What Does it Take to Control Global Temperatures?

A toolbox for testing and estimating the impact of economic policies on climate.

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April 16, 2024

Abstract

This paper tests the feasibility and estimates the cost of climate control through economic policies. It provides a toolbox for a statistical historical assessment of a Stochastic Integrated Model of Climate and the Economy, and its use in (possibly counterfactual) policy analysis. Recognizing that stabilization requires supressing a trend, we use an integrated-cointegrated Vector Autoregressive Model estimated using a newly compiled dataset ranging between years A.D. 1000-2008, extending previous results on Control Theory in nonstationary systems. We test statistically whether, and quantify to what extent, carbon abatement policies can effectively stabilize or reduce global temperatures. Our formal test of policy feasibility shows that carbon abatement can have a significant long run impact and policies can render temperatures stationary around a chosen long run mean. In a counterfactual empirical illustration of the possibilities of our modeling strategy, we show that the cost of carbon abatement for a retrospective policy aiming to keep global temperatures close to their 1900 historical level is about 75% of the observed 2008 level of world GDP, a cost equivalent to reverting to levels of output historically observed in the mid 1960s. This constitutes a measure of the putative opportunity cost of the lack of investment in carbon abatement technologies.

Keywords: Vector Autoregression, Cointegration, Control theory, Carbon abatement, Climate change, Counterfactual analysis, Integrated Assessment Models.

*The authors thank participants to the EC^2 conference in Aarhus, the IAAE at King's College, London, the ISF in Oxford, the Climate Econometrics online seminar, the SNDE at UCF, Orlando, the Barcelona Summer Forum, and the IWEEE at Bolzano for helpful comments and suggestions. They are in particular grateful to Jennifer Castle, Frank Diebold, Luca Fanelli, David Hendry, Robert Kaufmann, Frank Kleibergen, Andrew Martinez, Sophocles Mavroeidis, Zack Miller, John Muellbauer, Barbara Rossi and Glenn Rudebusch.

1 Introduction

This article presents a novel approach to testing the feasibility and evaluating the costs associated with global temperature control by drawing on historical data on the interaction between climate and the economy spanning over a thousand years. Our primary objective is to rely on empirically *estimated* relationships, acknowledging the criticism raised by Pindyck (2013) towards models that solely rely on theoretical assumptions, calibrations, or simulations, as they may create a "misleading perception of knowledge and precision". In pursuit of our objective, we construct a dataset and employ a new econometric methodology relying on stable long-run interactions to test the hypothesis of temperature controllability and evaluate its cost through counterfactual policy analysis.

Our approach relies on a model inspired by Stochastic Dynamic Integrated Models of Climate and the Economy (SDICE) proposed by Nordhaus (2017) that has become one of the main Integrated Assessment Models (IAMs) of climate and the economy. This model is often assessed through simulations and scenario analyses in order to evaluate the cost of carbon abatement policies required to achieve some specific temperature control objectives. The lack of historical carbon abatement policies, at least until the recent decades, renders empirical studies of the SDICE models limited. This is one issue we tackle using long historical time series dating back to AD 1000, i.e., prior to natural experiment that the industrial revolution constitutes. This allows us to prove, through a statistical test, that policies whose objectives focus on temperature control are empirically feasible.

The industrial revolution having accelerated the upward trend in economic activity, Greenhouse gas (GHG) emissions and temperatures, we recognize that temperature control requires suppressing the upward stochastic trend to render temperatures stable around a long run mean. For this we draw on the abundant literature studying the dynamic interactions between climate variables and human activity through an integrated-cointegrated Vector Autoregressive (VAR) model (see, inter alia, Stern and Kaufmann, 2014, and Chang *et al.,* 2020). We study this modeling strategy in light of work on nonstationary Control Theory developed by Johansen and Juselius (2001, JJ01 henceforth) who find cointegration properties to be the determining factor. We show that the effect of the control policy is to augment the VAR(p) into a VARMA($p, 1$) that reflects additional cointegration relations.

Hence, we assess within a cointegrated VAR model what policies are feasible to render "climate" variables stationary around a stated objective while retaining economic progress. We focus on very long series (despite the unavoidable mismeasurements) to capture long-run equilibria that are invariant to changes in policy, i.e., that are immune to the Lucas (1976) critique. Our empirical model can be used as a toolbox for evaluating the *statistical* feasibility and cost of policies. In an empirical application, we propose a formal test that carbon abatement or technology investment are capable of achieving temperature control. We entertain the counterfactual question whether a centralized authority could have implemented, in the 20th century, a policy aiming to maintain global temperatures at the level of 1900. We assess its cost using two distinct *indirect* examples, either (i) via a costing of the lack of carbon abatement policy, using a reduction of output and consumption as controls; or (ii) through the increased wealth that a costless reduction of the carbon content of production technology would generate. The estimated cost of the lack of abatement policy amounts to about 75% of the 2008 global level of output (equivalent to forestalling growth since the 1960s) together with a reduction of 45% in consumption. Investment in carbon neutral technology would by contrast achieve its objective and be profitable as long as it costs less than 50% of 2008 global GDP and 75% of consumption. Both policies show that, under the condition that the stated temperature control is achieved, the huge magnitude of investment in mitigating technologies that is required and is more profitable than a degrowth alternative. We state in the title that our analysis provides a toolbox as our model and methodology can be applied to other policies and choices of controls that climate scientists may prefer.

