The Return On Private Capital Across Countries: Rising And Diverging

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Abstract

Across 88 countries for 1970-2014, the return on private capital (ROK^{P}) has exhibited two phases, approximately constant from 1970-1990, but then rising dramatically from 1991-2014. This latter increase occurs for both Rich/Developed and Poor/Developing countries, though at an uneven pace; the Lucas Paradox seems to have become more pronounced in recent years. Despite falling real interest rates lowering ROK^{P}'s, 60% [163%] of the secular rise in $ROK^{P,POOR}$ [$ROK^{P,RICH}$] is explained by rising equity risk, depreciation, and markups and by the capital loss from expected decreases in the relative price of new capital. Policy implications are discussed.

JEL Codes:
F21 (International Capital Movements), E22 (Capital), O10 (Economic Development)

Keywords:
International Capital Allocations, Return On Private Capital

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Highlights

The return on private capital (ROK^P) has exhibited two phases between 1970-2014

Across 88 countries, ROK^P has been approximately constant from 1970-1990

But, from 1991-2014, ROK^P has risen dramatically, especially for Poor countries

The Lucas Paradox seems to have become more pronounced in recent years

A good part of ROK^P increases can be explained by the user cost of private capital

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> …the central concept in capital theory should be *the rate of return on investment*… almost any important planning question we wish to ask about the saving-investment process has an unambiguous if perhaps approximate answer in terms of rates of return…
>
> Solow, 1964, p. 16

> …the welfare effects of debt depend not only on how low the average safe rate is, but also on how high the average marginal product is.
>
> Blanchard, 2019, p. 1200

1. Introduction

The return on an investment in private capital is central to many ongoing academic and policy discussions. The Secular Stagnation hypothesis advanced by Summers (2015, 2020) links a persistent decline in investment demand to a persistent decline in the return on capital. The extent to which capital is misallocated globally and hence the scope for foreign aid and other policies to improve welfare depends on the return on capital (Caselli and Feyer, 2007). Along a balanced growth path, the "Golden Rule" (Phelps, 1961) and dynamic efficiency (Diamond, 1965; Abel, Mankiw, Summers, and Zeckhauser, 1989) depend on the return on capital. The ominous prediction by Piketty (2014) of a secular increase in wealth inequality is based on the return on capital exceeding the growth rate of income. Understanding capital formation is important for understanding the divergence in per capita income across countries (Mankiw, Romer, and Weil, 1992). The implications for fiscal and debt policies are highly dependent on the return on capital. Reis (2022) argues that the sustainable level of government borrowing increases with the return on capital relative to the return on a public bond. Blanchard (2019) emphasizes the link between the cost of public debt and the profitability of alternative uses of resources in capital formation. He finds that this return is low, and hence the cost of public debt is low as well. As indicated by the above two quotations, the centrality of the return on capital has endured for decades.

This paper examines the return on capital across countries. Based on data for 1970 to 2014, we find that return on private capital (ROK^P) has exhibited two phases, being more or less constant from 1970 to 1990, but then rising dramatically for the remaining part of the sample. This latter increase occurs for both Rich/Developed and Poor/Developing countries, though at different rates. Thus, the gap has widened in recent years; that is, the Lucas Paradox -- why the return on capital remains relatively high in Poor countries – seemingly has grown even more perplexing.[1] The secular rise in the price of output relative to the price of new capital is one of the primary reasons for the rising and diverging ROK^P's. A rising ROK^P may seem puzzling in light of the well-known dramatic fall in real interest rates. However, ROK^P and the real rate are separated by several wedges -- risk, depreciation, capital losses, markups, and taxes. To understand the quantitative importance of these wedges, we evaluate the sources of rising and diverging ROK^P's in a calibration exercise with the user cost of capital. Despite a falling real interest rate, 60% [163%] of the overall secular rise in ROK^P's for Poor [Rich] countries can be explained by rising equity risk premia, depreciation, and markups and by the capital loss from expected increases in the relative price of new capital. Policy implications are discussed in the concluding section.

[1] The Lucas Paradox (Lucas, 1990) is properly evaluated in terms of steady-state values of the ROK^P's. In the presence of costly capital adjustment and gradual transition paths, the steady-state ROK^P's depend on the "spot" ROK^P's studied in this paper, a model of capital adjustment, and adjustment parameters.

2. Measuring The Return On Private Capital (ROK^P)

We take the perspective of investors allocating private capital and compute the return they would enjoy. Relating ROK^P to observables follows from two identities (Poterba, 1998, eqns. 2 and 3),

$$(1) \qquad ROK^P \equiv CIS^P * \left(Y / K^P\right) * \left(PY / PK^P\right) = MPK^P * \left(PY / PK^P\right),$$

where CIS^P is the private capital income share and (Y/K^P) is the output/private capital ratio. Their product equals the marginal product of capital (ROK^P; this result follows from an application of Euler's Theorem; cf. (3) below). The latter component (PY/PK^P) is the inverted relative price of capital. This relative price, which we refer to as the relative price of output, is unity in a one-good model and, in this case, the ROK^P equals the ROK^P. However, as emphasized by Caselli and Feyrer (2007), this relative price is required in a two-good model in which the prices of capital and output may diverge. This divergence proves very important in the empirical work to follow.

Several challenges exist in estimating the ROK^P. While data are readily available for the price indices for capital and output, estimating ROK^P is made difficult by competing claims on capital income. This problem becomes clearer when analyzing a production function. Output in each country is determined by a constant returns to scale production function (F[.]) depending on labor (L) augmented by productivity increases (A), and three types of capital -- private (K^P), natural (K^N), and government (K^G),

$$(2) \qquad Y = F\left[AL, K^P, K^N, K^G\right],$$

where, for notational simplicity, country (i) and time indices (t) have been omitted.

The contributions of labor and private capital to GDP are well known, and their value added corresponds to increases in corporate profits or proprietors' income. Government capital is the accumulated and depreciated value of tangible government investments in equipment, structures, and intellectual property. Value added that flows from government capital adds to GDP in three different ways. Government capital can be operated within a government enterprise (which results in an increase in the government enterprise surplus) or within a private enterprise (which results in an increase in corporate profits or proprietors' income; this relationship was relevant, for example, in the United States before and during World War II (Gordon, 1969)). Additionally, government capital can be part of general government operations, and its implicit contribution to value added is the difference between the expenses a business does not need to incur as a result of the services associated with government capital (e.g., security) less the taxes needed to finance the capital acquisition. The net amount corresponds to an increase in corporate profits or proprietors' income in the GDP accounts.

Natural capital is the capitalized value of rents from non-reproduced assets -- oil, natural gas, coal (hard and soft), minerals, and forests. (Unimproved land is excluded from the natural capital stock.) These rents increase GDP irrespective of ownership. The impact on GDP of government ownership is the same as for government capital discussed in the above paragraph. Private ownership results in an increase in corporate profits or proprietors' income in the GDP accounts.

Returning to the production function, we apply Euler's Theorem of Homogeneous Functions to (2), we obtain,

(3) $$Y = A\,F_L\;L + F_{K^P}\,K^P + F_{K^N}\,K^N + F_{K^G}\,K^G$$

If labor is paid a wage equal to its marginal product ($w = A\,F_L$), (3) can be rewritten to equate the capital income share for private, natural, and government capital (CIS^{P+N+G}) to marginal products ($F_{K^h} = MPK^h, h = P, N, G$) for three types of capital weighted by their capital/output ratios,

$$(4) \quad Y - wL = \left(1 - \frac{wL}{Y}\right)Y \rightarrow CIS^{P+N+G} = MPK^P \frac{K^P}{Y} + MPK^N \frac{K^N}{Y} + MPK^G \frac{K^G}{Y}.$$

We assume that the flow of private capital is the operative margin along which capital is allocated internationally. Equation (4) highlights the fundamental problem in measuring ROK^P given the competing claims on CIS^{P+N+G}.[2]

This problem is addressed by stating the weighted marginal products for natural and government capital in terms of observables. Following Monge-Naranjo, Sánchez, and Santaeulàlia-Llopis (2019, MSS), we measure the income flow attributable to natural capital (CIS^N).[3] The MSS approach allows us to directly address an important aspect of the fundamental measurement problem, replacing (ROK^N K^N / Y) in equation (4) with data for CIS^N.

The impact of government capital is accounted for in a similar manner. Data published by the IMF provides separate estimates for K^P and K^G. This distinction is important because the ratio of government to private capital ($RK^{G,P}$) is large, and government capital formation has absorbed substantial resources in some developing countries (Gourinchas and Jeanne, 2013).[4] Dividing our sample into Poor and Rich groups of countries, we confirm that the average value

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[2] An important advantage of this formulation of ROK's is that it properly accounts for differences in productivity (A) across countries. In (4), the effects of A are incorporated into the labor share, and hence the capital share. This robustness is important in forming international comparisons because of the cross-country differences in A documented by Gourinchas and Jeanne (2006).

[3] Caselli and Feyrer (2007) initially proposed a similar adjustment for natural capital. However, their correction (using natural capital stocks) was limited by data availability at the time they wrote. The MSS approach using flows is to be preferred. We use flow data from the World Bank because of its longer time span.

[4] This adjustment separating government and private capital parallels the adjustment by Caselli and Feyrer (2007) separating natural from produced (government plus private) capital (cf. fn. 3).

for $RK^{G,P}$ is 1.28 and 0.56, respectively. Further dividing our sample into earlier (1970-1990) and later (1991-2014) periods, we find that, stated in terms of percentage changes, $RK^{G,P}$ falls modestly for Rich countries (13%) but sharply for Poor countries (53%). While these capital stock data are an important element in solving the fundamental measurement problem (4), comprehensive data for one component, ROK^{G}, is unavailable. Returning to the Euler representation in (4), we assume that the marginal product of government capital is proportional to that for private capital for Poor and Rich countries separately ($MPK^{G,g} = \varphi^{g} MPK^{P,g}$, $g = Poor, Rich$). With these adjustments for natural and government capital and rearrangements of (1) and (4), we obtain the following equation,

$$(5) \qquad ROK^{P} = \left(CIS^{P+N+G} \mathbf{- CIS^{N}}\right) * \left(Y / K^{P}\right) * \left(PY / PK^{P}\right) * \left(\mathbf{1 + \varphi * RK^{G,P}}\right)^{\mathbf{-1}}.$$

In computing international ROK^{P}'s, it is important to recognize that the extent of economic activity outside the market will vary systematically across countries, and hence will not be captured by officially reported output. This "shadow economy" adjustment adds to our measure of ROK^{P} legal, market-based production of goods and services that are deliberately concealed from public authorities. The share of economic activity in the shadow economy (SE, measured as a percentage of reported GDP) is twice as large for Poor countries than Rich countries, and this ratio is approximately constant over the two sub-periods (see Table 1). The ROK^{P}'s are modified by multiplying the output by one plus the share of shadow economic activity, [5]

$$(6) \quad ROK^{P} = \left(CIS^{P+N+G} - CIS^{N}\right) * \left(Y / K^{P}\right) * \left(PY / PK^{P}\right) * \left(1 + \varphi * RK^{G,P}\right)^{-1} * \mathbf{(1 + SE)}.$$

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[5] We do not adjust the capital stock for shadow economy activity because assets are more difficult to conceal than income. We are thus assuming that businesses per se are not hidden, just their value added.

Data needed to compute (6) for 88 countries are available from a variety of sources. The CIS^{P+N+G}, PY, and PK^P series are from the Penn World Tables (PWT) created by Summers and Heston (1991), revised several times (Feenstra, Inklaar, and Timmer, 2015), and currently updated (version PWT9) and further developed by the Groningen Growth and Development Centre (2019). Subsequently, the IMF has used these data as a starting point and added some refinements and extensions. We use data from the IMF for K^P, K^G, and Y. The CIS^N data are from the World Bank; the φ parameters from Lowe, Papageorgiou, and Perez-Sebastian (2012); shadow economy data from Schneider, Buehn, and Montenegro (2010). Further information about the data used in this study is presented in Appendix A, which contains a glossary and details concerning variable definitions and sources.

Table 1 contains descriptive statistics for the five objects defining the ROK^P in (6), as well as the price indices for output and private capital considered separately.

3. Movements In The ROK^P's

This section documents the rise and divergence in ROK^P's. The key results can be easily understood in terms of the time-series plots in Figure 1 (Appendix B contains the underlying time series for the baseline). Results are presented for 1970 to 2014 and are divided in two dimensions: 1) by sub-periods, 1970-1990 and 1991-2014 because of a clear break in the time-series in 1990 and 2) by income per capita, sorting countries into Poor and Rich groups and then, for the former, Upper Middle, Lower Middle, and Low income groups. The Poor/Rich divide is between Malaysia (the highest income Poor country) and Uruguay (the lowest income Rich country). (Appendix C contains a list of the countries and groups.) For a given group, the country ROK^P's are aggregated with capital weights defined as averages for the full sample.

3.1. The Rising And Diverging ROK^P's: Baseline Results

Figure 1.A shows that ROK^P, aggregated over all 88 countries, exhibits two phases. From 1970 to 1990, the ROK^P is approximately constant. However, starting in 1991, the ROK^P moves steadily upward. As shown in the baseline results reported in Table 2.A, the average ROK^P's for these two sub-periods is 0.13 and 0.19, respectively.

This aggregate figure hides an important result. While the rising pattern is shared by both Poor and Rich countries, it occurs at different rates. Figure 1.B shows that the $ROK^{P,Poor}$ is always greater than $ROK^{P,Rich}$ and that, since 1990, the two series have diverged. Gaps between Poor and Rich countries, stated in terms of percentage differentials, are 48% and 78% in the earlier and later periods, respectively.

Figure 1.C and Table 2.A document that the pattern of rising ROK^P's that diverge from the Rich countries also holds for the three sub-groups of Poor countries.

Table 2 offers a second perspective on the rising ROK^P. Column 4 contains the percentage change in the average ROK^P for 1991-2014 (column 2) relative to a starting point, defined as the four-year average for 1987-1990 (column 3). Poor countries grew 72.2%, substantially greater than the comparable growth of Rich countries of 15.5%.

3.2. Robustness

Before proceeding further, it is useful to explore the robustness of results in four dimensions. First, intangible capital is a productive factor growing in importance in recent years. National income accountants have recognized these developments, and the data we use capture several types of intangible capital -- R&D, mineral exploration, computer software and databases, and entertainment, literary and artistic originals. Nonetheless, there are other forms of intangible capital -- designs, financial innovations, and economic competencies such as advertising, market research, training, and organization capital -- excluded from our measure of capital.

