Fixation of inequality and emergence of the equal split norm: Approach from behavioral bargaining theory

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Abstract

Negotiation is at the heart of communication, social exchange, and economic transactions. Using the bargaining model as the unit of analysis, this study aims to deepen our understanding of negotiation and economic behavior based on the behavioral bargaining theory (BBT) developed by Kamijo and Yokote (2022). We introduce a key concept to analyze a bargaining situation: the *stability* of entitlements (people's expectations of distribution or sense of ownership) for a bargaining pie. When a pair of entitlements initially formed is stable, negotiations are expected to end immediately; when unstable, negotiations are more likely to end in delay or failure. We show the boundary condition for stable entitlements and find that some unequal distribution between two players can be stable even for a symmetric bargaining problem. By seeking stable entitlements for all members of society, it is possible to define a distribution norm mathematically. We show that the distribution norm that arises in a symmetric situation is the golden rule of distribution: 50–50 split of the pie. Finally, by examining the dynamic process of the formation of entitlements, we clarify the sufficient conditions under which the equal split norm emerges.

Keywords: Behavioral bargaining theory, Nash solution, Reference dependent utility, Stable entitlement, Inequality cap theorem, Distribution norm, Equal split norm

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1 Introduction

Negotiation is at the heart of communication, social exchange, and economic transactions. It occurs between families, friends, students and faculty, workers and employers, and businesses. A standard tool to analyze the bargaining problem is *Nash's bargaining model*, and it becomes the basis for the analysis of negotiations, both theoretically and in applications.

Nash's model are widely recognized for its generality and wealth of theoretical support.[1] However, there is skepticism about it from both practical and experimental perspectives (Raiffa, 1982). It is well known that the observed bargaining agreements in a laboratory are biased towards the equal split of the bargaining pie (Anbarci and Feltovich, 2013, 2018; Birkeland and Tungodden, 2014; Hoffman and Spitzer, 1982; Nydegger and Owen, 1974; Roth, 1995), but it is difficult to explain such a phenomenon from the Nash solution. It has been reported that the influence of disagreement point on the results of negotiations is smaller than one expected in the Nash solution (Anbarci and Feltovich, 2013; Kamijo, 2023a). Moreover, many studies (Baranski, 2016, 2019; Cappelen et al., 2007; Luhan et al., 2019; Takeuchi et al., 2022) have shown that experimental results differ significantly between cases in which bargaining pies are produced from the earnings of experiment participants and those in which they are given as gifts from heaven. It is challenging to understand these results from the framework of standard bargaining problems.

Kamijo and Yokote (2022) built a new bargaining theory, called the *behavioral bargaining theory* (BBT), by incorporating the *reference-dependent utility* a la Tversky and Kahneman (1979) into Nash's theory, and succeeded in explaining the deviation from the traditional model by considering the variety of different feeling of entitlement caused by the experimental manipulations. For example, the BBT can describe the bargaining outcome affected by an equal split norm, which differs from the one caused by the preference for equality. In addition, the new theory can explain how the bargaining results are different between the cases wherein the bargaining pie is given by the experimenter and produced from the participants' efforts. They also examined the empirical validity of the theory by borrowing the experimental data from Takeuchi et al. (2022) and confirmed the usefulness of BBT in explaining and interpreting the data.

This study aims to deepen our understanding of negotiation and economic behavior by using the framework of the BBT. While the BBT provides great applicability by incorporating entitlements theoretically, essential questions remain as to how people form entitlements and what entitlements are possible. We extend the BBT to shed light on these fundamental questions and address situations in which players form entitlements endogenously.

To this end, we consider a situation in which bargainers participate in bargaining repeatedly rather than only once. Suppose that, after the first round, a bargainer realizes that the amount of money she feels entitled to is largely different from the one the opponent proposes. Then, in the next round, she would revise her entitlement closer to the proposed amount in pursuit of an agreement. If this repetition occurs sufficiently many times, then the process would result in a steady state, or a fixed-point mathematically, where the subjective entitlements coincide with the objective bargaining outcome. The notion of stability formalizes this steady state: we say that an entitlement E_i for two players $i = 1, 2$ is stable if the Nash solution applied to the instance with total bargaining pie $M = E_1 + E_2$ returns back to the original entitlement (E_1, E_2).

A notion of stable entitlements is also helpful in understanding when negotiations begin and how they proceed. If players are rational enough to expect the consequence of their bargaining,

[1] Axiomatizations of the Nash solution are found in Nash (1950), Roth (1979) and Thomson (1994). Non-cooperative games whose equilibrium outcome coincides with the Nash solution are discussed in Nash (1953), Binmore et al. (1986), Anbarci and Feltovich (2013) and Kamijo (2023b). In addition to these theoretical supports to the Nash solution, its several interpretations are provided by Young (1993), Rubinstein et al. (1992) and Bastianello and LiCalzi (2019).

a negotiation might not begin when a pair of entitlements initially formed is stable because they cannot change the outcome from the initial proposal through the bargaining. Alternatively, even if the bargaining starts, it is expected to end immediately. In contrast, when the initial entitlements are unstable, the negotiation is more likely to end in delay or failure if there is a time limit.

We present three theorems concerning stable entitlement and its applications. First, we characterize stable outcomes in terms of the level of inequality. Our *inequality cap theorem* states the following: (i) entitlements are stable if and only if the level of inequality at the bargaining outcome is within a particular region, and (ii) the region is characterized by the parameter of loss aversion. In light of this theorem, an unequal distribution becomes stable when a loss-averse player in an advantageous position sticks to the distribution. It reveals an interesting relationship between equality and loss aversion; while strong loss aversion seems to promotes equality, it is suggested that inequality, once established between two parties, is more likely to sustain by the loss aversion of the advantageous player. Furthermore, this theorem is useful to understand why people often refrain from negotiation even though they seem to have a strategic advantage and why bargainers in the laboratory often fail to reach an agreement where they have contrasting self-serving fairness (Roth and Murnighan, 1982).

