market price of risk below zero, both the unconditional average and the conditional average are lower, but otherwise this variant of the model reproduces the pattern in the data from Finland equally well.

4.4 Predictability

In our model, the participation rate negatively predicts stock market returns as it is correlated with the consumption and wealth share of the participants, which are state variables for the interest rate and the market price of risk. Intuitively, times of particularly high stock market participation are times of rampant exuberance, especially by the young. As over time statistically more common shocks fade away the exuberance, we see lower realized stock market returns. Panel A in Table 3 confirms this intuition both for total and excess model-based returns for 12, 24 and 36 month horizons.

Panel B lends empirical support to the predictive power of the participation rate, which is the mechanism entertained in this paper. Here, We use detrended participation rates from the four countries (Finland, Germany, Norway and the USA) to run predictive panel regressions. As we can see, all coefficients of the participation rate are, consistent with the model, negative and are statistically significant at the 0.01 level for 12 and 24 month horizons for total as well as excess returns. For the 36 month horizon, the statistical significance of the predictive power of the participation rate declines to the 0.1 level. These regressions include country fixed effects with standard errors clustered by country. Further, the results are robust to including the price-dividend-ratio, see Table 7 in the Online Appendix.

Panel A: Simulation results						
	Total returns			**Excess returns**		
	(1)	(2)	(3)	(4)	(5)	(6)
	12m	24m	36m	12m	24m	36m
Participation rate	-0.022	-0.032	-0.039	-0.054	-0.076	-0.093

Panel B: International evidence						
	Total returns			**Excess returns**		
	(1)	(2)	(3)	(4)	(5)	(6)
	12m	24m	36m	12m	24m	36m
Participation rate	-0.350***	-0.374***	-0.323*	-0.378***	-0.406***	-0.361**
	(0.032)	(0.053)	(0.121)	(0.048)	(0.048)	(0.112)
R-squared	.1418	.1755	.1343	.1428	.1728	.1379
N	94	91	88	94	91	88

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3: **Participation rate and future returns - data versus model.** Panel A shows predictive regressions with 12, 24 and 36 month horizons, with model participation rate as explanatory variable, where columns (1) - (3) show regressions with total returns and columns (4) - (6) use excess returns. All the variables are standardized. The coefficients estimates are averaged from the regression coefficients obtain from 10000 paths, with non-overlapping annual data samples along each path, each with 200 data points. Panel B shows predictive regressions with 12, 24 and 36 month horizons, with detrended participation rate as explanatory variable from Finland, Germany, Norway and the USA, where columns (1) - (3) show regressions with total returns and columns (4) - (6) use excess returns. Interest rates are 12-month T-bill rates from Bloomberg for Finland, Bundesbank for Germany, Statistics Norway for Norway, and Amit Goyal's website for the USA. All the variables are standardized. The panel regressions include country fixed effects with standard errors clustered on country.

4.5 The Cost of Beliefs-driven Entry and Exit

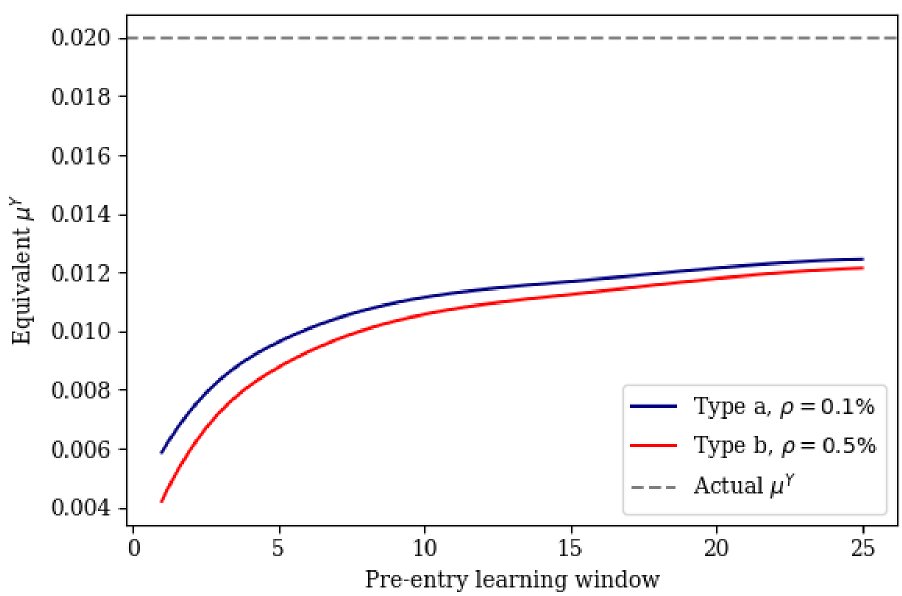
Above we have learned that entry and exit as a result of extrapolative beliefs formed based on a short sample can be costly, as agents tend to enter at times when expected returns are low and exit at times when expected returns are high.

To measure the cost of entry and exit, we back out the growth rate of aggregate consumption that yields the same realized average life-time utility in an economy with complete information as in our incomplete information economy, where in the complete information economy all cohorts participate.[30] This equivalent economic growth rate in the *reentry* scenario di￿ers for agents with di￿erent time-preference and depends on the pre-entry learning window. According to Figure 9 the cost of entry and exit appears very large in that even for the impatient type agent with a 25 year pre-entry learning the equivalent growth rate with complete information is only 1.3% compared to the actual 2% in our model. With a one-year pre-entry learning the equivalent consumption growth rate is as low as 0.6% for type-a agents and as low as 0.4% for type-b agents.

The documented costs of entry and exit expressed in terms of reduced consumption growth may be best compared to an equivalent increase in consumption volatility in the complete market benchmark. As an example, raising the aggregate consumption volatility in the frictionless benchmark economy from 3.3% to 10% yields the same expected utility as a reduction in consumption growth from 2% to 1.6%. As even with a long 25 year pre-entry learning window the equivalent growth rate relative to the complete information is already as low as 1.3%, we conclude that the welfare cost of the return-driven entry and exit that we study cannot be understated.

[30] For details of the derivations on the benchmark economy with complete information, refer to Subsection C.3 in the Online Appendix.

Figure 9: **Equivalent economic growth rate in the *reentry* scenario.** The blue and red lines denotes the equivalent economic growth rate in a benchmark OLG economy with complete information, which gives the same realized average life-time utility with a given number of years of pre-entry learning window, for an agent with time-preference ρ equals to 0.1% and 0.5%, respectively. The gray dash line denotes the actual economy growth rate in the economy with complete information. For details of the derivations on the benchmark economy with complete information, refer to Subsection C.3 in the Online Appendix. For parameters see Subsection 3.1.



5 Conclusion

In this paper, we study the endogenous entry into and exit from the stock market through the interplay between constrained learning from experience and an effectively binding constraint mimicking that most investors cannot or do not want to short. In our general equilibrium model, overlapping generations of agents form beliefs about the output growth rate through Bayes' rule and through a constrained rationality, which deviates from full rationality along two dimensions. First, agents are exogenously exposed to different parts of an economic path by birth and thus form different expectations about output growth rate accordingly. Second, the presence of the constraint endogenously distinguishes the agents as either participants who invest in the stock market, or nonparticipants who hold only the bond based on their beliefs. By assumption, an additional signal containing imperfect information about the fundamental is not observed by the nonparticipants. This captures the notion of reduced attention. As belief and wealth dynamics determine the equilibrium Sharpe ratio and participation, they in turn drive belief formation and wealth accumulation. Consequently, nonparticipation slows down the speed of learning in the aggregate economy.

Besides the large cross-section of cohorts, our model accommodates many agent types within a cohort, including unconstrained designated participants, designated nonparticipants who never invest in the stock market, and agents who can leave the stock market permanently when they are disappointed by the experienced stock returns. With these ingredients, our model produces a large cross section of beliefs, as well as time-varying participation as cohorts enter and exit the stock market endogenously.

Upon crossing the equilibrium cutoff belief between participants and non-participants, agents enter into or exit from the stock market. The cutoff belief corresponds to the negative of the market price of risk; it depends not only on learning of an individual agent or cohort but also on the entire belief and wealth distribution. Through the *beliefs-driven participation effect*, the participants consumption weighted estimation error and the consumption share of the participants, fully determine the real short rate of interest and the market price of risk.