Excluded intangible capital (EIK) is unlikely to affect our results for two reasons. Capital stock data presented by Corrado, Haskel, Jona-Lasinio, and Iommi (CHJI, 2016, Table 3, for 14 EU countries, the United States, and four formerly Communist countries for 2000-2013) show that the ratio of EIK to the reported capital stock is 6.9% and 9.8% for poorer and richer countries, respectively.[6] If we assume that no other ROK^P components are affected by introducing EIK and make the extreme assumption that the EIK ratio in the later period (1991-2014) is three times greater than its value in the initial interval (1987-1990), then the aggregate ROK^P would rise by 27.2% in the latter period (relative to 1987-1990), rather than the 33.9% increase reported in Table 2.A. Moreover, in the later period, the gap between Poor and Rich

[6] The richer countries in the CHJI dataset are the 14 EU+USA countries with Greece and Portugal removed and placed in the poorer group. If Ireland is excluded because of its peculiar tax system that attracts intangible capital, the EU+USA ratio falls to 9.3.

countries would widen slightly from 1.78 to 1.82. Our rising and diverging ROK^P results appear robust to EIK.

Second, for computing ROK^P, two of the adjustments – government capital and the shadow economy -- are clearly necessary for assessing the intertemporal and cross-sectional returns to private capital. However, they rely on data lacking the pedigree and longevity of the national income data measuring other ROK^P components. These two adjustments are removed from (6) by setting $RK^{G,P}$=0=SE, and these restricted ROK^P's are reported in Figure 1.D and Table 2.B. The patterns of rising ROK^P's remain. The gap is attenuated because of a decrease in $ROK^{P,POOR}$ and an increase in $ROK^{P,RICH}$ in both earlier and later periods. Nonetheless, the ROK^P's continue to diverge after 1990.

Third, the classification of countries into Poor and Rich groups is based on the four-way World Bank country classification (by income) – high, upper middle, lower middle, and low. Our baseline results define the Poor and Rich groups as follows: Poor = {upper middle, lower middle, low}; Rich = {high}. To examine the sensitivity of our results to this classification, we move the upper middle countries from the Poor to the Rich group. This alternative classification raises the averages for both groups in both periods, and it has a modest impact on reducing the Poor/Rich ratios from 1.48 to 1.36 in the earlier period and 1.78 to 1.62 in the later period.

Fourth, in evaluating the international allocation of capital, it is essential to aggregate the ROK^P's with capital weights, but the results could be influenced by some large countries. We consider the impact of skewed weights in two ways. First, we re-compute (6) with equal weights. The results in Figure 1.E and Table 2.C show that equal weighting has little effect relative to our baseline ROK^P results. $ROK^{P,RICH}$ is higher in both periods, more so in the later period. These results attenuate but do not eliminate the widening gap between Poor and Rich countries. Second, we re-compute (6) after removing the following large countries: Rich (USA, Japan, Germany), Poor (Brazil, China, India, Indonesia), Upper Middle (Brazil, China), Lower Middle (India, Indonesia), Low (Tanzania). As shown in Figure 1.F and Table 2.D, the results again are largely robust. While $ROK^{P,POOR}$ and $ROK^{P,RICH}$ are both higher in the later period, the gap is unaffected.

3.3. Decomposing Movements In The ROK^P's

We return on our preferred measure of ROK^P defined in (6) and explore which ROK^P components are responsible for the rise in the later period. Equation (6) is composed of five multiplicative components (indexed by j). For a given component, its contribution to the overall ROK^P growth rate (Γ^j) is computed as its growth rate -- holding the other four components fixed at their starting values – relative to the growth rate in ROK^P (see Appendix D, especially equation (D7), for details),

$$(7a) \qquad \Gamma^j = \frac{\sum_{i=1}^{88} \omega_i G_i^j}{\sum_{i=1}^{88} \omega_i \left(G_i^1 + G_i^2 + G_i^3 + G_i^4 + G_i^5\right)} = \frac{G^j}{\sum_{j=1}^{5} G^j} . \qquad j=1,5$$

$$(7b) \qquad G^j \equiv \sum_{i=1}^{88} \omega_i G_i^j . \qquad j=1,5$$

Country growth rates (G_i^j) are capital-weighted (ω_i) across the 88 countries to form group growth rates (G^j). When we evaluate sub-groups (e.g., Poor, Rich), some of the ω_i's are set to 0 and the positive ω_i's renormalized to sum to 1.0. The starting and ending values are the average values for 1987-1990 and 1991-2014, respectively.

The Γ^j's are reported in Table 2.A for the five components and six groups of countries. We begin with all 88 countries considered together (aggregate) in column 1. The rise in PY/PK^P (shown in Figure 2.A) is the dominant influence, explaining 52% of the growth in ROK^P. Increases in the capital income share have received significant attention in both the popular and academic literatures (e.g., Karabarbounis and Neiman (2014), Piketty (2014)), and it is responsible for 24% of the rise in aggregate ROK^P. The decline in the share of government capital, through the $RK^{G,P}$ variable and the $\left(1+\varphi * RK^{G,P}\right)^{-1}$ term appearing in (6), is the third important component affecting ROK^P, resulting in an increase of 19%. The share of economic activity in the shadow economy and capital shallowing (as measured by Y/K^P) have only modest effects on the ROK^P growth rate.

However, these aggregate results hide important differences between Poor and Rich countries. While the rise in the capital income share is important in both groups, it plays a much larger role in Rich countries (40%) than in Poor countries (16%) (as shown in Table 2.A, columns 2 and 3). The slowdown in private capital formation in Rich countries (Crouzet and Eberly, 2018; Gutiérrez and Philippon, 2017), and the resulting capital shallowing has contributed to a 29% rise in ROK^P in Rich countries; Y/K^P has had almost no effect (-2%) in Poor countries. The fall in government capital has a much larger impact in Poor countries (24%) than Rich countries (8%). (These results highlight the importance of adjusting for government capital in evaluating the return on private capital.) Lastly, the relative price of output, the PY/PK^P variable translating \$1 of capital investment into the value of output, plays a much more important role in Poor countries (63%) than Rich countries (25%), as shown in Figure 2.B.

These differential movements casts new light on the role of relative prices in international capital allocation (Easterly, 1993; Jones, 1994), and they supplement the well-known results of

Hsieh and Klenow (HK, 2007). HK documented that 1) the PY/PK^{P} ratio was lower in poorer countries, 2) this differential was due to relatively lower output prices in Poor countries, and 3) capital goods prices were approximately the same in both groups of countries.

The HK results were based on actual prices for 1980, 1985 and 1996. Since our study is based on price indices, we cannot evaluate levels, only changes over time. Our data indicate that there have been two major developments starting in 1990 that refine the HK results. First, while PY has increased in both Poor and Rich countries since 1990 (cf. Figure 2.C), the percentage increase has been somewhat greater for Poor countries (cf. Figure 2.D). Second, capital goods prices have moved in opposite directions, falling slightly in Poor countries, but rising in Rich countries (cf. Figures 2.E and 2.F). Both movements -- $\%\Delta PY^{POOR} > \%\Delta PY^{RICH}$ and $\%\Delta PK^{P,POOR} < \%\Delta PK^{P,RICH}$ – attenuate the HK results and provide a new perspective on the role of relative prices. In recent years, a \$1 investment in a Poor country allows investors to buy more capital and the return on that purchase is higher. Both channels contribute to a relatively higher $ROK^{P,POOR}$.

Our results documenting a rising and diverging ROK^{P}'s leads to a fundamental puzzle – why has this marked increase in ROK^{P}'s not led to a substantial flow of capital that would lower returns in both Poor and Rich countries and that would narrow the gap between the two? Does the rising and diverging ROK^{P}'s reflect a systematic misallocation of capital or an equilibrium response to cost factors? To examine the cost channel, we perform a calibration exercise with the user cost of capital in the next section.

4. Explaining Movements In The ROK^P's

This section utilizes the Jorgensonian user cost of capital to understand the fundamental factors driving the rise in the ROK^P. The first sub-section formally derives the relation between the ROK^P, the marginal product of private capital (ROK^P), and the user cost of private capital (UCC^P). The second sub-section calibrates the user cost and examines to what extent it explains the rise in the ROK^P's.

4.1. Deriving the User Cost Of Capital

We assume that a representative firm chooses private capital and labor inputs to maximize its value at time 0 ($V(0)$), the discounted value of its net-of-tax profits over an infinite horizon stated in continuous time,

$$(8) \qquad V(0) = \int_0^{\infty} e^{-\int_0^s (\rho(s)+\zeta(s))ds} \left[PY(t)Y(t) - PL(t)L(t) - PK^P(t)I^P(t) \right] * (1-\tau(t))\, dt,$$

where $\rho(t)$ is the real discount rate, $\zeta(t)$ is the equity risk premium, $PY(t)$ is the output price, $Y(t)$ is output, $PL(t)$ is the price of labor, $L(t)$ is labor input, $PK^P(t)$ is the price of new private capital, $I^P(t)$ is investment in new private capital, and $\tau(t)$ is the tax rate. (To preserve notational simplicity, we model only the private capital good; extension to a multi-capital model is straightforward.)

In choosing labor and private capital ($K^P(t)$) inputs that maximize (8), the firm faces three constraints,

- Technology: $Y(t) = F\left[L(t), K^P(t)\right]$, where output is determined by a neoclassical production function ($F[.]$) that depends on labor and private capital inputs;

- Capital accumulation: $\dot{K}^P(t) = I^P(t) - \delta^P K^P(t)$, where the capital stock is increased by new investment and decreased by depreciation/obsolescence that occurs at a geometric rate (δ^P).

- Market power: $PY(t) = G\left[Y(t), X(t)\right]$, where $G[.]$ is decreasing in $Y(t)$ with constant inverse price elasticity at a point in time ($-(1/\varepsilon(t)), \varepsilon(t) > 1$) and $X(t)$ is a set of exogenous factors.

Substituting the three constraints into (8), we obtain,

$$(9) \quad V(0) = \int_0^\infty e^{-\int_0^s (\rho(s)+\zeta(s))\,ds} \begin{bmatrix} G\left[F\left[L(t), K^P(t)\right], X(t)\right] F\left[L(t), K^P(t)\right] \\ - PL(t)\,L(t) - PK^P(t)(\dot{K}^P(t) + \delta^P K^P(t)) \end{bmatrix} * (1-\tau(t))\,dt,$$

The Euler equation for the optimal choice of private capital in (9) is as follows,

$$(10a) \quad MPK^P(t) = \left(\frac{PK^P(t)}{PY(t)}\right) * \frac{\left(\rho(t) + \zeta(t) + \delta^P + \lambda^P(t)\right)}{(1-\tau(t))} * \mu(t),$$

$$(10b) \quad \lambda^P(t) \equiv -\left(dPK^P(t)/PK^P(t) - dPY(t)/PY(t)\right),$$

$$(10c) \quad \mu(t) \equiv \frac{\varepsilon(t)}{\varepsilon(t)-1} \geq 1,$$

where $\mu(t)$ is the markup of price over marginal cost and $\lambda^P(t)$ is the capital loss when the private capital goods price does not rise as fast (or falls) as the rise in the output price. The $\lambda^P(t)$ term is often overlooked in studies using the user cost of capital. However, consider a situation where the price of a capital good (or any asset) is expected to fall next period. That expected loss is a cost that needs to be considered when purchasing the capital good today. This loss may be attenuated (or even reversed) by the expected fall in the price of output. If the fall (or rise) in the capital goods and output prices are equal (which can occur by happenstance or in a one-good model), then $\lambda^P = 0$. As will be documented in Table 4, $\lambda^P(t)$ can be particularly important when analyzing the intertemporal behavior of the ROK^P across countries, especially for Poor countries given the significant declines in their capital goods prices.

The $ROK^P(t)$ is the $MPK^P(t)$ adjusted for differences between the prices of private capital investment and output,

$$\text{(11)} \qquad ROK^P = MPK^P \frac{PY}{PK^P} = \frac{\left(\rho+\zeta+\delta^P+\lambda^P\right)\mu}{(1-\tau)} \equiv UCC^P ,$$

where time subscripts have been omitted for notational convenience. We assume that the U.S. market represents the alternative investment opportunity for international investors and measure ρ and ζ accordingly. The user cost of private capital is defined by the last equality in (11). Note that using this last equality to define the UCC^P creates some tension with traditional terminology.[7] These six variables – ρ, ζ, δ^P, λ^P, μ, $(1-\tau)^{-1}$ – are factors driving ROK^P and their roles in raising ROK^P's will be quantified in the next sub-section.

4.2 Relating The User Cost Of Capital To ROK^P

Equation (11) highlights that the real interest rate is only one of many factors determining the ROK^P. The real interest rate and ROK^P are separated by several wedges -- risk, depreciation, capital losses, markups, and taxes, and a proper assessment of ROK^P must be based on UCC^P. Data sources for the six factors are discussed in Appendix A, Section II. We use (11) to quantify the extent to which the factors explain the growth rate in ROK^P between 1987-1990 and 1991-2014. The procedure is similar to that used in the Section 3.3 decomposition with the exception that, since the first four factors enter additively, their impact is quantified by including an additional weight (υ_i^h) evaluated at the starting values,

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[7] The Jorgensonian user cost of capital (Jorgenson, 1963) is usually defined inclusive of the relative price of capital. In our calculations, the inverse of that variable has been multiplied by ROK^P to define ROK^P in (11).

(12a) $$\Omega^h = \frac{\sum_{i=1}^{88} \omega_i \upsilon_i^h G_i^h}{G^{ROK^P}} = \frac{\upsilon_i^h G^h}{G^{ROK^P}}, \qquad h=1,\ldots,6$$

(12b) $$\upsilon_i^h \equiv h/(\rho+\zeta+\delta^P+\lambda^P), \; h=\rho,\zeta,\delta^P,\lambda^P, \qquad \upsilon_i^h \equiv 1 \;\; h=\mu,(1-\tau)^{-1}$$

(12c) $$G^{ROK^P} \equiv \sum_{i=1}^{88} \omega_i G_i^{ROK^P} \qquad G_i^h \equiv \sum_{i=1}^{88} \omega_i G_i^h. \quad h=1,\ldots,6$$

where the six factors are indexed by h. All country growth rates (G_i^h) are capital-weighted (ω_i) across the 88 countries. For a given factor h, its contribution to the growth in ROK^P is given by Ω^h in (12a).

Results based on (12) are presented in Table 4 for six groups of countries. We begin with all 88 countries considered together (aggregate). The well-documented fall in the real rate leads to a fall in ROK^P of 45%. This decline is counterbalanced by the rise in the equity risk premium, contributing a 44% boost in $ROK^{P,AGG}$.[8] On balance, changes in the cost of funds ($\rho + \zeta$), as well as changes in the corporate tax rate, have not affected movements in the $ROK^{P,AGG}$.

By contrast, the remaining three factors each contributed to a roughly comparable rise in ROK^P. The largest impact was the rise in markups lifting $ROK^{P,AGG}$ by 39%.[9] The rising depreciation rate contributed 27% and the capital loss, due to the falling relative price of capital, contributed 26%. Taken together, these six factors account for nearly all of the rise in $ROK^{P,AGG}$ (94%).

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[8] In the accounting exercise by Caballero, Farhi, and Gourinchas (2017a), ζ is a dominant driver of the secular trends in the return to capital and three other macroeconomic stylized facts.