Second, we define a *distribution norm* by using the concepts of stable entitlements. A distribution norm in a society is one that is acceptable to all potentially existing members of the society. Here, we will consider a variety of people as entities with different loss aversion parameters. A distribution norm is one that, once accepted, will remain robust against bargaining between two players from such a diverse group of people. We show that for a symmetric bargaining situation in the sense that both disagreement outcomes are the same, the distribution norm should be an equal split of the bargaining pie. Therefore, only the equal split norm satisfies the requirements of a distribution norm in society.

Third, we investigate how the *equal split norm* emerges among players. We consider a society consisting of a fixed number of players and formalize the aforementioned entitlement-revision process: two members are matched randomly with some probability, conduct bargaining, and revise their entitlements. The question then arises: when this matching process occurs sufficiently many times, do the members of the society concur on the equal division of the total bargaining pie? To put it differently, does the equal split norm emerge among the society members? We provide two sufficient conditions for this claim to hold: (i) the bargaining is conducted for a pie that falls short of their requirements, or (ii) the society displays a certain form of homogeneity. These findings are consistent with observations in real-life allocation problems.

The paper is organized as follows. In the next section, we briefly explain the setting of the BBT. In Section 3, we introduce a notion of the stability of entitlements and show the inequality cap theorem, one of our main results. We discuss a distribution norm of society in Section 4, and we consider how an equal split norm arises in Section 5. Section 6 concludes the paper.

2 Model

2.1 Simple two-player bargaining game and Nash solution

A *bargaining problem* consists of players, a potential gain of agreement, and a disagreement outcome. Two players 1 and 2 bargain over a fixed amount M of some divisible good in pursuit of an agreed-upon outcome (x_1, x_2) with $x_1 + x_2 = M$. If the negotiation breaks down, player i gets $v_i \geqq 0$, of the divisible good for each $i = 1, 2$. A disagreement outcome is denoted by $v = (v_1, v_2)$. As the context makes clear, we will denote the bargaining problem by (M, v) without indicating the players.

One of the most prominent solutions to the bargaining problem is the *Nash solution*. The Nash solution selects the utility pair that maximizes the product of the players' utility differences between the agreed and disagreeable outcomes (this product is called the *Nash product*).

Let u_i $(i = 1, 2)$ denote i's utility function. Formally, the Nash solution chooses an allocation (x_1^*, x_2^*) that is a solution to the following maximization problem:

$$\max \ (u_1(x_1) - u_1(v_1)) \times (u_2(x_2) - u_2(v_2))$$
$$s.t. \ \ x_1 + x_2 = M, \ \ x_1 \geqq v_1, \quad x_2 \geqq v_2.$$

We refer to the pair (x_1^*, x_2^*) as the *Nash allocation* and write

$$x^*(M, v) = (x_1^*(M, v), x_2^*(M, v)).$$

2.2 Bargaining model with a reference-dependent utility

As in Kamijo and Yokote (2022), we consider a bargaining between two players having entitlements $E_1 \geqq 0$ and $E_2 \geqq 0$. We assume that a player has a reference-dependent utility wherein the entitlement has a role of the reference. We model the utility function as a monotonically increasing function considering asymmetry before and after the reference point. Let Ψ be a function on $[0, \infty)$ with $\Psi(0) = 0$, $\Psi'(x) > 0$, and $\Psi''(x) < 0$. Then, the utility function is defined as the result of its simple transformation to model a loss aversion. Given a loss aversion parameter $\lambda_i \geqq 1$, for any $x_i \geqq 0$, the utility function of player i is defined by

$$u_i(x_i; E_i) = \begin{cases} \Psi(x_i - E_i) & \text{if } x_i \geqq E_i, \\ -\lambda_i \Psi(E_i - x_i) & \text{if } x_i < E_i. \end{cases} \tag{1}$$

If needed, it can be possible to add some positive constant in order to assure that $u_i(x_i; E_i) \geqq 0$.

In the *behavioral bargaining theory* (BBT) by Kamijo and Yokote (2022), the Nash solution is calculated using the reference-dependent utility defined in (1). An allocation calculated in this way depends not only the bargaining problem (M, v) but also the pair of entitlements (E_1, E_2).[2] As a result, the Nash allocation becomes a function that associates $(M, v, (E_1, E_2))$ with the efficient agreement, and is denoted by

$$x^*(M, v, (E_1, E_2)) = (x_1^*(M, v, (E_1, E_2)), x_2^*(M, v, (E_1, E_2))).$$

Kamijo and Yokote (2022) find that various experimental results that deviate from predictions from conventional models can be explained by considering (E_1, E_2) that vary due to the experimental manipulations. For example, setting $E_1 = E_2 = M/2$ allows the BBT to describe bargaining outcomes affected by an equal split norm. It can also explain how bargaining outcomes differ when the experimenter gives the bargaining pie or when the participants' efforts generate it. The former can be captured by setting $E_1 = E_2 = 0$, and the latter can be modeled using an entitlement function that relates (M, v) and (E_1, E_2). They consider four types of entitlement functions and test which type can better explain the data of Takeuchi et al. (2022). They conclude that the type of entitlement function selected is consistent with Takeuchi et al. (2022) experimental manipulations and participants' responses in the post-experimental questionnaire.

3 Stability of entitlements and fixation of inequality

[2] Kamijo and Yokote (2022) considered the following types of entitlement function γ_i for each $i = 1, 2$: Equal Split Norm is $\gamma_i(M, v) = M/2$; Manna from Heaven is $\gamma_i(M, v) = 0$; Disagreement is $\gamma_i(M, v) = v_i$; Proportion is $\gamma_i(M, v) = \frac{v_i}{v_1 + v_2} M$.

3.1 Stable entitlements

The concept of entitlement examined in Kamijo and Yokote (2022) and other literature is a relatively objective one with some basis in evidence (e.g., input earned in the production stage, and the history-dependent status quo). However, the entitlement is essentially subjective one and the feeling of that is very fragile and even varies in the process of the bargaining. Some study attempts to analyze such a situation in a non-cooperative approach under a certain bargaining protocol (Compte and Jehiel, 2003). In contrast, Shalev (2002) focus on the property that the bargaining agreement should satisfy under the situation that the reference point is endogeneously formed, without specifying the bargaining protocol. Here, we follow the approach of Shalev (2002).

