

Democratic Victory and War Duration: Why Are Democracies Less Likely to Win Long Wars?*

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Abstract

Using costly-process models of war with democratic citizens and soldiers, this article explores two contrasting propositions on the negative association between the probability of democratic victory and the duration of war. As a proposition holds, democracies need a long time to terminate a war especially on unfavorable terms, because they incur high audience costs if they break a prewar commitment too hastily. According to the other, democracies are less likely to win long wars, because their battlefield effectiveness declines over time. Even though the two propositions sharply differ in underlying logic, they offer analogous predictions about the choices of military strategies consistent with empirical findings that while democracies could raise their chances of victory with a shortening strategy (e.g., maneuver), autocracies might have mixed incentives for shortening and protracting strategies (e.g., attrition). These results imply that both the mechanisms might be at work in a democracy's prosecution of war. (149 words)

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	Democracies'				Total
	Wins	Draws	Losses	Others	
Short wars (less than 18 months)	20	2	1	3	26
Percentage	76.9%	7.7%	3.8%	11.5%	
Long wars (18 months or more)	3	2	2	0	7
Percentage	42.9%	28.6%	28.6%	0%	
Short and long wars	23	4	3	3	33
Percentage	69.7%	12.1%	9.1%	9.1%	

Note: Draws include compromises, ties, and stalemates; Others denote wars transformed into another kind of wars (cf. Sarkees and Wayman 2010).

Table 1: Percentages of democracies' war outcomes by duration

1 Introduction

The negative association between the probability of democratic victory and the duration of war has been long recognized as a stylized fact (Bennett and Stam 1998). As it holds, democracies are more likely to win than are their autocratic opponents in the short run, but after roughly 18 months have passed, democracies become far more likely than autocracies to quit and more willing to settle for draws or losses. This fact can be confirmed by the record of all 33 interstate wars fought by democracies since 1816 through 2007 (Table 1)—the percentage of democracies' wins is larger in short wars ending in less than 18 months (76.9%) than in long wars lasting 18 months or more (42.9%).¹ Correspondingly, the percentages of democracies' draws and losses are much lower in short wars (7.7% and 3.8%, respectively) than in long wars (28.6% for each).

As democracies' high propensity for victory (i.e., “powerful pacifists”) is explained in part by their strategic selection of winnable targets (Bueno de Mesquita and Siverson 1995; Bueno de Mesquita et al. 1999; Clark and Reed 2003; Reed and Clark 2000), this fact could also be due to the selection effect toward the end of war—because a democracy's typical war plan is to win shortly, only democracies having failed their original war plans fight long wars (Koch 2009). Besides the selection effect, there are two propositions drawn from the literature that might explain the fact.

One was originally posed by Tocqueville ([1835, 1840] 2004) and later bolstered

¹The data on wars and democracies are drawn from Correlates of War (Sarkees and Wayman 2010) and Polity5 (Marshall and Gurr 2020). Following Bennett and Stam (1998), we deem a state with the Polity score 7 or higher on [-10,10] at the onset of war to be democratic. All the wars in Table 1 are listed in Online Appendix.

by Iklé (1971).² According to them, it is difficult for democracies to end a war especially in unfavorable terms, because restoring peace at the price of major concessions almost inevitably evokes a severe cleavage between hawks and doves within a government. In light of such an intense internal struggle, anyone who advocates peace must bear the risk of being labeled as a “traitor.” The fear of this taint naturally deters the leadership from taking initiatives toward peace and causes a delay in producing the decision to end a war. In Iklé’s words: “fighting often continues long past the point where a rational calculation would indicate the war should be ended” (p. ix). However, as battle deaths accumulate, public support for war declines, and thus policymakers become pressured to make peace rather than continuing the war (Mueller 1973; Gartner, Segura, and Wikenning 1997; Gartner and Segura 1998).

The other proposition is based on empirical findings by Reiter and Stam (1998, 2002). They found that the armies of democracies are superior in terms of leadership, initiative, and logistics. Moreover, all the three advantages dwindle as wars lengthen. These findings could be a powerful foundation for the stylized fact above, because it is straightforward from their findings to conclude that democracies are less likely to win long wars, as their effectiveness on the battlefield deteriorates.