The rest of this study consists of four sections. Section 2 reviews the SDICE model, the database we consolidate, and its framing in a cointegrated VAR system. Section 4 discusses the control theory of JJ01 and develops some of the results needed for a counterfactual analysis. Section 4 then assesses empirically the controllability of temperature through carbon abatement policies and performs some counterfactual costing analyses. Details, further explanations and empirical results are provided in an online Supplementary Appendix.

2 A linearized SDICE Model

We propose an empirical model for the climate-economy nexus and estimate over a thousand years. We introduce a log-linearized SDICE that has been sufficiently streamlined so it can shed light on long-run equilibria estimated later in the paper. There exist a multiplicity of IAM models but most of them can been seen as refinements or extensions of the main equations we consider here (see, e.g., Barnett *et al.*, 2022, and Hänsel *et al.*, 2022, for analyses of the uncertainty surrounding the models). By construction, log-linearization removes the nonlinearities that are inherent in the discussion surrounding possible future *tipping points* but these can easily be introduced through additional local trends.

We consider here a simplified version of the SDICE model of Nordhaus (2017) as studied, inter alia., in Ikefuji *et al.* (2020), see Figure 1 for a presentation of its general principles. The key feature is that the model introduces a negative feedback loop from climate to the economy. At each period t, human economic activity combines various production factors (such as labor force and capital) to generate real Gross Domestic Product (GDP, or world output), Y_t. This production generates externalities in the form of greenhouse gas (GHG) emissions – Carbon dioxide (CO_2) in particular. Humans may decide to mitigate these externalities via abatement, i.e. investment that reduces the emission producing (*brown*) content of economic activity. We simplify the model and only present the log-linearized version of that in Ikefuji *et al.* (2020), with lower case letters representing logarithms, see the Supplementary Appendix for more details.

Total CO_2 emissions consist of anthropogenic emissions (caused by human activity) and other – exogenous – types e_t^0. Total emissions e_t then result from

$$e_t = \sigma_t - \mu_t + y_t + e_t^0, \tag{1}$$

where σ_t is the emissions-to-output ratio for CO_2 and μ_t is the abatement fraction for CO_2. Regarding the presence of CO_2 in the atmosphere, the SDICE specifies that its concentration m_t accumulates through interactions with shallow and lower oceans. The lack of long historical series on the carbon contents of oceans, m_t can be seen as following a Markov state-space process with two hidden layers (latent variables). This amounts to specifying that m_t follows an autoregressive model with distributed lags of e_t, here an ARDL(4,4), $A(L)\, m_t = B(L)\, e_t$,where L denotes the lag operator such that $Lm_t = m_{t-1}$ and $A(\cdot)$, $B(\cdot)$ are polynomials of degree 4.

Now atmospheric temperatures, h_t, themselves relate dynamically to atmospheric gas concentrations, ocean temperatures, as well as extraneous radiative forcing, f_t^0, which in log form can be written as an equation linking $h_{t+1} - a_1 m_{t+1} - f_{t+1}^0$ to the lagged value of that expression together with h_t and h_{t-1}. Finally, we consider the impact of climate on economic growth: the fraction of GDP not spent on abatement is consumed, c_t, or invested, i_t, along the budget constraint: $y_t - \omega_t - \xi h_t = \tilde{c}_t$, where $\tilde{c}_t = c_t + i_t$, and the logarithm of the cost of abatement, ω_t, satisfies $\omega_t = \psi_t + \theta\mu_t$.with $\theta > 1$ so the cost of abatement increases faster than abatement itself. Parameter ξ represents damage induced by warming: it closes the feedback loop from climate to the economy shown in Figure 1. Considering ψ_t constant over the historical sample (but not over the future under tipping point scenarios), we identify ω_t to $\theta\mu_t$ below.

