

Exploring the Macroeconomic Interdependence of East Asian Countries: A GVAR Approach ^{*}

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Abstract

Over the past few decades, with rapid economic growth, East Asia has also experienced deepening economic integration with active intraregional economic activities such as trade and investments. This development has contributed to the complex and possibly time-varying macroeconomic interdependence of countries in the region. To understand this interdependence more fully, this study aims to employ the Global Vector Autoregressive (GVAR) model. This model is useful because it can comprehensively capture the interdependence of domestic macroeconomies in the region and quantitatively describe the impact of shocks that occur in East Asian countries on the macroeconomies of other countries. By incorporating changes in trade relations over time, the model can capture the temporal evolution of shock propagation through supply chains. Furthermore, the idea of connectedness measures, developed by Diebold and Yilmaz (2009), will be applied to quantify changes in the interdependence of East Asian countries over the past 40 years.

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1 Introduction

In recent years, there has been an increase in economic interdependence between countries. This can be attributed to various factors, such as the expansion of trade integration, the creation of global supply chains, the growth of international financial markets, and the implementation of coordinated fiscal and monetary policies by national governments and central banks. As a result, numerous empirical studies have examined the connection between international business cycles, including noteworthy works from authors such as Duval et al. (2016), Di Giovanni et al. (2017), Davis (2014), and Chiquiar and Ramos-Francia (2005). Various empirical analysis methods have been employed, such as the pairwise correlation of GDPs by Backus et al. (1995) and Baxter (1995), and the dynamic latent factor models proposed by Kose et al. (2003). In recent years, the measure of connectedness proposed by Diebold and Yilmaz (2015) has also become a powerful analytical method.

This study presents a novel analytical approach that builds upon Diebold and Yilmaz (2015), an influential contribution to the field. Our approach combines the GVAR model suggested by Pesaran et al. (2004) with the connectedness measure proposed by Diebold and Yilmaz (2014) to derive a new index from at least three viewpoints.

Our methodology has the advantage of expanding the number of countries that can be analyzed. With the growth of numerous emerging economies and the intricate blend of economies in close geographic proximity, it is advantageous to widen the analysis scope. To examine the connection between business cycles among the G7 countries (excluding Canada), Diebold and Yilmaz (2015) estimated the six-variable VAR model using monthly industrial production indices for the six nations, and computed a connectedness. However, the curse of dimensionality prevents the application of their approach when the sample size in the time dimension is limited (as data are only available quarterly) or when the sample size in the cross-sectional dimension is extensive (with an increasing number of countries). To address this issue, this study utilizes a GVAR model.*¹.

*¹ See Binder and Soofi-Siavash (2017) for a combined analysis of Diebold-Yilmaz's connectedness index and the GVAR model; Their paper is available at <https://drive.google.com/file/d/16NAw89kPCD4XbLoSIImHTz8veKMnqG1IA/view>

The second feature involves quantifying business cycle linkages and determining the direction of impact between different levels of units. While the FEVD has been commonly used in previous GVAR-related studies, the application of connectedness measures by Diebold and Yilmaz (2014) provides insight into unexplored interrelationships.

The third objective is introducing a novel approach to computing time-varying connectedness measures. To create a time-varying measure of connectedness, Diebold and Yilmaz (2014) uses rolling-sample estimation. While this approach is preferable for maintaining objectivity in the analysis^{*2}, it proves ineffective when dealing with a large number of sample countries and only quarterly data is available, resulting in a small sample size in the time direction as in this study. Thus, this study employs a time-varying weighted formulation, one of the GVAR's features. Specifically, we include changes in trade dependence. This formulation has been extensively utilized in other GVAR empirical applications. Moreover, this specification is suitable for this study since several previous studies have indicated that the intensity of the trade connection in business cycle synchronization is linked to the synchronization between domestic and foreign GDP (Frankel and Rose (1998), Inklaar et al. (2008), and Rana et al. (2012)).

As an example of this innovative approach, we investigate the economic interdependence in East Asia. The rationale for examining this region is that, as noted by Vu (2015), it distinguishes from the Latin American and African regions in that intra-regional trade and investment are thriving and, furthermore, the economies of each country are firmly connected through production relationships of intermediate goods. Therefore, this study is considered an exceptional case study for the analytical method proposed. Additionally, economic relations among East Asian countries have experienced significant transformations over the past four decades. In the 1980s, following the Plaza Accord, economic interdependence deepened, and supply chains were established via the overseas expansion of Japanese firms. Around 2000, the prominence of the Chinese economy grew, further transforming economic relations within the region. Therefore, analyzing the East Asian economies is an appropriate case study for

^{*2} As a reason for using rolling-sample estimation, they wrote "Our goal was always the empirical description of connectedness and its evolution, "getting the facts straight" with minimal assumptions." See Diebold and Yilmaz (2023).

the proposed analytical approach.

In the following, Section 2 explains how to construct the connectedness measures from the GVAR model. Section 3 describes the data. In particular, changes in the interrelationships of trade networks are important, and we provide a descriptive statistical overview of this point. Section 4 presents the estimation results and the various connectedness measures calculated, and Section 5 summarizes the results.

2 Connectedness Measures and GVAR Model

This section first presents the definitions of the various connectedness measures and then explains how to calculate the FEVD, which is essential information for their calculation.

2.1 Measures of Connectedness

Table 1 is a conceptual representation of the connectedness table in Diebold and Yilmaz (2014). The central part of this table consists of H -step-ahead forecast error variance decomposition matrix $D^H = [d_{ij}^H]$ computed from the VAR model and others. By d_{ij}^H , we denote the fraction of variable i 's H -step-ahead forecast error variance due to shocks in variable j . In addition, the connectedness table extends D^H with the rightmost column containing the row sums, the bottom row containing the column sums, and the bottom right element containing the grand mean in all cases of $i \neq j$.

Based on this information, Diebold and Yilmaz (2014) defines several connectedness measures, broadly categorized, that capture the connectedness of individuals to individuals, individuals to the rest of the whole, and connectedness as a whole. Below, we review the definitions in turn.

The first indicator expresses the connectedness between country i and country j and is the most basic connectedness measure. The “(gross) pairwise directional connectedness” from country j to country i is defined as

$$C_{i \leftarrow j}^H = d_{ij}^H \quad (1)$$

In general, $C_{i \leftarrow j}^H \neq C_{j \leftarrow i}^H$, so there are $N^2 - N$ pairwise directional connectedness. Then, from the asymmetry $C_{i \leftarrow j}^H \neq C_{j \leftarrow i}^H$, we use these to define “net pairwise directional connectedness”

Table 1: Connectedness Table Schematic

	x_1	x_2	\cdots	x_N	From Others to i
x_1	d_{11}^H	d_{12}^H	\cdots	d_{1N}^H	$C_{1\leftarrow G}^H$
x_2	d_{21}^H	d_{22}^H	\cdots	d_{2N}^H	$C_{2\leftarrow G}^H$
\vdots	\vdots	\vdots	\ddots	\vdots	\vdots
x_N	d_{N1}^H	d_{N2}^H	\cdots	d_{NN}^H	$C_{N\leftarrow G}^H$
To Others from j	$C_{G\leftarrow 1}^H$	$C_{G\leftarrow 2}^H$	\cdots	$C_{G\leftarrow N}^H$	C_G^H

Notes: Authors' modification based on Table 1 in Diebold and Yilmaz (2014). d_{ij}^H denotes the fraction of variable i 's H -step-ahead forecast error variable due to shocks in variable j . Hence, the H -step-ahead total directional connectedness from others to i is given by $C_{i\leftarrow G}^H = 100 \cdot (\sum_{j \in G, j \neq i} d_{ij}^H) / (\sum_{j \in G} d_{ij}^H)$, and the H -step-ahead total directional connectedness to other from j is given by $C_{G\leftarrow j}^H = 100 \cdot (\sum_{i \in G, i \neq j} d_{ij}^H) / (\sum_{i \in G} d_{ij}^H)$.

as follows

$$C_{ij}^H = C_{j\leftarrow i}^H - C_{i\leftarrow j}^H \quad (2)$$

In this study, this measure shows the net effect of shocks occurring in each of the two specific countries on the GDP forecast errors of the other country.

The second measure is the connectedness between country j and other countries. We now define "total directional connectedness from others to i " as follows.

$$C_{i\leftarrow G}^H = 100 \cdot \frac{\sum_{j \in G, j \neq i} d_{ij}^H}{\sum_{j \in G} d_{ij}^H} \quad (3)$$

This can be calculated for each country, so there are N possible values. Then, as an inverse relationship, the "total directional connectedness to others from j " is

$$C_{G\leftarrow j}^H = 100 \cdot \frac{\sum_{i \in G, i \neq j} d_{ij}^H}{\sum_{i \in G} d_{ij}^H} \quad (4)$$

This also exists in N ways. Based on these two types of connectedness measures, we define "net total directional connectedness" of i as follows:

$$C_{i,G}^H = C_{G\leftarrow i}^H - C_{i\leftarrow G}^H \quad (5)$$

This indicator shows whether, on a net basis, i countries have more impact on other countries or receive more impact from other countries.

The third indicator is the "total connectedness" of the entire sample. It is the sum of the off-diagonal elements of D^H divided by the sum of all elements of D^H .

$$C_G^H = 100 \cdot \frac{\sum_{i,j \in G, i \neq j} d_{ij}^H}{\sum_{i,j \in G} d_{ij}^H} \quad (6)$$

This paper proposes two new indicators. These are indicators that capture connectivity within a region.*³ The first is the connectedness of country i to the other countries in a group. This is a similar indicator to total directional connectedness but differs in that the countries that make up the others are the others in the group, not the whole of the others. The purpose here is to measure the possibility that the connectedness of country i to the whole is different from that of a particular group of countries when looking at the connectedness of country i to the rest of the group.