This feature implies an asymmetry between stock market participants and nonparticipants, as only the beliefs and consumption dynamics of the relatively optimistic agents who participate in the stock market are directly relevant for equilibrium asset pricing moments in our constrained economy. As the old have high consumption share, the cutoff belief gravitates around their beliefs, which leads to overall higher participation rate among the old.

We leverage on the large cross-section of agents in the model, and feed into the model the stock market shocks to produce simulated time-series of participation, entry and exit. The upshot is that the model produces realistic dynamics of participant rates as well as entry and exit vis-à-vis data from Finland, Germany, Norway and the US. In our model, fluctuations in participation, entry and exit are entirely driven by constrained learning from experience, and it captures the swings we observe in the data across the countries.

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A Proofs of Propositions

A.1 Proof of Proposition 1

Following standard filtering theory in Liptser and Shiryayev (2013), we have that the dynamics of the expected output growth rate and the posterior variance of the estimates perceived by an agent born at s being either N or P are given by Equations (5 - 6). From the definition of the standardized estimation error $\Delta_{s,t}$, and using Ito's lemma, we have

$$d\Delta_{s,t} = \frac{d\hat{\mu}_{s,t}}{\sigma_Y} = \begin{cases} \frac{\hat{V}_s'}{\sigma_Y^2 + \hat{V}_s'\left(t - t_s'\right)} dz_{s,t}^Y, & \text{if } N \text{, or} \\ \frac{(1-\phi^2)\hat{V}_s'}{\sigma_Y^2(1-\phi^2) + \hat{V}_s'\left(t - t_s'\right)} \left(dz_{s,t}^Y - \frac{\phi}{\sqrt{1-\phi^2}} dz_{s,t}^{SI} \right), & \text{if } P. \end{cases}$$

Solving the stochastic di erential equations gives the following expression for $\Delta_{s,t}$:

$$\Delta_{s,t} = \begin{cases} \frac{\sigma_Y^2}{\sigma_Y^2 + \hat{V}_s'\left(t - t_s'\right)} \Delta_{s,t_s'} + \frac{\hat{V}_s'}{\sigma_Y^2 + \hat{V}_s'\left(t - t_s'\right)} \left(z_t^Y - z_{t_s'}^Y \right), & \text{if } N \text{, or} \\ \frac{\sigma_Y^2(1-\phi^2)}{\sigma_Y^2(1-\phi^2) + \hat{V}_s'\left(t - t_s'\right)} \Delta_{s,t_s'} + \frac{\hat{V}_s'(1-\phi^2)}{\sigma_Y^2(1-\phi^2) + \hat{V}_s'\left(t - t_s'\right)} \left\{ \left(z_t^Y - z_{t_s'}^Y \right) - \frac{\phi}{\sqrt{1-\phi^2}} \left(z_t^{SI} - z_{t_s'}^{SI} \right) \right\}, & \text{if } P. \end{cases}$$

A.2 Proof of Proposition 2

The proof follows directly from Equation (19), when t = s, and $c_{i,s,s} = \frac{1}{\kappa_{i,s}(1+\tau)\xi_{s,s}}$. We also show that β_i is a constant through the initial budget constraint. Given that $\beta_i \equiv \frac{c_{i,s,t}}{W_{i,s,t}}$ for any $t > s$, we have

$$\begin{aligned} W_{i,s,s} &= E_{s,s}\left[\int_s^\infty e^{-\nu(t-s)} \frac{\xi_{s,t}}{\xi_{s,s}} \left(c_{i,s,t} + \tau c_{i,s,t}\right) dt\right] \\ &= (1+\tau)\, E_{s,s}\left[\int_s^\infty e^{-\nu(t-s)} \frac{\xi_{s,t}}{\xi_{s,s}} c_{i,s,s} e^{-\rho_i(t-s)} \frac{\xi_{s,s}}{\xi_{s,t}} dt\right] \\ &= (1+\tau)\, \frac{1}{\rho_i + \nu} c_{i,s,s} \\ &= \frac{c_{i,s,s}}{\beta_i}. \end{aligned}$$

We thus have $\beta_i = \frac{\rho_i + \nu}{1+\tau}$, which is a constant.

A.3 Proof of Proposition 3

From the market clearing condition for the consumption goods market, we find

$$
\begin{aligned}
Y_t &= \int_i \alpha_i \int_{-\infty}^{t} \nu e^{-\nu(t-s)} c_{i,s,t} ds di, \quad \text{a.s., and therefore,} \\
dY_t &= \left(\int_i \alpha_i \nu c_{i,t,t} di - \nu C_t \right) dt \\
&\quad + \int_i \alpha_i \int_{-\infty}^{t} \nu e^{-\nu(t-s)} c_{i,s,t} \left[\left(-\rho_i + r_t + \theta_{s,t}^{+} \theta_t \right) dt + \theta_{s,t}^{+} dz_t^Y \right] ds di \\
&= \left(\int_i \alpha_i \nu \beta_i W_{i,t,t} di - \nu C_t \right) dt + r_t C_t dt - \left(\int_i \alpha_i \rho_i \int_{-\infty}^{t} \nu e^{-\nu(t-s)} c_{i,s,t} ds di \right) dt \\
&\quad + \theta_t \left[\int_i \alpha_i \int_{-\infty}^{t} \nu e^{-\nu(t-s)} c_{i,s,t} \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right] dt \\
&\quad + \left[\int_i \alpha_i \int_{-\infty}^{t} \nu e^{-\nu(t-s)} c_{i,s,t} \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right] dz_t^Y \\
&= \left(\int_i \alpha_i \nu \beta_i \frac{\tau}{\nu} C_t di - \nu C_t \right) dt + r_t C_t dt - \left(\int_i \alpha_i \rho_i \int_{-\infty}^{t} f_{i,s,t}^{c} ds di \right) C_t dt \\
&\quad + \theta_t C_t \left[\int_i \alpha_i \int_{-\infty}^{t} f_{i,s,t}^{c} \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right] dt \\
&\quad + C_t \left[\int_i \alpha_i \int_{-\infty}^{t} f_{i,s,t}^{c} \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right] dz_t^Y \\
&= C_t \left\{ \left(\tau\beta - \nu + r_t - \bar{\rho}_t + \theta_t^2 \bar{\Phi}_t + \theta_t \bar{\Phi}_t \bar{\Delta}_t \right) dt + \bar{\Phi}_t \left(\theta_t + \bar{\Delta}_t \right) \right\} \\
&= Y_t \left(\mu_Y dt + \sigma_Y dz_t^Y \right),
\end{aligned}
$$

using the definitions of β, $\bar{\Delta}_t$ and $\bar{\Phi}_t$. By matching the drift and diffusion terms, we have the interest rate r_t and the market price of risk θ_t.

A.4 Proof of Proposition 4

As the bond has zero net supply, and the stock is the only form of wealth in the economy, we have,

$$
S_t = W_t = \int_i \alpha_i \int_{-\infty}^{t} \nu e^{-\nu(t-s)} W_{i,s,t} ds di.
$$

With log utility, $\frac{dW_{i,s,t}}{W_{i,s,t}} = \frac{dc_{i,s,t}}{c_{i,s,t}}$ holds for each type i. Using Proposition (20) and $\int_i \alpha_i \nu W_{i,t,t} di = \tau C_t$ from consumption tax, we have

$$
\begin{aligned}
dW_t &= d\int_i \alpha_i \int_{-\infty}^{t} \nu e^{-\nu(t-s)} W_{i,s,t} ds di \\
&= \int_i \alpha_i \nu W_{i,t,t} di dt - \nu W_t dt \\
&\quad + \int_i \alpha_i \int_{-\infty}^{t} \nu e^{-\nu(t-s)} W_{i,s,t} \left[\left(-\rho_i + r_t + \theta_{s,t}^{+} \theta_t \right) dt + \theta_{s,t}^{+} dz_t^Y \right] ds di \\
&= W_t \left\{ \tau \tilde{\beta}_t dt - \nu dt + \int_i \alpha_i \int_{-\infty}^{t} f_{i,s,t}^W \left[\left(-\rho_i + r_t + \theta_{s,t}^{+} \theta_t \right) dt + \theta_{s,t}^{+} dz_t^Y \right] ds di \right\} \\
&= W_t \left\{ \tau \tilde{\beta}_t - \nu - \tilde{\rho}_t + r_t + \theta_t \int_i \alpha_i \int_{-\infty}^{t} f_{i,s,t}^W \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right\} dt \\
&\quad + W_t \left\{ \int_i \alpha_i \int_{-\infty}^{t} f_{i,s,t}^W \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right\} dz_t^Y \\
&= W_t \left\{ \tau \tilde{\beta}_t - \nu - \tilde{\rho}_t + r_t + \theta_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right) \right\} dt + W_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right) dz_t^Y \\
&= dS_t = S_t \left(\mu_t^S dt + \sigma_t^S dz_t^Y \right) - Y_t dt,
\end{aligned}
$$

