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Excess shocks can limit the economic interpretation ☆

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ABSTRACT

When a model has more shocks than observed variables (excess shocks) the Estimated Model (EM) shock innovations will be correlated. These correlations limit the usefulness of variance and variable decompositions, as the latter use the EM shock innovations. Furthermore, any such correlations limit the ability to directly test the assumptions made about the Assumed Model (AM) shocks. These need to be correct in order to interpret impulse responses. A partial interpretation of the data may be possible if some AM shocks can be recovered. An approach to determining which shocks can be recovered when using either the current and past or all data is presented, unifying the existing methods for assessing recovery. It is applicable to a wide range of macroeconomic models.

1. Introduction

Modern macroeconomics interprets the macroeconomy using a model and its structural shocks. Shocks in models are given names that have economic meanings. These names could stem from theory, as in Dynamic Stochastic General Equilibrium (DSGE) models, but may also just be provided based on a particular structural equation, as with Structural VARs (SVARs) models. The dynamics of the economy are then studied through the impulse responses to perturbations in the shocks, and an understanding of past economic developments is obtained by decomposing the observed data into the contributions of the shocks. This paper discusses a common situation in which the ability to interpret the macroeconomy using structural shocks can be limited. We explain why these limitations arise and provide applied economists with a simple approach to determining which shocks are affected. This approach can be applied after estimation of a wide variety of structural macroeconomic models, including DSGE and SVARs, with or without Time-Varying Parameters (TVP). The approach can also be applied to other macroeconomic models which involve shocks, but which have less structural interpretation, such as Unobserved Components (UC) and some Factor Models (FMs).

The situation which can limit the ability to interpret the economy is when the number of shocks in the model exceeds the number of observed variables. We term this situation one of *excess shocks*; alternatively (Forni et al., 2019) call such systems "short" . The presence of excess shocks is widespread. For UC and FMs it is invariably the case. A common example with structural models is when DSGE models incorporate measurement error. However, there are many other possible sources, such as indeterminacies (resulting in sunspots), stochastically time-varying coefficients, and Markov switching. Excess shocks are common in macroeconomic structural modeling; for example, they occur in around one quarter of the estimated DSGE models of the U.S. economy in the Macroeconomic

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Model Database.[1] This includes models published in leading journals, such as the *American Economic Review*, the *Economic Journal* and the *Review of Economics and Statistics*.[2] Excess shocks are present in the large-scale DSGE models developed at central banks and other policy institutions, such as RAMSES II (Sveriges Riksbank; Adolfson et al., 2013) and the Multi-Sector Model (Reserve Bank of Australia; Rees et al., 2016).

As part of giving these shocks names assumptions are made about their nature. There are parallels with regression analysis. In a regression we make assumptions about the error term — for example, that it is uncorrelated and homoskedastic. In the same way many DSGE models have shocks, u_t, that are *assumed* to follow a first-order autoregressive process of the form $u_t = \rho u_{t-1} + \varepsilon_t$. In time-series analysis the ε_t are called innovations as they are assumed to be unpredictable using the past history of u_t, i.e. the ε_t are uncorrelated with ε_{t-j} $(j > 0)$. Usually there are a number of shocks in a DGSE model, each of which is driven by innovations, and it is assumed that they are uncorrelated with each other. We will label these shocks in the structural model as *Assumed Model (AM) shocks*. In the case of DSGE models the names of the shocks come from theory, and so they are often said to be "theoretical shocks".

There are two reasons why assumptions are made about the nature of the structural shocks in DSGE models. One is to perform estimation of the model's parameters, exactly like a regression. But there is a second reason which relates to *interpretation*. Assuming that the innovations driving the structural shocks are *uncorrelated* with each other allows one innovation to be changed and thereby for the effects of that perturbation to be traced out by holding constant the innovations in other shocks, i.e. it produces a *ceteris paribus* assumption. If instead these innovations to the structural shocks are correlated then the one being perturbed will have an effect on the other shocks, making interpretation of the impulse response functions problematic. Indeed, Ramey (2016) says that one of the characteristics a shock should have is to be " ...uncorrelated with other exogenous shocks; otherwise we cannot identify the unique causal effects of one exogenous shock relative to another..." (p. 75).

A model can be used together with data to produce the *Estimated Model (EM) shocks*. In a regression they are called residuals. The residuals need not satisfy the properties investigators *assumed* about the error term of the regression. As is well known such a failure can have consequences for the subsequent inferences drawn from the regression, and so many procedures have been developed for detecting these failures. Exactly the same situation should occur in structural macroeconomic models, such as DSGEs and SVARs. In a regression it is straightforward; one uses the residuals to check the assumptions made about the errors. But in the case of structural models with excess shocks it is not so simple, and we detail why in the paper. Yet it needs to be done.

To illustrate these issues in this introduction we consider the recent work of Farmer and Nicolò (2021) (FN). We return to a more detailed analysis of it later in the paper. FN estimate a model developed in Farmer and Nicolò (2018) for several countries. They assess its fit relative to some other popular models, as well as commenting on the role of shocks in explaining cross-country divergences in economic performance. We will focus on their estimates for the U.S. The FN model is a New Keynesian (NK) model with inflation, output and an interest rate driven by supply, demand and monetary shocks. But it lacks a Phillips curve, and instead the system is closed with beliefs about nominal GDP growth. That can lead to static and dynamic indeterminacies. As a consequence there will be a sunspot solution which can be represented as relating to the three "fundamental" shocks above and a fourth shock, which we will refer to as animal spirits. Consequently, there are three observed variables and four shocks in this model, resulting in excess shocks. As one of the conclusions of FN is that differences in shocks (and the responses of central banks to them) explain divergences in economic performance, one would like to check if these can be recovered in order to assess their properties.

A first complication in checking the properties of the *AM* shocks arises from the fact that with excess shocks the *EM shock innovations must be correlated*, even when this is not the case for *AM* shock innovations. This means that one cannot simply infer properties of the *AM* shock innovations from the *EM* shocks, as one would do with the residuals from a regression.

A second complication is that the commonly used interpretation procedures of variance and variable decompositions use the *EM* shocks. In the presence of excess shocks the *EM* shock innovations *are always correlated* and these decompositions do not apply. For example, in FN's work the correlation between the *estimated* supply and monetary shock innovations is .34; this is despite the assumption that the fundamental *AM* shock innovations are maintained to be uncorrelated. Thus one could not use a variance decomposition to find which shock is particularly important in driving the business cycle.

We are not the first to draw attention to the fact that there are consequences of having excess shocks. Ravenna (2007, p. 2051), in the context of understanding the relationship between DSGE and SVARs, stated "...it will not be possible to map y_t into a higher-dimension vector of orthogonal shocks". Forni et al. (2019, p. 226) point out that their recovered (estimated) shock innovations would be "linearly dependent", i.e. they would be correlated, even if the *AM* shock innovations are not. Plagborg-Moller and Wolf (2020) and Chahrour and Jurado (2022) show that excess shocks will mean that not all of the *AM* shocks are recoverable. Indeed, Chahrour and Jurado (2022, p.8) say "...a necessary condition for all the structural shocks to be recoverable is that there be at least as many observable variables as disturbances...". So the consensus of the existing literature is that the *EM* and *AM* shock innovations are different when there is an excess of shocks, and this needs to be accounted for.

In this paper we add to the existing literature by developing an approach to determine the consequences of there being excess shocks. It is intended to help applied researchers know along what dimensions any of their models which feature excess shocks can be used to interpret the macroeconomy, and when they cannot. This is done in three stages. First, we ask which of the *AM* shock innovations can be recovered from the data, that is, from the *EM* shock innovations. This simply involves asking whether the *AM* shock innovations are the same as the *EM* ones. In order to avoid secondary issues such as sample size we take the parameters of

[1] Version 3.1.14. The Macroeconomic Model Database is an outcome of the Macroeconomic Model Comparison Initiative. In turn this is a product of the Hoover Institution at Stanford University and the Institute for Monetary and Financial Stability at the Goethe University Frankfurt. For further information see https://www.macromodelbase.com/. When multiple variants of an individual model are included we use only one to avoid double counting.

[2] See Christiano et al. (2014), Chen et al. (2012) and Ireland (2011).

the assumed model as known. In a regression this would make the assumed error and the residuals the same. Applying our approach to the FN model we find that the supply and sunspot shock innovations cannot be recovered.

Second, for those *AM* shock innovations that can be recovered we ask whether they exhibit the assumptions made about them by looking at their estimated counterparts. In the FN model it was assumed that monetary policy shock innovations had zero serial correlation. As these can be recovered from the EM shock innovations the properties of that AM shock's innovations can be assessed directly; they are found to be heavily serially correlated and so are not truly innovations.[3]

Third, we provide a way of assessing the properties of those shocks that cannot be recovered, utilizing indirect inference. It proceeds by determining the correlation in the *EM* shock innovations that the presence of excess shocks induces when there is none in the *AM* shock innovations. This is then compared to the actual correlations in the *EM* shock innovations. These should not differ if the assumptions made about the model shocks were valid.

This article is structured as follows. Section 2 makes the basic point that, when the number of shocks equals the number of observables, one can generally treat the *EM* shocks from the model as the *AM* shocks, i.e. the latter are recoverable.[4] Section 3 then turns to determining whether it is possible to recover *some* of the *AM* shock innovations where there is an excess of them. The existing methods for assessing this are discussed, which highlights that different types of information can be used for recovery; Forni et al. (2019) use current and past information, which aligns with the traditional concept of invertibility, whereas Chahrour and Jurado (2022) additionally use future information.

An intuitive way to assess if an assumed shock innovation can be recovered is to ask if the variance (P) of the divergence between it and its estimated counterpart is zero. We show in Sections 3.1 and 3.2 that P can be computed using the steady-state Kalman filter and smoother. There will be two Ps, depending on which of the two information sets mentioned above are used.[5] Our approach is easy to implement and unifies the existing literature.

The Kalman filter is also useful for understanding why there is a divergence between *AM* and *EM* shock innovations when there is an excess. The filter enables the computation of an estimate of the one-step ahead prediction errors of the observed variables, η_t. When there are no excess shocks the number of η_t is the same as the number of assumed shocks, but with excess shocks there are less, and therefore knowledge of η_t alone cannot be used to recover all the *AM* shock innovations.[6]

In Section 3.3 we demonstrate our state-space approach with a simple one variable and two shock UC model. Such models are often used to generate filters to recover quantities that are functions of the shocks, such as an output gap, a credit gap or a NAIRU.[7] The properties of the estimated and assumed gaps may differ considerably. This is known in the literature; we demonstrate that it arises due to excess shocks. Section 3.4 looks at the problems that correlated innovations create for impulse response computations. Generally, there is no unique response.

Our approach for assessing the properties of the non-recoverable *AM* shocks uses indirect inference and is presented in Section 3.5. We then consider in Section 3.5.1 an influential model where the source of excess shocks is the introduction of a sunspot due to dynamic indeterminacy, namely Lubik and Schorfheide (LS) (2004). It is found that the shock termed a sunspot cannot be recovered. Applying our indirect inference approach points to the assumed sunspot innovation being significantly correlated with the assumed monetary and supply shock innovations. This is followed in Section 3.5.2 by further analysis of Farmer and Nicolò (2021), which we have used throughout the introduction.

Section 3.6 provides a demonstration of other ways in which excess shocks can impact on the ability to interpret the macroeconomy. Two examples are used; one is the risk paper of Christiano et al. (2014), where excess shocks come from "news", and the other a simple TVP model. It is found in the former that the variance of interest rate spreads in the data is almost a third of what their model predicts. This is due to the fact that the EM shock innovations are correlated. Variance decompositions about the importance of risk shocks therefore are not reliable.

