Superstar Firms and Aggregate Fluctuations*

Qazi Haque
The University of Adelaide & CAMA

Oscar Pavlov†
University of Tasmania & CAMA

Mark Weder
Aarhus University & CAMA

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Abstract

The rise of market power in the last decades is primarily driven by the largest (superstar) firms. The current paper examines the impact on aggregate fluctuations of these multi-product superstars through their interaction with smaller competitors. Market share reallocation and product scope adjustments generate heterogeneous markup dynamics across firms and time. Moreover, the prevalence of superstar firms increases the parameter space for macroeconomic indeterminacy. The endogenous amplification mechanism of product creation improves the fit of the estimated general equilibrium model and implies animal spirits play a non-trivial role in driving U.S. business cycles.

1 Introduction

Firms are not identical. Many markets are polarized and populated by a few relatively big firms mixed in with a greater number of smaller firms that extort less market power. Empirically such dispersion is well documented.[1] In the last thirty years, this polarization of markets has become more accentuated. De Loecker et al. (2020) report a steady and significant increase of market power but this increase of the average markup in the U.S. was foremost driven by the firms in the top percentiles.

What are the effects of increasing product market concentration on the workings of the macroeconomy? And what is the role of the competition among big and smaller firms? This paper proposes a dynamic economy that emphasizes the role of firm heterogeneity

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†Tasmanian School of Business and Economics, University of Tasmania, Sandy Bay TAS 7005, Australia. oscar.pavlov@utas.edu.au.

[1] See for example Bartelsman et al. (2013), De Loecker et al. (2020), Baqaee and Farhi (2020), Kehrig and Vincent (2021) and Edmond et al. (2023).

and, in particular, it examines the influence of superstar firms on aggregate fluctuations. What makes superstar firms special? In our artificial economy, large firms have access to different technologies and markets that results in more market power and larger market shares. Neary suggests the following characteristics of superstars:

> "So far, the advantage of superstar firms has not been specified exactly. One interesting and important case is where the superstar technology involves the ability to produce a large number of products. In that case, the small number of superstar firms are multi-product firms, while the remaining insiders which constitute the competitive fringe are single-product firms. This configuration is consistent with the empirical evidence [...]." [Neary, 2010, p 15]

We follow Neary's suggestion and propose a model in which firm heterogeneity manifests itself in a grouping of firms being able to produce multiple products and to gain larger market power relative to mono-product firms. This characterization parallels empirical work as in Broda et al. (2010) and particular Bernard et al. (2010) who report that a considerable fraction of U.S. manufacturing firms produces in multiple five-digit SIC industries and these firms account for well over 80 percent of total sales. The theoretical framework then allows to shed light on the effects of the superstar firm environment involving changes in market concentration and also of the dispersion of market power. Of particular importance to us is explaining in a unified way the observed market power by a restricted group of firms, their interactions with conventional firms and the impact of that environment on the macroeconomic dynamics – either via the propagation of impulses or multiple equilibria. We show that a rising market share of superstar firms allows for greater divergence in markups between large and small firms. Our theory predicts that superstars charge a higher price, they set larger markups and grab a larger market share. Our paper, thus, provides a quantitative theory of superstar firms' divergence from the rest and it can explain various empirical findings such as in De Loecker et al. (2020). We also show how this divergence increases the parametric space for macroeconomic indeterminacy. This indeterminacy implies that profit-seeking businessmen' animal spirits can lead to self-fulfilling macroeconomic outcomes. This indeterminacy mechanism is novel as it comes about from the superstars' endogenous and time-varying product creation even when we keep constant the number of firms. The estimated version of the model suggests that the endogenous amplification mechanism of product creation within superstar firms is empirically important and that a non-trivial portion of U.S. aggregate fluctuations is driven by realized animal spirits, i.e. non-fundamental swings between euphoria and pessimism.

This article is in five parts. It begins by presenting the baseline model from which we have stripped off various bells and whistles that we insert into the full model when estimating it. This approach allows us to highlight the main mechanisms that drive our results. Section 3 analyses the local dynamics by presenting the parametric zones for indeterminacy and impulse responses of macroeconomic variables to animal spirits (i.e. to a animal spirits or expectational error shock). The fourth Section presents the Bayesian estimation of the fuller model. We end the article by listing our conclusions.

2 Model

The economy is populated by two groups of firms. One group consists of smaller mono-product firms. We will coin them ordinary firms. The other grouping are superstars: they produce multiple products, and consequently, have more market power. Both groups of firms produce differentiated goods and adjust their markups according to fluctuations in their market shares. The firms' goods are bought by perfectly competitive firms that weld the varieties together into the final good that is used for consumption purposes or added to the capital stock. People rent out labor and capital services. Firms and households are price takers on factor markets.

2.1 Final goods

Similar to Shimomura and Thisse (2012), final output is a combination of products produced by M ordinary firms and N superstar firms.[2] M and N are constant for now but this will be relaxed later. With this assumption on firm numbers, we can pinpoint to the role of time-varying product scopes as opposed to firm dynamics of entry and exit. Final output Y is then

$$Y = \left(\sum_{i=1}^{M} x(i)^{\frac{\sigma-1}{\sigma}} + \sum_{j=1}^{N} Y(j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

in which $\sigma > 1$ stands for the elasticity of substitution and $x(i)$ is the amount produced by mono-product firm i. Since superstar firms are multi-product firms, $Y(j)$ is a composite good

$$Y(j) = \left(\int_0^{S(j)} x(j,s)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

in which $S(j)$ stands for the product scope and $x(j,s)$ denotes variety s of superstar j. The CES aggregators imply a love of variety effect of $1/(\sigma-1)$. This variety effect in (2) provides the benefit of product creation for the superstar firm.[3] The final profit

[2] We suppress the time index in these static equations for notational ease.

[3] More broadly, the love of variety can be interpreted as a stand-in for other efficiency gains of product creation within multi-product firms. Pavlov (2021) discusses an alternative way of modelling efficiency gains of product creation without the love of variety.

maximization problem yields two demand functions

$$x(i) = \left(\frac{p(i)}{P}\right)^{-\sigma} Y,$$

$$x(j,s) = \left(\frac{p(j,s)}{P}\right)^{-\sigma} Y$$

and the aggregate price index

$$P = \left(\sum_{j=1}^{M} p(i)^{1-\sigma} + \sum_{j=1}^{N} P(j)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

with

$$P(j) = \left(\int_0^{S_t(j)} p(j,s)^{1-\sigma} ds\right)^{\frac{1}{1-\sigma}}.$$

2.2 Intermediate good firms

Varieties supplied by superstar firms are produced using labor $h(j,s)$ and capital services $Uk(j,s)$. U stands for the utilization rate set by the owners of physical capital. Superstar firm j maximizes profits

$$\pi(j) = \int_0^{S(j)} p(j,s)x(j,s) - wh(j,s) - rUk(j,s)ds$$

subject to the production technology

$$\int_0^{S(j)} x(j,s)ds = \int_0^{S(j)} [U^\alpha k(j,s)^\alpha h(j,s)^{1-\alpha} - \phi_s]ds - \phi_f \qquad 0 < \alpha < 1.$$