We first define “total directional connectedness from others in group R to i ” as:

$$C_{i \leftarrow R}^H = 100 \cdot \frac{\sum_{j \in R, j \neq i} d_{ij}^H}{\sum_{j \in R} d_{ij}^H} \quad (7)$$

This can be calculated for each country in group R . Similarly, “total directional connectedness to others in group R from j ” can be defined. Using these, we define “net pairwise within-group R directional connectedness” as

$$C_{i,R}^H = C_{R \leftarrow i}^H - C_{i \leftarrow R}^H. \quad (8)$$

The second new indicator is the total connectedness within group R , which is the sum of the non-diagonal elements of D^H in group R divided by the sum of all elements of D^H also in group R .

$$C_R^H = 100 \cdot \frac{\sum_{i,j \in R, i \neq j} d_{ij}^H}{\sum_{i,j \in R} d_{ij}^H} \quad (9)$$

In the following, we use these indicators to analyze connectedness among East Asian countries.

2.2 Specification of GVAR Model

The GVAR model comprises VARX* models estimated individually for each economy. These VARX* models are essentially VAR models that contain distinct exogenous variables (also known as star variables, denoted by *), where X stands for the exogenous variable. An explanation of the star variables is provided below.

In this initial study, we assume the domestic variables to comprise only real GDP. In contrast, a typical empirical analysis within the GVAR framework assumes a VARX* model

*³ This is similar to the way the 150 bank connections were analyzed by Demirer et al. (2018), which aggregated the D matrix by country to provide an overview of the relationship.

consisting of approximately five domestic variables and an equivalent number of foreign variables for each of over 30 economies. A more recent and common approach is to include an international commodity market model in the system. The formulation used in this study is ARX instead of VARX, strictly speaking. However, for future expandability purposes, we shall refer to it as GVAR and further elaborate on it below.^{*4}

The present ARX* model represents the dynamic behavior of GDP x_{it} in the economy of country i

$$x_{it} = a_{i0} + a_{i1}t + \phi_{i1}x_{i,t-1} + \phi_{i2}x_{i,t-2} + \lambda_{i0}x_{it}^* + \lambda_{i1}x_{i,t-1}^* + \psi_{i0}\omega_t + \psi_{i1}\omega_{t-1} + u_{it} \quad (10)$$

where $a_{i0}, a_{i1}, \phi_{i1}, \phi_{i2}, \lambda_{i0}, \lambda_{i1}, \psi_{i0}, \psi_{i1}$ represents the regression coefficients. The variable t denotes the time trend, x_{it}^* denotes the foreign GDP, ω_t represents the crude oil price, and u_{it} is the error term. Now define the following for $i = 1, \dots, N$.

$$\Theta_i = \{a_{i0}, a_{i1}, \phi_{i1}, \phi_{i2}, \lambda_{i0}, \lambda_{i1}, \psi_{i0}, \psi_{i1}\}, \quad E(u_{it}^2) = \sigma_{u_i}^2$$

In VAR models, which are frequently employed in empirical analyses of macroeconomic policy, the model is typically formulated solely with the domestic macroeconomic variable x_{it} or with the addition of the oil price as a foreign factor. In contrast, the GVAR is distinguished by the incorporation of the star variable x_{it}^* , enabling the examination of how domestic shocks spread to foreign countries or affect domestic macroeconomic variables due to shocks from other countries. Given the economic susceptibility of small open economies like East Asian nations, accounting for fluctuations originating abroad in the domestic economy is imperative. As such, the GVAR serves as an effective analytical framework for examining the region's economies.

Next, we also consider the ARX* model to explain the behavior of the oil price ω_t

$$\omega_t = \mu_0 + \mu_1t + \phi_1\omega_{t-1} + \phi_2\omega_{t-2} + \lambda_1\tilde{x}_{t-1} + \eta_t. \quad (11)$$

Here, ω_t represents the oil price, and the coefficients of the VAR are denoted by μ_0, μ_1, ϕ_1 , and λ_1 , while η_t signifies the error vector. Additionally, \tilde{x}_{t-1} serves as a feedback variable for

^{*4} In this section, we assume that the lags of home GDP extend up to two periods, while the lags of foreign GDP and oil prices extend up to one period. This simplifies the explanation; however, it is important to note that selecting the appropriate lag order is also a crucial aspect of estimation.

capturing exogenous factors that might impact commodity price changes. As in the country models, the following is defined here.

$$\Theta_{du} = \{\mu_0, \mu_1, \phi_1, \phi_2, \lambda_1\}, \quad E(\eta_t^2) = \sigma_\eta^2.$$

The GVAR model includes a feedback variable, \tilde{x}_t . This variable allows for the consideration of global economic fluctuations on international commodity markets, using world GDP based on the GDP of the country analyzed. Endogenous treatment of oil prices in the domestic model is feasible for large open economies such as the US economy by including oil prices. However, the oil price is primarily an externally determined factor for small open economies such as those in East Asia. In this regard, the initial generation of GVAR models considered the oil price an endogenous variable in the US economic model, while in the VAR models of other small open economies, it was deemed exogenous. However, when examining the potential for rapid expansion of the Chinese economy during the analysis period and its effect on global commodity prices, such as oil prices, it is uncertain whether treating commodity prices as an endogenous variable solely in the US model is valid. Therefore, in the second-generation GVAR model, the international market block, encompassing crude oil prices, is depicted by an independent VAR model. A variable designating global factors is further included to render international commodity prices an endogenous variable in the system's entirety.

2.3 Derivation of Generalized FEVD

In the following, GFEVD is derived from Eq.(10) and Eq.(11). Initially, a variable vector \mathbf{z}_{it} including home GDP x_{it} and foreign GDP x_{it}^* for country i is defined.

$$\mathbf{z}_{it} = \begin{pmatrix} x_{it} \\ x_{it}^* \end{pmatrix}$$

Furthermore, the ARX* model for country i described in Eq.(10) can be expressed in the following manner.

$$\mathbf{G}_{i0}\mathbf{z}_{it} = a_{i0} + a_{i1}t + \mathbf{G}_{i1}\mathbf{z}_{i,t-1} + \mathbf{G}_{i2}\mathbf{z}_{i,t-2} + \psi_{i0}\omega_t + \psi_{i1}\omega_{t-1} + u_{it} \quad (12)$$

where $\mathbf{G}_{i0} = (1, -\lambda_{i0})$, $\mathbf{G}_{i1} = (\phi_{i1}, \lambda_{i1})$, and $\mathbf{G}_{i2} = (\phi_{i2}, \lambda_{i2})$.

Then, two identities between \mathbf{z}_{it} , \tilde{x}_t , and $\mathbf{x}_t = (x_{1t}, \dots, x_{Nt})'$ is established by means of the link matrix \mathbf{W}_i and $\widetilde{\mathbf{W}}$ outlined in Section 3.

$$\mathbf{z}_{it} = \mathbf{W}_i\mathbf{x}_t, \quad \tilde{x}_t = \widetilde{\mathbf{W}}\mathbf{x}_t \quad (13)$$

Then, Eq.(12) is rewritten as

$$\mathbf{G}_{i0}\mathbf{W}_i\mathbf{x}_t = a_{i0} + a_{i1}t + \mathbf{G}_{i1}\mathbf{W}_i\mathbf{x}_{t-1} + \mathbf{G}_{i2}\mathbf{W}_i\mathbf{x}_{t-2} + \psi_{i0}\omega_t + \psi_{i1}\omega_{t-1} + u_{it},$$

So, if we stack up the ARX models for N countries, we obtain

$$\mathbf{G}_0\mathbf{x}_t = \mathbf{a}_0 + \mathbf{a}_1t + \mathbf{G}_1\mathbf{x}_{t-1} + \mathbf{G}_2\mathbf{x}_{t-2} + \Psi_0\omega_t + \Psi_1\omega_{t-1} + \mathbf{u}_t. \quad (14)$$

where the corresponding relevant coefficient matrices are defined below.

$$\mathbf{a}_j = \begin{pmatrix} a_{1j} \\ \vdots \\ a_{Nj} \end{pmatrix}, \mathbf{G}_j = \begin{pmatrix} \mathbf{G}_{1j}\mathbf{W}_1 \\ \vdots \\ \mathbf{G}_{Nj}\mathbf{W}_N \end{pmatrix}, \Psi_j = \begin{pmatrix} \psi_{1j} \\ \vdots \\ \psi_{Nj} \end{pmatrix}, \mathbf{u}_t = \begin{pmatrix} u_{1t} \\ \vdots \\ u_{Nt} \end{pmatrix}.$$

The ARX* model for oil prices can also be reformulated through the use of a link matrix.

$$\omega_t = \mu_0 + \mu_1t + \phi_1\omega_{t-1} + \phi_2\omega_{t-2} + \lambda_1\widetilde{\mathbf{W}}\mathbf{x}_{t-1} + \eta_t. \quad (15)$$

Next, we define variable vector \mathbf{y}_t as:

$$\mathbf{y}_t = \begin{pmatrix} \mathbf{x}_t \\ \omega_t \end{pmatrix}.$$

With this, the global model combining Eq.(15) and Eq.(14) can be formulated as follows.