Section 4 closely examines a New-Keynesian model, Ireland (2011). We use it to demonstrate our method and some strategies that applied researchers can use to deal with excess shocks. This model was chosen as it uses standard techniques that are representative of much of the DSGE literature. It has four *AM* shocks but only three observed variables, and therefore has excess shocks. There are no sunspots or news shocks, as was the case in our earlier examples. We find that two of the four *AM* shock innovations are close to being recoverable. Consequently, assuming that they are, we can assess the validity of the assumptions made about them. Ultimately, these shock innovations are found to be uncorrelated but autocorrelated, so the ability to compute impulse responses is to some extent limited. The section also deals with some issues raised by referees concerning factor models and the assumption we made that parameters were known when, in practice, they must often be estimated.

In summary, the *AM* structural shock innovations provide information about the nature of the model but they do not interpret the data unless they equal the *EM* shock innovations, and this cannot be the case for all of them when there are excess shocks. We believe that it is important for applied researchers to be cognizant of the resulting limitations of their models when making interpretations in this context. In particular, excess shocks create problems in two ways. First, the correlation they induce in the *EM*

[3] There is a semantic issue here. We should really refer to "the estimate of the random variable assumed to be an innovation", rather than "EM shock innovations", as the latter may not be innovation at all, i.e. it may have serial correlation.

[4] Of course it has to be that the model is not mis-specified and there are no identification issues.

[5] In order to perform these operations the model must be placed into a State-Space Form (SSF), which is very common for DSGE and UC models. For the FN model illustration we used smoothed shocks, which is typical for DSGE models.

[6] This is distinct from parameter estimation. Using the Kalman filter, the likelihood for the model is expressed in terms of η_t alone, and consequently as long as the parameters are identified, their estimation is not an issue.

[7] More complex UC models, including the Hodrick–Prescott filter, are assessed in an appendix. The conclusions are the same.

shock innovations makes it difficult to examine the assumptions made about the *AM* shock innovations. This paper addresses how that difficulty can be overcome. We provide an approach by which the properties of the *AM* shock innovations can be assessed for a wide variety of models, either directly and indirectly. Second, we demonstrate that the correlation in the *EM* shock innovations limits the usefulness of variance and variable decompositions.

2. Recovering the *AM* shocks when there is no excess

For simplicity, in order to consider the nature of the shocks when there is no excess, let us first assume that the solution for a model is a SVAR in n observable variables y_t and the n AM shock innovations ε_t

$$A_0 y_t = A_1 y_{t-1} + \varepsilon_t,$$

where A_0 and A_1 are matrices. We assume that these matrices are known. The assumption that the parameters are known means that the filtered EM innovations, $E_t\varepsilon_t \equiv \mathbb{E}(\varepsilon_t|y_1...y_t)$ are $E_t\varepsilon_t = A_0 y_t - A_1 y_{t-1}$, and these *coincide with the realization of the AM shock innovations* ε_t. This enables us to use the EM innovations to test the assumptions made about the AM shock innovations. If instead the parameters were not known, then these two sets of shocks would not necessarily coincide as, for example, the model may be mis-specified. Throughout this paper we work with known parameters so as to focus upon the impacts of excess shocks alone.

We further assume that A_0^{-1} exists so that the Moving-Average (MA) representation is

$$y_t = C_0\varepsilon_t + C_1\varepsilon_{t-1} + \cdots \equiv C(L)\varepsilon_t. \tag{1}$$

There is an underlying VAR process for y_t of the form

$$y_t = B_1 y_{t-1} + e_t,$$

and $C_0 \equiv A_0^{-1}, C_j \equiv B_1 C_{j-1}$ $(j > 1)$.[8] Suppose the *AM* shock innovations ε_t are assumed to be uncorrelated with covariance matrix I_n. Then C_0^{ij} is the contemporaneous impulse response of the i'th variable to the j'th *AM* shock.

Whether the AM shock innovations are uncorrelated is an assumption which we wish to evaluate. If the parameters of the SVAR were known there are no excess shocks and the EM shock innovations will be the AM shock innovations. In practice, however the parameters are estimated and, as the SVAR is exactly identified, the *EM* shock innovations will be uncorrelated by construction, regardless of the nature of the AM shock innovations. This is just like a regression where the residuals must be orthogonal to the covariates. Because of this variance decompositions can be done with the EM shocks.

Most DSGE models can be represented as a SVAR, in which case A_0 and A_1 are functions of the deep parameters of the DSGE model. However, the SVAR implied by a DSGE model is rarely exactly identified; see Liu et al. (2018). One example of a source of overidentifying restrictions in DSGE models is the common assumption of rational expectations. Thus the EM shock innovations from DSGE models do not have zero correlation after their parameters are estimated and so, *when there are no excess shocks*, assumptions made about the AM shocks can be potentially checked using the EM shocks.

As mentioned in the introduction, our analysis distinguishes between two types of *EM* shocks. These differ in the information sets used for their conditioning. If, as above, information up to time t is used, then we would have *filtered* estimates $E_t\varepsilon_t \equiv \mathbb{E}(\varepsilon_t|y_1...y_t)$, while the *smoothed* estimates are $E_T\varepsilon_t \equiv \mathbb{E}(\varepsilon_t|y_1...y_T)$, and use all the data (i.e. including future observations). If there are no excess shocks $E_t\varepsilon_t = A_0 y_t - A_1 y_{t-1} = E_T\varepsilon_t$, i.e. the AM and EM shocks are the same.

3. Recovering the *AM* shocks if there is an excess

As not all models used in macroeconometrics have a SVAR structure, we need to allow for these more general cases. This will be done with a State Space Form (SSF). The SSF we adopt follows Nimark (2015) and is:

$$z_t = D_1\psi_t + D_2\psi_{t-1} + R\varepsilon_t \tag{2}$$

$$\psi_t = M\psi_{t-1} + C\varepsilon_t. \tag{3}$$

Here Eqs. (2) and (3) are the observation and state equations respectively, z_t are the $n\times 1$ observed variables, ψ_t the $p\times 1$ core model variables, and ε_t the $m\times 1$ vector of shock innovations. The latter are assumed to be $N(0, I_m)$. By "core" model variables we mean those that cannot be substituted out. DSGE models, for example, often have variables that can be substituted out using identities included in the model. It must be that $p \geq n$ and with excess shocks $m > n$. Also, to reiterate, frequently in DSGE models the shocks are assumed autocorrelated; we are therefore focusing on their innovations and to study them we will assume they are placed as the last elements in ψ_t.[9] In most cases $p > m$. D_1, D_2, R and C are matrices of parameters.

We now turn to the case where there may be excess shocks. In many instances macroeconometric models will include variables that are not observed; some are latent. One example of this occurring is when auxiliary shocks like measurement error are added; each observed variable will have an unobserved counterpart. Another example is when the DSGE includes stock variables that are

[8] The reduced-form e_t are sometimes referred to as innovations in the literature. They are combinations of the structural shock innovations ε_t.

[9] This will generally mean that $R = 0$.

treated as unobserved, such as the capital stock. These are easily handled by using the SSF. Assessing shock recoverability through the SSF is also useful as it readily accommodates $I(1)$ variables.

If the parameters are known we can clearly find the impulse responses to the AM shock innovations ε_t. This is mostly what researchers do. But for these responses to be relevant it must be the case that the assumptions about ε_t being uncorrelated (and also not serially correlated) are correct, and that is why we want to check these assumptions. However, the only innovations available to perform an assessment are those recovered from the data – the EM ones – and so, ideally, we will want those to align with the AM shock innovations, as was the case in the previous section.

3.1. Tools of recovery — the Kalman filter and smoother

The Kalman filter can be applied to the SSF with known parameters to obtain the filtered EM shock innovations.[10] Nimark (2015) shows that the filtered estimate of ψ_t, $E_t\psi_t$, given the data available to time t, and the system in Eqs. (2) and (3), evolves as

$$
\begin{aligned}
E_t\psi_t &= \Phi_t E_{t-1}\psi_{t-1} + K_t z_t &(4)\\
K_t &= [MP_{t-1|t-1}\Psi' + CC'D_1' + CR']F_t^{-1} &(5)\\
F_t &= \Psi P_{t-1|t-1}\Psi' + \Lambda\Lambda' &(6)\\
P_{t|t} &= P_{t|t-1} - K_t[\Psi P_{t-1|t-1}\Psi' + \Lambda\Lambda']K_t' &(7)\\
P_{t+1|t} &= MP_{t|t}M' + CC' &(8)\\
\Psi &\equiv D_1 M + D_2, \Lambda \equiv D_1 C + R, \Phi_t \equiv M - K_t\Psi, &(9)
\end{aligned}
$$

where K_t is the gain of the Kalman filter, $P_{t|t}$ the filtered variance of $E_t\psi_t - \psi_t$ and $P_{t+1|t}$ provides the one-step-ahead predictor of the latter. As we observed above, to recover the filtered shock innovations the AM shock innovations are made elements of ψ_t. In some applications, this is naturally the case; in others, such as DSGE models, they must be added to the endogenous state variables. This can be done by simply appending identities to the state equations (Eq. (2)), and padding the matrices D_1 and D_2 in the observation equation, Eq. (1), with appropriate zeros, so as to be conformable with the expanded ψ_t.

What happens if we try to recover the AM shocks using all the information? Kurz (2018, Equation 4.11) shows that the smoothed states, $\psi_{t|T}$, are obtained from the recursion

$$
\begin{aligned}
E_T\psi_t &= \psi_{t|T} = \psi_{t|t} + P_{t|t}\tau_t, &(10)\\
\tau_t &= \Psi' F_{t+1}^{-1}\eta_{t+1} + L_{t+1}'\tau_{t+1}, &(11)\\
L_t &\equiv M - K_t\Psi, &(12)
\end{aligned}
$$

where η_{t+1} is the one-step ahead prediction error of z_{t+1} and $\tau_T = 0$.

To capture the closeness of the AM innovations ε_t with the smoothed EM innovations $E_T\varepsilon_t$, we naturally use the variance of their difference. These shocks are a sub-set of the ψ_t so we denote the part of $P_{t|T}$ corresponding to them as $P^*_{t|T}$. Kurz (Equation 4.13) gives the following expression for computing $P_{t|T}$

$$
\begin{aligned}
P_{t|T} &= P_{t|t} - P_{t|t}N_t P_{t|t}\\
N_t &= \Psi' F_{t+1}^{-1}\Psi + L_{t+1}'N_{t+1}L_{t+1}, N_T = 0.
\end{aligned}
$$

Eqs. (10) and (11) can be used to investigate the nature of the smoothed shock innovations. Firstly, if $P^*_{t|t} = 0$ then we can recover the AM shock innovations from the filtered shocks. Indeed, we see that if $P^*_{t|t} = 0$, Eq. (10) shows there will be no difference between filtered and smoothed shocks and they will both have whatever properties the AM shock innovations have been assigned. Alternatively, if $P^*_{t|t} \neq 0$, then at $t = T$ the smoothed and filtered innovations are the same but, when $t < T$, they will differ. Moreover the difference depends on τ_t and, even if the one step predictions (η_t) are white noise, the fact that the τ_t are accumulated means there will likely be serial correlation in the smoothed shock innovations. Thus the $E_T\varepsilon_t$ will inherit some of the properties that $E_t\varepsilon_t$ has, but may also have more serial correlation.[11]

[10] This variant of the Kalman filter, due to Nimark (2015), allows for a lag of the state in the measurement equations. This is useful for DSGE models which include permanent shocks. In that case typically the observed non-stationary variables will be expressed as growth rates while the model variables will have been normalized by another $I(1)$ variable to make them $I(0)$, i.e. they have been stationized. To give a concrete example, if the production function is $X_t = A_t N_t$, where X_t is output, A_t a permanent technology shock and N_t labor (hours), then the model will be expressed in terms of $I(0)$ variables by using stationized output, $\frac{X_t}{A_t}$. It is the log of this variable that will appear in the model as ψ_t, i.e. $\psi_t = x_t - a_t$ (lowercase denoting logs). These stationized model variables need to be related to observables, which are growth rates for the $I(1)$ variables, i.e. $z_t = \Delta x_t$. Hence $\Delta x_t = \Delta\psi_t + a_t$, and this adds a lagged state ψ_{t-1} to the observation equation. Alternatively, one could expand the state to include a lag and use the standard Kalman filter recursions.