The model presented above can be expressed in terms of six endogenous variables ($y_t, \tilde{c}_t, m_t, h_t, \mu_t, \sigma_t$) and two exogenous $\left(e_t^0, f_t^0\right)$. This leads (removing constant terms and introducing two lag polynomials $D(\cdot)$ and $G(\cdot)$) to the following equations:

$$\text{Economy-Climate nexus:} \quad A(L)\, m_t = B(L)\left(\sigma_t - \mu_t + y_t + e_t^0\right), \tag{2a}$$

$$\text{Carbon-Temperature:} \quad D(L)\, h_t = G(L)\left(a_1 m_t + f_t^0\right), \tag{2b}$$

$$\text{Damage loop:} \quad y_t = \tilde{c}_t + \theta\mu_t + \xi h_t. \tag{2c}$$

The SDICE model is typically solved by the Central Planner who sets the policy variables: level of abatement, μ_t, or its cost ω_t, and carbon content of technology σ_t to achieve a welfare objective in terms of $\tilde{c}_t$.[1] We consider this objective from an empirical perspective below through

[1] By specifying a production function for y_t as a function of labor and capital, as well as a utility function, the central planner may consider the tradeoff between consumption and leisure. We abstain from it for simplification.

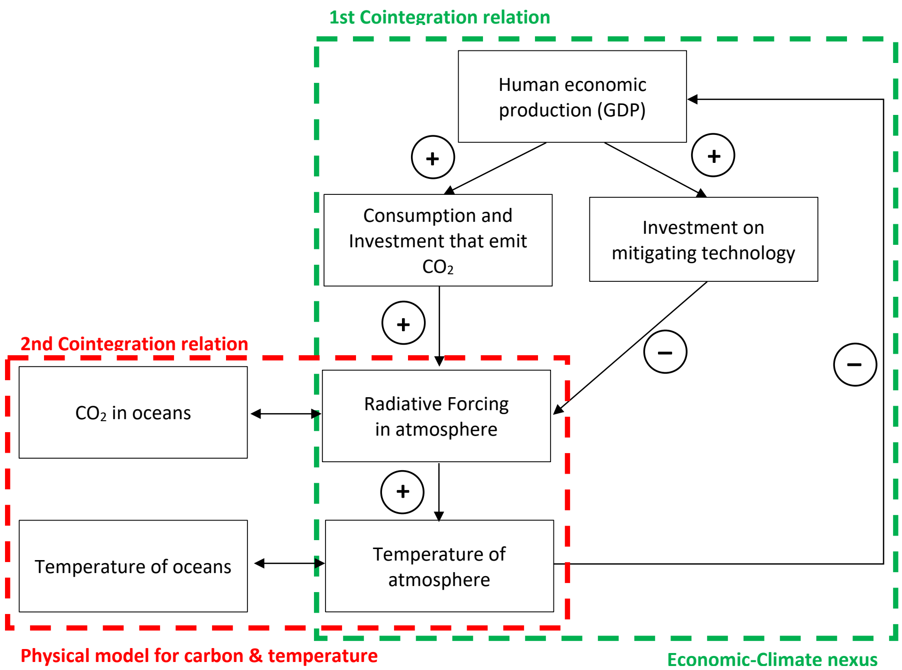

Figure 1: Principle of SDICE Model. The solved out empirical model finds two long term equilibria ('cointegration relations') corresponding to (i) the economic-climate nexus and (ii) the physical equilibria between global temperatures and radiative forcings linked to concentrations of carbon in the atmosphere and the oceans.

the concept of controllability, where we ask whether and what policies can generate a stable equilibrium between economic activity and climate.

3 An Empirical Model for Climate and the Economy

The dynamic model described above can be assessed using historical empirical evidence through a cointegrated vector autoregressive model (CVAR) developed by Johansen (1988), i.e., a model for the dynamic interactions and long run equilibria between climate of and the economy. For this we first construct a new dataset compiling and extending various sources over the second millennium AD.

Constructing a long dataset. The data were obtained and reconstructed from various sources that are presented in the Supplementary Appendix, and whose online links and interpolation details are provided in separate code file (*Data Preparation*). The data is presented in Figure **??**: it comprises measures of economic activity (real world output and consumption) as well as variables describing temperature anomalies, carbon concentrations in the atmosphere, radiative forcings of non-anthropogenic origin (solar, volcanic...). We consolidate this data at the annual frequency, dating back to year 1000 AD.

It is obvious that the data we consider must necessarily be subject to mismeasurement as we consider global variables over extended periods of time. Yet, following Duffy and Hendry (2017), we expect that this mismeasurement does not affect inference on the presence of cointegration. In Figure **??**, we see that all variables except radiative forcings of volcanic origin exhibit upward trends towards the end of the sample.

The Cointegrated VAR approach. We perform a cointegration analysis within a VAR(8) model for the four variables $X_t = (y_t, c_t, m_t, h_t)$ measured as log GDP and log consumption, log CO_2 atmospheric concentration and actual (non-logged) temperature anomalies. The impact of the Industrial Revolution is captured by a broken linear trend that starts in 1800 (that is restricted to the cointegrating space). We prefer to model long term population patterns through this trend for statistical reasons, as conditioning on the actual data impacts estimator distributions in a non-standard way. For technical reasons, we also need to include the change in the broken trend, i.e., a step dummy starting in 1800. Solving the model for concentrations and radiative forcings as proxies for emissions allows to extend the data set to cover the years 1000-2008. Our analysis leads to focusing on X_t, with radiative forcings of volcanic origin, f_t^{Vol} entering as unrestricted exogenous regressors.