$$\mathbf{H}_0\mathbf{y}_t = \mathbf{h}_0 + \mathbf{h}_1t + \mathbf{H}_1\mathbf{y}_{t-1} + \mathbf{H}_2\mathbf{y}_{t-2} + \zeta_t \quad (16)$$

where

$$\mathbf{H}_0 = \begin{pmatrix} \mathbf{G}_0 & -\Psi_0 \\ \mathbf{0} & 1 \end{pmatrix}, \mathbf{h}_0 = \begin{pmatrix} \mathbf{a}_0 \\ \mu_0 \end{pmatrix}, \mathbf{h}_1 = \begin{pmatrix} \mathbf{a}_1 \\ \mu_1 \end{pmatrix}, \mathbf{H}_1 = \begin{pmatrix} \mathbf{G}_1 & \Psi_1 \\ \lambda_1\widetilde{\mathbf{W}} & \phi_1 \end{pmatrix}, \mathbf{H}_2 = \begin{pmatrix} \mathbf{G}_2 & \mathbf{0} \\ \mathbf{0} & \phi_2 \end{pmatrix}, \zeta_t = \begin{pmatrix} \mathbf{u}_t \\ \eta_t \end{pmatrix}.$$

Note that we assume the covariance matrix of ζ_t to be diagonal. We then multiply both sides of Eq.(16) by \mathbf{H}_0^{-1} from the left to derive the first-order autoregressive expression for \mathbf{y}_t .

$$\begin{aligned} \mathbf{y}_t &= \mathbf{H}_0^{-1}\mathbf{h}_0 + \mathbf{H}_0^{-1}\mathbf{h}_1t + \mathbf{H}_0^{-1}\mathbf{H}_1\mathbf{y}_{t-1} + \mathbf{H}_0^{-1}\mathbf{H}_2\mathbf{y}_{t-2} + \mathbf{H}_0^{-1}\zeta_t \\ &= \mathbf{c}_0 + \mathbf{c}_1t + \mathbf{C}_1\mathbf{y}_{t-1} + \mathbf{C}_2\mathbf{y}_{t-2} + \epsilon_t. \end{aligned} \quad (17)$$

The moving average expression corresponding to Eq.(17) is

$$\mathbf{y}_t = \mathbf{d}_t + \sum_{s=0}^{\infty} \mathbf{B}_s\epsilon_{t-s} = \mathbf{d}_t + \mathbf{B}_0\epsilon_t + \mathbf{B}_1\epsilon_{t-1} + \mathbf{B}_2\epsilon_{t-2} + \dots \quad (18)$$

where \mathbf{d}_t is the deterministic components, and the coefficient matrices \mathbf{B} are recursively defined as

$$\mathbf{B}_s = \begin{cases} \mathbf{C}_1\mathbf{B}_{s-1} + \mathbf{C}_2\mathbf{B}_{s-2} & s = 1, 2, \dots \\ \mathbf{I} & s = 0 \\ \mathbf{0} & s < 0 \end{cases}.$$

Therefore, for instance, the coefficient of impulse response H terms ahead to a shock to ζ_t can be calculated using the formula $\mathbf{B}_h \mathbf{H}_0^{-1}$.

Lastly, the Generalised FEVD is calculated as follows.

$$d_{ij}^H = \frac{(\sigma_{jj})^{-1} \sum_{h=0}^H (\mathbf{e}_i' \mathbf{B}_h (\mathbf{H}_0)^{-1} \boldsymbol{\Sigma}_\zeta \mathbf{e}_j)^2}{\sum_{h=0}^H \mathbf{e}_i' \mathbf{B}_h (\mathbf{H}_0)^{-1} \boldsymbol{\Sigma}_\zeta ((\mathbf{H}_0)^{-1})' (\mathbf{B}_h)' \mathbf{e}_i} \quad (19)$$

This shows us how much a shock occurring in the j -th variable affects the H -step-ahead forecast error variance of the i -th variable. \mathbf{e}_i is the selection vector whose i th element is 1 and the rest are 0, and $\sigma_{jj}^{(s)}$ is the variance of the disturbance term in the j th expression (or the j th diagonal element of $\boldsymbol{\Sigma}_\zeta^{(s)}$). Note that in GFEVD, the shocks are not orthogonalized, so the sum of the relative variance contribution (RVC) of the forecast error variance is not necessarily equal to 1. Therefore, the following standardization is used.

$$\tilde{d}_{ij}^g = \frac{d_{ij}^g}{\sum_{j=1}^N d_{ij}^g} \quad (20)$$

In the following, GFEVD is denoted by d_{ij} to simplify notation, but all connectedness measures in this study are calculated using the standardized value $\tilde{D}^H = [\tilde{d}_{ij}^H]$.

3 Data

In this study, real GDP by country (log-transformed values) and oil prices (also log-transformed values) from the Mohaddes and Raissi (2020) data set are used in the analysis.*⁵ Specifically, 10 countries from East Asia and the Pacific region: Australia (AUS), China (CHN), Indonesia (IDN), Japan (JPN), Korea (KOR), Malaysia (MYS), New Zealand (NZL), Philippines (PHL), Singapore (SGP), Thailand (THA); 13 countries from Europe and Central Asia region: Austria (AUT), Belgium (BEL), Finland (FIN), France (FRA), Germany (DEU), Italy (ITA), Netherlands (NLD), Norway (NOR), Spain (ESP), Sweden (SWE), Switzerland (CHE), Turkey (TUR), United Kingdom (GBR); five countries from the Latin American region: Argentina (ARG), Brazil (BRA), Chile (CHL), Mexico (MEX), Peru (PER); 2 countries from North American region: Canada (CAN), United States (USA). The other three countries

*⁵ Mohaddes, K. and M. Raissi (2020). Compilation, Revision and Updating of the Global VAR (GVAR) Database, 1979Q2-2019Q4. University of Cambridge: Judge Business School (mimeo). <https://www.mohaddes.org/gvar>. The data set covers the period from the fourth quarter of 1979 to the fourth quarter of 2019. In addition to the data used in this paper, the data set also includes the long-term interest rate, stock prices, and the bilateral exchange rate against the dollar.

are: Saudi Arabia (SAU) in the Middle East and North Africa region; India (IND) in South Asia; and South Africa (ZAF) in Sub-Saharan Africa. Total number of countries is 33.

The foreign GDP x_{it}^* was calculated using the matrix \mathbf{W}_i , which depicts the relations between countries. This matrix was derived from the annual trade flow data provided by Mohaddes and Raissi (2020).^{*6} Based on annual trade flow data, the linkage coefficient w_{ij} between countries i and j is calculated as follows.

$$w_{ij} = \frac{\text{Trade volume between country } i \text{ and country } j}{\sum_{k=1}^N \text{Trade volume between country } i \text{ and country } k} \quad (21)$$

where trade volume is the sum of annual exports and imports between the two countries. For example, foreign GDP for Japan is defined in the following way.

$$x_{jpn,t}^* = \sum_{j=1, j \neq jpn}^{33} w_{jpn,j} x_{j,t} \quad (22)$$

By the way, it is widely acknowledged that China's economy has experienced significant growth over the last 30 years, resulting in substantial alterations in its economic partnerships with neighboring countries, specifically those in East Asia. Figure 1 displays individual countries' linkage coefficients and GDP at four selected time points from the sample period, namely, 1985, 1995, 2005, and 2015. The circle's center depicts the USA, while the circles on the circumference represent 32 sample countries excluding the USA. Each circle's size corresponds to the respective economy's size, while the arrows indicate the influencing relationships. Arrows with lower than 20.5 percent linkage coefficient values have been disregarded to emphasize the more critical relationships.

From Figure 1, it is evident that the trade relationship, which is measured by the linkage coefficient as defined in Eq.(21), has been dynamically evolving over time. To account for these changes, \mathbf{W}_i in Eq.(13) is altered to be time-varying. This is done to accurately capture the modifications occurring in the trade relationship over time. The linkage coefficient is computed based on the previous year's trade volume. In accordance with this, the weighted coefficient of foreign GDP changes annually and the GFEVD, as defined in Eq.(C.3), also

^{*6} Values for the three most recent years were updated using rates of change calculated from annual export and import values (in U.S. dollars) obtained from the latest Direction of Trade Statistics (DOTS). Because the interest of this study is real economic interrelationships rather than financial ones, and because of the availability of data, trade volume data were employed. Another approach is to use capital investment and asset portfolio data.

