An Experiment for Dilation Property in Ambiguity*

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Abstract

In standard models of ambiguity, receiving information is theoretically predicted to dilate ambiguity. This study experimentally investigates whether this dilation property can be observed. Subjects placed bets on colors in ambiguous urns before and after observing a draw, revealing their certainty equivalents. Our observations contradict the theoretical prediction, which suggests decreasing values after information for ambiguity-averse subjects and increasing values for ambiguity-seeking subjects. We examined the possibility that participants perceived a correlation between pre- and post-observation draws, finding that in a two-stage updating experiment, more participants reported feeling confused after the second update compared to a single-stage update. While direct evidence of the dilation property was not observed, our findings suggest that individuals in ambiguous settings may, when given information, integrate decisions and consider joint probabilities, possibly serving as an indirect indication of the dilation property.

JEL codes: C91; D81; D90

Keywords: Updating; Dilation; Joint ambiguity; Independent ambiguity

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1 Introduction

When a decision maker faces an ambiguous situation in multiple periods, the range of conditional probabilities becomes wider compared to those from previous periods (Nishimura and Ozaki 2017; Kato, Nishimura, and Ozaki 2019; Shishkin and Ortoleva 2023). This phenomenon is called the 'dilation property.'The concept of dilation was first introduced by Seidenfeld and Wasserman (1993), who explored how uncertainty in probability assessments can expand with the acquisition of new information.

Although several applications appear (Bose and Renou 2014, Beasuchene, Li and Li 2019), whether this property is empirically true received little attention. One exception is the experiment in Shishkin and Ortoleva (2023), which tests dilation property by asking subjects certainty equivalents of bets on colors of picked balls from urns before and after ambiguous information is shown and investigating differences of them called information premium. They discover that for ambiguity averse and neutral subjects there is no robust relation between ambiguity attitude and information premium, namely, dilation property is not observed though ambiguity seeking ones exhibit positive information premium, which accords with dilation property.

Our paper aims to investigate, following the foundational motivation of Shishkin and Ortoleva (2023), whether the dilation property exists across a wide range of ambiguity attitudes. To achieve this, our experiment includes urns R,A,A_b, and A_w, which we will explain in detail later.

Urn R is a risk urn that contains two colors, red and yellow, in equal quantities. An ambiguous urn contains up to two colors of balls, but the proportions of these colors are unknown. To help subjects distinguish the urns, the ambiguous urn contains blue and/or white balls.

From Urn A, a ball is drawn only once, and the subject receives a reward only if the color of the ball matches the one specified by the experimenter.

To observe dilation under ambiguity, we elicit differences in subjects' certainty equivalents for the ambiguous urn, both without and with receiving information from drawing one ball. The ambiguous urn without information is referred to as Urn A, which is equivalent to a single draw without any updates or prior knowledge.

The dilation property involves observing how the prior belief about the composition of colors in an ambiguous urn changes with updates. To test this, we include an operation where a ball is drawn from an urn containing blue and white balls and then returned. Subjects must form priors corresponding to each state in the set $\Omega = \{BB, BW, WB, WW\}$.

We compare a case where the subject does not receive information about the color of the first ball drawn to one where they do. In the latter case, the prior belief is updated conditionally based on the information received, while in the former, it remains unchanged.

To evaluate how ambiguity preferences influence updating based on information, by observing the difference between certainty equivalent of lotteries tied for the draw of Urn A with and without information. Theoretical predictions suggest that the certainty equivalent for ambiguity-averse individuals should be lower after receiving information, a proposition we test in our experiment.

Following Shishkin and Ortoleva (2023), we define an individual as ambiguity averse if their certainty equivalent of a risky urn, R, is larger than that of an ambiguous urn, A; as ambiguity seeking if the difference is positive; and as ambiguity neutral if the certainty equivalent of urn R is equal to that of urn A.

Subjects are asked for their certainty equivalent of ambiguous urns in the different way in the first and second experiment.

In the first experiment, all ambiguous urns are created transparently after subjects make their decisions. This is an approach that avoids any perception of manipulation by experimenter in the composition of the urns, which could influence subjects' rewards. The compositions of the ambiguous urns are determined using the method of Hayashi and Wada (2010), which employs multiple dice and complex rules, making it practically impossible for subjects to calculate the probabilities.

Since the composition of ambiguous urn A is determined and physically created only after the experiment, we ask for the certainty equivalent of the ambiguous urn conditionally: subjects are asked to imagine that a ball (either blue or white) is drawn and then returned to the original ambiguous urn A, preserving the color composition. We denote this urn as A_b (if a blue ball is imagined) or A_w (if a white ball is imagined).

The dilation property is identified by the sign of the difference between the certainty equivalent of a lottery tied to a single draw from the ambiguous urn A and that tied to the second draw from the ambiguous urn. A negative difference implies that subjects dilate their range of priors.

In the first experiment, subjects express the certainty equivalent of lotteries tied to the risky urn, R, and three ambiguous urns: A, A_b, A_w. For A_b and A_w, subjects are payed 1500 yen only when the second draw results in blue. In the first experiment, these ambiguous urn's compositions are independently made by web-dice after the decision making, partly because we hypothesize that individual's ambiguity preferences is constant for any ambiguous urn and partly because subjects expressed their certainty equivalence only conditionally and it is impossible to see the dilation directly. Two lotteries are selected to be rewarded. Certainty equivalents of all urns are measured using the methodology developed by Becker, DeGroot, and Marschak (1964) (hereafter BDM mechanism), which is a well-known truth-telling mechanism.

Despite testing the theory that dilation should be observed among ambiguity-averse individuals, our results, obtained in a different experimental setting, align with Shishkin and Ortoleva (2023), showing that dilation occurs only among ambiguity seekers.

The possible interpretation is that some subjects integrate each bet and expect that there is a correlation between probabilities of drawn colors before and after observation in which the same colors are likely or unlikely to be drawn.

Let the blue color be the winning bet. When a subject considers that the blue color is likely to be drawn after the blue is observed, they tend to reveal high certainty equivalent of A_b, and when blue is expected to be drawn after the white is drawn, the high A_w is observed. That feature makes the results liable to the opposite ones. On the other hand, the subjects who consider no correlation are unlikely to exhibit the opposite result compared to those who expect such correlation. These hypothesis are tested statistically.

Based on the aforementioned reasons, we conducted a second experiments in which an ambiguous urn was actually created in front of the subjects before their eliciting the certainty equivalents of all urns. Additionally, we drew a ball twice from the ambiguous urn, asking for the certainty equivalents of each urn after observing the color at each draw. Furthermore, we also posed qualitative questions to the subjects, such as how they felt about the bets after each observation. The quantitative results were largely consistent with the previous experiments: the certainty equivalents for ambiguity-averse subjects tended to increase, with similar patterns observed for ambiguity-seeking and ambiguity-neutral subjects.

