Foreign Reserves and Capital Controls: Role of Financial Development*

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Abstract

This paper develops a small-open-economy model to study optimal capital controls and foreign reserve policy. Private agents hold reserves to prepare for a liquidity shock that requires them to repay a part of outstanding foreign debt before new borrowing. A fire-sale externality associated with asset liquidation induces private agents to overborrow and accumulate too little reserves. The optimal policy calls for a tax on debt and either of a subsidy on private reserves or public foreign reserve accumulation. We show that the optimal debt tax rate becomes higher as the size of a potential liquidity shock becomes larger, but the optimal amount of foreign reserves is non-monotonic and maximized when the size of a potential liquidity shock is intermediate. This pattern is consistent with the observed cross-country relationships across financial development, capital controls, and foreign reserves.

Keywords: Capital Controls; Foreign Reserves; Sudden Stops; Liquidity Crisis.

JEL: F32, F41, F44.

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1 Introduction

Emerging economies have been integrated into the global financial market since the 1990s and actively engaged in international financial transactions. As a result, these economies are subject to volatile capital flows driven by changes in the global financial conditions and policies by major advanced economies (Rey 2015). As of 2022, aggressive monetary tightening by the Federal Reserve and other major central banks to combat high inflation have been causing capital outflows and currency depreciation in many emerging economies. For these economies, capital controls and foreign reserves are two important policy tools to smooth impacts from the global financial market. Accordingly, there is a growing literature on these policy tools (Basu et al. 2020). However, there is a wide cross-country variation in how actively each country uses these policy tools, and important questions remain unanswered: What is the optimal combination of capital controls and reserve policy? What is the key determinant of the optimal combination of these policies? What explains the cross-country variation in capital controls and reserve holdings?

In this paper, we develop a small-open-economy model to address these questions. To motivate our study, we first show empirical facts about capital controls and foreign reserves, focusing on cross-country differences in the stage of financial development. There are two main findings. First, the cross-country relationship between financial development and the reserve-to-GDP ratio is non-monotonic. In particular, countries with an intermediate level of financial development tend to have a higher reserve-to-GDP ratio than countries with either high or low financial development. Second, financial development also affects capital controls policy. In the data, countries with high financial development use capital controls less actively. Therefore, financial development is an important factor to understand both capital controls and reserve policies.

We then develop a dynamic small-open-economy model to understand these empirical facts. Our main contribution to the literature is twofold. First, we deviate from preceding models of sudden stops and capital controls by highlighting the role of asset fire sales that cause persistent slowdown of economic growth in shaping the optimal policies. In our model, capital controls and reserve policies are jointly required to correct a fire-sale externality, which is triggered by costly asset liquidation responding to a liquidity shock. Second, we show that the size of a liquidity shock, which we interpret as the measure of financial development, can explain the observed cross-country patterns in capital controls and foreign reserves.

In the model economy, households produce and consume tradable goods, borrow from abroad, and invest tradable goods to accumulate productive assets. Production is linear in the amount of assets, and the model features semi-endogenous growth.[1] The key element of the model is that at the beginning of each period before new borrowing and production, households may be required to repay a certain fraction of outstanding foreign debt with an exogenous probability. We call it a liquidity shock. When this happens, households can liquidate a part of their productive assets and use the proceeds for repayment. However, liquidation is costly because liquidated assets can be sold only at a fire-sale price. To reduce this costly liquidation, households have an incentive to hold foreign reserves as a liquidity buffer. Our model thus highlights a liquidity role of foreign reserves.[2]

The amount of asset liquidation is determined to cover the liquidity shortage, which is the amount of required repayment minus reserve holdings. Therefore, households can reduce liquidation by either reducing debt or increasing reserves ex ante. An important difference between these two is that holding reserves has a relative advantage in liquidity management over reducing debt. If households increase reserve holdings by one unit, it will reduce the next-period liquidity shortage by one unit if a liquidity shock occurs. If households alternatively reduce debt by one unit, it will reduce the next-period liquidity shortage by units equal to the fraction of debt subject to a liquidity shock, which can be less than one. This relative advantage of holding reserves motivates households to hold reserves. On the other hand, reserve holdings are associated with an opportunity cost because the interest rate on reserves is lower than the interest rate on debt. Households thus choose debt and reserves to achieve a balance between the relative advantage in liquidity management and the opportunity cost of holding reserve.

The need for policy interventions comes from a fire-sale externality associated with asset liquidation. There are foreign buyers who buy liquidated assets and use them to produce tradable goods. The price at which they buy assets is equal to their marginal product of assets, which declines with the aggregate amount of liquidated assets. However, individual domestic households take this liquidation price as given, and thus a fire-sale externality arises. This externality distorts households' decisions on debt and reserves as follows. When a liquidity shock hits the economy, households need to liquidate assets to cover the liquidity shortage, which is the amount of required repayment minus reserve holdings. If households marginally reduce debt or increase reserves in the previous period, the liquidity shortage and

[1] What makes growth semi-endogenous instead of fully endogenous is the existence of foreign buyers who own foreign assets that grow at a fixed rate.

[2] A survey conducted by IMF (2013) shows that about 75% of the country authorities answer that precautionary liquidity holding is the main motivation for holding foreign reserves.

the necessary amount of liquidation become smaller, which increases the liquidation price and reduces liquidation even further. However, individual households do not internalize the impact of their debt and reserve decisions on the liquidation price. As a result, they borrow socially too much and hold socially too little reserves. The optimal policy to correct these distortions is a tax on foreign debt and either a subsidy on reserve holdings or official foreign reserve accumulation by the public sector.

We show that the potential size of a liquidity shock, which is a fraction of debt to be repaid when a liquidity shock occurs, is the key determinant of reserve holdings and capital controls. On the one hand, if the potential size of a liquidity shock is small, the potential size of a liquidity shortage is also small, and there is no need to hold a large amount of reserves. On the other hand, if the potential size of a liquidity shock is large, the relative advantage of reserves in liquidity management over debt becomes small. This means that reducing debt is as effective as accumulating reserves in reducing the liquidity shortage. In this case, households simply reduce debt rather than holding reserves. Therefore, the amount of reserve holdings becomes the largest when the size of a liquidity shock is intermediate. By contrast, the optimal debt tax rate monotonically increases in the size of a liquidity shock. We interpret the potential size of a liquidity shock as a measure of financial development. Then these model features are consistent with our empirical findings.

In the quantitative analysis, we calibrate the model to the average of 47 emerging economies and solve it numerically using a global method. The severity of asset fire sales is calibrated using the empirical estimates by Aguiar and Gopinath (2005). In a stochastic simulation with interest rate and liquidity shocks, a liquidity shock triggers a sharp current account reversal, asset liquidation, and persistently low output, consumption, and investment. These persistent impacts are due to semi-endogenous growth and are consistent with the empirical regularities of sudden stops. Compared with the decentralized economy, the social planner holds a larger amount of reserves to reduce liquidation. As a result, the share of liquidated assets is only 0.7% in contrast to 2.6% in the decentralized economy, and output after a crisis is persistently higher by more than 1% than in the decentralized economy.

Finally, we solve the model with a wide range in the size of a liquidity shock and compute the average debt tax rate and the amount of reserves. Consistent with our empirical findings, the amount of reserves is maximized at 33% of GDP when the size of a liquidity shock is intermediate, whereas the tax rate is monotonically increasing in the size of a liquidity shock. The expected welfare gain by these policies is also non-monotonic in the size of a liquidity shock and reaches 0.4% of a permanent consumption at the peak. The size of a welfare gain is substantially higher than welfare gains suggested by many preceding models, because in our model the optimal policy mitigates persistent negative impacts of crises on the economy. This result points to the importance of the persistent impacts of crises often observed in the data when we consider the optimal policy design.

Literature Review This paper contributes to a broad literature on capital controls and foreign reserve policy by emerging economies for a precautionary motive. One strand of literature focuses on capital controls to correct excessive foreign borrowing. A typical assumption in this literature is that a pecuniary externality through a drop in the collateral asset price induces over-borrowing by private agents and calls for a tax on private debt. Papers in this literature include Bianchi (2011), Benigno et al. (2013, 2016), Bianchi and Mendoza (2018), and Jeanne and Korinek (2020), among others. Ma (2020) introduces endogenous growth into the model and studies how capital controls should be designed when it affects growth.

Another strand of literature focuses on foreign reserve policies. Papers in this literature introduce different assumptions to motivate reserve accumulation, such as a shock to borrowing limit (Jeanne and Rancière 2011, Céspedes and Chang 2020, Matsumoto 2022), a liquidity shock (Hur and Kondo 2016), capital flow shocks (Cavallino 2019), sovereign default and endogenous borrowing cost (Hernández 2017, Bianchi et al. 2018, Bianchi and Sosa-Padilla 2020), self-fulfilling currency crisis (Bocola and Lorenzoni 2020), and collateral constraint on foreign borrowing (Shousha 2017). Jeanne and Sandri (2020) develop a model with a liquidity shock and a pecuniary externality to show that foreign reserve policy is necessary only for countries with intermediate levels of financial development, which is similar to our result. In contrast to these papers that study capital controls and reserve policies separately, we study the optimal combination of capital controls and reserve policy in a unified framework.

There are several papers that study the relationship between capital controls and reserve policies. Arce et al. (2019) is the first to show that public reserve accumulation can be used as a macroprudential policy tool similar to capital controls against sudden stops. Davis et al. (2021a), Davis et al. (2021b), and Fanelli and Straub (2021) assume financial frictions on private foreign debt similar to Gabaix and Maggiori (2015) and show that foreign reserve policy can be used as a substitute for capital controls to manage private capital flows. In these papers, reserve policy is at best a perfect substitute for capital controls, and if reserve policy is associated with a higher cost than capital controls, due to an interest gap between debt and reserves for example, then reserve policy would be dominated by capital controls.

Lutz and Zessner-Spitzenberg (2020) assume that a working capital payment for production is subject to a liquidity shock and show that both capital controls and reserve policies are necessary to achieve constrained efficient allocation. Our model deviates from these papers by highlighting the role of liquidity to reduce asset fire sales and mitigate slowdown of economic growth in shaping the optimal combination of capital controls and reserve policy. In this regard, our model is also related to Lorenzoni (2008) and Dávila and Korinek (2018) who focus on fire-sale externalities. In addition, our model can explain the observed cross-country patterns of financial development, capital controls, and reserve accumulation, which is novel in the literature.

The rest of the paper is organized as follows. Section 2 shows empirical facts about foreign reserves and capital controls. Section 3 lays out the model. Section 4 calibrates the model and conducts quantitative analyses, and Section 5 shows how the size of a liquidity shock affects the optimal policy. Section 6 concludes.

2 Motivating Empirical Facts

In this section, we present motivating empirical facts on financial development, foreign reserves, and capital controls. We use data for 88 countries (economies) in $1980 - 2019$.[3] The data source is described in Appendix A.

Fact 1: *The cross-country relationship between the level of financial development and the foreign reserve-to-GDP ratio is non-monotonic. Countries with an intermediate level of financial development tend to have a higher reserve-to-GDP ratio than countries with either high or low financial development.* Panel A in Figure 1 shows this result. We are interested in the cross-sectional long-run relationship. In the sample period of 1980 to 2019, we take the time average of the financial development index from the IMF and the reserve-to-GDP ratio from the dataset constructed by Lane and Milesi-Ferretti (2007). There is a

[3] Advanced economies (31) include Germany, Italy, Australia, United States, Czech Republic, Taiwan, Cyprus, Canada, Japan, Iceland, Estonia, Sweden, Denmark, France, Spain, United Kingdom, South Korea, Ireland, Slovenia, Finland, Norway, New Zealand, Latvia, Belgium, Austria, Lithuania, Portugal, Netherlands, Greece, Slovak Republic, and San Marino. Emerging economies (31) include Brazil, Nigeria, Vietnam, Paraguay, Thailand, Indonesia, Venezuela, Costa Rica, Morocco, India, Chile, South Africa, Romania, Russia, Uruguay, Mexico, Peru, Bulgaria, Philippines, Pakistan, Croatia, Malaysia, Colombia, Ukraine, Belarus, Poland, Serbia, Egypt, Turkey, Ecuador, and Hungary. Low-income economies (26) include Sierra Leone, Kenya, Mozambique, Burundi, Cameroon, Ghana, Zambia, Tanzania, Uganda, Eritrea, Haiti, Comoros, Rwanda, Ethiopia, Nicaragua, Guinea, Guyana, Central African Republic, Somalia, Mauritania, Madagascar, Chad, Malawi, Honduras, Sudan, and Bolivia.

non-linear relationship between these two variables. The red curve is a fitted curve obtained by regressing each country's reserve-to-GDP ratio on a linear and a quadratic term of the financial development index. Countries with an intermediate level of financial development tend to have a higher reserve-to-GDP ratio than countries with either high or low financial development. We also conduct a regressional analysis in Table 1 by conditioning on other important country-level variables such as population (a proxy for country size), GDP per capita (a proxy for economic development), private credit (a proxy for the size of the financial sector), and trade (a proxy for the economic openness). The non-linear correlation is robust to adding those country-level control variables. We also check whether this long-run relationship was driven by a specific decade in our sample. Our sample includes 80s, 90s, 00s, and 10s. It turns out that this non-linear relationship holds for each of the sub-sample periods (See Appendix A for a detailed analysis).

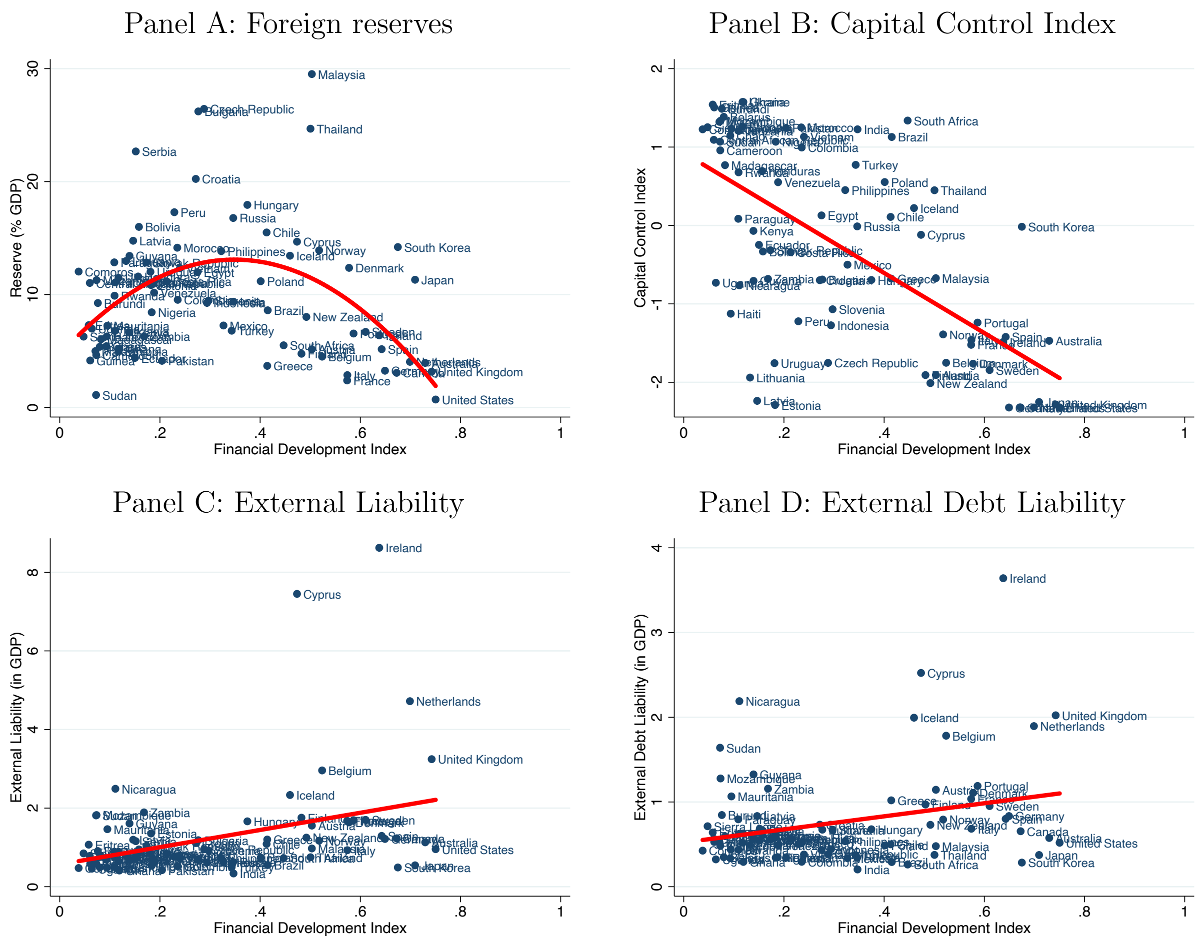
Fact 2: *Countries with high financial development use capital controls less actively.* Panel B in Figure 1 plots a similar scatter plot for the same 88 economies, this time with the average capital control index, i.e. the inverse of the Chinn-Ito index constructed by Chinn and Ito (2006), on the vertical axis. We observe a clear pattern that countries with high financial development tend to use capital controls less actively. Once we control for country-level variables, the negative correlation is still statistically significant.

Fact 3: *Countries with high financial development tend to have a high external liability-to-GDP ratio.* Panel C in Figure 1 plots this result with the same sample countries as above. The vertical axis is each country's average external liability-to-GDP ratio in $1980-2019$. The right panel looks at the external debt liability. We observe a clear pattern in both panels that countries with high financial development tend to have a high external liability-to-GDP ratio and debt liability-to-GDP ratio. Moreover, this relationship is robust in controlling for other important country-level characteristics.