[9] The importance of markups in analyzing the return to capital has been noted in several studies: Farhi and Gourio (2018), Eggertsson, Mehrotra, and Robbins (2019), and Barkai (2020). By contrast, Karabarbounis and Neiman (2018) conclude that markups do not play a significant role.

As an aside, we note that UCC^P is a combination of additive and multiplicative components. The latter introduce second-order and third-order terms to the Table 4 computations (cf. Appendix D, (D4) to (D7)). These terms are presented at the bottom of Panel B and explain a further 7% and 0%, respectively, of increases in ROK^P. If included in the calculations by being distributed across the six factors proportionally, the contribution of each factor would be raised by 7.4%, and the movement in the UCC^P factors would then explain slightly more than 100% (101.3%) of the movement in the $ROK^{P,AGG}$. Since we do not have a clear procedure for allocating these interactions to the individual factors and their impact is modest, they will be ignored in the remaining calculations.

Returning to the six first-order terms, we find that the results change markedly when we divide the sample into Poor and Rich countries. While the movements in ρ and ζ continue to approximately cancel and $(1-\tau)^{-1}$ has a very modest effect, λ^P, δ^P, and μ have differential impacts.[10] For Poor countries, the sharply falling relative price of capital creates a capital loss that contributes to 35% of the rise in UCC^P, and hence $ROK^{P,Poor}$. By contrast, for Rich countries, the capital loss has little impact (7%). However, increases in the depreciation rate (due to greater investment in equipment and intangible capital) and markups (due to greater firm concentration and market power) explain 62% and 94%, respectively, of the rise in $ROK^{P,Rich}$.

A further difference is that, taken together, the six user cost factors imply that ROK^P has risen in the latter period either too much or too little. For Poor countries, the movement in the user cost accounts for 60% of the rise in ROK^P. This result suggests that there is some factor(s) not included in our user cost definition that should increase $UCC^{P,Poor}$ so that, in equilibrium, it

[10] Since the percentage change in ROK^P to be explained is much greater for Poor countries (cf. row A), the contribution of a factor common to both Poor and Rich countries (e.g., the real interest rate) will be much less for Poor countries relative to Rich countries.

would approximately equal $ROK^{P,Poor}$. The opposite result applies to Rich countries, where the six user cost factors account for a 163% rise in ROK^{P}, and thus there is some additional factor(s) that should decrease $UCC^{P,Rich}$ to the estimated $ROK^{P,Rich}$. These omitted factors would be represented by additional terms (θ 's) to the financial cost $(\rho + \zeta + \delta + \lambda + \theta)$ only in the latter period. Based on the results in Table 4, $\theta^{POOR} = +0.041$ and $\theta^{RICH} = -0.015$.[11]

One candidate omitted factor is a "reach for safety." As noted by Caballero, Farhi, and Gourinchas (2017b, p. 43), the supply of safe assets issued by credible governments became scarce relative to demands due to "central banks' international reserve accumulation, regulatory changes, and demographic factors." Much of this demand emanated from emerging market countries. In response, firms in Rich countries had an incentive to become safer by building a sturdier capital structure. These changes are consistent with the data presented by Chen, Karabarbounis, and Neiman (2017, Table 4) -- 1) a substantial increase in the surplus of saving over capital formation in the corporate sector[12] and 2) a use of this surplus to increase cash holdings by 32 cents and reduce debt by 40 cents per $1 of surplus. As a result, investing in Rich countries would be perceived as relatively safer in more recent years. This channel would be reflected in negative [positive] θ's for $UCC^{P,Rich}$ [$UCC^{P,Poor}$], a flow of funds to Rich [from Poor] countries, and downward [upward] pressure on $ROK^{P,RICH}$ [$ROK^{P,POOR}$].

It is beyond is beyond the scope of this paper to rigorously evaluate the "reach for safety" channel, but two pieces of evidence cast doubts about its plausibility. For Poor countries, the increase in the default risk premium in the latter period of 410 basis points seems implausibly

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[11] These calculated θ's are similar in spirit to the capital wedges computed by Gourinchas and Jeanne (2013).

[12] For the earlier and later periods, corporate saving rose from 9.8 to 12.2 percent of world GDP. Household and government saving fell: 12.3 to 8.6 and 1.4 to 0.7, respectively. The earlier period is for 1980 to 1990, and the latter period for 1991 to 2013 due to available data.

large. For Rich countries, the required fall of 150 basis points is more plausible. But data for the United States indicates that default risk premia have changed little in the latter period. The risk premium for Baa bonds has fallen only 20 basis points. The Aaa bond risk premium change is in the opposite direction and equals a modest +20 basis points.[13]

Demographic changes may be an alternative candidate factor that has differential effects on Rich and Poor countries. Population aging is a global phenomenon particularly evident in Rich countries (Sudo and Takizuka, forthcoming). The increased dependency ratio lowers the effective labor force that, in turn, puts downward pressure on $ROK^{P,RICH}$. Goodhart and Pradham (2020) advance a novel theory that increased globalization had a substantial impact on increasing the effective labor force by integrating workers from Poor countries into the global economy. The mobilized labor force puts upward pressure on $ROK^{P,POOR}$. Introducing the differential effects of aging and integration and their dynamics on the effective labor force explicitly into our model may well prove to be useful in understanding the evolution of the ROK^{P}'s.

In sum, our analysis of user costs provides a good although partial accounting of the movements in the return to capital in Poor and Rich countries. Fuller explanations for the evolutions of the ROK^{P}'s over time and across countries remain for future research.

[13] The risk premia is the yield on Aaa [Baa] corporate bonds less the yield on 20-year constant maturity Treasury bonds. The data are from FRED; the corporate bond data are supplemented with information from Council of Economic Advisers (1999, Table B-73) for 1970-1985.

5. Summary And Conclusions

This paper has examined the secular evolution of the return to private capital (ROK^P) across 88 countries for 1970-2014. We document that ROK^P has exhibited two phases, being approximately constant from 1970-1990, but then rising dramatically for 1991-2014 (cf. Figure 1.A and Table 2.A). There is important heterogeneity in this increase (cf. Figure 1.B). While the 1991-2014 rise occurs for both Rich/Developed and Poor/Developing countries, the increases are at an uneven pace, 72% for Poor countries and 15% for Rich countries. These differences are driven by movements in capital income shares, capital shallowing, government capital, and the price of output relative to the price of investment (cf. Table 3). The Lucas Paradox seems to have become more pronounced in recent years.

Apart from these results, the paper introduced two refinements in the computation of ROK^P – explicit adjustments for government capital and shadow economic activity. Both vary systematically across Poor and Rich countries and proved important in the quantitative comparisons of these different groups of countries (cf. Figures 1.B and 1.D and Table 2.B), though did not alter the qualitative results of rising and diverging ROK^P's.

A good part of the increases in ROK^P are an optimal response to movements in the user cost of private capital. Despite a falling real interest rate lowering ROK^P, 60% of the secular rise in $ROK^{P,POOR}$ is explained by rising equity risk, depreciation, and markups and by the capital loss from expected decreases in the relative price of new private capital (cf. Table 4). The same factors explain 163% of the rise for Rich countries.

Our rising and diverging results are relevant to several policy issues.

Secular Stagnation. As emphasized by Summers (2015, 2020), a combination of a downward shift in the supply of capital schedule due to the "saving glut" (Bernanke, 2005) affecting Rich countries combined with an insufficient upward shift in the demand for capital results in a negative full-employment real interest rate. However, the increase in $ROK^{P,RICH}$ in the latter period that is more than fully explained by an increase in the user cost of capital raises questions about weak capital demand and the relevance of the secular stagnation perspective.

Wealth Inequality. As emphasized by Piketty (2014), the ROK^{P} (relative to income growth) is crucial for continuing and expanding wealth inequality.[14] Thus our results highlighting rising ROK^{P}'s suggest that wealth inequality will continue and possibly accelerate.

Public Debt. As emphasized by Blanchard (2019), the ROK^{P} is the opportunity cost of public debt. The rising ROK^{P}'s documented in this paper suggest that the cost associated with government debt has been growing in recent years.

Misallocated Capital. As emphasized by Caselli and Feyer (2007), proper measurement of the ROK^{P}'s is paramount and, in their work, leads to the conclusion of equality between the returns to capital in Poor and Rich countries.[15] This equality suggests that the global capital stock is allocated efficiently. Based on new data for a longer sample, our results of a divergence between $ROK^{P,POOR}$ and $ROK^{P,RICH}$ suggests misallocation. Before proposing corrective policies, it is important to know whether this divergence represents an optimal response to heretofore hidden, country-specific factors entering the user cost, micro level misallocations and TFP (Hsieh and Klenow, 2009, 2010), or frictions such as home bias (French and Poterba, 1991; Coeurdacier and Rey, 2013), political bias (Kempf, Luo, Schäfer, and Tsoutsoura, 2022), or home currency bias (Maggiori, Neiman, and Schreger, 2020) that impede international capital flows. Moreover, the

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[14] Piketty's model depends crucially on an elasticity of substitution between capital and labor exceeding 1.0. Our reading of the evidence does not support this view and suggests a value of 0.5 is more appropriate (Chirinko, 2008; Chirinko and Mallick, 2017).

[15] As a matter of notation, Caselli and Feyer's marginal products of capital are equivalent to our ROK^{P}'s. See equation (11) for the definitions used in the current paper.

structural break that occurred in 1990 hints that, in countries with weak financial systems and substantial domestic financial frictions, financial liberalizations may have amplified misallocations (Gertler and Rogoff, 1990; Gourinchas and Jeanne, 2013) and hence led to the sharp upward movement in $ROK^{P,POOR}$. Further research is needed to determine the roles played by these and other factors in the international allocation of capital.

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The paper includes two additional sections, "Addendum 1: Estimating Steady-State Values" and "Addendum 2: The Optimal Reallocation Of Private Capital." They represent material relevant to the analysis of ROK but that has not yet been incorporated into the paper. They are discussed briefly here.

"Addendum 1: Estimating Steady-State Values" highlights a bias in the above computations. We have assumed so far that the steady-state ROK can be estimated as an average of ROK's over a sample period. This Addendum presents two cases where a wide-discrepancy can exist. The possibility of a quantitatively important bias is documented by the hypothetical relationships between observed ROK data and the unobserved steady-state in Figures Addendum 1.A and 1.B. The data are generated by a deterministic partial adjustment model; see table notes for details. Figure Addendum 1.A is based on the assumptions that the initial conditions and ultimate steady-states are identical for poor and rich countries but that the transition speed to the steady-state is relatively slower for poor countries ($\lambda^{poor} = 0.10 < \lambda^{rich} = 0.25$). Using average spot ROK's as a measure of steady-state capital allocation leads to a bias that can be misleading. In this case, the Poor/Rich ratio computed from average spot ROK's is 1.33. The bias disappears as T gets very large but, even over a 20-year interval, the bias can be substantial.

The analysis in Figure Addendum 1.B reveals a similar bias, though in this case the differentiating factor between poor and rich countries is the initial conditions. For countries approaching the steady-state from above by accumulating more capital, the bias is positive. Even with equal transition speeds, estimates of the steady-state based on spot ROK averages overstate the true value. Again, the Poor/Rich ratio computed from average spot ROK's is 1.33, thus suggesting an LP when none exists.

Addendum 1 shows how a partial equilibrium model can generate unbiased estimates of the state-state. The procedure maps the "spot" ROK's discussed in this paper into steady-state values using estimated parameter values (equation A.1.6). Future work will estimate the steady-state ROK's using this procedure, replacing the current procedure of using sample averages.

"Addendum 2: The Optimal Reallocation Of Private Capital" takes a global perspective and quantifies the additional output that would be forthcoming if the existing capital stock was allocated optimally across all countries. As is well known, the capital allocation that maximizes global output equates the ROK's across all countries to a single ROK, ROK^{**}. This global optimum ROK is calculated according to an iterative scheme; details are provided in Appendix Addendum 2.A. For the full sample period (1970-2014), global output would increase by 1.2% if allocated optimally and if the elasticity of substitution is 0.5 (Table Addendum 2.1, panel A). This figure represents a transfer of capital from the relatively low ROK Rich countries to the Poor countries, and is a weighted sum of the output increase in Poor countries of 4.9% and the output decrease in Rich countries of 1.1%.

As shown in Table Addendum 2.1, panel B, if the allocation is constrained so that Rich countries as a whole do not suffer any output loss, the output increase for Poor countries falls from 4.9% to 3.2%.

The results are sensitive to the value of the substitution elasticity. If it is raised to the Cobb-Douglas value of 1.0, all of the above results double. See Table Addendum 2.1 for details.

Future work will incorporate these results into a separate section.

Addendum 1: Estimating Steady-State Values

I. Bias Due To Transition Dynamics

While the above results are interesting, these spot ROK's may not be fully informative. In evaluating capital allocations, we are interested in the steady-state ROK and how quickly an economy is moving towards this direct measure of capital allocation/misallocation. Even if the steady-states for poor and rich countries are equal, dynamics of the capital accumulation process from initial conditions toward steady-states distort the mapping of spot ROK's into the steady-state ROK's. As we shall discuss in the next section, transition speeds depend on, *inter alia*, the costs of misallocation, adjustment, and finance. Different transition speeds, as well as different initial conditions, are consistent with average spot ROK's showing an LP even if the steady-states are in fact equal.

The possibility of a quantitatively important bias is documented by the hypothetical relationships between observed ROK data and the unobserved steady-state in Figures Addendum 1.A and 1.B. The data are generated by a deterministic partial adjustment model; see table notes for details. Figure Addendum 1.A is based on the assumptions that the initial conditions and ultimate steady-states are identical for poor and rich countries but that the transition speed to the steady-state is relatively slower for poor countries ($\lambda^{\text{poor}} = 0.10 < \lambda^{\text{rich}} = 0.25$). Using average spot ROK's as a measure of steady-state capital allocation leads to a bias that can be misleading. In this case, the Poor/Rich ratio computed from average spot ROK's is 1.33. The bias disappears as T gets very large but, even over a 20-year interval, the bias can be substantial.

The analysis in Figure Addendum 1.B reveals a similar bias, though in this case the differentiating factor between poor and rich countries is the initial conditions. For countries approaching the steady-state from above by accumulating more capital, the bias is positive. Even with equal transition speeds, estimates of the steady-state based on spot ROK averages overstate the true value. Again, the Poor/Rich ratio computed from average spot ROK's is 1.33, thus suggesting an LP when none exists.

The important point to be drawn from the above analyses is that an averaging procedure of spot ROK's does not give due allowance to transition dynamics. To support more accurate inferences about capital allocations, steady-states, and transitions thereto we need to rely on the formal model developed in the next section.

II. A New Framework

A. Derivation Of The Estimating Equation

Our estimating equation is derived from an explicit optimization problem. It is considered "semi-structural" because the choice variables are not the policy variables per se. Rather, the means by which policymakers influence the ROK is left in the background, and we assume that policymakers directly choose ROK's. While a more structural model could be easily constructed, it is not easy to estimate. The semi-structural approach pursued here allows us to derive explicitly a linear estimating equation that will prove useful in pursuing our objective of generating an unbiased estimate of the steady-state ROK that accounts for transition dynamics.