Although theoretically dilation increases as the drawing accumulates, expansion of ambiguity is not observed statistically. On the other hand, the answers to qualitative questions exhibit 'more confused' on the bets after second observation than first observation, which indirectly suggests expansion of ambiguity.

Mathematically, dilation property arises if a decision maker is unsure about probability defined on the product of each marginal state space. We call this usual kind of ambiguity 'joint ambiguity'. On the other hand, if each marginal state space is treated separately and the marginal probability on each state is ambiguous, dilation property does not occur when the degree of ambiguity in each marginal probability is the same (e.g. the number of balls whose color is unknown is the same in each urn). We name this sort of ambiguity 'independent ambiguity'. To expect a correlation of probability on each state implies that the subjects consider a situation as joint ambiguity. Although dilation property is not directly observed, agents tend to integrate each decision and envisage a set of joint probabilities, which is indirect manifestation of dilation property.

In summary, while the dilation property was not directly observable, our findings suggest an implicit presence of dilation through decision patterns reflecting joint ambiguity and participants' increasing confusion with subsequent draws. Therefore, it is possible that the dilation property is not pronounced enough to be directly observed, but it might still exist implicitly.

This paper is organized as follows. The next section explains the content of experiment providing the theory based on ε-contamination and its prediction on the result of our experiment. In the third section, the procedure and results of experiment are presented. The fourth section discusses additional experiments. The fifth concludes the paper.

2 Theory

In this section, we explain the content of experiment and provide mathematical background of it. Let the prize be set as 1500 yen, Japanese monetary unit, if the bet on the color of a ball drawn from a urn is correct and let the prize be zero otherwise. Consider the urn containing balls that could be either red or yellow. Formally, envition drawing a ball consecutively twice from the urn in which the first ball is returned before the second draw. Then the state space can be described as $\Omega = \{RR, RY, YR, YY\}$ and its partition is denoted by $\{E_1, E_2\}$ for $E_1 := \{RR, RY\}$ (a red ball is drawn first), $E_2 := \{YR, YY\}$ (a yellow ball is drawn first). We describe the choices of a decision maker by employing the idea of ε-contamination (Nishimura and Ozaki 2017, Kato, Nishimura and Ozaki 2019). Let p^0 denote a probability on Ω that decision makers specify with $(1-\varepsilon) \times 100\%$ confidence, which we call principal probability. Define ε-contamination of p^0 by

$$\begin{aligned}&\{p^0\}^\varepsilon \\ := &\left\{ \, (1-\varepsilon)(p_1^0, p_2^0, p_3^0, p_4^0) + \varepsilon(q_1, q_2, q_3, q_4) \,\middle|\, (q_1, q_2, q_3, q_4) \text{ is any probability on } \Omega \, \right\} .\end{aligned}$$

The set of first marginal probabilities is written by

$$\mathscr{P}_1 := \{ \ (p_1 + p_2, \ p_3 + p_4) \mid (p_1, p_2, p_3, p_4) \in \{p^0\}^\varepsilon \ \}$$

and the set of conditional probabilities is denoted by

$$\mathscr{P}\big|_{E_1} := \left\{ \ \left(\frac{p_1}{p_1 + p_2}, \frac{p_2}{p_1 + p_2} \right) \ \middle| \ (p_1, p_2, p_3, p_4) \in \{p^0\}^\varepsilon \ \right\}$$

and

$$\mathscr{P}\big|_{E_2} := \left\{ \left(\frac{p_3}{p_3+p_4}, \frac{p_4}{p_3+p_4}\right) \;\middle|\; (p_1,p_2,p_3,p_4) \in \{p^0\}^{\varepsilon} \right\}.$$

Let $u : \mathbb{R} \to \mathbb{R}$ be a utility function. So $u(1500)$ represents the utility gotten from winning the bet and $u(0)$ is obtained when losing the bet. In the following subsections, we explain each part of experiment as in Introduction.

2.1 The urn R (single risk)

Consider the urn consisting of balls that could be either red or yellow but the composition of color is known (set up as twenty red balls and twenty yellow ones). When the winning bet is the case that a red ball is drawn. Its value is expressed by

$$(p_1^0 + p_2^0)u(1500) + (p_3^0 + p_4^0)u(0). \tag{A}$$

The certainty equivalent of the above is written (for abusing a little bit of notation) by A, namely $u(A) = (p_1^0 + p_2^0)u(1500) + (p_3^0 + p_4^0)u(0)$.

2.2 The urn A (single ambiguity)

Set up the urn consisting of red or yellow balls totaling forty balls where the composition of color is unknown. Suppose that red is the winning color. For ambiguity averse subject we have

$$\begin{aligned}&\min_{(p_1+p_2,\, p_3+p_4)\in\mathscr{P}_1} \Big[\ (p_1+p_2)u(1500) + (p_3+p_4)u(0)\ \Big] \\ &= (1-\varepsilon)(p_1^0+p_2^0)u(1500) + \{(1-\varepsilon)(p_3^0+p_4^0)+\varepsilon\}u(0) =: u(B).\end{aligned} \tag{B-a}$$

because it holds that

$$(1-\varepsilon)(p_1^0+p_2^0) = \min_{(q_1,q_2,q_3,q_4)} (1-\varepsilon)(p_1^0+p_2^0) + \varepsilon(q_1+q_2)$$

which is attained by $(q_1, q_2) = (0, 0)$ and $q_3 + q_4 = 1$,

$$(1-\varepsilon)(p_3^0+p_4^0) + \varepsilon = \max_{(q_1,q_2,q_3,q_4)} (1-\varepsilon)(p_3^0+p_4^0) + \varepsilon(q_3+q_4)$$

with $(q_3, q_4) = (1, 0)$ or $(0, 1)$ and $q_1 + q_2 = 0$. Note that B means (again abusing the notation) the certainty equivalent of the value of the urn B. For ambiguity seeking subjects, min is replaced with max and the similar arguments follow. For ambiguity seeking subject we have

$$\begin{aligned}&\max_{(p_1+p_2,\, p_3+p_4)\in\mathscr{P}_1} \Big[\ (p_1+p_2)u(1500) + (p_3+p_4)u(0)\ \Big] \\ &= \{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon\}u(1500) + (1-\varepsilon)(p_3^0+p_4^0)u(0) =: u(B).\end{aligned} \tag{B-s}$$

2.3 The urn A_b and A_w (the two way representation of the second ambiguity urn)

The colors in our experiment are randomized to prevent subjects from developing a preference for a specific color. In this step, we change the colors from the red and yellow to the blue and white (so the state space becomes $\Omega = \{BB, BW, WB, WW\}$ and its partition is represented by $E_1 = \{BB, BW\}$ and $E_2 = \{WB, WW\}$).