[11] Henceforth, for brevity we will describe the smoothed EM shocks as correlated, rather than contemporaneously and serially correlated.

3.2. Can we recover the AM shocks?

The Kalman filter produces filtered *EM* shock innovations while the Kalman smoother will give the smoothed ones, $E_T\varepsilon_t$. Chahrour and Jurado (2022) give an excellent discussion of the history of the literature on whether AM shock innovations can be recovered. They note that early work on this looked at whether the EM shocks could perfectly predict the AM shocks with the information used being just current and past. If this could be done then it was said that the model was *invertible and the shocks were fundamental*.[12] Chahrour and Jurado (2022) then suggest that, for example, when expectations are involved in a model one would probably want to *use all the data* rather than just current and past, i.e. to ask the same question but now using smoothed EM shock innovations. This gives rise to their definition of *recoverabilty*. It is clearly wider than that of invertibility as it uses more information.[13]

Definition 2 of recoverability in Chahrour and Jurado (2022) involves finding a smoothed estimate of ε_t, defined as the best linear estimate of ε_t given all the data. That is what $E_T\varepsilon_t$ from the Kalman smoother is. In the proof of Theorem 1 in their appendix they state the condition for recoverability as $\|\varepsilon_t - E_T\varepsilon_t\|^2$ equaling zero. The Kalman smoother provides $P^*_{t|T} = var(\varepsilon_t - E_T\varepsilon_t)$, so if $P^*_{t|T} = 0$, $E_T\varepsilon_t$ satisfies the condition of Chahrour and Jurado's definition of recoverability. We note that they explain that the condition involves "....the filtered variance of the disturbance of interest is zero in population". So they are essentially working with $P^*_{t|T}$.

Rather than using the Kalman smoother estimate of $P^*_{t|T}$, Chahrour and Jurado (2022) express $\|\varepsilon_t - E_T\varepsilon_t\|^2$ as functions of the coefficients of the moving average (MA) representation and obtain a condition on the MA coefficients that they then test. Directly computing $P^*_{t|T}$ from the SSF and checking if it is zero is a straightforward alternative way of capturing this variance.[14] Because we will have purely stationary variables in the SSF after any stationizing transformations (which is why Chahrour and Jurado can work with a MA representation), it is known that there is a steady-state Kalman filter, and so we will ask whether the *steady-state counterpart* of $P^*_{t|T}$ is zero.[15]

As an illustration of this approach to assessing whether an *AM* shock is recoverable, consider the three equation model in Chahrour and Jurado (2022). They show that only the third shock is recoverable, and this is only when future information is used. Computing the diagonal of the steady-state $P^*_{t|T}$, when their example is placed in a SSF, produces $\text{diag}(P^*_{t|T}) = \begin{bmatrix} .0243 \\ .9757 \\ 0 \end{bmatrix}$. Consequently our approach also shows that only the third shock is recoverable. This is what we would expect given the equivalence of the steady-state $P_{t|T}$ with what they focus on.

Returning to working with filtered *EM* shocks, it will be the *covariance matrix of the variable* $\psi_t - E_t\psi_t$, i.e. the relevant elements of $P_{t|t}$ being zero, that shows recoverability of AM shock innovations from filtered EM shock innovations. Often this is called "invertibility"; it is possible for some shocks to be invertible (or fundamental) when the model is not invertible, but not all of them. Forni et al. (2019) described the situation when there is sufficient information in a SVAR to recover a particular structural shock as one involving partial fundamentalness, and developed a measure of informational deficiency (δ_i) to check for this. It follows Sims and Zha (2006) and looks at the unexplained variance of the best linear projection of the shock of interest onto the residuals (in population). That quantity is the variance of $\varepsilon_t - E_t(\varepsilon_t)$ from the Kalman filter. Their deficiency index δ_i has to be zero for the i'th shock if the i'th AM shock innovation is to be recovered when using only current and past data. In the same way the (i,i) element of $P^*_{t|t}$ in our approach will have to be zero. As an illustration of this we look at Forni et al.'s Example 1. This is a two equation system in an output gap (z_{1t}), an interest rate (z_{2t}), demand shocks (ε_{1t}), and monetary shocks (ε_{2t}) of the form

$$\begin{aligned} (1+\gamma\beta L)z_{1t} &= (1+\gamma L)\varepsilon_{1t} - \beta\varepsilon_{2t} \\ (1+\gamma\beta L)z_{2t} &= \gamma(1+\alpha L)\varepsilon_{1t} + \varepsilon_{2t}. \end{aligned}$$

Using their parameter values of $\alpha = 3,\ \gamma = .4$, and $\beta = 1$ we find that $\text{diag}(P^*_{t|t}) = \begin{bmatrix} .8889 \\ 0 \end{bmatrix}$, which is identical to what they give for δ_i in their Table 3. So using $P^*_{t|t}$ is a simple way of computing their deficiency index δ_i.[16] It is noteworthy that, for this example, only the second shock would be recoverable and so not all shocks may be recoverable, even where there is not an excess of them. But having an excess of shocks does mean that not all shocks can be recovered.

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[12] Closely related is the concept of informational sufficiency (Forni and Gambetti, 2014); see also Forni et al. (2019). Forni and Gambetti (2014) developed a test for informational sufficiency in a SVAR using Granger causality and principal components; a related approach for testing for non-fundamentalness was developed by Canova and Sahneh (2018).

[13] Chahrour and Jurado (2022) provide a discussion of the differences between invertibility and recoverability.

[14] There may be useful information in the MA representation that is not in $P_{t|T}$ about specification issues so there is an argument for computing that as well as $P_{t|T}$.

[15] A referee commented that a non-invertible MA(1) would mean shocks are not recoverable and then this might be taken as "evidence of excess shocks when none exists". The check on recoverability is *not a test for the existence of excess shocks.* One knows if there are excess shocks simply by counting the number of observable variables and shocks.

[16] Note that in the SSF we compute $P^*_{t|t}$ from the shocks ε_t having unit variance, just as in Forni et al. (2019). The answers they obtain depend on how many simulations they do and the .8889 they report comes from 1000 simulations. It might be mentioned that the computed δ_i could be used to find the covariance of $E_T(\varepsilon_t)$ with ε_t. This may be a more informative way of capturing the magnitude of the discrepancy between the AM and EM shock innovations.

To reiterate, in our approach there two different measures of P. $P_{t|t}$, obtained from the Kalman filter, assesses whether shocks are recoverable in the sense of invertibility, namely using past and current information. It allows us to assess if a particular shock is fundamental, even if the model is not invertible. The second measure, $P_{t|T}$, instead relates to the wider concept of recoverability in Chahrour and Jurado (2022); it additionally uses future information and therefore can be computed from the Kalman smoother.

Which definition of recoverability would one use? A referee asked this and proposed that we look at a non-invertible MA(1) process of the form

$$z_t = \varepsilon_t + \theta\varepsilon_{t-1},\ \theta > 1, \tag{13}$$

saying that ε_t is not recoverable. This can be placed in our SSF, Eqs. (2) and (3), by defining $\zeta_t = \varepsilon_t$, $D_1 = 1, D_2 = \theta$ and $C = 1$. With a value for $\theta = 3$, $P^*_{t|t} = .8889$, so one cannot recover ε_t with current and past information. However $P^*_{t|T} = 0$, meaning that the shock is recoverable using future information. This was shown by Amisano and Giannini (1997). As they note, because the first order autocorrelation of z_t is $\frac{\theta}{1+\theta^2}$, for any given sample estimate there are two solutions for θ, and so there two observationally equivalent MA(1) processes for z_t, namely Eq. (13) and

$$z_t = \omega_t + \frac{1}{\theta}\omega_{t-1}. \tag{14}$$

So the question is which shock would be recovered in practice if a $MA(1)$ is fitted to the data on z_t? When the MA parameter is estimated the likelihood has two maxima, and one can get either of θ or $\frac{1}{\theta}$, depending on starting values. Many existing computer programs however *impose* invertibility constraints, so using them one finds $\frac{1}{\theta}$. As Amisano and Giannini (1997) point out, while the two MA models are observationally equivalent their impulse responses are different and this is due to the different shocks characterizing them.

Another example of non-invertibility given to us by a referee was that in Leeper et al. (2013), which comes from a DSGE model with tax rates (τ_t) and the capital stock (k_t). The model is solved under an assumption that agents receive news $\varepsilon_{\tau,t}$ that tells them the tax rate two periods ahead, and results in the following two equations

$$\tau_t = \varepsilon_{\tau,t-2} \tag{15}$$

$$k_t = \alpha k_{t-1} + \varepsilon_{A,t} - \kappa(\varepsilon_{\tau,t-1} + \theta\varepsilon_{\tau,t}). \tag{16}$$

If all one has is k_t then there are excess shocks and both AM shocks cannot be recovered. What happens if τ_t is observed as well? Can both AM shocks be recovered then? Leeper et al. (2013) showed that AM shock recovery was not possible using filtered EM shocks, due to a lack of invertibility. Indeed using parameter values of $\alpha = .3, \beta = .99, \tau = .4, \kappa = .77$; and $\theta = \alpha\beta(1-\tau)$ we find that when τ_t is observed $P^*_{t|t}$ is not zero. However, when future information is used, $P^*_{t|T} = 0$. So, in this case one would want to use smoothed shocks to recover the AM shocks; using it also gave a correct answer for the non-invertible MA discussed earlier.[17] In the context of DSGE models, while it will depend on the research question, it seems likely that in most instances using the smoothed shocks will be the appropriate choice. Indeed, in the widely used program Dynare it is the smoothed, rather than filtered, shocks that are available as output.

3.3. Example: A simple unobserved-components model

To illustrate our approach of using the SSF to assess recoverability more explicitly, and the consequences of having excess shocks, it is useful to consider a simple UC model:

$$\begin{aligned} y_t &= y_{1t} + y_{2t} \\ y_{1t} &= \rho y_{1t-1} + \varepsilon_{1t} \\ y_{2t} &= \varepsilon_{2t}, \end{aligned}$$

where the innovations ε_{1t} and ε_{2t} are assumed to be $n.i.d.(0, I_2)$. When the second variable y_{2t} is not observed we will have excess shocks. With ρ known the model can be written as[18]

$$z_t = (1-\rho L)y_t = \varepsilon_{1t} + \varepsilon_{2t} - \rho\varepsilon_{2t-1}. \tag{17}$$

To find filtered *EM* innovations $E_t\varepsilon_{1t}$ and $E_t\varepsilon_{2t}$ from the one observed variable z_t we express the model in an SSF. Therefore ψ_t equals $\begin{bmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\end{bmatrix}$ and the matrices involved in the Kalman filter are $M = 0, D_1 = \begin{bmatrix}1 & 1\end{bmatrix}, D_2 = \begin{bmatrix}0 & -\rho\end{bmatrix}, C = I$, and $R = 0$.

Using Eq. (4), the recovered filtered innovations evolve as

$$E_t\psi_t = -K_t D_2 E_{t-1}\psi_{t-1} + K_t z_t.$$

[17] The referee also commented on this example as showing that conditioning on more variables will lead to "severe problems". But the "severe problems" do not come from having more observed variables. It is the *properties of the model* which came from the use of the tax equation, Eq. (15), that Leeper et al. found unacceptable. Those properties do not change with the number of observed variables. All that extra observables do is to expand our ability to recover the shocks. One has to be content with the model before one would ask if the shocks are recoverable.