We also study capital and reserve flow over business cycles. For this analysis, we use data for 47 emerging market economies in $1987-2019$.[4] Data for capital and reserve flows

[4] The sample consists of 47 countries: Argentina, Belarus, Belize, Brazil, Bulgaria, Chile, China, Colombia, Cote d'Ivoire, Croatia, Dominican Republic, Ecuador, Egypt, El Salvador, Gabon, Georgia, Ghana, Hungary, Indonesia, Iraq, Jamaica, Jordan, Kazakhstan, Lebanon, Lithuania, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Panama, Peru, Philippines, Poland, Russia, Senegal, South Africa, South Korea, Sri Lanka, Thailand, Trinidad and Tobago, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, Vietnam.

Figure 1 Reserve, Capital Control Index and External Debt: Long-run Relationship with Financial Development Index in 1980-2019



Note. The cross-sectional regression is conducted by taking the time average of all variables from 1980 to 2019 for each economy. The data on international reserves, external liability and external debt liability is from Lane and Milesi-Ferretti (2007). Capital control index is the inverse of the Chinn-Ito index from Chinn and Ito (2006).

Table 1 Reserve, Capital Control Index and External Debt: Long-run Relationship with Financial Development Index in 1980-2019

	Reserve/GDP		Capital Control Index		External Liability/GDP		External Debt Liability/GDP	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Financial Development	0.48***	0.43***	-3.83***	-3.78***	2.19***	2.19***	0.77***	0.77***
	(0.12)	(0.12)	(0.52)	(0.44)	(0.61)	(0.52)	(0.28)	(0.24)
Financial Development^2	-0.69***	-0.62***						
	(0.15)	(0.16)						
Pop (log)		-0.00		0.09		-0.34***		-0.21***
		(0.01)		(0.09)		(0.10)		(0.05)
GDP per capita (log)		-0.01		-0.57***		-0.33**		-0.14**
		(0.01)		(0.13)		(0.15)		(0.07)
Private credit		0.03		0.39		0.25		-0.11
		(0.02)		(0.40)		(0.48)		(0.22)
Trade		0.05***		-0.01		1.11***		0.32*
		(0.02)		(0.32)		(0.38)		(0.18)
Constant	0.05***	0.05***	0.93***	0.90***	0.57**	0.57***	0.52***	0.52***
	(0.02)	(0.02)	(0.19)	(0.16)	(0.23)	(0.19)	(0.10)	(0.09)
Observations	85	85	83	83	85	85	85	85
Adjusted R-squared	0.186	0.282	0.396	0.570	0.124	0.368	0.074	0.333

Note. The cross-sectional regression is conducted by taking the time average of all variables from 1980 to 2019 for each economy. As our main focus is on the financial development index which is correlated with other country-level variables, i.e. population (log), GDP per capita (log), private credit, and trade, we first orthogonalize those variables on the financial development index and use the residual values in the regression. Robust standard errors are reported in parentheses. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. All variable constructions are reported in Appendix.

is taken from the updated dataset in Alfaro, Kalemli-Ozcan, and Volosovych (2014). The borrowing cost data is EMBI spread, which is from the World Bank's Global Economic Monitor. Capital control measure is from Chinn and Ito (2006).

Fact 4: *Capital flows are positively correlated with reserve accumulation.* Column (1) and (2) in Table 2 show that on average, 1% increase in reserve flows is associated with 0.57% increase in total capital flows. This is economically significant considering the average total capital flows of 6.2% in the sample. Moreover, the relationship holds even when we include other important country-level characteristics such as population (measuring country size), per capita GDP (measuring economic development), trade-to-GDP ratio (a common measure of trade openness), and domestic credit to the private sector as a share of GDP (a common measure of domestic financial development). The point estimate barely changes

Table 2 CAPITAL FLOWS, RESERVE FLOWS AND BORROWING COST

Dep. Variables	Capital flows (% GDP)				Reserve flows (% GDP)	
	(1)	(2)	(3)	(4)	(5)	(6)
Reserve flows (% GDP)	0.57***	0.56**				
	(0.19)	(0.21)				
EMBI spread			-0.30***	-0.20***	-0.05**	-0.06**
			(0.09)	(0.07)	(0.02)	(0.03)
Population		13.76*		46.29**		2.10
		(7.01)		(19.22)		(4.94)
GDP per capita		7.33**		15.05***		-0.06
		(3.00)		(4.97)		(0.93)
Trade		-0.41		8.75*		5.97**
		(2.99)		(4.97)		(2.42)
Private credit		4.23		-7.83		-7.76***
		(3.23)		(7.92)		(2.68)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1269	961	663	574	664	575
Adjusted R^2	0.143	0.183	0.202	0.250	0.112	0.150

NOTE. Robust standard errors clustered at the country level are reported in parentheses. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. All variable constructions are reported in Appendix.

with additional controls.

Fact 5: *Capital flows and reserve flows are negatively correlated with borrowing costs.* We measure external borrowing costs using the Emerging Markets Bond Index (EMBI) spreads, calculated as the premium paid by an emerging market over a U.S. government bond with comparable maturities. Economically, it also captures the *cost* difference for a country between external borrowing and holding reserves. Columns (3)-(6) in Table 2 present the relationship between the EMBI spreads and those flows. When the spread is high, both the capital flows and the reserve flows decline. Again, the relationship holds even after controlling for other important country-level characteristics. In terms of economic importance, the effect of EMBI spreads on capital flows and reserve flows is about the same. Even though the point estimates for EMBI spreads in column (4) and (6) differ by an order of magnitude, the average capital flows and reserve flows in the sample are also different (see Table A1). Based on the estimation results in column (4) and (6), a 1% increase in EMBI spreads is associated with 0.2% and 0.06% decline in capital flows and reserve flows respectively, both of which are about 3-4% of their historical mean in the sample.

3 Model

3.1 Setup

The model is a small open economy inhabited by a unit measure of identical infinitely-lived households. They produce and consume tradable goods, borrow from abroad, hold liquidity in safe foreign assets (reserves), and invest in productive assets. Their utility is given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{1}$$

where β is the discount factor and the utility function u is strictly increasing and strictly concave. The budget constraint is

$$c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t = a_t + b_{t-1} + s_{t-1} + q_t a_t^{\ell} \tag{2}$$

where c_t denotes consumption of tradable goods, b_t is foreign bond holdings, s_t is reserve holdings, and z_t is investment in productive assets. R_t is the gross interest rate on foreign bonds, and R^s is the fixed gross interest rate on reserves. Households are not allowed to borrow using reserves, implying that s_t cannot be negative. In the quantitative analysis below, we set parameter values such that b_t is always negative. Therefore we call b_t foreign debt or simply debt henceforth. a_t is productive asset holdings and also output, because we assume a linear production function $y_t = a_t L$ with fixed labor supply $L = 1$. $q_t a_t^{\ell}$ is the amount of resource obtained by liquidating a part of asset holdings when a liquidity shock hits the economy. This will be explained in detail below.

Productive assets in the model are broad assets that include capital and technology used for production. It can also be interpreted as the productivity level of the economy. The amount of productive assets a_t grows endogenously through households' investment. The law of motion for a_t is given as follows:

$$a_t = a_{t-1} + \eta(z_{t-1})^{\gamma}\left[(1-\kappa)a_{t-1} + \kappa a^*_{t-1}\right]^{1-\gamma} - a^{\ell}_t \tag{3}$$

a^*_{t-1} is the level of foreign productive assets, which is assumed to grow at a fixed rate $1+\bar{g}$. The bracketed term implies that both domestic and foreign assets, a_{t-1} and a^*_{t-1}, promote accumulation of productive assets, and κ captures the degree of technological spillover from foreign technology.[5] By introducing exogenously-growing a^*_t, the model features semi-endogenous growth, in the sense that the domestic productivity a_t endogenously fluctuates around the exogenous path of a^*_t, but will not deviate far from it, and the long-run average growth rate is exogenously given at $1+\bar{g}$.[6] We assume that households internalize that current assets a_t facilitate future growth through (3), so that there is no externality associated with growth. a^{ℓ}_t denotes the amount of liquidated assets. As explained below, households may need to liquidate a part of their assets to repay foreign debt when a liquidity shock hits the economy.

There are two stochastic shocks to the economy. One is a shock to the interest rate on foreign debt. The interest rate R_t is given as follows:

$$R_t = R^b \exp(\varepsilon^R_t) + \psi^b\left[\exp\left(-\frac{b_t}{a_t} - \bar{b}\right) - 1\right] \tag{4}$$

where R^b is the baseline interest rate, ε^R_t is a stochastic shock, and the second term is a debt-elastic component with $\psi^b > 0$ as in Schmitt-Grohé and Uribe (2003). The debt-elastic component is not essential to the model mechanism, but it improves the quantitative performance of the model.[7] Households internalize that both b_t and a_t affect the interest rate R_t through this equation, and thus there is no externality resulting from this equation.

The other shock is liquidity shock, which is the key component of the model. At the beginning of each period and before households obtain new borrowing b_t and production a_t, households may need to repay a $\theta_t \in [0,1]$ fraction of the previous debt b_{t-1}, where θ_t is a stochastic variable. The stochastic process of θ_t indicates the size of a roll-over risk because

[5] This spillover formulation is similar to one adopted in Gavazzoni and Santacreu (2020) among others.

[6] Even without the spillover, a^*_t appears in the liquidation price (8), and the model still features semi-endogenous growth. But in this case, a_t would deviate substantially far from a^*_t, and the liquidation price q_t would be extremely volatile. Therefore, we introduce the spillover to discipline the volatility of q_t, and κ is calibrated to target the price and elasticity of q_t.

[7] Without a debt-elastic component of R_t, foreign debt b_t and reserves s_t would become substantially more volatile than the case with a debt-elastic component.

the liquidity shock implies that a $(1-\theta_t)$ fraction of debt is rolled over without any frictions and the remaining θ_t fraction cannot be rolled over. The size of a roll-over risk may potentially depend on various domestic and global factors, but domestic financial development is obviously important because a well-developed financial market provides alternative financial channels and thereby reduces a roll-over risk.[8] Therefore, we interpret that the stochastic process of θ_t is a measure of financial development. In particular, in countries with high financial development, θ_t is likely to take a low value and the roll-over risk is low, and vice versa.[9]

When a liquidity shock hits the economy, households can use reserve holdings s_{t-1} for the early repayment. However, if s_{t-1} is not enough to repay the entire early repayment $-\theta_t b_{t-1}$, households need to liquidate some of their assets to cover the liquidity shortage $-\theta_t b_{t-1} - s_{t-1} > 0$. Let q_t denote the price of liquidated assets. Then the amount of liquidated assets a_t^ℓ needs to satisfy the following inequality:

$$q_t a_t^\ell \geq -\theta_t b_{t-1} - s_{t-1} \tag{5}$$

We call this inequality a liquidity constraint. When there is no liquidity shock ($\theta_t = 0$), or households have enough reserves ($-\theta_t b_{t-1} < s_{t-1}$), the right-hand side of (5) is negative. But we do not allow households to choose negative a_t^ℓ, which would imply that households buy assets at the price q_t ex post. We thus impose a non-negativity constraint on liquidation:

$$q_t a_t^\ell \geq 0 \tag{6}$$

Because domestic households are homogeneous and equally short of liquidity, foreign agents are the only possible buyers of liquidated assets. We assume that foreign agents are competitive and less efficient than domestic agents in producing goods from domestic assets. In particular, they combine liquidated assets a_t^ℓ with their own assets a_t^* to produce tradable goods. Their profit maximization problem is given as follows:

$$\pi_t^* = \max_{a_t^\ell} (a_t^*)^\zeta (a_t^\ell)^{1-\zeta} - F a_t^* - q_t a_t^\ell \tag{7}$$

[8] Chang and Velasco (2001) develop a model where domestic banks' inability to commit to debt repayment prevents them from additional borrowing when a liquidity shock hits them, and triggers a roll-over crisis.

[9] In the quantitative analysis, we assume that θ_t takes a fixed positive value θ with an exogenous probability. In this case, the value of θ indicates the degree of financial development, and a low value of θ corresponds to high financial development.

with $0 < \zeta < 1$. Fa_t^* is an entry cost to enter the market of liquidated assets, which grows along with a_t^*. q_t denotes the price of liquidated assets. The first-order condition determines the price of liquidated assets:

$$q_t = (1-\zeta)\left(\frac{a_t^*}{a_t^\ell}\right)^\zeta \tag{8}$$

This equation implies that the liquidation price q_t goes down as liquidation a_t^ℓ increases, indicating a downward-sloping demand by foreign agents. This is meant to capture asset fire sales during crises, documented by Aguiar and Gopinath (2005) regarding Asian currency crises in the 1990s among others. However, atomistic households take the liquidation price q_t as given and do not internalize the effect of their liquidation on its price through (8). This is a fire-sale externality, and the only source of an externality in the model. As shown below, this externality distorts the debt and reserve decisions by households, and calls for policy interventions.

The price function (8) also implies that q_t becomes very high when a_t^ℓ is very small. In this case, domestic households have an incentive to sell a small amount of their assets even if there is no liquidity shock. To avoid such asset sales in normal times, we set the entry cost parameter F such that foreign buyers are willing to buy liquidated assets only when a liquidity shock hits the domestic economy and a_t^ℓ is large enough to cover the fixed cost.

3.2 Decentralized Equilibrium

In the decentralized equilibrium, atomistic households choose c_t, b_t, s_t, z_t, and a_t^ℓ to maximize their expected utility (1) subject to the budget constraint (2), the law of motion for productive assets (3), debt-elastic interest rate (4), the liquidity constraint (5), the non-negativity constraint on liquidation (6), and the non-negativity constraint on reserves, taking the liquidation price q_t as given, but it is determined by (8). The recursive maximization problem by households is set up as follows:

$$
\begin{aligned}
V(b_{t-1}, s_{t-1}, z_{t-1}, a_{t-1}; \Theta_t, a^*_{t-1}) = & \max_{c_t, b_t, s_t, z_t, a^\ell_t, a_t} u(c_t) + \beta \mathbb{E}_t V(b_t, s_t, z_t, a_t; \Theta_{t+1}, a^*_t) && (9) \\
& - \lambda_t \left[c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t - a_t - b_{t-1} - s_{t-1} - q_t a^\ell_t \right] && (10) \\
& - \xi_t \left[a_t - a_{t-1} - \eta z^\gamma_{t-1} \left[(1-\kappa) a_{t-1} + \kappa a^*_{t-1} \right]^{1-\gamma} + a^\ell_t \right] && (11) \\
& + \psi_t \left[q_t a^\ell_t + \theta_t b_{t-1} + s_{t-1} \right] && (12) \\
& + \varphi_t q_t a^\ell_t && (13) \\
& + \nu_t \frac{s_t}{R^s} && (14)
\end{aligned}
$$

Θ_t is a set of stochastic shocks $\Theta_t = \left\{ \theta_t, \varepsilon^R_t \right\}$. Foreign assets a^*_t follows an exogenous path $a^*_t = (1 + \bar{g}) a^*_{t-1}$. The last term (14) is the non-negativity constraint on reserve holdings.

The first-order conditions and the definition of the decentralized equilibrium are given in Appendix B.1. Arranging the first-order conditions leads to the following equations:

$$
u'(c_t) = \beta \mathbb{E}_t \left[\xi_{t+1} \eta \gamma \left(\frac{z_t}{(1-\kappa) a_t + \kappa a^*_t} \right)^{\gamma - 1} \right] \tag{15}
$$

$$
\psi_t + \varphi_t = \frac{\xi_t}{q_t} - u'(c_t) \tag{16}
$$

$$
\begin{aligned}
\xi_t = & u'(c_t) \left[1 + \left(\frac{b_t / a_t}{R_t} \right)^2 \psi_b \exp \left(-\frac{b_t}{a_t} - \bar{b} \right) \right] \\
& + \beta \mathbb{E}_t \left[\xi_{t+1} \left\{ 1 + \eta (1-\gamma)(1-\kappa) \left(\frac{z_t}{(1-\kappa) a_t + \kappa a^*_t} \right)^\gamma \right\} \right]
\end{aligned} \tag{17}
$$

$$
u'(c_t) = \beta \frac{R_t}{1 + \psi_b \exp \left(-\frac{b_t}{a_t} - \bar{b} \right) \frac{b_t / a_t}{R_t}} \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1} \theta_{t+1} \right] \tag{18}
$$

$$
u'(c_t) = \beta R^s \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1} \right] + \nu_t \tag{19}
$$

(15) is the Euler equation regarding investment z_t. (16) is the first-order condition regarding liquidation a^ℓ_t. (17) is the first-order condition regarding assets a_t, and ξ_t captures a shadow value of one unit of asset. It consists of a marginal utility of an additional consumption and a contribution to next-period asset accumulation. (18) and (19) are the Euler equations regarding debt and reserves respectively. The interest rate on debt is adjusted to internalize the effect of debt on the interest rate through (4). We denote this adjusted interest rate by

$\tilde{R}_t$ henceforth.

The first-order condition regarding liquidation (16) needs some explanation. We allow households to liquidate their assets more than necessary. When a liquidity shock hits the economy, households may liquidate more than the amount necessary for the early repayment. Households may also liquidate even when there is no liquidity shortage or liquidity shock. In this case, households liquidate until the right-hand side of (16) becomes zero, implying $\psi_t = \varphi_t = 0$ and neither the liquidity constraint (5) nor the non-negativity constraint (6) binds. However, in the quantitative analysis below, we set the parameter values such that liquidation is costly and the right-hand side of (16) is always strictly positive.[10] This condition is guaranteed by a low liquidation price q_t relative to a value of assets ξ_t. Under this condition, a reduction in liquidation always improves welfare, and households never liquidate more than necessary. It follows that either (5) or (6) binds in each period. In particular,

(1) When the liquidity constraint (5) binds, $a_t^\ell > 0$ units of assets are liquidated to cover the liquidity shortage. Then $\varphi_t = 0$ and ψ_t capture the shadow value of an additional unit of liquidity, which is the right-hand side of (16). One unit of additional liquidity enables households to reduce asset liquidation by $1/q_t$ units, whose value is the first term in the right-hand side of (16). At the same time, a $1/q_t$-unit reduction in asset liquidation reduces households' available budget by 1 unit, whose value is the second term. We call this ψ_t a private value of liquidity to distinguish it from a social value of liquidity discussed below in Section 3.4.