The Tocqueville-Iklé proposition is sharply contrasting to the Reiter-Stam one, because according to the former, the war outcomes influence the duration, whereas the latter indicates vice versa. Although both the propositions sound sensible, they entail some empirical shortcomings. Namely, the former seems to contradict the historical tendency that wars initiated by democracies are more likely to end swiftly (Bennett and Stam 1996), whereas the latter might suffer the selection effect in the closing phase of war (Koch 2009). Therefore, instead of further pursuing empirical investigation, we undertake theoretical assessments of whether these propositions could withstand the rigor of microfoundation. For this end, we develop a series of game-theoretic models of war as a costly process (Wagner 2000).

The models are presented in three steps. In the first step, a baseline model is built upon the model of war of attrition (Maynard Smith 1974). The baseline model depicts a war fought between a democracy and an autocracy across discrete time periods. It critically differs from the canonical war-of-attrition model in that both

²In Tocqueville’s words: “There are two things that will always be very difficult for a democratic people to do: to start a war and to finish it” (p. 765). Iklé concurs: “Warfare is such an all-absorbing enterprise [...] that after starting one, a government may lose sight of ending it” (p. viii).

the belligerents are uncertain about the relative strength denoted by a state variable, which determines their risks of being defeated. As the model presumes, the democracy is militarily advantageous at the war's onset (by the selection effect). However, the democracy's failure to bring about a victory in a timely manner indicates that the democracy is weaker than the prior estimates. This change in the beliefs as to the relative strength induces the democracy to terminate the war.

In the second step, we advance the baseline model in two directions: one is to delineate the Tocqueville-Iklé proposition by incorporating audience costs (Fearon 1994; Schultz 2001; Smith 1998a); and the other to capture the Reiter-Stam proposition by endogenizing battlefield effectiveness. The model of audience costs highlights democratic citizens, who decide to support or oppose a war in every period. In doing so, they can exert collective influence on their leader's decisions to continue fighting or not (Croco 2011). At the war's onset when citizens are largely optimistic about the war's prospects, they place a high value on a prewar commitment to the issue at stake. That means, the political cost of compromising original war aims is substantially large for the leader. However, as the democracy is revealed to be less advantageous over time, the citizenry support for the war withers. Accordingly, the political cost of surrendering inflicted by the domestic audience decreases as the war lengthens. If this cost is so large and persistent, the decision to end the war could be delayed.³ Moreover, because longer wars are likely to be more costly, this delay in turn enhances the democracy's deterrability of an opponent. Therefore, unlike what the audience-costs literature commonly presumes, our model suggests that the deterrence effect of audience costs depends not just on the prewar civilian influence, but also on how tenaciously the audience keeps their support for the war after its outbreak. The durability of audience costs could matter for deterrence.

In the model of battlefield effectiveness, there are democratic soldiers, who decide to undertake or evade their military missions. The success in a mission depends on the relative strength, but conversely the soldiers can also influence the democracy's effectiveness on the battlefield. As the democracy fails to accomplish a short-run victory, less and less soldiers undertake their missions, and its military effectiveness gradually declines. In contrast to the audience-costs model, according to which democratic cit-

³Audience costs do not necessarily make a war longer. The audience might also urge their leader to stop fighting as soon as they perceive it hopeless (Bennett and Stam 1996). In this sense, audience costs empower the public in shaping foreign policies, while reducing the leader's discretion.

izens can directly influence their leader's decisions, the battlefield-effectiveness model implies that direct influence by soldiers is limited to battle outcomes although they can indirectly influence whether to continue or end a war. Both the models render some clues as to why democracies are less likely to win long wars. They suggest that domestic politics and military effectiveness can matter for the outcomes and duration of wars fought by democracies.