Here, w is the wage and r the rental rate for labor and capital services. The variety-level fixed cost ϕ_s is paid each period and it restricts the amount of varieties the firm produces. The firm-level fixed cost ϕ_f provides economies of scope and helps to pin down steady state profits. Ordinary firm i only produces a single variety and its production technology is

$$x(i) = U^\alpha k(i)^\alpha h(i)^{1-\alpha} - \phi_o$$

in which the fixed cost ϕ_o is calibrated so that regular firms have zero profits at the steady state. The first-order conditions are

$$w = (1-\alpha)\Lambda U^\alpha k(j,s)^\alpha h(j,s)^{-\alpha} = (1-\alpha)\Lambda U^\alpha k(i)^\alpha h(i)^{-\alpha} \tag{3}$$

$$r = \alpha\Lambda U^{\alpha-1} k(j,s)^{\alpha-1} h(j,s)^{1-\alpha} = \alpha\Lambda U^{\alpha-1} k(i)^{\alpha-1} h(i)^{1-\alpha} \tag{4}$$

where

$$\Lambda \equiv \alpha^{-\alpha}(1-\alpha)^{\alpha-1} r^\alpha w^{1-\alpha} \tag{5}$$

are the marginal costs that are the same for both firm types. In the spirit of Yang and Heijdra (1993), firms take into account the effect of their prices on the aggregate price index P.[4] Due to each variety having the same production technology, superstar j charges the same price for all of its varieties i.e. $p(j,s) = p(j,k) = p(j)$. The markups are then

$$\mu(j) \equiv \frac{p(j)}{\Lambda} = \frac{\sigma\left(1 - \left(\frac{P(j)}{P}\right)^{1-\sigma}\right)}{\sigma\left(1 - \left(\frac{P(j)}{P}\right)^{1-\sigma}\right) - 1}$$

and

$$\mu(i) \equiv \frac{p(i)}{\Lambda} = \frac{\sigma\left(1 - \left(\frac{p(i)}{P}\right)^{1-\sigma}\right)}{\sigma\left(1 - \left(\frac{p(i)}{P}\right)^{1-\sigma}\right) - 1}.$$

From the above demand functions we find that

$$\left(\frac{P(j)}{P}\right)^{1-\sigma} = \frac{P(j)Y(j)}{PY} \equiv \epsilon(j)$$

and

$$\left(\frac{p(i)}{P}\right)^{1-\sigma} = \frac{p(i)x(i)}{PY} \equiv \epsilon(i).$$

The markups are thus positively related to the firms' market shares and for the superstar firm this market share is increasing in the number of varieties $S(j)$.

Superstar j maximizes profits

$$\pi(j) = \left(\frac{p(j) - \Lambda}{p(j)}\right) PY\epsilon(j) - \Lambda[S(j)\phi_s + \phi_f]$$

with respect $S(j)$ and takes into account the effect of its product scope on its own prices, prices of other firms, and the aggregate price index. The first-order condition, $\frac{\partial \pi(j)}{\partial S(j)} = 0$, implies

$$\Lambda\phi_s = \sigma PY \left(\frac{p(j) - \Lambda}{p(j)}\right)^2 \frac{\partial \epsilon(j)}{\partial S(j)} + Y\epsilon(j)\left(\frac{p(j) - \Lambda}{p(j)}\right) \frac{\partial P}{\partial S(j)}. \tag{6}$$

It can be shown that $\frac{\partial \epsilon(j)}{\partial S(j)} > 0$ and $\frac{\partial P}{\partial S(j)} < 0$. The term on the left-hand side represents the direct cost of expanding the product scope. The first term on the right-hand side represents the gain to market share due to the love of variety in the CES aggregator (2). The second term indicates that profits are reduced due to the higher product scope reducing the aggregate price index. Again, as demonstrated in Pavlov and Weder (2017), given the Dixit and Stiglitz (1977) demand system and fixed costs structure, a form of variety effect is required for multi-product firms to exist.

[4] Our economy can also be interpreted as a representative sector where firms take into account the effect of their prices on the sectoral price index. We abstract from explicitly modelling sectors to keep the presentation tidy.

2.3 Symmetric equilibrium

In the symmetric equilibrium each superstar firm produces the same number of varieties $S(j) = S$, charges the same price $p(j) = p_s$, and has the same market share $\epsilon(j) = \epsilon_s$. Similarly, for the ordinary firm $p(i) = p_o$ and $\epsilon(i) = \epsilon_o$ hold. The markups arrange to

$$\mu_s = \frac{\sigma(1-\epsilon_s)}{\sigma(1-\epsilon_s)-1} > \mu_o = \frac{\sigma(1-\epsilon_o)}{\sigma(1-\epsilon_o)-1} > \frac{\sigma}{\sigma-1} \tag{7}$$

and

$$\epsilon_z = Sp_s^{1-\sigma} > \epsilon_o = p_o^{1-\sigma}$$

with the final good set as the numeraire $P = 1$. Superstar firms have larger market shares and markups than regular firms due to producing multiple products. Since both ordinary and superstar firms hire labor and capital services from the same factor markets and both have constant returns to scale (abstracting from fixed costs) production functions, from (3) and (4) we obtain

$$\frac{w}{r} = \frac{1-\alpha}{\alpha}\frac{K_oU}{H_o} = \frac{1-\alpha}{\alpha}\frac{K_sU}{H_s}$$

in which $H_s = NSh_s$ and $H_o = Mh_o$. Therefore, all firms choose an identical capital-labor intensities

$$\frac{K_o}{H_o} = \frac{K_s}{H_s}.$$

From the marginal costs (5) we get

$$p_s = \mu_s\alpha^{-\alpha}(1-\alpha)^{\alpha-1}r^\alpha w^{1-\alpha} > p_o$$

thus, superstars charge a higher price than their ordinary counterparts. Moreover, summing production and demand functions of ordinary firms

$$\sum_{i=0}^{M} x(i) = \sum_{i=0}^{M}\left(\frac{p(i)}{P}\right)^{-\sigma} Y = \sum_{i=0}^{M}\left(U^\alpha k(i)^\alpha h(i)^{1-\alpha} - \phi_o\right)$$

then applying symmetry yields

$$Y = \frac{p_o}{M\epsilon_o}\left(U^\alpha K_o^\alpha H_o^{1-\alpha} - M\phi_o\right).$$

Similarly, for superstar firms

$$\sum_{j=1}^{N}\int_0^{S(j)} x(j,s)ds = \sum_{j=1}^{N}\int_0^{S(j)}\left(\frac{p(j,s)}{P}\right)^{-\sigma} Y ds = \sum_{j=1}^{N}\left(\int_0^{S(j)}\left[U^\alpha k(j,s)^\alpha h(j,s)^{1-\alpha} - \phi_s\right]ds - \phi_f\right)$$

which yields

$$Y = \frac{p_s}{N\epsilon_s}(U^\alpha K_s^\alpha H_s^{1-\alpha} - NS\phi_s - N\phi_f).$$

Lastly, the first-order condition (6) rearranges for the product scope

$$S = f(\mu_s, \mu_o, N, M, \sigma)\frac{Y}{\phi_s p_s}$$

where the function $f > 0$. We delegate details on the derivation of the product scope to Appendix A.1. Finally, the capital and labor markets must be in equilibrium, that is $K = K_o + K_s$ and $H = H_o + H_s$.