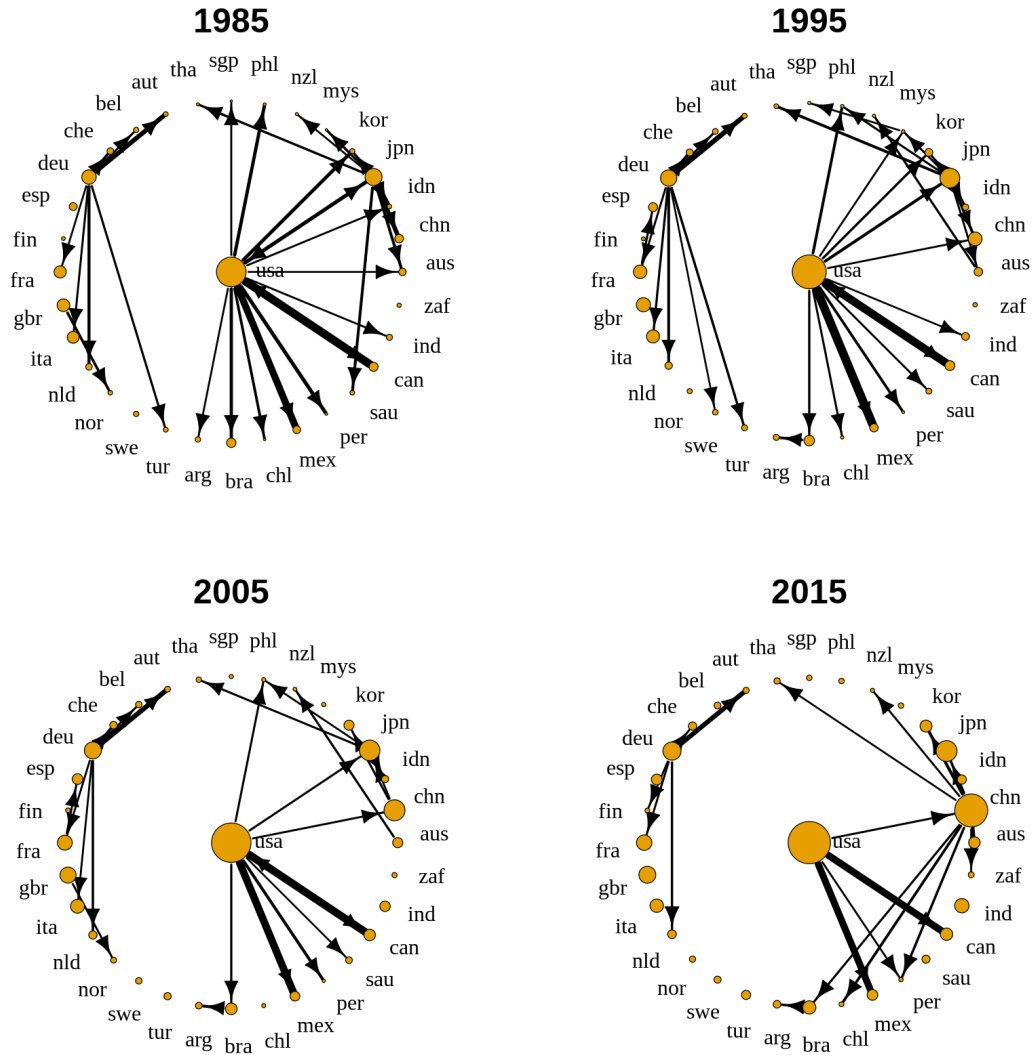


Fig. 1: Trade networks

Note: See Table A.1 in Appendix for country codes. The circles on the circumference indicate the size of the economy based on real GDP, and arrows are marked if the relevance calculated from trade flows is 20.5% or more, while the thickness of the arrows indicates the strength of the relevance.

fluctuates to reflect diverse evaluation points in time. Thus, measures of connectedness will also vary over time.

Finally, the feedback variable $\tilde{\mathbf{x}}_t$ in the oil price model is a weighted average of each country's log-transformed GDP value. The weights to determine the average (represented by the $\tilde{\mathbf{W}}$ matrix in Eq.(13)) were calculated using the share obtained from the 2014-2016 average of nominal GDP in PPP (in current international \$) for each respective country, as reported by

the World Bank's WDI. This variable serves as a proxy for global business cycle fluctuations and covers aspects of oil demand. Examining crude oil supply factors is also valuable for price fluctuations, but this study does not include them.

4 Results

4.1 Estimation of GVAR Model

Estimation is conducted on a model-by-model basis, following the standard VAR model. The number of lags for home and foreign variables included in the model is set at $p = 4$ and $q = 4$, respectively. In the case of the oil price model, the number of lags is also set at four periods for both the own and feedback variables.

When multiple countries are analyzed simultaneously, the presence of outliers can cause the system to become unstable and divergent. The sample period encompasses the Asian currency crisis, the substantial devaluation of currencies in Thailand and other Asian nations during 1997-1998, and the worldwide financial crisis between 2007-2008. Other local shocks include the Severe Acute Respiratory Syndrome (SARS) in China from approximately November 2002 to the first half of 2003, the Great East Japan Earthquake in Japan in March 2011, and the significant flooding in Thailand in the latter half of the same year. These events not only impacted domestic economic activities in each country but also affected supply chains. The concern lies in the impact of these events not just on domestic economic activities in each country but also on the economies of other countries connected through supply chains and other means.

Normally, historical data should be employed to identify outliers. However, due to the vast number of countries covered, statistical criteria were used to identify outliers, and we assigned dummy variables to manage them. Specifically, if the maximum absolute value of the residuals did not fall within three standard deviations of the error variance, dummy variables were allocated to the residuals of the ARX* model computed for each country. This procedure was performed iteratively until all anomalies were identified for each country. The same approach was followed for the model of crude oil prices.

4.2 Various Measures of Business Cycle Connectedness

The connectedness measures presented in this study are computed using generalized forecast error variance decomposition for the next 12 quarters (three years). Estimated tables depicting connectedness in 1985, 1995, 2005, and 2015 are presented in the Appendix.

Let us begin by examining the Own effect, represented by the diagonal element in the Connectedness Table. Table 2 presents the effect of each country in 10-year intervals from 1985. The eight countries with low own effects (1/4 of the total sample) are highlighted for clarity. The table reveals that several European countries have low own effects, which is a comprehensible phenomenon owing to their near economic relations. The influence of the USA also extends to Canada. Notably, the East Asian region is not among the countries highlighted. Peru and Brazil in South America have experienced a noteworthy diminution of home-country effects from 1985 to 2015 (i.e., they have become more vulnerable to foreign influences). Meanwhile, the value of China, Indonesia, and South Korea in East Asia has increased, albeit only slightly. On the contrary, the Philippines and Japan have experienced a decrease of approximately 9 and 5 percent, respectively.

4.2.1 Total Connectedness

Now, we shall assess connectivity. Firstly, let us examine the overall connectivity. Please refer to Figure 2, highlighting the Total Connectedness for all 33 countries considered. The Total Connectedness value decreased from 18% in 1981 to around 15% in 1985. It remained around 15% until approximately 2000, after which it increased and attained over 22% in 2019. For reference, excluding China, the calculation of total connectedness demonstrated a long-term decline over the analyzed period. This implies that the increase in global connectedness could be attributed to the Chinese economy.

Next, let us examine the total connectedness of sub-groups, with the first group being classified based on geographical location. The left panel of Figure 3 illustrates the following three regions: (1) East Asia & Pacific, (2) North America, Latin America & Caribbean, and (3) Europe & Central Asia. The regional variations are evident from Figure 3: whilst Europe & Central Asia has the highest level over the period, it shows a decreasing trend with an

	1985		1995		2005		2015	
	own	rank	own	rank	own	rank	own	rank
ARG	0.755	18	0.752	18	0.756	18	0.759	16
AUS	0.933	1	0.935	2	0.932	2	0.910	4
AUT	0.359	31	0.321	32	0.341	32	0.336	32
BEL	0.299	33	0.280	33	0.317	33	0.320	33
BRA	0.818	13	0.796	13	0.771	15	0.688	18
CAN	0.616	27	0.614	25	0.599	27	0.565	25
CHN	0.825	11	0.834	11	0.846	9	0.840	9
CHL	0.933	2	0.932	3	0.932	3	0.919	3
FIN	0.658	24	0.634	24	0.624	24	0.574	24
FRA	0.596	28	0.574	29	0.593	28	0.562	26
DEU	0.701	20	0.695	20	0.693	19	0.644	19
IND	0.593	29	0.586	28	0.632	22	0.596	22
IDN	0.914	4	0.919	4	0.920	4	0.925	2
ITA	0.357	32	0.330	31	0.357	31	0.369	31
JPN	0.870	8	0.851	9	0.845	10	0.815	10
KOR	0.824	12	0.823	12	0.837	11	0.840	8
MYS	0.665	23	0.667	23	0.668	21	0.603	21
MEX	0.850	10	0.837	10	0.830	12	0.808	12
NLD	0.497	30	0.488	30	0.521	30	0.540	29
NOR	0.890	6	0.887	6	0.892	5	0.873	6
NZL	0.861	9	0.862	8	0.868	7	0.882	5
PER	0.678	22	0.672	22	0.586	29	0.460	30
PHL	0.905	5	0.900	5	0.863	8	0.814	11
ZAF	0.679	21	0.676	21	0.628	23	0.545	28
SAU	0.926	3	0.937	1	0.937	1	0.934	1
SGP	0.886	7	0.876	7	0.880	6	0.855	7
ESP	0.775	17	0.756	17	0.762	17	0.775	15
SWE	0.628	25	0.591	27	0.610	25	0.558	27
CHE	0.786	15	0.759	16	0.769	16	0.745	17
THA	0.816	14	0.788	14	0.815	13	0.794	14
TUR	0.785	16	0.777	15	0.793	14	0.798	13
GBR	0.617	26	0.599	26	0.606	26	0.593	23
USA	0.738	19	0.727	19	0.689	20	0.633	20
poil	0.969		0.966		0.960		0.953	

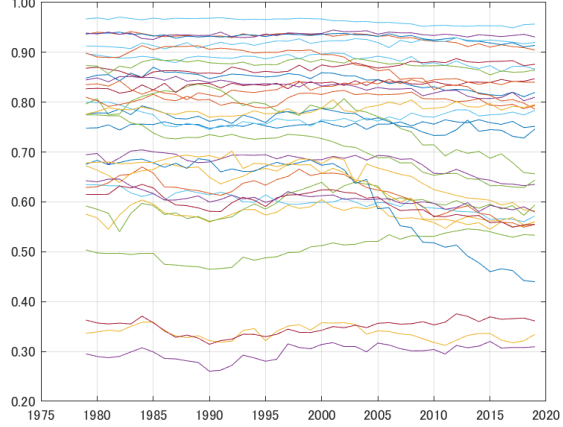


Table 2: Own effect: 1985, 1995, 2005, and 2015

Note: Each year, the eight countries with small home country effect values (corresponding to 1/4 of the total sample) are highlighted.