In the first experiment, subjects make their decisions before the urns are constructed. The rules for constructing the ambiguous urn are clearly explained in advance, ensuring there is no room for manipulation or deception. However, it is not possible to ask questions after observing the color of the ball drawn from the urn, so information updates cannot be realistically incorporated. To address this limitation, we define two conditional urns (cond.-urn): A_b represents the scenario where a blue ball is drawn and returned to the urn, and A_w represents the scenario where a white ball is drawn and returned. Once the ambiguous urn's composition of blue and/or white balls is determined, the experimenter draws a ball and shows its color to the subjects. This color determines which of the two conditional urns is relevant for the bet. Afterward, subjects receive payment based solely on the color of the second draw, and they take a comprehension test before the experiment begins to ensure they understand the setup.

For ambiguity averse subjects, the value of the urn after blue is drawn is described as

$$\min_{\left(\frac{p_1}{p_1+p_2}, \frac{p_2}{p_1+p_2}\right) \in \mathscr{P}|_{E_1}} \left(\frac{p_1}{p_1+p_2} u(1500) + \frac{p_2}{p_1+p_2} u(0) \right)$$

and the value after white is represented by

$$\min_{\left(\frac{p_3}{p_3+p_4}, \frac{p_4}{p_3+p_4}\right) \in \mathscr{P}|_{E_2}} \left(\frac{p_3}{p_3+p_4} u(1500) + \frac{p_4}{p_3+p_4} u(0) \right)$$

where $E_1 = \{BB, BW\}$ (The blue is firstly drawn) and $E_2 = \{WB, WW\}$ (The white is firstly drawn). Concerning the urn after blue, we can see that

$$\begin{aligned} &\min_{\left(\frac{p_1}{p_1+p_2}, \frac{p_2}{p_1+p_2}\right) \in \mathscr{P}\big|_{E_1}} \left(\frac{p_1}{p_1+p_2} u(1500) + \frac{p_2}{p_1+p_2} u(0) \right) \\ &= \frac{(1-\varepsilon)p_1^0}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon} u(1500) + \frac{(1-\varepsilon)p_2^0+\varepsilon}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon} u(0) = u(A_b) \end{aligned} \tag{A\_b-a}$$

because it holds that

$$\frac{(1-\varepsilon)p_1^0}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon} = \min_{(q_1,q_2)} \frac{(1-\varepsilon)p_1^0+\varepsilon q_1}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon(q_1+q_2)}$$

with $(q_1, q_2) = (0, 1)$, and

$$\frac{(1-\varepsilon)p_2^0+\varepsilon}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon} = \max_{(q_1,q_2)} \frac{(1-\varepsilon)p_2^0+\varepsilon q_2}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon(q_1+q_2)}$$

with the same $(q_1, q_2) = (0, 1)$ noticing that

$$\frac{(1-\varepsilon)p_2^0+\varepsilon q_2}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon(q_1+q_2)} \text{ is increasing with respect to } q_2.$$

Note A_b represents, again abusing the notation, the value (certainty equivalent) of the urn A_b. Similarly let A_w denote the value of the urn A_w. We can also calculate the value after white is drawn by

$$\begin{aligned}&\min_{\left(\frac{p_3}{p_3+p_4},\frac{p_4}{p_3+p_4}\right)\in\mathscr{P}|_{E_2}} \left(\frac{p_3}{p_3+p_4}u(1500)+\frac{p_4}{p_3+p_4}u(0)\right)\\ &= \frac{(1-\varepsilon)p_3^0}{(1-\varepsilon)(p_3^0+p_4^0)+\varepsilon}u(1500)+\frac{(1-\varepsilon)p_4^0+\varepsilon}{(1-\varepsilon)(p_3^0+p_4^0)+\varepsilon}u(0)=u(A\_w).\end{aligned} \tag{A\_w - a}$$

The objective function of ambiguity 'seeking' one is defined by replacing 'min' operator with 'max'. Thus for ambiguity seeking subjects the value of A_b is written as

$$\begin{aligned}&\max_{\left(\frac{p_1}{p_1+p_2},\frac{p_2}{p_1+p_2}\right)\in\mathscr{P}|_{E_1}} \left(\frac{p_1}{p_1+p_2}u(1500)+\frac{p_2}{p_1+p_2}u(0)\right)\\ &= \frac{(1-\varepsilon)p_1^0+\varepsilon}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon}u(1500)+\frac{(1-\varepsilon)p_2^0}{(1-\varepsilon)(p_1^0+p_2^0)+\varepsilon}u(0)=u(A\_b)\end{aligned} \tag{A\_b - s}$$

and the value of A_w is

$$\begin{aligned}&\max_{\left(\frac{p_3}{p_3+p_4},\frac{p_4}{p_3+p_4}\right)\in\mathscr{P}|_{E_2}} \left(\frac{p_3}{p_3+p_4}u(1500)+\frac{p_4}{p_3+p_4}u(0)\right)\\ &= \frac{(1-\varepsilon)p_3^0+\varepsilon}{(1-\varepsilon)(p_3^0+p_4^0)+\varepsilon}u(1500)+\frac{(1-\varepsilon)p_4^0}{(1-\varepsilon)(p_3^0+p_4^0)+\varepsilon}u(0)=u(A\_w).\end{aligned} \tag{A\_w - s}$$

The following theorem, which states the prediction of the experiment, can be seen. The proof of which is delegated to Appendix.

Theorem 1. Assume that the principal probability is symmetry: p_i^0, $i = 1, 2, 3, 4$ are the same (so $A\_b = A\_w =: A$). If a subject is ambiguity averse, we should observe

$$R > A > A\_b = A\_w$$

if ambiguity seeking it follows that

$$R < A < A\_b = A\_w$$

and if neutral one has $R = A = A\_b = A\_w$.

2.4 The procedure and results of the experiment

In order to elicit the certainty equivalents of bets on all urns, we applied BDM mechanism as follows. Each subject answers their values x such that they get y yen if $y \geq x$ for uniformly randomly determined value $y \in [0, 1000]$ and faces the bet on specific color drawing from the urn R, A, A_b, A_w if $y < x$. Subjects are explained the x is selling price of lotteries tied for the risky and ambiguous urns for the experimenter and y shows the experimenter's buying prices of these lotteries. For each subject, their own buying prices are given.

Subjects are tested to determine whether they understand how to express their selling price for the lotteries, and the experiment begins only when all subjects pass the test. Each procedure is clearly explained to all subjects.

The concrete instruction is as follows.

1. `You are asked to provide the selling prices for lotteries that will be assigned to you. Although you need to set selling prices for four lotteries, you will acquire only three of them. Among these three lotteries, two will be paid if your selling prices are equal to or lower than the buying prices offered by the experimenter. The buying prices are randomly determined by the computer.`
2. `If your lottery is bought by the experimenter, you will receive the experimenter's buying price and will not draw the lottery. The color of the ball drawn from the urn`

will not affect your reward in this case. However, if the lottery is not bought by the experimenter and remains with you, you must draw the lottery. If the color drawn from the urn matches the winning color, you will receive 1,500 yen. If it does not, you will receive nothing other than the 500 yen participation fee.