(2) When the non-negativity constraint (6) binds, there is no liquidation, $a_t^\ell = 0$. In this case, $\psi_t = 0$ and φ_t captures the shadow value of negative liquidation if possible.

The Euler equation regarding debt (18) can be understood as follows. By giving up one unit of consumption at period t, households can reduce debt by $\tilde{R}_t$ units and increase the resource at period $t+1$ by the same amount. This will bring the expected utility given by the first term in the right-hand side. In addition, when there is a liquidity shortage $-\theta_{t+1} b_t - s_t > 0$ at period $t+1$, a reduction in debt by $\tilde{R}_t$ units will reduce the liquidity shortage by $\tilde{R}_t \theta_{t+1}$ units. This will bring the expected utility given by the last term in (18).

The Euler equation regarding reserves (19) can be understood in a similar way. By giving up one unit of consumption at period t, households can increase reserves and the resource at

[10] To determine the value of φ_t, the liquidation price q_t needs to be defined even when $a_t^\ell = 0$. We assume that there are other types of foreign agents who are willing to buy domestic assets at a very low q_t. This assumption affects only the value of φ_t and does not affect any other allocations or prices.

period $t+1$ by R^s units, whose value is captured by the first term in the right-hand side. The difference from (18) is the second term in the right-hand side. R^s units of reserves at period $t+1$ will reduce the liquidity shortage by R^s units, and therefore ψ_{t+1} is not multiplied by θ_{t+1}, in contrast to $\psi_{t+1}\theta_{t+1}$ in (18). This difference between (18) and (19) plays a critical role in the model, which is explained in detail in the next subsection.

3.3 Reserve Holdings

In this subsection, we examine the mechanism of how households choose reserve holdings in detail. Combining the two Euler equations (18) and (19), we obtain the following equation, which is the key equation of the model:

$$\beta(\tilde{R}_t - R^s)\mathbb{E}_t[u'(c_{t+1})] = \beta\mathbb{E}_t[(R^s - \theta_{t+1}\tilde{R}_t)\psi_{t+1}] + \nu_t \tag{20}$$

We set the parameter values such that $\tilde{R}_t > R^s$ always holds, as is typically the case in emerging economies.[11] Then the left-hand side is the opportunity cost of holding reserves. Households can use one unit of tradable good to either buy R^s units of reserves or reduce $\tilde{R}_t$ units of debt. If households choose the former, they receive R^s units at period $t+1$, but lose an opportunity to reduce the interest payment on debt $\tilde{R}_t$. This gap is the opportunity cost of holding reserves.

The first term in the right-hand side captures the relative advantage of holding reserves over reducing debt in liquidity management. By buying R^s units of reserves, households can reduce the potential liquidity shortage $-\theta_{t+1}b_t - s_t$ by R^s units in the next period, and its expected value is given by $\beta\mathbb{E}_t[R^s\psi_{t+1}]$. In contrast, by reducing $\tilde{R}_t$ units of debt, households can reduce the potential liquidity shortage by $\theta_{t+1}\tilde{R}_t$ units in the next period, and its expected value is $\beta\mathbb{E}_t[\theta_{t+1}\tilde{R}_t\psi_{t+1}]$. The gap between these two is the relative advantage of holding reserves over reducing debt in liquidity management. Recall that ψ_{t+1} is a private value of liquidity, and it is positive only when there is a liquidity shortage that requires a positive amount of liquidation. It follows that this relative advantage has a positive value only when a realized liquidity shock θ_{t+1} triggers a liquidity shortage $-\theta_{t+1}b_t - s_t > 0$ and requires a positive amount of liquidation $a_t^{\ell} > 0$.

In principle, households choose debt and reserves to equalize the opportunity cost and

[11] If $\tilde{R}_t = R^s$, then households choose debt and reserves such that there is no liquidity shortage in any states at $t+1$, implying $\psi_{t+1} = 0$. Then all the terms in (20) become zero. In this case, the model reduces to a standard open-economy model in which only the net foreign asset position $b_t + s_t$ matters.

the benefit from the relative advantage of holding reserves. However, when the cost is high and/or the benefit is low, and it is not possible to equalize these two, households choose $s_t = 0$, and the Lagrange multiplier for the non-negativity constraint on reserves $\nu_t > 0$ fills the gap between the cost and benefit in (20). The value of θ_{t+1} is a critical determinant of the relative advantage of holding reserves and thus the amount of reserve holdings. We can derive the following three propositions regarding reserve holdings from (20).

Proposition 1. *If $\theta_{t+1} = 0$ for all states at $t+1$, households do not hold reserves, $s_t = 0$.*

Proof: If $\theta_{t+1} = 0$ for all states at $t+1$, then there is no liquidity shortage and $\psi_{t+1} = 0$ for all states. This implies that the first term in the right-hand side is zero, implying $\nu_t > 0$ to satisfy (20) and thus $s_t = 0$.

The intuition is straightforward: if there is no liquidity risk at all, there is no reason to hold reserves because it comes with an opportunity cost.

Proposition 2. *If $\theta_{t+1} \geq R^s/\tilde{R}_t$ for all states with $\theta_{t+1} > 0$ at $t+1$, households do not hold reserves, $s_t = 0$.*

Proof: If this condition holds, the first term in the right-hand side of (20) is non-positive. Because the left-hand side is positive, $\nu_t > 0$ and thus $s_t = 0$.

Intuitively, if θ_{t+1} is very high, holding reserves does not have a relative advantage over reducing debt in liquidity management. In this case, households simply reduce debt to manage a liquidity shortage without holding reserves.

Proposition 3. *Households never hold enough reserves to cover the entire early repayment $\theta_{t+1} b_t$ for all possible states at period $t+1$.*

Proof: Suppose to the contrary that households hold enough reserves to cover the entire early repayment for all possible states at $t+1$. Then there is no liquidity shortage and $\psi_{t+1} = 0$ in all states at $t+1$. In addition, $\nu_t = 0$ because $s_t > 0$. This implies that the right-hand side of (20) is zero. But the left-hand side is positive, and this cannot be an equilibrium.

This proposition implies that if households hold a positive amount of reserves and $\nu_t = 0$, then there is at least one state at $t+1$ in which there is a positive liquidity shortage and $\psi_{t+1} > 0$. Intuitively, if the amount of reserves is enough to cover the entire early repayment for any states at $t+1$, there is no additional benefit of holding reserves by reducing liquidation at the margin, which implies $\psi_{t+1} = 0$. This is too much reserve holdings, and households never hold reserves up to this amount.

An important implication of these propositions is that households hold a positive amount of reserves only if θ_{t+1} may take intermediate values between 0 and $R^s/\tilde{R}_t$. More generally, the amount of reserves is small when θ_{t+1} takes only low values close to 0 or only high values close to $R^s/\tilde{R}_t$, and the amount becomes larger if θ_{t+1} can take intermediate values. On the one hand, if θ_{t+1} can take only low values close to 0, then the size of the early repayment $-\theta_{t+1}b_t$ is small. Proposition 3 says that households never hold enough reserves to cover the entire early repayment, which implies $-\theta_{t+1}b_t > s_t$ for at least one possible state at $t+1$. Then reserve holdings s_t become small. On the other hand, if θ_{t+1} can take only high values close to $R^s/\tilde{R}_t$, the size of the early repayment can be large, but the relative advantage of holding reserves in liquidity management is small. In this case, households choose low reserve holdings to increase the private value of liquidity ψ_{t+1} to achieve the balance between the cost and the benefit of reserves. As households reduce reserve holdings at period t, the size of a potential liquidity shortage and asset liquidation at period $t+1$ becomes larger. This lowers the liquidation price q_{t+1} and increases ξ_{t+1}/q_{t+1}, which increases the private value of liquidity ψ_{t+1}. In this way, households choose low reserves if θ_{t+1} can take only high values close to $R^s/\tilde{R}_t$.

In contrast to these two cases, when θ_{t+1} may take intermediate values, there are both a certain amount of the early repayment to be covered by reserves and also the relative advantage of holding reserves in liquidity management. In this case, reserve holdings can become large. Therefore, this mechanism leads to a non-monotonic relationship between reserve holdings and the value of θ_t. As we interpret the stochastic process of θ_t as the degree of financial development, this non-monotonic relationship is consistent with the empirical finding in Section 2.

A similar discussion can be applied to the interest rate gap in the left-hand side of (20). When the interest rate on debt $\tilde{R}_t$ becomes high, it does not just discourage households' borrowing, but also increases the opportunity cost of holding reserves. In this case, households reduce reserve holdings to increase ψ_{t+1} in the right-hand side and restore the balance between the cost and benefit in (20). In this way, households reduce both debt and reserves when the interest rate on debt is high, and vice versa. This correlation across debt, reserves, and the interest rate is also consistent with the empirical finding in Section 2.

3.4 Social Planner's Solution

In this section, we examine the social planner's solution to characterize externalities in the model and policies to correct them. The only difference between the decentralized economy and the social planner's solution is that the planner internalizes that the price of liquidated assets q_t is decreasing in the size of liquidation a_t^ℓ as in (8). The setup of the maximization problem and the first-order conditions are provided in Appendix B.2. The key first-order condition is the one regarding liquidation a_t^ℓ, which corresponds to (16) in the decentralized equilibrium:

$$\psi_t^{SP} + \varphi_t^{SP} = \frac{\xi_t}{q_t - \zeta q_t} - u'(c_t) \tag{21}$$

$-\zeta q_t$ in the denominator is obtained by $(\partial q_t/\partial a_t^\ell)a_t^\ell = -\zeta q_t$, and this is the difference from (16) in the decentralized equilibrium. Because $-\zeta q_t < 0$, this term implies that the social value of liquidity ψ_t^{SP} is greater than the private value of liquidity ψ_t when the liquidity constraint binds and there is liquidation $a_t^\ell > 0$. The intuition is that the planner internalizes that an additional unit of liquidity will reduce liquidation and increase the liquidation price q_t, which will reduce liquidation even more. Therefore the social value of liquidity ψ_t^{SP} is greater than the private value ψ_t.

Although this externality appears in the first-order condition regarding liquidation a_t^ℓ, the decision on a_t^ℓ itself is not distorted by this externality. This is because the amount of liquidation a_t^ℓ is determined by the binding liquidity constraint (5), given debt b_{t-1} and reserves s_{t-1} in the previous period. As discussed above, the social value of liquidity ψ_t^{SP} is greater than the private value ψ_t, implying that the planner has a higher incentive to hold liquidity and reduce liquidation than decentralized households. This means that whenever the liquidity constraint (5) binds and a positive amount of liquidation is required in the decentralized equilibrium, the planner has no incentive to liquidate more, and thus chooses the same liquidation a_t^ℓ.

What is actually distorted by this externality is households' decisions on debt and reserves. To see this, recall that the private value of liquidity ψ_{t+1} appears in the right-hand sides of the Euler equations (18) and (19), and affects how much households borrow and hold reserves. The corresponding Euler equations by the planner are given as follows:

$$u'(c_t) = \beta \tilde{R}_t \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}^{SP} \theta_{t+1} \right] \tag{22}$$

$$u'(c_t) = \beta R^s \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}^{SP} \right] + \nu_t \tag{23}$$

where ψ_{t+1}^{SP} is given by (21) one period ahead. The only difference from the decentralized Euler equations is ψ_{t+1}^{SP} instead of ψ_{t+1}. ψ_{t+1}^{SP} being greater than ψ_{t+1} implies that individual households underestimate the value of liquidity when the liquidity constraint binds. As a result of this externality, households borrow too much and hold too little reserves compared with the planner's allocation.

The social planner's allocation can be decentralized by the following policy instruments. Overborrowing can be corrected by a tax on foreign debt, which is capital control. Too little reserves can be corrected by a subsidy on reserve holdings, or alternatively by official foreign reserve accumulation, as discussed in Appendix B.3. The optimal tax and subsidies, denoted by τ_t^b and τ_t^s respectively, are given as follows:

$$u'(c_t) = \beta(1+\tau_t^b) \tilde{R}_t \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1} \theta_{t+1} \right] \tag{24}$$

$$u'(c_t) = \beta(1+\tau_t^s) R^s \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1} \right] + \nu_t \tag{25}$$

with

$$1+\tau_t^b = \frac{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}^{SP} \theta_{t+1} \right]}{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1} \theta_{t+1} \right]} \tag{26}$$

$$1+\tau_t^s = \frac{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}^{SP} \right]}{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1} \right]} \tag{27}$$

where ψ_{t+1} and ψ_{t+1}^{SP} are given by (16) and (21) respectively. Because $\psi_{t+1}^{SP} > \psi_{t+1}$ as shown above, both τ_t^b and τ_t^s are positive.

Combining the two Euler equations (22) and (23), we obtain the equation similar to the key equation (20). All the discussions on reserve holdings in Section 3.3 apply to the social planner's solution. In particular, the planner holds a relatively large amount of reserves when θ_t may take an intermediate value, and holds a small amount when θ_t can take only low values or high values.

We also show in Appendix C that given everything else equal, the debt tax rate τ_t^b becomes higher when θ_t is likely to take higher values. The intuition is as follows. As shown in (26), the tax rate is determined by the relative size of the social value of liquidity ψ_{t+1}^{SP} versus the private value ψ_{t+1}. When θ_t takes a high value, the size of liquidation a_t^ℓ becomes large given everything else equal. This leads to a low liquidation price q_t, which increases the value of both ψ_t and ψ_t^{SP} as shown in (16) and (21). Because the planner internalizes that the liquidation price drops more when liquidation becomes larger, an increase in ψ_t^{SP} is larger than an increase in ψ_t, which increases the tax rate τ_t^b. We now turn to the quantitative analysis of our model.

4 Quantitative Analysis

4.1 Calibration

One period in the model is meant to be one year. We assume log utility $u(c_t) = \ln(c_t)$ for the households' utility function. We first set the standard parameters to the conventional values in the literature. Table 3 summarizes these externally determined parameter values. The discount factor β is set to 0.91 following Bianchi (2011). The baseline gross interest rate on foreign debt R^b is set to 1.06, which is standard in the literature. We assume no interest on reserves and set $R^s = 1$. The curvature of investment γ is set to 0.8 following Comin and Gertler (2006). The exogenous growth rate of foreign asset $\bar{g}$ is set 2.61%, which is the average growth rate of the 47 sample emerging economies over the sample period in the empirical analysis in Section 2.

For the stochastic shock process, we assume that a liquidity shock θ_t takes either a fixed value $\theta \in [0, 1]$ or 0 with an exogenous probability, and calibrate the value of θ below. Later in Section 5, we change the value of θ and examine how it affects the optimal policy. For the interest rate shock, we follow Mendoza (2010) and assume that ε_t^R takes two values $\pm\varepsilon^R$. We also assume that when a liquidity shock hits the economy, the interest rate shock takes a high value ε^R. This means that there are three possible realizations of shocks, $(\varepsilon_t^R, \theta_t) = \{(\varepsilon^R, 0), (-\varepsilon^R, 0), (\varepsilon^R, \theta)\}$. We assume that this shock follows a three-state Markov process, and the probability transition matrix is based on Mendoza (2010) and Bianchi and Mendoza (2018). Specifically, in normal times with no liquidity shock, the same interest shock continues with probability 0.54, the interest rate shock changes with probability 0.36, and a liquidity shock occurs with probability 0.1. When a liquidity shock occurs, the economy goes back to a normal state with $\varepsilon_t^R = \varepsilon^R$ with probability 0.9, and a liquidity shock occurs again with probability 0.1.

Table 3 Externally Determined Parameters

	Parameter	Value	Source
β	Discount factor	0.91	Bianchi (2011)
R^b	Gross interest rate on debt	1.06	Standard
R^s	Gross interest rate on reserves	1	Standard
γ	Investment curvature	0.8	Comin and Gertler (2006)
$\bar{g}$	Foreign growth rate	0.0261	Data
ε^R	Interest rate shock	0.0196	Mendoza (2010)

Table 4 Calibrated Parameters

	Parameter	Value	Target		Model
η	Investment efficiency	0.1085	Mean CA-to-GDP	−0.017	−0.017
κ	Productivity spillover	0.25	Fire-sale price/normal price	0.37	0.36
ζ	Share of foreign assets	0.46	Elasticity of fire-sale price	1.74	1.87
ψ_b	Debt-elasticity of spread	0.01	S.D. of CA-to-GDP	0.063	0.064
$\bar{b}$	Baseline debt-to-GDP	0.8	Mean debt-to-GDP	0.53	0.53
θ	Size of liquidity shock	0.45	Mean reserve-to-GDP	0.17	0.17

Given the externally determined parameter values, the remaining six parameter values are jointly determined to match the simulation moments of the decentralized economy to the corresponding data in Section 2 or empirical estimates in other papers. These parameters are (1) the investment efficiency η, (2) the productivity spillover coefficient κ, (3) the share of foreign assets in foreign production ζ, (4) the debt-elasticity of the spread ψ_b, (5) the baseline debt-to-GDP ratio for the debt-elastic component of the spread $\bar{b}$, and (6) the size of a liquidity shock θ. η and $\bar{b}$ determine the households' incentive to borrow from abroad, and thus are set to match the simulation mean of the current account and the debt-to-GDP ratio to the data. ψ_b is set to target the standard deviation of the current account. κ and ζ are closely related to the liquidation price and its elasticity with respect to liquidity. Aguiar and Gopinath (2005) show that during Asian currency crisis, firms were acquired by foreign investors at a price on average equal to 37% of the market price before the crisis.[12] They also estimate that the elasticity of this price regarding firm liquidity holdings is 1.74. We target these numbers in determining the values for κ and ζ. Finally, θ is determined to match the mean reserve-to-GDP ratio in the model simulation to the data in Section 2. The parameter values and the calibration results are summarized in Table 4.