[18] The process for y_t is an ARMA(1,1) of the form $(1-\rho L)y_t = e_t + \alpha e_{t-1}$, so three parameters can be estimated — the variance of e_t, ρ and α. These can be used to estimate ρ and the variances of the two shocks when they are not known. Consequently the model is identified.

Eq. (5) then gives the Kalman filter gain as

$$K_t = D_1'[D_2 P_{t-1|t-1} D_2' + D_1 D_1']^{-1},$$

and

$$(D_2 P_{t-1|t-1} D_2' + D_1 D_1')^{-1} = (\rho^2 \sigma^2 P_{22,t-1|t-1} + 1 + \sigma^2)^{-1}.$$

So this is a scalar, c_t, and it means that $K_t = \begin{pmatrix} c_t \\ c_t \end{pmatrix}$. Consequently, $\Phi = -K_t D_2 = \begin{bmatrix} 0 & c_t \rho \\ 0 & c_t \rho \end{bmatrix}$, and $E_t \psi_t$ evolves as

$$E_t \psi_{1t} = c_t \rho E_{t-1} \psi_{2t-1} + c_t z_t \quad (18)$$

$$E_t \psi_{2t} = c_t \rho E_{t-1} \psi_{2t-1} + c_t z_t, \quad (19)$$

giving $E_t \psi_{1t} = E_t \psi_{2t}$. This implies that the *EM* shocks – the recovered filtered innovations $E_t \varepsilon_{1t}$ and $E_t \varepsilon_{2t}$ – are *the same*, unlike the *AM* innovations. The presence of excess shocks means we cannot recover all the *AM* shocks from the *EM* shocks.[19]

Nelson (1975) and McDonald and Darroch (1983) pointed out that the one-step prediction errors η_t will generally be a combination of *all* $\{\varepsilon_{k,t-j}\}_{k=1,2}$. This implies that both of these shock innovations cannot be recovered. As Eq. (19) shows, the filtered shock $E_t \psi_{2t}$ will be a function of all z_{t-j} and so depends upon the histories of the *AM* innovations ε_{1t} and ε_{2t}. Canova and Ferroni (2022) make this same point about the *EM* shocks, but in the context of a DSGE model whose solution is a Vector AutoRegression Moving-Average (VARMA) structure, referring to this as "deformation", and saying (p.2) " Deformation makes identified shocks mongrels with little economic interpretation for two reasons. Identified shocks are unlikely to combine structural disturbances of the same type...the shocks one can identify will be, in general, linear combinations of current and *past* structural disturbances".[20]

Is it possible to recover one of the *AM* shocks with just current and past data? As we have argued, the answer to this resides in the variance $P_{t|t}$ computed by the Kalman filter. To parameterize the model, let $\rho = .9$. In the limit, using the steady-state Kalman filter, the steady-state value for $P_{t|t}$ is $P = \begin{bmatrix} .5974 & -.4026 \\ -.4026 & .5974 \end{bmatrix}$. The variance of $E_t \psi_{jt} - \psi_{jt}$ is P_{jj}; if it was zero then we could recover the *AM* shocks from the *EM* ones, but this is not the case.

What other differences might we see in the *EM* shocks that are not in the assumed ones? One is that there can be serial correlation in the *EM* "innovations". This is because $\Phi_t = -K_t D_2$ and it may not be zero.[21] When $\rho = .9$ we find the steady-state value of Φ_t is $\Phi = \begin{bmatrix} 0 & .36 \\ 0 & .36 \end{bmatrix}$, so the *EM* innovations follow a VAR process, in contrast to the *AM* innovations which are assumed to have no serial correlation. It is the inability to separate the innovations $\varepsilon_{1t|t}$ and $\varepsilon_{2t|t}$ when there is only one observed variable which results in serial correlation in the recovered shock innovations.

The reason for these different properties can be seen by looking at the one-step prediction error for z_t, η_t. From Kurz (2018), this is a linear combination of all the shocks ε_{t-j}. Now the log likelihood depends directly on η_t so that, once we know $\eta_1, \ldots, \eta_T$, we know the likelihood. Because η_t depends in a linear form on $\{\varepsilon_k\}_{k=1}^t$, when there is an excess of model shocks, i.e. more ε_t than observables y_t, we would need to recover $E_t \varepsilon_t$ with a g-inverse.

3.4. Implications of correlated innovations for impulse responses

To us one motivation and appeal of SVAR and DSGE models is that one works with truly exogenous variables; that is what the shocks are designed to be. This is done so as to remove a problem encountered with regressions in macroeconomics, namely that regressors can rarely be assumed uncorrelated due to simultaneity. But what can be done if a researcher finds their *EM* shock innovations to be correlated? One possible approach is to "orthogonalize" them. Apart from the issue of what economic label one would attach to such shocks, it needs to be understood that this strategy does not deliver a unique set of impulse responses. Consider Eq. (3), the state equation, written as

$$\begin{aligned} \psi_t &= M\psi_{t-1} + CRR'\varepsilon_t \\ &= M\psi_{t-1} + CRv_t, \end{aligned}$$

where $Rcov(\varepsilon_t)R' = I$, and R is an orthonormal matrix, i.e. $R'R = I$. A Cholesky factorization of the variance–covariance matrix of ε_t works by choosing one such R, and this delivers new uncorrelated shock innovations v_t whose impulse responses are different to those for ε_t, but with the same fit to the data. To be clear, the Cholesky factorization here is being applied to the innovations of the *EM* structural shocks, not the reduced-form innovations, in order to produce a new set of shocks that are orthogonal by construction. This is distinct from applying the Cholesky factorization to the reduced-form innovations, as is sometimes done to identify a SVAR. The impulse responses to these new shocks are CR, so they have changed. However, because the orthonormal matrix is not unique, this is also true of the impulse responses — there is a *set of responses*.[22] Effectively we obtain a set of impulse

[19] In Appendix A we provide an examination of some UC trend-cycle decompositions, including that used to generate the Hodrick–Prescott (1997) filter.

[20] Identified shocks in Canova and Ferroni (2022) are what we term as *EM*, i.e. compatible with the data.

[21] This was also noted by Harvey (1992) for this type of model.

[22] This is different to the bounds in Plagborg-Moller and Wolf (2020) which arise from model parameters not being identified. Our identified set arises strictly due to correlation in shocks, since the model parameters are known. There are also many variance decompositions and not just one. The range of impulse responses that are found as R varies can be very large — see for example Liu et al. (2018).

responses as there are a range of models that are transformations of the DSGE that we started with, all of which fit the data equally well. This has come up in the sign restrictions literature and the range may be very large.

To analyze this further, suppose we had the moving-average representation of Eq. (1). Then C_j can be computed once C_0 and A_1 are known. If the *AM* innovations ε_t have no serial correlation and are uncorrelated with each other then we can recover ε_{jt} from $E_t\varepsilon_{jt}$ i.e. regressing z_{it} on $E_t\varepsilon_{jt}$ will correctly give C^0_{ij}. This is because all the other regressors in Eq. (1) will be uncorrelated with ε_{jt} and so can be omitted. This is not true if the assumptions made about the ε_t are incorrect. Suppose ε_{jt} is correlated with ε_{kt}. In this case omitting ε_{kt} from the regression will cause a bias in the estimate of C^0_{ij}. Of course it may be small if the correlation of the regressors is small. But it points to why we would need to check that. Cast in this way we consider the Leeper et al. (2013) model of Section 3.2. There were two observables and two shocks. The AM shocks could not be recovered using the filtered *EM* shocks, but could be using the smoothed EM shocks. Thus regressing z_{it} on $E_T\varepsilon_{jt}$ will correctly give C^0_{ij}.

3.5. Assessing the properties of the AM shocks that cannot be recovered with indirect inference

If the AM shocks are not recoverable can we still find some information about the correlation properties of the AM shock innovations using the EM shocks, as the latter are all we have? One is not seeking information about all observations on the AM shock innovations but about parameters of their density, as that is what the correlations depend on, and that is much easier to find. One approach to doing this is to conduct indirect inference.

Suppose we took the i'th smoothed EM shock innovation $\varepsilon_{it|T}$ and regress it against $\varepsilon_{jt|T}$ $(j \neq i)$ to get an estimated coefficient $\hat{\theta}$. The rationale for this is that if the *AM* shock innovations did have zero correlations then the regression of ε_{it} against ε_{jt} would give a zero coefficient (in large samples). However, because the EM shocks are not the AM shocks when there is no recoverability, we would expect that the $\hat{\theta}$ would not be zero, even in large samples. So we need to ask what the value of $\hat{\theta}$ would be in large samples when there is a valid zero correlation assumption among the AM shock innovations, and to then compare that with what is seen when the regression uses the EM shock innovations. This is an example of indirect inference.

To implement this approach, take the parameter values of a model, say a DSGE model, and simulate a large number of observations (N). Then compute EM shock innovations using the simulated data. Call these EMS, and use a superscript s to distinguish them. The regression will now be of $\varepsilon^s_{it|N}$ on $\varepsilon^s_{jt|T}$ to get θ^s. That value will be what we expect to get from using the EM shock innovations when the assumptions made about the AM shock innovations are correct. So testing if $\hat{\theta} = \theta^s$ is an indirect test of the assumptions made about the AM shock innovations. We use a HAC-adjusted t test to guard against any autocorrelation. Of course one can also use a similar approach to test if the innovations are really that, i.e. to test if they have no serial correlation.

To illustrate this we use the UC model of Section 3.3 and suppose the AM shock innovations are correlated with a correlation of ϕ. Because the variances of the innovations are unity we can write

$$\varepsilon_{1t} = \phi\varepsilon_{2t} + v_t,$$

with $var(v_t) = 1 - \phi^2$. Therefore

$$\begin{aligned} var(\varepsilon_{1t}) &= 1 = \phi^2 var(\varepsilon_{2t}) - \phi^2 \\ &= var(\varepsilon_{2t}). \end{aligned}$$

In order to simulate z_t when there is correlation among the *AM* shock innovations Eq. (17) will become

$$\begin{aligned} z_t &= (1-\rho L)y_t = \phi\varepsilon_{2t} + v_t + \varepsilon_{2t} - \rho\varepsilon_{2t-1} \\ &= (1+\phi)\varepsilon_{2t} + v_t - \rho\varepsilon_{2t-1}, \end{aligned}$$

where $var(v_t) = 1 - \phi^2$. Hence this provides a nested model for a direct test of $\phi = 0$.

We look at what happens for two cases, namely when $\{\phi = 0, \rho = .6\}$ and $\{\phi = .5, \rho = .6\}$. In the first case, when there is no correlation between the *AM* shock innovations, the regression of $\varepsilon_{1t|T}$ against $\varepsilon_{2|T}$ (10,000 observations) gives a coefficient of .81. When $\phi = .5$ this becomes .55. So the presence of correlation causes a *decrease* in the correlation of *EM* shock innovations from what we expect it to be when the *AM* shocks are uncorrelated.

To test if there is correlation between the AM shock innovations we use the EM shock innovations computed from the smaller set of data (200 observations) and get an estimated coefficient of .545. Comparing this to .81 we get a (HAC-adjusted) t ratio of −21.30, so there is strong evidence of correlation in the *AM* shock innovations, as there should be.