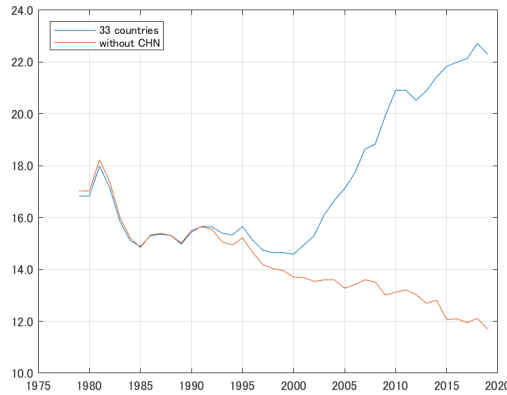


Fig. 2: Total Connectedness

Note: Total Connectedness is calculated as $C_G^H = 100 \cdot (\sum_{i,j \in G, i \neq j} d_{ij}^H) / (\sum_{i,j \in G} d_{ij}^H)$. This calculation excludes the part related to oil prices. The forecasting horizon, H , is set to 12.

amplitude of 20-year cycles. In contrast, North America, Latin America & the Caribbean, and East Asia & the Pacific display lower absolute levels, although the magnitude of East Asia & the Pacific has exhibited a consistent increase since 2000.

How about another grouping: the G7, BRICs, Next 11*7, ASEAN and ASEAN+3? The

*7 The Next Eleven are the eleven countries - Bangladesh, Egypt, Indonesia, Iran, Mexico, Nigeria, Pakistan,

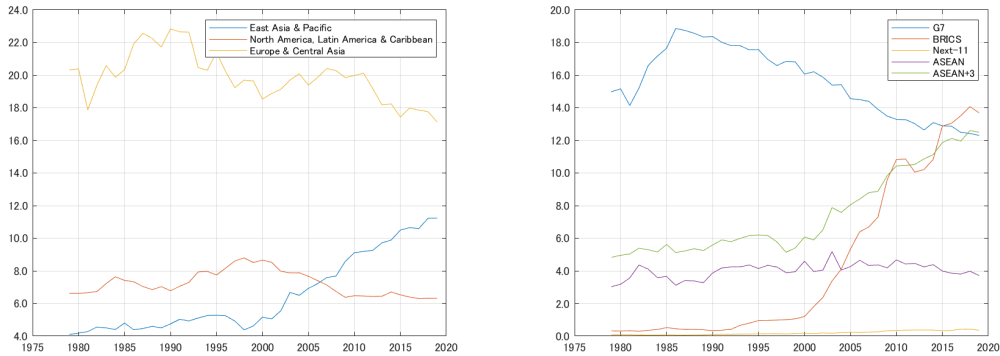


Fig. 3: Total Connectedness within Different Groups

Note: Within-region total connectedness is calculated as $C_R^H = 100 \cdot (\sum_{i,j \in R, i \neq j} d_{ij}^H) / (\sum_{i,j \in R} d_{ij}^H)$. The figure on the left depicts different geographic regions, whilst the figure on the right shows various groups, including G7, BRICS, ASEAN, and Next-11. The forecasting horizon, H , is set to 12.

G7 group peaked in the mid-1980s and experienced a gradual decline until the end of the sample period, ultimately reaching the same level as the other groups. Conversely, the BRICS group displayed a considerable increase starting in 2000, which aligns with the growth of the Chinese economy. On the other hand, the level of total connectedness observed in the Next-11 remains notably low, despite an upward trend that began around 1985. Turning to Asia, the connectedness of the ASEAN countries encompasses the Asian currency crisis in the middle of the sample period. However, no significant changes were observed during this time, and the level has remained stable. Conversely, ASEAN+3, incorporating China, Japan and Korea alongside ASEAN, displayed a considerable decline during the Asian currency crisis, followed by a consistent upward trend.

4.2.2 Total Directional Connectedness

Let us focus on East Asian nations and examine their directional connectedness of business cycles across countries within and beyond the region.

Figure 4 displays the directional connectedness between the 10 East Asian countries in the dataset and the other 32 countries. The graph illustrates the changes over time in terms of “connectedness to others ($C_{G \leftarrow i}^H$)”, “connectedness from others ($C_{i \leftarrow G}^H$)”, and “net connectedness to others (C_i^H)”. Here denoted by the acronym G for Global, meaning the whole

the Philippines, South Korea, Turkey, and Vietnam

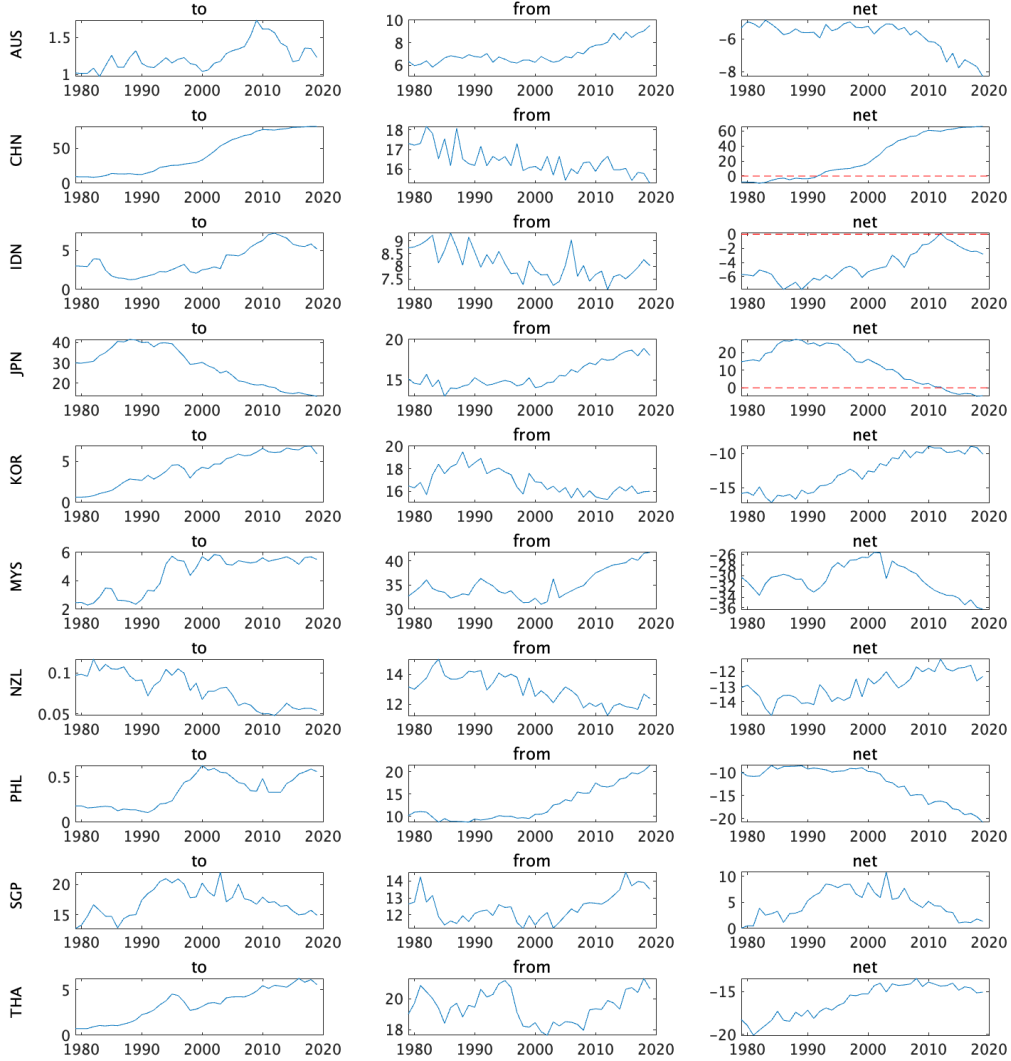


Fig. 4: Directional Connectedness: $C_{G \leftarrow i}^H$, $C_{i \leftarrow G}^H$, and C_i^H

Note: The forecasting horizon, H , is set to 12.

world.

Only three countries (Japan, China and Singapore) displayed positive values for net connectedness across the sample period. This implies that these three countries were net transmitters of business cycle shocks. During the mid-1980s to mid-1990s, Figure 4, column 3 indicates that Japan was responsible for net connectedness. During this period, Japan's gross connectedness with other countries reached approximately 40%, while the connectedness received by Japan from other nations was around 15%, resulting in a net connectedness of up to 25% from Japan. Japan's net connectedness started to decline from mid-1995. Contrarily, China's