where $\tilde{\rho}_t = \int_i \rho_i \int_{-\infty}^{t} f_{i,s,t}^W ds di$ is the wealth weighted average time preference, and $\tilde{\beta}_t \equiv \frac{C_t}{W_t}$ is the aggregate consumption-to-wealth ratio. Matching the drift and diffusion terms, we then have μ_t^S and σ_t^S. Specifically, with the expression of β_i from A.2, we have

$$
\begin{aligned}
\tilde{\beta}_t = \frac{C_t}{W_t} = \frac{\int_i \int_{-\infty}^{t} \nu e^{-\nu(t-s)} c_{i,s,t} ds di}{W_t} &= \int_i \int_{-\infty}^{t} \frac{c_{i,s,t}}{W_{i,s,t}} \frac{W_{i,s,t}}{W_t} ds di \\
&= \int_i \beta_i \int_{-\infty}^{t} f_{i,s,t}^W ds di = \int_i \frac{\rho_i + \nu}{1+\tau} \int_{-\infty}^{t} f_{i,s,t}^W ds di \\
&= \frac{\tilde{\rho}_i + \nu}{1+\tau}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\mu_t^S - r_t &= \tau \tilde{\beta}_t - \nu - \tilde{\rho}_t + \theta_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right) + \frac{Y_t}{S_t} = \tau \tilde{\beta}_t - \nu - \tilde{\rho}_t + \theta_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right) + \tilde{\beta}_t \\
&= (\tau + 1) \tilde{\beta}_t - \nu - \tilde{\rho}_t + \theta_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right) \\
&= \theta_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right).
\end{aligned}
$$

A.5 Proof of Proposition 5

The proof is obtained by matching the diffusion terms of the wealth process as in Equation (17) and the relation between wealth and consumption, as well as the fact that for participants we have, $\theta^+_{s,t} = \theta_{s,t} = \theta_t + \Delta_{s,t}$,

$$\begin{aligned}
W_{i,s,t} =& \frac{c_{i,s,t}}{\beta_i} = \frac{c_{i,s,s}e^{-\rho_i(t-s)}}{\beta_i}\frac{\eta_{i,s,t}}{\eta_{i,s,s}}\frac{\xi_s}{\xi_t}, \\
\therefore dW_{i,s,t}/W_{i,s,t} =& \left[-\rho_i + r_t + \theta_t^2 - \left(\theta^+_{s,t} - \theta_t\right)\theta_t\right]dt + \left(\theta_t + \theta^+_{s,t} - \theta_t\right)dz^Y_t \\
=& \left(-\rho_i + r_t + \theta_t^2 - \Delta_{s,t}\theta_t\right)dt + \left(\theta_t + \Delta_{s,t}\right)dz^Y_t,
\end{aligned}$$

and thus, $\pi_{i,s,t} = \frac{\Delta_{s,t}+\theta_t}{\sigma^S_t}W_{i,s,t}$ for stock market participants. The optimal stock investment also confirms that an agent born at s would have non-negative demand for risk at time t if $\Delta_{s,t} \geq -\theta_t$. Otherwise, she would be subject to the short-sale constraint and not participate in the stock market.

A.6 Proof of Proposition 6

From the expression for optimal consumption as described in Proposition 2, and using Ito's lemma, we have

$$dc_{i,s,t}/c_{i,s,t} = (-\rho_i + r_t + \theta^+_{s,t}\theta_t)dt + \theta^+_{s,t}dz^Y_t,$$

which implies that

$$\begin{aligned}
d\log(c_{i,s,t}) =& \left(-\rho_i + r_t + \theta^+_{s,t}\theta_t - \frac{1}{2}(\theta^+_{s,t})^2\right)dt + \theta^+_{s,t}dz^Y_t \\
=& \begin{cases} (-\rho_i + r_t)dt, & \text{if } N\text{, or} \\ (-\rho_i + r_t + \frac{1}{2}\theta_t^2 - \frac{1}{2}\Delta^2_{s,t})dt + (\theta_t + \Delta_{s,t})dz^Y_t, & \text{if } P. \end{cases}
\end{aligned}$$

A.7 Proof of Proposition 7

From the definition of $f_{i,s,t}$, we have

$$\begin{aligned}
df^c_{i,s,t} =& d\frac{\alpha_i \nu e^{-\nu(t-s)}c_{i,s,t}}{C_t} \\
df^c_{i,s,t}/f^c_{i,s,t} =& -\nu dt + \frac{dc_{i,s,t}}{c_{i,s,t}} - \frac{dC_t}{C_t} + \frac{dC_t^2}{C_t^2} - \frac{dc_{i,s,t}dC_t}{c_{i,s,t}C_t} \\
=& \left\{-\nu - \rho_i + r_t + \theta^+_{s,t}\theta_t - \mu_Y + \sigma_Y^2 - \theta^+_{s,t}\sigma_Y\right\}dt + \left(\theta^+_{s,t} - \sigma_Y\right)dz^Y_t \\
=& -\tau\beta dt + (\theta_t - \sigma_Y)(\theta^+_{s,t} - \sigma_Y)dt + (\theta^+_{s,t} - \sigma_Y)dz^Y_t.
\end{aligned}$$

Recall that $\theta^+_{s,t} = 0$ for N and $\theta^+_{s,t} = \theta_t + \Delta_{s,t}$ for P. Thus, the dynamics of consumption share follows

$$
\begin{aligned}
df^c_{i,s,t}/f^c_{i,s,t} &= \left(-\tau\beta + \bar{\rho}_t - \rho_i\right) dt + \left(\theta_t - \sigma_Y\right)\left(\theta^+_{s,t} - \sigma_Y\right) dt + \left(\theta^+_{s,t} - \sigma_Y\right) dz^Y_t \\
&= \begin{cases} \left[-\tau\beta + \bar{\rho}_t - \rho_i - \sigma_Y\left(\theta_t - \sigma_Y\right)\right] dt - \sigma_Y dz^Y_t, & \text{if } N, \text{ or} \\ \left[-\tau\beta + \bar{\rho}_t - \rho_i - \sigma_Y\left(\theta_t - \sigma_Y\right) + \left(\theta_t + \Delta_{s,t}\right)\left(\theta_t - \sigma_Y\right)\right] dt & \\ \qquad + \left(\theta_t + \Delta_{s,t} - \sigma_Y\right) dz^Y_t, & \text{if } P. \end{cases}
\end{aligned}
$$

For wealth share, since $df^W_{i,s,t} = d\frac{\alpha_i \nu e^{-\nu(t-s)} W_{i,s,t}}{W_t}$, we have

$$
\begin{aligned}
df^W_{i,s,t}/f^W_{i,s,t} &= -\nu dt + \frac{dW_{i,s,t}}{W_{i,s,t}} - \frac{dW_t}{W_t} + \frac{dW_t^2}{W_t^2} - \frac{dW_{i,s,t}dW_t}{W_{i,s,t}W_t} \\
&= -\nu dt + \frac{dc_{i,s,t}}{c_{i,s,t}} - \frac{dW_t}{W_t} + \frac{dW_t^2}{W_t^2} - \frac{dc_{i,s,t}dW_t}{c_{i,s,t}W_t} \\
&= \left\{-\nu - \rho_i + r_t + \theta^+_{s,t}\theta_t - \mu^S_t + \frac{Y_t}{W_t} + \left(\sigma^S_t\right)^2 - \theta^+_{s,t}\sigma^S_t\right\} dt + \left(\theta^+_{s,t} - \sigma^S_t\right) dz^Y_t \\
&= \left\{-\nu - \rho_i + \frac{Y_t}{W_t} + \theta^+_{s,t}\theta_t - \sigma^S_t\theta_t + \left(\sigma^S_t\right)^2 - \theta^+_{s,t}\sigma^S_t\right\} dt + \left(\theta^+_{s,t} - \sigma^S_t\right) dz^Y_t \\
&= \begin{cases} \left[-\nu - \rho_i + \tilde{\beta}_t - \sigma^S_t\left(\theta_t - \sigma^S_t\right)\right] dt - \sigma^S_t dz^Y_t, & \text{if } N, \text{ or} \\ \left[-\nu - \rho_i + \tilde{\beta}_t - \sigma^S_t\left(\theta_t - \sigma^S_t\right) + \left(\theta_t + \Delta_{s,t}\right)\left(\theta_t - \sigma^S_t\right)\right] dt & \\ \qquad + \left(\theta_t + \Delta_{s,t} - \sigma^S_t\right) dz^Y_t, & \text{if } P. \end{cases}
\end{aligned}
$$