[12] In the model, we divide the average asset fire-sale price by the simulation mean of the domestic value of an asset ξ_t. We calibrate the model so that this value becomes close to 37%.

Table 5 Stochastic simulation moments

	Decentralized economy		Social planner	
	Mean	S.D.	Mean	S.D.
Consumption	0.807	0.035	0.811	0.036
Investment	0.181	0.164	0.171	0.172
Debt	−0.535	0.370	−0.530	0.342
Reserve	0.168	0.579	0.209	0.406
Current account	−0.017	0.065	−0.008	0.056
Mean tax on debt	...		4.78%	
Mean subsidy on reserve	...		10.49%	
Crisis probability	3.57%		0.27%	

Note: Consumption, investment, debt, reserves, and the current account are the ratios to GDP. Standard deviations are divided by the mean of each variable.

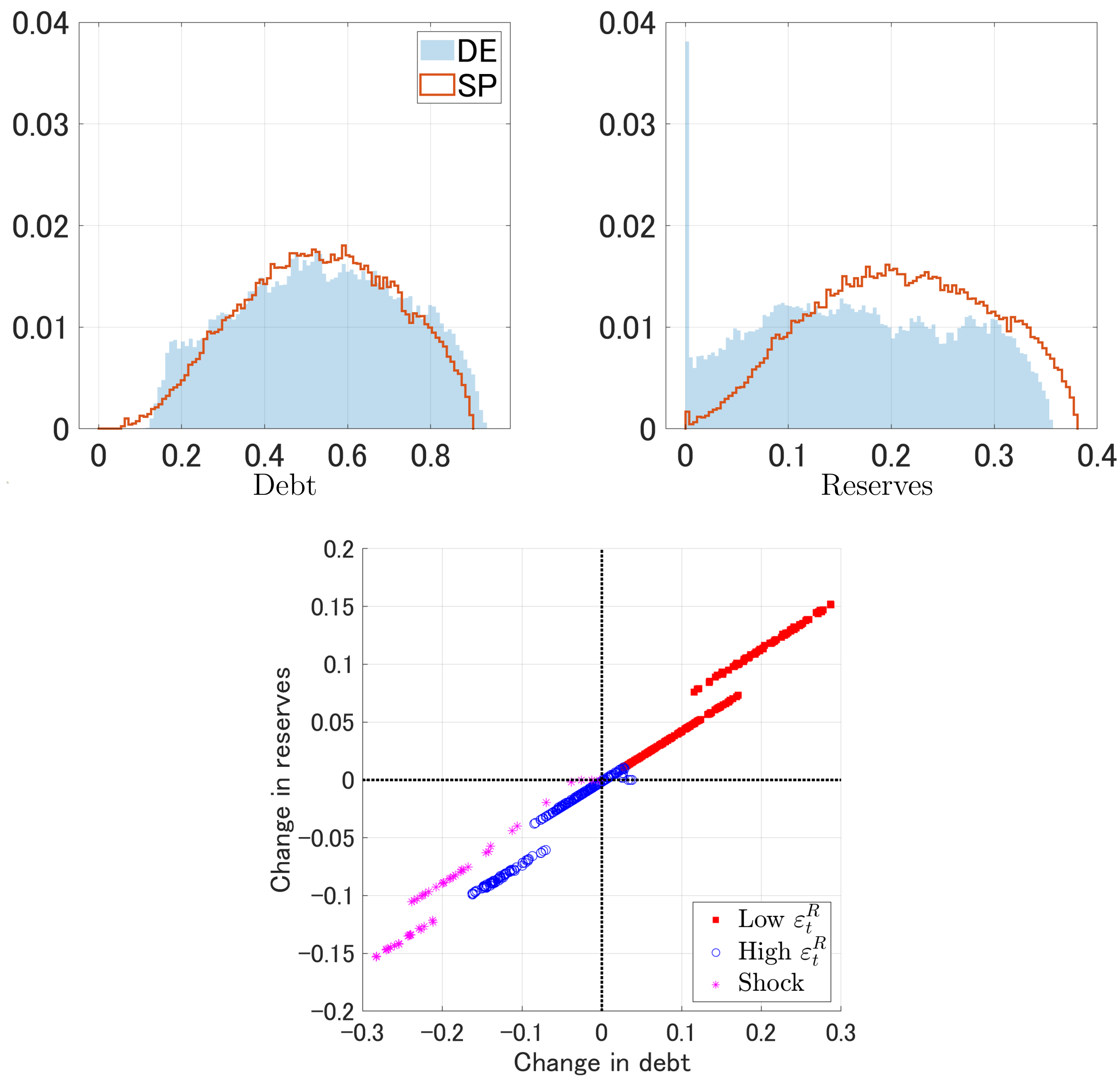
4.2 Business Cycle Moments

We simulate the model with stochastic shocks for 100,000 periods and compute the business cycle moments. Table 5 displays the mean and standard deviations of the key variables under the decentralized economy (DE) and the social planner's allocation (SP). Consumption, investment, debt, reserves, and current account are expressed in the ratios to GDP. Standard deviations are divided by the mean of each variable. Comparing debt and reserves between DE and SP, we observe that debt is slightly larger in DE, but reserve is substantially larger in SP. As a result, the mean net foreign asset position is −0.367 in DE and −0.321 in SP. The current account deficit is also larger in DE than in SP, and it is more volatile in DE than in SP. A small gap in debt does not mean that a debt tax is not important. If a subsidy on reserves is provided but a tax on debt is not imposed, decentralized households would hold larger debt, offsetting the stabilization effect of a subsidy on reserves. The crisis probability is substantially lower in SP.[13]

Figure 2 plots the distribution and dynamics of debt and reserves under DE and SP. Debt is plotted as positive values for better visibility. The top two panels show the histograms of debt and reserves, with the vertical axis indicating the frequency in a stochastic simulation. The light blue plot is the histogram under DE, and the red is that under SP. We observe that the gap in debt between DE and SP is very small, but there is a clear gap in reserves and the planner holds more reserves than the decentralized households. The frequency of

[13]Following the literature, a crisis is identified when the current account is more than two standard deviations above its mean.

Figure 2 Distributions of debt and reserves



Note: The top two panels display the histogram of debt and reserves in a stochastic simulation under the decentralized economy (DE, light blue) and the planner's solution (SP, red). The horizontal axis is the ratios of debt and reserves to GDP, and the vertical axis is the frequency. The bottom panel plots the joint distribution of changes in debt and reserves in the decentralized economy.

zero reserves, indicated by far-left bars, is substantially lower under SP.

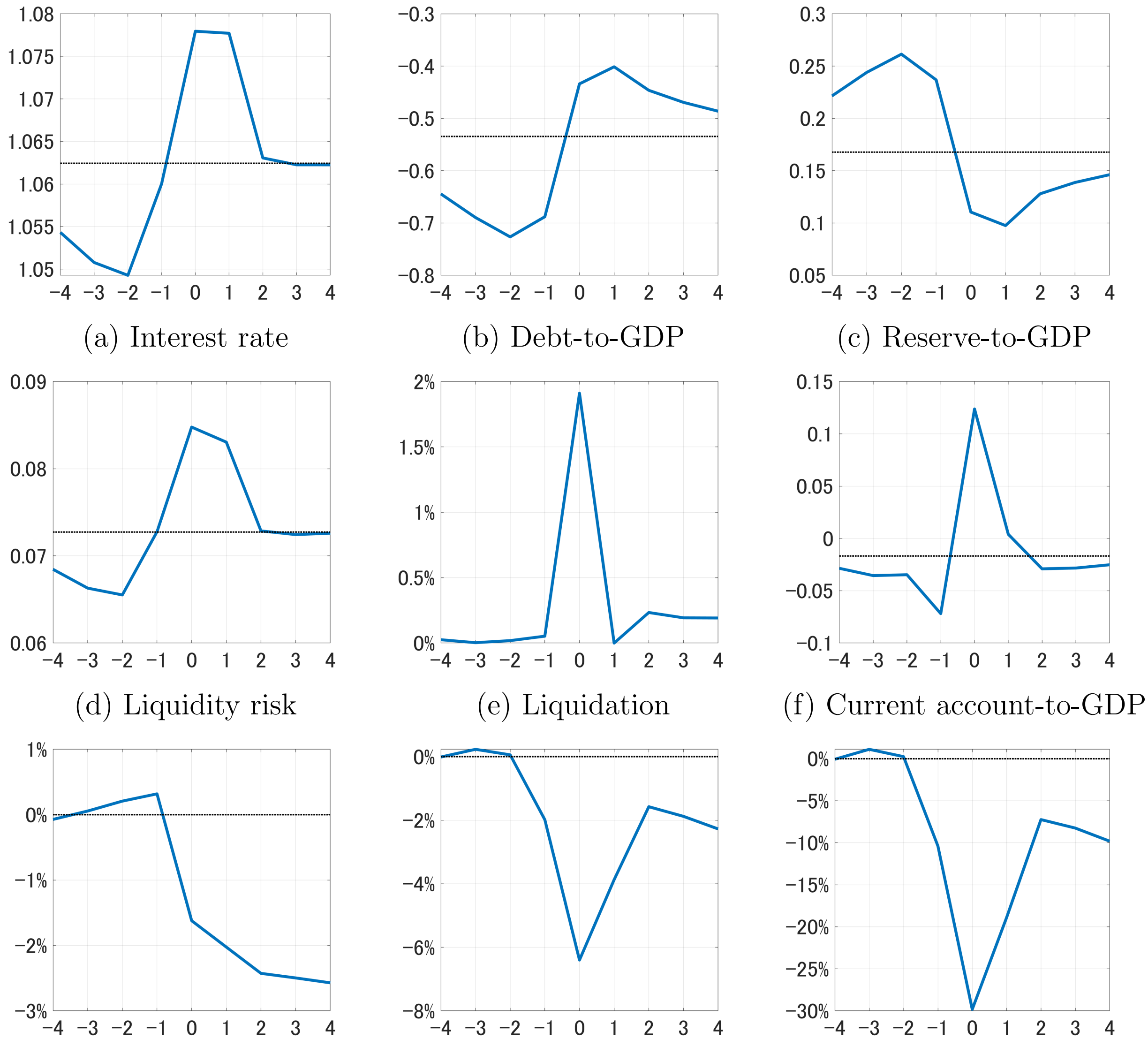
The bottom panel plots the joint distribution of changes in debt and reserves over a stochastic simulation under DE. Dots in the top-right quadrant indicate periods when both debt and reserves increase, and dots in the bottom-left quadrant are periods when both debt and reserves decrease. There are two observations. First, changes in debt and reserves are positively correlated, indicated by the fact that most of the dots are located in the top-right and bottom-left quadrants. This is consistent with Fact 4 in our empirical analysis. The correlation between changes in debt and reserves over a stochastic simulation is 0.99 in our model. Second, both debt and reserves increase when the interest rate is low, indicated by red dots, and both decrease when the interest rate is high, indicated by blue and purple dots. This is consistent with Fact 5. The correlation between the interest rate and changes in debt is -0.83, and the correlation between the interest rate and changes in reserves is -0.82. This result can be understood as follows. When the interest rate is low, households borrow more, and it is natural that the correlation between the interest rate and changes in debt is negative. There are two reasons why households increase reserves when the interest rate is low. First, as households increase debt, the potential liquidity shortage $-\theta_{t+1}b_t - s_t$ becomes larger, and the value of liquidity ψ_{t+1} becomes higher. This induces households to hold more reserves. Second, a low interest rate on debt implies that the opportunity cost of holding reserves is low. This also induces households to hold more reserves. The same logic with an opposite direction is applied when the interest rate is high. The dynamics under SP show essentially the same patterns.

4.3 Crisis Dynamics

We next show the crisis dynamics of the model. We simulate the model for 100,000 periods with stochastic shocks, pick up all crisis events and the dynamics four periods before and after each crisis, and take the average dynamics of each variable. Figure 3 plots the average crisis dynamics under the decentralized economy obtained in this way. A dotted horizontal line in Panels (a)-(f) indicates the simulation mean of each variable.

Panel (a) plots the interest rate dynamics. The interest rate is low from period -4 to -2, and increases one period before a crisis at period -1. Responding to these interest dynamics, Panels (b) and (c) show that debt and reserves are large up to period -2, and slightly shrink at period -1. Panel (d) shows the ratio of the risk of a liquidity shortage $-\theta b_t - s_t$ to GDP, which is the size of a liquidity shortage if a liquidity shock occurs in the next period. The risk increases at period -1 as the interest rate increases and reserve shrinks. Given this heightened risk, a liquidity shock at period 0 triggers large liquidation and a severe crisis. Panel (e) shows that about 2% of assets are liquidated upon a liquidity shock, and the current account sharply reverses in Panel (f). Panels (g), (h), and (i) show the

Figure 3 Crisis dynamics



(a) Interest rate (b) Debt-to-GDP (c) Reserve-to-GDP

(d) Liquidity risk (e) Liquidation (f) Current account-to-GDP

(g) Output (h) Consumption (i) Investment

Note: These panels plot the average crisis dynamics under the decentralized economy obtained from a 100,000-period stochastic simulation. Debt, reserves, liquidity risk, and the current account are the ratios to output. Output, consumption, and investment are percentage deviations from the 10-period pre-crisis trend of each variable.

dynamics of output, consumption, and investment. Because these variables grow over time, we compute the 10-period pre-crisis trend of each variable, and plot the percentage deviations from this trend. Panel (g) shows that output drops by about 2% through liquidation, and the deviation from the pre-crisis trend even widens in the following periods. This widening output loss is due to the fact that asset liquidation slows down future asset accumulation by lowing the investment efficiency in the law of motion (3).[14] Panels (h) and (i) show that consumption and investment fall below the pre-crisis trend by 6% and 30% respectively. Although there is a partial recovery from the bottom, both consumption and investment stay lower than the pre-crisis trend in the following periods. These persistent negative effects on output and consumption are often observed in empirical studies, but many of business cycle models without growth fail to capture these facts. This result points to the importance of incorporating endogenous growth into models of crisis, as Ma (2020) and Benguria et al. (2022) do.

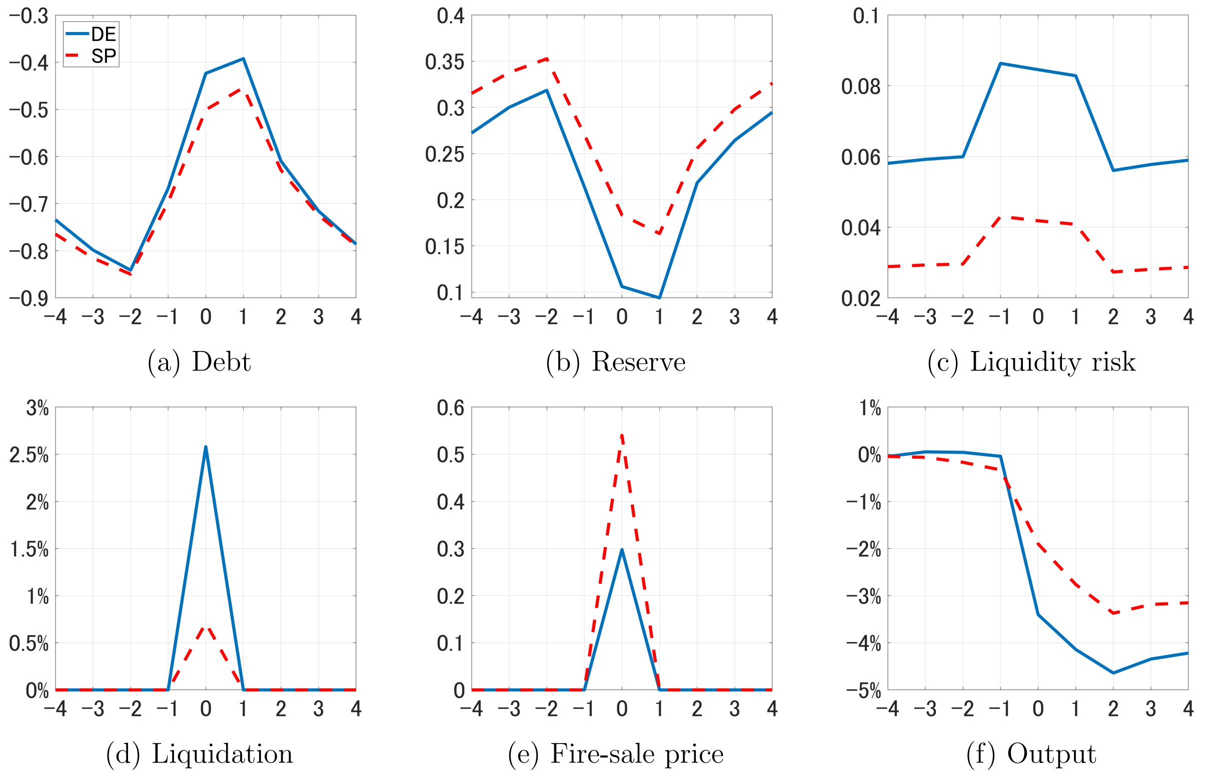
Next, we compare the crisis dynamics under the decentralized economy and the social planner's allocation. We follow Bianchi and Mendoza (2018) and conduct the following exercise: we set the initial state of the economy at period -4 of the crisis dynamics plotted in Figure 3. Then we feed the mean path of stochastic shocks from period -4 to period 4 to these two economies.[15]

Figure 4 plots the result of this exercise. The blue solid lines are the dynamics under the decentralized economy (DE), and the red dashed lines are those under the planner's solution (SP). Panels (a) shows that the planner borrows slightly more than decentralized households. As explained above, this does not mean that a debt tax is not important. If the planner does not impose a debt tax, decentralized households would borrow substantially more, given the larger amount of reserves. Panel (b) shows that the planner holds more reserves than decentralized households. Due to this gap in reserves, Panel (c) shows that the liquidity risk $-\theta b_t - s_t$ of the planner's allocation is roughly half of that in the decentralized economy, implying that the planner chooses a substantially safer position. As a result, there is a substantial gap in asset liquidation as shown in Panel (d). Decentralized households sell 2.6% of assets during a crisis, whereas the planner sells only 0.7%. There is also a sizable gap in the asset fire-sale price. Panel (e) plots the price at which assets are sold, divided by the simulation mean of the domestic value of an asset ξ_t. The numbers in Panel (e) thus indicate the severity of fire sales. It shows that decentralized households sell assets at the price equivalent to 30% of the domestic value, whereas the planner sells at the price equal to 54% of it. This sizable gap in the fire-sale price also contributes to the large gap in

[14] The level of asset and output will recover the pre-crisis trend in the long run through a spillover from foreign assets.