2.4 People

People are personified by a representative agent who chooses sequences of consumption C_t and hours worked H_t to maximize lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \upsilon \frac{H_t^{1+\chi}}{1+\chi} \right) \qquad \beta > 0, \upsilon > 0, \chi \geq 0$$

in which β is the discount rate, υ denotes the disutility of working and χ is the inverse of the Frisch labor supply elasticity. The agent owns all firms and receives the profits Π_t that they potentially generate. The budget is thus constrained by

$$w_t H_t + r_t U_t K_t + \Pi_t \geq I_t + C_t$$

in which I_t is investment that adds to the capital stock

$$K_{t+1} = (1 - \delta_t) K_t + I_t$$

and the depreciation rate varies according to

$$\delta_t = \frac{1}{\theta} U_t^{\theta} \qquad \theta > 1.$$

The first-order conditions from the agent's maximization problem combine for the labor supply

$$\upsilon H_t^{\chi} C_t = w_t$$

the Euler equation

$$\frac{1}{C_t} = \frac{1}{C_{t+1}} \beta \left(r_{t+1} U_{t+1} + 1 - \delta_t \right)$$

and the optimal rate of capital utilization

$$r_t = U_t^{\theta - 1}.$$

The steady state versions of these equations then pin down $\theta = (1/\beta - 1 + \delta)/\delta$.

3 Dynamics

Let us now analyze the local dynamic properties of the model. The equilibrium conditions are log-linearized and the dynamical system is arranged to

$$\begin{bmatrix} \widehat{K}_{t+1} \\ \widehat{C}_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \widehat{K}_t \\ \widehat{C}_t \end{bmatrix}.$$

Hatted variables denote percent deviations from their steady-state values and $\mathbf{J}$ is the 2×2 Jacobian matrix of partial derivatives. Note that C_t is a non-predetermined variable and that K_t is predetermined. Indeterminacy, and the potential presence of animal spirits, requires both roots of $\mathbf{J}$ to be inside the unit circle. For easier comparison to previous studies, the standard parameters are calibrated at a quarterly frequency as $\alpha = 0.3$, $\beta = 0.99$, $\delta = 0.025$ and $\chi = 0$. We directly calibrate the steady state markups and the elasticity of substitution σ. From (7), these parameters then determine the market shares

$$\epsilon_s = 1 - \frac{\mu_s}{\mu_s - 1}\frac{1}{\sigma} > \epsilon_o = 1 - \frac{\mu_o}{\mu_o - 1}\frac{1}{\sigma}.$$

Since market shares sum to unity, $M\epsilon_o + N\epsilon_s = 1$, we can then calibrate the market share of superstar firms, $N\epsilon_s$, to pin down the number of firms at the steady state

$$M = \frac{1 - N\epsilon_s}{\epsilon_o}$$

$$N = \frac{1}{\epsilon_s\left(1 + \frac{M\epsilon_o}{N\epsilon_s}\right)}.$$

It is then straightforward to show that for each calibration of μ_o, the lower bound on σ is $\mu_o/(\mu_o - 1)$. As σ approaches this lower bound, the number of regular firms approaches infinity and their markups become constant at $\sigma/(\sigma - 1)$ as in the monopolistic competition version of the model. This is also where the love of variety $1/(\sigma - 1)$ implied by the CES aggregator (2) is at its maximum. For the upper bound, σ must be smaller than both $\mu_o/(\mu_o - 1)(1 + \frac{M\epsilon_o}{N\epsilon_s})$ and $\mu_s/(\mu_s - 1)(1 + \frac{N\epsilon_s}{M\epsilon_o})$ to guarantee $M > 1$ and $N > 1$, respectively.

Figure 1 plots the feasible parameter space for indeterminacy and determinacy. We can clearly see that a rise in the market share of superstars increases both zones and allows for greater differences between the markups of superstar and regular firms. Animal spirits arise more easily in situations where substantial markup differences, as documented by many studies mentioned previously, exist between small and large firms. When markups approach unity, the implied variety effect $1/\left(\sigma - 1\right)$ becomes too small and indeterminacy cannot arise. That is, low markups imply a small gain to product creation and a weak endogenous amplification mechanism.

The way indeterminacy arises is best explained via the equilibrium wage-hours locus. Product creation within superstar firms makes this locus upwardly sloping and indeterminacy arises when it becomes steeper than the labor supply curve. The reason is the presence of love of variety in the CES aggregator (2). The composite good from each superstar can be created more efficiently the greater the product scope. Endogenous variations in superstars' product scopes thus generates an endogenous efficiency wedge that expands production possibilities. If people feel optimistic about the future path of income and consumption, the labor supply curve shifts up along the upwardly sloping wage-hours locus, thereby raising employment and output, validating the initial optimistic expectation.

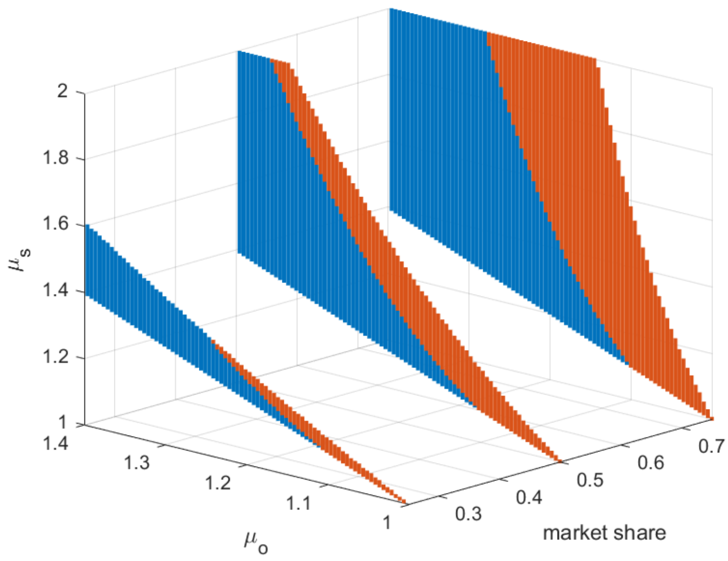

Figure 1: Indeterminacy (blue) and determinacy (orange) zones.

Figure 2 illustrates the dynamics of the model in response to an animal spirits shock that raises output one percent above its steady state. We set the two markups at 1.4 and 1.8 as well as the market share of superstars at 60 percent which, as we argue below, matches broadly the reported values in De Loecker et al. (2020) and Bernard et al. (2010). Conditional on an animal spirits shock, product creation and the markups of large firms are procyclical whereas smaller firms' markups are countercyclical. This pattern is not unlike the unconditional correlations of markups and output that are reported by Burstein et al. (2023). Unlike, Minniti and Turino (2013) and Pavlov and Weder (2017), where markups are countercyclical, the number of firms here is constant. Product scope adjustments together with firm heterogeneity thus provide a novel indeterminacy mechanism for market share reallocation and markup dynamics without entry and exit. The model's multiplicity carries over to environments with endogenous entry and exit as in the following section.