B Other Proofs

B.1 Proof of the Static Budget Constraint

With Equations (17) and (13), and applying Ito's lemma, we have

$$
\begin{aligned}
d\left(\xi_{s,t}W_{i,s,t}\right) &= -\xi_{s,t}W_{i,s,t}\left(r_t dt + \theta_{s,t}dz^Y_{s,t}\right) \\
&\quad + \xi_{s,t}\left\{\left[\left(r_t + \nu - \beta\right)W_{i,s,t} + \pi_{i,s,t}\sigma^S_t\theta_{s,t} - c_{i,s,t}\right] dt + \pi_{i,s,t}\sigma^S_t dz^Y_{s,t}\right\} \\
&\quad - \xi_{s,t}\theta_{s,t}\sigma^S_t\pi_{i,s,t}dt \\
&= \xi_{s,t}\left((\nu - \beta_i)W_{i,s,t} - c_{i,s,t}\right) dt + \xi_{s,t}\left(-W_{i,s,t}\theta_{s,t} + \pi_{i,s,t}\sigma^S_t\right) dz^Y_{s,t}.
\end{aligned}
$$

With stochastic time of death T, we have

$$\xi_{s,t}W_{i,s,\tau} + \int_s^T \xi_{s,t}c_{i,s,t}dt + \int_s^T \xi_{s,t}\beta_i W_{i,s,t}dt - \int_s^T \xi_{s,t}\nu W_{i,s,t}dt$$
$$= \xi_{s,s}W_{i,s,s} + \int_s^T \xi_{s,t}\left(\pi_{i,s,t}\sigma_t^S - W_{i,s,t}\theta_{s,t}\right)dz_{s,t}^Y.$$

Internet Appendix

C Online Appendix (Not for publication)

C.1 Wealth Tax

Alternative to consumption tax, we can impose a wealth tax on all living agents to finance the consumption of the new-born cohort. With wealth tax at rate τ^W, the dynamic budget constraint becomes

$$dW_{i,s,t} = \begin{cases} \left(r_t W_{i,s,t} + \nu W_{i,s,t} - \tau^W W_{i,s,t} - c_{i,s,t}\right) dt, & \text{if } N, \quad \text{or} \\ \left[r_t W_{i,s,t} + \pi_{i,s,t}\left(\mu^S_{s,t} - r_t\right) + \nu W_{i,s,t} - \tau^W W_{i,s,t} - c_{i,s,t}\right] dt + \pi_{i,s,t}\sigma^S_t dz^Y_{s,t}, & \text{if } P, \end{cases}$$

which implies that the static budget constraint is

$$E_{s,s}\left[\int_s^\infty e^{-v(t-s)}\xi_{s,t}(c_{i,s,t} + \tau^W W_{i,s,t})dt\right] = W_{i,s,s}.$$

We conjecture and verify that $\beta_i = \frac{c_{i,s,t}}{W_{i,s,t}}$ is a constant. Using (2), we have $\frac{1}{\rho_i+\nu}c_{i,s,t}\left(1 + \frac{\tau^W}{\beta_i}\right) = W_{i,s,t} = \frac{c_{i,s,t}}{\beta_i}$. Rearranging, we have $\beta_i = \rho_i + \nu - \tau^W$, which is a constant. It then follows that the equilibrium real short rate of interest and the market price of risk with wealth tax are

$$r_t = \nu - \tau\frac{\beta}{\tilde{\beta}_t} + \bar{\rho}_t + \mu_Y - \sigma_Y\left(\sigma_Y\frac{1}{\bar{\Phi}_t} - \bar{\Delta}_t\right) = \nu - \tau^W\frac{\beta}{\tilde{\beta}_t} + \bar{\rho}_t - \sigma^2_Y\frac{1}{\bar{\Phi}_t} + \bar{\mu}_t, \tag{A1}$$

and the market price of risk is

$$\theta_t = \sigma_Y\frac{1}{\bar{\Phi}_t} - \bar{\Delta}_t = \sigma_Y\frac{1}{\bar{\Phi}_t} - \frac{\bar{\mu}_t - \mu_Y}{\sigma_Y}, \tag{A2}$$

where $\tilde{\beta}_t \equiv \frac{Y_t}{W_t} = \int_i \beta_i \int_{-\infty}^t f^W_{i,s,t} ds di$ is the aggregate consumption-wealth ratio at time t.[31] Imposing a wealth tax instead of a consumption tax does not change the expressions of the stock volatility σ^S_t, the risky portfolio $\pi_{i,s,t}$, as well as the dynamics of log consumption $log(c_{i,s,t})$ in Propositions (4), (5), and (6). The consumption share at time t of type i agents

[31] Equivalently, $\tilde{\beta}_t$ is also the wealth-weighted average consumption-wealth ratio in the economy, as $\frac{Y_t}{W_t} = \frac{\int_i \int_{-\infty}^t \alpha_i v e^{-v(t-s)} c_{i,s,t} ds di}{W_t} = \frac{\int_i \int_{-\infty}^t \alpha_i \beta_i v e^{-v(t-s)} W_{i,s,t} ds di}{W_t} = \int_i \beta_i \int_{-\infty}^t f^W_{i,s,t} ds di$. Therefore, $\tilde{\beta}_t = \tilde{\rho}_t + \nu - \tau^W$.

born at s follows the process

$$
\begin{aligned}
df^{c}_{i,s,t}/f^{c}_{i,s,t} &= \left(-\tau^{W}\frac{\beta}{\tilde{\beta}_t}+\bar{\rho}_t-\rho_i\right)dt+\left(\theta_t-\sigma_Y\right)\left(\theta^{+}_{s,t}-\sigma_Y\right)dt+\left(\theta^{+}_{s,t}-\sigma_Y\right)dz^{Y}_{t} \\
&= \begin{cases}
\left[-\tau^{W}\frac{\beta}{\tilde{\beta}_t}+\bar{\rho}_t-\rho_i-\sigma_Y\left(\theta_t-\sigma_Y\right)\right]dt-\sigma_Y dz^{Y}_{t}, & \text{if } N\text{, or} \\
\left[-\tau^{W}\frac{\beta}{\tilde{\beta}_t}+\bar{\rho}_t-\rho_i-\sigma_Y\left(\theta_t-\sigma_Y\right)+\left(\theta_t+\Delta_{s,t}\right)\left(\theta_t-\sigma_Y\right)\right]dt & \\
\qquad +\left(\theta_t+\Delta_{s,t}-\sigma_Y\right)dz^{Y}_{t}, & \text{if } P,
\end{cases}
\end{aligned}
$$

and the wealth share at time t of type i agents born at s follows the process

$$
df^{W}_{i,s,t}/f^{W}_{i,s,t} = \begin{cases}
\left[-\tau^{W}+\tilde{\rho}_t-\rho_i-\sigma^{S}_{t}\left(\theta_t-\sigma^{S}_{t}\right)\right]dt-\sigma^{S}_{t}dz^{Y}_{t}, & \text{if } N\text{, or} \\
\left[-\tau^{W}+\tilde{\rho}_t-\rho_i-\sigma^{S}_{t}\left(\theta_t-\sigma^{S}_{t}\right)+\left(\theta_t+\Delta_{s,t}\right)\left(\theta_t-\sigma^{S}_{t}\right)\right]dt & \\
\qquad +\left(\theta_t+\Delta_{s,t}-\sigma^{S}_{t}\right)dz^{Y}_{t}, & \text{if } P.
\end{cases}
$$