In the third step, to assess these models' predictive validity, we extend the models by allowing the belligerents to choose a military strategy before they initiate fighting. Their options are those archetypal in the literature (Mearsheimer 1983; Reiter and Meek 1999; Stam 1996). One is to shorten the war by making battles more decisive (e.g., blitzkrieg or maneuver), and the other to protract the war by making battles less decisive (e.g., attrition).⁴ Predictions based on our extended models are consistent with empirical findings that democracies tend to adopt the maneuver strategy, whereas the choices of autocracies are rather ambiguous (Reiter and Meek 1999; Reiter and Stam 2002). These results imply that both the mechanisms of audience costs and of battlefield effectiveness might be at work in a democracy's prosecution of war. Moreover, our theory renders a rationalist explanation for the question as to why democratic and autocratic belligerents choose their military strategies in such ways when the maneuver strategy is highly correlated with victory (Reiter and Meek 1999). With the maneuver strategy, a democracy could raise the chances of victory by terminating war before it loses public support for fighting or before its forces on the battlefield become ineffective. On the other hand, an autocracy might have mixed incentives in that while the maneuver strategy shortens a war and thus reduces the cost of fighting, the attrition strategy could play on democracy's long-run weakness of losing public support and deteriorating military effectiveness in protracted warfare.

The rest of the article proceeds as follows. The second section reviews the related literature. The third section presents and solves the baseline model. Models in the fourth and fifth sections incorporate democratic citizens and soldiers into the baseline model. The sixth further extends these models to explore the relationship between regime types and military strategies. The seventh section concludes. Technical materials are left in Appendix.

⁴Blitzkrieg is frequently used to describe a quick victory on the battlefield, whereas with attrition, victory follows a series of set-piece battles and is not expected to be quick (Mearsheimer 1983: 34-35).

2 Theoretical Literature Review

Our theory pertains to (i) war termination, (ii) regime types, and (iii) military strategies. In the series of models presented in this article, one stream (Model I and its extension) relates regime types to (a) domestic politics, while another stream (Model II and its extension) relates them to (b) battlefield effectiveness. To the best of our knowledge, ours is the first theoretical attempt to incorporate these aspects in formal models of war termination.

As to (i) war termination, extant models have focused mostly on bargaining in dyadic contexts (Filson and Werner 2002; Langlois and Langlois 2009, 2012; Powell 2012; Slantchev 2003a, 2003b; Wolford, Reiter, and Carrubba 2011). According to them, two major causes of lengthy wars are the commitment problem (Fearon 2004, 2007) and private information (Powell 2004; Smith and Stam 2004; Wagner 2000). Our theory suggests that regime types can also matter for the outcomes and duration of war, holding either that democratic citizens affect their leader's decision to continue or end a war or that democratic soldiers influence battle outcomes through their military engagements.

As to (ii-a) domestic politics, a number of theoretical studies explain peace among democracies by institutional constraints (Baliga, Lucca, and Sjöström 2008; Debs and Goemans 2010; Bueno de Mesquita et al. 1999; Jackson and Morelli 2007) or by informational advantage (Fearon 1994; Guisinger and Smith 2002; Schultz 1998, 2001; Slantchev 2006; Smith 1998a; Tarar and Leventoglu 2009, 2013). Others are concerned with wars caused by diversionary purposes (Smith 1996; Tarar 2006) or the agency problem (Downs and Rocke 1994). As shown above, the theoretical literature extensively investigates the influence of domestic politics on war's outbreak, but formal studies on the domestic influence on war termination remain limited. There are few exceptions (Filson and Werner 2004; Smith 1998b), but they capture the differences among regime types only with parameter values. For further theoretical sophistication, ours explicitly portrays democratic citizens as players in costly-process models of war.

As to (ii-b) battlefield effectiveness, some costly-process models of war portray shifts of the military balance as a result of developments on the battlefield (Langlois and Langlois 2009, 2012; Slantchev 2003b; Smith 1998; Smith and Stam 2003, 2004). We seek microfoundation for the shifts by incorporating individual soldiers who can

influence the military balance, as they decide to undertake or evade military missions.