3. Now I, the experimenter, am going to fill the urn with 20 red balls and yellow balls in front of you. Let the winning bet be on red balls. After all of you decide on the selling price of the lottery tied to the draw from urn R, the assistant of the experimenter will draw a ball without looking inside.
4. The composition of urn A will be determined in the following way, after you decide on the selling price of the lottery associated with this urn. You can simulate this process on the website by yourself from now on, and try it as many times as you like.

   When you click the button, the computer program will randomly select a number $\alpha \in \{1,2,3,4,5,6\}$. If α is odd, it will randomly select a number $a \in \{1,2,\cdots,12\}$, if α is even, it will randomly select a number $a \in \{1,2,\cdots,8\}$. Next, it will randomly pick a number $b \in \{1,2,\cdots,10\}$.

   If $a \times b > 40$ define $d := a \times b - 40n^*$ where $n^* := \max\{\, n \in \mathbb{N} \mid (a \times b - 40n) \leq 40 \,\}$, if $a \times b \leq 40$ let $d := a \times b$.

   If d is odd, urn A will contain d red balls and $40-d$ yellow balls. If d is even, it will contain $40-d$ red balls and d yellow balls. **The winning bet will be on the red balls.**
5. Please indicate the minimum price at which you are willing to sell these lotteries to the experimenter. At the end of the experiment, the final composition of the urn A will be determined for all of you in this laboratory by the experimenter's assistant, who will click the button in front of everyone to set it. The actual urn A will be physically constructed in front of all of you, and the assistant will them draw a ball from it while all of you watch.

2.5 Results and interpretation of the first experiment

In January 2024, a total of 66 undergraduate students participated in the study at Keiai University.

Results for $A - R$, the difference between the certainty equivalent of the ambiguous urn A and the risky urn R, showed that 17 subjects (28%) reported a negative value (ambiguity averse), 35 (53%) a positive value (ambiguity seeking), and 14 (21%) a zero value (ambiguity neutral).

Theory predicts a positive relationship between $A-R$ and both A_b-A and $A_w - A$. However, the observed relationships are opposite. In the following sections, figures and tests are conducted using Python.

An OLS regression finds a negative relationship between $A-R$ and A_b-A $(t = -4.919, p = 0.000)$ and between $A - R$ and $A_w - A$ $(t = -4.016, p = 0.000)$.

In Figure 1, the horizontal axis (ambiguity1) represents $A - R$, and the vertical axis (ambiguity2) represents $A_b - A$. Figure 2 similarly shows $A - R$ on the horizontal axis and $A_w - A$ on the vertical axis.

Interpretation: Prediction crucially depends on the symmetry of the principal probability, especially

$$(p_1^0 + p_2^0, p_3^0 + p_4^0) = (p_1^0/(p_1^0 + p_2^0), p_2^0/(p_1^0 + p_2^0)) = (p_3^0/(p_3^0 + p_4^0), p_4^0/(p_3^0 + p_4^0))$$

Some subjects may integrate each bet and assume a correlation between the probabilities of consecutively drawn colors, expecting that the same color is either more or less likely to be drawn. If a subject believes that blue is likely to be drawn after observing blue, then $p_1^0 > p_2^0$, leading to a high certainty equivalent for A_b. Alternatively, if blue is expected after white, a high A_w is observed, with $p_3^0 > p_4^0$. This feature can lead to outcomes opposite to theoretical predictions.

Among the subjects, 30 (about 45%) responded with $A_b = A_w =: A$ (no correlation), 14 (about 21%) with $A_b < A_w$, and 22 (about 33%) with $A_b > A_w$.

Within the group of subjects who answered $A_b = A_w =: A$, 18 out of 30 (60%) answered either $A - R \geq 0$ and $A\_b - A \geq 0$' or $A - R \leq 0$ and $A_s - A \leq 0$'. Conversely, among those who answered $A_b \neq A_w$, 9 out of 36 (25%) provided responses that satisfy either $A - R \geq 0$ and $\min\{A\_b - A, A\_w - A\} \geq 0$' or $A - R \leq 0$ and $\max A_b - A, A_w - A \leq 0$'.

We tested whether the two proportions differ statistically using a χ-square test, which yielded a domain value of 8.292 and a p-value of 0.003, indicating a statistically significant difference between the proportions.

Subjects who consider no correlation are unlikely to produce results opposite to theory, compared to those who assume correlated principal probabilities. Thus, outcomes contrary to theoretical predictions tend to arise from subjects expecting correlated principal probabilities.

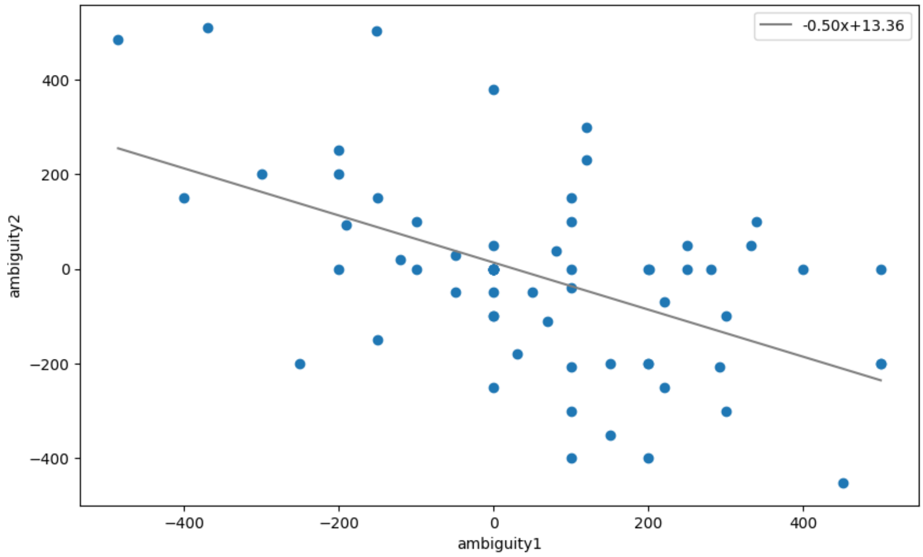

Figure 1: The relation between $A{-}R$ (ambiguity1) and $A\_b{-}A$ (ambiguity2)