We introduce the notion of stable entitlements, which arise after the endogenous formation of entitlements. Stable entitlements are those in which the subjective sense of entitlement of the two parties through negotiation is consistent with the outcome of the agreement. Therefore, no matter the negotiation process, stable entitlements can be regarded as an agreement that both parties will not withdraw later.

Formally, the stability of entitlements is defined as follows.

Definition 1. A pair of entitlements $E = (E_1, E_2)$ is *stable* to a simple bargaining problem (M, v) if and only if

$$x_i^*(M, v, E) = E_i$$

holds for any $i = 1, 2$.

Therefore, the pair of entitlements is stable when it is very the bargaining agreement under the condition that they have such entitlements. Mathematically, this is the fixed point of the bargaining function $x^{**}(E) := x^*(M, v, E)$. Since x^{**} is a continuous function defined on the compact set, the existence of a stable entitlement is guaranteed. By its definition, $E_1 + E_2 = M$ holds if (E_1, E_2) is stable to (M, v).

A notion of stability of entitlements is useful to understand when the bargaining begins or is delayed. Suppose that negotiations needs transaction cost $\epsilon > 0$ for both players, it starts when one of them call for it, and the results of the bargaining are well described by the Nash bargaining solution applied to the reference-dependent utility. Then, if an initial pair of entitlements (E_1, E_2) is stable and this is an initial proposal chosen by some way (e.g., status quo, one proposed by third party, etc.), they understand that the actual negotiation is useless and they immediately agree on (E_1, E_2) without tough negotiation. In later subsections, we discuss which pair of entitlements can be stable.

3.2 Inequality cap theorem

Some may think that if we admit the subjective assessment of an entitlement, we can explain almost everything. In other words, does our theory of the subjective entitlement have falsifiability? As the following theorem states, we can provide positive answer to this question by analyzing the agreement that can be justifiable to the entitlements. In particular, an excessively unequal outcome cannot be justified as a pair of stable entitlements. Thus, requiring the stability of entitlements caps the possible inequality between the bargainers.

Theorem 1 (Inequality cap theorem)**.** Suppose that $v_1 + v_2 < M$, $v < E$ and $\Psi'(0) < \infty$. Then, E with $E_1 + E_2 = M$ is stable to (M, v) if and only if

$$\frac{1}{\lambda_1} \leqq \frac{u_2(E_2; E_2) - u_2(v_2; E_2)}{u_1(E_1; E_1) - u_1(v_1; E_1)} \leqq \lambda_2$$

holds true.

Proof. To simplify the notations, we write $u_i(x_i)$ to denote $u_i(x_i; E_i)$ for $i = 1, 2$.

We first show "only if" part. In this case, the Nash products at (E_1, E_2) is $(u_1(E_1) - u_1(v_1))(u(E_2) - u_2(v_2))$. Consider that the share of player 1 is slightly increased by x, and then, the Nash product is $f(x) = (u_1(E_1 + x) - u_1(v_1))(u_2(E_2 - x) - u_2(v_2))$. Then, $f'(x) = u_1'(E_1 + x)(u_2(E_2 - x) - u_2(v_2)) - (u_1(E_1 + x) - u_1(v_1))u_2'(E_2 - x)$. Since f is at least locally maximized at (E_1, E_2),

$$\lim_{x_i \to +0} f'(x) \leqq 0 \iff \Psi'(0)(u_2(E_2) - u_2(v_2)) - (u_1(E_1) - u_1(v_1))\lambda_2 \Psi'(0) \leqq 0$$

which is equivalent to the right-hand inequality relationship of the condition specified in this theorem since $E > v$. We obtain the left-side inequality by considering the case that share of player 1 is slightly decreased from the point E_1.

Next, we show the "if" part. Consider again the situation that the share of player 1 increases by x from the point of E_1. Then, again, the Nash product is $f(x)$. By the definition of u_i, for some $\Delta u > 0$, $u_1(E_1+x) = u_1(E_1)+\Delta u$, $u_2(E_2-x) = u_2(E_2)-\lambda_2\Delta u$, and $u_2'(E_2-x) = \lambda_2 u_1'(E_1+x)$. Since $f'(x) = u_1'(E_1 + x)(u_2(E_2 - x) - u_2(v_2)) - (u_1(E_1 + x) - u_1(v_1))u_2'(E_2 - x)$, we have

$$f'(x) = u_1'(E_1 + x)\Big(u_2(E_2) - \lambda_2\Delta u - u_2(v_2)\Big) - \Big(u_1(E_1) + \Delta u - u_1(v_1)\Big)\lambda_2 u_1'(E_1 + x)$$

$$= u_1'(E_1 + x)\Big(u_2(E_2) - u_2(v_2) - \lambda_2(u_1(E_1) - u_1(v_1)) - 2\lambda_2\Delta u\Big) < 0.$$

The inequality holds by the right-side inequality of the condition of this theorem. Thus, the increase of player 1's share from E_1 decreases the Nash product.