We rely on a partial adjustment model developed in the literature in several places, staring with Eisner and Strotz (1959) and Lucas (1967). Our formulation closely follows the derivation in Kennan (1979), though the policymaker's problem is defined in disaggregate terms over a set of industries within a country.

Policymakers choose ROK's to minimize costs. Costs arise from misallocation and adjustment for country i for each of its j industries. Misallocation costs occur because the current ROK deviates from ROK*. These deviations are squared and then multiplied by a coefficient, ζ^{M}, that translates squared deviations into pecuniary costs. The second cost arises from the adjustment of the ROK's (and the underlying capital stocks). These costs represent lost output from disruptions to the existing production process as the ROK is altered and capital allocated. These installation and "teething" costs are a standard element in modeling input demands (see the surveys by Chirinko (1993), Caballero (1999), and Bond and van Reenen (2007)). Adjustment costs are modeled as the change in the ROK, squared, and then multiplied by a coefficient, ζ^{A}, that translates squared changes into pecuniary costs.

Misallocation and adjustment costs are embedded in the following dynamic cost minimization problem for the j^{th} industry in country i, discounted over time t by a constant discount factor (R), and summed across industries,

(A.1.1)

$$L_{i,j} = \underset{\{ROK_{i,j,t}\}}{MIN} \sum_{t=1}^{\infty} R^{t} \left\{ \zeta^{M} \left(ROK_{i,j,t} - ROK^{*}_{i,j} \right)^{2} + (\zeta^{A}/2) \left(ROK_{i,j,t} - ROK_{i,j,t-1} \right)^{2} \right\} wt_{i,j,t}$$

where $R \equiv (1+r)^{-1} < 1\ (0 < r < 1)$ and $wt_{i,j,t}$ is a fixed weight representing the relative importance of industry j in country i at time t. Optimal choices of the ROK lead to the well-known partial adjustment model. Differentiating equation (A.1.1) with respect to $ROK_{i,j,t}$ and rearranging, we obtain the following equation,

$$\text{(A.1.2a)} \quad \Delta ROK_{i,j,t}\, wt_{i,j,t} = \lambda^g \left(ROK^*_{i,j} - ROK_{i,j,t-1} \right) wt_{i,j,t} = \left(\alpha_{i,j} - \lambda^g ROK_{i,j,t-1} \right) wt_{i,j,t}$$

$$\text{(A.1.2b)} \qquad \alpha_{i,j} \equiv \lambda^g ROK^*_{i,j}$$

where λ^g is the stable root solving the second-order difference equation generated by the first-order conditions and determines the speed of adjustment to the steady-state. Summing across the J industries and defining the aggregates as weighted averages of the industry components, we obtain the following equation for the aggregate ROK's for a given country,

$$\text{(A.1.3)} \qquad \Delta ROK_{i,t} = \alpha_i - \lambda^g ROK_{i,t-1}.$$

The econometric estimates are based on a panel of all countries, and equation (A.1.3) is supplemented with time fixed effects (τ_t) and a stochastic error term ($\varepsilon_{i,t}$),

$$\text{(A.1.4)} \qquad \Delta ROK_{i,t} = \alpha_i - \lambda^g ROK_{i,t-1} + \tau_t + \varepsilon_{i,t},$$

where the adjustment speed parameter (λ^g) varies by poor and rich groups of countries. As is common in the literature starting with at least Sims (1974), the model variables are specified as logs. Thus, percentage changes in the ROK are proportional to the percentage difference between the steady-state and current ROK's.

B. Adjustment Speed (λ^g)

The adjustment speed parameter is a complicated function of the primitives in the optimization problem – ζ^M, ζ^A, and R,

$$\text{(A.1.5)} \quad \lambda^g = \lambda\left[\zeta^M, \zeta^A, R[r]\right] = 1 - \left\{ \frac{(1 + R + \zeta^M / \zeta^A) - \sqrt{(1 + R + \zeta^M / \zeta^A)^2 - 4R}}{2R} \right\}.$$

Differentiation of equation (A.1.5) with respect to each of the arguments and some tedious manipulations yield some interesting insights:

- $\lambda^{g}_{\zeta^{M}} > 0$ implies that the higher the cost of misallocation, the faster countries will converge to the long-run desired value because, ceteris paribus, they wish to eliminate losses as quickly as possible.
- $\lambda^{g}_{\zeta^{A}} < 0$ implies the obvious result that, with larger adjustment costs, the approach to the steady-state is slower.
- $\lambda^{g}_{r} < 0$ implies the higher the interest rate (or the lower the discount factor), the slower adjustment. Higher r could reflect finance constraints. With higher interest rates, the discounted costs of misallocation are lower, so they are eliminated more slowly by optimizing countries. In a sense, this result is the mirror image of that for ζ^{M} above.

C. Country-Specific Steady-State ROK (ROK_i^*)

An appealing property of the partial adjustment specification is that it readily yields an estimate of the country-specific steady-state ROK in terms of estimated parameters. The steady-state is defined by $ROK_{i,t} = ROK_{i,t-1}$, $\varepsilon_{i,t} = 0$, and $\phi_t = 0 \;\forall t$. The latter assumption effectively treats the ϕ_t's as incidental parameters that we remove from α_i by subtracting the mean of the time fixed effects ($\overline{\phi}$). With these restrictions, logarithmic versions of equations (A.1.4) and (A.1.2b) imply that the steady-state marginal product of capital is as follows,

$$\text{(A.1.6)} \qquad ROK_i^* = EXP\left[\frac{\alpha_i[\lambda^g] + \overline{\phi}}{\lambda^g}\right],$$

where $\overline{\phi}$ is the average of the time fixed effects and the dependence of $\alpha_i[\lambda_g]$ on λ_g (through the second term on the right-side of equation (A.1.6)) is indicated explicitly.[16]

D. Correcting For The Nickell Bias

In panel models with country-specific effects and a "small" number of observations in the time dimension, the coefficient on the lagged dependent variable is biased. The conventional analysis of this bias is conducted in terms of an AR(1) model in which the positive coefficient on

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[16] In the two-way fixed effects model, one time fixed effect (ϕ_t) is dropped to avoid singularity during estimation. The $\overline{\phi}$ term is included in (A.1.6) so that the computation ROK_i^* is invariant to which time fixed effect is dropped.

the lagged dependent variable is biased downward. In our first-difference model, this effect will result in an upwardly biased estimate of λ^g. One solution to this problem is to transform the model appropriately and then find suitable instruments. We do not follow this approach because of the general difficulty of finding suitable instruments, a challenge that is even more daunting in our panel of 88 countries for 45 years. Instead, we exploit the simple structure of our model that excludes additional endogenous regressors and correct the bias directly with the formula developed by Nickell (1981) in his seminal article (see the Table 4 note for details). Correcting for the Nickell bias has a very small impact on our estimates of the steady-state ROK because λ^g has a similar effect on both the numerator and denominator of ROK_i^* in equation (A.1.6).

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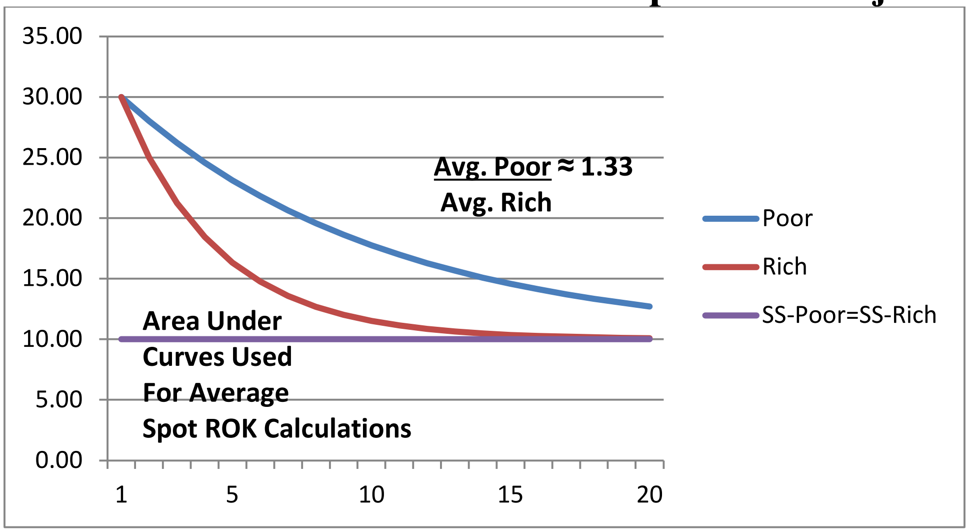
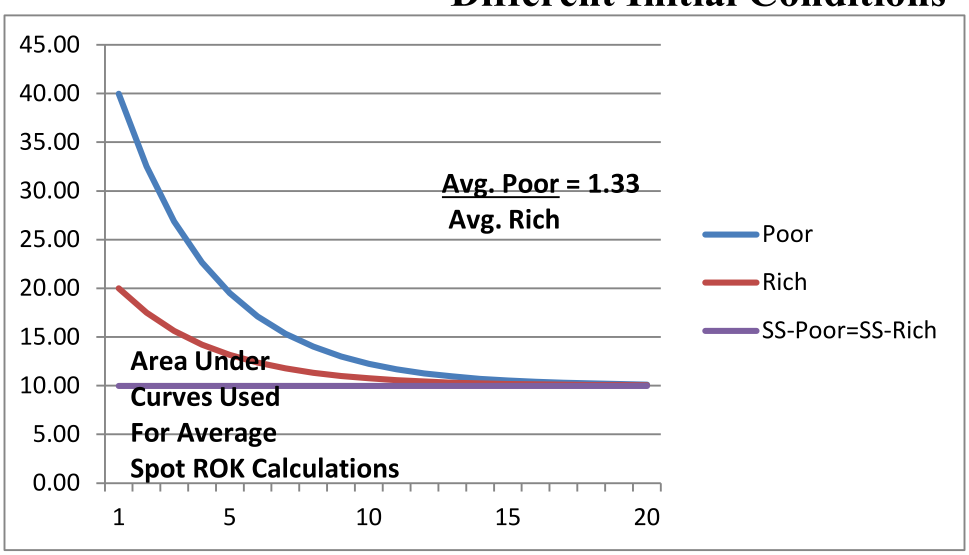
Figure Addendum 1.A: Hypothetical Bias In Estimating Steady-State ROK's Different Speeds Of Adjustment



Notes: Hypothetical data are generated by the following partial adjustment model, $\Delta ROK_t = \lambda^g (ROK^* - ROK_{t-1})$, where $\lambda^{poor} = 0.10$, $\lambda^{rich} = 0.25$, $ROK_0^g = 30.0$, $ROK^{*g} = 10.0$, $g = \{poor, rich\}$.

Figure Addendum 1.B: Hypothetical Bias In Estimating Steady-State ROK's Different Initial Conditions



Notes: Hypothetical data are generated by the following partial adjustment model, $\Delta ROK_t = \lambda^g (ROK^* - ROK_{t-1})$, where $\lambda^{poor} = \lambda^{rich} = 0.25$, $ROK_0^{poor} = 40.0$, $ROK_0^{rich} = 20.0$, $ROK^{*g} = 10.0$, $g = \{poor, rich\}$.

Addendum 2: The Optimal Reallocation Of Private Capital

This section takes a global perspective and quantifies the additional output that would be forthcoming if the existing capital stock was allocated optimally across all countries. As is well known, the capital allocation that maximizes global output equates the ROK's across all countries to a single ROK, ROK^{**}. This global optimum ROK is calculated according to an iterative scheme; details are provided in Appendix Addendum 2.A. We begin by assuming an arbitrary initial value for ROK^{**}. This initial estimate determines for the first iteration the percentage changes in the ROK's (from the country-specific steady-state ROK's, the ROK_i^{*}'s) that equate ROK's across all countries. In turn, this percentage change determines the private capital stocks with a discrete version of equation (~~1~~9) normalized to K_i^P. We then compute the deviation between the sum of these estimated K_i^P and the fixed amount of global private capital and use this deviation to compute a new estimate of the global optimum ROK. The process is repeated until the deviation is close to zero. Given the negative and monotonic relation between ROK's and capital stocks, the process is guaranteed to converge to a unique solution. The change in global output due to this globally optimal allocation of private capital is computed with an output-weighted average of a discrete version of equation (~~1~~8) with dK_i^P / K_i^P defined by equation (~~1~~9).

The percentage changes in global output are presented in Table Addendum 2.1, row 1, columns 1 to 3 and range from 1.0% to 1.5%.[17] Our estimate of the change in output increases with the elasticity of substitution; with an overly large elasticity of 1.5, the increase in global output is 4.4%. The 1.2% increase is our preferred estimate and represents an increase in output of $1,047 billion (2019 first quarter U.S. dollars) (see Appendix Addendum 2.B for details about the dollar figure computations reported in this paragraph). By way of comparison, the fiscal stimulus programs enacted by the United States (ARRA) and China in response to the global financial crisis amounted to $774 and $677 billion (2019 first quarter U.S. dollars).

[17] ~~CF (2007, Table VI) and MSS (2019, Table 6, average of QROK) estimate capital reallocation benefits of 0.1% and 3.0%. Since these estimates are based on a Cobb-Douglas production function, they are most comparable to those in our Table 7 with $\sigma = 1$ of 2.4%. Our estimate is much larger than that of CF because they found near equalization of ROK's for poor and rich countries and hence little scope for beneficial reallocation. Note that the estimates are not strictly comparable because of differences in spot ROK's for produced capital (CF and MSS) vs. steady-state ROK's for private capital (this study), as well as differences in samples.~~

The remaining entries in panel A separate the change in global output between poor and rich countries. The $ROK_i^{*Rich} = 0.144$, which is very close to $ROK^{**} = 0.156$. This result indicates that the rich countries as a group are fairly close to the globally optimal capital allocation. By contrast, the $ROK_i^{*Poor} = 0.235$ is very far from ROK^{**}, and the scope for improvement lies in reallocating capital from rich to poor countries. For $\sigma = 0.5$ for the full sample, output in poor countries increases by 4.9%, while it decreases by -1.1% in rich countries. These figures are consistent with the modest gain in global output discussed above because a disproportionate amount of world capital and world output is concentrated in the rich countries.

The above results may be somewhat unrealistic since it involves the rich countries suffering a loss in output and the rich countries control the international economic organizations that might lead the effort at such a redistribution. A second scenario imposes an additional constraint that the rich countries do not suffer any loss in output due to an ex-post transfer of output from the poor countries under the new ROK** (i.e., a "compensating variation," a concept used frequently in public economic analyzes). In this case, the results in columns 1 to 3 are identical in panels A and B, and the percentage increase in output for poor countries drops from 4.9% to 3.2%. Insofar as incremental income has very high utility in poor countries, this change may have meaningful welfare consequences.