4 Estimation

So far, we have shown that superstar firms can lead to macroeconomic instability which opens the possibility of animal spirits driving business cycles. The current section estimates both the indeterminate and the determinate model to assess the importance of the product creation mechanism within superstars on explaining aggregate fluctuations. In doing so, we examine the importance of animal spirits versus other fundamental shocks and see whether they help to replicate the basic business cycle facts by comparing the model's second moments to the U.S. quarterly time series counterparts. Appendix A.4

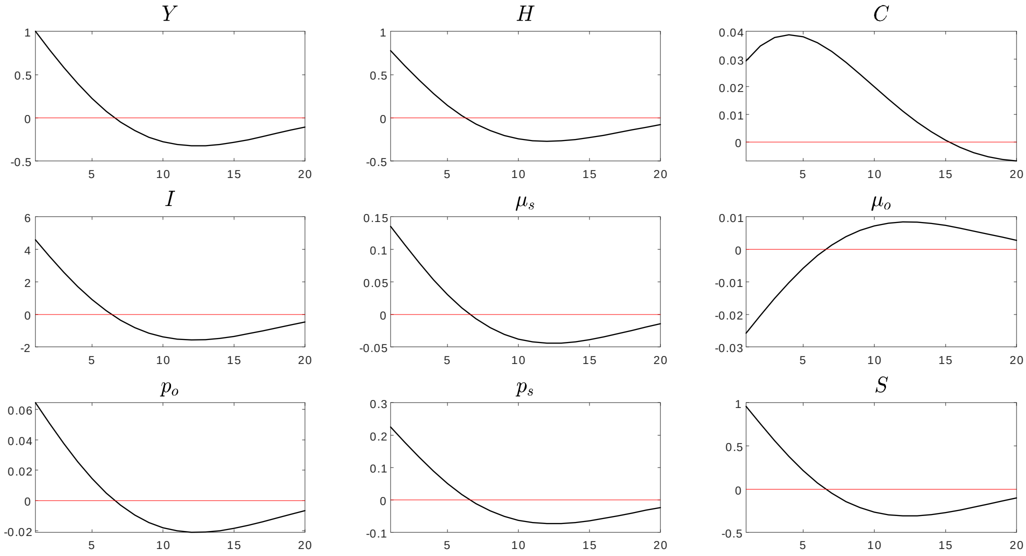

Figure 2: Impulse responses to an ouput sunspot shock, percent deviations from the steady state, $\mu_s = 1.8$, $\mu_o = 1.4$, $\sigma = 4.5$.

sets out the exact data sources. The models used here are extended by exogenous growth, fundamental aggregate supply and demand shocks, external habits in consumption, separable love of variety, and endogenous entry and exit of ordinary firms.

4.1 The extended model

Since entry and exit of superstar firms is not likely to be significant at business cycle frequencies, we have continued with the assumption that N is constant, but M_t now adjusts via free entry of ordinary firms that forces their profits to zero. That is, each period firm i's profits are

$$\pi_t(i) = \left(\frac{p_t(i) - \Lambda_t}{p_t(i)} \right) P_t Y_t \epsilon_t(i) - \Lambda_t \phi_o = 0$$

which in symmetric equilibrium boils down to

$$p_{o,t} = (\mu_{o,t} - 1) \frac{\epsilon_{o,t} Y_t}{\phi_o}$$

to determine the number of ordinary firms. Similar to product scope adjustments, entry and exit enlarges the indeterminacy region due to the efficiency gains in product aggregation from the love of variety in the CES aggregator (1).[5]

The love of variety governs the gain to product creation for superstar firms and is the central amplification mechanism in our model, leading to equilibrium indeterminacy as

[5] The entry decision is static to keep the model tractable. Appendix A.2 shows that indeterminacy remains when we introduce dynamic entry as in Bilbiie et al. (2012).

explained in Section 3. Similar to Pavlov and Weder (2017), here we separate the variety effect from the elasticity of substitution σ and we do it for two main reasons. First, isolating the variety effect allows us to estimate the direct benefit of product creation separately from the parameter that primarily determines the markup elasticities and the steady state number of firms. Second, it gives us an option to estimate a determinate version of the model where the gain to product creation is too small for animal spirits to play a role. Specifically, the CES aggregators are now

$$Y_t = \left(M_t^{\frac{\nu(\sigma-1)-1}{\sigma}} \sum_{i=1}^{M_t} x_t(i)^{\frac{\sigma-1}{\sigma}} + N^{\frac{\nu(\sigma-1)-1}{\sigma}} \sum_{j=1}^{N} Y_t(j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

and

$$Y_t(j) = \left(S_t(j)^{\frac{\nu(\sigma-1)-1}{\sigma}} \int_0^{S_t(j)} x_t(j,s)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}}$$

in which $\nu > 0$ debotes the love of variety. Setting $\nu = 1/(\sigma - 1)$ would bring back the CES aggregators from Section 2. As ν is a key parameter that transports the economy between the determinacy and indeterminacy zones, in the following estimation we will let data decide its magnitude.

We include a mix of fundamental supply and demand disturbances to the model. The first such fundamental shock takes the form of labor augmenting technological progress A_t and it affects all firms equally. Aggregate output is now

$$Y_t = \frac{p_{s,t}}{N\epsilon_{s,t}}[(U_t K_{s,t})^\alpha (A_t H_{s,t})^{1-\alpha} - \phi_s N S_t - \phi_f N] = \frac{p_{o,t}}{M_t \epsilon_{o,t}}[(U_t K_{o,t})^\alpha (A_t H_{o,t})^{1-\alpha} - M_t \phi_o].$$

Technological progress is non-stationary and follows the process

$$\ln A_t = \ln A_{t-1} + \ln a_t$$

$$\ln a_t = (1 - \psi_A) \ln a + \psi_A \ln a_{t-1} + \varepsilon_t^A$$

where $0 \leq \psi_A < 1$ governs the persistence of the shock, $\ln a$ is the average growth rate and ε_t^A is an i.i.d. disturbance with variance σ_A^2. Next, shifts of marginal efficiency of investment z_t affect the transformation of investment to physical capital as in Greenwood et al. (1988)

$$K_{t+1} = (1 - \delta_t) K_t + z_t I_t.$$

The technological shifter follow the exogenous process

$$\ln z_t = \psi_z \ln z_{t-1} + \varepsilon_t^z.$$

As laid out by Justiniano et al. (2011), this shock can be a stand-in for disturbances of financial markets on investment behavior. Intuitively, a positive shock to z_t represents a boom in financial markets that reduces borrowing costs for firms, leading to a rise in investment.

The first fundamental demand disturbance is a taste shock Δ_t to the agent's period utility that increases the marginal utility of consumption as in Christiano (1988). Lifetime utility becomes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\Delta_t \ln(C_t - bC_{t-1}) - \upsilon \frac{H_t^{1+\chi}}{1+\chi} \right)$$

where E_0 denotes the expectations operator and parameter $0 \leq b < 1$ determines the degree of external habits in consumption. The taste shock follows the process

$$\Delta_t = \psi_\Delta \Delta_{t-1} + \varepsilon_t^\Delta.$$

This shock affects the economy's labor wedge, i.e. the gap between the marginal rate of consumption-leisure substitution and the marginal product of labor and hence can be interpreted as a stand-in for other shocks that affect this wedge. The second demand shock is to government expenditures, G_t, financed by lump sum taxes. Consequently, the economy's resource constraint becomes

$$Y_t = C_t + I_t + G_t.$$

Lastly, government spending G_t follows a stochastic trend

$$A_t^g = (A_{t-1}^g)^{\psi_{ag}} (A_{t-1})^{1-\psi_{ag}}$$

where ψ_{ag} governs the smoothness of the trend relative to the trend in output. Then, detrended government spending is $g_t \equiv G_t / A_t^g$ and follows

$$\ln g_t = \psi_g \ln g_{t-1} + \varepsilon_t^g.$$

As in Pavlov and Weder (2017), the non-fundamental animal spirits shock is modelled as an expectation error to output that is unrelated to any fundamental changes in the economy[6]. Under indeterminacy, the economy's response to fundamentals is not uniquely determined, and we model the behavior of output as

$$\widehat{Y}_t = E_{t-1}\widehat{Y}_t + \Omega_A \varepsilon_t^A + \Omega_z \varepsilon_t^z + \Omega_\Delta \varepsilon_t^\Delta + \Omega_g \varepsilon_t^g + \varepsilon_t^s$$

where the parameters Ω_A, Ω_z, Ω_Δ and Ω_g determine the effect of technology, investment, preference and government shocks on output. The term ε_t^s is i.i.d., independent of fundamentals and comes with variance σ_s^2. It can be thought of profit-seeking businessmen exercising their animal spirits.