C.2 Equilibrium with *Disappointment* Type

Here, we consider an alternative economy, which has an identical setting with the economy described in the main model section (Section 2), except that a cohort leaves the stock market for good once the short-sale constraint binds (and to simply the exposition we turn o heterogeneity in time preferences). In this setting, the same learning mechanism continues to hold, while the disagreement process contains an indicator variable, $I_{s,t}$:

$$I_{s,t} \equiv \begin{cases} 1, & \text{if } t \leq t_s^{'}, \\ 0, & \text{otherwise,} \end{cases}$$

where $t_s^{'}$ denotes the first time when cohort s transitions from P to N. For a cohort to participate in the stock market, both the two conditions $I_{s,t} = 1$ and $\theta_{s,t} \geq 0$ have to be satisfied. Therefore, the disagreement process follows

$$d\eta_{s,t}^{'}/\eta_{s,t}^{'} = (\theta_{s,t}^{+} \cdot I_{s,t} - \theta_t^{'})dz_t^Y.$$

In equilibrium, the real short rate of interest is

$$r_t^{'} = \nu - \tau + \rho - \sigma_Y^2 \frac{1}{\Phi_t^{'}} + \bar{\mu}_t^{'},$$

and the market price of risk is

$$\theta_t^{'} = \sigma_Y \frac{1}{\Phi_t^{'}} - \frac{\bar{\mu}_t^{'} - \mu_Y}{\sigma_Y},$$

where $\Phi_t^{'} \equiv \int_{-\infty}^{t} f_{s,t} \mid (\Delta_{s,t} \geq -\theta_t^{'} \cap I_{s,t} = 1)ds$ is the total wealth share of the participants in the stock market, and $\bar{\mu}_t^{'} \equiv \frac{\int_{-\infty}^{t} f_{s,t}\hat{\mu}_{s,t} | (\Delta_{s,t} \geq -\theta_t^{'} \cap I_{s,t}=1)ds}{\Phi_t^{'}}$ denotes the wealth-weighted average expected growth rate conditional on stock market participation. Except for the additional component $I_{s,t}$, the real short rate of interest and the market price of risk have the same construction as in the *reentry* scenario. Construction of the equilibrium in the *mix* scenario follows analogous steps.

C.3 Benchmark OLG Economy with Complete Information

In Subsection 4.5, we compare the welfare loss from entry to and exit from the stock market using a benchmark overlapping generations economy with complete information. Clearly, complete information does not change the form of the individual or cohort specific optimization problems. Hence, the interest rate r_t and the market price of risk θ_t can be written as,

$$r_t = \nu - \tau\beta + \bar{\rho}_t + \mu_Y - \sigma_Y^2,$$
$$\theta_t = \sigma_Y,$$

where $\bar{\rho}_t = \int_t \rho_i \int_{-\infty}^t f_{i,s,t}^c ds di$ is the consumption weighted average time preference. These two equilibrium quantities have identical structure to Proposition (3), with $\bar{\Delta}_t = 0$ and $\bar{\Phi}_t = 1$ implying zero estimation error and participation by all cohorts. Furthermore, also here we conjecture and verify that $\bar{\rho}_t = \bar{\rho}$ is a constant. That is, the consumption share of type-i agents, $f_i^c \equiv \int_{-\infty}^t f_{i,s}^c ds = \int_{-\infty}^t \frac{\alpha_i \nu e^{-\nu(t-s)} c_{i,s,t}}{C_t} ds$ is a constant for any t. For f_i^c to be a constant,

$$\begin{aligned} df_i^c &= d\int_{-\infty}^t \frac{\alpha_i \nu e^{-\nu(t-s)} c_{i,s,t}}{C_t} ds \\ &= f_i^c \left(\bar{\rho} - \tau\beta - \rho_i\right) dt + \alpha_i \nu \frac{c_{i,t,t}}{C_t} dt \\ &= f_i^c \left(\bar{\rho} - \tau\beta - \rho_i\right) dt + \alpha_i \tau \beta_i dt = 0 \quad \text{satisfies,} \end{aligned}$$

which implies $f_i^c = \frac{\alpha_i \tau \beta_i}{\tau\beta + \rho_i - \bar{\rho}}$ is a constant. In our simulations with two types of time preference, ρ_a and ρ_b (let $\rho_a < \rho_b$), we can further write out $\bar{\rho} = \rho_a f_a^c + \rho_b f_b^c$. Therefore, we have

$$\begin{aligned} \bar{\rho} &= \rho_a f_a^c + \rho_b f_b^c = \rho_a f_a^c + \rho_b \left(1 - f_b^c\right) \\ &= \rho_a \frac{\alpha_a \tau \beta_a}{\tau\beta + \rho_a - \bar{\rho}} + \rho_b \left(1 - \frac{\alpha_a \tau \beta_a}{\tau\beta + \rho_a - \bar{\rho}}\right). \end{aligned}$$

Rearranging, we have $\bar{\rho}^2 - (\tau\beta + \rho_a + \rho_b) + [\alpha_a \tau \beta_a (\rho_a - \rho_b) - \tau\beta\rho_b + \rho_a\rho_b] = 0$, for which $\bar{\rho} = \frac{(\tau\beta+\rho_a+\rho_b) - \sqrt{(\tau\beta+\rho_a+\rho_b)^2 - 4\cdot[\alpha_a\tau\beta_a(\rho_a-\rho_b) - \tau\beta\rho_b + \rho_a\rho_b]}}{2} \in (\rho_a, \rho_b)$ is the only economically meaningful solution.

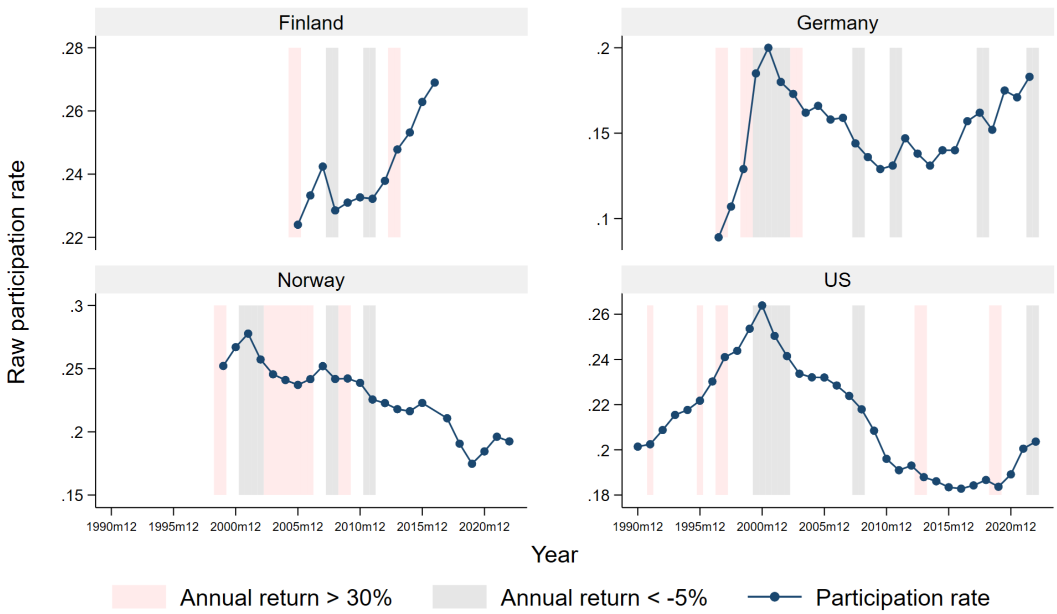
C.4 Participation Rate Time-series

In Figure 10, we present the original and the detrended time-series of participation rates for Finland, Germany, Norway and the USA.

And in Figure 11, we present the original and the detrended time-series of entry and exit rates for Finland and Norway.

Figure 10: **Time-series of stock market participation rate - raw and detrended.** The top plots show the time-series of participation rate in Finland, Germany, Norway and the USA. The bottom plots show the detrended time-series of participation rate in these countries. We detrend any significant country-specific time trends, and then add back the time-series average. We remove significant time-trends in Finland, Norway and the USA. The gray (red) shaded areas are the years when the total stock returns are lower than -5% (higher than 30%).