In contrast, let consider the situation that the share of player 1 decreases by x from the point of E_1. The Nash product in this case is $g(x) = (u_1(E_1 - x) - u_1(v_1))(u_2(E_2 + x) - u_2(v_2))$. By the definition of u_i, for some $\Delta u > 0$, $u_1(E_1 - x) = u_1(E_1) - \lambda_1\Delta u$, $u_2(E_2 + x) = u_2(E_2) + \Delta u$, and $u_1'(E_1 - x) = \lambda_1 u_2'(E_2 + x)$. Thus,

$$g'(x) = -u_1'(E_1 - x)(u_2(E_2 + x) - u_2(v_2)) + (u_1(E_1 - x) - u_1(v_1))u_2'(E_2 + x)$$

$$= -\lambda_1 u_2'(E_2 + x)\Big(u_2(E_2) + \Delta u - u_2(v_2)\Big) + \Big(u_1(E_1) - \lambda_1\Delta u - u_1(v_1)\Big)u_2'(E_2 + x)$$

$$= u_2'(E_2 + x)\Big(u_1(E_1) - u_1(v_1) - \lambda_1(u_2(E_2) - u_2(v_1)) - 2\lambda_1\Delta u\Big) < 0,$$

where the last inequality is because of the left-side inequality of the condition of this theorem. Thus, the Nash product is maximized at E_1. □

This theorem asserts that there is a reasonable range of the stable entitlements. Interestingly, we can use the loss aversion parameter λ to describe that range. Clearly, when $v_1 = v_2 = 0$, the equal division is only the stable entitlement in the absence of loss aversion (i.e., $\lambda_i = 1$ for $i = 1, 2$). On the other hand, as the loss aversion parameter increases, more and more entitlements become stable to $(M, (0, 0))$ (see Figure 1).

This theorem indicates that a certain unequal distribution among two people are sustained once it happens to be established by some reasons even in a symmetric problem. This may explain why some degree of unequal distribution of tasks and resources is ubiquitous even for small size group without any strategic reason. In contrast, if a considerable size of inequality is kept, there should exist some strategic reason behind it.

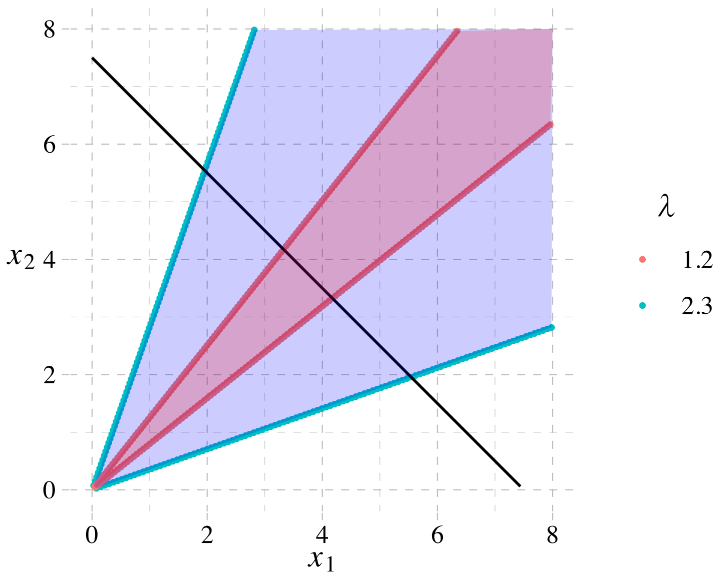

Figure 1: Regions of Stable Entitlements. We assume $\Psi(x) = x^{0.8}$.

The intuitive understanding of the theorem may seem easy at first glance, but it requires attention. Simply put, loss aversion is related to the status quo bias, making it easier for inequality to persist once it is established. However, such an interpretation is incomplete because it does not identify whether the status quo bias is at work for the strong or the weak. The theorem's statement arises because in cases where loss aversion is significant, the utility decrease of the strong due to the correction of inequality is relatively larger than the utility increase of the weak. Thus, the inequality prevails due to the status quo bias of the strong.

In the literature, Shalev (2002) discussed the similar property to stability (called the self-supporting property) in Nash's bargaining theory with endogenous formation of reference points, and obtained the similar results (Theorem 3.1 in his paper) to our inequality cap theorem. Moreover, he imposed an additional property, the robustness of the bargaining outcome to the manipulations of the opponent's reference point, to select the unique outcome from the region of stable ones. While the mathematical setup of Shalev (2002) is slightly different from this study (e.g., in the definitions of a reference-dependent utility), the real difference lies in the positioning and the goal. Shalev (2002) is positioning in the solution theory of bargaining. Thus, it sticks to obtaining the unique outcome and tries to provide axiomatic and non-cooperative foundations for using an extension of the Nash solution. In contrast, this study comes from the behavioral bargaining theory, which tries to understand the observed bargaining outcome in the laboratory and provide useful insights on the economic activity related to the bargaining. Indeed, in the later subsections we will show that the inequality cap theorem is quite useful for understanding the diversity of bargaining outcomes. Moreover, in Sections 4 and 5, we will apply the concept of stability beyond the level of two-person bargaining and discuss the distribution norm and the equal split norm, which are essential for understanding our economic phenomena.

3.3 Empirical support to the inequality cap theorem

In addition to providing some rationale to sustained inequality, the inequality cap theorem helps us understand the limits of inequality that can be tolerated from the standpoint of the disadvantaged party. To simplify the situation, assume that Ψ is close to linear within the bargaining range and therefore $\Psi(x) \approx x$. Then, the condition of the theorem is rearranged as follows,

$$\frac{E_s}{E_w} \leqq \lambda_s,$$

where E_s, E_w with $E_s \geqq E_w$ are the shares of the strong and the weak, respectively, and λ_s is the loss aversion parameter of the strong. Thus, the theorem states that if the ratio of inequality between the strong and the weak is greater than or equal to λ_s, then such inequality is unacceptable from the weak perspective. In fact, in the extensive literature on experimental ultimatum bargaining, offers of 20 ~ 30% of the total pie (i.e., inequality ratio $\frac{x_{\text{proposer}}}{x_{\text{responder}}}$ is around 7/3 ~ 4) are often rejected by the responders (for review, see Güth and Kocher (2014)).

While an interesting consistency, this is not a good example of the application of our theorem because the ultimatum bargaining game is a rule-based bargaining game. More direct evidence is found from the experiment of Takeuchi et al. (2022). In their experiment, experimental participants earn a disagreement payoff through their efforts in the first stage, and in the second stage, paired participants freely negotiate the distribution of the pie produced with their disagreement payoffs as input. The relation between a bargaining pie M and their disagreement payoffs (v_1, v_2) is manipulated, and in one treatment, termed the proportional surplus treatment, it is explained as $M = \beta(v_1 + v_2)$ where $\beta > 1$ is changed across rounds according to certain rules.