[6] Farmer et al. (2015) show that estimation results are robust to the choice of expectation error.

4.2 Bayesian estimation

The model is estimated via Bayesian methods with the procedure largely following Farmer et al. (2015) and Pavlov and Weder (2017). U.S. data includes quarterly real per capita growth rates of output, consumption, investment, government spending and the logarithm of per capita hours worked from 1990:I-2019:IV as observables. We use credit spread data to identify investment shocks $\hat{z}_t$ as in Justiniano et al. (2011). Concretely, we adopt the credit spread between BAA corporate bonds and the market yield on 30 year Treasury securities to identify disturbances to the marginal efficiency of investment

$$spread_t = \varkappa \widehat{z}_t \qquad \varkappa < 0.$$

We focus on the period post 1990 to coincide with the rise of superstar firms as reported by De Loecker et al. (2020) and abstract from the COVID-19 pandemic as our small scale model is not designed to deal with its complexities.

We follow Bilbiie et al. (2012) and deflate Y_t, C_t, I_t, and G_t in the model by a data-consistent price index to obtain variables that are more comparable to the data, which does not take into account the welfare improvements of product variety at quarterly frequency. For example, data-consistent output is

$$Y_t^d \equiv \frac{P_t Y_t}{p_t} \equiv \frac{P_t Y_t}{p_{o,t}} M_t \epsilon_{t,o} + \frac{P_t Y_t}{p_{s,t}} N \epsilon_{t,s}$$

which removes the welfare gains coming from entry and product scope adjustments. The measurement equation is thus

$$\begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t - \ln C_{t-1} \\ \ln I_t - \ln I_{t-1} \\ \ln G_t - \ln G_{t-1} \\ \ln H_t - \ln H \\ spread_t \end{bmatrix} = \begin{bmatrix} \widehat{Y^d}_t - \widehat{Y^d}_{t-1} + \widehat{a}_t \\ \widehat{C^d}_t - \widehat{C^d}_{t-1} + \widehat{a}_t \\ \widehat{I^d}_t - \widehat{I^d}_{t-1} + \widehat{a}_t \\ \widehat{G^d}_t - \widehat{G^d}_{t-1} + \widehat{a^g}_t - \widehat{a^g}_{t-1} + \widehat{a}_t \\ \widehat{H}_t \\ \varkappa \widehat{z}_t \end{bmatrix} + \begin{bmatrix} \ln a \\ \ln a \\ \ln a \\ \ln a \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{m.e.} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $a_t^g = A_t^g / A_t = (a_{t-1}^g)^{\psi_{ag}} a_t^{-1}$, $\varepsilon_t^{m.e.}$ is a measurement error restricted to account for not more than ten percent of output growth and $\ln H$ is the logarithm of the average hours worked over the sample period. We calibrate a subset of the model parameters to avoid identification issues. We set the quarterly growth rate of labor augmenting technological progress to 0.34 percent to be consistent with the growth rate of per capita real GDP over the sample period. The share of government expenditures in output G/Y is 0.19 and the values of standard parameters remain the same as in previous sections. In line with Barkai (2020), we calibrate the share of fixed costs in output so that the steady state profits are ten percent. We calibrate μ_s, μ_o, σ, and $N\epsilon_s$ as follows. A large portion of firms are multi-product producers. Bernard et al. (2010) report that close

Table 1: Prior and posterior distributions

Prior					Posterior	
Name	Range	Density	Mean	Std. Dev.	Mean	90% Interval
ν	R^+	Normal	0.075	0.05	0.19	[0.16,0.22]
b	[0,1)	Beta	0.5	0.1	0.52	[0.43,0.60]
ψ_A	[0,1)	Beta	0.5	0.2	0.00	[0.00,0.01]
ψ_z	[0,1)	Beta	0.5	0.2	0.79	[0.73,0.86]
ψ_Δ	[0,1)	Beta	0.5	0.2	0.95	[0.93,0.97]
ψ_g	[0,1)	Beta	0.5	0.2	0.99	[0.98,0.99]
ψ_{ag}	[0,1)	Beta	0.5	0.2	0.86	[0.65,0.99]
σ_s	R^+	Inverse Gamma	0.1	Inf	0.26	[0.23,0.29]
σ_A	R^+	Inverse Gamma	0.1	Inf	0.70	[0.63,0.78]
σ_z	R^+	Inverse Gamma	0.1	Inf	0.07	[0.04,0.10]
σ_Δ	R^+	Inverse Gamma	0.1	Inf	0.47	[0.42,0.52]
σ_g	R^+	Inverse Gamma	0.1	Inf	0.81	[0.72,0.89]
$\sigma^{m.e.}$	$[0, 0.18]$	Uniform	0.09	0.05	0.18	[0.18,0.18]
Ω_A	[-3,3]	Uniform	0	1.73	-0.53	[-0.63,-0.42]
Ω_z	[-3,3]	Uniform	0	1.73	1.60	[0.72,2.57]
Ω_Δ	[-3,3]	Uniform	0	1.73	0.67	[0.54,0.79]
Ω_g	[-3,3]	Uniform	0	1.73	0.09	[-0.01,0.19]
$\varkappa$	[-20,0]	Uniform	-10	5.77	-4.71	[-6.73,-2.75]

This table presents the prior and posterior distributions for model parameters and shocks. Inf implies two degrees of freedom for the inverse gamma distribution. Standard deviations are in percent terms. Log-data density (modified harmonic mean): -780.60.

to half of U.S. manufacturing firms produce in multiple five-digit SIC industries. These firms account for well over 80 percent of total sales. We accordingly calibrate the market share of superstars to 60 percent, i.e. $N\epsilon_s = 0.6$. A conservative interpretation of the composition of markups reported in De Loecker et al. (2020) suggests a markup of large firms to be around $\mu_s = 1.8$ as this is the rough average for the (revenue weighted) top 75 to top 90 percentiles of firms. Smaller firms, say the top 50 percentile, have seen a steady markup at around $\mu_o = 1.3$. Lastly, we agnostically pick a number for $\sigma = 5$ that falls in the middle of its admissible values. We take these numbers as our benchmark calibration. We can demonstrate robustness.