(a) **Raw participation rate time-series**



(b) **Detrended participation rate time-series**

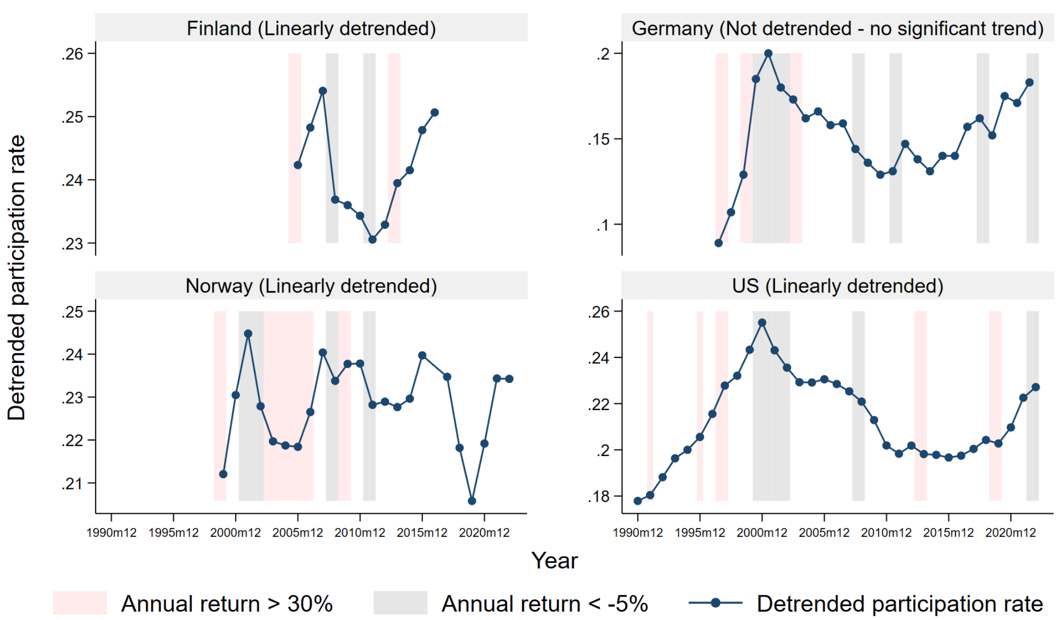
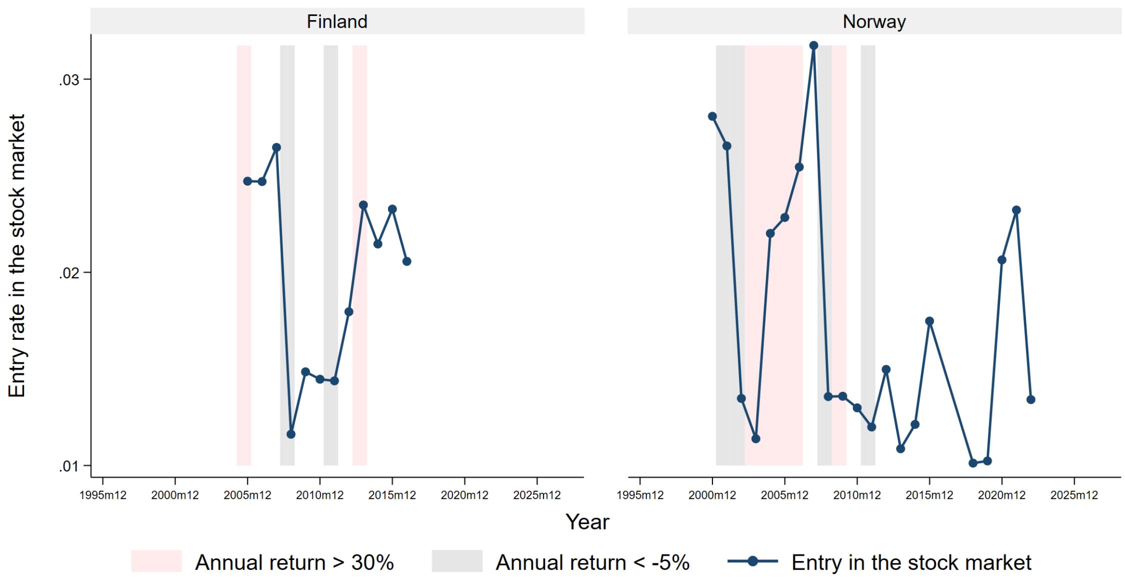
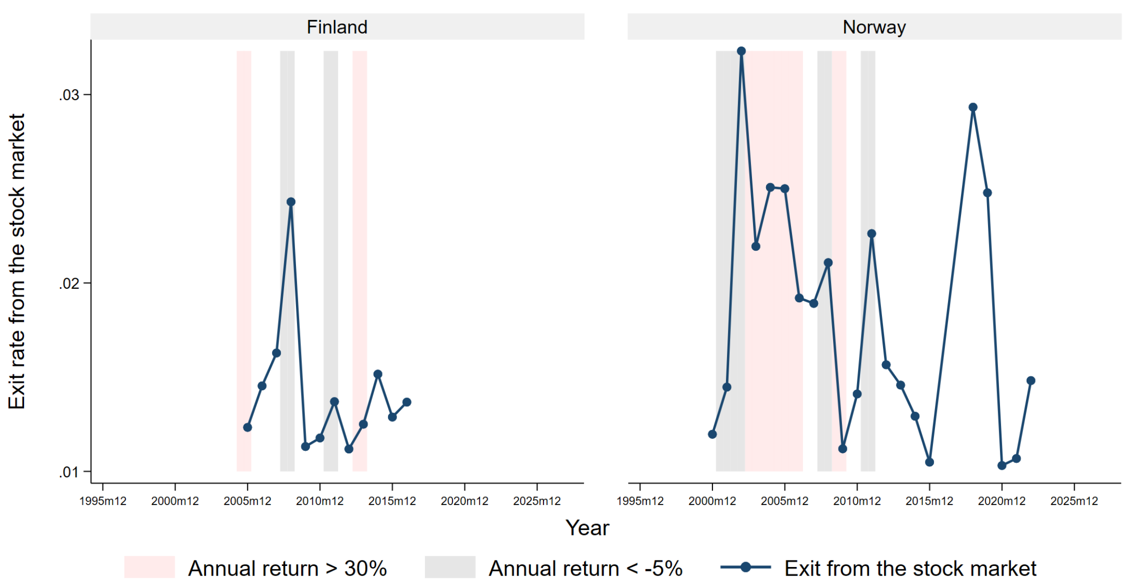


Figure 11: **Stock market and entry and exit.** The top plots show the time-series of entry rate in the stock market in Finland and Norway. The bottom plots show the time series of exit rate from the stock market in Finland and Norway. The gray shaded areas are the years when the total stock returns are lower than -5%, and the red shaded areas are the years when the total stock returns are higher than 30%. Comparable entry and exit rate in Norway for the year 2016 is missing in the tax registry due to changes in the tax definition in that year.

(a) **Entry rate in Finland and Norway**



(b) **Exit rate in Finland and Norway**



C.5 Asset Pricing Moments

Table 4 summarizes the mean and standard deviation of standard asset pricing quantities and correlations between the two shocks and the market price of risk and shocks to output and the diffusion coefficient of the stock market in Panel A. Panel B summarizes the mean and standard deviation of the participation rate and entry and exit rates (in the model and the data) along with various correlations.