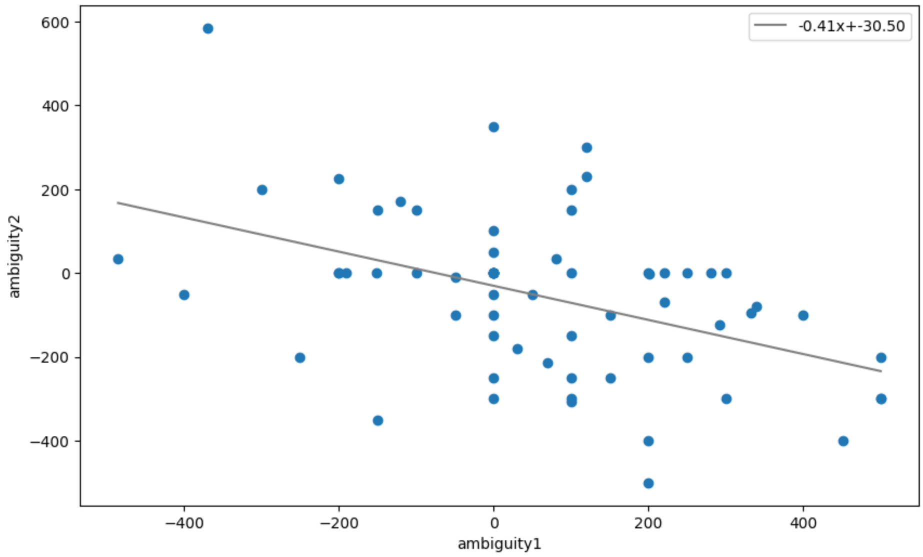

Figure 2: The relation between $A - R$ (ambiguity1) and $A\_w - A$ (ambiguity2)

Mathematically, the dilation property arises if a decision maker is uncertain about probabilities defined on the product of each marginal state space. We call this type of ambiguity 'joint ambiguity'. Conversely, if each marginal state space is treated *separately* and the marginal probability on each state is ambiguous, the dilation property does not occur when the degree of ambiguity in each marginal probability is the same (e.g., when the number of unknown-colored balls is the same in each urn). We refer to this kind of ambiguity as 'independent ambiguity'.

To illustrate this, let p^0 be the product of identical marginal probabilities defined by p^{00} over B, W, namely,

$$\begin{aligned}&(p_1^0, p_2^0, p_3^0, p_4^0)\\&= (p^{00}(B)p^{00}(B), p^{00}(B)p^{00}(W), p^{00}(W)p^{00}(B), p^{00}(W)p^{00}(W))\end{aligned}$$

and define

$$\begin{aligned}&\{p^0\}^\varepsilon\\&:= \Bigg\{ \Big(((1-\varepsilon)p^{00}(\omega_1) + \varepsilon q_{\omega_1})((1-\varepsilon)p^{00}(\omega_2) + \varepsilon q_{\omega_2}) \Big)_{\omega_1, \omega_2 \in \{B,W\}}\\&\qquad\qquad \Bigg| \sum_{\omega_1 \in \{B,W\}} q_{\omega_1} = \sum_{\omega_2 \in \{B,W\}} q_{\omega_2} = 1 \Bigg\}.\end{aligned}$$

We easily see that

$$\mathscr{P}_1 = \mathscr{P}\big|_{E_1} = \mathscr{P}\big|_{E_2}.$$

Hence it follows that

$$A = A\_b = A\_w,$$

which does not exhibit the dilation property. Therefore, expecting a correlation of probabilities across states implies that subjects perceive the situation as joint ambiguity. Although the dilation property is not directly observed, agents tend to integrate each decision and envision a set of joint probabilities, which indirectly manifests the dilation property.

3 The second experiment: Twice withdrawing and qualitative questions

We conduct an additional experiment in which an ambiguous urn is constructed in front of participants before asking questions. The urn is created by a randomly selected participant in a specific way, completely eliminating any potential manipulation by the experimenter. Details on how concrete ambiguity is maintained will be explained later. After withdrawing a ball twice from the ambiguous urn, we ask subjects for their certainty equivalents of lotteries tied to each urn, depending on the color of the drawn balls after each observation. In this experiment, subjects elicit their certainty equivalents of lotteries using a multiple price list. Subjects are asked in a binary manner whether they would prefer to receive a specified amount of cash or each lottery.

3.1 Theory

Theoretically, dilation becomes larger as observations accumulate (Kato et al. 2019). We demonstrate this as follows. Under the state space $\Omega := \{B, W\}^3$ and the principal probability p^0 defined on it, we define the ε-contamination of p^0 as:

$$\left\{p^0\right\}^\varepsilon := \Big\{(1-\varepsilon)(p^0_{a_1a_2a_3})_{(a_1,a_2,a_3)\in\Omega} + \varepsilon(q^0_{a_1a_2a_3})_{(a_1,a_2,a_3)\in\Omega} \;\Big|\; \text{where } (q^0_{a_1a_2a_3})_{(a_1,a_2,a_3)\in\Omega} \text{ is any probability on } \Omega\Big\}.$$

The set of first marginal probabilities (blue or white probabilities at the first draw) is written by

$$\mathscr{P}_1 := \left\{ \Big(\sum_{(a_2,a_3)\in\{B,W\}^2} p_{Ba_2a_3}, \sum_{(c_2,c_3)\in\{B,W\}^2} p_{Wa_2a_3}\Big) \;\Big|\; (p_{a_1a_2a_3})_{(a_1,a_2,a_3)\in\Omega} \in \left\{p^0\right\}^\varepsilon \right\}$$

and the set of conditional probabilities after the first observation is blue is denoted by

$$\mathscr{P}\big|_{E_B} := \left\{ \Big(\frac{\sum_{a_3\in\{B,W\}} p_{BBa_3}}{\sum_{(a_2,a_3)\in\{B,W\}^2} p_{Bc_2c_3}}, \frac{\sum_{c_a\in\{B,W\}} p_{BWa_3}}{\sum_{(a_2,a_3)\in\{B,W\}^2} p_{Ba_2a_3}}\Big) \;\Big|\; (p_{a_1a_2a_3})_{(a_1,a_2,a_3)\in\Omega} \in \left\{p^0\right\}^\varepsilon \right\}$$

where $E_B := \{B\} \times \{B, W\}^2$. The set of conditional probabilities after the first observation is white, $\mathscr{P}\big|_{E_W}$, is defined similarly. The set of conditional probabilities after twice observation, say blue and blue, is defined by

$$\mathscr{P}\big|_{E_{BB}} := \left\{ \Big(\frac{p_{BBB}}{\sum_{a_3\in\{B,W\}} p_{BBa_3}}, \frac{p_{BBW}}{\sum_{a_3\in\{B,W\}} p_{BBa_3}}\Big) \;\Big|\; (p_{a_1a_2a_3})_{(a_1,a_2,a_3)\in\Omega} \in \left\{p^0\right\}^\varepsilon \right\}$$

where $E_{BB} := \{B\} \times \{B\} \times \{B, W\}$. The other cases $\mathscr{P}\big|_{E_{BW}}$, $\mathscr{P}\big|_{E_{WB}}$ and $\mathscr{P}\big|_{E_{WW}}$ are defined similarly.