The remaining parameters are estimated. These include the love of variety, ν, external habits, b, the coefficient mapping the credit spread to investment shocks, $\varkappa$, and parameters that govern the stochastic processes: ψ_A, ψ_z, ψ_Δ, ψ_g, ψ_{ag}, σ_s, σ_A, σ_z, σ_Δ, σ_g, Ω_A, Ω_z, Ω_Δ, Ω_g, and $\sigma^{m.e.}$. Table 1 presents the initial prior and posterior distributions. We employ a normal distribution, truncated at zero, for the variety effect ν. Since this parameter is central to our amplification mechanism which generates indeterminacy, we set the mean to 0.075 to give a prior probability of determinacy of about 50 percent. A wide uniform distribution is employed for the expectation error parameters Ω_A, Ω_z, Ω_Δ, Ω_g and the credit spread coefficient $\varkappa$. The shock processes follow the standard inverse gamma distribution. The Metropolis-Hastings algorithm is employed to obtain

Table 2: Business cycle dynamics

	Data			Model		
x	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF
$\ln(Y_t/Y_{t-1})$	0.58	1	0.29	0.79	1	0.49
$\ln(C_t/C_{t-1})$	0.47	0.67	0.38	0.63	0.49	0.48
$\ln(I_t/I_{t-1})$	1.66	0.79	0.62	2.97	0.82	0.62
$\ln(G_t/G_{t-1})$	0.77	0.25	0.24	0.83	0.13	0.04
$\ln(H_t/H)$	6.16	0.20	0.99	4.80	0.12	0.99
spread$_t$	0.60	-0.58	0.85	0.56	-0.25	0.79

Business cycle statistics for the artificial economy are calculated at the posterior mean. σ_x denotes the standard deviation of variable x, $\rho(x, \ln(Y_t/Y_{t-1}))$ is the correlation of variable x and output growth, and ACF is the first order autocorrelation coefficient.

$500,000$ draws from the posterior mean for each of the two chains. Half of the draws are discarded and the scale in the jumping distribution is adjusted to achieve a $25-30$ percent acceptance rate for each chain. The table shows that the parameters are precisely estimated and consistent with previous studies. The love of variety is estimated at 0.19, which suggests a strong amplification mechanism of product creation within superstars that guarantees indeterminacy but is below the value that would be implied under the original CES configuration in Section 2, i.e. $1/(\sigma-1) = 0.25$. The persistence of the permanent technology shock is essentially zero and, consistent with a determinate real business cycle model, a positive shock causes a fall in detrended output. The investment shock is moderately persistent and as expected, raises output on impact. Finally, both demand shocks are highly persistent and also cause an increase in output.

The choice of priors leads to a prior predictive probability of indeterminacy of 0.50 and indicates no prior bias toward either determinacy or indeterminacy. The first main result is that, through the lens of our model, the post 1990 period, is best characterized by the indeterminacy version of our model. Specifically, the log data densities come in as -780.60 for the indeterminacy model versus -914.96 for its competitor (see Appendix A.3). Thus, since data prefers the indeterminacy, and in winner takes it all fashion, the below will present the results for the animal spirits driven model.

Table 2 reports the theoretical second moments of the main macroeconomic aggregates and the credit spread at the posterior mean. Our admittedly small scale model replicates the behavior of the considered U.S. macroeconomic variables quite well. The relative volatilities and correlations are consistent with the data. The model slightly overpredicts the volatilities of output and consumption but underpredicts for hours worked. Government expenditures and the credit spread are matched well. The sole outlier is investment, where the model strongly overpredicts its variance. As a result of the richer internal propagation mechanism under indeterminacy, the autocorrelation functions are able to show persistence in the growth rates despite the lack of the many real frictions employed in the literature.

Table 3: Unconditional variance decomposition (in percent)

	$\ln\left(\frac{Y_t}{Y_{t-1}}\right)$	$\ln\left(\frac{C_t}{C_{t-1}}\right)$	$\ln\left(\frac{X_t}{X_{t-1}}\right)$	$\ln\left(\frac{G_t}{G_{t-1}}\right)$	$\ln\left(\frac{H_t}{H}\right)$	$spread_t$
ε_t^s	13.18	0.38	20.49	0	2.81	0
ε_t^A	30.26	34.74	18.65	5.42	16.46	0
$\varepsilon_t^{\tilde{z}}$	19.23	0.69	29.00	0	20.21	100
ε_t^Δ	34.80	64.12	28.10	0	51.29	0
ε_t^g	2.53	0.08	3.76	94.58	9.23	0

Variance decompositions are performed at the posterior mean.

Table 3 displays the variance decomposition which reveals the relative contribution of each of the shocks to the macroeconomic aggregates. As discussed earlier, the non-fundamental animal spirits propagate the fundamental disturbances, while also causing fluctuations on their own. The effect of "pure" animal spirits on the U.S. business cycle is non-trivial: they drive a modest fraction of output and a sizeable portion of investment. We see this as success as these shocks primarily stand for business' expectations and their alternations between euphoric and pessimistic states. The technology shock offers a good explanation of output and consumption, while the marginal efficiency of investment shock best explains investment data. In fact, its importance shrinks considerably when compared to Justiniano et al. (2011). The preference shock is the dominant shock and explains most of consumption, half of hours worked, and about a third of output and investment. Finally, the government expenditure shock is a negligible source of business cycles.

5 Concluding remarks

Recent empirical research has highlighted the significant rise of market power in the last few decades primarily driven by the largest (superstar) firms. The current paper examines the impact on aggregate fluctuations of these multi-product superstars through their interaction with smaller competitors. We find that the rising market share of superstar firms increases the parameter space for indeterminacy and allows for greater divergence between markups of large and small firms. Market share reallocation between superstars and ordinary firms via product scope adjustments generates heterogeneous markup dynamics across firms and time. The endogenous amplification mechanism of product creation within superstars improves the fit of the estimated general equilibrium model and implies animal spirits play a non-trivial role in driving U.S. business cycles.

References

[1] Baqaee, D. and E. Farhi (2020): "Productivity and Misallocation in General Equilibrium", *Quarterly Journal of Economics* **135**, 105-163.

[2] Barkai, S. (2020): "Declining Labor and Capital Shares", *Journal of Finance* **75**, 2421–2463.

[3] Bartelsman, E., J. Haltiwanger and S. Scarpetta (2013): "Cross-Country Differences in Productivity: The Role of Allocation and Selection," *American Economic Review* **103**, 305-334.

[4] Bilbiie, F., F. Ghironi and M. Melitz (2012); "Endogenous Entry, Product Variety, and Business Cycles", *Journal of Political Economy* **120**, 304-345.

[5] Bernard, A., S. Redding and P. Schott (2010): "Multi-product Firms and Product Switching", *American Economic Review* **100**, 70-97.

[6] Broda, C. and D. Weinstein (2010): "Product Creation and Destruction: Evidence and Price Implications", *American Economic Review* **100**, 691-732.

[7] Burstein, A., V. Carvalho and B. Grassi (2023): "Bottom-up Markup Fluctuations", UCLA mimeo.

[8] Christiano, L. (1988): "Why Does Inventory Investment Fluctuate So Much?", *Journal of Monetary Economics* **21**, 247-280.

[9] De Loecker J., J. Eeckhout and G. Unger (2020): "The Rise of Market Power and the Macroeconomic Implications," *Quarterly Journal of Economics* **135**, 561-644.

[10] Dixit, A. and J. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity", *American Economic Review* **67**, 297-308.

[11] Edmond, C., V. Midrigan and D. Xu (2023): "How Costly Are Markups?", *Journal of Political Economy* **131**, 1619–1675.

[12] Farmer, R., V. Khramov and G. Nicolò (2015): "Solving and Estimating Indeterminate DSGE Models", *Journal of Economic Dynamics and Control* **54**, 17-36.

[13] Greenwood, J., Z. Hercowitz and G. Huffman (1988): "Investment, Capacity Utilization, and the Real Business Cycle", *American Economic Review* **78**, 402-417.