Lastly, we examine the impact of the shadow economy and government capital adjustments on reallocations by repeating the analysis in panel A with steady-state ROK that does not incorporate the government capital and shadow economy adjustments replacing an ROK that does include these adjustments. As shown in panel C, the estimated benefits of optimally reallocating capital are now approximately one-third lower than the values reported in panel A for steady-state ROK with the two adjustments. These lower values are due to the Poor/Rich ratio based on the unadjusted ROK's being smaller than the Poor/Rich ratio based on the adjusted ROK's and hence the scope for beneficial reallocations being smaller. The shadow economy and government capital adjustments introduced in Section II matter for quantitative evaluations.

Table Addendum 2.1: Reallocation Of Private Capital
Equalizing ROK's Subject To A Capital Constraint
Percentage Changes In Output

Adjusting ROK's	**1970 2014**	**1970 1990**	**1991 2014**	**1970-2014**		**1970-1990**		**1991-2014**	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	------- Global -------			Poor	Rich	Poor	Rich	Poor	Rich
A. Maximize Global Output				$(\text{ROK}^{**} = 0.156)$		$(\text{ROK}^{**} = 0.124)$		$(\text{ROK}^{**} = 0.187)$	
$\sigma = 0.5$	1.2	1.0	1.5	4.9	-1.1	4.2	-0.4	5.7	-1.6
$\sigma = 1.0$	2.4	2.1	2.9	9.9	-2.2	8.5	-0.9	11.4	-3.1
$\sigma = 1.5$	3.7	3.1	4.4	14.8	-3.4	12.7	-1.3	17.1	-4.7
B. Maximize Global Output With No Loss to Rich				$(\text{ROK}^{**} = 0.156)$		$(\text{ROK}^{**} = 0.124)$		$(\text{ROK}^{**} = 0.187)$	
$\sigma = 0.5$	1.2	1.0	1.5	3.2	0.0	3.3	0.0	3.5	0.0
$\sigma = 1.0$	2.4	2.1	2.9	6.3	0.0	6.6	0.0	7.1	0.0
$\sigma = 1.5$	3.7	3.1	4.4	9.5	0.0	9.9	0.0	10.6	0.0
C. Maximize Global Output (~~ROK^3~~) (no government capital or shadow economy adjustments)				$(\text{ROK}^{**} = 0.163)$		$(\text{ROK}^{**} = 0.127)$		$(\text{ROK}^{**} = 0.200)$	
$\sigma = 0.5$	0.8	0.6	1.0	2.6	-0.3	1.0	0.4	3.4	-0.6
$\sigma = 1.0$	1.6	1.1	2.1	5.1	-0.5	2.0	0.7	6.7	-1.3
$\sigma = 1.5$	2.5	1.7	3.1	7.7	-0.8	2.9	1.1	10.1	-1.9

Notes: See Section VIII for a discussion of these results. ROK** is chosen as follows: Panel A, to maximize global output by equating ROK's across all countries subject to a capital constraint; Panel B, in a manner similar to that in Panel A with the additional constraint that output levels in rich countries as a group do not change by a transfer of output from poor to rich countries; Panel C, in a manner similar to that in Panel A with ROK^3 used in place of ROK^6. The ROK**'s vary across the three intervals and between Panels A, B, and C and are presented in columns (4)/(5), (6)/(7), and (8)/(9). The percentage changes in output reported in this table are based on the percentage changes in ROK^{**} relative to country-specific steady-state ROK's (ROK_i^*'s).

Appendix Addendum 2.1.A: Iterative Scheme For Computing The Global Reallocation Of Private Capital And The Increase In Global Output

This global optimum ROK is calculated according to the following four-step iterative scheme. First, we begin by assuming an arbitrary initial value for ROK^{**}, $\widehat{ROK_1}^{**}$, where the subscript 1 indicates the first iteration. Second, this initial estimate determines for the first iteration the percentage changes in the ROK's (from the country-specific steady-state ROK's, the ROK_i^{*}'s) that equate ROK's across all countries,

(D-1) $$\alpha_{i,1} = \frac{\widehat{ROK_1}^{**} - ROK_i^{*}}{ROK_i^{*}} = dROK_i \,/\, ROK_i \,.$$

Third, a discrete version of equation (~~19~~) determines the private capital stock associated with $\alpha_{i,1}$,

(D-2) $$K_{i,1}^{P} \;=\; \left(1-\left(\sigma\;\alpha_{i,1}\right)/\left(1-CIS_i^{P}\right)\right)\,K_i^{P}\,.$$

Fourth, we compute the extent to which the sum of the country capital stocks in the first iteration ($\bar{K}_1^{P}$) deviates from the fixed amount of global private capital ($\bar{K}^{P}$),

(D-3a) $$\Delta_2 \equiv \left(\bar{K}_1^{P} - \bar{K}^{P}\right)/\,\bar{K}^{P},$$

(D-3b) $$\bar{K}_1^{P} \equiv \sum_i K_{i,1}^{P}\,,$$

(D-3c) $$\bar{K}^{P} \equiv \sum_i K_{i.0}^{P}\,,$$

(D-3d) $$\Gamma_2 = 1 + \Delta_2\,,$$

where $K_{i.0}$ is the initial value of country i's private capital stock.

We begin the second iteration by using this deviation to compute a new estimate of the global optimum ROK,

(D-4) $$\widehat{ROK_2}^{**} = \Gamma_2 \widehat{ROK_1}^{**}\,,$$

return to step 1, and insert $\widehat{ROK_2}^{**}$ into equation (D-1). The process continues until Δ_N is sufficiently close to 0.

Convergence is assured. Consider, for example, the situation where the total amount of private capital consistent with $\widehat{ROK}_n^{**}$ in the n[th] iteration exceeds the fixed, global, private capital stock. In this case, Δ_{n+1} is positive, Γ_{n+1} is greater than one, $\widehat{ROK}_{n+1}^{**}$ rises, and $\bar{K}_{n+1}^{P}$ falls. Since the negative relation between $\widehat{ROK}_{n+1}^{**}$ and $\bar{K}_{n+1}^{P}$ is monotonic, the process is guaranteed to converge to a unique solution, $\left\{K_i^{P**},\, i=1,88\right\}$.

The change in output from this globally optimal allocation of private capital is computed with a discrete version of equation (18), weighted by country-specific output shares,

$$\text{(E-5)} \qquad \frac{d\bar{Y}^{**}}{\bar{Y}} = \frac{\sum_i \left(Y_i^{**} - Y_i\right)}{\sum_i Y_i} = \sum_i \left(C\,IS_i^P \frac{dK_i^{P**}}{K_i^P} \right) \frac{Y_i}{\sum_i Y_i} .$$

Appendix Addendum 2.1.B: Computing The Gains From Capital Reallocations In Terms Of 2019 (First Quarter) U.S. Dollars

This appendix provides details about the computations of the gains from reallocating capital from rich to poor countries discussed in Addendum 2. All dollar figures are in billions of U.S. dollars in 2019.1 (the first quarter of 2019). The Renminbi figure is in billions.

1. Increase in world GDP.

$1,047 = 0.012 [Table 7, panel A, column 1] * $87,270 [IMF-WEO]
$873 = 0.010 [Table 7, panel A, column 2] * $87,270 [IMF-WEO]
$1,309 = 0.015 [Table 7, panel A, column 3] * $87,270 [IMF-WEO]

2. United States' stimulus program, The American Recovery and Reinvestment Act (ARRA). Outlays occurred over several years. We assume all outlays were made in 2009.

$774 = $663 [CBO] * 1.168 [1 + percentage change in the GDP price deflator from 2009 to 2019.1, FRED-PGDP].

3. China's stimulus program, We assume all outlays are made in 2008.

$677 = 4,000 Renminbi [CGPY] / 6.9477 [Renimbi/Dollar exchange rate in 2008, FRED-ER] * 1.176 [1 + the percentage change in the GDP price deflator from 2008 to 2019.1, FRED-GDP].

LEGEND

CBO: Congressional Budget Office, "Estimated Impact of the American Recovery and Reinvestment Act on Employment and Economic Output in 2014," (February 2015), Table 1.

CGPY: Cong, William Lin, Gao, Haoyu, Ponticelli, Jacopo, and Yang, Xiaoguang, "Credit Allocation under Economic Stimulus: Evidence from China," Chicago Booth (November 2018), p. 1.

FRED-ER: Federal Reserve Economic Data, "China / U.S. Foreign Exchange Rate (DEXCHUS)," https://fred.stlouisfed.org/series/DEXCHUS#0 .

FRED-PGDP: Federal Reserve Economic Data, "GDP Implicit Price Deflator in United States (USAGDPDEFAISMEI)," https://fred.stlouisfed.org/series/USAGDPDEFAISMEI .

IMF-WEO: International Monetary Fund, "IMF DataMapper," https://www.imf.org/external/datamapper/NGDPD@WEO/WEOWORLD .

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Appendix A: Glossary, Variable Definitions, And Sources

Section I discusses data needed to construct the return on private capital analyzed in Sections 2 and 3 of the text. Section II discusses data needed to construct the components of the user cost of capital and other variables analyzed in Section 4 of the text.

I. Data For The Return On Private Capital Analyzed In Sections 2 and 3

Notes:

1. Roman letters define variables. Greek letters define econometric parameters and calibrated parameters or variables.

2. International prices are purchasing power parity prices. "A country's PPP gives the number of local currency units (e.g. euro's) that are needed to buy a bundle of products worth one dollar in the US. Dividing the PPP by the nominal exchange rate (also in local currency units per dollar) then gives the "price level" of that country relative to the US. A price level of 0.5, for example, indicates that local prices converted to US dollars with the nominal exchange rate are ½ as high on average as in the United States, as might be the case for a developing country" (PWT-USER, p. 2, fn. 2).

3. Some of the text below is taken directly from government documents that are in the public domain.

Roman Letters

$CIS^{N}_{i,t}$ Capital income share for natural capital. Source: WB-WDI (NY.GDP.TOTL.TR.ZS). Capital income for natural capital is the sum of rents from oil, natural gas, coal (hard and soft), minerals, and forests. For many countries, especially the developing countries, the rent share data are not available for the entire 1970-2014 period. However, data are usually missing at the tails. The missing data for beginning periods are replaced by the value for the earliest available period, and the missing data for end periods are replaced by the value for the most recently available period.

These rents increase GDP irrespective of ownership. The impact on GDP of government ownership is the same as for government capital discussed in the entry for $K^{G}_{i,t}$. Private ownership results in an increase in the corporate profits or proprietors' income categories in the GDP accounts.

Natural capital is the capitalized value of these rents. (Unimproved land is excluded from the natural capital stock.)

$CIS_{i,t}^{P+G}$ Capital income share for private and government (public) capital (also referred to as produced capital) based on an adjustment using flow data. Transformation: $CIS_{i,t}^{P+G+N} - CIS_{i,t}^{N}$.

$CIS_{i,t}^{P+G+N}$ Capital income share for private, government, and natural capital. Transformation: $(1 - LIS_{i,t})$.

EIK Excluded intangible capital. Source: CHJI
Intangible capital not included in our measures of capital -- designs, financial innovations, and economic competencies such as advertising, market research, training, and organization capital. Included in our capital data are several types of intangible capital -- R&D, mineral exploration, computer software and databases, and entertainment, literary and artistic originals. The richer countries in the CHJI dataset are the 14 EU+USA countries with Greece and Portugal removed and placed in the poorer group.

F[.] Neoclassical production function.

g Subscript indexing a group of countries. Two groupings are employed in this study based on the four-way World Bank country classification (by income) defined by WB-CLG: high, upper middle, lower middle, low. For the 2018 fiscal year, high / upper middle / lower middle / low economies are defined as having Gross National Income (formerly Gross National Product) per capita falling in the following intervals: greater than $12,236 / $12,235 to $3,956 / $3,955 to $1,006 / less than $1,005. These figures are calculated with 2016 data using the method in WB-ATLAS. See Appendix C for the countries included in each of the groupings.

The first grouping defines the rich group as the high income countries and the poor group as the sum of the upper middle, lower middle, and low income countries.

The second grouping defines the rich group as the sum of the high income and upper middle countries and the poor group as the sum of the lower middle and low income countries.

i Subscript indexing a country. See Appendix C for a list of the countries, organized by groups. See the entry above for "g" for further information about the groups.

$K^{G}_{i,t}$ Government (public) capital stock, billions of constant 2011 international dollars. Source: IMF (kgov_rppp). Government capital is the accumulated and depreciated value of tangible government investments in equipment, structures, and intellectual property. Value added that flows from government capital adds to GDP in three different ways. Government capital can be operated within a government enterprise (which results in an increase in the government enterprise surplus) or within a private enterprise (which results in an increase in corporate profits or proprietors' income; this relationship (Gordon, 1969) was relevant, for example, in the United States before and during World War II). Additionally, government capital can be part of general government operations, and its implicit contribution to value added is the difference between the expenses a business does not need to incur as a result of the services associated with government capital (e.g., security) less the taxes needed to finance the capital acquisition. The net amount corresponds to an increase in corporate profits or proprietors' income in the GDP accounts.

$K^{N}_{i,t}$ Natural capital stock. Rather than computing this stock, this paper utilizes the rents from non-reproduced assets -- oil, natural gas, coal (hard and soft), minerals, and forests. (Unimproved land is excluded from the natural capital stock.) Natural capital is the capitalized value of these rents, which increase GDP irrespective of ownership. The impact on GDP of government ownership is the same as for government capital discussed in the entry for $K^{G}_{i,t}$. Private ownership results in an increase in corporate profits or proprietors' income in the GDP accounts.

$K^{P}_{i,t}$ Private capital stock, billions of constant 2011 international dollars. Source: IMF (kpriv_rppp).

$K^{P+G}_{i,t}$ Private plus government (public) capital stock, billions of constant 2011 international dollars. Also referred to as the produced capital stock. Transformation: $K^{P}_{i,t} + K^{G}_{i,t}$.

$L_{i,t}$ Labor input.

$LIS_{i,t}$ Labor income share, ratio of labor compensation to GDP all in current national prices. Source: PWT-9 (labsh).

$PK^{P}_{i,t}$ Price index for private capital, 2011 base year. Transformation: equal to $P^{K^{P+G}}_{i,t}$.

$PK_{i,t}^{P+G}$ Price index for private plus government capital, 2011 base year. Source: PWT-9 (pl_k). This variable is measured as the price index for new capital goods (i.e., investment).

$PY_{i,t}$ Price index for output, 2011 base year. Source: PWT-9 (pl_gdpo).

$ROK_{i,t}^{p}$ The return on private capital. Transformation: See equation 6 reproduced here:

$$\left(CIS^{P+N+G} - CIS^{N}\right)*\left(Y / K^{P}\right)*\left(PY / PK^{P}\right)*\left(1+\varphi*RK^{G,P}\right)^{-1}*(1+SE).$$

$RK_{i,t}^{G,P}$ Ratio of capital stocks, government relative to private. Transformation: $K_{i,t}^{G} / K_{i,t}^{P}$.

