

# Fair Allocation in Hierarchies: A Compromise between Marginalism and Egalitarianism

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## Abstract

This paper explores the fair allocation of economic surplus among individuals within hierarchical social structures, incorporating the seemingly conflicting principles of marginalism and egalitarianism. We formalize this situation as cooperative games with permission structures and introduce a novel class of allocation rules called *egalitarian permission values*, extending the traditional egalitarian Shapley values. Our main contribution lies in establishing an axiomatic foundation for this class of rules through a monotonicity axiom. Additionally, we offer a monotonicity-based foundation for the permission value as a special case. Our results also uncover that a monotonicity property alone cannot justify the use of linear allocation rules in hierarchical structures among individuals, which is in sharp contrast with the conventional knowledge in the literature.

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*Keywords:* fairness, monotonicity, Permission value, Shapley value, TU-game with permission structures.

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