[15] More specifically, the interest rate is low from period -4 to -2, high in period -1, and a liquidity shock occurs at period 0. In the following periods, the interest rate is high in period 1, and low from period 2 to 4.

Figure 4 Crisis dynamics under decentralized economy and planner's solution



Note: These panels plot the crisis dynamics under the decentralized economy (blue) and the planner's allocation (red dashed). Fire-sale price in (e) is the price at which assets are sold, divided by the simulation mean of the domestic value of one unit of asset ξ_t. Output in (f) is percentage deviations from the pre-crisis trend under the decentralized economy.

liquidation in Panel (d), in addition to the gap in the liquidity shortage in Panel (c).

Finally, the substantial gap in liquidation in Panel (d) leads to a large and persistent gap in output after the crisis, as shown in Panel (f). Panel (f) plots the dynamics of output in the decentralized economy and in the planner's allocation, both in terms of a percentage deviation from the pre-crisis trend in the decentralized economy. It shows that output in the planner's allocation is persistently higher than that in the decentralized economy by more than 1%, even 4 periods after the crisis. This large and persistent gap in output suggests a sizable welfare gain by the policy intervention, which we study in Section 5.2.

5 Financial Development and Optimal Policy

In the quantitative analysis above, the size of a liquidity shock θ is calibrated and fixed at 0.45. In this section, we change the value of θ and examine how financial development measured by θ affects the optimal capital controls and reserve holdings. We also study the welfare implications of these policies.

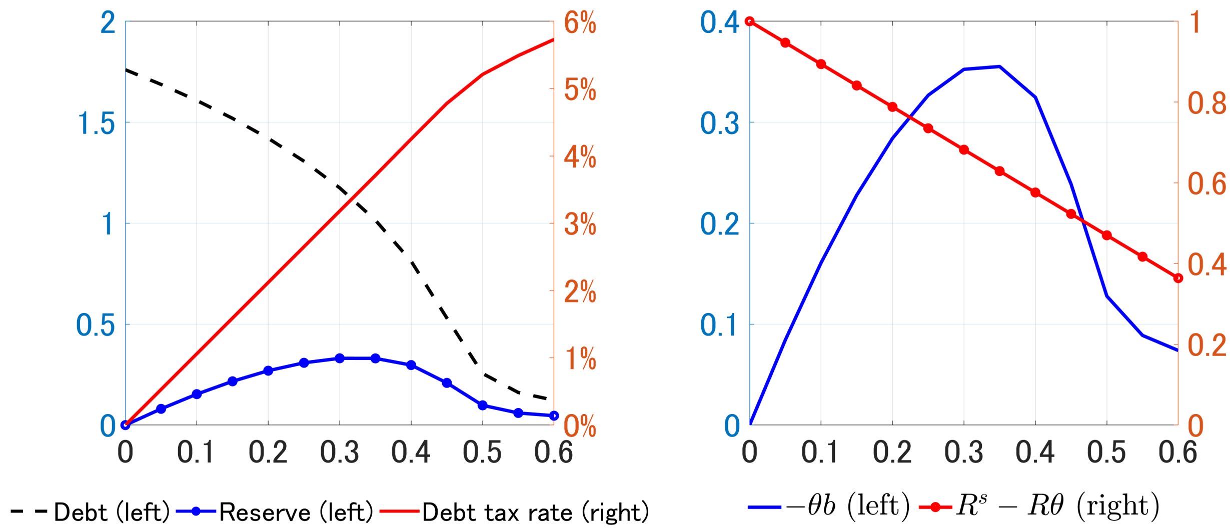
5.1 Capital Controls and Reserve Holdings

We numerically solve the model under the planner's allocation with different θ, and simulate the model for 100,000 periods with stochastic shocks. We then compute the mean of debt, reserves, and the debt tax rate. The left panel of Figure 5 plots the result, with the value of θ on the horizontal axis from 0 to 0.6 with a 0.05 grid. Debt and reserves are the ratios to GDP and scaled on the left axis. The debt tax rate is scaled on the right axis. The key observation is that the tax rate monotonically increases as θ becomes higher, whereas the amount of reserve holdings shows a non-monotonic pattern regarding θ. Reserve holdings are small when θ is either low or high, and reach the maximum of 33% of GDP when $\theta = 0.30$. Our interpretation of θ is that a low value of θ corresponds to high financial development, because low θ implies a low risk of roll-over and a liquidity crisis. The model therefore successfully replicates the cross-country differences in the use of capital controls and reserve holdings over different degrees of financial development shown in Section 2. Another observation in this panel is that the debt-to-GDP ratio monotonically decreases in θ, which is also consistent with the empirical finding in Section 2.

We discussed in Section 3.3 the key mechanism of the non-monotonic relation between reserves and θ. We now show this mechanism quantitatively in the right panel of Figure 5. The blue curve, scaled on the left axis, is $-\theta b$ for each value of θ, where b is the simulation mean of debt by the planner's allocation. $-\theta b$ indicates the size of a liquidity risk without reserves. When it is high, the planner has an incentive to hold large reserves to reduce a liquidity risk. The red line, scaled on the right axis, is $R^s - R\theta$ for each value of θ. This indicates the size of the relative advantage of increasing reserves over debt in liquidity management.

The right panel of Figure 5 shows that when θ is low, the relative advantage is high, but the liquidity risk is low. Therefore, the planner holds a small amount of reserves. As θ becomes higher, the liquidity risk $-\theta b$ increases and peaks when $\theta = 0.35$. The relative advantage decreases in θ, but it is still not very low when $\theta = 0.35$. A high liquidity risk

Figure 5 Debt, reserves, and capital control across different θ



Note: The horizontal axis is θ in both panels. The left panel plots the stochastic simulation mean of debt, reserves, and the debt tax rate. Debt and reserves are the ratios to GDP and scaled on the left axis. The debt tax rate is scaled on the right axis. The right panel plots $-\theta b$ and $R^s - R\theta$ across different θ, where b is the simulation mean of the debt-to-GDP ratio. $-\theta b$ is scaled on the left axis, and $R^s - R\theta$ is scaled on the right axis.

and some relative advantage induce the planner to hold a large amount of reserves when θ is an intermediate value around 0.35. As θ becomes even higher, the relative advantage keeps decreasing. This implies a low incentive to hold reserves, and reserve holdings actually decrease as θ exceeds 0.35. As reserve holdings become smaller, large debt with high θ is extremely risky, and thus debt quickly shrinks and so does $-\theta b$. One may wonder if low reserves when θ is high is simply due to low $-\theta b$, and debt is small because high θ implies a high risk of debt. To see the impact of low relative advantage when θ is high, it is useful to compare the case of $\theta = 0.05$ and $\theta = 0.55$. The value of $-\theta b$ is very close in these two cases; 0.084 when $\theta = 0.05$, and 0.088 when $\theta = 0.55$. However, reserve holding are quite different; 0.081 when $\theta = 0.05$, and 0.060 when $\theta = 0.55$. As a result, the liquidity risk $-\theta b - s$ is 0.004 when $\theta = 0.05$, and 0.029 when $\theta = 0.55$. The liquidity risk when $\theta = 0.55$ is seven times as high as the risk when $\theta = 0.05$. These large gaps in reserves and the liquidity rick are driven by the gap in the relative advantage of reserves, which is captured by $R^s - R\theta$. The gap in the liquidity risk also has implications for the welfare gain by policy, which is studied in the next subsection.

5.2 Welfare Analysis

Finally, we study welfare implications of the optimal policy across different θ. We compute a welfare gain by the optimal policy given θ in two steps. First, we create grid points over the state space, and on each grid point, we compute a welfare gain by the planner's allocation relative to the decentralized economy in terms of a permanent consumption gain. The relevant state variables are households' wealth w_t and foreign assets a_t^*, with $w_t = b_{t-1} + s_{t-1} + q_t a_t^\ell$.[16] Given a grid point on the state space (w_0, a_0^*, Θ_0), we compute a welfare gain γ that satisfies the following equation:

$$U_{SP}(w_0, a_0^*, \Theta_0; \theta) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta u \left([1 + \gamma(w_0, a_0^*, \Theta_0; \theta)] c_t \right) \right]$$

where the left-hand side is the expected utility under the planner's solution given the initial state (w_0, a_0^*, Θ_0). The right-hand side is the expected utility in the decentralized economy, and $\gamma(w_0, a_0^*, \Theta_0; \theta)$ is a permanent consumption gain that makes households indifferent between the planner's allocation and the decentralized allocation. In this way, we compute state-contingent γ across different initial states (w_0, a_0^*, Θ_0) given θ.

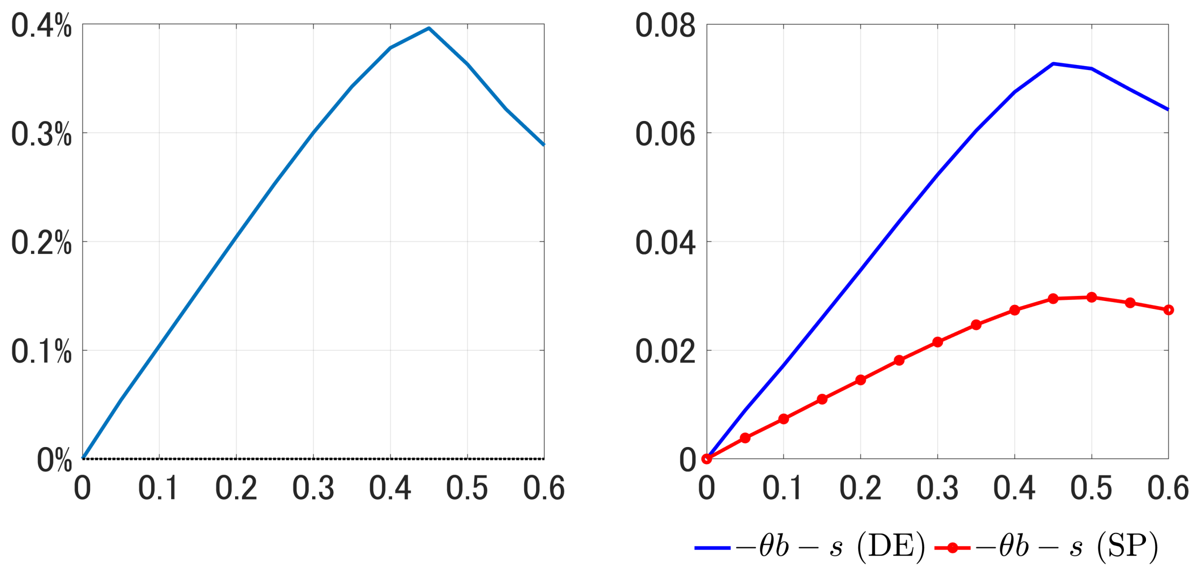
In the second step, we simulate the decentralized economy with stochastic shocks for 100,000 periods and compute the ergodic distribution of the state variables (w_0, a_0^*, Θ_0) given θ. We then compute the weighted average of γ, which is the average of state-dependent welfare gains $\gamma(w_0, a_0^*, \Theta_0; \theta)$ weighted by the ergodic distribution of the state variables (w_0, a_0^*, Θ_0). We compute the expected welfare gain for different θ by this method.

The blue line in the left panel of Figure 6 plots the expected welfare gains computed by this method across different θ on the horizontal axis. It shows that welfare gain is the largest at 0.4% when $\theta = 0.45$. The size of this welfare gain is substantially greater than welfare gains in preceding papers in the literature. The key difference from preceding papers is that policy interventions reduce liquidation and thereby mitigate persistent negative impacts of crises on output, whereas in many preceding papers the economy quickly recovers from a crisis even without policy interventions. The panel also shows that the welfare gain becomes smaller as θ becomes either higher or lower from $\theta = 0.45$.

The right panel of Figure 6 explains why the welfare gain is the largest at $\theta = 0.45$. The blue line plots the liquidity risk $-\theta b - s$ across different θ under the decentralized economy, where b and s are the simulation mean of debt and reserves in terms of the ratio to GDP. The

[16] w_t and a_t^* are the relevant state variables in the numerical solution of our model.

Figure 6 Welfare analysis and risk taking



Note: The horizontal axis is θ in both panels. The left panel plots a permanent consumption gain in percentage point on the vertical axis. In the right panel, the blue line plots the liquidity risk $-\theta b - s$ in the decentralized economy, where b and s are the simulation mean of debt and reserves in terms of the ratio to GDP. The red line plots the liquidity risk in the planner's allocation.

red line plots the liquidity risk under the planner's allocation. The liquidity risk under the decentralized economy peaks at 0.072 when $\theta = 0.45$, whereas the risk under the planner's allocation peaks at 0.030 when $\theta = 0.50$. The risk is substantially higher under the decentralized economy for any θ, because the fire-sale externality induces individual households to take an excessive risk. The gap between the two lines can be interpreted as 'excessive risk taking' by the decentralized economy, because the gap indicates how much larger liquidity risk individual households take relative to the socially optimal level of liquidity risk chosen by the planner. The gap also indicates the size of excessive asset liquidation, because $-\theta b - s$ is the liquidity shortage when a shock hits the economy, and it needs to be covered by asset liquidation. The gap becomes the largest at 0.043 when $\theta = 0.45$. This means that the size of excessive risk taking. which is corrected by capital controls and the reserve policy, is the largest when $\theta = 0.45$. This explains why the welfare gain is the largest when $\theta = 0.45$.

6 Conclusion

In this paper, we first show empirical facts about the cross-country relationships between financial development, capital controls, and reserve holdings. We show that the relationship between financial development and reserve holdings is non-monotonic, and countries with the intermediate level of financial development accumulate more reserves than countries with higher or lower financial development. We also show that countries with higher financial development are likely to use capital controls less actively, and vice versa.

We then develop a small-open-economy model to rationalize these findings. The model is featured by a liquidity shock that requires households to repay a part of outstanding foreign debt before new borrowing. This assumption motivates households to hold reserves for debt repayment because a liquidity shortage forces households into costly liquidation of productive assets. Because asset liquidation is associated with a fire-sale externality, individual households over-borrow and hold too little reserves. The optimal policy calls for a subsidy on reserve holdings or public foreign reserve accumulation, along with a tax on debt.

In the quantitative analysis, we show that the crisis dynamics of our model are in line with the empirical regularities of sudden stops in emerging economies. In particular, crises have persistent negative impacts on output, consumption, and investment, in contrast to many preceding paper without growth in which the economy quickly recovers from crises. The social planner internalizes the fire-sale externality and accumulates more reserves than decentralized households to prepare for a liquidity shock. As a result, asset liquidation is substantially smaller in the planner's allocation, and persistent negative impacts on the economy are mitigated.

The key determinant of the optimal policy is the size of a liquidity shock, which we interpret as the degree of financial development. We show that our model can replicate the empirical relationships across financial development, reserve holdings, and capital controls. In particular, the amount of reserve holdings becomes the largest when the level of financial development is intermediate. We also show that the welfare gain by the optimal policy can be as large as 0.4% of a permanent consumption, which is substantially larger than the welfare gain in many preceding papers without growth.

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A Data Appendix

Both the reserve and external liability data are from Lane and Milesi-Ferretti (2007). Key country-level control variables are from the World Development Indicators. Our country/economy sample is the same as in Bianchi and Lorenzoni (forthcoming). We exclude countries with extreme reserve-to-GDP ratios and external liability-to-GDP ratios such as Lebanon, Liberia, Luxembourg, Malta, Saudi Arabia, Singapore, and Switzerland. Our final sample thus includes 88 economies from 1980 to 2019. We focus on the following variables.