Let the winning prize be 2000 yen. For ambiguity averse subjects, the value of the urn after blue A_b is drawn is described as

$$\min_{(x,\,1-x)\in\mathscr{P}_1}\Big[\; xu(2000)+(1-x)u(A)\;\Big]$$
$$=(1-\varepsilon)\sum_{(a_2,a_3)\in\{B,W\}^2} p^0_{Ba_2a_3}u(2000)+\Big\{(1-\varepsilon)\sum_{(a_2,a_3)\in\{B,W\}^2} p^0_{Wa_2a_3}+\varepsilon\Big\}u(0)=:u(B'). \tag{A\_b-a}$$

For ambiguity seeking case, similar definition to the previous one applies. The value after the first observation, say blue, is written by

$$\min_{(y,1-y)\in\mathscr{P}|_{E_B}}\Big[\; yu(2000)+(1-y)u(0)\;\Big]$$
$$=\frac{(1-\varepsilon)\sum_{a_3\in\{B,W\}} p^0_{BBa_3}}{(1-\varepsilon)\sum_{(a_2,a_3)\in\{B,W\}^2} p^0_{Ba_2a_3}+\varepsilon}u(2000)+\frac{(1-\varepsilon)\sum_{a_3\in\{B,W\}} p^0_{BWa_3}+\varepsilon}{(1-\varepsilon)\sum_{(a_2,a_3)\in\{B,W\}^2} p^0_{Ba_2a_3}+\varepsilon}u(0)$$
$$=u(A\_b). \tag{A\_b-s}$$

The white case A_w for ambiguity averse and for ambiguity seeking case is defined in the same way, the similar argument proceeds. Let A_α, $\alpha=\{bb,bw,wb,ww\}$ denote the urn after twice withdrawing. Its betting value for ambiguity averse ones, say*blue and blue*, is written by

$$\min_{(z,1-z)\in\mathscr{P}|_{E_{BB}}}\Big[\; zu(2000)+(1-z)u(0)\;\Big]$$
$$=\frac{(1-\varepsilon)p^0_{BBB}}{(1-\varepsilon)\sum_{a_3\in\{B,W\}} p^0_{BBa_3}+\varepsilon}u(2000)+\frac{(1-\varepsilon)p^0_{BBW}+\varepsilon}{(1-\varepsilon)\sum_{a_3\in\{B,W\}} p^0_{BBa_3}+\varepsilon}u(0)$$
$$=u(A\_bb). \tag{A_bb-a}$$

The other cases, $A\_a_2a_3$, $(a_2,a_3)\in\{B,W\}$, are also defined the same way, and for ambiguity seeking case, the similar argument applies. Since

$$\frac{(1-\varepsilon)\sum_{a_3\in\{B,W\}} p^0_{BBa_3}}{(1-\varepsilon)\sum_{(a_2,a_3)\in\{B,W\}^2} p^0_{Ba_2a_3}+\varepsilon}>\frac{(1-\varepsilon)p^0_{BBB}}{(1-\varepsilon)\sum_{a_3\in\{B,W\}} p^0_{BBa_3}+\varepsilon}$$

the following theorem holds.

Theorem 2. Assume that the principal probability p^0 is symmetry. Thus $A\_b=A\_w=:A$ and $A\_bb=A\_bw=A\_wb=A\_ww=:A$. If a subject is ambiguity averse, we should observe

$$R>A>A\_b=A\_w>A\_bb=A\_bw=A\_wb=A\_ww$$

if ambiguity seeking it follows that

$$R < A < A\_b = A\_w < A\_bb = A\_bw = A\_wb = A\_ww$$

and if neutral one has

$$R > A = A\_b = A\_w = A\_bb = A\_bw = A\_wb = A\_ww$$

.

3.2 Procedure of the second experiment

We explain the procedure of the additional experiment.

All questions focus on the certainty equivalents of bets and are conducted as binary comparisons using a Multiple Price List (MPL). This list presents choices between receiving 'x yen for sure' versus 'the bet on the urn α', with values $x = 100, 200, \ldots, 1100$ in increments of 100 yen, where $\alpha = A, A\_b, A\_w, A\_bb, A\_bw, A\_wb, A\_ww$, as described above.

The steps to construct the ambiguous urn are as follows.

- On the day of the experiment, participant numbers are assigned by lottery, so neither the experimenter nor the subjects know in advance who will be selected to create the ambiguous urn.
- We prepare transparent bags, each containing either 5 blue balls or 5 white balls. A total of 40 bags (20 with blue balls and 20 with white balls) are shown to the subjects. All bags, except one containing blue balls and one containing white balls, are placed into a large transparent box. Afterward, the balls from the two remaining bags are removed and placed directly into the box to establish an initial known composition of 5 blue balls and 5 white balls.
- The box is then covered with black paper and further concealed with a black cloth to prevent any visibility from above.
- Six subjects are randomly selected by lottery on the day of the experiment. Each selected subject, while blindfolded and assisted by an experimenter, reaches into the box, opens a randomly chosen bag, and scatters its balls into the box. Afterward, they display the empty plastic bag to confirm it is empty.
- This process continues until a total of 30 balls (from six bags with 5 balls each) are added to the box. Note that selected subjects practice this procedure before the actual experiment.

- This process results in an ambiguous urn with possible distributions of balls: $(B, W) = (5, 35), (10, 30), (15, 25), (20, 20), (25, 15), (30, 10), (35, 5)$. Subjects are fully informed of these probabilities and required to answer quizzes to confirm their understanding. The experiment only begins once all subjects provide correct answers.

We prepare transparent bags each containing either 5 blue balls or 5 white balls. We show subjects 20 such bags and place them into one large urn and cover the urn with a black plastic board. First, each color's bags are opened, and all balls are placed into the covered urn, so that 5 blue and 5 white balls exist in the urn. Secondly six subjects are randomly selected and each subject is required to come to the urn, open one bag and scatter balls in it into the urn while wearing an eye-mask. After that, let them take off the empty transparent bag and show it to everyone. Then 30 balls (6 subjects open one bag containing 5 balls) are added in the urn to the end. Note that selected subjects practice this conduct before execution. It turns out that the balls taken out of bags create an ambiguous urn with possible distributions: $(B, W) = (5, 35), (10, 30), (15, 25), (20, 20), (25, 15), (30, 10), (35, 5)$.

[1] Subjects are clearly informed of this fact and are required to complete quizzes to confirm their understanding; the experiment begins only once all subjects answer correctly. Each step is clearly explained to all participants.