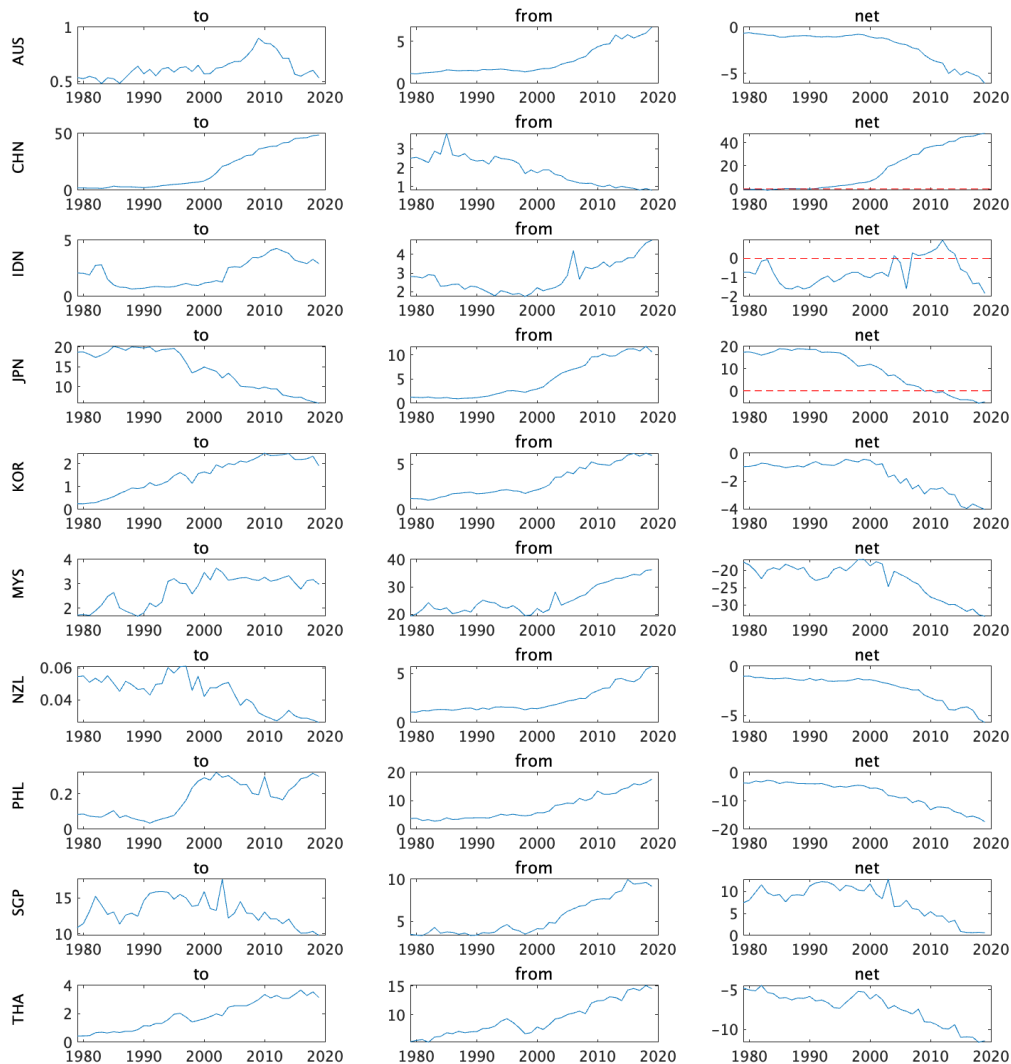


Fig. 5: Directional Connectedness within the East Asian and Pacific Region

Note: The forecasting horizon, H , is set to 12.

influence grew rapidly, reaching 60% by around 2010.

What happens if we limit the measurement of directional connectedness to East Asian countries? Based on the data, there is no significant alteration in the trends observed in Japan and China. However, a few countries show noteworthy variations. Korea, New Zealand, and Thailand are net recipients of business cycle shocks, as net connectedness is negative throughout the sample period. Nonetheless, Figure 4 indicates a tendency for the negative value to reduce towards the latter half of the period, while Figure 5 shows a tendency to increase. This trend may indicate a growing influence of the economies of these three countries

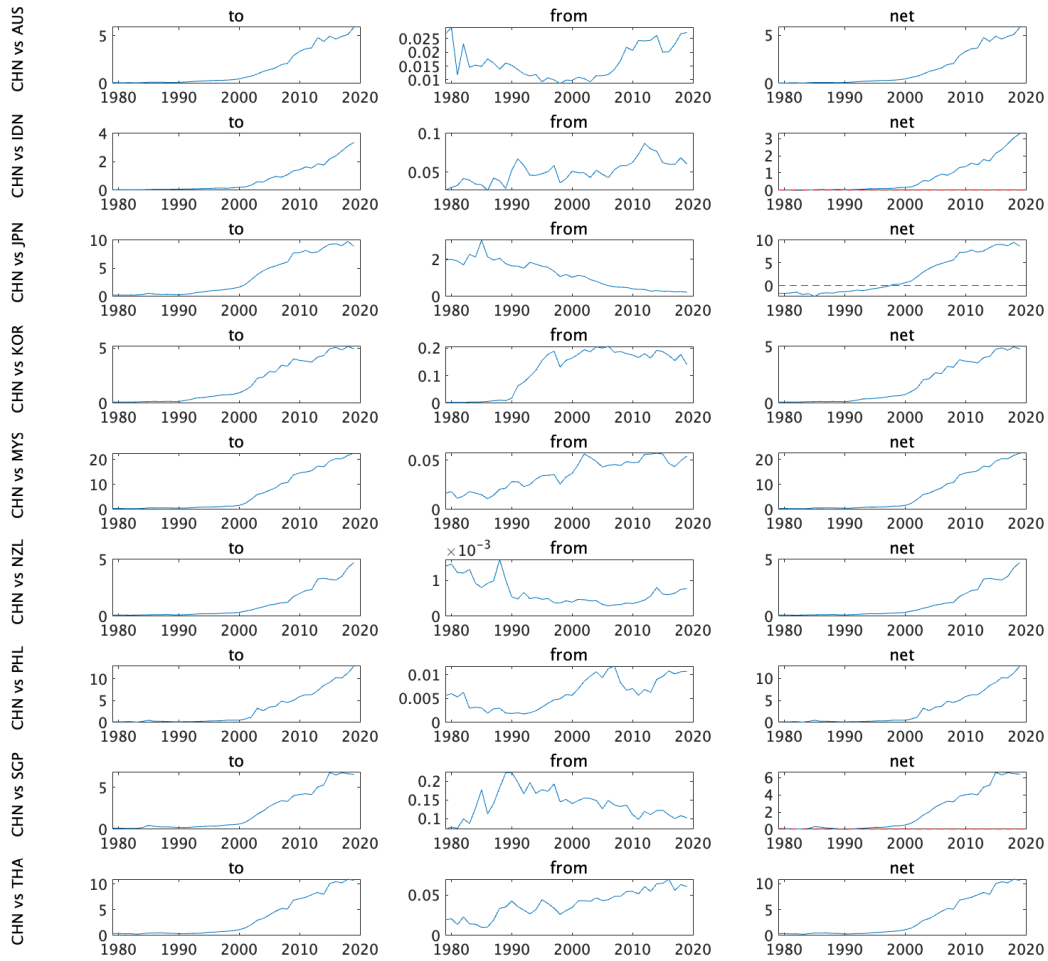


Fig. 6: China’s Pairwise Directional Connectedness within East Asian and Pacific

Note: The forecasting horizon, H , is set to 12.

by the East Asian region.

4.2.3 Pairwise Directional Connectedness

Finally, let us examine the interrelationships among East Asian nations. As this study’s primary focus is on the waning impact of the Japanese economy and the growing influence of the Chinese economy, let us consider China, Japan, and Korea’s relationship with other Asian states. Figure 6 shows the pairwise directional connectedness of China, Figure 7 shows the pairwise directional connectedness of Japan, and Figure 8 shows the pairwise directional connectedness of South Korea.

Figure 6 shows that China has been the net transmitter of business cycle shock for East Asian countries since 2000. According to the latest assessment, Malaysia has more than 20

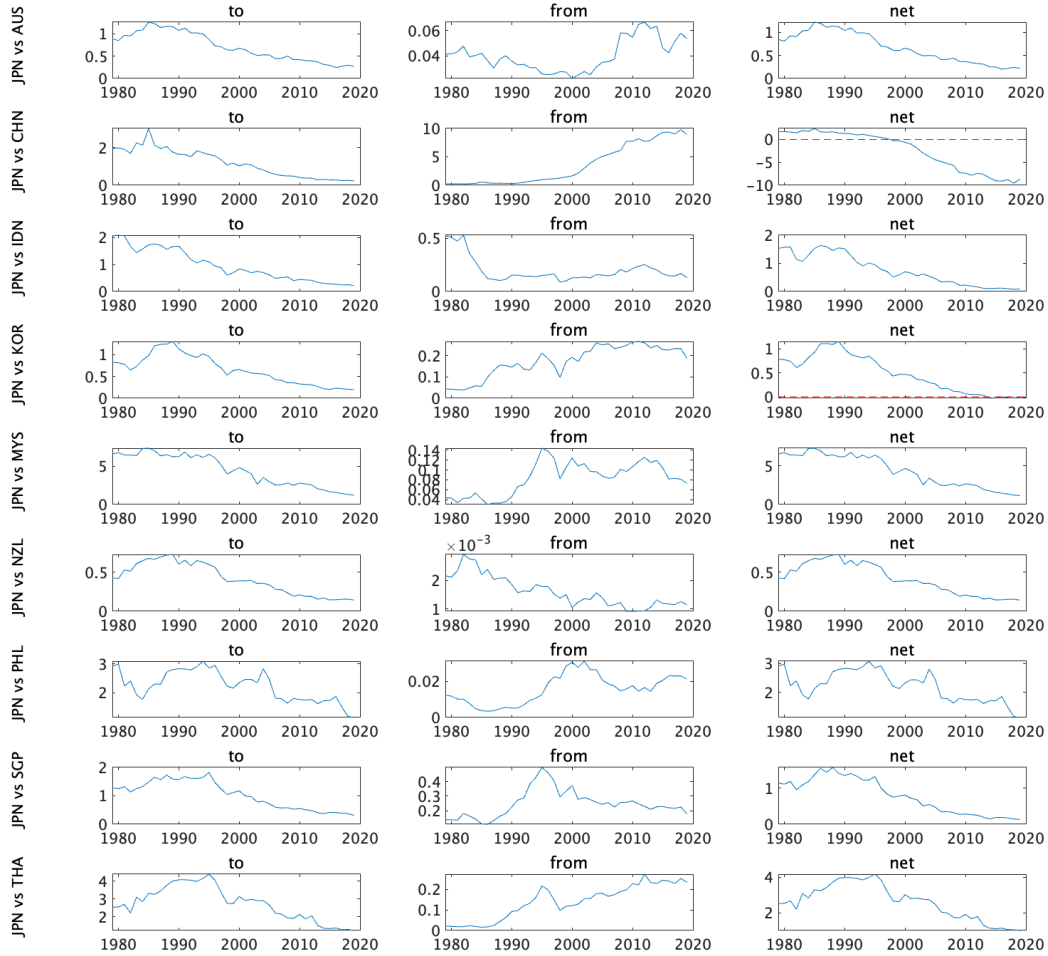


Fig. 7: Japan's Pairwise Directional Connectedness within East Asian and Pacific

Note: The forecasting horizon, H , is set to 12.

percent of the influence from China; Japan, the Philippines, and Thailand have about 10 percent, followed by Singapore and South Korea.