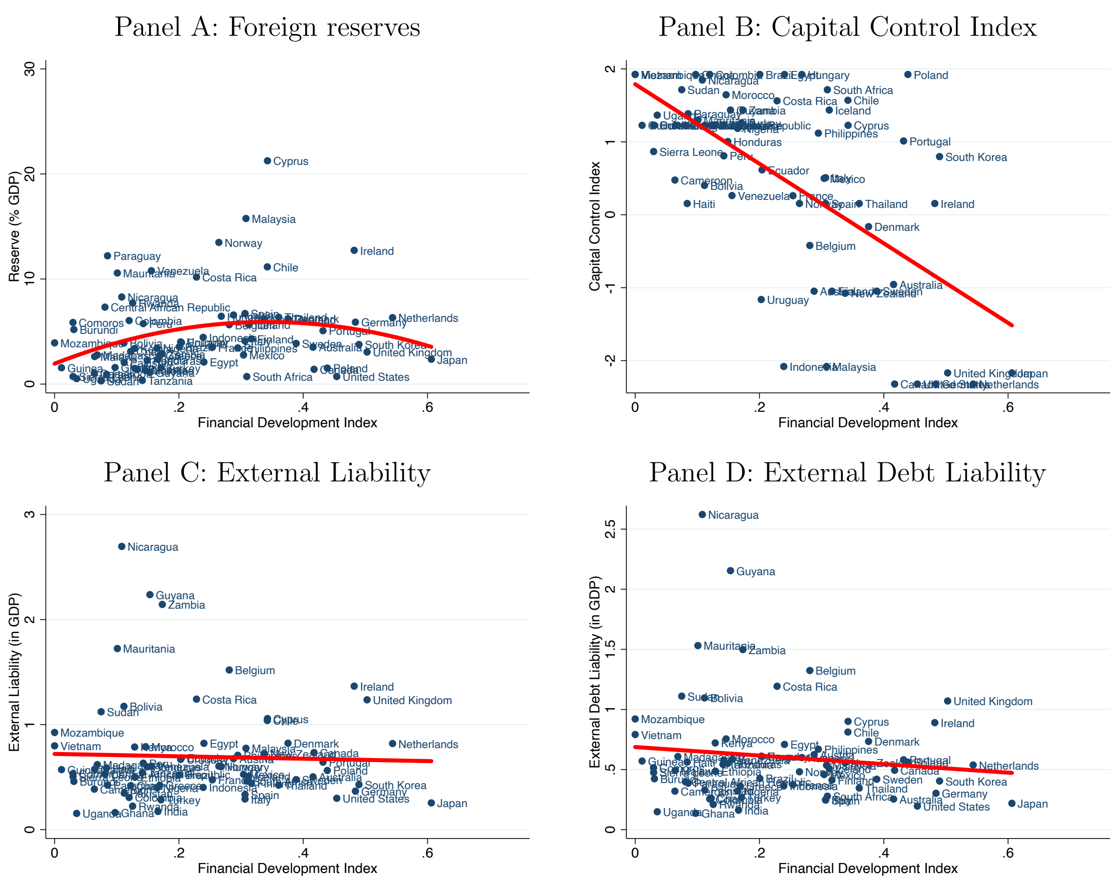
Reserves (% GDP) Foreign reserves (excluding gold) divided by GDP. Source: Lane and Milesi-Ferretti (2007).

External liability (% GDP) Total external liability divided by GDP. Source: Lane and Milesi-Ferretti (2007).
External debt liability (% GDP) Total external debt liability divided by GDP. Source: Lane and Milesi-Ferretti (2007).

Capital controls index Inverse of Chinn-Ito index from Chinn and Ito (2006).

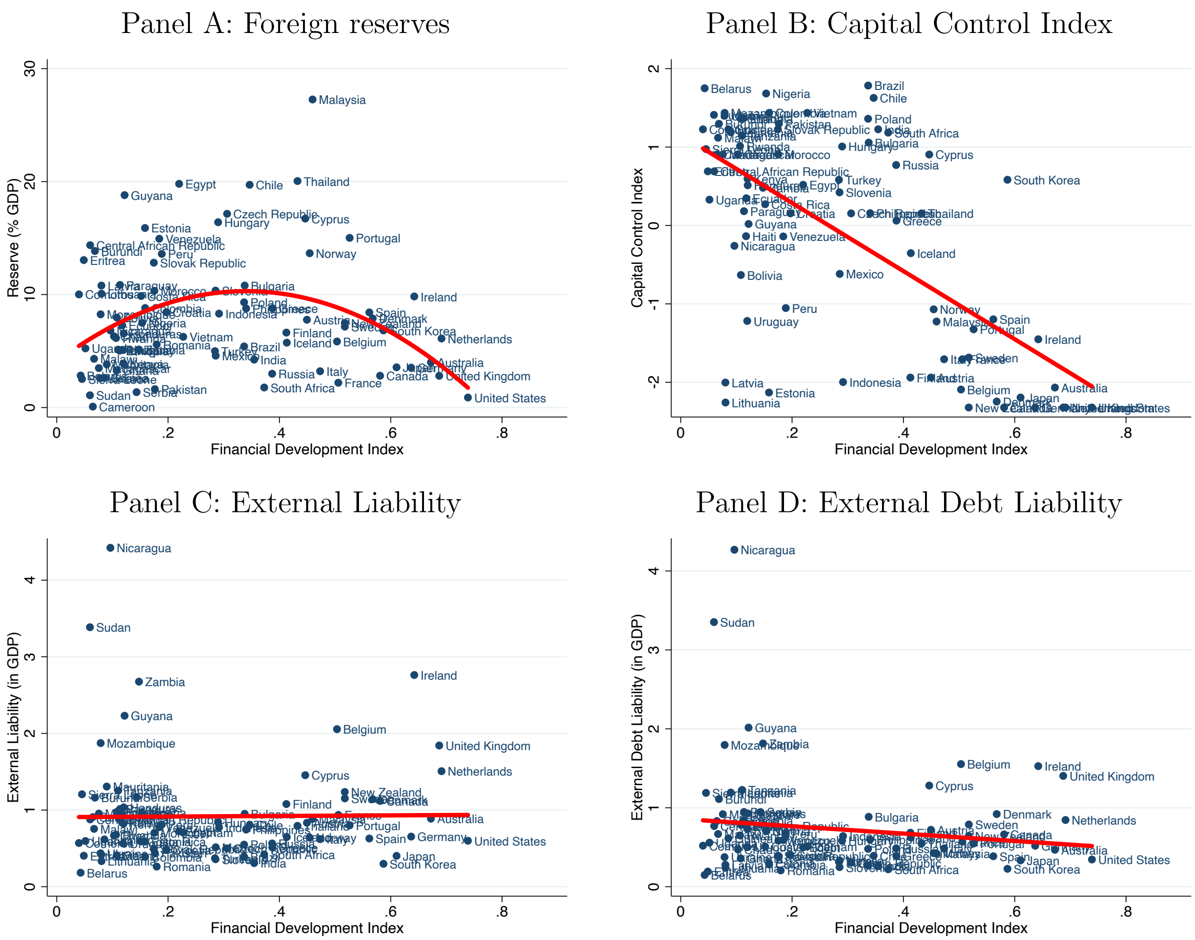
Financial development index Measures how developed financial institutions and financial markets are in each country. Source: IMF.

Figure A1 Reserve, Capital Control Index and External Debt: Long-run Relationship with Financial Development Index in 1980-1989



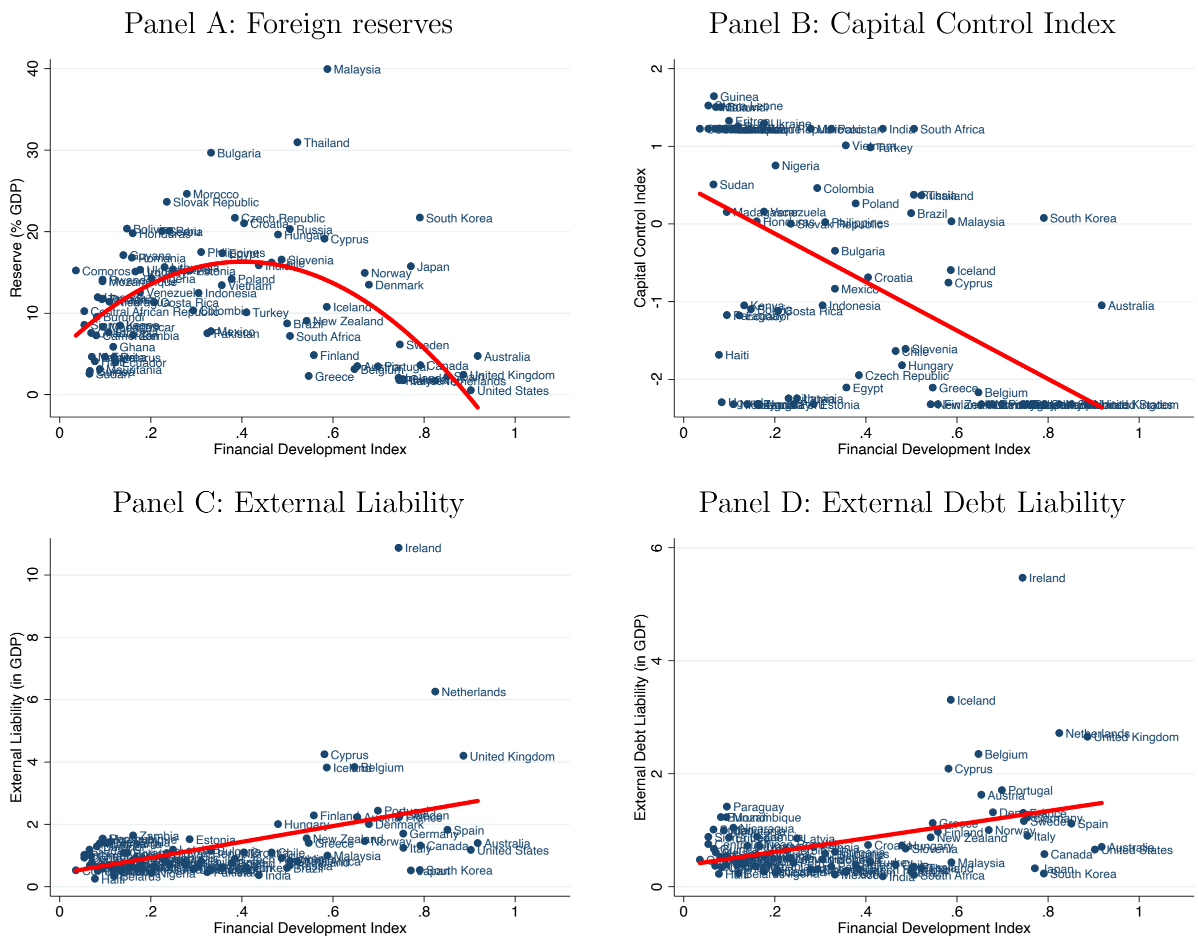
Note. The cross-sectional regression is conducted by taking the time-average of all variables from 1980 to 1989 for each economy. The data on international reserves, external liability and external debt liability is from Lane and Milesi-Ferretti (2007). Capital control index is the inverse of the Chinn-Ito index from Chinn and Ito (2006).

Figure A2 Reserve, Capital Control Index and External Debt: Long-run Relationship with Financial Development Index in 1990-1999



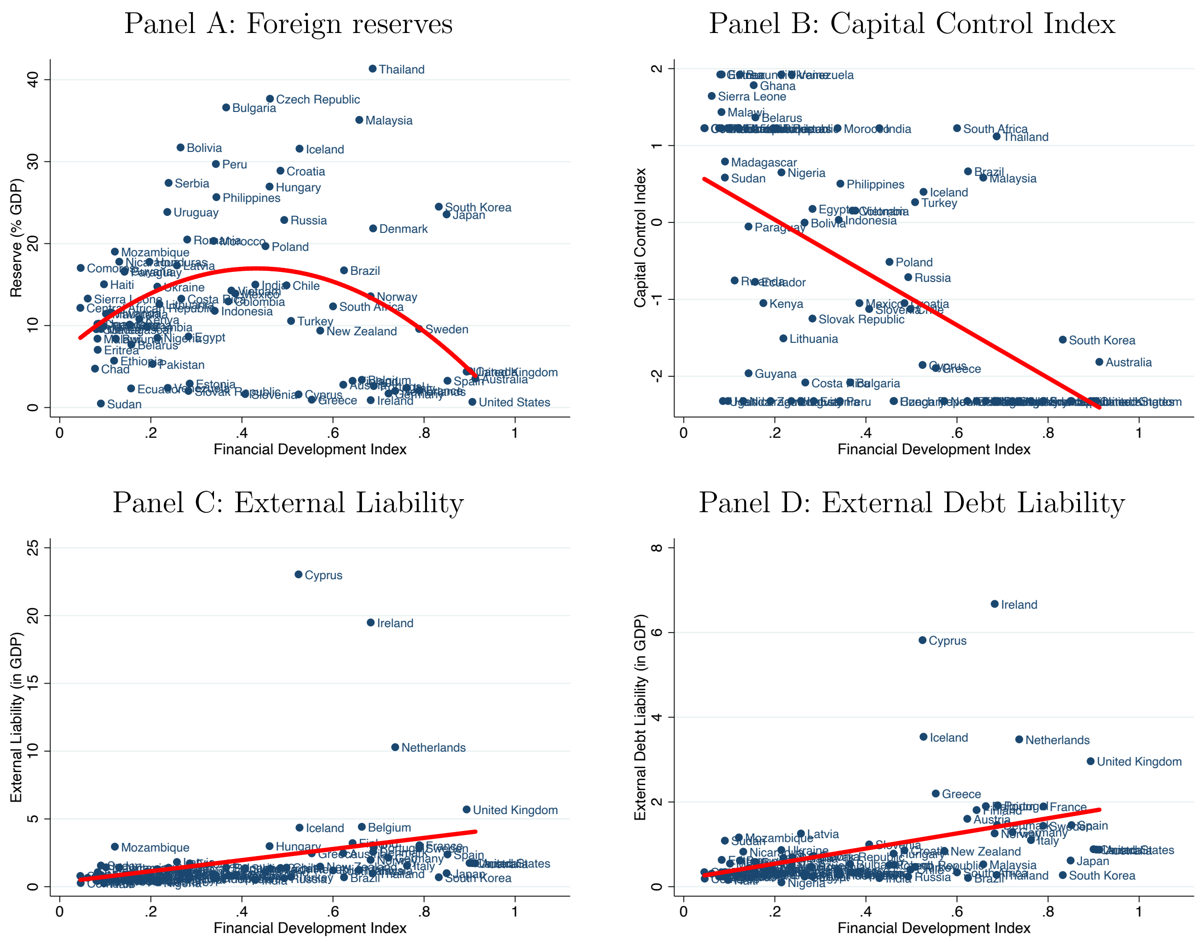
Note. The cross-sectional regression is conducted by taking the time average of all variables from 1990 to 1999 for each economy. The data on international reserves, external liability, and external debt liability is from Lane and Milesi-Ferretti (2007). The capital control index is the inverse of the Chinn-Ito index from Chinn and Ito (2006).

Figure A3 Reserve, Capital Control Index and External Debt: Long-run Relationship with Financial Development Index in 2000-2009



Note. The cross-sectional regression is conducted by taking the time-average of all variables from2000 to 2009 for each economy. The data on international reserves, external liability and external debt liability is from Lane and Milesi-Ferretti (2007). Capital control index is the inverse of the Chinn-Ito index from Chinn and Ito (2006).

Figure A4 Reserve, Capital Control Index and External Debt: Long-run Relationship with Financial Development Index in 2010-2019



Note. The cross-sectional regression is conducted by taking the time-average of all variables from2010 to 2019 for each economy. The data on international reserves, external liability and external debt liability is from Lane and Milesi-Ferretti (2007). Capital control index is the inverse of the Chinn-Ito index from Chinn and Ito (2006).

Table A1 Summary Statistics

	Obs	Mean	S.D.	Min	Max
Capital flows (% GDP)	1275	6.24	12.91	-161.68	126.09
Reserve flows (% GDP)	1274	1.51	3.70	-17.11	26.27
EMBI spreads (%)	844	4.84	6.51	0.00	98.54
Population (log)	1551	16.78	1.58	12.07	21.06
GDP per capita (log)	1499	8.03	0.99	4.55	10.42
Trade/GDP	1471	0.74	0.36	0.00	2.20
Private credit/GDP	1180	0.47	0.35	0.01	1.65
Chinn-Ito index	1459	0.44	0.33	0.00	1.00
Reserves (% GDP)	969	16.99	10.61	0.05	53.07
Total external debt (% GDP)	1241	53.15	35.80	1.22	467.98
GDP per capita growth %	1489	2.61	5.41	-64.99	53.97
Current account balances (% GDP)	1414	-1.69	6.30	-26.21	38.79

Note. The sample consists of 47 countries: Argentina, Belarus, Belize, Brazil, Bulgaria, Chile, China, Colombia, Cote d'Ivoire, Croatia, Dominican Republic, Ecuador, Egypt, El Salvador, Gabon, Georgia, Ghana, Hungary, Indonesia, Iraq, Jamaica, Jordan, Kazakhstan, Lebanon, Lithuania, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Panama, Peru, Philippines, Poland, Russia, Senegal, South Africa, South Korea, Sri Lanka, Thailand, Trinidad and Tobago, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, Vietnam. We require countries to have more than 5 years data for EMBI spread.

Table A2 Reserve, Capital Control Index and External Debt: Long-run Relationship with Financial Development Index in 1980-1989

	Reserve/GDP		Capital Control Index		External Liability/GDP		External Debt Liability/GDP	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Financial Development	0.23**	0.01	-5.46***	-5.98***	-0.11	0.17	-0.36	0.00
	(0.11)	(0.19)	(0.81)	(1.08)	(0.38)	(0.35)	(0.35)	(0.35)
Financial Development^2	-0.34	-0.05						
	(0.20)	(0.32)						
Pop (log)		0.00		0.03		0.03		0.02
		(0.01)		(0.15)		(0.05)		(0.05)
GDP per capita (log)		0.01		-0.24		0.03		0.02
		(0.01)		(0.23)		(0.07)		(0.07)
Private credit		0.01		-1.58*		-0.11		-0.12
		(0.04)		(0.92)		(0.30)		(0.30)
Trade		0.06		-0.10		1.25***		1.13***
		(0.03)		(0.83)		(0.27)		(0.27)
Constant	0.02	0.05**	1.79***	1.75***	0.72***	0.65***	0.69***	0.60***
	(0.01)	(0.02)	(0.22)	(0.29)	(0.10)	(0.09)	(0.09)	(0.09)
Observations	71	37	71	37	71	37	71	37
Adjusted R-squared	0.045	0.057	0.390	0.492	-0.013	0.423	0.001	0.379

Note. The cross-sectional regression is conducted by taking the time average of all variables from 1980 to 1989 for each economy. As our main focus is on the financial development index which is correlated with other country-level variables, i.e. population (log), GDP per capita (log), private credit, and trade, we first orthogonalize those variables on the financial development index and use the residual values in the regression. Robust standard errors are reported in parentheses. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. All variable constructions are reported in Appendix.