- The risky urn R consists of twenty red and yellow balls, with the composition known to all subjects. The winning bet is set to red balls. A ball is drawn without looking inside, and the color is shown to the subjects.
- The ambiguous urn A is constructed as described above.
- A ball is drawn by the assistant without looking inside, reaching behind them, and the color is shown to the subjects.
- In the previous step, if a blue ball is drawn, urn A_b is created by returning the ball to urn A. If a white ball is drawn, urn A_w is created by returning the ball to urn A. The assistant then draws another ball, again reaching behind without looking inside, and shows the color to the subjects.
- One of the urns, A_bb or A_bw, is created by returning the blue ball drawn from urn A_b, or one of the urns, A_wb or A_ww, is created

[1] From the construction, the probabilities of each possible combination $(5+5k, 35-5k)$ are ${}_6\mathrm{C}_k/2^6$ for $k = 0, 1, 2, \ldots, 6$. The highest probability is ${}_6\mathrm{C}_3/2^6$ at $(20, 20)$, making the probability of drawing a blue ball $1/2$.

by returning the white ball from urn A_w. The assistant then draws another ball, again without looking inside and reaching behind them, and shows the color to the subjects. Subjects are able to recognize which urn is realized.

- Additionally, we ask qualitative questions regarding how subjects perceive the composition of the urns, allowing them to select their responses from provided options.

3.3 Results and interpretation of the second Experiment

Results: The result for $A - R$, the value of the certainty equivalent of urn B minus that of urn A, shows that 21 subjects (about 51%) answered negative (ambiguity averse), 20 (about 49%) positive (ambiguity seeking), and 0 (0%) zero (ambiguity neutral).

In step (2), a blue ball was drawn, so we consider the urn C after observing blue, labeled as A_b. In step (3), a blue ball was drawn again, so the ambiguous urn after two blue draws is labeled A_bb. According to theoretical predictions, both the relationship between $A - R$ and $A\_b - A$ and between $A\_b - A$ and $A\_bb - A\_b$ should be positive.

However, the results are opposite. In the following analyses, figures and tests are conducted using Python. An OLS regression between $A - R$ and $A\_b - A$ reveals a negative relationship ($t = -4.937, p = 0.000$), as does the relationship between $A\_b - A$ and $A\_bb - A\_b$ ($t = -4.188, p = 0.000$).

In Figure 3, the horizontal axis (ambiguity1) represents $A - R$, and the vertical axis (ambiguity2) represents $A\_b - A$. Figure 4 shows ambiguity2 ($A\_b - A$) on the horizontal axis and ambiguity3 ($A\_bb - A\_b$) on the vertical axis. Despite these negative correlations, one might expect a positive correlation between $A - R$ and $A\_bb - A_b$. However, as shown in Figure 5, this is not the case (OLS regression: $t = 0.464, p = 0.646$). Table 1 presents the sign patterns for $A - R$, $A\_b - A$, and $A\_bb - A_b$, with notable instances of 'zigzag' changes in sign.

We also asked subjects qualitative questions about their perception of the urns A_b (after the first observation) and A_bb (after the second observation). Note that, due to a limited sample size, only the urns A_b, A_bw, and A_bb were actually observed during the experiment.[2] The specific questions and corresponding responses are shown in Table 2. Notably, the number of subjects who chose "Observing the color made it harder to predict" increased

[2] We plan to continue this experiment to observe all possible cases.

The sign pattern	The number of subjects
$- - -$	0
$- - +$	6
$- + -$	10
$- + +$	5
$+ - +$	13
$+ - -$	4
$+ + -$	3
$+ + +$	0

Table 1: From left: the sign of $A - R$, $A\_b - A$ and $A\_bb - A\_b$

significantly, indirectly suggesting that ambiguity perception increases after each observation.

Interpretation: The implications of this additional experiment are similar to the previous one, with a notable feature being the 'zigzag' type changes in certainty equivalents (see Figures 3, 4, and Table 1). In the qualitative responses, many subjects selected the option 'The opposite color to the previously observed one would be more likely to appear', which aligns with the tendency for subjects to predict outcomes by integrating the urns.

Selected Options as Closest Feelings about the Urns	A_b	A_bb &A_bw
Observing the color and prediction are unrelated	21	17
Observing the color made it harder to predict	1	9
The opposite color to the previously observed one would be more likely to appear	16	12
The same color as the previously observed one would be more likely to appear	2	3
Other thought	1	0

Table 2: Qualitative questions and the corresponding number of subjects

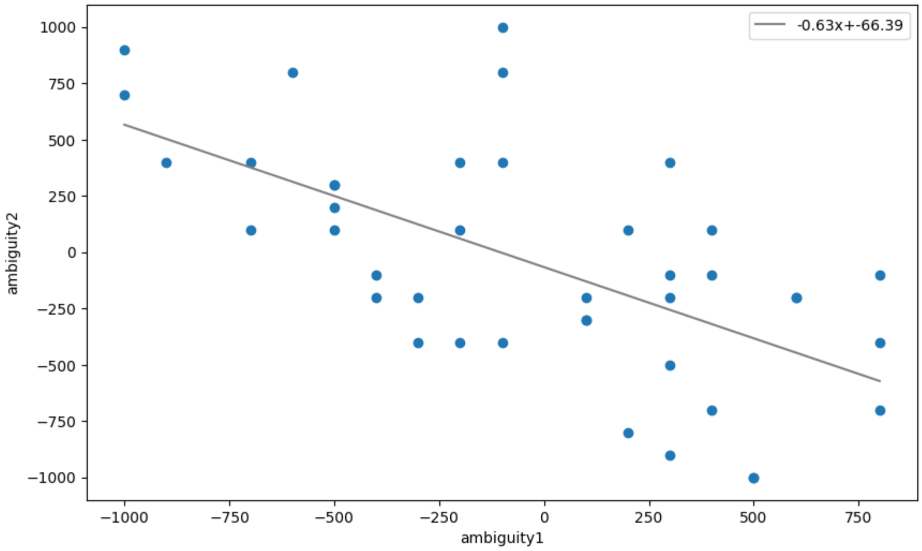

Figure 3: The relation between $A - R$ (ambiguity1) and $A_b - A$ (ambiguity2)