$SE_{i,t}$ Share of economic activity in the shadow economy, measured as a percentage of reported GDP. Source: SBM (Table 2, pp. 454-456). Definition: "The shadow economy includes all market-based legal production of goods and services that are deliberately concealed from public authorities for any of the following reasons: (1) to avoid payment of income, value added or other taxes, (2) to avoid payment of social security contributions, (3) to avoid having to meet certain legal labor market standards, such as minimum wages, maximum working hours, safety standards, etc., and (4) to avoid complying with certain administrative procedures, such as completing statistical questionnaires or other administrative forms" (p. 444). Data for 1970-1998 have been extrapolated by the 1999 value. Data for 2008-2014 have been extrapolated by the 2007 value.

t Subscript indexing calendar time. $t = \{1970, 2014\}$.

$Y_{i,t}$ Output, gross domestic product, billions of constant 2011 international dollars. Source: IMF (GDP_rppp).

Greek Letter

φ^{g} Ratio of government ROK's to private ROK's for group g. Source: Computed from LLP (2012, Table 1). Data are available only as a cross-section. For a given country, we compute the ratio of the marginal product of government capital to the marginal product of private capital. These country-specific ratios are then sorted into poor and rich groups, and averaged: $\varphi^{POOR} = 1.2$ and $\varphi^{RICH} = 1.9$. These estimates reflect two economic forces. The general phenomenon of the underprovision of government goods is consistent with both ratios being greater than one. The greater proportions of government to private capital in poor countries relative to rich countries (cf. $RK_{i,t}^{G,P}$ in Table 1, panel 4) is consistent with $\varphi^{POOR} < \varphi^{RICH}$.

Legend

CHJI: Corrado, Carol, Haskel, Jonathan, Jona-Lasinio, Cecilia, and Iommi, Massimiliano, "Intangible investment in the EU and US before and since the Great Recession and its contribution to productivity growth," EIB 2016/8 (European Investment Bank) (August 2016).

GORDON: Gordon, Robert J., "$45 Billion of U.S. Private Investment Has Been Mislaid," *American Economic Review* 59 (June 1969), 221-238.

IMF: International Monetary Fund, *The IMF and Public Investment Management.* http://www.imf.org/external/np/fad/publicinvestment/data/data122216.xlsx

LLP: Lowe, Matt, Papageorgiou, Chris, and Perez-Sebastian, Fidel, "The Public and Private ROK," International Monetary Fund (July 2012).

PWT-9: Groningen Growth and Development Centre. *Penn World Table, Version 9*. www.ggdc.net/pwt .

PWT-USER: Feenstra, Robert C., Inklaar, Robert, and Timmer, Marcel, "PWT 8.0 – a user guide." https://irs.princeton.edu/sites/irs/files/event/uploads/PWT%2080%20-%20a%20user%20guide.pdf .

SBM: Schneider, Friedrich , Buehn, Andreas, and Montenegro, Claudio E.,"New Estimates for the Shadow Economies all over the World," *International Economic Journal* 24/4 (December 2010), 443-461. DOI: 10.1080/10168737.2010.525974.

WB-ATLAS: World Bank, *The World Bank Atlas method – detailed methodology*. https://datahelpdesk.worldbank.org/knowledgebase/articles/378832-what-is-the-world-bank-atlas-method .

WB-CLG: World Bank, Country and Lending Groups, *Data*. https://datahelpdesk.worldbank.org /knowledgebase/articles/906519-world-bank-country-and-lending-groups .

WB-WDI: World Bank, *World Development Indicators*. https://databank.worldbank.org/data/source/world-development-indicators .

II. Data For The Components Of The User Cost Of Capital And Other Variables Analyzed in Section 4

$\delta^{P}_{i,t}$ Depreciation rate for private capital. Source: IMF-FAD. The depreciation rates are only available for high, middle, and low income countries as a group, and they are available only for 1960 (Low = 0.0425, Middle = 0.0628, High = 0.0751) and 2015 (Low = 0.0425, Middle = 0.0628, High = 0.0751). Values between each pair of years are linearly interpolated.

ζ_t The equity risk premium for the United States (we assume that the U.S. market represents the alternative investment opportunity for international investors). Source: DR. The data are the first principal component from monthly estimates of the equity risk premium from 20 models.

$\lambda^{P}_{i,t}$ Capital loss on private capital. Transformation: the percentage change in PK^{P}/PY multiplied by minus one. Unusual volatility in the data led us to truncate the earlier period from 1986-1990 to 1987-1990 and to replace Mozambique (with an unreasonable increase in the 1987-1990 value of -17.3) with the average for all Low countries.

$\mu_{i,t}$ Markup of price over marginal cost. Source: DE. Markups are only available for continents.

ρ_t Risk-free real discount rate for the United States (we assume that the U.S. market represents the alternative investment opportunity for international investors). Transformation: the 7-Year Constant Maturity Rate For U.S. Treasury Bonds (FRED, DGS7) less the expected inflation rate seven years into the future. Expected inflation is equated to actual inflation. Inflation is measured by the Consumer Price Index for All Urban Consumers: All Items in U.S. City Average (FRED, CPIAUCSL). Owing to the unprecedented variation due to the pandemic, expected inflation in 2014 is measured only over six future years.

$\tau_{i,t}$ Corporate tax rate: Transformation: ITAX / (GOS – DEPC). Source: UN -- ITAX: current taxes on income, wealth, etc.; GOS: gross operating surplus; DEPC: consumption of fixed capital.

The following countries appear in the UN dataset but have been removed because of an inadequate amount of data: Rich: Bahrain and Kuwait; Upper Middle: Botswana, Costa Rica, Dominican Republic, Ecuador; Lower Middle: India; Low, Guinea.

Relative to the 88 countries in our study, the UN data are incomplete in both the time and cross-country dimensions. A balanced panel has been constructed as follows. For a given country, missing values are set equal to the "nearest neighbor," that is, the $\tau_{i,t}$ in the year closest to the missing value year. The procedure results in a balanced panel of 29 countries.

There are 32 Rich countries in our study. For 21 Rich countries, there is sufficient data to compute $\tau_{i,t}$. For the remaining 11 Rich countries, the $\tau_{i,t}$'s equal the unweighted average of the 21 $\tau_{i,t}$'s. There are 56 Poor countries in our study. There is sufficient data to compute the $\tau_{i,t}$'s for 8 countries. The $\tau_{i,t}$'s for all 56 Poor countries equal the unweighted average of the 8 $\tau_{i,t}$'s. Thus, all Poor countries have the same $\tau_{i,t}$'s.

Legend

DE: De Loecker, Jan and Eeckhout, Jan, "Global Market Power," NBER Working Paper No. 24768 (June 2018). Figure 5; underlying data kindly provided by the authors.

DR: Duarte, Fernando and Rosa, Carlo, "The Equity Risk Premium: A Review of Models," *FRBNY Economic Policy Review* 21/2 (December 2015), 39-57. Chart 1; underlying data kindly provided by the authors.

FRED: Federal Reserve Economic Data. Published by the Federal Reserve Bank of St. Louis. https://fred.stlouisfed.org/

IMF-FAD: International Monetary Fund, *FAD Investment And Capital Stock Database 2019: Manual & FAQ - Estimating Public, Private, And PPP Capital Stocks* (no date), p. 4. https://www.imf.org/external/np/fad/publicinvestment/data/info080219.pdf .

UN: *United Nations National Accounts*, Table 4.3, Non-financial Corporations. https://unstats.un.org/unsd/nationalaccount/madt.asp .

Appendix B: Underlying Data for The Baseline ROK^P's In Figure 1

Year	Aggregate	Rich	Poor	Upper Middle	Lower Middle	Low
1970	0.142	0.118	0.200	0.190	0.220	0.174
1971	0.141	0.117	0.202	0.196	0.216	0.166
1972	0.140	0.116	0.201	0.201	0.200	0.188
1973	0.143	0.116	0.212	0.218	0.201	0.166
1974	0.135	0.109	0.200	0.208	0.187	0.163
1975	0.131	0.106	0.190	0.196	0.179	0.159
1976	0.131	0.109	0.186	0.190	0.180	0.158
1977	0.134	0.112	0.187	0.185	0.191	0.146
1978	0.137	0.117	0.187	0.180	0.203	0.150
1979	0.127	0.115	0.156	0.158	0.152	0.155
1980	0.125	0.109	0.165	0.164	0.168	0.157
1981	0.127	0.113	0.164	0.157	0.180	0.149
1982	0.127	0.112	0.164	0.153	0.188	0.137
1983	0.125	0.114	0.152	0.145	0.166	0.149
1984	0.129	0.119	0.154	0.154	0.153	0.138
1985	0.131	0.121	0.156	0.161	0.148	0.112
1986	0.139	0.126	0.169	0.173	0.163	0.135
1987	0.136	0.127	0.158	0.161	0.151	0.146
1988	0.140	0.132	0.160	0.160	0.159	0.138
1989	0.141	0.135	0.156	0.155	0.157	0.156
1990	0.141	0.134	0.157	0.158	0.154	0.160
1991	0.147	0.134	0.179	0.185	0.166	0.202
1992	0.146	0.132	0.179	0.185	0.168	0.186
1993	0.151	0.136	0.188	0.200	0.167	0.182
1994	0.160	0.143	0.202	0.213	0.181	0.189
1995	0.166	0.149	0.209	0.214	0.200	0.174
1996	0.170	0.150	0.220	0.224	0.211	0.197
1997	0.177	0.149	0.245	0.239	0.257	0.209
1998	0.174	0.146	0.242	0.241	0.244	0.210
1999	0.169	0.144	0.232	0.238	0.221	0.255
2000	0.167	0.141	0.233	0.236	0.228	0.204
2001	0.166	0.138	0.237	0.234	0.244	0.225
2002	0.168	0.138	0.244	0.234	0.263	0.234
2003	0.174	0.141	0.255	0.243	0.279	0.241
2004	0.182	0.148	0.266	0.256	0.284	0.259

--continued--

Appendix B: Underlying Data for The Baseline ROK^{P}'s In Figure 1 (continued)

Year	Aggregate	Rich	Poor	Upper Middle	Lower Middle	Low
2005	0.190	0.155	0.277	0.262	0.307	0.275
2006	0.202	0.163	0.300	0.281	0.336	0.292
2007	0.212	0.169	0.317	0.298	0.354	0.305
2008	0.205	0.165	0.304	0.290	0.333	0.310
2009	0.207	0.159	0.327	0.309	0.362	0.340
2010	0.217	0.167	0.340	0.325	0.368	0.356
2011	0.222	0.170	0.350	0.335	0.377	0.380
2012	0.227	0.173	0.360	0.345	0.388	0.392
2013	0.239	0.176	0.394	0.369	0.443	0.416
2014	0.245	0.179	0.407	0.373	0.471	0.407

Appendix C: List Of 88 Countries

Country	Country Code	Income Group (4 Categories; World Bank)	Rich/Poor (2 Categories)
(1)	(2)	(3)	(4)
A. 32 Rich Countries 32 High Income Countries			
Australia	AUS	High	Rich
Austria	AUT	High	Rich
Bahamas	BHS	High	Rich
Bahrain	BHR	High	Rich
Belgium	BEL	High	Rich
Canada	CAN	High	Rich
Chile	CHL	High	Rich
China, Hong Kong SAR	HKG	High	Rich
Cyprus	CYP	High	Rich
Finland	FIN	High	Rich
France	FRA	High	Rich
Germany	DEU	High	Rich
Iceland	ISL	High	Rich
Ireland	IRL	High	Rich
Israel	ISR	High	Rich
Italy	ITA	High	Rich
Japan	JPN	High	Rich
Kuwait	KWT	High	Rich
Luxembourg	LUX	High	Rich
Malta	MLT	High	Rich
Netherlands	NLD	High	Rich
New Zealand	NZL	High	Rich
Norway	NOR	High	Rich
Oman	OMN	High	Rich
Republic of Korea	KOR	High	Rich
Singapore	SGP	High	Rich
Sweden	SWE	High	Rich
Switzerland	CHE	High	Rich
Trinidad and Tobago	TTO	High	Rich
United Kingdom	GBR	High	Rich
United States	USA	High	Rich
Uruguay	URY	High	Rich
-- continued –			

Country	Country Code	Income Group (4 Categories; World Bank)	Rich/Poor (2 Categories)
(1)	(2)	(3)	(4)
B.1. 56 Poor Countries 23 Upper Middle Income Countries			
Argentina	ARG	Upper Middle	Poor
Botswana	BWA	Upper Middle	Poor
Brazil	BRA	Upper Middle	Poor
China	CHN	Upper Middle	Poor
Colombia	COL	Upper Middle	Poor
Costa Rica	CRI	Upper Middle	Poor
Dominican Republic	DOM	Upper Middle	Poor
Ecuador	ECU	Upper Middle	Poor
Fiji	FJI	Upper Middle	Poor
Gabon	GAB	Upper Middle	Poor
Iran (Islamic Republic of)	IRN	Upper Middle	Poor
Lebanon	LBN	Upper Middle	Poor
Malaysia	MYS	Upper Middle	Poor
Mauritius	MUS	Upper Middle	Poor
Mexico	MEX	Upper Middle	Poor
Namibia	NAM	Upper Middle	Poor
Panama	PAN	Upper Middle	Poor
Paraguay	PRY	Upper Middle	Poor
Peru	PER	Upper Middle	Poor
South Africa	ZAF	Upper Middle	Poor
Suriname	SUR	Upper Middle	Poor
Thailand	THA	Upper Middle	Poor
Venezuela	VEN	Upper Middle	Poor

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Country	Country Code	Income Group (4 Categories; World Bank)	Rich/Poor (2 Categories)
(1)	(2)	(3)	(4)
B.2. 56 Poor Countries **22 Lower Middle Income Countries**			
Bolivia	BOL	Lower Middle	Poor
Cameroon	CMR	Lower Middle	Poor
Côte d'Ivoire	CIV	Lower Middle	Poor
Egypt	EGY	Lower Middle	Poor
Guatemala	GTM	Lower Middle	Poor
Honduras	HND	Lower Middle	Poor
India	IND	Lower Middle	Poor
Indonesia	IDN	Lower Middle	Poor
Jordan	JOR	Lower Middle	Poor
Kenya	KEN	Lower Middle	Poor
Lao People's DR	LAO	Lower Middle	Poor
Lesotho	LSO	Lower Middle	Poor
Mauritania	MRT	Lower Middle	Poor
Mongolia	MNG	Lower Middle	Poor
Morocco	MAR	Lower Middle	Poor
Nicaragua	NIC	Lower Middle	Poor
Nigeria	NGA	Lower Middle	Poor
Philippines	PHL	Lower Middle	Poor
Sri Lanka	LKA	Lower Middle	Poor
Sudan (Former)	SDN	Lower Middle	Poor
Swaziland	SWZ	Lower Middle	Poor
Tunisia	TUN	Lower Middle	Poor

-- continued --

Country	**Country Code**	**Income Group (4 Categories; World Bank)**	**Rich/Poor (2 Categories)**
(1)	(2)	(3)	(4)
B.3. 56 Poor Countries 11 Low Income Countries			
Benin	BEN	Low	Poor
Burkina Faso	BFA	Low	Poor
Central African Republic	CAF	Low	Poor
Chad	TCD	Low	Poor
Guinea	GIN	Low	Poor
Mozambique	MOZ	Low	Poor
Niger	NER	Low	Poor
Rwanda	RWA	Low	Poor
Senegal	SEN	Low	Poor
Sierra Leone	SLE	Low	Poor
U.R. Tanzania: Mainland	TZA	Low	Poor

Notes: Countries are excluded in the sample of 88 countries used in this paper if they (i) have produced capital stock (K^{P+G}), labor income share (LIS), or output (Y) data missing during any part of the 1970-2014 sample period; (ii) are ex-communist countries; or (iii) have negative values for ROK^{P} during any part of the sample period. Zimbabwe is excluded because of its extremely high values for ROK^{P}. The Rich/Poor groups are defined in terms of the four income groups taken from the World Bank classification: Rich ={High}, Poor = {Upper Middle, Lower Middle, Low}.