Panel A: Asset pricing quantities						
	Complete Market		*Reentry*		*Mix*	
	Mean	Std	Mean	Std	Mean	Std
θ_t	0.0330	0.0761	-0.0029	0.0947	0.0458	0.0841
r_t	0.0354	0.0025	0.0367	0.0031	0.0350	0.0028
μ_t^S	0.0365	0.0003	0.0366	0.0001	0.0366	0.0001
σ_t^S	0.0330	0.0013	0.0329	0.0005	0.0330	0.0007
Correlations						
Corr(dz_t^Y, θ_t)	-0.0812		-0.0744		-0.0793	
Corr(dz_t^Y, σ_t^S)	-0.0975		-0.0768		-0.1107	
Corr(dz_t^{SI}, θ_t)	-0.0975		-0.0768		-0.1107	

Panel B: Stock market participation rate					
	Reentry	*Mix*	SCF	Finland	Norway
P_t	0.5224	0.4673	0.4798	0.2412	0.2282
Entry rate	0.0637	0.0101		0.0198	0.0175
Exit rate	0.0451	0.0112		0.0141	0.0182
Corr$(\bar{\Phi}_t, P_t)$	0.2758	0.6532			
Corr$(\tilde{\Phi}_t, P_t)$	0.2573	0.6384			
Corr$(R_{t-2,t}, P_t)$	0.0698	0.1056			
Corr$(R_{t-2,t}^{high}, \text{Entry}_t)$	0.6395	0.5624			
Corr$(R_{t-2,t}^{low}, \text{Exit}_t)$	0.4454	0.4968			

Table 4: **Mean and standard deviation of the asset pricing quantities.** For Panel A, the top section of shows the mean and standard deviation of the market price of risk θ_t, the real short rate of interest r_t, as well as the drift μ_t^S and diffusion σ_t^S of the stock return, for the complete market benchmark, the *reentry* scenario, and the *mix* scenario. The 2^{nd} section shows the average covariance coefficients between shocks to the fundamental dz_t^Y and θ_t, dz_t^Y and σ_t^S, shocks to the signal dz_t^{SI} and θ_t, the consumption share of participants $\bar{\Phi}_t$ and the participation rate P_t, and the wealth share of participants $\tilde{\Phi}_t$ and P_t. Panel B shows the average participation rate, entry rate and exit rate for the complete market benchmark, the *reentry* scenario and the *mix* scenario, as well as the average correlation between participation rate in the *reentry* scenario and that in the *mix* scenario, given the same joint path of shocks. To compute the averages, we simulate the economies to generate data based on 10,000 simulations, each with 6000 periods or 500 years. For parameters see Subsection 3.1. For implied shocks see Subsection 3.3.

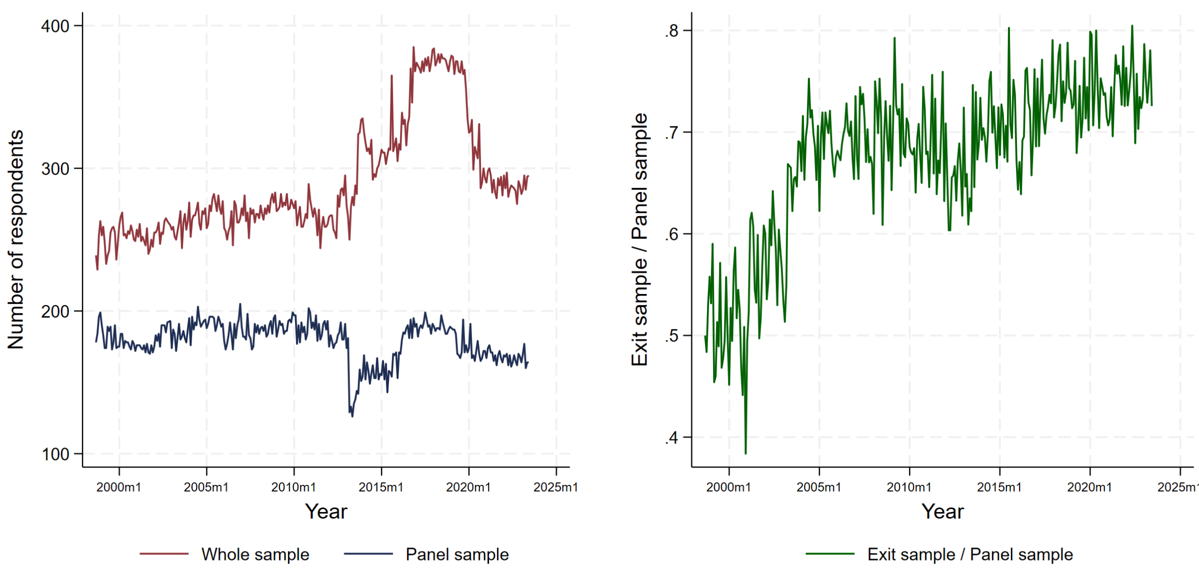
C.6 Michigan Data

The Michigan Surveys of Consumers select a sample of 400 respondents each month. From August 1998, the survey contains question on investment portfolio. A subsample of respondents are interviewed a second time after 6 months. We utilize this panel structure to also study entry and exit dynamics in the USA. Figure 12 shows the monthly time-series of the number of respondents who are selected for interview, as well as the number of respondents who are subsequently interviewed for a second time after 6 months.

Figure 12: **Time-series of sample size and sample composition.**

(a) Whole sample and panel sample

(b) Exit sample / panel sample



Panel A: Characteristics - Reported in first interview

Whole Sample	Count	Mean	Std	p25	p50	p75
Age	85864	50	17	37	50	63
Income	85864	84199	78909	35000	62000	105000
Total wealth	85864	341872	768933	0	125000	385000
Investment amount	45937	311672	772213	30000	100000	300000
Panel sample - Interviewed Twice						
Age	53002	52	17	39	53	65
Income	53002	84793	77914	35000	65000	105000
Total wealth	53002	368337	813122	1000	146000	400000
Investment amount	30395	330547	808473	30000	100000	300000
Entry Sample - Not Participating in Round 1						
Age	17212	52	18	37	53	67
Income	17212	44763	43885	19200	34000	56000
Total wealth	6524	199233	230942	80000	145000	250000
Exit Sample - Participating in Round 1						
Age	35790	52	15	40	52	64
Income	35790	104044	83202	50000	80000	125000
Total wealth	33270	547725	975296	100000	290000	600000
Investment amount	30395	330547	808473	30000	100000	300000

Panel B: Control variables

		Whole Sample	Sub Sample	Entry Sample	Exit Sample
Gender	Female	47.83	47.60	53.70	44.67
	Male	52.17	52.40	46.30	55.33
Region	North Central	25.87	26.62	25.41	27.21
	Northeast	17.11	17.16	15.66	17.88
	South	35.32	34.33	37.15	32.97
	West	21.71	21.89	21.78	21.94
Political affiliation	Democrat	12.67	12.19	10.37	13.07
	Indep / Not answered	76.18	77.34	82.04	75.09
	Republican	11.16	10.46	7.59	11.84
Education	Grade 8	1.91	1.54	4.19	0.27
	Grade 12 - no diploma	3.63	2.99	7.07	1.03
	Grade 12 - diploma	22.70	21.26	33.35	15.44
	Grade 16 - college diploma	25.55	26.65	16.61	31.48
	Grade 17 - no college diploma	18.18	20.48	8.76	26.12
	Grade 17 - no diploma	28.04	27.07	30.02	25.66
Home ownership	Own	74.00	78.18	62.28	85.83
	Rent	26.00	21.82	37.72	14.17

Panel C: Entry and exit frequencies

Entry sample	Observations	Exit sample	Observations
Count of entries	3196	Count of exits	3040
Entry rate	18.57%	Exit rate	8.49%

Table 5: **Summary Statistics, Whole Sample vs. Subsample.**

Entry among non-participants	(1)	(2)	(3)
6m High Return	0.055*	0.082**	0.069**
	(0.032)	(0.034)	(0.033)
High Income	0.460***	0.509***	0.460***
	(0.057)	(0.060)	(0.057)
High Wealth	1.305***	1.306***	1.357***
	(0.076)	(0.076)	(0.080)
High Income × 6m High Return		-0.245**	
		(0.104)	
High Wealth × 6m High Return			-0.258*
			(0.141)
Controls	Y	Y	Y
N	40442	40442	40442

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Exit among participants	(4)	(5)	(6)
6m Low Return	0.108***	0.104***	0.099***
	(0.025)	(0.028)	(0.026)
High Income	-0.078***	-0.083**	-0.078***
	(0.030)	(0.034)	(0.030)
High Wealth	-0.839***	-0.839***	-0.871***
	(0.042)	(0.042)	(0.051)
High Income × 6m Low Return		0.017	
		(0.057)	
High Wealth × 6m Low Return			0.099
			(0.082)
Controls	Y	Y	Y
N	40442	40442	40442

Table 6: **Michigan data: excluding survey respondents who report change in income quartile.** We use a probit model with sample selection (Heckman two-step estimator) to regress the probability of a household entering (exiting) the stock market on high (low) 6-month stock return, where we use the panel structure of the Michigan survey to define stock market entry (exit). We report second-step coe□cients and standard errors. High (low) 6-month return is a dummy variable that equals to 1 if the return is within the top (bottom) 25% between 1998 and 2023. High income and high wealth are dummy variables, which equal to 1 if a household has income or wealth above the 75^{th} percentile in an interview wave. We use the following control variables in both first and second steps: high income, high wealth, age group which equal to 1 if an individual has age above 65, level of education (from lowest to highest grade completed), the housing situation of the household (own or rent), the region of residence of the household (North, South, Northeast, Central North), gender of respondent, and political a□liation (democrat, republican, non a□liated). We include a linear time trend in the first-step. Returns on the stock market are from Amit Goyal's website.