The cointegrated model writes as

$$\begin{aligned} \Delta X_t &= \tau + \alpha \left(\beta' X_{t-1} + \delta_1 t 1_{\{t \geq 1800\}} \right) \\ &\quad + \sum_{i=1}^{7} \Gamma_i \Delta X_{t-i} + \gamma' \Delta f_{t-1}^{Vol} + \delta_0 1_{\{t \geq 1800\}} + \epsilon_t, \end{aligned} \tag{3}$$

where $\beta' X_{t-1}$ captures the two long run equilibria taking the form of "cointegration" relations are stationary although the individual variables are not.

In the empirical model, the data indicates the presence of two stochastic trends, and two cointegration relations. Based on the SDICE model, we make the following assumption on the common stochastic trends.

Assumption 1 (S) *The sources of nonstationarity in the empirical system come from two latent common stochastic trends:*
(i) *a measure of the* carbon content of technology progress*, which in the model amounts to the logarithm of emissions-to-output ratio of CO_2, σ_t;*
(ii) *a* wealth effect *that combines global population and capital accumulation through investment, i_t.*

Remark 1 *We could alternatively interpret a linear combination of them as wealth and technology of* green *(carbon free) and* brown *(carbon intensive) origin and impact noting that these* *are not orthogonal in the sample.*

Assumption S helps us identify the two stable (i.e. stationary) long run cointegration relations (see the Supplementary Appendix for a description of the estimated model) as

$$\begin{aligned} c_{1,t} &= y_t - 1.54c_t - .81h_t - .036t \times 1_{\{t \geq 1800\}}, \\ c_{2,t} &= h_t - 4.12m_t. \end{aligned}$$

We interpret them with the help of the SDICE model.

1. The first equation, $c_{1,t}$, corresponds to an interaction between temperature and human activity. In the SDICE, this corresponds to equation (2c). The broken trend grows at an annual rate of 3.6%, corresponding to the long run growth rate of population, total factor productivity and capital intensity over the industrial era (for the latter, see Baumol, 1986 and references therein).

2. The second cointegrating relation, $c_{2,t}$, corresponds to the interplay between CO_2 concentrations in the atmosphere and temperature, as in equation (2b).

3. The data does not support equation (2a) as a cointegration relation. This is in line with Assumption S.(i) that includes the nonstationary σ_t.

The second cointegration relation does not involves human activity although the technology mix has strong evolved over the millennium. Yet, y_t is close to significant in this equation so we might have preferred to retain it. In Smil (2017, in his last table on page 458) who reports estimates of per capita annual consumption of primary energy, these have only started increasing by an order of magnitude when countries started their industrial revolution (the estimates are explicitly uncertain). Hence, it is likely that the technology mix, prior to 1800, has not modified much the elasticity of temperature to CO2 emissions, yet given the ensuing changes, we prefer to leave human activity outside this equation, to reflect only physical equilibria. For robustness, we reestimated the model starting in AD 1750, and found the estimates to be very similar, with a test of overidentifying restrictions that has a p-value of 0.41.

In the model above, 100% green growth is feasible if a downward trend in σ_t is achieved, tending towards $-\infty$ so actual carbon content $\Sigma_t = \exp(\sigma_t)$ is driven to zero. This constitutes an alternative to reducing growth altogether. In practice, given the current technologies, a combination of green investment and restrained *brown* growth is required in our model to achieve stability in temperature, let alone the counterfactual analysis we perform below.

Comparison with estimates in the literature. To assess the plausibility of our estimated model, we compare its implications with meta analyses based on the existing literature. The long run impact of climate change on the economy has been considered nonlinear so calibrated parameter uncertainty is an important issue. The table below summarizes the results for three main indicators used in the climate-economy literature (see the Appendix for derivations and explanations).

	Ranges in Literature	Our results (*s.e.*)
Temperature damage on GDP, ξ	$[.68, 1.34]^*$	.81 (.40)
200 year temperature increase due to CO_2	$[.7, 2.1]^\dagger$	1.35 (.66)
GDP loss due to CO_2, γ	$[.27, 10.4]^\dagger$	4.25 (.38)

*Nordhaus & Moffat (2017); †Hassler, Krusell & Olovsson (2018).

Overall, these results (with reported standard errors) show that our estimates are very much in the low to mid-range of those reported in the literature, so we are confident that our analysis neither severely underestimate nor overestimate the relative impacts of climate and the economy.