In contrast, Figure 7, which shows the influence of Japan, shows that its role as a transmitter of business cycle shocks is gradually diminishing. It has been a net recipient of China since 2000. However, except for Korea, the other countries remain net transmitters, although their influence has diminished.

Korea's role is more complex: Figure 8 shows that it is mostly a net recipient for China and Japan over the period. However, it is a net transmitter for Australia, Malaysia, New Zealand, the Philippines, and Thailand, and its role for Indonesia and Singapore varies from period to period.

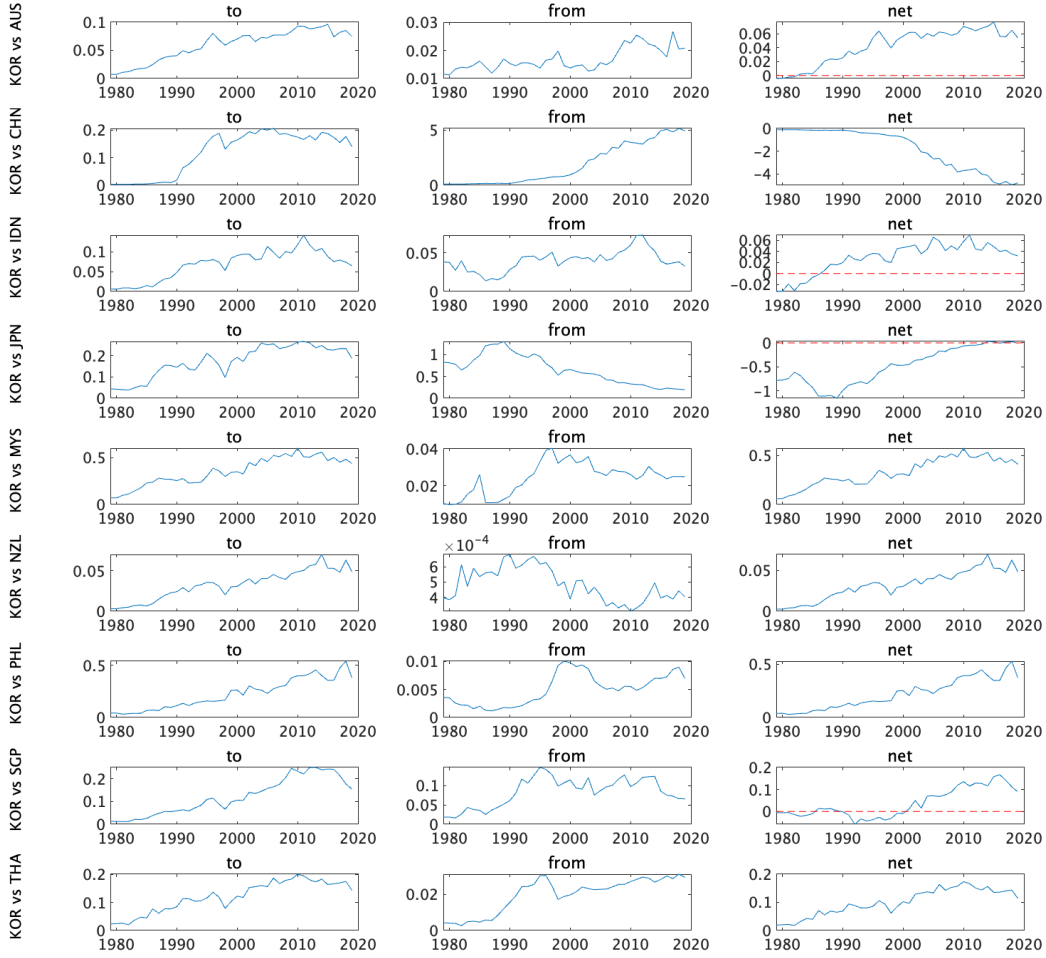


Fig. 8: Korea's Pairwise Directional Connectedness within East Asian and Pacific

Note: The forecasting horizon, H , is set to 12.

Finally, it is essential to mention the relationship between trade relations and connectedness measures. The time-varying connectedness measure proposed in this study is due to using time-varying trade weights for the foreign variables in the GVAR. What is the relationship between the GVAR and the trade weights, which is the most important information in calculating the connectedness measure? In Figure 9, the variable on the vertical axis is the (gross) pairwise directional connectedness from country j to country i , and the horizontal axis is the trade ratio for the country in question. For example, in the second column, the vertical variable is the influence of China on country i ($C_{i \leftarrow CHN}^{12}$), and the horizontal variable is the trade ratio of country i with China in the previous year. A noticeable quadratic correlation exists between the two variables in the second column of Figure 9. However, for other nations, the

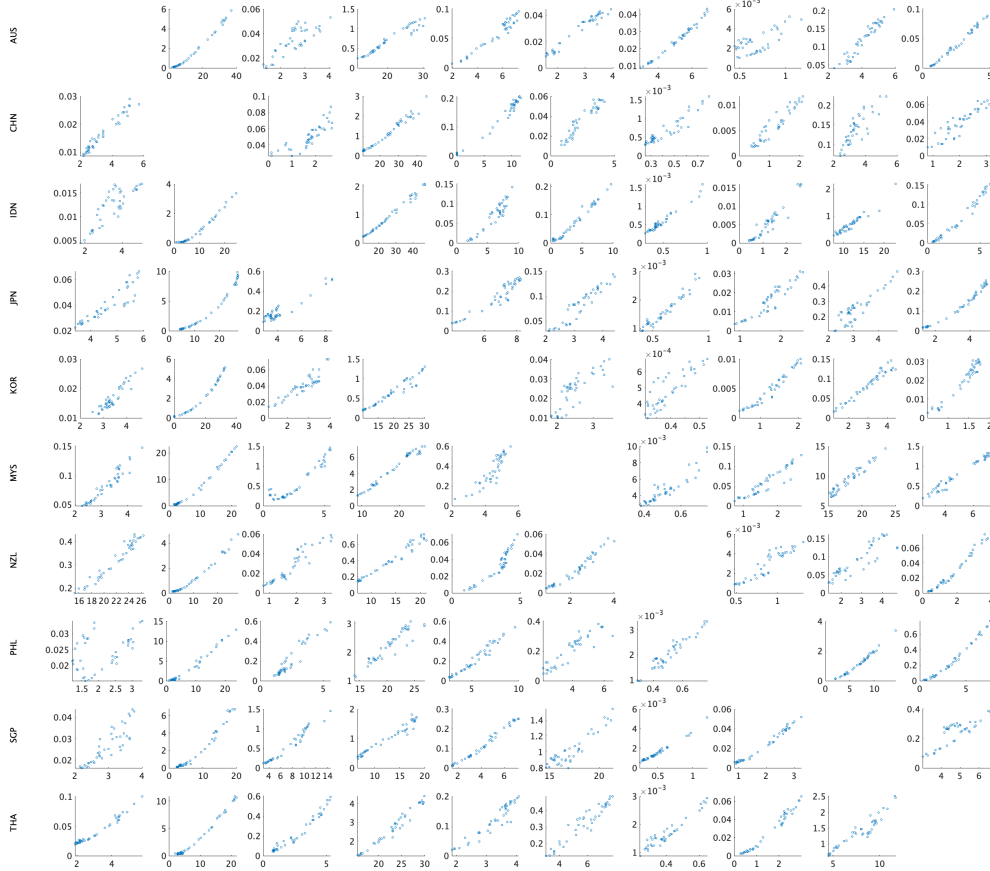


Fig. 9: Pairwise directional connectedness vs trade ratio of the East Asian and Pacific Countries

Note: The variable on the vertical axis is the (gross) pairwise directional connectedness from country j to country i , and the horizontal axis is the trade ratio for the country in question. The forecasting horizon, H , is set to 12.

correlation is frequently uncertain. This phenomenon verifies that the connectedness metric is linked to the trade ratio but doesn't solely rely on information concerning the trade ratio.

5 Conclusion

Over the past few decades, with rapid economic growth, East Asia has also experienced deepening economic integration with active intraregional economic activities such as trade and investments. This development has contributed to the complex and possibly time-varying macroeconomic interdependence of countries in the region.

To understand this interdependence more fully, we present a novel analytical approach that builds upon Diebold and Yilmaz (2015) as a novel contribution to the field. Our approach combines the GVAR model suggested by Pesaran et al. (2004) with the connectedness measure

proposed by Diebold and Yilmaz (2014) to derive a new index which is used to analyze the business cycle shock transmission.

Using quarterly GDP and oil price data for 33 countries from 1979 to 2019, we analyzed the growing interdependence of business cycles and dependencies between regions and countries in East Asia from various perspectives. Notably, the study employed the GVAR model to investigate the flow of stock transmissions, yielding intriguing findings.

One of the remaining concerns is to ensure the results' robustness from varying viewpoints. For this analysis, the forecasting period of the GFEVD was fixed at 12 quarters, and other periods were not investigated. It may also be necessary to consider the formulation of the GVAR model since it can be converted to a differenced VAR or a cointegrating VAR. The impact of the change in the formulation on the results must be analyzed. These issues will be the subject of future work.