C.7 Alternative Measures of Participation Rate

Figure 13: **Annual average participation rates from simulation.**

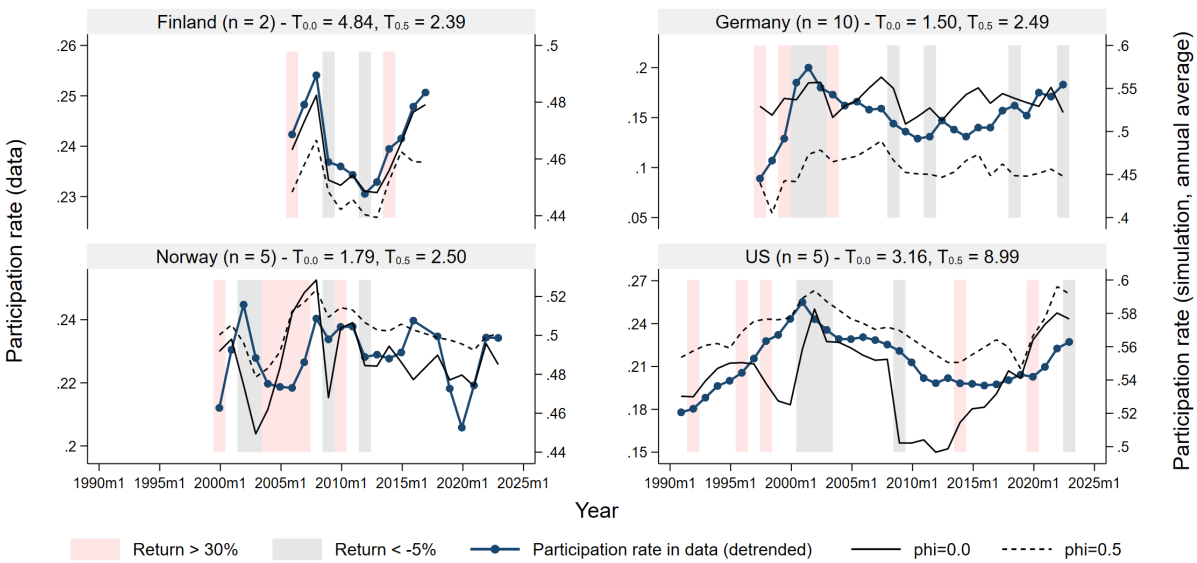
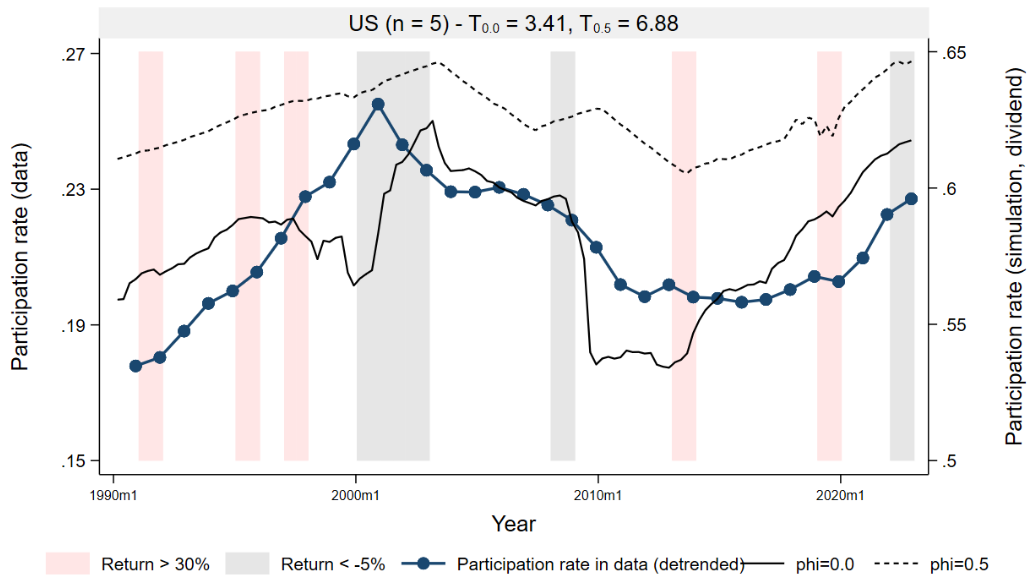


Figure 14: **Alternative measure of participation rate from simulation.** In simulation, we define participants as agents who have received dividend or who have been in the stock market at any point within the past 12 months. This definition is analogous to the tax measure in the USA, which is based on dividend income.



D Robustness Tests

Table 7 shows predictive regressions that include the price-dividend-ratio on top of the participation rate as explanatory variables.

	Total returns			Excess returns		
	(1)	(2)	(3)	(4)	(5)	(6)
	12m	24m	36m	12m	24m	36m
Participation rate	-0.340***	-0.330**	-0.268*	-0.362**	-0.351**	-0.290*
	(0.055)	(0.085)	(0.094)	(0.070)	(0.097)	(0.101)
Price-dividend ratio	-0.056	-0.209	-0.273	-0.086	-0.262	-0.347*
	(0.088)	(0.129)	(0.117)	(0.092)	(0.156)	(0.137)
R_squared	.1443	.2108	.1947	.1478	.219	.2185
N	94	91	88	94	91	88

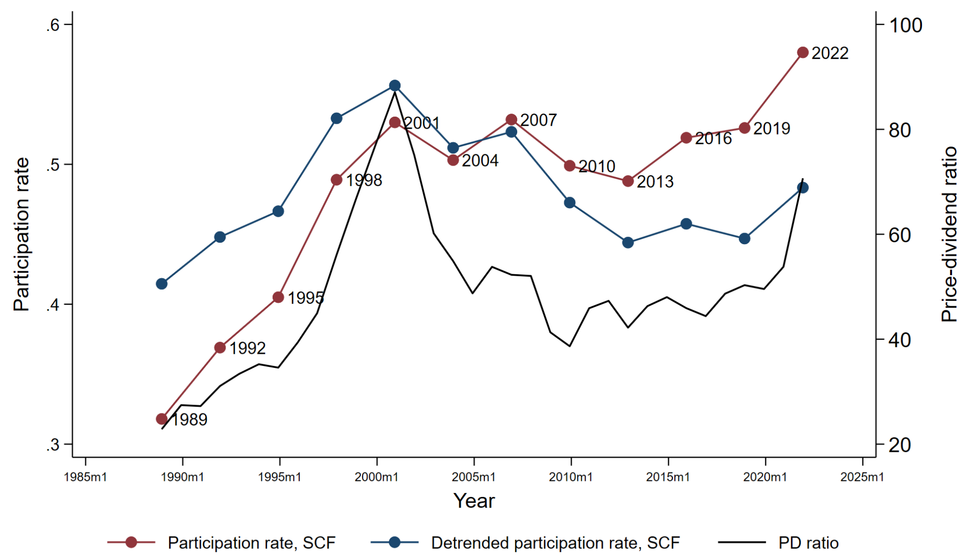
Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 7: **Participation Rate and Future Returns - Data versus Model.** The table shows predictive regressions with 12, 24 and 36 month horizons, with detrended participation rate and the price-dividend ratio as explanatory variables for Finland, Germany, Norway and the USA, where columns (1) - (3) show regressions with total returns and columns (4) - (6) use excess returns. Interest rates are 12-month T-bill rates from Bloomberg for Finland, Bundesbank for Germany, Statistics Norway for Norway, and Amit Goyal's website for the USA. Price-dividend ratio are calculated using rolling 12-month returns from Ken French's website. All the variables are standardized. Price-dividend ratio is used at time t. Including country fixed e�ects, standard error clustered on country.

Figure 15: **Alternative measure of participation rate in the USA - Survey of Consumer Finances.** We use participation rate from Survey of Consumer Finances as a measure for the USA.

(a) **Raw and detrended participation rate time-series**



(b) **Participation rate, Data versus Model.**