4 Control theory in a cointegrated system

We now review the non-stationary control theory derived by JJ01 and show how it can be used to understand the issue of climate control through carbon abatement, reinterpreting the objective of the policy as suppressing the stochastic trend in temperature through use of cointegration properties. For the sake of expositional simplicity, setting $k = 1$ in (3) and removing deterministic terms reduces the model to

$$\Delta X_t = \alpha \left(\beta' X_{t-1} - \mu\right) + \epsilon_t, \tag{4}$$

and its Granger-Johansen moving average representation is

$$X_t = C\textstyle\sum_{j=1}^{t}\epsilon_j + C\left(L\right)\epsilon_t + A_0, \tag{5}$$

where the long-run impact matrix C governs the nonstationary stochastic trends of the system: it latter plays a key role in the theory of controllability. In the expression above, $C\left(L\right)\epsilon_t$ represents a stationary series, A_0 depends upon the initial values and μ such that $\beta' A_0 = \mu$. It follows from $\beta' C = 0$ that $\beta' X_t - \mu = \beta' C\left(L\right)\epsilon_t$ is stationary so β are cointegration vectors. The long-run expected value of X_t is defined as $X_\infty = \lim_{\tau\to\infty} \mathsf{E}\left(X_\tau \left| X_0\right.\right) = CX_0 + \alpha\left(\beta'\alpha\right)^{-1}\mu$. We now consider a Control Theory derived from Preston and Pagan (1982, Chapter 4, and Section 5.8 in particular) and adapted to the nonstationary context as follows.

Definition 1 *The* Control Policy *consists in two selection matrices* (a, b)*, an objective* b^* *and a contemporaneous control rule* $\nu\left(\cdot\right)$ *such that, in the system (4),*

(i) Policy controls $a'X_t$ *can be changed by intervention using control rule* $\nu(X_t)$:

$$\mathsf{ctr} : a'X_t^{ctr} = a'X_t + a'\nu(X_t);$$

(ii) *The* objective *is the desired value* b^* *of a* targeted *combination of variables,* $b'X_t$*. It is defined as the long-run conditional expectation*

$$b^* = \lim_{h\to\infty} \mathsf{E}\left(b'X_{t+h}^{new}|X_t^{ctr}\right),$$

where $b'X_{t+1}^{new}$ *is the ecosystem outcome for* X_{t+1} *(now made stationary) given by expression (4) when the control has been applied to* X_t*.*

We assume in this paper that the policy objective is to control temperature so that $b'X_{t+1}^{new} = h_{t+1}^{new}$ and this process becomes stationary around a mean b^*.

The control policy defined above is explicitly written as a new equation to the system, one that necessitates the "authority" that implements it must be able to modify $a'X_t$ via $a'\nu(X_t)$. This may require extra controls that are outside the system but interact with it, though for simplicity here we disregard this possibility. Indeed, our aim is not to assess *how* to implement a policy, but *whether* it can be effective.

With continuous monitoring and control, the procedure delineated above works as follows (for a policy that starts at time $t = 0$)

$$X_0 \to \underbrace{X_0^{ctr} = X_0 + \nu(X_0)}_{\text{(Policy)}} \to \underbrace{X_1^{new} = \left(I_p + \alpha\beta'\right)X_0^{ctr} - \alpha'\mu + \varepsilon_1}_{\text{(Ecosystem)}} \to \underbrace{X_1^{ctr} = X_1^{new} + \nu(X_1)}_{\text{(Policy)}} \to ....$$

where we define the projector onto the space spanned by a as $\overline{a} = a(a'a)^{-1}$.

Invariance. Success of the above approach relies on the notion of *invariance* of the system (4) to interventions of the type $a'X_t \to a'X_t^{ctr}$ so that we can assume that the potential outcome is generated through mechanism (4) that is not affected by interventions. For invariance, we require here that parameters remain unaffected by the introduction of the new policy so we can be assured that $X_{t+1}^{new} = X_t^{ctr} + \alpha\left(\beta'X_t^{ctr} - \mu\right) + \epsilon_{t+1}$.

This relates to the notion of "super-exogeneity" proposed by Engle, Hendry and Richard (1983), and studied by Pretis (2021) in the context of climate. Yet, super-exogeneity is about invariance of conditional equations, while we consider here a system approach. While invariance cannot be ascertained with certainty, we follow in the Supplementary Appendix two approaches to ensure it constitutes a plausible assumption. These rely on (i) estimating the model using extra long historical data to capture stable relations pre- and post-industrial revolution, treating the latter as an historical "natural expirement" in climate change; and (ii) using statistical tests used in the context of super-exogeneity, inter alia by Castle et al., (2017).