It is natural to think that experimental participants in the proportional surplus treatment will form a proportional entitlement defined by

$$E_i^{prop} = \beta v_i = M\frac{v_i}{v_1 + v_2} \quad \text{for } i = 1, 2$$

at least before negotiations begin. Two pieces of evidence reinforce this conjecture. First, after the experimental task, Takeuchi et al. (2022) ask participants to answer which way of division is most preferred, the equal split division $(M/2, M/2)$, the equal surplus division $(\frac{M-v_1-v_2}{2} + v_1, \frac{M-v_1-v_2}{2} + v_2)$, or the proportional surplus division $(\frac{Mv_1}{v_1+v_2}, \frac{Mv_2}{v_1+v_2})$. They answered that the proportional surplus division is the most favorable in proportional surplus treatment. Second, Kamijo and Yokote (2022) conducted the data fitting to the data of Takeuchi et al. (2022), and showed that among the several types of entitlements, the proportional entitlements explain the data in the proportional surplus treatment the best.

Even though they have proportional entitlements initially, their actual agreements show some deviation from the proportional surplus division toward reducing inequality. To describe this point, we introduce two notions, the disagreement ratio DR and the agreement ratio AR defined by

$$DR = \frac{v_2}{v_1}, \quad AR = \frac{x_2 - v_2}{x_1 - v_1}$$

where player 2 is in stronger position than player 1 (i.e., $v_2 > v_1$) and (x_1, x_2) is their agreed allocation. If they agree on the proportional surplus division, $AR = DR$ should hold. Concerning the inequality cap theorem, AR should be no more than λ, under the assumption of the neutral component function. The relation between DR and AR in the data of Takeuchi et al. (2022) are summarized in Table 1. It is shown that even when DR is greater than or equal to 3, the means and the medians of AR are about 2.[3] Also, in this case, the value of AR is less than 3 for most of the data. Therefore, the majority of the agreed allocation belongs to the region that the inequality cap theorem suggests, even though they (probably) have very unequal entitlements at the beginning of the negotiation.

[3]Only the exception is mean of AR for $DR = 8$ where the agreements in this case show a considerable variance.

DR	Num of data	AR					
		Mean	Median	SD	% of $\leqq 2$	% of $\leqq 3$	% of $\leqq 4$
1.5	34	1.2627	1.3155	0.3977	97.06	100	100
2	80	1.5310	1.6275	0.5265	93.75	100	100
3	11	2.0936	2.0769	0.9213	45.45	81.82	100
4	53	2.5581	2.7500	1.4520	39.62	56.60	98.11
6	17	2.3758	1.8000	2.1755	70.59	76.47	76.47
8	20	4.5965	1.9615	4.5983	55.00	60.00	60.00

Table 1: Summary of the data in Takeuchi et al. (2022)
Note: $DR = v_2/v_1$ and $AR = (x_2 - v_2)/(x_1 - v_1)$ where (v_1, v_2) is pair of disagreement payoff satisfying $v_2 > v_1$ and (x_1, x_2) is the agreed allocation.

3.4 Some implications to the failure of the negotiation

The inequality cap theorem also gives us insight into whether bargainers can reach some agreement. Suppose an "unstable" proposal is being made, and one party (the disadvantaged party) reluctantly tries to accept it. Then, immediately after subjectively accepting the distribution proposal and it becoming the new status quo, they realize that there is still room for further negotiation (because the proposed allocation is unstable). Such renegotiation is expected to increase the likelihood of prolonged negotiations and a breakdown if there is a time limit.

An "unstable" proposal is likely to be made when two bargainers have different perspectives on fairness. In a series of binary lottery bargaining experiments by Roth and Malouf (1979), Malouf and Roth (1981), and Roth and Murnighan (1982), two subjects need to divide a certain number of tokens by negotiation. The split tokens correspond to the probability of winning a prize, and the amount of the prize varies between the two.[4]. Here, there are two types of fairness. One fairness criterion calls for the "equal probability agreement", and the other is for the "equal expected value agreement" (Murnighan et al., 1988). Indeed, the bargainer with a larger prize prefers the former and one with a smaller prefers the latter.

In the experiment of Roth and Murnighan (1982), the number of tokens is 100, and the larger and the smaller prizes are \$20 and \$5, respectively. Thus, the player with a larger prize calls for a 50:50 split, and one with a smaller prize calls for a 20:80 split. If they negotiate these as initial entitlements, it is expected that both sides will make certain compromises. However, when equity conflicts, often their bargaining becomes an argument about which equity is right, and they stick to one outcome or the other. If one bargainer sticks to the 20:80 split of tokens, the difference in tokens between the two sides is almost four times, and such a disparity may be more than the loss aversion parameter. Agreement on such a proposal is often tricky, and disagreement happens more often. In fact, Roth and Murnighan (1982) observed that under such conditions, about 22% of negotiations were not completed in time, even though they had more than 10 minutes for bargaining.[5]

4 Distribution norms

Stable entitlements can be considered as allocations that are reached in repeated negotiations for the same amount of the pie because once they reach some agreement, it becomes the status quo

[4]In this framework, the utility function of player i is $u_i(x_i; E_i) = P_i \times (x_i - E_i)$ when $x_i \geqq E_i$ and $P_i \times \lambda(x_i - E_i)$ when $x_i < E_i$, where x_i is the probability of obtaining the prize and P_i is the utility gained from the prize. Due to the independence of the Nash solutions by a linear transformation, it is possible to ignore the effect of the prize in this setting, and our inequality cap theorem holds.

[5]From Table IV of Roth and Murnighan (1982). In the conditions that both players know both prizes, the frequency of the disagreement is 5 out of 30 when their knowledge about their prizes becomes common knowledge and 9 out of 35 when this is not common knowledge.

and affects subsequent negotiations. When the agreement is stable, such a change in reference point cannot change the negotiation outcome. In this light, it is possible to regard stability as a closely related concept to establishing norms regarding distribution.