3.5.1. Example: Lubik and Schorfheide (2004)

Another example of excess shocks is indeterminacy in a DSGE model leading to sunspots. In an influential study (Lubik and Schorfheide, 2004) (LS) reported this when estimating a simple New-Keynesian (NK) model for the pre-Volcker period. Their NK model had three observable variables – deviations of output, inflation and interest rates from their steady states – and three basic ("fundamental") shocks ε_t - monetary (ε^R_t), demand (ε^d_t) and supply (ε^s_t). With indeterminacy there is a fourth; a sunspot shock (ε^{su}_t), and therefore excess shocks. To look at this case we need to solve the model under indeterminacy and we use the approach of Bianchi and Nicolo (2021).[23] The solution from that can be expressed in terms of our SSF

$$z_t = D_1\zeta_t \tag{20}$$

[23] Their Dynare program for the LS model is available from https://sites.google.com/view/francescobianchi/home/codes.

$$\zeta_t = M\zeta_{t-1} + C\varepsilon_t, \quad (21)$$

and we use the values for D_1, M and C coming from the parameter values in Bianchi and Nicolo's Dynare code.[24]

Can we recover the AM shock innovations? Arranging these as $\left(\varepsilon_t^s,\ \varepsilon_t^d,\ \varepsilon_t^R,\ \varepsilon_t^{su}\right)'$, it is found that

$$P^*_{t|T} = \begin{bmatrix} .00 & -.03 & .02 & -.01 \\ -.03 & .48 & -.11 & -.18 \\ 0.01 & -.12 & .03 & .06 \\ -.01 & -.18 & .06 & .49 \end{bmatrix},$$

so the supply and monetary policy shocks are recoverable, or seem nearly so. The demand shock and the sunspot are not.

A noteworthy aspect of LS is that they allow the innovations to the demand and supply AM shocks to be correlated; they estimated it to be .14. The solution to the model remains the same regardless of this correlation, i.e. D_1, M and C are invariant to any correlations between the ε_t. It is only in the computation of impulse responses to these shocks that any correlation has to be taken into account. They proceed to then orthogonalize the demand and supply shocks. As we have shown in Section 3.4 this does not produce unique impulse responses.

As the AM demand and sunspot shock innovations are not recoverable from their EM counterparts, in order to assess their nature we use our indirect inference approach. The matrices below show the correlations in the EM shocks (left-hand side), and those based on data simulated under the assumption of uncorrelated AM shocks (right-hand side). Pairwise testing suggests that the sunspot shock is significantly correlated with both the AM monetary and demand shocks (e.g. for the former the HAC-adjusted t ratio is -4.31).[25]

$$\underset{EM\text{ Shock Correlations in Data}}{\begin{bmatrix} 1 & .31 & .39 & -.29 \\ .31 & 1 & .34 & .35 \\ .39 & .34 & 1 & -.28 \\ -.29 & .35 & -.28 & 1 \end{bmatrix}}; \qquad \underset{EM\text{ Shock Correlations Assuming Uncorrelated }AM\text{ Shocks}}{\begin{bmatrix} 1 & .02 & -.05 & -.04 \\ .02 & 1 & .86 & .17 \\ -.05 & .86 & 1 & .33 \\ -.04 & -.17 & .33 & 1 \end{bmatrix}}.$$

3.5.2. Example: The Farmer and Nicolò (2021) beliefs-based model

Farmer and Nicolò (2018) set out a NK system like LS but instead of a Phillips curve being present there is a beliefs function about nominal GDP growth, x_t, and the expectations about it are assumed to be

$$E_t(x_{t+1}) = x_t. \quad (22)$$

In Farmer and Nicolò (2021) they estimated the model with data from three countries. Using parameter values for their model in Table 4A and 4B of their paper for the U.S. shows indeterminacy. The solution can be represented as having three fundamental shocks – monetary m, demand d, and supply s – as well as an animal spirits shock, a.[26] The demand and supply shocks are assumed to be AR(1) processes with innovations ε_t^d and ε_t^s, whereas the monetary and animal spirit shocks are white noise, and therefore equal their innovations ε_t^m and ε_t^a. There are three observables — inflation π_t, an interest rate R_t, and linearly de-trended log GDP y_t. All these variables are $I(1)$ in the model and there are two error correction terms $ec_{1t} = R_t - \pi_t$ and $ec_{2t} = (1-\lambda)\pi_t - \mu y_t$, where λ and μ are parameters in the interest rate rule.

The solved FN system can be placed in a SSF, as was done for LS, and this is used to check recovery of the shocks, yielding:

$$P^*_{t|T} = \begin{bmatrix} .35 & -.37 & -.16 & .14 \\ -.37 & .46 & .12 & -.16 \\ -.16 & .12 & .14 & -.06 \\ .14 & -.16 & -.06 & .06 \end{bmatrix},$$

where the shock innovations are in the order $\varepsilon_t^a,\ \varepsilon_t^s,\ \varepsilon_t^m$ and ε_t^d. Thus the sunspot and the supply-side shock cannot be recovered. The serial correlation in the innovations to the EM shocks of animal spirits and monetary shocks are substantial, namely .53 and .58 respectively, i.e. they are not really innovations. Turning to the nature of the EM shock innovations, which we have said will be correlated even if this was not true of the AM shock innovations, we find the correlation between $(\varepsilon^a_{t|T}, \varepsilon^s_{t|T})$ is 0.80, whereas $(\varepsilon^d_{t|T}, \varepsilon^s_{t|T})$ is -0.07.

3.6. Implications for variance and variable decompositions

To analyze this, suppose that there are only two shocks and we use the identity involving data $z_t = G(L)E_T\varepsilon_t$, giving

$$z_t = G_1(L)E_T\varepsilon_{1t} + G_2(L)E_T\varepsilon_{2t}$$

[24] Eq. (20) abstracts from the constants in the LS measurement equations. To calculate D_1, M and C we use simulated observations on z_t and ε_t and fit the identities, Eqs. (20) and (21). The method was used in Liu et al. (2018).

[25] This is done by regressing the monetary shock on the sunspot.

[26] FN actually work with a sunspot variable that is a linear combination of the four innovations.

$$= z^s_{1t} + z^s_{2t},$$

where $G(L) = [G_1(L)\ G_2(L)]$. If z^s_{1t} and z^s_{2t} are uncorrelated, which occurs when the *AM* innovations are recoverable and found to be uncorrelated among each other and over time, one can find the contribution to the *data variance* from each shock. This is *not* the case if the AM shocks are correlated or there are excess shocks, because the *EM* shocks are then correlated. Specifically, in those instances z^s_{1t} and z^s_{2t} are correlated and so $var(z_t)$ is not $var(z^s_{1t}) + var(z^s_{2t})$.

Variance decompositions are often presented in applied papers where it is implicitly assumed that $var(z_t)$ equals $var(z^s_{1t})+var(z^s_{2t})$, as it is maintained that the contribution to the variance of z_t of (say) the first shock is $\frac{var(z^s_{1t})}{var(z^s_{1t})+var(z^s_{2t})}$. That is true in fitted exactly-identified SVARs, since the zero correlation between *EM* shock innovations is enforced in estimation. But it will *not* be true of models with excess shocks.

3.6.1. Example: Christiano et al. (2014)

To give one example from the literature where this issue arises we consider the contribution of risk shocks to the spread between short and long rates in Christiano et al. (2014). In that model the data standard deviation for the spread is .38, while using their model the standard deviation of the component of the spread due to risk shocks alone is almost *three times higher* (1.085).[27] The only way this can happen is if there is a correlation between the risk and non-risk *AM* shock innovations. Essentially, it is *not* a decomposition of the *data* variance that is presented but the fraction of the *synthetic quantity* $var(z^s_{1t}) + var(z^s_{2t})$, and that is not $var(z_t)$.

The same issue arises with variable (i.e. historical) decompositions. In these the second shock above is set to zero and that leaves z^s_{1t} as the contribution. When we have zero correlation between the innovations ε_t this gives a correct measure. But when the innovations ε_t are correlated, moving ε_{2t} to zero changes ε_{1t} so that the total impact due to a change in ε_{1t} has a direct effect of $G_{0,i1}$ and an indirect effect of $G_{0,i2}$.

3.6.2. Example: A Time-Varying Parameter (TVP) autoregressive process

There are many models where parameters are allowed to be stochastically time varying. Prominent examples are TVP SVARs - Primiceri (2005) is an influential reference. Mostly these models involve excess shocks. To see the implications of excess shocks for variance decompositions in TVP models, consider an AR(1) where the autoregressive coefficient is allowed to be time varying:

$$z_t = \alpha_t z_{t-1} + \varepsilon_{1t}$$
$$\alpha_t = \rho\alpha_{t-1} + \varepsilon_{2t},$$

with the *AM* shock innovations ε_{1t} and ε_{2t} uncorrelated with unit variances. This can be placed in the SSF above, with the states $\zeta_t = \begin{bmatrix}\alpha_t \\ \varepsilon_{1t} \\ \varepsilon_{2t},\end{bmatrix}$. D_1 in this instance is time-varying, with $D_{1t} = z_{t-1}$. Note it varies in a known way, since z_{t-1} is observed. The Kalman filter and smoother remain the same, except for D_1 being time varying.

The above model was fitted to the inflation data used by Lubik and Schorfheide (2004). The resulting estimated model was

$$z_t = 2.404 + \alpha_t z_{t-1} + 1.249\varepsilon_{1t}$$
$$\alpha_t = .969\alpha_{t-1} + .1264\varepsilon_{2t},$$

showing time variation in the persistence of inflation. Turning to the implications of excess shocks, the smoothed EM estimates of the innovations from the estimated SSF have a correlation of .54. This has important implications for variance decompositions, which are constructed using the smoothed shocks and are reported in many TVP-SVAR papers. A common methodology is to compute a variance decomposition at time t by setting $\alpha_t = E_T\alpha_t$. With α_t fixed the impulse responses underlying the variance decomposition are just computed with respect to ε_{1t}. Thus, the hypothetical experiment performed involves raising $E_T\varepsilon_{1t}$. However, since $E_T\varepsilon_{1t}$ and $E_T\varepsilon_{2t}$ are correlated, this would also raise $E_T\alpha_t$ *above what it would be if* $E_T\varepsilon_{1t}$ *had not been raised.*

To assume $\alpha_t = E_T\alpha_t$ when constructing variance decompositions one needs to have a zero correlation between $E_T\varepsilon_{1t}$ and $E_T\varepsilon_{2t}$, which will not occur due to excess shocks. Essentially it is often ignored that there is an additional shock in the model and it is instead presumed that the assumption made about the *AM* shocks being uncorrelated translates to the smoothed shocks being used in the variance decompositions.

4. Excess shocks in DSGE models: Consequences and strategies

Excess shocks can occur in a wide variety of macroeconomic models. In this section we demonstrate assessing shock recoverability in a DSGE model, Ireland (2011). This is a very standard DSGE model that has no sunspots etc. To reiterate, for an impulse response from any model to a particular AM shock to be relevant, ideally it must be recoverable, and it needs to be uncorrelated with the others. To be clear, *we are not advocating constructing impulse responses to the EM* shocks, as they are correlated when excess shocks exist. Instead, the application below demonstrates which AM shocks can be recovered. After that, testing whether they are uncorrelated can be done for a DSGE model in a straightforward manner. We then turn to two strategies for dealing with excess shocks, which are demonstrated using the Ireland (2011) model.

[27] This is taken from Pagan and Wickens (2022, Table 1). One can see the divergence between the variances in Figure 1 of Christiano et al. (2014).

4.1. The New-Keynesian DSGE model of Ireland (2011)

The methodology used by Ireland (2011) is typical within the DSGE literature. It is assumed that the shocks are driven by uncorrelated innovations, parameters of the model are then estimated, and impulse responses and variance decompositions to these *AM* shocks are computed. We treat the estimated parameters as true ones and so assume that the model Ireland estimates is not mis-specified.