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Appendix

A List of Countries

iso3	Code	Country	Region	G7	OECD	ASEAN	N-11
ARG	213	Argentina	Latin America & Caribbean				
AUS	193	Australia	East Asia & Pacific		X		
AUT	122	Austria	Europe & Central Asia		X		
BEL	124	Belgium	Europe & Central Asia		X		
BRA	223	Brazil	Latin America & Caribbean				
CAN	156	Canada	North America	X	X		
CHN	160	China	East Asia & Pacific				
CHL	228	Chile	Latin America & Caribbean		X		
FIN	172	Finland	Europe & Central Asia		X		
FRA	132	France	Europe & Central Asia	X	X		
DEU	134	Germany	Europe & Central Asia	X	X		
IND	534	India	South Asia				
IDN	536	Indonesia	East Asia & Pacific			X	X
ITA	136	Italy	Europe & Central Asia	X	X		
JPN	158	Japan	East Asia & Pacific	X	X		
KOR	542	Korea	East Asia & Pacific		X		X
MYS	548	Malaysia	East Asia & Pacific			X	
MEX	273	Mexico	Latin America & Caribbean		X		
NLD	138	Netherlands	Europe & Central Asia		X		
NOR	142	Norway	Europe & Central Asia		X		
NZL	196	New Zealand	East Asia & Pacific		X		
PER	293	Peru	Latin America & Caribbean				
PHL	566	Philippines	East Asia & Pacific			X	X
ZAF	199	South Africa	Sub-Saharan Africa				
SAU	456	Saudi Arabia	Middle East & North Africa				
SGP	576	Singapore	East Asia & Pacific			X	
ESP	184	Spain	Europe & Central Asia		X		
SWE	144	Sweden	Europe & Central Asia		X		
CHE	146	Switzerland	Europe & Central Asia		X		
THA	578	Thailand	East Asia & Pacific			X	
TUR	186	Turkey	Europe & Central Asia		X		X
GBR	112	United Kingdom	Europe & Central Asia	X	X		
USA	110	United States	North America	X	X		

Table A.1: ISO3 Country codes, Country names, Geographical Regions, and Associations of Countries

B Connectedness Tables

Tables B.2, B.3, B.4, and B.5 are the connectedness tables as of 1985, 1995, 2005, 2015, respectively. Cells with a value greater than 0.005 are highlighted because of the small font size. The diagonal cells in the table are highlighted because they are their own effect, so it is obvious that their values are larger than the other cells. We can also see that for all countries, the impact of oil price shocks is relatively non-negligible compared to shocks from other countries.

C Calculation Procedure of Generalized FEVD

Forecast Error Variance Decomposition (FEVD) was calculated in the following way.

1. The data are used to estimate the coefficient matrix $\hat{\Theta}_i$ of the country-specific VARX model and the variance of the error term $\hat{\sigma}_i^2$ ($i = 1, \dots, N$), and the $\hat{\Theta}_{\text{du}}$ and $\hat{\sigma}_{\text{du}}^2$ are estimated. The diagonal matrices are assumed for the covariance matrix of the disturbance vector of Eq.(16).

$$\zeta_t = \begin{pmatrix} \mathbf{u}_t \\ \eta_t \end{pmatrix}$$

It seems safe to assume no correlation for the error terms among the VARXs for each country since the foreign variables are included at the same time as the explained variable. On the other hand, it is not clear from the formulation whether the error terms between the country models and the international market model at the same time are uncorrelated or not. However, when $N \rightarrow \infty$, ω_t can function as a proxy variable for a common factor that cannot be directly observed and can be approximated by x^* , so we judged that assuming uncorrelation would pose few problems.

2. Next, the error variance σ_i^2 is generated for each $i = 1, \dots, N$ separately using the inverse Wishart distribution and the coefficient matrix is generated based on the normal distribution

$$N\left(\text{vec}(\hat{\Theta}_i), \hat{\sigma}_i^2 \otimes (\mathbf{X}_i' \mathbf{X}_i)^{-1}\right)$$

subject to this covariance matrix and the data matrix, \mathbf{X}_i , for each VARX. The error variance and regression coefficient matrices are generated in a similar manner for the oil price model.

3. Using the generated $\sigma_i^{2(s)}$ and $\sigma_{\text{du}}^{2(s)}$, construct the covariance matrix:

$$\Sigma_{\zeta}^{(s)} = \text{diag}\left(\sigma_1^{2(s)}, \dots, \sigma_N^{2(s)}, \sigma_{\text{du}}^{2(s)}\right) = \begin{bmatrix} \sigma_1^{2(s)} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma_2^{2(s)} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \sigma_N^{2(s)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \sigma_{\text{du}}^{2(s)} \end{bmatrix} \quad (\text{C.1})$$

$\mathbf{B}_j^{(s)}$ ($j = 1, 2, \dots$) are calculated from $\Theta_i^{(s)}$ and $\Theta_{\text{du}}^{(s)}$ as follows.

(a) First, using

$$\begin{aligned}\Theta_i^{(s)} &= \{a_{i0}^{(s)}, a_{i1}^{(s)}, \phi_{i1}^{(s)}, \phi_{i2}^{(s)}, \lambda_{i0}^{(s)}, \lambda_{i1}^{(s)}, \psi_{i0}^{(s)}, \psi_{i1}^{(s)}\} \\ \Theta_{\text{du}}^{(s)} &= \{\mu_0^{(s)}, \mu_1^{(s)}, \phi_1^{(s)}, \phi_2^{(s)}, \lambda_1^{(s)}\},\end{aligned}$$

construct

$$\mathbf{G}_{i0}^{(s)} = (1, -\lambda_{i0}^{(s)}), \quad \mathbf{G}_{i1} = (\phi_{i1}^{(s)}, \lambda_{i1}^{(s)}), \quad \mathbf{G}_{i2} = (\phi_{i2}^{(s)}, \lambda_{i2}^{(s)}).$$

Then, from these vectors and the trade weight matrix, \mathbf{W} , we construct

$$\mathbf{G}_j^{(s)} = \begin{pmatrix} \mathbf{G}_{1j}^{(s)} \mathbf{W}_1 \\ \vdots \\ \mathbf{G}_{Nj}^{(s)} \mathbf{W}_N \end{pmatrix}.$$

Then, construct $\mathbf{H}_0^{(s)}$ and $\mathbf{H}_1^{(s)}$.

$$\mathbf{H}_0^{(s)} = \begin{pmatrix} \mathbf{G}_0^{(s)} & -\Psi_0^{(s)} \\ \mathbf{0} & 1 \end{pmatrix}, \quad \mathbf{H}_1^{(s)} = \begin{pmatrix} \mathbf{G}_1^{(s)} & \Psi_1^{(s)} \\ \lambda_1^{(s)} \widetilde{\mathbf{W}} & \phi_1^{(s)} \end{pmatrix}, \quad \mathbf{H}_2^{(s)} = \begin{pmatrix} \mathbf{G}_2^{(s)} & \mathbf{0} \\ \mathbf{0} & \phi_2^{(s)} \end{pmatrix}$$

(b) Next, construct $\mathbf{C}_1^{(s)}$ and $\mathbf{C}_2^{(s)}$ as follows.

$$\mathbf{C}_1^{(s)} = (\mathbf{H}_0^{(s)})^{-1} \mathbf{H}_1^{(s)}, \quad \mathbf{C}_2^{(s)} = (\mathbf{H}_0^{(s)})^{-1} \mathbf{H}_2^{(s)}$$

(c) Lastly, construct $\mathbf{B}_j^{(s)}$ ($j = 1, 2, \dots$).^{*8}

$$\mathbf{B}_j^{(s)} = \begin{cases} \mathbf{C}_1^{(s)} \mathbf{B}_{j-1}^{(s)} + \mathbf{C}_2^{(s)} \mathbf{B}_{j-2}^{(s)} & j = 1, 2, \dots \\ \mathbf{I} & j = 0 \\ \mathbf{0} & j < 0 \end{cases}$$

4. Generalized FEVD is calculated as:

$$d_{ij}^H = \frac{(\sigma_{jj}^{(s)})^{-1} \sum_{h=0}^H (\mathbf{e}_i' \mathbf{B}_h^{(s)} (\mathbf{H}_0^{(s)})^{-1} \boldsymbol{\Sigma}_\zeta^{(s)} \mathbf{e}_j)^2}{\sum_{h=0}^H \mathbf{e}_i' \mathbf{B}_h^{(s)} (\mathbf{H}_0^{(s)})^{-1} \boldsymbol{\Sigma}_\zeta^{(s)} ((\mathbf{H}_0^{(s)})^{-1})' (\mathbf{B}_h^{(s)})' \mathbf{e}_i} \quad (\text{C.3})$$

This shows us how much a shock occurring in the j -th variable affects the H -step-ahead forecast error variance of the i -th variable. \mathbf{e}_i is the selection vector whose i th element is 1 and the rest are 0, and $\sigma_{jj}^{(s)}$ is the variance of the disturbance term in the j th expression (or the j th diagonal element of $\boldsymbol{\Sigma}_\zeta^{(s)}$). Note that in GFEVD, the

^{*8} The eigenvalue of Eq.(17) can be obtained from the following companion form

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{y}_{t-1} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_1 \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{y}_{t-2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \mathbf{0} \end{bmatrix}. \quad (\text{C.2})$$

shocks are not orthogonalized, so the sum of the relative variance contribution (RVC) of the forecast error variance is not necessarily equal to 1. Therefore, the following standardization is used.

$$\tilde{d}_{ij}^H = \frac{d_{ij}^H}{\sum_{j=1}^N d_{ij}^H} \quad (\text{C.4})$$

The connectedness measures calculated in this paper are based on \tilde{d}_{ij}^H , not d_{ij}^H .

5. Repeat steps 2. through 4. a sufficient number of times. In this study, it was repeated 1,000 times.