In the traditional textbook of von Neumann and Morgenstern (1944), they argue that the formation of norms (the standard of behavior in their terminology) occurs in the process of each person modifying themselves in response to experience. They consider the standard of behaviors as the results of such a process and define their solution concepts (called the stable set) as the system of allocations. In contrast, we model the process as adjusting the reference point or status quo in repetitive bargaining environments and focus on the property of an allocation that is invariant in the dynamic process.

Let's define distribution norms by adding one more point to their considerations of the standard of behavior. The new perspective is that distribution norms in society need to be acceptable to all potentially existing members of society. Here, we will consider various people as entities with different loss aversion parameters. A distribution norm is one that, once accepted, will remain robust against bargaining between two players from such a diverse group of people.

To clarify the dependency of Nash allocation to the loss aversion parameters, we denote it by $x_i^*(M, v, E; \lambda_1, \lambda_2)$ for $i = 1, 2$. Then, we define a distribution norm as follows.

Definition 2. A pair of entitlements $E = (E_1, E_2)$ is a *distribution norm* for a simple bargaining problem (M, v) if and only if

$$E_i = x_i^*(M, v, E; \lambda_1, \lambda_2)$$

holds for any $i = 1, 2$ and for *any* $\lambda_1 \geqq 1$ and $\lambda_2 \geqq 1$.

In other words, E is a distribution norm when it is a stable entitlement for any two bargainers in society. Thus, once some allocation E is considered a distribution norm to (M, v), any two players in the society facing the same bargaining problem agree on allocation E. In the definition, by taking any value of $\lambda_i \geqq 1$, we model a situation where there are many people having different values of loss aversion parameters.

It should be emphasized that in the definition of a distribution norm, players $i = 1, 2$ are treated as the role of a simple bargaining game, and any two individuals of the society are assigned one of these roles before the bargaining. Therefore, we assume that a distribution norm is anonymous in the sense that it does not depend on the identity of the bargainer, but it is role-dependent. Role dependency is an essential property of the norm that spreads consistently and widely (e.g., a norm regarding the division of housework between men and women). However, it may be useful to consider identity-based entitlement when we try to answer how a norm arises. This point is discussed in the next section.

By directly applying Theorem 1, we obtain the following theorem showing that an equal split is the only distribution norm to a simple bargaining problem $(M, (0, 0))$.

Theorem 2. $(M/2, M/2)$ is the only distribution norm to a simple bargaining problem $(M, (0, 0))$.

Proof. Applying Theorem 1 to the case that $\lambda_1 = \lambda_2 = 1$, we have $u_2(E_2; E_2) \leqq u_1(E_1; E_1)$, implying $\Psi(E_2) \leqq \Psi(E_1)$. But since $E_1 \leqq E_2$ and Ψ is increasing, we should conclude that $E_1 = E_2$.

It is easily checked that $(M/2, M/2)$ is a stable entitlement for any $\lambda_1 \geqq 1$ and $\lambda_2 \geqq 1$. □

One remark of this theorem is that it is easily extended to the statement that for any symmetric bargaining game $(M, (a, a))$ with $0 \leqq a \leqq \frac{M}{2}$, $(M/2, M/2)$ is the only distribution norm.

This theorem provides a rationale for why an equal split norm plays such an essential role in everyday life and the laboratory. This happens because the equal split allocation is a stable entitlement for any value of $\lambda_i \geqq 1$. Thus, this becomes the robust distribution by the negotiation between any bargainers with different loss aversion parameters.

Once a distribution norm is established, one can form a rational expectation to the bargaining result. Practically, this reduces the transaction cost regarding the bargaining, and they immediately agree on the one that the distribution norm suggests. In other words, they can avoid negotiations to save time. Theorem 2 provides a good explanation of the phenomena in the laboratory: experimental participants immediately agree on the equal split of the bargaining pie.

5 Emergence of the equal split norm

Theorem 2 states that once a norm of equal distribution is spread in society, this norm cannot be overridden by any other distribution. In reality, information transmission processes such as education, culture, and history play a role in sharing some norms among the members of society—however, the myth of how these norms came to be remains.

To demonstrate the process of the birth of the equal split norm, we consider the dynamic process of revising the entitlements of players through the repetition of bargaining. Let $N = \{1, 2, ..., n\}$ with $n \geqq 2$ be the finite set of people in a society. We consider the simple bargaining problem $(M, (0, 0))$ between two players in this society. For simplicity, we use the following notation

$$(x_i^*(M, (0, 0), (E_i, E_j); \lambda_i, \lambda_j), x_j^*(M, (0, 0), (E_i, E_j); \lambda_i, \lambda_j))$$

to denote the Nash allocation of $(M, (0, 0))$ between $i \in N$ and $j \in N, j \neq i$, where a pair of their entitlements is (E_i, E_j) and their loss aversion parameters are λ_i and λ_j, respectively.

For any i, let $\lambda_i \geqq 1$ be the loss aversion parameter of i, which is assumed to be fixed in this process. Let E_i^1 be the initial entitlement of i that will be updated through the bargaining experience. For each period $t = 1, 2, ...$, two players $i \in N$ and $j \in N, j \neq i$ are randomly chosen with fixed probability $p_{ij} > 0$, and they face the bargaining game $(M, (0, 0))$ with their entitlements E_i^t and E_j^t at that time period. Through this bargaining, they agree on the Nash bargaining allocation $(x_i^*(M, (0, 0), (E_i^t, E_j^t); \lambda_i, \lambda_j), x_j^*(M, (0, 0), (E_i^t, E_j^t); \lambda_i, \lambda_j))$ and this becomes their entitlements E_i^{t+1} and E_j^{t+1} in the next period. In contrast, if a player is not chosen at t, they keep the same entitlement to the next period. Let $(E_i^t)_{i \in N}$ be the vector of the society members' entitlements, which will be repeatedly updated due to the above dynamic rule as time goes on.