As to (iii) military strategies, theoretical studies are still sparse, although there are some formal work on indirect strategy (Lindsey 2015), *fait accompli* (Tarar 2016), punishment (Intriligator and Brito 1984; Nakao 2022), and concealment of strength (Baliga and Sjötröm 2008; Meirowitz and Sartori 2008; Slantchev 2010). Unlike those above, our models explore belligerents' choices between shortening and protracting strategies (i.e., maneuver and attrition) to assess whether our theoretical predictions are consistent with empirical findings (Reiter and Meek 1999; Reiter and Stam 2002). In doing so, we seek the rationales for each regime type to choose a certain military strategy.

3 Baseline Model

The baseline model is build upon the canonical war-of-attrition model (Maynard Smith 1974), but ours critically differs in threefold: (i) players can be defeated with some probabilities; (ii) these probabilities are determined by a state variable; (iii) the true state is unknown to both the players.⁵ Subsequently, the baseline model is advanced to endogenize the cost of surrendering (in Model I) and the probabilities of defeats (in Model II).

3.1 War of Attrition with Exogenous Defeats

Across discrete time periods $t \in \{1, 2, 3, \dots\}$, a war is prosecuted between two belligerents—a democracy and an autocracy—indexed by $i, j \in \{D, A\}$. The belligerents simultaneously decide to “fight” or “surrender” in every period. If they both choose to “fight,” a battle takes place and results in one of D ’s “winning,” “losing,” and “indecisive” outcomes. In a battle, i ’s “win” is identical to its opponent j ’s “loss.”

If the outcome is “indecisive”, the war continues to the next period, and another battle is fought unless either i surrenders. The war ends with i ’s victory either when i “wins” a battle or when its opponent j “surrenders.” If i brings about a victory, i seizes a lump-sum benefit $W_i > 0$, while j gains nothing. If both D and A “surrender” simultaneously, they both gain nothing. Throughout the war, each belligerent i incurs a per-period cost of fighting $c_i > 0$.

3.2 Two-Sided Uncertainty about the Relative Strength

The probability distribution of battle outcomes is determined by a state variable θ that is binary, favoring either D or A ($\theta \in \{d, a\}$). Neither D nor A knows whether they are in state d or in a , but they share the common prior probabilities $\Pr(d) > 0$ and $\Pr(a) > 0$ such that $\Pr(d) + \Pr(a) = 1$.⁶

⁵Unlike typical models of war with private information (Fearon 2007; Filson and Werner 2002, 2004; Powell 2004; Slantchev 2003b), our models do not assume informational asymmetry—neither side entertains informational advantage despite uncertainty (Smith and Stam 2004).

⁶In the model, the priors are set to be common merely for simplicity, but they can differ (Smith and Stam 2004).

The probabilities of battle outcomes are defined as:

$$\begin{aligned}\Pr(\text{win}_i|\theta) &\equiv (1 - \delta^\theta) p_i^\theta \\ \Pr(\text{ind}|\theta) &\equiv \delta^\theta,\end{aligned}$$

where $\delta^\theta \in (0, 1)$ is the per-period probability that a battle is “indecisive” in state θ , and $p_i^\theta \in (0, 1)$ the per-period probability of i ’s “win” in θ given a battle is decisive (i.e., not “indecisive”), satisfying that $p_D^\theta + p_A^\theta = 1$ for each $\theta \in \{d, a\}$. Each p_i^θ represents belligerent i ’s relative military strength in state θ . It is presumed that $p_D^d > p_D^a$ and $p_A^a > p_A^d$, so that each i is more advantageous in its own state. To further refer to the states, $\theta \in \{d, a\}$ may be added to the superscript of relevant symbols, as with δ^θ and p_i^θ above.