Appendix D: Formula For Decomposing Movements In the ROK^{P}'s

This appendix provides details on the decomposition of ROK^{P} discussed in Section 3.3 and Table 3. For a given country, $ROK^{P}_{i,t}$ is determined by five multiplicative components detailed in (6):

$$\text{(D1)} \qquad ROK^{P}_{i,t} = X^{1}_{i,t} * X^{2}_{i,t} * X^{3}_{i,t} * X^{4}_{i,t} * X^{5}_{i,t}, \qquad t=1970, 2014$$

where

$$X^{1}_{i,t} \equiv \left(CIS^{P+N+G}_{i,t} - CIS^{N}_{i,t}\right),$$
$$X^{2}_{i,t} \equiv \left(Y_{i,t} / K^{P}_{i,t}\right),$$
$$X^{3}_{i,t} \equiv \left(PY_{i,t} / PK^{P}_{i,t}\right),$$
$$X^{4}_{i,t} \equiv \left(1+\varphi_{i} * RK^{G,P}_{i,t}\right)^{-1},$$
$$X^{5}_{i,t} \equiv (1+SE_{i,t}).$$

For a given group of countries defined by income (Aggregate, Rich, Poor, Upper Middle, Lower Middle, Low), the $ROK^{P}_{i,t}$'s are aggregated with a fixed set of capital weights (ω_{i}),

$$\text{(D2)} \qquad ROK^{P}_{t} = \sum_{i=1}^{88} \omega_{i} ROK^{P}_{i,t} = \sum_{i=1}^{88} \omega_{i} \left(X^{1}_{i,t} * X^{2}_{i,t} * X^{3}_{i,t} * X^{4}_{i,t} * X^{5}_{i,t}\right) \qquad t=1970, 2014$$

When we evaluate sub-groups, some of the ω_{i}'s are set to 0 and the positive ω_{i}'s renormalized to sum to 1.0.

We are interested in quantifying how much of the growth rate in ROK^{P} from 1990 to 2014 is explained by each of the five components. The ROK^{P} for 1990 can be written as follows and redefined in terms of gross growth rates, (D3a) $$ROK^{P}_{2014} = \sum_{i=1}^{88} \omega_i \left(X^{1}_{i,2014} * X^{2}_{i,2014} * X^{3}_{i,2014} * X^{4}_{i,2014} * X^{5}_{i,2014} \right),$$

(D3b) $$ROK^{P}_{2014} = \sum_{i=1}^{88} \omega_i \begin{pmatrix} \frac{X^{1}_{i,2014}}{X^{1}_{i,1990}} X^{1}_{i,1990} * \frac{X^{2}_{i,2014}}{X^{2}_{i,1990}} X^{2}_{i,1990} * \frac{X^{3}_{i,2014}}{X^{3}_{i,1990}} X^{3}_{i,1990} * \\ \frac{X^{4}_{i,2014}}{X^{4}_{i,1990}} X^{4}_{i,1990} * \frac{X^{5}_{i,2014}}{X^{5}_{i,1990}} X^{5}_{i,1990} \end{pmatrix},$$

(D3c) $$ROK^{P}_{2014} = \sum_{i=1}^{88} \omega_i \begin{pmatrix} (1+G^{1}_{i}) * X^{1}_{i,1990} * (1+G^{2}_{i}) * X^{2}_{i,1990} * (1+G^{3}_{i}) * X^{3}_{i,1990} * \\ (1+G^{4}_{i}) * X^{4}_{i,1990} * (1+G^{5}_{i}) * X^{5}_{i,1990} \end{pmatrix},$$

(D3d) $$ROK^{P}_{2014} = \sum_{i=1}^{88} \omega_i \left((1+G^{1}_{i}) * (1+G^{2}_{i}) * (1+G^{3}_{i}) * (1+G^{4}_{i}) * (1+G^{5}_{i}) \right) * ROK^{P}_{1990},$$

where $\left(1+G^{j}_{i}\right)$ is the gross growth rate in component j between 1990 and 2014,

(D3e) $$\left(1+G^{j}_{i}\right) \equiv \left(X^{j}_{i,2014} / X^{j}_{i,1990} \right). \qquad j=1,5$$

Multiplying-out the five gross growth rates on the right-side of (D3d), we obtain the following equation,

(D4a) $$\begin{aligned} ROK^{P}_{2014} = {} & ROK^{P}_{1990} + \sum_{i=1}^{88} \omega_i \left(G^{1}_{i} + G^{2}_{i} + G^{3}_{i} + G^{4}_{i} + G^{5}_{i} \right) ROK^{P}_{1990} \\ & + \sum_{i=1}^{88} \omega_i \, Z_i \, ROK^{P}_{1990} , \end{aligned}$$

(D4b) $$\frac{ROK^{P}_{2014}}{ROK^{P}_{1990}} - 1 - \sum_{i=1}^{88} \omega_i \, Z_i = \sum_{i=1}^{88} \omega_i \left(G^{1}_{i} + G^{2}_{i} + G^{3}_{i} + G^{4}_{i} + G^{5}_{i} \right) ,$$

where Z_i represents the set of second-order, third-order, fourth-order, and fifth-order interaction terms from multiplying-out the five gross growth rates on the right-side of (D3d).

There are two equivalent ways to treat this term and to make (D4b) empirically operational. The $\sum_{i=1}^{88} \omega_i \, Z_i$ term can be moved to the right-side of (D4b) and allocated to the five components in proportion to the size of the component's first-order terms, (D5a) $$\sum_{i=1}^{88} \omega_i Z_i = \sum_{j=1}^{5} \left(\Omega^j \sum_{i=1}^{88} \omega_i Z_i \right) \sum_{i=1}^{88} \omega_i G_i^j \ ,$$

(D5b) $$\Omega^j \equiv \sum_{i=1}^{88} \omega_i G_i^j \ / \ \sum_{j=1}^{5} \left(\sum_{i=1}^{88} \omega_i G_i^j \right),$$

(D5c) $$\sum_{j=1}^{5} \Omega^j = 1 \, .$$

This allocation can be implemented by multiplying all terms in (D4b) by a proportionality constant so that they sum to $\frac{ROK_{2014}^{P}}{ROK_{1990}^{P}} - 1$.

Alternatively, the adjusted growth rate of ROK^P (the left-side of (D4b)) can be defined as follows,

(D6) $$G^{ROK^P} \equiv \frac{ROK_{2014}^{P}}{ROK_{1990}^{P}} - 1 - \sum_{i=1}^{88} \omega_i Z_i \ ,$$

and measured by the five terms on the right-side of (D4b). The percentage contribution of any of the five components to the change in ROK^P from 1990 to 2014, Γ^j, is given by the following equation, the ratio of the weighted-average growth rates (for the countries in a group) for the given component to the growth rate of ROK^P for that group,

(D7a) $$\Gamma^j = \frac{\sum_{i=1}^{88} \omega_i G_i^j}{G^{ROK^P}} \ , \quad j=1,5$$

(D7b) $$\Gamma^j = \frac{\sum_{i=1}^{88} \omega_i G_i^j}{\sum_{i=1}^{88} \omega_i \left(G_i^1 + G_i^2 + G_i^3 + G_i^4 + G_i^5 \right)} \ , \quad j=1,5$$

(D7c) $$\Gamma^j = \frac{G^j}{\sum_{j=1}^{5} G^j} \ , \quad j=1,5$$

(D7d) $$G_i^j \equiv \sum_{i=1}^{88} \omega_i G_i^j, \quad j=1,5$$

(D7e) $$\sum_{j=1}^{5} \Gamma^j = 1.$$

The Γ^j's are reported in Table 3 for the five components and six groups.

Table 1: Descriptive Statistics

	1970-2014	1970-1990	1991-2014
	(1)	(2)	(3)
1. Capital Income Share For Produced Capital (CIS^{P+G} = CIS^{P+N+G} – CIS^{N})			
Poor	0.489 [0.487] {0.233}	0.473 [0.472] {0.260}	0.503 [0.500] {0.223}
Rich	0.450 [0.419] {0.254}	0.434 [0.391] {0.281}	0.465 [0.452] {0.241}
Poor / Rich	1.086 [1.161]	1.090 [1.207]	1.082 [1.105]
2. Output/Private Capital Ratio (Y / K^{P})			
Poor	1.168 [0.840] {0.893}	1.309 [0.795] {1.345}	1.045 [0.871] {0.586}
Rich	0.750 [0.543] {0.993}	0.790 [0.491] {1.403}	0.715 [0.569] {0.689}
Poor / Rich	1.558 [1.547]	1.657 [1.618]	1.462 [1.530]
3. Relative Price Of Output To The Price Of Private Capital (PY / PK^{P})			
Poor	0.799 [0.809] {0.208}	0.642 [0.657] {0.363}	0.937 [0.942] {0.170}
Rich	1.002 [0.990] {0.156}	0.910 [0.959] {0.207}	1.081 [1.042] {0.158}
Poor / Rich	0.798 [0.816]	0.705 [0.686]	0.866 [0.904]
4. Ratio Of Capital Stocks, Government Relative To Private ($RK^{G,P}$)			
Poor	1.278 [0.547] {3.000}	1.776 [0.498] {4.188}	0.842 [0.576] {1.060}
Rich	0.557 [0.353] {1.130}	0.599 [0.359] {1.256}	0.521 [0.342] {1.046}
Poor / Rich	2.292 [1.550]	2.964 [1.385]	1.615 [1.684]
5. Share Of Economic Activity In The Shadow Economy (SE)			
Poor	0.378 [0.380] {0.305}	0.383 [0.384] {0.304}	0.373 [0.370] {0.307}
Rich	0.190 [0.172] {0.446}	0.193 [0.177] {0.445}	0.187 [0.168] {0.447}
Poor / Rich	1.988 [2.203]	1.986 [2.167]	1.990 [2.200]
6. Price Of Output (PY)			
Poor	0.334 [0.325] {0.283}	0.264 [0.248] {0.444}	0.395 [0.370] {0.267}
Rich	0.656 [0.669] {0.238}	0.417 [0.431] {0.245}	0.864 [0.877] {0.247}
Poor / Rich	0.509 [0.487]	0.632 [0.575]	0.457 [0.422]
7. Price Of Private Capital (PK^{P})			
Poor	0.485 [0.431] {0.516}	0.526 [0.379] {0.836}	0.449 [0.422] {0.307}
Rich	0.662 [0.661] {0.254}	0.469 [0.465] {0.288}	0.831 [0.857] {0.274}
Poor / Rich	0.732 [0.652]	1.120 [0.814]	0.540 [0.492]

Notes: A glossary and details concerning variable definitions and sources can be found in Appendix A. The figures are the mean [median] {coefficient of variation = standard deviation / absolute value of the mean}. For a given $X_{i,t}$, we first compute the time mean ($\bar{X}_i$), and then compute the mean, median, and standard deviation reported in Table 1 based on $\bar{X}_i$.

Table 2: Return On Private Capital (ROK^P), 1970-2014

Income Group	1970-1990	1991-2014		1987-1990	% Change
	(1)	(2)		(3)	(4)
A. Baseline ROK^P					
Aggregate	0.134	0.187		0.139	33.90
Poor	0.175	0.271		0.158	72.16
Rich	0.118	0.153		0.132	15.48
Poor/Rich	1.484	1.777		1.192	
Upper Middle	0.174	0.264		0.159	66.27
Lower Middle	0.177	0.285		0.156	83.58
Low	0.153	0.268		0.150	79.09
B. Restricted ROK^P ($RK^{G,P} = 0 = SE$)					
Aggregate	0.135	0.184		0.142	29.77
Poor	0.141	0.222		0.129	72.52
Rich	0.132	0.168		0.147	14.59
Poor/Rich	1.072	1.324		0.880	
Upper Middle	0.139	0.215		0.128	67.72
Lower Middle	0.148	0.237		0.131	81.47
Low	0.110	0.193		0.108	79.20
C. Equal Weighted ROK^P					
Aggregate	0.168	0.255		0.164	55.66
Poor	0.184	0.288		0.172	66.93
Rich	0.139	0.197		0.148	32.75
Poor/Rich	1.327	1.461		1.162	
Upper Middle	0.164	0.289		0.161	79.44
Lower Middle	0.187	0.289		0.169	70.97
Low	0.220	0.283		0.203	39.42
D. Trimmed ROK^P With Large Countries Removed					
Aggregate	0.136	0.210		0.144	45.89
Poor	0.175	0.312		0.162	92.33
Rich	0.122	0.172		0.137	25.36
Poor/Rich	1.435	1.818		1.185	
Upper Middle	0.175	0.293		0.165	78.03
Lower Middle	0.174	0.370		0.155	138.33
Low	0.197	0.250		0.164	52.17

Notes: The entries in columns 1, 2, and 3 are rates of return, except for the "Poor/Rich" rows, which are ratios of the two entries immediately above. See equation (6) and sections 3.1 and 3.2 for further details. Column 4 is the ratio of column 2 to column 3, less 1 times 100. Entries in Panels A, B, and D are capital weighted.

Table 3: Decomposing Movements In The Return On Private Capital (ROK^P) Average (1987-1990) To Average (1991-2014)

	Aggregate		Poor	Rich		Upper Middle	Lower Middle	Low
	(1)		(2)	(3)		(4)	(5)	(6)
A. All Five Components (%) [arithmetic]	100.0 [0.046]		100.0 [0.114]	100.0 [0.018]		100.0 [0.106]	100.0 [0.131]	100.0 [0.109]
CIS^{P+N+G} - CIS^N	23.6		16.2	39.8		13.6	20.9	4.4
Y/K^P	7.6		-2.0	28.7		-1.4	-3.5	20.7
$RK^{G,P}$	18.8		23.5	8.4		29.1	14.5	4.8
SE	-1.5		-1.1	-2.3		-1.2	-0.9	-1.2
PY / PK^P	51.5		63.4	25.4		59.9	69.0	71.3
B. Relative Price Components								
PY	99.1		63.3	155.8		83.7	21.8	37.6
PK^P	0.9		36.7	-55.8		16.3	78.2	62.4

Table 3 Notes: See Section 3.3 and Appendix D, equation (D7) for the formula decomposing ROK^P growth. All computations based on equation (6), which is composed of five multiplicative components. For a given component, its contribution to the overall ROK^P growth rate is computed as its growth rate -- holding the other four components fixed at their starting values – relative to the growth rate in ROK^P. Entries are the percentage of the overall ROK^P change explained by a given ROK^P component. Entries in brackets are the arithmetic change in ROK^P. The starting and ending values are the average values for 1987-1990 and 1991-2014, respectively. Note that $RK^{G,P}$ enters inversely. All entries are capital weighted.