We say that the *equal split norm* emerges in the society if and only if $(E_i^t)_{i \in N}$ with $E_i^t = M/2$ for any $i \in N$ is realized and is never overwritten by another entitlement vector. From this framework of the dynamic adjustment of the entitlements, we can say that from Theorem 2, the "equal split" entitlement vector is a stable state of this dynamics in the sense that this vector will not be changed forever if it is realized. Our question here is whether the process converges to this state. The following theorem reveals the sufficient conditions of the convergence to the equal split norm.

Theorem 3 (Emergence of equal split norm)**.** Suppose that $\Psi(x) = x$ for any $x \geqq 0$. The dynamic process converges to the equal split norm in probability 1 if either of the following conditions is satisfied;

(i) for any $i \in N$, $E_i^1 \geqq M/2$, and

(ii) $n \geqq 3$, and for any $i \in N$, there exists another $j \in N$ with $j \neq i$ such that $\lambda_i = \lambda_j$.

Proof. Let ϵ be defined by $\epsilon = \min_{i,j\in N, i\neq j} p_{ij} > 0$.

(i) It suffices to show that for any state (the vector of entitlements) that can be reached from the initial state $(E_i^1)_{i\in N}$, there exists a finite sequence of bargaining pairs that leads to the equal split norm.

We first show the following claim:

Claim A. When $E_i \geqq \frac{M}{2}$ and $E_j \geqq \frac{M}{2}$, the Nash allocation is $(M/2, M/2)$.

Let x be the share of player i. Then, the Nash product is one of the following threes:[6]

$$\begin{cases} f_{LG}(x) = (-\lambda_i(E_i - x) + \lambda_i E_i) \times ((M - x - E_j) + \lambda_j E_j) & \text{if } x < M - E_j, \\ f_{LL}(x) = (-\lambda_i(E_i - x) + \lambda_i E_i) \times (-\lambda_j(E_j - (M - x)) + \lambda_j E_j) & \text{if } M - E_j \leqq x < E_i, \\ f_{GL}(x) = ((x - E_i) + \lambda_i E_i) \times (-\lambda_j(E_j - (M - x)) + \lambda_j E_j) & \text{if } x \geqq E_i. \end{cases}$$

Then, it is easily checked that $f_{LG}(x)$ is increasing in $[0, M - E_j)$, $f_{LL}(x)$ is maximized at $x = M/2$, and $f_{GL}(x)$ is decreasing in $[E_i, M]$.[7] Thus, it is maximized at $x = M/2$.

From this claim, we can derive two consequences. First, if $E_i^t \geqq M/2$ for any $i \in N$ at some period t, the same condition still holds true at the next period $t + 1$. Second, after some finite sequence of bargaining pairs, the entitlement of every player i becomes $M/2$. If n is even number, $(1, 2), (3, 4), ..., (n - 1, n)$ is such sequence and if n is odd number, $(1, 2), (3, 4), ..., (n - 2, n - 1), (n, 1)$ is the one. In both cases, the probability of obtaining such sequence is greater than or equal to $\epsilon^{\lceil n/2 \rceil}$.[8]

(ii) To prove this, we need to show the following claim:

Claim B. Suppose $\lambda_i = \lambda_j$ and $E_i < E_j = \frac{M}{2}$. Then, the Nash allocation is $(M/2, M/2)$.

Let x be the share of player i, and let $\lambda = \lambda_i = \lambda_j$. Then, the Nash product is one of the following threes:

$$\begin{cases} g_{LG}(x) = (-\lambda(E_i - x) + \lambda E_i) \times ((M - x - M/2) + \lambda M/2) & \text{if } x < E_i, \\ g_{GG}(x) = ((x - E_i) + \lambda E_i) \times ((M - x - M/2) + \lambda M/2) & \text{if } E_i \leqq x < M/2, \\ g_{GL}(x) = ((x - E_i) + \lambda E_i) \times (-\lambda(M/2 - (M - x)) + \lambda M/2) & \text{if } x \geqq M/2. \end{cases}$$

Then, it is easily checked that $g_{LG}(x)$ is increasing in $[0, E_i)$, $g_{GG}(x)$ is increasing in $[E_i, M/2)$, and $g_{GL}(x)$ is decreasing in $[M/2, M]$.[9] Thus, it is maximized at $x = M/2$.

To compete the proof of (ii), with the aid of (i) of this theorem, it suffices to prove that for any vector of the entitlements, there exists a finite sequence of bargaining pairs that leads to an entitlement vector $(E_i^t)_{i\in N}$ such that $E_i^t \geqq M/2$ for any $i \in N$. To describe such sequence, let the set of strong players and the set of weak players at some period t be defined as follows.

$$S = \{i \in N | E_i^t \geqq M/2\}, \quad \text{and} \quad W = \{i \in N | E_i^t < M/2\}.$$

We will show that after some repetitions of bargaining, $S = N$ and $W = \emptyset$ hold.

[6] This classification is due to the domain to which the each bargainer belongs. For example, $f_{LG}(x)$ is the Nash product when player 1 is in a loss domain and player 2 is in a gain domain and

[7] These properties are proved as follows. Any of the three functions are concave and quadratic function. So, it is enough to say that $f_{LG}(x)$ is maximized at $M/2 + (\lambda_j - 1)E_j/2$, which is greater than or equal to $M/2 \geqq M - E_j$; $f_{GL}(x)$ is maximized at $M/2 - (\lambda_i - 1)E_i/2$, which is less than or equal to $M/2 \leqq E_i$.

[8] $\lceil \cdot \rceil$ is a ceiling function defined by $\lceil x \rceil = \min\{z \in Z | x \leqq z\}$.

[9] These properties are proved as follows. Any of the three functions are concave and quadratic function. So, it is enough to say that $g_{LG}(x)$ is maximized at $(\lambda + 1)M/4$, which is greater than or equal to $M/2 > E_i$; $g_{GG}(x)$ is maximized at $(\lambda + 1)M/4 - (\lambda - 1)E_i/2$, which is greater than or equal to $M/2$; $g_{GL}(x)$ is maximized at $M/2 - (\lambda - 1)E_i/2$, which is less than or equal to $M/2$.