[14] Justiniano A., G. Primiceri and A. Tambalotti (2011): "Investment Shocks and the Relative Price of Investment," *Review of Economic Dynamics* **14**, 102-121.

[15] Kehrig, M. and N. Vincent (2021): "The Micro-Level Anatomy of the Labor Share Decline", *Quarterly Journal of Economics* **136**, 1031-1087.

[16] Minniti, A. and F. Turino (2013): "Multi-product Firms and Business Cycle Dynamics", *European Economic Review* **57**, 75-97.

[17] Neary, P. (2010): “Two and a Half Theories of Trade,“ *The World Economy* **33**, 1-19.

[18] Pavlov, O. (2021): “Multi-product Firms and Increasing Marginal Costs,“ *Journal of Economic Dynamics and Control* **133**, 104239.

[19] Pavlov, O. and M. Weder (2017): “Product Scope and Endogenous Fluctuations“, *Review of Economic Dynamics* **24**, 175-191.

[20] Shimomura, K and J. Thisse (2012): “Competition Among the Big and the Small“, *RAND Journal of Economics* **43**, 329-347.

[21] Yang, X. and B. Heijdra (1993): “Monopolistic Competition and Optimum Product Diversity: Comment“, *American Economic Review* **83**, 295–301.

A Appendix

A.1 Product scope

This Appendix derives the firm’s optimal product scope. Product scope determination largely follows the approach of Minniti and Turino (2013) and Pavlov and Weder (2017). Firm j maximizes profits with respect to $S(j)$ and takes into account the effect of its product scope on its own prices, prices of other firms, and the aggregate price index. First we rewrite profits as

$$\pi(j) = \left(\frac{p(j)-\Lambda}{p(j)}\right) PY\epsilon(j) - \Lambda[S(j)\phi_s + \phi_f]$$

and obtain the first-order condition

$$\frac{\partial \pi(j)}{\partial S(j)} = \sigma PY \left(\frac{p(j)-\Lambda}{p(j)}\right)^2 \frac{\partial \epsilon(j)}{\partial S(j)} + Y\epsilon(j)\left(\frac{p(j)-\Lambda}{p(j)}\right)\frac{\partial P}{\partial S(j)} - \Lambda\phi_s = 0.$$

Then

$$\frac{\partial \epsilon(j)}{\partial S(j)} = \frac{\epsilon(j)}{S(j)} - (\sigma-1)\epsilon(j)\left[\frac{1}{p(j)}\frac{\partial p(j)}{\partial S(j)} - \frac{1}{P}\frac{\partial P}{\partial S(j)}\right]$$

and for other multi-product firms

$$\frac{\partial \epsilon(k)}{\partial S(j)} = -(\sigma-1)\epsilon(k)\left[\frac{1}{p(k)}\frac{\partial p(k)}{\partial S(j)} - \frac{1}{P}\frac{\partial P}{\partial S(j)}\right]$$

and regular firms

$$\frac{\partial \epsilon(i)}{\partial S(j)} = -(\sigma-1)\epsilon(i)\left[\frac{1}{p(i)}\frac{\partial p(i)}{\partial S(j)} - \frac{1}{P}\frac{\partial P}{\partial S(j)}\right].$$

Next, rewrite the aggregate price index as

$$P = \left(\sum_{i=1}^{M} p(i)^{1-\sigma} di + \sum_{k=1}^{N} S(k)p(k)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

From here, we use symmetry to simplify. After some algebra $\partial P/\partial S(j)$ can be expressed as

$$\frac{\partial P}{\partial S(j)} = M\frac{\epsilon_o}{p_o}\frac{\partial p(i)}{\partial S(j)} + \frac{\epsilon_s}{p_s}\left((N-1)\frac{\partial p(k)}{\partial S(j)} + \frac{\partial p(j)}{\partial S(j)}\right) + \frac{1}{1-\sigma}\frac{\epsilon_s}{S}$$

where

$$\frac{\partial p(i)}{\partial S(j)} = -\sigma(\sigma-1)(\mu_o-1)(1-1/\mu_o)\epsilon_o\left[\frac{\partial p(i)}{\partial S(j)} - p_o\frac{\partial P}{\partial S(j)}\right]$$

$$\frac{\partial p(k)}{\partial S(j)} = -\sigma(\sigma-1)(\mu_s-1)(1-1/\mu_s)\epsilon_s\left[\frac{\partial p(k)}{\partial S(j)} - p_s\frac{\partial P}{\partial S(j)}\right]$$

$$\frac{\partial p(j)}{\partial S(j)} = \sigma(\mu_s-1)(1-1/\mu_s)\left(p_s\frac{\epsilon_s}{S} - (\sigma-1)\epsilon_s\left[\frac{\partial p(j)}{\partial S(j)} - p_s\frac{\partial P}{\partial S(j)}\right]\right).$$

Putting all these together, it can then be shown that $\frac{\partial P}{\partial S(j)} < 0$, $\frac{\partial p(k)}{\partial S(j)} < 0$, $\frac{\partial p(i)}{\partial S(j)} < 0$, $\frac{\partial p(j)}{\partial S(j)} > 0$, $\frac{\partial \epsilon(j)}{\partial S(j)} > 0$, $\frac{\partial \epsilon(k)}{\partial S(j)} < 0$, and $\frac{\partial \epsilon(i)}{\partial S(j)} < 0$. Finally, $\frac{\partial \epsilon(j)}{\partial S(j)}$, $\frac{\partial P}{\partial S(j)}$, and $\frac{\partial p(j)}{\partial S(j)}$ can be substituted in the first-order condition $\frac{\partial \pi(j)}{\partial S(j)} = 0$ to find the product scope

$$S = f(\mu_s, \mu_o, N, M, \sigma)\frac{Y}{\phi_s p_s}$$

where $f > 0$.

A.2 Dynamic entry

This Appendix presents the version of the model where the entry of mono-product firms is dynamic as in Bilbiie et al. (2012) and shows that indeterminacy remains. A prospective entrant i computes their expected value

$$v_t(i) = E_t\sum_{s=1}^{\infty} Q_{t,s}\pi_{o,t+s}(i)$$

where $Q_{t,s}$ is the stochastic discount factor and $\pi_{o,t}(i)$ denotes profits of ordinary firms. There is a time-to-build lag in that period t entrants begin operating in period $t+1$ and the number of firms evolves according to

$$M_t = (1-\delta_M)(M_{t-1} + M_{E,t-1})$$

where δ_M is the exogenous exit probability and $M_{E,t}$ is the number of entrants. Entry occurs until the expected value, $v_t(i)$, is equal to the sunk cost of entry. To enter, f_E amount of labor needs to be hired and since labor is paid the real wage w_t, this sunk cost is equal to

$$v_t(i) = w_t f_E.$$

The production function for new firms is thus

$$M_{E,t} = \frac{H_{E,t}}{f_E}$$

where $H_{E,t}$ is the amount of labor hired for the production of new firms. In a symmetric equilibrium, a representative household enters period t with mutual fund share holdings x_t and has the budget constraint

$$C_t + I_t + v_t(M_t + M_{E,t})x_{t+1} = (\pi_{o,t} + v_t)M_t x_t + w_t H_t + r_t U_t K_t + N\pi_{s,t}$$

where $\pi_{s,t}$ are profits from a constant number of superstar firms and $H_t = H_{E,t}+H_{o,t}+H_{s,t}$. The Euler equation for share holding is then

$$v_t = E_t \beta(1-\delta_M)\frac{C_t}{C_{t+1}}(\pi_{o,t+1} + v_{t+1}).$$

Imposing the equilibrium condition $x_{t+1} = x_t = 1$ for all t gives

$$C_t + I_t + v_t M_{E,t} = \pi_{o,t}M_t + w_t H_t + r_t U_t K_t + N\pi_{s,t} \equiv Y_t$$

where Y_t is GDP consisting of consumption, investment in capital, and investment in new firms. Total investment is then

$$X_t \equiv I_t + v_t M_{E,t}$$

and the CES aggregator is now

$$Y_{g,t} = C_t + I_t = \left(\sum_{i=1}^{M} x_t(i)^{\frac{\sigma-1}{\sigma}} + \sum_{j=1}^{N} Y_t(j)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