The model consists of three key equations: an IS equation, a NK Phillips Curve, and a Taylor rule. Ireland assumed four uncorrelated AM shocks: preferences a_t, technology Z_t, cost push $\hat{e}_t$ and monetary policy ε_{rt}. Using three observables on inflation, output growth and the interest rate, he estimated the model and computed impulse responses. There are therefore excess shocks. The technology shock is permanent, while the remainder are transitory. For completeness, the model equations he used are:

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_{at}, \tag{23}$$

$$(z-\beta\gamma)(z-\gamma)\hat{\lambda}_t = \gamma z \hat{y}_{t-1} - (z^2+\beta\gamma^2)\hat{y}_t + \beta\gamma z \mathbb{E}_t(\hat{y}_{t+1}) + (z-\beta\gamma\rho_a)(z-\gamma)\hat{a}_t - \gamma z \hat{z}_t, \tag{24}$$

$$\hat{\lambda}_t = \hat{r}_t + \mathbb{E}_t(\hat{\lambda}_{t+1}) - \mathbb{E}_t(\hat{\pi}_{t+1}), \tag{25}$$

$$\hat{e}_t = \rho_e \hat{e}_{t-1} + \varepsilon_{et}, \tag{26}$$

$$\hat{z}_t = \varepsilon_{zt}, \tag{27}$$

$$(1+\beta\alpha)\hat{\pi}_t = \alpha\hat{\pi}_{t-1} + \beta\mathbb{E}_t(\hat{\pi}_{t+1}) - \psi\hat{\lambda}_t + \psi\hat{a}_t + \hat{e}_t, \tag{28}$$

$$\hat{r}_t - \hat{r}_{t-1} = \rho_\pi \hat{\pi}_t + \rho_g \hat{g}_t + \varepsilon_{rt}, \tag{29}$$

$$\hat{g}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t, \tag{30}$$

$$0 = \gamma z \hat{q}_{t-1} - (z^2+\beta\gamma^2)\hat{q}_t + \beta\gamma z\mathbb{E}_t(\hat{q}_{t+1}) + \beta\gamma(z-\gamma)(1-\rho_a)\hat{a}_t - \gamma z \hat{z}_t, \tag{31}$$

and

$$\hat{x}_t = \hat{y}_t - \hat{q}_t. \tag{32}$$

The variables in the model are a Lagrange multiplier coming from the consumer's budget constraint λ_t; output y_t; the growth rate of technology z_t; inflation π_t; interest rate r_t; output growth g_t; the efficient level of output q_t and the output gap x_t. Some of these variables have been normalized by the non-stationary technology Z_t so as to have a well-defined path to log-linearize about. $\hat{}$ denotes a log deviation from steady-state.

The parameters are the autocorrelation coefficients for the respective shock processes (ρ_a and ρ_e); the steady-state growth rate of technology z; a discount factor β; an internal habits intensity γ; the degree of forward-looking behavior in price setting α; and interest rate rule parameters on inflation and growth (ρ_π and ρ_g).

Eqs. (23), (26) and (27) represent three of the model innovations processes and the fourth shock ε_{rt} is that for monetary policy. The *AM* innovations $\varepsilon_{rt}, \varepsilon_{zt}$ etc., are all assumed to be uncorrelated and to have no serial correlation. Eqs. (24) and (25) together produce the NK IS curve. Eq. (28) is a NK Phillips Curve. Eq. (29) is the interest rate rule. Finally, Eq. (32) is the output gap, with the natural rate of output defined by Eq. (31).

There is another equation in the model that describes hours (h_t). This is

$$\hat{h}_t = \hat{y}_t = \hat{g}_t + \hat{y}_{t-1} - \hat{z}_t \tag{33}$$

Because it does not feed back into the earlier equations, and h_t was treated as being not observable, it was ignored, but later in our work it will become important, as it provides extra information.

4.2. Which AM shocks of Ireland's model can be recovered and what are their properties?

To examine the implications of excess shocks in this model we define the state vector ψ_t in our SSF as consisting of the three observed variables, the unobserved variable, hours, and the four shock innovations (preference, cost-push, technology and monetary). Doing so enables us to compute the steady-state $P_{t|T}$ and therefore to check which of the AM shocks can be recovered from the data. As excess shocks exist we know not all of them can be recovered.

Treating Ireland's parameter estimates as the "true" ones it is found that the diagonal elements of $P^*_{t|T}$ corresponding to the shocks are $\begin{bmatrix} .36 \\ .57 \\ .00 \\ .07 \end{bmatrix}$. Accordingly, only the AM technology shock innovation can be recovered, although one might think that the monetary shock seems quite close to being so. The AM preference and cost push shock innovations quite clearly cannot be.

Suppose we decided that, based on the above evidence, the AM technology and monetary shock innovations are recoverable. Then they are equal to the equivalent smoothed *EM* shock innovations. So to look at the correlation between the *AM* shock innovations we can compute the correlation between the *EM* shock innovations. For the technology and monetary policy shocks we find that it is −.20; generally the monetary EM shock innovation is only weakly correlated with all the other *EM* shocks. Hence this points to the impulse responses for it being valid. A caveat, however, is that Ireland also assumes that the AM shock innovation has no serial correlation while it actually has non-trivial first-order autocorrelation (0.55).

As the preference shock is not recoverable we apply the indirect inference approach. Interestingly, the smoothed EM preference shock innovation is found to not be significantly correlated with the others beyond that induced by excess shocks.

The sizable bivariate correlations that exist amongst some of the *EM* shock innovations mean that variable and variance decompositions are not appropriate, regardless of whether the *AM* shock innovations can be recovered, as these decompositions use the *EM* shocks.

4.3. Excess shocks and factor models

There is a literature on factor models in macroeconometrics where there are some common factors and also idiosyncratic shocks connected to all observables. As a referee noted, it is well known that when the number of observables tends to infinity one can recover all shocks even though this is a situation of excess shocks — see Bai and Ng (2002). This seems to contradict what we have said in previous sections. To appreciate the issues raised here, consider a simple example of such a factor model:

$$z_{it} = \Lambda_i F_t + \varepsilon_{it}, i = 1, \dots, n, \tag{34}$$

$$F_t = \eta_t, \tag{35}$$

$$\Longrightarrow z_{it} = \Lambda_i \eta_t + \varepsilon_{it}, \tag{36}$$

where there are n observed variables z_{it} and a factor F_t. The loadings are Λ_i and there are two types of *AM* shocks - ε_{it} and the factor shock η_t. There is one more shock than observables and we look at whether shocks are recoverable.

Consider estimating the factor with some weighted average $\tilde{F}_t = \frac{1}{\delta}\sum_{i=1}^n \omega_i z_{it}$, where $\sum_{i=1}^n \omega_i^2 < D$ and $\delta = \sum_{i=1}^n \omega_i \Lambda_i$.[28] Then

$$\tilde{F}_t = F_t + \frac{1}{\delta}\sum_{i=1}^n \omega_i \varepsilon_{it}.$$

A simple approach is to choose $\omega_i = \frac{1}{n}$, but there are many other combinations. For example, both the smoothed and filtered shocks are constructed in this way. Then as $n \to \infty$ $\tilde{F}_t \to F_t$ and the factor can be recovered. Using the formula for $\tilde{F}_t$ we then have

$$z_{it} = \Lambda_i \tilde{F}_t + \tilde{\varepsilon}_{it}. \tag{37}$$

Subtracting Eq. (37) from (36) produces

$$\Lambda_i(\tilde{F}_t - F_t) + (\tilde{\varepsilon}_{it} - \varepsilon_{it}) = 0.$$

So $\tilde{\varepsilon}_{it} \to \varepsilon_{it}$ as $n \to \infty$. But for any finite n there is linear dependence between the estimated shocks. In empirical examples n is finite, so the same issues arise in dynamic factor models as in DSGE models. Consequently, it makes sense to check for *AM* shock recovery and the assumptions made about them in the same way as was described earlier.

Often factor models are used in a different way, particularly in the context of a SVAR. The factors are replaced by a weighted average of a large set of observables, mostly the principal components. A finite number of these are then added to an SVAR which has some other observable variables. Thus in Bernanke et al. (2005) a few principal components are found from a large set of observables. These are then included in a SVAR that also has an interest rate. The monetary shock impulses are subsequently found by using a recursive ordering. So there are no excess shocks in those type of Factor-Augmented VAR (FAVAR) models.

So what happens in a DSGE model as the number of observables increases? This is context dependent, but we note that Liu et al. (2018) reviewed the Reserve Bank of Australia's Multi-Sector Model (Rees et al., 2016). This is a relatively large model, with 17 observed variables and, with measurement error included for most variables, 31 shocks. Correlations amongst some of the EM shock innovations were found to be sizable — for example, the correlation between cost-push innovations in the non-traded and import sectors was 0.78. Sizable correlations also existed between some of the measurement error and structural shock innovations. Our summary is that one needs to check any DSGE model with excess shocks and factor models with the methods we have shown in order to determine issues when interpreting outcomes.

4.4. Strategies for dealing with excess shocks

To look at this question one needs to know why there are excess shocks and what one wants to learn from the modeling exercise. Sometimes the situation arises from shocks being added on to a model to improve the fit with the data. Examples of that would

[28] We are assuming that the parameters Λ_i are known for expository purposes.

Table 1
Maximum likelihood estimates of Ireland's model parameters under different shock and observable specifications.

Parameter	Replication estimate	Excluding preference shock estimate	Adding hours worked estimate
γ	0.390	0.942	0.647
	(0.077)	(0.042)	(0.054)
ρ_π	0.415	0.419	0.439
	(0.046)	(0.044)	0.050
ρ_g	0.127	0.015	0.043
	(0.025)	(0.013)	(0.022)
ρ_a	0.980	0.000	0.861
	(0.025)	–	0.043
ρ_e	0.000	0.927	0.000
	–	(0.030)	–
Standard deviations of the shocks			
σ_a	0.087	0.000	0.033
	(0.102)	–	0.005
σ_e	0.002	0.004	0.009
	(0.0002)	(0.001)	0.001
$\sigma_{\hat{z}}$	0.010	0.110	0.005
	(0.002)	(0.077)	(0.0004)
σ_r	0.001	0.001	0.001
	(0.0001)	(0.0001)	(0.0001)

Standard errors are shown in parentheses.

be measurement error and time-varying parameters. This is a noble intention but there are costs — one cannot recover all the *AM* shock innovations, and the *EM* shock innovations will be correlated. So, if the argument being advanced from the research does not necessarily need such information, one might want to question whether their addition incurs more of a cost than a benefit. However, in other cases they are part of the model, and so there will *have* to be excess shocks. In such cases we have to recognize the limitations this imposes on using the model: we would need to ask exactly what can be learned from it, and that requires us to ask what *AM* shock innovations are recoverable, and whether the properties assumed about them are correct. More intermediate cases arise where the excess shocks are in the maintained model but, for some reason, not enough observable data is used. Ireland's model is an example of that. So a range of situations need to be dealt with, and no one strategy will work in all situations. Hence, we canvass some possible responses by researchers in the next sub-sections before returning to an assumption we have made in our analysis that the parameters of models exhibiting excess shocks were known.