Suppose $S \neq N$. We first show that there is a finite sequence of bargaining pairs that leads to $|W| = 1$. To show this, it is important to notice that if two players in W bargain over M, their agreement should be efficient and thus, their updated entitlements should satisfy either (a) that both become strong players or (b) that one becomes strong and the other becomes weak. Thus, after the bargaining between weak players, the size of W should decrease at least by 1. Therefore, at most $|W| - 1 \leqq n - 1$ repetitions of bargaining, we obtain the entitlement vector with $|S| = n - 1$.

Next, we explain how we can obtain the entitlement vector satisfying $S = N$. Let $W = \{i\}$ at some period t. Then, by the assumption of the theorem, there exists some $j \in S$ such that $\lambda_j = \lambda_i$ and some $k \in S, k \neq j$. Consider the bargaining sequence such that (j, k) first, and then (j, i). The first bargaining leads to $E_j^{t+1} = E_k^{t+1} = M/2$ by Claim A and the second implies $E_j^{t+2} = E_i^{t+2} = M/2$ from Claim B. Hence, all players become the strong at period $t + 2$.

Finally, from the proof of (i) of this theorem, there exists a finite sequence with length at most $\lceil n/2 \rceil$ that leads to the equal split norm from any entitlement vector satisfying $S = N$. In sum, for any entitlement vector, there exists a finite sequence of bargaining realizing the equal split norm, and the probability of obtaining such sequence is greater than or equal to $\epsilon^{(n-1)+2+\lceil n/2 \rceil}$. □

Examining the sufficient conditions of the theorem is essential for understanding the emergence of the equal split norm. The first sufficient condition implies that the members' initial sense of entitlement is relatively large compared with the bargaining pie. This situation arises in loss-domain bargaining, where the bargaining pie is smaller than the expected level of their shares. The norms established in shelters after a disaster are an example of this situation.[10] Even in gain-domain bargaining, this sufficient condition is more likely to hold when people tend to overestimate their contribution (Kahneman and Tversky, 1996).

On the other hand, the second condition requires that the population size be somewhat large, and for any player, another player must have the same loss aversion parameter. Let us consider the opposite cases to understand why this condition is important. For example, when the group size is two, and the initial entitlement is stable between them, it will continue to sustain even if it is not an equal split. Also, even if there are three or more people, there can be cases where only one person with different loss aversion parameters continues to be treated poorly, and the equal split norm is not realized.[11] In other words, a certain amount of group size and a certain degree of homogeneity concerning their characteristics are essential for establishing the norm of equal division.

Finally, since the theorem assumes $\Psi(x) = x$, it applies to distribution problems concerning goods with small and medium stake sizes where the effects of diminishing marginal utility are negligible. Also, since this theorem considers the establishment of an equal-split norm through negotiation, it assumes that the society size is not too large so that one person can interact and communicate with every other group member.

We can also think about another type of updating process of entitlements. If players learn the feeling of entitlement from others, it is natural that they learn from the person who obtains most

[10]The implication that the norm of equal split arises in loss domain situations is consistent with the anthropological literature describing communal sharing systems. They explain that sharing systems exist in response to resource-poor environments (Kaplan et al., 1985; Blurton-Jones, 1984).

[11]An example of this situation is three player society wherein $\lambda_1 = \lambda_2 = 2$ and $\lambda_3 = 4$, and their initial entitlements vector is $E^0 = (0.5, 0.5, 0.4)$ with $M = 1$. Then, the bargaining between players 1 and 3 gives the allocation $(0.6, 0.4)$, which is a stable entitlement for these players. Thus, player 3's entitlement does not change from the initial value even after repeated bargaining because his bargaining partner's entitlement is either 0.5 or 0.6.

from the bargaining problem. Suppose that unmatched players in a certain period update their entitlements to the highest value in society. Then, the situation described in (i) of Theorem 3 quickly occurs, and the equal split norm will be born immediately. Therefore, social learning facilitates the emergence of the equal split norm.

The emergence of the equal split norm has been discussed in the literature on the evolutionary bargaining theory (Young, 1993; Binmore et al., 2003). Ellingsen (1997) found that the equal split norm is the stable state of the evolutionary dynamic process of the Nash demand game. Our theorem offers an alternative explanation to the evolutionary model with at least two strengths. First, the evolutionary models usually assume a perfectly homogeneous group of people, while our model allows for a diverse group of people described by different loss aversion parameters and entitlements. Second, as Dawid and Dermietzel (2006) argue, the emergence of the equal split norm depends on the type of evolutionary dynamic process and the parameters selection, but their economic interpretation needs to be more obvious. In contrast, the sufficient conditions of the theorem have clear interpretations and provide valuable insights into when the norm emerges.

6 Concluding remarks

The analysis in this paper provides deep insights into social exchange and economic transactions through negotiation. The discussion of stability provides perspective on the progress of the negotiations: will they begin, be protracted, or be likely to break down? The inequality cap theorem also reveals, on the one hand, that a certain degree of inequality can be maintained in a symmetric bilateral relationship (in terms of an objective bargaining problem) and, on the other hand, that equal outcomes can be accepted in an asymmetric bilateral relationship. The latter point was discussed by Kamijo and Yokote (2022) as the bargaining outcome affected by the equal split norm.

By applying the stability argument to groups, we formulated a distribution norm mathematically and showed that the distribution norm should be an equal split in a reasonably large group. Furthermore, by considering a model in which entitlements of members in society are dynamically updated, we found a sufficient condition for an equal split norm to be self-generating in society. The results of these analyses provide a series of mechanisms by which equally splitting a pie, the golden rule of distribution problems, is born and spread.

We end the paper with one remark about future extensions inspired by the inequality cap theorem. The inequality cap theorem indicates that our fairness consideration is closely related to our propensity for loss aversion. Reference dependence, the basis for the definition of loss aversion, appears to be a more fundamental element in human systems than the pursuit of fairness. Thus, it may be promising to construct a new theory of equity and fairness from this aspect.

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