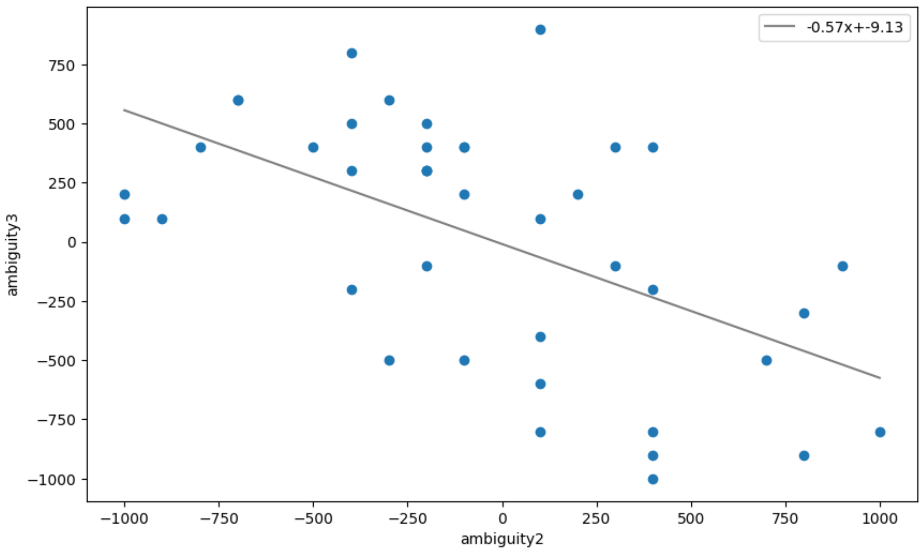

Figure 4: The relation between $A\_b - A$ (ambiguity2) and $A\_bb - A\_b$ (ambiguity3)

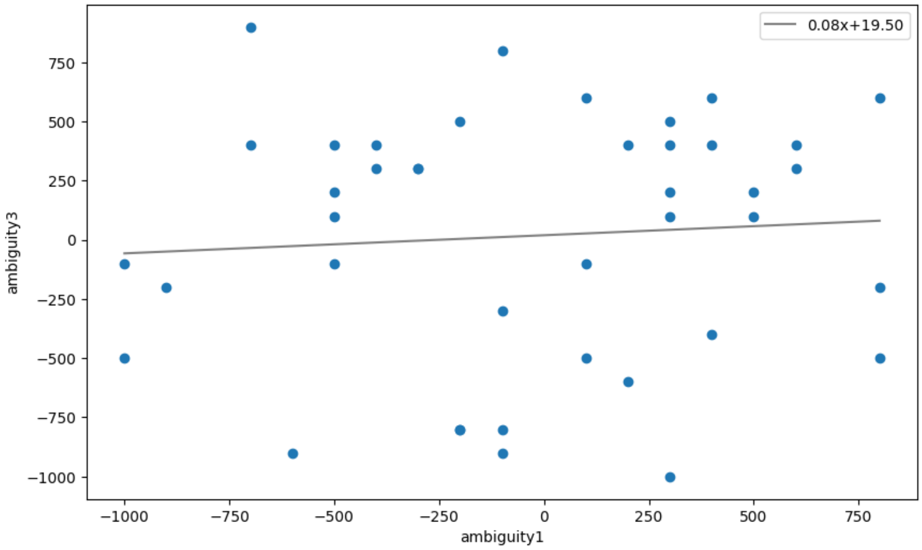

Figure 5: The relation between $A - R$ (ambiguity1) and $A\_bb - A\_b$ (ambiguity3)

4 Conclusion

This paper experimentally investigates the observability of the dilation property in ambiguity. Subjects were asked to bet on colors in two ambiguous urns consecutively and to reveal their certainty equivalents. Although theoretical predictions suggest that certainty equivalents should decrease for ambiguity-averse subjects and increase for ambiguity-seeking ones, our experimental results show the opposite.

A possible interpretation is that some subjects integrate each bet and perceive a correlation between the probabilities of consecutively drawn colors, anticipating that certain colors are more or less likely to reappear. Our findings show that subjects who perceive no correlation do not exhibit the opposite result as strongly as those who expect such correlation. While the dilation property is not directly observed, subjects tend to integrate each decision and consider joint probabilities, which serves as an indirect indication of the dilation property. The additional experiments align with the primary findings, and responses to qualitative questions suggest an increasing difficulty in prediction, further supporting the indirect presence of dilation.

Appendix

Proof of Theorem. We only proved the ambiguity averse case. $R > A$ is obviously seen because

$$\big(p_1^0 + p_2^0,\ p_3^0 + p_4^0\big) \subsetneq \big((1-\varepsilon)(p_1^0 + p_2^0),\ (1-\varepsilon)(p_3^0 + p_4^0) + \varepsilon\big).$$

In what follows, we show $A > A\_x$. $x = \{b, w\}$.

Under the symmetry, $(p_1^0 + p_2^0, p_3^0 + p_4^0) = (p_1^0/(p_1^0 + p_2^0), p_2^0/(p_1^0 + p_2^0)) = (p_3^0/(p_3^0 + p_4^0), p_4^0/(p_3^0 + p_4^0))$, it holds that

$$\begin{aligned}
&\frac{(1-\varepsilon)p_1^0}{(1-\varepsilon)(p_1^0 + p_2^0) + \varepsilon}\\
&= \left(1 - \frac{\varepsilon}{(1-\varepsilon)(p_1^0 + p_2^0) + \varepsilon}\right)\frac{p_1^0}{p_1^0 + p_2^0} < (1-\varepsilon)(p_1^0 + p_2^0)
\end{aligned}$$

and

$$\begin{aligned}
&\frac{(1-\varepsilon)p_2^0 + \varepsilon}{(1-\varepsilon)(p_1^0 + p_2^0) + \varepsilon}\\
&= \left(1 - \frac{\varepsilon}{(1-\varepsilon)(p_1^0 + p_2^0) + \varepsilon}\right)\frac{p_2^0}{p_1^0 + p_2^0} + \frac{\varepsilon}{(1-\varepsilon)(p_1^0 + p_2^0) + \varepsilon}\\
&> (1-\varepsilon)(p_1^0 + p_2^0) + \varepsilon = (1-\varepsilon)(p_3^0 + p_4^0) + \varepsilon
\end{aligned}$$

because

$$\frac{\varepsilon}{(1-\varepsilon)(p_1^0 + p_2^0) + \varepsilon} > \varepsilon.$$

Hence we proved

$$\big((1-\varepsilon)(p_1^0+p_2^0),\ (1-\varepsilon)(p_1^0+p_2^0)+\varepsilon\big) \subsetneq \left(\frac{(1-\varepsilon)p_1^0}{(1-\varepsilon)(p_1^0 + p_2^0) + \varepsilon},\ \ \frac{(1-\varepsilon)p_2^0}{(1-\varepsilon)(p_1^0 + p_2^0) + \varepsilon}\right),$$

and

$$\big((1-\varepsilon)(p_3^0+p_4^0),\ (1-\varepsilon)(p_3^0+p_4^0)+\varepsilon\big) \subsetneq \left(\frac{(1-\varepsilon)p_3^0}{(1-\varepsilon)(p_3^0 + p_4^0) + \varepsilon},\ \ \frac{(1-\varepsilon)p_4^0}{(1-\varepsilon)(p_3^0 + p_4^0) + \varepsilon}\right).$$

That leads to the conclusion. $\square$

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