4.4.1. Eliminating excess shocks

One possibility is to use more data. Ireland (2011) is an example of a DSGE model where it is straightforward to eliminate excess shocks by adding in the extra observed variable of hours. The key requirement to do this is that the model has a readily available counterpart in the data.[29] In the event that the excess shocks are measurement error it may be that one can use a summary of this extra data – such as principal components – to compute items such as impulse responses. Forni et al. (2020) seems to be such an example in a SVAR. Another alternative would be to remove one of the excess shocks — such as the preference shock. To compare these strategies we do both here. This does not represent a belief that the preference shocks are unimportant. Because we are using Ireland's data it is necessary to first replicate his parameter estimates. Table 1 does this and it shows the implications for the parameter estimates of the DSGE model under the two strategies. It is seen that, relative to his estimates with excess shocks, in both cases the habits parameter, γ increases significantly while, when the preference shock is excluded, the cost-push shock becomes highly persistent.[30]

Table 2 shows the smoothed EM shock innovation correlations. As we have argued earlier the preference AM shock cannot be recovered from its EM equivalent, but the other shocks probably can, and there are several sizable correlations among the *EM* shock innovations — for example, that between the smoothed cost push (ε_e) and technology growth shocks ($\varepsilon_{\hat{z}}$) is −0.56. When the preference shock is dropped – so there is no excess now – the evidence is that the three remaining *AM* shock innovations seem to be uncorrelated although the innovations are found to have substantial autocorrelations and so are not really innovations. Consequently, this latter assumption used to compute impulse responses is inappropriate. Allowing for four shocks by using the extra series on hours worked does produce, with the exception of money and cost-push, some moderate correlations between the EM (and hence *AM*) shocks innovations, but large degrees of serial correlation. Both sets of results point to mis-specification in the model. Indeed, if we do the same analysis as above but with data simulated from the models – so that there is no mis-specification – and no excess shocks, the estimated correlations and autocorrelations of the assumed innovations are essentially zero.[31]

[29] Adding hours worked was done by Pagan and Wickens (2022) for a related model, Ireland (2004). The hours worked data are from Kulish et al. (2017).
[30] The standard errors in the replication can differ from Ireland (2011) as he bootstrapped the model, and we do not.
[31] These results are available upon request.

Table 2
Properties of the estimated shock innovations.

	Replication				Excluding preference shock				Adding hours worked			
Shock innovation correlations												
Filtered												
	ε_a	ε_e	$\varepsilon_{\dot{z}}$	ε_r	ε_a	ε_e	$\varepsilon_{\dot{z}}$	ε_r	ε_a	ε_e	$\varepsilon_{\dot{z}}$	ε_r
ε_a	1	0.46	0.38	0.09	–	–	–	–	1	−0.27	0.29	0.14
ε_e	0.46	1	−0.64	0.22	–	1	0.09	0.12	−0.27	1	−0.09	−0.40
$\varepsilon_{\dot{z}}$	0.38	−0.64	1	−0.15	–	0.09	1	0.15	0.29	−0.09	1	0.05
ε_r	0.09	0.22	−0.15	1	–	0.12	0.15	1	0.14	−0.40	0.05	1
Smoothed												
ε_a	1	0.15	0.36	0.13	–	–	–	–	1	−0.27	0.29	0.14
ε_e	0.15	1	−0.56	0.00	–	1	0.09	0.12	−0.27	1	−0.09	−0.40
$\varepsilon_{\dot{z}}$	0.36	−0.56	1	−0.20	–	0.09	1	0.15	0.29	−0.09	1	0.05
ε_r	0.13	0.00	−0.20	1	–	0.12	0.15	1	0.14	−0.40	0.05	1

Shock innovation first-order autocorrelations	Filtered	Smoothed	Filtered	Smoothed	Filtered	Smoothed
ε_a	0.32	0.39	–	–	0.02	0.02
ε_e	−0.14	−0.53	−0.09	−0.09	0.81	0.81
$\varepsilon_{\dot{z}}$	−0.15	−0.05	−0.46	−0.46	−0.03	−0.03
ε_r	0.55	0.55	0.59	0.59	0.58	0.58

In Ireland's model it is important that the extra variable which can be measured was in the model. There may be cases, however, when that is not possible. In such cases one may have to just admit that there will be excess shocks and to ask what that means for any interpretation.

A comment we have received is that there are many more shocks in the economy than in any model. However, there are also more observed variables, so this does not mean that excess shocks will always be needed. The issue with any model is whether one can simplify the economy so that we can work with a smaller number of shocks and variables than are actually present in it so as to analyze the particular aspect of interest. If we cannot make this simplification then the shocks we retain will be correlated, making the assumption that they are not more implausible. This is a reason why we would want to check if there is a correlation between the *AM* shock innovations. Doing so checks whether we have actually set up a model with enough structure and shocks so that it can capture the observable variables being used, and therefore whether any particular aspect of interest can be studied in a data-coherent way.

4.4.2. Packages of shocks

In Ireland's model it was shown that we could recover at most two shocks. Of these the monetary shock innovation shows no correlation with the others, while that is not true of the technology shocks. The preference shock and cost-push cannot be recovered. Reflecting the correlations existing in the *EM* shock innovations, one way to proceed with analyzing the dynamics of the model might be to study *packages of recovered shocks*.

Consequently, the first package would be the monetary shock. Had the cost-push shock been recoverable, a second might be termed a *business shock* as it would combine the cost-push and technology shocks together. For any given combination we can compute an impulse response to this business shock. Of course this combination is not unique, but there may be some evidence to suggest what weights might be applied to construct it, or one could consider a range of weights.

There are limitations to working with packages of correlated shocks, most notably that an economically meaningful name to the package may not exist. This is likely to be more important in larger models; for example shocks pertaining to different agents in the model could be correlated.

4.5. The issue of parameter values

In our analysis we have assumed that the model being used had known parameter values. An editorial question was what happens when you do not know them? Ultimately, questions of recoverability of *AM* shock innovations will be answered with the parameter values that are being used by the investigators and these may have been estimated. It could be the case that using different parameter values alters whether *AM* shock innovations can be recovered. However, all of the *AM* shock innovations cannot be recovered when there are excess shocks. So clearly before starting to look at recovery of *AM* shock innovations, and testing the assumptions made about them, one would want to make every effort to ensure that the model being used is not mis-specified, i.e. the parameter estimates are appropriate. Applying our material involves assuming that this is the case, and then the question asked is what one can learn about the *AM* shock innovations given these parameter values. In the case of Ireland (2011), we assumed the estimated parameters were the true ones and found, from the recoverable shocks (or those that may nearly be) that those *AM* shock innovations did not satisfy some of their assumed properties.

The *EM* shock are more complex; these will be correlated when there are excess shocks. The indirect inference approach is presented as a way of determining whether any correlation evident in the innovations is statistically different from that induced by excess shocks even with uncorrelated *AM* shock innovations. However, it is not necessary to know that in order to conclude that variance and variable decompositions are inappropriate when excess shocks exist, as they use the *EM* shock innovations.

Our assumption of treating the parameter values as known is essentially considering the best possible scenario when analyzing models with excess shocks.

5. Conclusion

Estimating structural shocks is a key aspect of applied macroeconomics, but today often one sees models with more shocks than observed variables, i.e. "excess shocks". Models with excess shocks are widespread, and come with a variety of different modeling approaches. This paper shows both how the shocks in such models can easily be assessed and the difficulties that excess shocks can create when interpreting commonly reported items produced from these models.

Excess shocks complicate the interpretation of impulse responses. The investigator typically maintains that the assumed model shock innovations can be recovered and are uncorrelated. This is done in order to implement the *ceteris paribus* assumption that impulse responses are found by varying one shock and keeping the others constant. That the shock innovations are uncorrelated is an *assumption* and, as the Gershwins' song says, "It Ain't Necessarily So". Because of the importance of this assumption we believe one should ask if it is contrary to the data. This paper demonstrates how it can be assessed when excess shocks exist.

If excess shocks exist the first problem is to determine whether one can recover the *AM* shock innovations from the data, while the second is whether the recoverable shock innovations are uncorrelated. We describe a method using the Kalman filter and smoother that enables one to identify which are the recoverable shocks. The results from the Kalman filter coincide with the method suggested in Forni et al. (2019) for the same purpose in SVARs. Forni et al. focus upon using filtered *EM* shocks. Our measure extends this to allow for smoothed *EM* shocks and so provides information about the ability to recover the assumed shocks with the complete data sample. It agrees with the definition of recoverability in Chahrour and Jurado (2022), unifying these two approaches. Our measure is straightforward to compute as it is an output from the Kalman smoother.

If a shock is not recoverable then its correlation with others cannot be directly assessed. We propose a way it can be checked using indirect inference, and this method was demonstrated with the Lubik and Schorfheide (2004) model.

There are three major implications of our analysis for applied macroeconomic research. First, when working with excess shocks researchers should check which of the assumed shocks can be recovered, and then ask about the validity of the assumptions they are making about them. Those that cannot be recovered can be assessed using indirect inference. Checking the shock properties is rarely done in the literature, even when there are no excess shocks. Maintaining that the assumptions hold when they can be checked is undesirable.

Secondly, while the decision to work with excess shocks is one made by the researchers, they should demonstrate to the reader the actual properties of the assumed shocks as revealed by the data, so the reader can assess which of the impulse responses presented are informative. In many ways excess shocks are best avoided if their inclusion is not necessary in order to answer the research question being investigated. For example, one might use more and/or better observed data. In the case of Ireland's (2011) model, an additional variable – hours worked – could readily be observed, thereby eliminating excess shocks.

Finally, the third implication of excess shocks relates to the common practice of assessing the importance of various shocks using variable and variance decompositions of the data. These use the *EM* shock innovations and not the *AM* shock innovations. When there are excess shocks the innovations into the *EM* shocks are contemporaneously correlated, and may also be serially correlated, and so one cannot perform these decompositions in a unique way. More generally, if it is necessary to work with excess shocks the analysis should be conducted recognizing the limitations that having an excess of shocks imposes.

In essence, excess shocks can limit our ability to validly interpret many of the common items constructed from a wide range of macroeconomic models.

Appendix A. Further applications of assessing shock recoverability

A.1. A permanent/transitory components model

To illustrate the consequences of excess shocks in models with I(1) variables we use a variant of the simple UC model analyzed previously. Such a model has been used to measure output gaps; see, for example, Orphanides and van Norden (2002). These models decompose an observed series y_t as $y_t = y_t^p + y_t^c$, where y_t^p is a permanent or "trend" component of y_t and y_t^c a transitory component, often called the cycle (or output gap). Assumptions have to be made about how these evolve, and we look at the simplest set:

$$\begin{aligned}\psi_{1t} &= \Delta y_t^p = \varepsilon_{1t}\\ \psi_{2t} &= y_t^c = \varepsilon_{2t}\\ z_t &= \Delta y_t = \varepsilon_{1t} + \Delta\varepsilon_{2t},\end{aligned}$$

where ε_{1t} and ε_{2t} are $n.i.d.(0,1)$ and independent of one another.[32]

[32] This model implies that Δy_t is a MA(1), $\Delta y_t = (1+\alpha L)u_t$, and so there are two parameters that can be estimated from the data – α and σ_u^2 – to give estimates of the variances of ε_{1t} and ε_{2t}. However, we will assume that all of the variance parameters are known so that no issues of identification arise.

To see the consequences of excess shocks, notice that this is the same model as analyzed in Section 3.4, but now with $\rho = 1$, so the conclusions about singularity of the covariance matrix of the filtered *EM* shock innovations and serial correlation in the smoothed *EM* shock innovations still hold.

The perfect correlation between the estimated filtered shocks has economic implications. In this model the cyclical component – the output gap – is ε_{2t}. Consequently the estimated output gap and innovations to trend growth are perfectly correlated. Therefore, placing an economic interpretation on trend growth, such as that it reflects increases in aggregate supply, is problematic. This will occur even if a distinction between the shocks driving the cycle and the trend exists in the assumed shocks.[33]

This correlation has been noted before. Morley et al. (2003) compared the Beveridge–Nelson (BN) definition of the cycle with what one would obtain from a UC model. Drawing on Watson (1986), and using a UC model with uncorrelated innovations, they noted that the BN decomposition led to a filtered estimate of the cycle, $E_t y_t^c$, identical to that from the Kalman filter of the UC model, since it was already known that the BN trend and cycle innovations were perfectly correlated. They also show that, although a BN decomposition based on an ARIMA model of the observed data implies a different UC model (and trend estimates), the estimated innovations to the trend and cycle components of this latter model are perfectly correlated.[34] Our contribution is to point out that the perfect correlation arises from excess shocks, and so it is a wider issue than just filtering.[35] Morley et al. (2003) note that Wallis (1995) made the point that the correlations between the DGP innovations and those evident in the estimated innovations can differ because of estimation.