Policy Impact. A question raised by JJ01 is that of controllability of $b'X_t$ via $a'X_t$. Since the objective is formulated in terms of a conditional expectation for $b'X_{t+h}^{new}$, the choice of controls and policy rule must ensure that $b'X_{t+h}^{new}$ is indeed stationary around b^*. In the cointegrated VAR(1), JJ01 show that

$$b^* = b' \lim_{h\to\infty} \mathsf{E}\left(X_{t+h}^{new} \left| X_t^{ctr}\right.\right) = b'\left(C\left[X_t + \nu\left(X_t\right)\right] + \alpha\left(\beta'\alpha\right)^{-1}\mu\right),$$

so that the condition for controllability writes as follows.

Condition 1 (Controllability, C) *The Policy objective is achievable using the chosen controls if*

$$\det\left(b'Ca\right) \neq 0. \tag{6}$$

where C is the matrix measuring the long run impact of the stochastic trends in the Granger-Johansen moving average representation (5).

Notice that Controllability is a property of the system (under invariance) for the targeted variables and chosen controls. It does not depend on the actual rule $\nu\left(X_t\right)$. JJ01 show that if Controllability applies, then a linear rule is possible:

$$\nu\left(X_t\right) = a\left(b'Ca\right)^{-1}\left[\underbrace{\left(b^* - b'X_t\right)}_{\text{policy discrepancy}} + b'\alpha\left(\beta'\alpha\right)^{-1}\underbrace{\left(\beta'X_t - \mu\right)}_{\text{system discrepancy}}\right]. \tag{7}$$

The rule consists of a weighted average of $b^* - b'X_t$, a "policy" discrepancy between the desired objective and the current value at t, and $\beta'X_t - \mu$ is a "system" deviation from the steady state at t. Policy becomes, here, fully endogenous and does not constitute an exogenous shock, as is often modelled in economics via structural VARs. The reason for the effectiveness of the policy, and hence the channel through which it operates, relies on $\nu\left(X_t\right)$ generating an extra linear cointegration relation. The *new* augmented system writes,

$$\Delta X_{t+1}^{new} = \alpha\left(\beta'X_t^{new} - \mu\right) + \left(I_p + \alpha\beta'\right)\nu\left(X_t^{new}\right) + \varepsilon_{t+1}. \tag{8}$$

Following the most recent literature on treatments in time series and macroeconometrics, the policy controls and objective above define a Direct Potential Outcome System in the sense of Rambachan and Shephard (2021). In there framework, the policy consists in an assignment $W_{t+1} = \nu\left(X_t^{new}\right)$ that is uncorrelated with ε_{t+1} so $X_{t+1}^{new} = -\alpha\mu + \left(I + \alpha\beta'\right)X_t^{new} + \left(I + \alpha\beta'\right)W_{t+1} + \varepsilon_{t+1}$ constitutes a "*Non-anticipating Potential Outcome*" (see the Supplementary Appendix for a discussion of our framework and theirs).

In the context of a VAR(1) dynamic system, it can be shown that $\nu\left(X_t^{new}\right) = \overline{a}\left(\kappa' X_t^{new} - \kappa_0\right) = \overline{a}\kappa'\varepsilon_t$, so the impact of the policy is to augment the VAR(1) into a VARMA$(1, 1)$:

$$X_{t+1}^{new} = -\alpha\mu + \left(I + \alpha\beta'\right) X_t^{new} + \varepsilon_{t+1} + \left(I + \alpha\beta'\right) \overline{a}\kappa'\varepsilon_t. \tag{9}$$

This results also holds for higher order VAR(p) dynamics that are modified into VARMA$(p, 1)$ (for a careful choice of policy parameters among those that achieve the stated objective, see JJ01, Theorem 6). Equation (9) shows that the policy can be identified in practice for its parameters $\overline{a}\kappa'$ through a Structural VARMA or VMA and associated response functions. This is not our objective here though, since our aim is to study the impact of implementing a given policy.

Testing for Climate Controllability. Our first question is whether economic activity, y_t or c_t, can be used as an instrument for a policy that aims to control temperature. Based on the analysis above which can be extended to cover $k > 1$, this amounts to testing the significance of the element in the long run C matrix corresponding to equation (6). In the empirical model, the estimated $\hat{C}$, which is reported in the Supplementary Appendix, shows that coefficient for the long run impact of y_t on temperature is 1.56, with a highly significant t-statistic of 3.33.

The matrix estimate and corresponding t-statistics show that the null of non controllability of temperature h_t by policy controls y_t or c_t, i.e. $b'Ca = 0$ for either variable in equation (6), or a linear combination thereof, can safely be rejected at conventional levels. This shows that carbon abatement can achieve its purpose of controlling temperature, i.e. rendering it stationary around a chosen mean. The levels of significance indicate that any linear combination of y_t and c_t can also be used as a policy. Note that direct capture of carbon dioxide in the atmosphere, provided the technology develops sufficiently fast, would also achieve its objective.