Table A3 Reserve, Capital Control Index and External Debt: Long-run Relationship with Financial Development Index in 1990-1999

	Reserve/GDP		Capital Control Index		External Liability/GDP		External Debt Liability/GDP	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Financial Development	0.37***	0.26	-4.34***	-4.63***	0.04	-0.26	-0.47	-0.63
	(0.11)	(0.16)	(0.58)	(0.59)	(0.38)	(0.41)	(0.34)	(0.40)
Financial Development^2	-0.54***	-0.39*						
	(0.16)	(0.22)						
Pop (log)		-0.00		0.13		-0.07		-0.08
		(0.01)		(0.10)		(0.07)		(0.07)
GDP per capita (log)		-0.00		-0.29*		-0.09		-0.10
		(0.01)		(0.16)		(0.11)		(0.10)
Private credit		0.02		-0.40		-0.14		-0.11
		(0.03)		(0.47)		(0.32)		(0.32)
Trade		0.09***		-0.12		0.18		0.05
		(0.02)		(0.37)		(0.26)		(0.25)
Constant	0.04***	0.06**	1.16***	1.34***	0.91***	0.94***	0.86***	0.87***
	(0.01)	(0.02)	(0.20)	(0.20)	(0.13)	(0.13)	(0.11)	(0.13)
Observations	85	50	83	48	85	50	85	50
Adjusted R-squared	0.097	0.368	0.404	0.599	-0.012	-0.024	0.010	0.016

Note. The cross-sectional regression is conducted by taking the time average of all variables from 1990 to 1999 for each economy. As our main focus is on the financial development index which is correlated with other country-level variables, i.e. population (log), GDP per capita (log), private credit, and trade, we first orthogonalize those variables on the financial development index and use the residual values in the regression. Robust standard errors are reported in parentheses. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. All variable constructions are reported in Appendix.

Table A4 Reserve, Capital Control Index and External Debt: Long-run Relationship with Financial Development Index in 2000-2009

	Reserve/GDP		Capital Control Index		External Liability/GDP		External Debt Liability/GDP	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Financial Development	0.54***	0.49***	-3.12***	-3.37***	2.54***	2.59***	1.21***	1.23***
	(0.11)	(0.13)	(0.52)	(0.43)	(0.53)	(0.50)	(0.30)	(0.28)
Financial Development^2	-0.67***	-0.61***						
	(0.13)	(0.14)						
Pop (log)		0.00		0.08		-0.40***		-0.28***
		(0.01)		(0.11)		(0.12)		(0.07)
GDP per capita (log)		-0.01		-0.68***		-0.19		-0.12
		(0.01)		(0.17)		(0.19)		(0.10)
Private credit		0.05		0.34		-0.14		0.07
		(0.03)		(0.48)		(0.56)		(0.31)
Trade		0.06**		0.04		0.76*		0.13
		(0.02)		(0.34)		(0.40)		(0.22)
Constant	0.05***	0.06***	0.50**	0.67***	0.42*	0.41*	0.38***	0.38***
	(0.02)	(0.02)	(0.24)	(0.20)	(0.24)	(0.23)	(0.13)	(0.13)
Observations	85	78	83	76	85	78	85	78
Adjusted R-squared	0.258	0.327	0.295	0.527	0.205	0.369	0.154	0.349

Note. The cross-sectional regression is conducted by taking the time average of all variables from 2000 to 2009 for each economy. As our main focus is on the financial development index which is correlated with other country-level variables, i.e. population (log), GDP per capita (log), private credit, and trade, we first orthogonalize those variables on the financial development index and use the residual values in the regression. Robust standard errors are reported in parentheses. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. All variable constructions are reported in Appendix.

Table A5 Reserve, Capital Control Index and External Debt: Long-run Relationship with Financial Development Index in 2010-2019

	Reserve/GDP		Capital Control Index		External Liability/GDP		External Debt Liability/GDP	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Financial Development	0.49***	0.69***	-3.43***	-3.42***	4.09***	4.26***	1.79***	1.86***
	(0.16)	(0.20)	(0.55)	(0.50)	(1.33)	(1.24)	(0.41)	(0.37)
Financial Development^2	-0.57***	-0.79***						
	(0.17)	(0.22)						
Pop (log)		-0.02		0.16		-0.41		-0.25***
		(0.01)		(0.12)		(0.30)		(0.09)
GDP per capita (log)		-0.04**		-0.65***		-0.14		-0.03
		(0.02)		(0.20)		(0.48)		(0.15)
Private credit		-0.01		0.49		2.39*		0.27
		(0.04)		(0.53)		(1.31)		(0.40)
Trade		0.00		0.14		2.49***		0.57**
		(0.03)		(0.36)		(0.89)		(0.27)
Constant	0.06**	0.03	0.72***	0.72***	0.32	0.28	0.19	0.17
	(0.03)	(0.03)	(0.26)	(0.23)	(0.62)	(0.57)	(0.19)	(0.17)
Observations	85	82	83	80	85	82	85	82
Adjusted R-squared	0.095	0.112	0.314	0.472	0.092	0.288	0.175	0.382

Note. The cross-sectional regression is conducted by taking the time average of all variables from 2010 to 2019 for each economy. As our main focus is on the financial development index which is correlated with other country-level variables, i.e. population (log), GDP per capita (log), private credit, and trade, we first orthogonalize those variables on the financial development index and use the residual values in the regression. Robust standard errors are reported in parentheses. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. All variable constructions are reported in Appendix.

B Model Appendix

B.1 Decentralized Equilibrium

The recursive maximization problem by households is set up as follows:

$$
\begin{aligned}
&V(b_{t-1}, s_{t-1}, z_{t-1}, a_{t-1}; \Theta_t, a^*_{t-1}) \\
&= \max_{c_t, b_t, s_t, z_t, a^\ell_t, a_t} u(c_t) + \beta \mathbb{E}_t V(b_t, s_t, z_t, a_t; \Theta_{t+1}, a^*_t) \\
&- \lambda_t \left[c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t - a_t - b_{t-1} - s_{t-1} - q_t a^\ell_t \right] \\
&- \xi_t \left[a_t - a_{t-1} - \eta (z_{t-1})^\gamma (a_{t-1} + \kappa(a^*_{t-1} - a_{t-1}))^{1-\gamma} + a^\ell_t \right] \\
&+ \psi_t \left[q_t a^\ell_t + \theta_t b_{t-1} + s_{t-1} \right] \\
&+ \varphi_t q_t a^\ell_t \\
&+ \nu_t \frac{s_t}{R^s}
\end{aligned}
\tag{A1}
$$

The first-order conditions are as follows:

$$c_t : \lambda_t = u'(c_t) \tag{A2}$$

$$b_t : \lambda_t = \beta \frac{R_t}{1 + \psi_b \exp\left(-\frac{b_t}{a_t} - \bar{b}\right) \frac{b_t / a_t}{R_t}} \mathbb{E}_t V_b(t+1) \tag{A3}$$

$$s_t : \lambda_t - \nu_t = \beta R^s \mathbb{E}_t V_s(t+1) \tag{A4}$$

$$z_t : \lambda_t = \beta \mathbb{E}_t V_z(t+1) \tag{A5}$$

$$a^\ell_t : \psi_t q_t + \varphi_t q_t = \xi_t - q_t \lambda_t \tag{A6}$$

$$a_t : \xi_t = \lambda_t \left[1 + \left(\frac{b_t / a_t}{R_t} \right)^2 \psi_b \exp\left(-\frac{b_t}{a_t} - \bar{b} \right) \right] + \beta \mathbb{E}_t V_a(t+1) \tag{A7}$$

$$\psi_t \left[q_t a^\ell_t + \theta_t b_{t-1} + s_{t-1} \right] = 0,\ \psi_t \geq 0 \tag{A8}$$

$$\varphi_t q_t a^\ell_t = 0,\ \varphi_t \geq 0 \tag{A9}$$

$$\nu_t \frac{s_t}{R^s} = 0,\ \nu_t \geq 0 \tag{A10}$$

The envelope conditions are given as follows:

$$V_b(t) = \lambda_t + \psi_t \theta_t \tag{A11}$$

$$V_s(t) = \lambda_t + \psi_t \tag{A12}$$

$$V_z(t) = \xi_t \eta \gamma \left(\frac{z_{t-1}}{(1-\kappa)a_{t-1} + \kappa a^*_{t-1}} \right)^{\gamma-1} \tag{A13}$$

$$V_a(t) = \xi_t \left[1 + \eta(1-\gamma)(1-\kappa) \left(\frac{z_{t-1}}{(1-\kappa)a_{t-1} + \kappa a^*_{t-1}} \right)^{\gamma} \right] \tag{A14}$$

Forwarding the envelope conditions one period and plugging them into the first-order conditions, we obtain the equilibrium conditions in the text, (15), (16), (17), (18), and (19). Foreign asset a^*_t follows the exogenous law of motion $a^*_t = (1+\bar{g})a^*_{t-1}$.

The decentralized equilibrium is defined by allocations $\{c_t,\, b_t,\, s_t,\, z_t,\, a^\ell_t,\, a_t,\, a^*_t\}_{t=0}^{\infty}$, the Lagrange multipliers $\{\psi_t,\, \varphi_t,\, \xi_t,\, \nu_t\}_{t=0}^{\infty}$, and the liquidation price $\{q_t\}_{t=0}^{\infty}$ that satisfy (2), (3), (5), (8), (15), (17), (18), (19), (A8), (A9), (A10), and the law of motion for a^*_t.

B.2 Social Planner's Problem

The only difference from the decentralized equilibrium is that the planner internalizes that the liquidation price q_t is a function of liquidation a^ℓ_t as in (8). Accordingly, the setup of the planner's problem is identical to the maximization problem in the decentralized economy, except that q_t is a function of a^ℓ_t. Formally,

$$\begin{aligned} &V(b_{t-1}, s_{t-1}, z_{t-1}, a_{t-1}; \Theta_t, a^*_{t-1}) \\ &= \max_{c_t, b_t, s_t, z_t, a^\ell_t, a_t} u(c_t) + \beta \mathbb{E}_t V(b_t, s_t, z_t, a_t; \Theta_{t+1}, a^*_t) \\ &- \lambda_t \left[c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t - a_t - b_{t-1} - s_{t-1} - q_t(a^\ell_t; a^*_{t-1}) a^\ell_t \right] \\ &- \xi_t \left[a_t - a_{t-1} - \eta (z_{t-1})^{\gamma} (a_{t-1} + \kappa(a^*_{t-1} - a_{t-1}))^{1-\gamma} + a^\ell_t \right] \\ &+ \psi^{SP}_t \left[q_t(a^\ell_t; a^*_{t-1}) a^\ell_t + \theta_t b_{t-1} + s_{t-1} \right] \\ &+ \varphi^{SP}_t q_t(a^\ell_t; a^*_{t-1}) a^\ell_t \\ &+ \nu_t \frac{s_t}{R^s} \end{aligned} \tag{A15}$$

The first-order condition regarding liquidation a_t^ℓ is:

$$\psi_t^{SP} + \varphi_t^{SP} = \frac{\xi_t}{q_t + (\partial q_t/\partial a_t^\ell)a_t^\ell} - u'(c_t) \tag{A16}$$

$$= \frac{\xi_t}{(1-\zeta)q_t} - u'(c_t) \tag{A17}$$

The Lagrange multipliers have superscripts SP to emphasize that they are different from the multipliers in the decentralized equilibrium. This is (21) in the main text. As explained in the main text, the shadow value of liquidity ψ_t^{SP} is higher than that in the decentralized equilibrium ψ_t because the planner internalizes the effect of liquidation a_t^ℓ on the liquidation price q_t, which is captured by $\partial q_t/\partial a_t^\ell < 0$.

B.3 Public Reserve Holdings

In Section 3.4, we show that the planner's allocation can be decentralized by a tax on debt and subsides on reserves and investment. In this subsection, we show that public reserve holdings instead of a subsidy on private reserve holdings can achieve the same allocation.

Consider a Ramsey planner's problem whose policy instruments are a tax on debt and direct reserve holdings. In particular, this planner can collect a tax from households in lump sum, buy foreign reserves with the interest R^s, and rebate the proceeds to households in lump sum. The planner can also provide reserves to households when a liquidity shock hits the economy. Let $\hat{s}_t$ denotes the planner's reserve holdings. Given this setup, the households' budget constraint can be written as follows:

$$c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + \frac{\hat{s}_t}{R^s} + z_t = a_t + b_{t-1} + s_{t-1} + \hat{s}_{t-1} + q_t a_t^\ell + T_t \tag{A18}$$

where T_t is a lump-sum transfer by the planner to rebate/finance the tax on debt and the subsidy on investment. Liquidation is subject to the liquidity constraint:

$$q_t a_t^\ell \geq -\theta_t b_{t-1} - s_{t-1} - \hat{s}_{t-1} \tag{A19}$$

Private and public reserves are subject to the non-negative constraint respectively:

$$s_t \geq 0 \tag{A20}$$

$$\hat{s}_t \geq 0 \tag{A21}$$

Because public reserve holdings do not distort the households' decisions, the first-order conditions are the same as those in the decentralized equilibrium, except the Euler equation regarding debt because the tax on debt is available. The Ramsey planner chooses the tax on debt τ_t^b and public reserve holdings $\hat{s}_t$ to maximize the households' expected utility, given the households' first-order conditions as the implementability constraints, and internalizing the effect of liquidation a_t^ℓ on the liquidation price q_t through (8).

In the budget constraint (A18) and the liquidation inequality (A19), private and public reserve holdings appear only in the form of $s_t + \hat{s}_t$. This means that households only care about the sum of private and public reserves, and each of s_t and $\hat{s}_t$ is individually irrelevant for the households' decisions. Let s_t^0 denote reserve holdings chosen by households when $\hat{s}_t = 0\ \forall t$, i.e. no public reserves. As the planner increases public reserves from zero, households reduce private reserves to satisfy $s_t + \hat{s}_t = s_t^0$, because s_t^0 is the individually optimal amount of total reserves. But this is possible only if $s_t \geq 0$ and equivalently $\hat{s}_t \leq s_t^0$. Once public reserves $\hat{s}_t$ exceed the individually optimal total reserves s_t^0, households choose $s_t = 0$, and the planner can choose any arbitrary amount of total reserves, which is only public reserves in this case.

Now, suppose the planner introduces the optimal tax on debt τ_t^b given by (26). Given this policy, private reserves chosen by households are smaller than the planner's optimal amount of reserves because of the gap between ψ_{t+1} and ψ_{t+1}^{SP} discussed in Section 3.4. This means that the planner can choose the optimal amount of reserves, and private reserves are zero. These policies achieve the socially optimal allocation discussed in 3.4.

C Two-Period Model

In this section, we introduce a two-period model and show analytical results to facilitate understanding of the full model in the main text.

C.1 Model Setup

We consider a two-period model with $t = 0, 1$. At $t = 0$, households choose consumption c_0, foreign bond b_0 (negative b_0 is borrowing), and reserves s_0. Investment in capital is exogenously fixed at k_0. There is no production or endowment at $t = 0$, and households need to borrow to consume and buy reserves, implying $b_0 < 0$.

At $t = 1$, households produce goods using capital. However, at the beginning of period

1 before production, a liquidity shock may hit the economy with probability π. A liquidity shock forces households to repay a θ fraction of the period-0 debt before production. Households make this repayment by reserves and liquidating some of their investment if reserves are not enough for the repayment. Liquidation of investment is costly because liquidated investment can be sold only at a fire-sale price. We assume that the liquidation price q_1 decreases in the amount of liquidation k_1^ℓ, satisfying $q_1'(\bullet) < 0$ and $q_1''(\bullet) < 0$. We also assume that liquidation proceeds $q_1(k_1^\ell)k_1^\ell$ increase in k_1^ℓ, at least for $0 \leq k_1^\ell \leq \bar{k}$ with some positive $\bar{k}$. This assumption implies that the elasticity of q_1 is not too high, so that households can make the repayment by liquidation when reserves are not enough. Another important assumption is that individual households take q_1 as given when they make decisions.

Later at $t = 1$, households produce goods and consume. Production is given by $y = f(k_0 - k_1^\ell)$ with the concave production function satisfying $f'(\bullet) > 0$ and $f''(\bullet) < 0$.

The period budget constraints are given as follows:

$$t = 0 : c_0 + \frac{b_0}{R_b} + \frac{s_0}{R_s} = 0 \tag{A22}$$

$$t = 1 : c_1 = f(k_0 - k_1^\ell) + b_0 + s_0 + q_1 k_1^\ell \tag{A23}$$

Liquidation k_1^ℓ needs to satisfy:

$$q_1 k_1^\ell \geq -\theta_1 b_0 - s_0$$
$$q_1 k_1^\ell \geq 0$$

where θ_1 is stochastic and takes θ with probability π and 0 with probability $1 - \pi$.

We assume the households' utility function as follows:

$$U(c_0, c_1^N, c_1^L) = \log(c_0) + \beta\left[(1-\pi)c_1^N + \pi c_1^L\right]$$

β is a discount factor, c_1^N is consumption when no liquidity shock at $t = 1$, and c_1^L is consumption when a liquidity shock hits at $t = 1$. We assume a linear utility at period 1 for analytical tractability.