A.2. The Hodrick–Prescott filter

The Hodrick–Prescott (HP) filter (Hodrick and Prescott, 1997) is extensively used in applied macroeconomic work and has been criticized along several dimensions (see, for, example, Hamilton, 2018; Cogley and Nason, 1995 and Fukač and Pagan, 2010). In this sub-section we demonstrate the inability to recover the hypothesized shocks in determining some of the properties of the HP filter. To do so we exploit the fact that it can be found from a UC model (see, for example, Harvey and Jaeger, 1993). Doing so enables our previous analysis to be readily applied to the HP filter. The corresponding UC model is

$$\begin{aligned} (1-L)^2 y_t^p &= \varepsilon_{1t} \\ y_t^c &= \phi \varepsilon_{2t} \\ y_t &= y_t^p + y_t^c. \end{aligned}$$

Note the cyclical component is a multiple (ϕ) of the innovation ε_{2t}. The parameter ϕ equals $\sqrt{\lambda}$ and $\lambda = 1{,}600$ is common when the HP filter is applied to quarterly data.

The system can be expressed as

$$\begin{aligned} z_t &= (1-L)^2 y_t \\ &= \varepsilon_{1t} + \phi(\varepsilon_{2t} - 2\varepsilon_{2t-1} + \varepsilon_{2t-2}). \end{aligned}$$

Defining $\psi_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{2t-1} \end{bmatrix}$ the SSF has the matrices $M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$, $D_1 = \begin{bmatrix} 1 & \phi & 0 \end{bmatrix}$, and $D_2 = \begin{bmatrix} 0 & -2\phi & \phi \end{bmatrix}$. This leads to filtered *EM* innovations $E_t\varepsilon_{2t} = \phi E_t\varepsilon_{1t}$, so the filtered trend innovations and the cycle are perfectly correlated. The *AM* shocks are not recoverable with past and current data; the steady-state $P_{t|t}$ has non-zero diagonal elements for the one-sided HP filter.

The original HP filter was two sided and therefore generated by smoothed, rather than filtered, shocks. In Section 3.3 it was shown that the smoothed shocks will have serial correlation even if the filtered shocks do not. That is, even if the hypothesized UC model used to generate the HP filter is correct there will still be serial correlation in the smoothed cycle innovations. This feature has often been observed in the literature for estimates of the cycle given by the HP filter and it has been interpreted as implying that the assumed UC model is mis-specified. However, *the serial correlation in the smoothed cycle innovations arises as the AM shocks are not recoverable; it occurs even if the HP filter is the correct UC model.* The steady-state $P_{t|T}$ does not have a zero diagonal.

As an experiment we simulated data using the model above as the DGP with $\phi = 40$, i.e. $\lambda = 1600$. Consequently the HP filter with a choice of $\lambda = 1600$ is the optimal filter. As predicted, applying it produces estimated innovations that are perfectly correlated, with $E_t\varepsilon_{2t} = 40E_t\varepsilon_{1t}$. The steady-state $diag(P_{t|t}) = \begin{bmatrix} .9995 \\ .2006 \\ .1608 \end{bmatrix}$. There is little serial correlation in the filtered cycle innovation but large amounts in the smoothed one. The steady-state $diag(P_{t|T}) = \begin{bmatrix} .9439 \\ .0561 \\ .0561 \end{bmatrix}$. As mentioned above, when this serial correlation in the

[33] If one uses the smoothed shocks they will still be correlated and are likely to be serially correlated as well.

[34] Anderson et al. (2006) demonstrate how the BN decomposition can be obtained from a “single source of error” state-space model (i.e. where the measurement and state equations are driven by a common innovation; see Snyder, 1985). This utilizes the perfect correlation between the innovations. Morley (2002) provides an alternative state-space approach to computing the BN decomposition, which was adopted in Morley et al. (2003).

[35] The UC model of Clark (1987) has the same feature.

smoothed estimates of the cycle ($E_T\varepsilon_{2t}$) has been seen in data, it has often been said it shows that the trend has not been correctly estimated, and so the cyclical component has to be purged of its serial correlation. But here the HP filter's UC model is the optimal filter and yet there is a large amount of persistence in the smoothed innovations.

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.euroecorev.2022.104120.

References

Adolfson, M., Laséen, S., Christiano, L., Trabandt, M., Walentin, K., 2013. Ramses II - Model Description. Sveriges Riksbank Occasional Paper Series, 12.
Amisano, G., Giannini, C., 1997. The problem of non-fundamental representations. In: Topics in Structural VAR Econometrics. Springer-Verlag, Berlin Heidelberg, ch 8 in G. Amisano and C. Giannini.
Anderson, H.M., Low, C.N., Snyder, R., 2006. Single source of error state space approach to beveridge Nelson decomposition. Econom. Lett. 91, 104–109.
Bai, J., Ng, S., 2002. Determining the number of factors in approximate factor models. Econometrica 70, 191–192.
Bernanke, B.S., Boivin, J., Eliasz, P.S., 2005. Measuring the effects of monetary policy: A factor-augmented vector autoregressive (FAVAR) approach. Q. J. Econ. 120, 387–442.
Bianchi, F., Nicolo, G., 2021. A generalized approach to indeterminancy in linear rational expectations models. Quant. Econ. 12, 843–868.
Canova, F., Ferroni, F., 2022. Mind the gap! Stylized dynamic facts and structural models. Am. Econ. J. Macroecon. forthcoming.
Canova, F., Sahneh, M.H., 2018. Are small-scale SVARs useful for business cyle analysis? Revisiting nonfundamentalness. J. Eur. Econom. Assoc. 16 (4), 1069–1083.
Chahrour, R., Jurado, K., 2022. Recoverability and expectations-driven fluctuations. Rev. Econom. Stud. 89 (1), 214–239.
Chen, H., Cúrdia, V., Ferrero, A., 2012. The macroeconomic effects of large-scale asset purchase programmes. Econ. J. 122 (564), 289–315.
Christiano, L.J., Motto, R., Rostagno, M., 2014. Risk shocks. Amer. Econ. Rev. 104 (1), 27–65.
Clark, P.K., 1987. The cyclical component of U.S. economic activity. Q. J. Econ. 102, 797–814.
Cogley, T., Nason, J.M., 1995. Effects of the Hodrick–Prescott filter on trend and difference stationary time series implications for business cycle research. J. Econom. Dynam. Control 19 (1–2), 253–278.
Farmer, R., Nicolò, G., 2018. Keynesian economics without the phillips curve. J. Econom. Dynam. Control 89, 137–150.
Farmer, R., Nicolò, G., 2021. Some international evidence for Keynesian economics without the phillips curve. Manchester School 89 (S1), 1–22.
Forni, M., Gambetti, L., 2014. Sufficient information in structural VARs. J. Monetary Econ. 66, 124–136.
Forni, M., Gambetti, L., Lippi, M., Sala, L., 2020. Common Component Structural VARs. CEPR Discussion Papers, 15529.
Forni, M., Gambetti, L., Sala, L., 2019. Structural VARs and noninvertible macroeconomic models. J. Appl. Econometrics 34, 221–246.
Fukač, M., Pagan, A.R., 2010. Limited information estimation and evaluation of DSGE models. J. Appl. Econometrics 25 (1), 55–70.
Hamilton, J.D., 2018. Why you should never use the Hodrick–Prescott filter. Rev. Econ. Stat. 100 (5), 831–843.
Harvey, A.C., 1992. On trend extraction models: Comments on a paper by garcia-ferrer and del hoyo. J. Forecast. 11, 669–672.
Harvey, A.C., Jaeger, A., 1993. Detrending, stylized facts and the business cycle. J. Appl. Econometrics 8 (3), 231–247.
Hodrick, R., Prescott, E., 1997. Post-war business cycles: An empirical investigation. J. Money Credit Bank. 29, 1–16.
Ireland, P.N., 2004. Technology shocks in the new Keynesian model. Rev. Econ. Stat. 86, 923–936.
Ireland, P.N., 2011. A new Keynesian perspective on the great recession. J. Money Credit Bank. 43, 31–54.
Kulish, M., Morley, J., Robinson, T., 2017. Estimating DSGE models with zero interest rate policy. J. Monetary Econ. 88, 34–49.
Kurz, M.S., 2018. A note on low-dimensional Kalman smoothers for systems with lagged states in the measurement equation. Econ. Lett. 168, 42–45.
Leeper, E.M., Todd, T., Walker, B., Yang, S-C.S., 2013. Fiscal foresight and information flows. Econometrica 81, 1115–1145.
Liu, X., Pagan, A.R., Robinson, T., 2018. Critically assessing estimated DSGE models: A case study of a multi-sector model. Econ. Rec. 94, 349–371.
Lubik, T.A., Schorfheide, F., 2004. Testing for indeterminacy: An application to U.S. monetary policy. Am. Econ. Rev. 94 (1), 190–217.
McDonald, J., Darroch, J., 1983. Consistent estimation of equations with composite moving average disturbance terms. J. Econometrics 23, 253–267.
Morley, J.C., 2002. A state-space approach to calculating the Beveridge–Nelson decomposition. Econom. Lett. 75, 123–127.
Morley, J.C., Nelson, C.R., Zivot, E., 2003. Why are the Beveridge–Nelson and unobserved components decompositions of GDP so different? Rev. Econ. Stat. LXXXV, 235–243.
Nelson, C.R., 1975. Rational expectations and the predictive efficiency of economic models. J. Bus. 43, 331–343.
Nimark, K.P., 2015. A low dimensional Kalman filter for systems with lagged states in the measurement equation. Econom. Lett. 127 (C), 10–13.
Orphanides, A., van Norden, S., 2002. The unreliability of output-gap estimates in real time. Rev. Econ. Stat. 84 (4), 569–583.
Pagan, A.R., Robinson, T., 2019. Implications of Partial Information for Applied Macroeconomic Modelling. Melbourne Institute: Applied Economic and Social Research Working Paper, University of Melbourne, 12/19.
Pagan, A.R., Wickens, M., 2022. Checking if the straitjacket fits? In: Chudik, A., Hsiao, C., Timmermann, A. (Eds.), Essays in Honor of M. Hashem Pesaran: Prediction and Macro Modeling. In: Advances in Econometrics, 43, (A), pp. 269–290.
Plagborg-Moller, M., Wolf, C.K., 2020. Instrumental Variable Identification of Dynamic Variance Decompositions. Working Paper, Princeton University.
Primiceri, G.E., 2005. Time varying structural vector autoregressions and monetary policy. Rev. Econom. Stud. 72 (3), 821–852.
Ramey, V.A., 2016. Macroeconomic shocks and their propagation. In: Taylor, J.B., Uhlig, H. (Eds.), Handbook of Macroeconomics. Elsevier, pp. 71–162, 2A.
Ravenna, F., 2007. Vector autoregressions and reduced form representations of DSGE models. J. Monetary Econ. 54 (7), 2048–2064.
Rees, D.M., Smith, P., Hall, J., 2016. A multi-sector model of the Australian economy. Econ. Rec. 92 (298), 374–408.
Sims, C.A., Zha, T., 2006. Does monetary policy generate recessions? Macroecon. Dyn. 10 (2), 231–272.
Snyder, R.D., 1985. Recursive estimation of dynamic linear models. J. R. Stat. Soc. Ser. B Stat. Methodol. 47, 272–276.
Wallis, K.F., 1995. Discussion of "Trend Extraction: A Practitioner's Guide" By M.R. Wickens, Annex E. Government Economic Service Working Paper, HM Treasury, London, p. 125.
Watson, M., 1986. Detrending with stochastic trends. J. Monetary Econ. 18, 49–75.