

# An over-rejection puzzle of bootstrap average tests for the no-threshold-effect hypothesis

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## Abstract

When testing the null hypothesis of no threshold effects based on threshold autoregressive models, wild-bootstrap supremum, average, and exponential tests are routinely used to handle an identification issue under the null. In this note, we demonstrate via Monte Carlo simulations that the bootstrap average tests lose control for the type-I error rate when the threshold variable is persistent and the delay parameter is chosen from more than a handful of choices. In some cases, the average tests reject the correct null hypothesis with probability exceeding nominal size by more than 10%. The size distortion is present even in large samples, indicating the average tests may not converge to the intended asymptotic null distribution. Supremum and exponential tests achieve correct type-I error rates, posing a puzzle why only the average tests suffer from over-rejections.

**JEL codes:** C12, C22.

**Keywords:** Asymmetry, nonlinear time series analysis, size distortion, Threshold Autoregression (TAR), type-I error, wild bootstrap.

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