Small firms no longer have firm-level fixed costs and the symmetric equilibrium goods production is then

$$Y_{g,t} = \frac{p_{o,t}U_t^{\alpha}K_{o,t}^{\alpha}H_{o,t}^{1-\alpha}}{M_t\epsilon_{o,t}} = \frac{p_{s,t}U_t^{\alpha}K_{s,t}^{\alpha}H_{s,t}^{1-\alpha} - p_{s,t}NS_t\phi_s - p_{s,t}N\phi_f}{N\epsilon_{s,t}}.$$

We calibrate the model as in Section 3 and additionally set $\delta_M = 0.025$. Analogous to Figure 1, Figure A1 plots the indeterminacy and determinacy zones for different levels of superstar market shares. Indeterminacy not only remains but now exists for a greater range of parameters. The reason is due to the interaction between entry of ordinary firms and the product scope of superstars. Entry pushes the market shares and markups of both firm types downwards. However, superstar firms are able to defend their market shares by increasing their product scopes. Since higher product scopes and more small firms both increase efficiency via the love of variety effect, the wage-hours locus becomes steeper relative to the labor supply curve and indeterminacy becomes more plausible.

A.3 Determinacy model

Table 4 presents the prior and posterior distributions of the estimated determinate version of the model. The main difference here is that the animal spirits shock is no longer

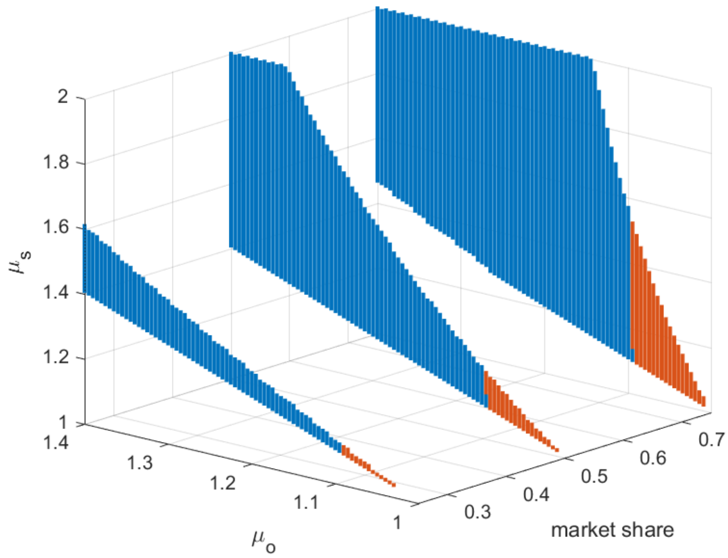

Figure A1: Dynamic entry model: indeterminacy (blue) and determinacy (orange).

available and in order to keep the number of shocks equal to observables, we introduce a temporary technology shock, A_t^T, that affects all firms equally with persistence ψ_T and variance σ_T. For example, the output of an ordinary firm is now

$$x_t(i) = A_t^T k_t(i)^\alpha \left[A_t h_t(i)\right]^{1-\alpha} - \phi_{o,t}$$

where

$$\ln A_t^T = \psi_T \ln A_{t-1}^T + \varepsilon_t^T.$$

Comparing the log-data densities between Tables 1 and 4, data clearly favors the indeterminate model with a strong product creation mechanism and animal spirits shocks.

A.4 Data sources

This Appendix details the source and construction of the U.S. data used in Section 4. All data is quarterly and for the period 1990:I-2019:IV.

1. Gross Domestic Product. Seasonally adjusted at annual rates, billions of chained (2009) dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.6.

2. Gross Domestic Product. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Personal Consumption Expenditures, Nondurable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

4. Personal Consumption Expenditures, Services. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

Table 4: Prior and posterior distributions for the determinate model

Prior					Posterior	
Name	Range	Density	Mean	Std. Dev.	Mean	90% Interval
ν	R^+	Normal	0.075	0.05	0.01	[0.00,0.02]
b	[0,1)	Beta	0.5	0.1	0.42	[0.34,0.48]
ψ_T	[0,1)	Beta	0.5	0.2	0.90	[0.87,0.92]
ψ_A	[0,1)	Beta	0.5	0.2	0.07	[0.05,0.09]
ψ_z	[0,1)	Beta	0.5	0.2	0.90	[0.83,0.98]
ψ_Δ	[0,1)	Beta	0.5	0.2	0.99	[0.98,0.99]
ψ_g	[0,1)	Beta	0.5	0.2	0.98	[0.97,0.99]
ψ_{ag}	[0,1)	Beta	0.5	0.2	0.99	[0.99,0.99]
σ_T	R^+	Inverse Gamma	0.1	Inf	0.12	[0.10,0.14]
σ_A	R^+	Inverse Gamma	0.1	Inf	0.78	[0.69,0.87]
σ_z	R^+	Inverse Gamma	0.1	Inf	0.02	[0.02,0.03]
σ_Δ	R^+	Inverse Gamma	0.1	Inf	0.47	[0.41,0.53]
σ_g	R^+	Inverse Gamma	0.1	Inf	0.80	[0.71,0.89]
$\sigma^{m.e.}$	$[0, 0.18]$	Uniform	0.09	0.05	0.18	[0.18,0.18]
$\varkappa$	[-20,0]	Uniform	-10	5.77	-15.03	[-19.20,-11.37]

This table presents the prior and posterior distributions for model parameters and shocks of the determinate model. Log-data density (modified harmonic mean): -914.66.

5. Personal Consumption Expenditures, Durable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

6. Gross Private Domestic Investment, Fixed Investment, Residential. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

7. Gross Private Domestic Investment, Fixed Investment, Nonresidential. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

8. Government consumption expenditures and gross investment. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

9. Nonfarm Business Hours. Index 2009=100, seasonally adjusted. Source: Bureau of Labor Statistics, Series Id: PRS85006033.

10. Civilian Noninstitutional Population. 16 years and over, thousands. Source: Bureau of Labor Statistics, Series Id: LNU00000000Q.

11. GDP Deflator = (2)/(1).

12. Real Per Capita Output, $Y_t = (1)/(10)$.

13. Real Per Capita Consumption, $C_t = [(3) + (4)]/(11)/(10)$.

14. Real Per Capita Investment, $X_t = [(5) + (6) + (7)]/(11)/(10)$.

15. Real Per Capita Government Expenditures, $G_t = (8)/(11)/(10)$.

16. Per Capita Hours Worked, $H_t = (9)/(10)$.