5 Counterfactual Policy Analysis

Now that we have established that a policy of carbon abatement is capable of controlling global temperatures, the natural follow-up question is at what cost. To this end, this section considers the retrospective and prospective costs of a policy. For this, we design, and then simulate based on the empirical model, a counterfactual path for the endogenous variables.

Policy design. We now consider JJ01's analysis from the perspective of the policy maker and econometrician who aim to perform a historical counterfactual analysis. We assume that the policy is actioned by a central authority, which may represent international coordination, or might be fictitious. For instance, a counterfactual analysis that may be of interest consists in deriving the development path that would have arisen if mitigating policies had been put in place through carbon abatement at some point in the past.

The methodology and model above allow for a variety of policy objectives and instruments. Here, we consider retrospective policies that would have aimed to control global temperatures to render them stationary around a long run mean equal to their 1900 level (assuming h_t were observed then), i.e. stable over the 20th century at 0.7°C below their 2008 level. Naturally, the ensuing cost depends on the timing of the policy initiation as well as on the choice of controls.

We contemplate two distinct policies that provide different approaches to measuring the opportunity cost of inaction on green investment.

1. One that mixes investment and consumption, with more weight accrued to the latter (since our model does not specify the increase in abatement effectiveness that would have arisen from higher investment on mitigating technologies – yet this can be easily embedded via specific assumptions). Our baseline choice for the control is $a'X_t = y_t + c_t/2$ with target $b'X_t = h_t$. The weights are arbitrary but the modeler has the flexibility to consider policies of their choice, as our study provides a *toolbox* that is adaptable.

2. Another that targets the same object through a reduction of the carbon content of technology, σ_t, in equation (2a), assuming that it can be put in place to control emissions and concentrations $a'X_t = m_t$ directly. This amounts, for a given level of economic activity, to reducing emissions and, possibly, to capturing atmospheric CO_2. Notwithstanding the actual cost of developing these technological solutions – which is assumed to be zero in a baseline scenario, but could be introduced as a fraction of y_t or $\tilde{c}_t$ – such a policy would restrain the economic damage caused by increasing temperatures, hence increasing y_t and $\tilde{c}_t$. As we do not model the cost of such a reduction, the ensuing impact on y_t and c_t can help the policy maker assess what costs they are willing to endure for a specific objective.

Policy 1: Cost of inaction on abatement technologies. Assuming an authority has modified at will both world GDP and consumption to achieve its objective, Figure 3 reports the resulting dynamics, where to avoid a sudden shock in the early 20th century, we introduce the policy progressively. We also include a forecast over the first half of the 21st century, both as obtained unconditionally from the empirical model and with a policy that aims to maintain the same temperature level. We also produce the bootstrap mean prediction and associated confidence intervals, restricting ourselves to using residuals post 1900.

Temperature control is achieved in this exercise via stabilizing atmospheric carbon concentrations to a level about 20% below that of 2008. The ensuing cost in terms of foregone GDP in 2008 is about 75% so the observed counterfactual GDP in 2008 would have been that which we have known in the 1960s (the uncertainty is large), i.e., a cost of about 40 years of growth. The cost in terms of world consumption is 45% of the 2008 level, foregoing the growth observed since the mid-1970s. When looking at the bootstrap distributions (at each step forecasting the next period using 500 bootstrap samples), we see that the historical sequence of shocks imposed that most of the gains, in the counterfactual experiments, where obtained in the second half of the 20th century.

In order to assess the cost of inaction in the face of climate change, we also perform a complementary analysis where the objective in terms of temperature control remains the same, but the policy only starts in 1950. Corresponding counterfactual outcomes are presented in the Supplementary Appendix. The ensuing cost becomes 90% of the GDP of 2008, i.e. essentially no growth since 1950. In terms of consumption, the counterfactual stands at 75% below the observe level of 2008, i.e. an additional 20% reduction to the baseline scenario.

Projecting our experiment over the 21st century in either policy, we see that the efforts will have to be sustained, reinforcing abatement policies. Clearly, these projections are contingent on specific assumptions over the forecast period and nonlinearities due to major climate change (see, e.g., Diebold *et al.*, 2022, and Lenton *et al.*, 2019).

Policy 2: Reducing the carbon content of technology Assuming the technological improvement is available freely but activated progressively to reach a reduction in 20% of atmospheric CO_2 concentrations, the resulting gain in economic activity is potentially massive: Figure 3 shows that the bootstrap interval ranges from -3% to $+160\%$, with a mean gain of $+50\%$, and where the realized policy is at the upper bound. Similar values hold for $\tilde{c}_t$, with a wider bootstrap range and higher mean.