C.2 Period 1 with No Liquidity Shock ($\theta_1 = 0$)

We solve the decentralized equilibrium of the model backward from period 1. The state variables at period 1 are debt b_0, reserves s_0, and a realization of a liquidity shock θ_1. We first consider the case of no liquidity shock, $\theta_1 = 0$. The value function is given by

$$\begin{aligned}
V^N(b_0, s_0) = & \max_{c_1^N, k_1^\ell} c_1^N \\
& - \lambda_1^N \left[c_1^N - f(k_0 - k_1^\ell) - b_0 - s_0 - q_1 k_1^\ell \right] \\
& + \psi_1^N \left[q_1 k_1^\ell + s_0 \right] \\
& + \varphi_1^N \left[q_1 k_1^\ell \right]
\end{aligned}$$

Combining the first-order conditions regarding c_1^N and k_1^ℓ leads to the following equation:

$$\psi_1^N + \varphi_1^N = \frac{f'(k_0 - k_1^\ell)}{q_1} - 1$$

Note that the liquidity constraint never binds because $\theta_1 = 0$ and $s_0 \geq 0$, implying $\psi_1^N = 0$. We assume $f'(k_0 - k_1^\ell) > q_1$ for any k_1^ℓ, implying $\varphi_1^N > 0$ and $k_1^\ell = 0$. This condition is guaranteed by assuming costly liquidation (low q_1) even for very small liquidation. This implies that households do not liquidate unless it is necessary.

Envelope conditions are given by

$$V_b^N(b_0, s_0) = 1 \tag{A24}$$

$$V_s^N(b_0, s_0) = 1 \tag{A25}$$

where the subscripts indicate the variable with which a partial derivative is taken. The second condition utilizes $\psi_1^N = 0$.

C.3 Period 1 with Liquidity Shock ($\theta_1 = \theta$)

Next, we consider the case of a liquidity shock hitting the economy, $\theta_1 = \theta$. The value function is given by

$$\begin{aligned}
V^L(b_0, s_0) = & \max_{c_1^L, k_1^\ell} c_1^L \\
& - \lambda_1^L \left[c_1^L - f(k_0 - k_1^\ell) - b_0 - s_0 - q_1 k_1^\ell \right] \\
& + \psi_1^L \left[q_1 k_1^\ell + \theta b_0 + s_0 \right] \\
& + \varphi_1^L \left[q_1 k_1^\ell \right]
\end{aligned}$$

Again, combining the first-order conditions leads to the same equation:

$$\psi_1^L + \varphi_1^L = \frac{f'(k_0 - k_1^\ell)}{q_1} - 1 \tag{A26}$$

If $-\theta b_0 > s_0$ and reserves are not enough to cover the early repayment (which is always the case as shown below), there is positive liquidation $k_1^\ell > 0$ and $\varphi_1^L = 0$. Assuming $f'(k_0 - k_1^\ell) > q_1$, ψ_1^L is given by (A26), and households liquidate investment just enough to cover the liquidity shortage, implying $q_1 k_1^\ell = -\theta b_0 - s_0$.

Instead, if $\theta b_0 \leq s_0$ and reserves are enough to cover the early repayment (which does not happen in the equilibrium as shown below), then $k_1^\ell = 0$ and $\psi_1^L = 0$.

Envelope conditions are given as follows:

$$V_b^L(b_0, s_0) = 1 + \psi_1^L \theta \tag{A27}$$

$$V_s^L(b_0, s_0) = 1 + \psi_1^L \tag{A28}$$

C.4 Period 0

We go back to period $t = 0$. The value function is given by

$$\begin{aligned} V_0 = \max_{c_0, b_0, s_0} & \log(c_0) + \beta\left[(1-\pi)V^N(b_0, s_0) + \pi V^L(b_0, s_0)\right] \\ & - \lambda_0 \left[c_0 + \frac{b_0}{R_b} + \frac{s_0}{R_s}\right] \\ & + \nu_0 \frac{s_0}{R_s} \end{aligned}$$

The first-order conditions are given as follows:

$$c_0 : \frac{1}{c_0} = \lambda_0 \tag{A29}$$

$$b_0 : \lambda_0 = \beta R_b \left[(1-\pi)V_b^N(b_0, s_0) + \pi V_b^L(b_0, s_0)\right] \tag{A30}$$

$$s_0 : \lambda_0 - \nu_0 = \beta R_s \left[(1-\pi)V_s^N(b_0, s_0) + \pi V_s^L(b_0, s_0)\right] \tag{A31}$$

C.5 Decentralized Equilibrium

Plugging the envelope conditions (A24), (A25), (A27), and (A28) into the Euler equations (A30) and (A31), we obtain the explicit expressions for the Euler equations regarding debt and reserves:

$$\frac{1}{c_0} = \beta R_b \left[(1-\pi) + \pi \left\{1 + \theta\psi_1^L\right\}\right] \tag{A32}$$

$$\frac{1}{c_0} - \nu_0 = \beta R_s \left[(1-\pi) + \pi \left\{1 + \psi_1^L\right\}\right] \tag{A33}$$

and ψ_1^L is given by

$$\psi_1^L = \begin{cases} \frac{f'(k_0-k_1^\ell)}{q_1} - 1 & \text{if } \theta b_0 - s_0 > 0 \text{ and } k_1^\ell > 0. \\ 0 & \text{if } \theta b_0 - s_0 \leq 0 \text{ and } k_1^\ell = 0. \end{cases} \tag{A34}$$

If $k_1^\ell > 0$, it is implicitly given by:

$$q_1(k_1^\ell)k_1^\ell = -\theta b_0 - s_0 \tag{A35}$$

which is the binding liquidity constraint. Combining the two Euler equations (A32) and (A33),

$$\beta(R_b - R_s) = \pi\beta\psi_1^L(R_s - \theta R_b) + \nu_0 \tag{A36}$$

which is a simplified version of the key equation (20) in the main text. It is straightforward to prove the three propositions in Section 3.3. In particular, Proposition 3 says that households never hold enough reserves to cover the entire early repayment. This means that ψ_1^L takes a positive value in (A34), implying

$$\psi_1^L = \frac{f'(k_0 - k_1^\ell)}{q_1} - 1 \tag{A37}$$

As explained in the main text, ψ_1^L is the net *private* benefit of a reduction in liquidity shortage.

The decentralized equilibrium of this model is the six variables $\{c_0, b_0, s_0, k_1^\ell, \psi_1^L, \nu_0\}$ that satisfy the six equations (A22), (A32), (A33), (A35), (A37), and $\nu_0 s_0 = 0$ with $\nu_0 \geq 0$. c_1^N and c_1^L are given by the resource constraint (A23).

C.6 Analysis

Plugging ψ_1^L in (A37) into the key equation (A36) leads to the following equation:

$$\beta(R_b - R_s) = \pi\beta\left(\frac{f'(k_0 - k_1^\ell)}{q_1(k_1^\ell)} - 1\right)(R_s - \theta R_b) + \nu_0 \tag{A38}$$

When the parameter values are such that $s_0 > 0$ and $\nu_0 = 0$ in the equilibrium, (A38) with $\nu_0 = 0$ alone pins down k_1^ℓ. In this case, we can compute how k_1^ℓ changes as θ increases by the implicit function theorem. Formally,

$$\frac{\partial k_1^\ell}{\partial \theta} = -\frac{\pi\beta R_b\left(\frac{f'}{q_1} - 1\right)}{-\pi\beta(R_s - \theta R_b)\frac{-f''q_1 - f'q'}{q_1^2}} > 0$$

It shows that when $s_0 > 0$, the size of liquidation k_1^ℓ increases as θ becomes greater.

When the parameter values are such that $s_0 = 0$ and $\nu_0 > 0$ in the equilibrium, the liquidation equation (A35) reduces to:

$$q_1(k_1^\ell)k_1^\ell = -\theta b_0 \tag{A39}$$

Assuming the liquidation proceeds $q_1 k_1^\ell$ increase in k_1^ℓ, k_1^ℓ and b_0 have a one-to-one relation. Plugging this equation into the Euler equation regarding debt (A32),

$$\frac{\theta R_b}{q_1(k_1^\ell)k_1^\ell} = \beta R_b\left[(1-\pi) + \pi\left\{1 + \theta\left(\frac{f'(k_0 - k_1^\ell)}{q_1(k_1^\ell)} - 1\right)\right\}\right]$$

This equation pins down k_1^ℓ. Applying the implicit function theorem,

$$\frac{\partial k_1^\ell}{\partial \theta} = -\frac{\frac{1}{\theta}\beta R_b}{R_b\theta\frac{-(q_1 + q_1' k_1^\ell)}{(q_1 k_1^\ell)^2} - \beta R_b \pi\theta\left(\frac{-f''q_1 - f'q_1'}{q_1^2}\right)} > 0$$

The inequality comes from that the denominator is negative and the numerator is positive. Therefore, whether $s_0 > 0$ or $s_0 = 0$ in the equilibrium, the size of liquidation k_1^ℓ increases as θ becomes greater.

C.7 Social Planner' Problem

The only difference from the decentralized equilibrium is that the planner internalizes that the liquidation price q_1 is decreasing in the amount of liquidation k_1^ℓ, i.e. $q_1'(k_1^\ell) < 0$. This affects the problem of period 1 with liquidity shock. The setup is the same as above, but the first-order condition w.r.t. k_1^ℓ leads to the following equation:

$$\psi_1^{SP} + \varphi_1^{SP} = \frac{f'(k_0 - k_1^\ell)}{q_1 + q_1'(k_1^\ell)k_1^\ell} - 1$$

Accordingly, the Euler equations by the planner are given as follows:

$$\frac{1}{c_0} = \beta R_b \left[(1-\pi)1 + \pi \left\{1 + \psi_1^{SP}\theta\right\}\right]$$
$$\frac{1}{c_0} - \nu_0 = \beta R_s \left[(1-\pi)1 + \pi \left\{1 + \psi_1^{SP}\right\}\right]$$

and ψ_1^{SP} is given by

$$\psi_1^{SP} = \frac{f'(k_0 - k_1^\ell)}{q_1 + q_1'(k_1^\ell)k_1^\ell} - 1 \tag{A40}$$

Because we assume $q'(k_1^\ell) < 0$, we have $\psi_1^{SP} > \psi_1^L$ given the same k_1^ℓ.

Comparing the Euler equations under the decentralized equilibrium and the planner's allocation, the optimal tax on debt and subsidy on reserves are characterized as follows:

$$\frac{1}{c_0} = \beta R_b(1+\tau_b)\left[1 - \pi\theta + \pi\theta\frac{f'(k_0 - k_1^\ell)}{q_1}\right] \tag{A41}$$
$$\frac{1}{c_0} - \nu_0 = \beta R_s(1+\tau_s)\left[1 - \pi + \pi\frac{f'(k_0 - k_1^\ell)}{q_1}\right] \tag{A42}$$

with the tax and subsidy given by

$$1 + \tau_b = \frac{1 - \pi\theta + \pi\theta\frac{f'(k_0 - k_1^\ell)}{q_1 + q_1'(k_1^\ell)k_1^\ell}}{1 - \pi\theta + \pi\theta\frac{f'(k_0 - k_1^\ell)}{q_1}} \tag{A43}$$
$$1 + \tau_s = \frac{1 - \pi + \pi\frac{f'(k_0 - k_1^\ell)}{q_1 + q_1'(k_1^\ell)k_1^\ell}}{1 - \pi + \pi\frac{f'(k_0 - k_1^\ell)}{q_1}} \tag{A44}$$

C.8 Effect of θ on τ_b

We now show how τ_b changes as θ changes. As is the case in the decentralized equilibrium, it is straightforward to show that the amount of liquidation k_1^ℓ increases in θ. Because τ_b in (A43) is a function of only k_1^ℓ given θ, we can take the derivative of τ_b with respect to θ, taking into account the positive effect of θ on k_1^ℓ.

Let us denote the numerator of the right-hand side of (A43) by F^{SP}, and the denominator by F^{DE}. Taking the derivative of the numerator regarding θ,

$$\frac{dF^{SP}}{d\theta} = \pi \left(\frac{f'}{q_1 + q_1' k_1^\ell} - 1 \right) + \pi\theta \left\{ \frac{-f''(q_1 + q_1' k_1^\ell) - f'(q_1' + q_1' + q_1'' k_1^\ell)}{(q_1 + q_1' k_1^\ell)^2} \right\} \frac{\partial k_1^\ell}{\partial \theta}$$

Taking the derivative of the denominator,

$$\frac{dF^{DE}}{d\theta} = \pi \left(\frac{f'}{q_1} - 1 \right) + \pi\theta \left\{ \frac{-f'' q_1 - f' q_1'}{(q_1)^2} \right\} \frac{\partial k_1^\ell}{\partial \theta}$$

Applying the chain rule, the sign of $d\tau_b/d\theta$ is the same as the sign of the following:

$$\frac{dF^{SP}}{d\theta} F^{DE} - \frac{dF^{DE}}{d\theta} F^{SP}$$

Writing out explicitly, the first term is

$$\left[\pi \left(\frac{f'}{q_1 + q_1' k_1^\ell} - 1 \right) + \pi\theta \left\{ \frac{-f''(q_1 + q_1' k_1^\ell) - f'(q_1' + q_1' + q_1'' k_1^\ell)}{(q_1 + q_1' k_1^\ell)^2} \right\} \frac{\partial k_1^\ell}{\partial \theta} \right] \times \left[1 + \pi\theta \left(\frac{f'}{q_1} - 1 \right) \right] \tag{A45}$$

The second term is

$$\left[\pi \left(\frac{f'}{q_1} - 1 \right) + \pi\theta \left\{ \frac{-f'' q_1 - f' q_1'}{(q_1)^2} \right\} \frac{\partial k_1^\ell}{\partial \theta} \right] \times \left[1 + \pi\theta \left(\frac{f'}{q_1 + q_1' k_1^\ell} - 1 \right) \right] \tag{A46}$$

We want to show (A45) minus (A46) is positive. First, compare the product of the first term in the first bracket and the second bracket in each of (A45) and (A46). Taking the gap,

$$\begin{aligned}
&\pi\left(\frac{f'}{q_1+q_1'k_1^\ell}-1\right)\times\left[1+\pi\theta\left(\frac{f'}{q_1}-1\right)\right]-\pi\left(\frac{f'}{q_1}-1\right)\times\left[1+\pi\theta\left(\frac{f'}{q_1+q_1'k_1^\ell}-1\right)\right]\\
=&\pi\left(\frac{f'}{q_1+q_1'k_1^\ell}-1\right)+\pi\left(\frac{f'}{q_1+q_1'k_1^\ell}-1\right)\left[\pi\theta\left(\frac{f'}{q_1}-1\right)\right]\\
-&\pi\left(\frac{f'}{q_1}-1\right)-\pi\left(\frac{f'}{q_1}-1\right)\left[\pi\theta\left(\frac{f'}{q_1+q_1'k_1^\ell}-1\right)\right]\\
=&\pi\left(\frac{f'}{q_1+q_1'k_1^\ell}-\frac{f'}{q_1}\right)>0
\end{aligned}$$

Next, compare the product of the second term in the first bracket and the second bracket. We further decompose the second bracket into $(1-\pi\theta)$ and the fraction term and examine them separately. First, $(1-\pi\theta)$ is common for both (A45) and (A46), so we simply compare the second term in the first bracket. Taking the gap,

$$\begin{aligned}
&\pi\theta\left\{\frac{-f''(q_1+q_1'k_1^\ell)-f'(q_1'+q_1'+q_1''k_1^\ell)}{(q_1+q_1'k_1^\ell)^2}\right\}\frac{\partial k_1^\ell}{\partial\theta}-\pi\theta\left\{\frac{-f''q_1-f'q_1'}{(q_1)^2}\right\}\frac{\partial k_1^\ell}{\partial\theta}\\
=&\pi\theta\frac{\partial k_1^\ell}{\partial\theta}\left[\frac{-f''(q_1+q_1'k_1^\ell)-f'(q_1'+q_1'+q_1''k_1^\ell)}{(q_1+q_1'k_1^\ell)^2}-\frac{-f''q_1-f'q_1'}{(q_1)^2}\right]
\end{aligned}$$

To determine the sign of the bracketed term, we compare the first and the second term in the fraction separately.

$$\begin{aligned}
\frac{-f''(q_1+q_1'k_1^\ell)}{(q_1+q_1'k_1^\ell)^2}=\frac{-f''}{q_1+q_1'k_1^\ell}>\frac{-f''q_1}{(q_1)^2}=\frac{-f''}{q_1}\\
\frac{-f'(q_1'+q_1'+q_1''k_1^\ell)}{(q_1+q_1'k_1^\ell)^2}>\frac{-f'q_1'}{(q_1)^2}
\end{aligned}$$

To derive these results, we use the assumptions for the functions $f''(\bullet)<0$, $q_1'(\bullet)<0$, $q_1''(\bullet)<0$, and $(q_1+q_1'k_1^\ell)>0$. We also use $\partial k_1^\ell/\partial\theta>0$. These results show that the product of the second term in the first bracket and $(1-\pi\theta)$ is greater in (A45) than in (A46).

Finally, we compare the product of the second term in the first bracket and the fraction term in the second bracket. Taking the gap,

$$
\begin{aligned}
&\pi\theta\frac{\partial k_1^\ell}{\partial\theta}\left[\frac{-f''(q_1+q_1'k_1^\ell)-f'(q_1'+q_1'+q_1''k_1^\ell)}{(q_1+q_1'k_1^\ell)^2}\times\pi\theta\frac{f'}{q_1}-\frac{-f''q_1-f'q_1'}{(q_1)^2}\times\pi\theta\frac{f'}{q_1+q_1'k_1^\ell}\right]\\
=&\pi^2\theta^2\frac{\partial k_1^\ell}{\partial\theta}\frac{f'}{q_1(q_1+q_1'k_1^\ell)}\left[\frac{-f''(q_1+q_1'k_1^\ell)-f'(q_1'+q_1'+q_1''k_1^\ell)}{q_1+q_1'k_1^\ell}-\frac{-f''q_1-f'q_1'}{q_1}\right]\\
=&\pi^2\theta^2\frac{\partial k_1^\ell}{\partial\theta}\frac{f'}{q_1(q_1+q_1'k_1^\ell)}\left[\frac{-f'(q_1'+q_1'+q_1''k_1^\ell)}{q_1+q_1'k_1^\ell}-\frac{-f'q_1'}{q_1}\right]>0
\end{aligned}
$$

Therefore, (A45) minus (A46) is positive, which proves $d\tau_b/d\theta > 0$.