Table 4 Notes: See Section 4, equation (11) for the formula determining the calculations. The entries are the percentage change of the factor between the earlier and later periods. The earlier [later] period is the average value for 1987-1990 [1991-2014]. Panel A entries are the movement in ROK^P. Panel B entries are the percentage of the ROK^P movement explained by a given factor. The UCC^P entry is the summation of the entries for the six factors. The second-order and third-order terms are the interactions among the additive factors in the UCC formula (11). Panel C entries are the values of the average value of the factors in [1987-1990] and {1991-2014}. The penultimate entry is for UCC^P constrained to equal the six factors, thus preserving the adding-up property between UCC^P and its six factors. The final entry does not impose this constraint. The percentage changes are computed by country and then capital weighted to create the group entries in Table 4.

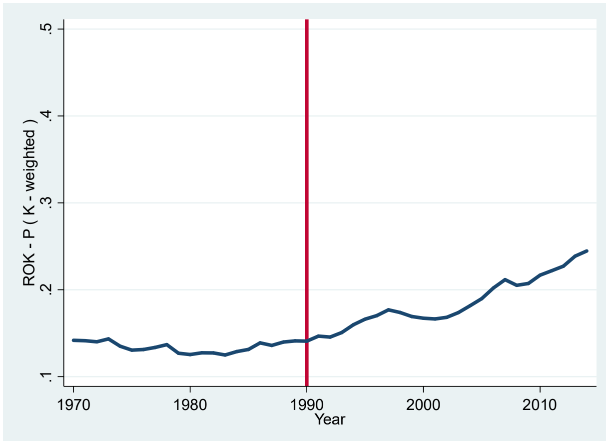
Table 4: Explaining Movements In The Return On Private Capital (ROK^P) Average (1987-1990) To Average (1991-2014)

	All		**Poor**	**Rich**		**Upper Middle**	**Lower Middle**	**Low**
	(1)		(2)	(3)		(4)	(5)	(6)
A. ROK^P % Change	35.3		82.0	16.5		68.0	108.8	93.7
B. Contribution Of Factors (% Of Movement In ROK^P)								
Real Interest Rate (ρ)	-45.1		-21.6	-92.2		-25.2	-17.3	-15.7
Equity Risk Premium (ζ)	43.5		20.8	89.1		24.4	16.7	15.1
Depreciation Rate (δ^P)	26.6		8.9	62.0		10.5	7.2	0.0
Capital Loss (λ^P)	25.8		35.2	6.9		31.5	40.8	-7.4
$\rho + \zeta + \delta^P + \lambda^P$	50.9		43.4	65.8		41.1	47.4	-7.9
Markup (μ)	39.4		12.0	94.3		10.3	14.0	17.1
Corporate Taxes ($(1-\tau)^{-1}$)	4.0		4.6	3.0		5.7	3.3	4.0
Summation (UCC^P)	**94.3**		**60.0**	**163.1**		**57.1**	**64.7**	**13.2**
Second-Order Terms	6.9		5.4	10.0		2.1	9.5	-0.9
Third-Order Terms	0.1		0.2	-0.1		0.1	0.3	-0.0
C. Values Of The Factors, [Average (1987-1990)] and {Average (1991-2014)}								
Real Interest Rate (ρ)	[0.047] {0.023}		[0.047] {0.023}	[0.047] {0.023}		[0.047] {0.023}	[0.047] {0.023}	[0.047] {0.023}
Equity Risk Premium (ζ)	[0.035] {0.059}		[0.035] {0.059}	[0.035] {0.059}		[0.035] {0.059}	[0.035] {0.059}	[0.035] {0.059}
Depreciation Rate (δ^P)	[0.073] {0.087}		[0.063] {0.073}	[0.076] {0.093}		[0.063] {0.074}	[0.063] {0.074}	[0.043] {0.043}
Capital Loss (λ^P)	[0.007] {0.013}		[0.004] {0.033}	[0.008] {0.005}		[0.009] {0.028}	[-0.006] {0.041}	[0.059] {0.032}
$\rho + \zeta + \delta^P + \lambda^P$	[0.161] {0.181}		[0.149] {0.188}	[0.166] {0.179}		[0.154] {0.183}	[0.139] {0.196}	[0.183] {0.156}
Markup (μ)	[1.237] {1.401}		[1.273] {1.379}	[1.222] {1.410}		[1.362] {1.435}	[1.107] {1.273}	[1.089] {1.264}
Corporate Taxes ($(1-\tau)^{-1}$)	[1.237] {1.251}		[1.163] {1.206}	[1.267] {1.270}		[1.162] {1.206}	[1.165] {1.207}	[1.166] {1.210}
User Cost Of Capital (UCC^P) Constrained	[0.246] {0.317}		[0.217] {0.311}	[0.257] {0.320}		[0.237] {0.317}	[0.180] {0.302}	[0.226] {0.239}
User Cost Of Capital (UCC^P) Unconstrained	[0.255] {0.319}		[0.257] {0.313}	[0.254] {0.321}		[0.292] {0.318}	[0.190] {0.304}	[0.214] {0.248}

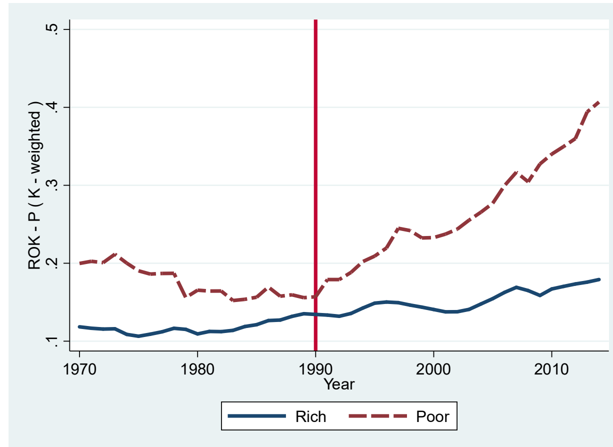
Notes: See the bottom of Table 3.

Figure 1: Return On Private Capital (ROK^P), 1970-2014

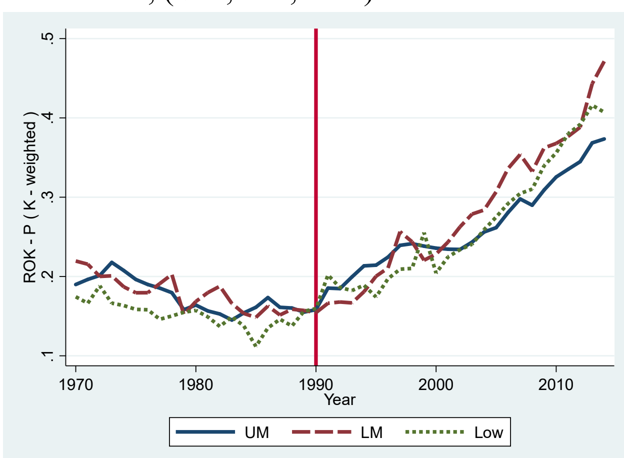
1.A: ROK^P (Aggregate)



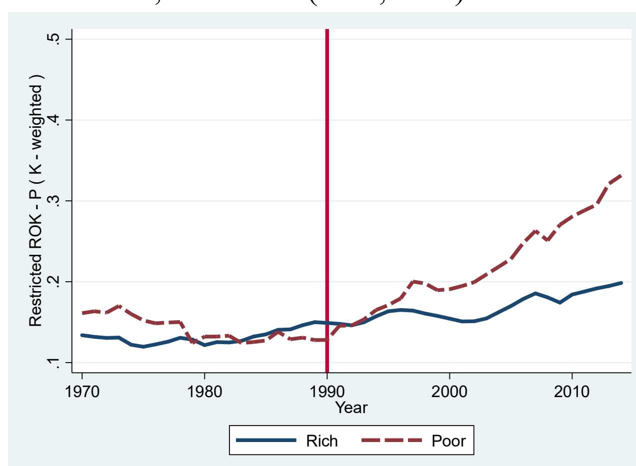
1.B: ROK^P (Rich, Poor)



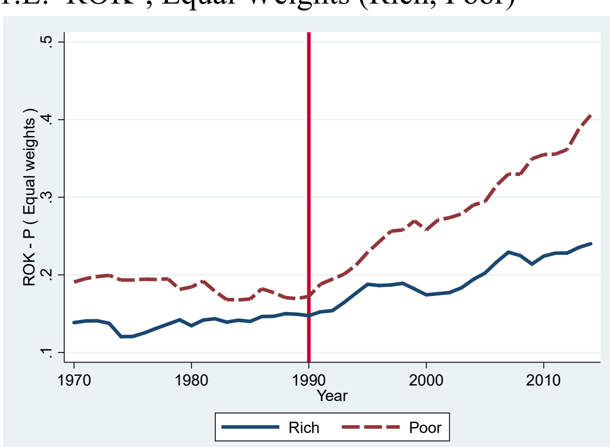
1.C: ROK^P, (UM, LM, Low)



1.D: ROK^P, Restricted (Rich, Poor)



1.E: ROK^P, Equal Weights (Rich, Poor)



1.F: ROK^P, Trimmed Weights (Rich, Poor)

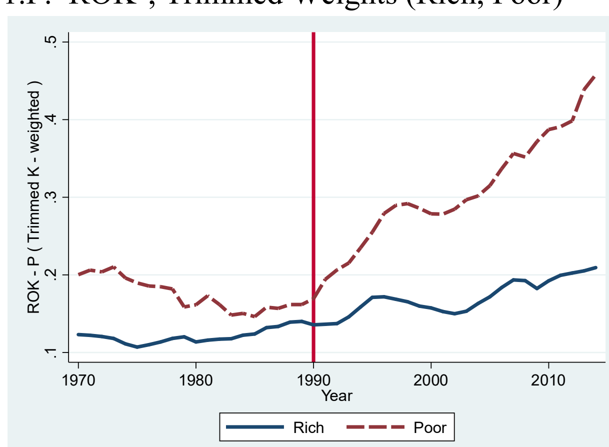
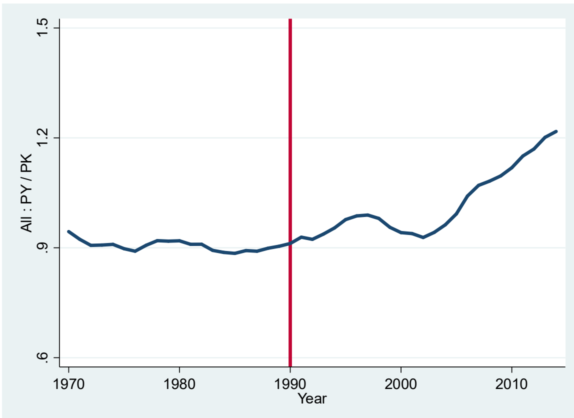
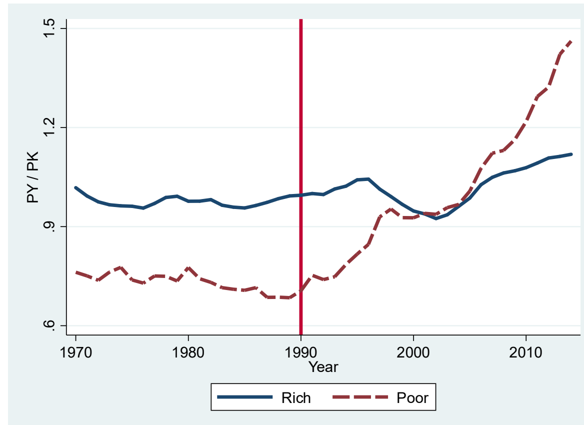


Figure 2: Relative Price Of Output (PY/PK^P) And Output Price (PY) And Capital Price (PK^P) Deflators, 1970-2014

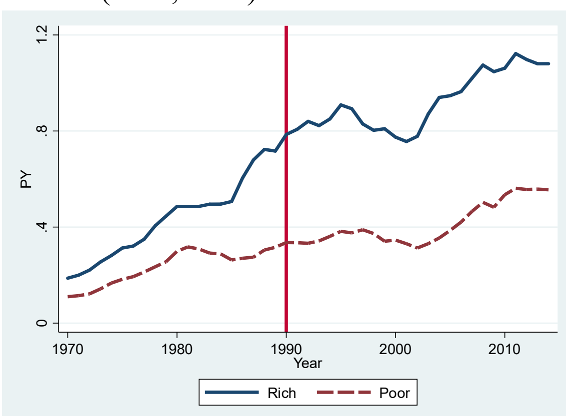
2.A: PY/PK^P (Aggregate)



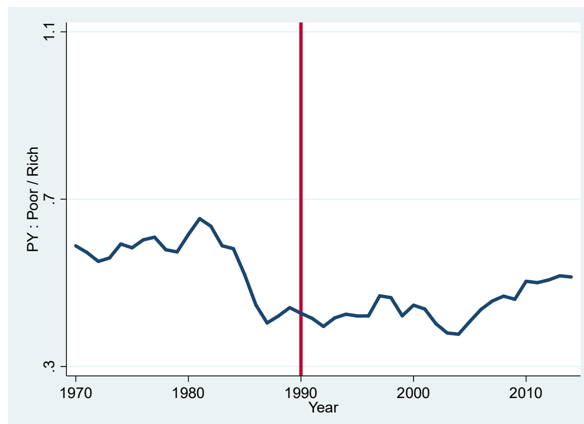
2.B: PY/PK^P (Rich, Poor)



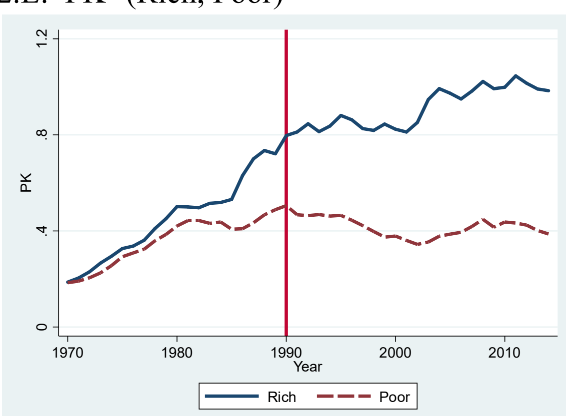
2C: PY (Rich, Poor)



2.D: PY^{Poor}/PY^{Rich}



2.E: PK^P (Rich, Poor)



2.F: $PK^{P,Poor}/PK^{P,Rich}$