6 Conclusions

This paper assesses the feasibility and quantifies the cost of carbon abatement policies using long series of economic and climate data compiled for the second millennium AD. By means of a cointegrated VAR modeling strategy that matches a simple linearized SDICE model, we test whether and show how a policy that aims to render temperatures stationary around a given long run mean can be achieved. In an empirical application, we tested that carbon abatement is indeed significantly capable of such a policy. We assessed its counterfactual cost, if a centralized authority had been able to implement such a policy in the 20th centuries in order to maintain the temperature level (observed ex post) in 1900. The estimated cost of the policy leads to a reduction of about 75% of the 2008 world level of output (equivalent to foregoing growth since historical levels of the 1960s) together with a reduction in about 45% in global consumption. These costs are assessed under the assumption of a constant carbon content of technology, so they show the opportunity cost of the lack of investment in mitigating technologies.

The analysis in this paper constitutes an exercise where we deliberately chose simple policy controls, but economically meaningful alternatives are also possible (say the discounted wealth rather than spot GDP and consumption). Indeed we see the collected data, model and results above as a toolbox for policy analysis where refinements on possible projected scenarios and

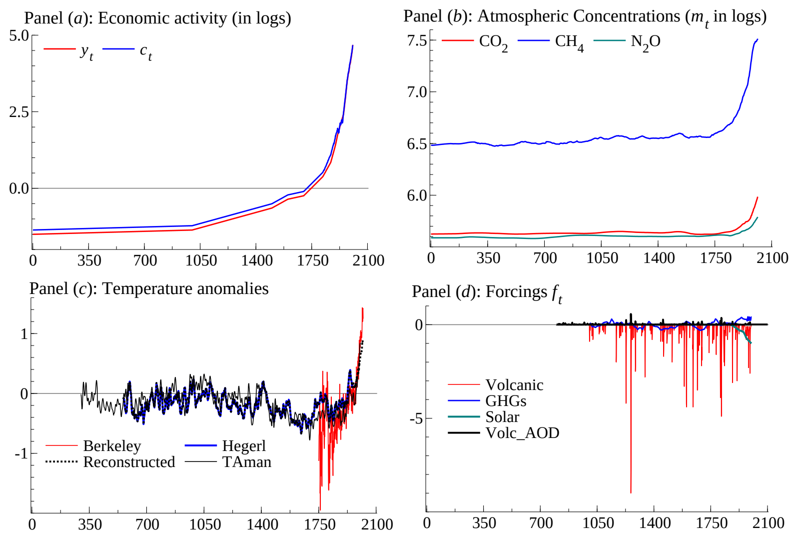

Figure 2: Data series obtained from various sources. All references and explanations are the Supplementary Appendix.

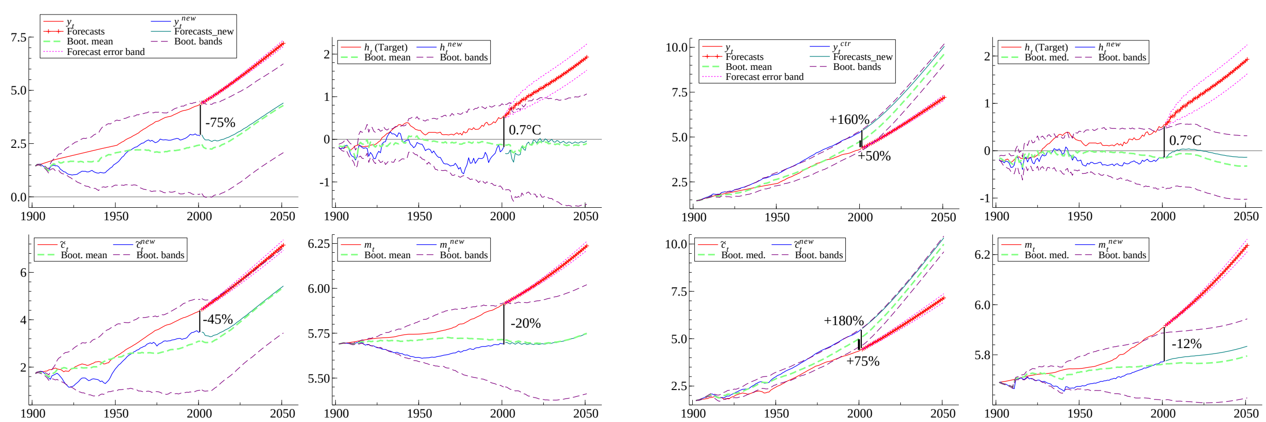

Figure 3: Counterfactual cost of a policy starting in 1900 aiming at controling global temperatures and rendering them stationary around a long run mean equal to their 1900 level (as it is known now). Panel (a): the control is $y_t + \tilde{c}_t/2$. Panel (b): the control is m_t.

abatement policies need to be assessed. This constitutes one step into statistical analyses of the feasibility of temperature control, and cost assessment of such policies.

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