3.3 Erosion of Democracy’s Confidence

As the war proceeds, the belligerents use Bayes’ rule to learn the true state from developments on the battlefield. Put formally, as both D and A choose to “fight,” they update their beliefs as to the two states, shown as:

$$\begin{aligned}\Pr(d|\text{ind}^T) &\equiv \frac{\Pr(d) \Pr(\text{ind}|d)^T}{\Pr(d) \Pr(\text{ind}|d)^T + \Pr(a) \Pr(\text{ind}|a)^T} \\ &= \frac{1}{1 + \frac{\Pr(a)}{\Pr(d)} \left(\frac{\Pr(\text{ind}|a)}{\Pr(\text{ind}|d)} \right)^T} \\ \Pr(a|\text{ind}^T) &\equiv 1 - \Pr(d|\text{ind}^T),\end{aligned}\tag{1}$$

where ind^T denotes $T \in \{0, 1, 2, \dots\}$ times of “indecisive” battle outcomes. As democracies strategically select winnable targets when they begin wars, this war breaks out with the democracy’s advantage, but lengthy battles indicate the match to be more equal than originally estimated.⁷ The following assumption guarantees that the tide of war shifts against the democracy through fighting:

Assumption 1 *A battle is more likely to remain indecisive in state a than in d, or $\delta^d < \delta^a$.*

⁷It is unnecessary but sounds reasonable to hold “powerful pacifists” (Lake 1992), which imply $\Pr(d) > 1/2$ and $p_D^d > 1/2$; In words, it is more likely that the democracy is advantageous to the autocracy than vice versa.

By Assumption 1, $\Pr(ind|a)/\Pr(ind|d) > 1$, so that $\Pr(d|ind^T)$ decreases with T (Equation (1)).

3.4 Payoff Analyses and Equilibrium

Based on the beliefs updated from T periods of past “indecisive” outcomes, D calculates its current-period payoff from fighting if A also fights:

$$u_D(ind^T) \equiv \sum_{\theta \in \{d,a\}} \Pr(\theta|ind^T) u_D^\theta,$$

where for $i \in \{D, A\}$, u_i^θ is i 's per-period payoff from fighting in state θ :

$$u_i^\theta \equiv (1 - \delta^\theta) p_i^\theta W_i - c_i.$$

Particularly for the first period when no battles are preceded, D 's payoff from fighting is denoted as: $u_D \equiv u_D(ind^0)$.

As D fights longer, it becomes more pessimistic about the war's prospect, and thus $u_D(ind^T)$ decreases with T .⁸ For the state variable to influence D 's behavior, the following restriction is put on D 's payoff:

Assumption 2 W_D is in the range where:

$$\frac{c_D}{\sum_{\theta \in \{d,a\}} \Pr(\theta) (1 - \delta^\theta) p_D^\theta} < W_D < \frac{c_D}{(1 - \delta^a) p_D^a}.$$

By Assumption 2, D is willing to fight at least once ($u_D > 0$) and unwilling to fight in state a ($u_D^a < 0$). With Assumptions 1 and 2, D 's sequentially rational strategy against A 's fight forever can be uniquely identified:

Lemma 1 *There exists a unique $T^* \geq 1$ such that $u_D(ind^{T^*-1}) > 0$ and $u_D(ind^{T^*}) \leq 0$.*⁹

⁸Trivially, $u_D^d > u_D^a$, because $p_D^d > p_D^a$ and $\delta^d < \delta^a$ (Assumption 1).

⁹To rule out uninteresting mixed-strategy equilibria that may arise when either i 's expected payoff from fighting equals its payoff from surrendering, the model adopts the tie-breaking rule that i chooses to surrender if it is indifferent between fighting and surrendering. This rule will hold for the rest of the article.

Proof. When $T = 0$, $u_D(ind^T)$ is positive (Assumption 2). As T rises, $u_D(ind^T)$ monotonically decreases, because $\Pr(d|ind^T)$ decreases and $u_D^d > u_D^a$ (Assumption 1). As $T \rightarrow \infty$, $\Pr(a|ind^T)$ converges to one, so that $\lim_{T \rightarrow \infty} u_D(ind^T) = u_D^a$, which is negative (Assumption 2). Therefore, there must exist a unique time period $T^* \geq 1$ around which the sign of $u_D(ind^T)$ changes from being positive to become zero or negative. ■

Given T^* , D chooses to fight for T^* periods (based on up to $T^* - 1$ times of indecisive outcomes) and then surrenders in period $T^* + 1$ (based on T^* times of indecisive outcomes).

Anticipating D 's surrender in period $T^* + 1$, A estimates its continuation payoff from fighting, based on $T \in \{0, 1, 2, \dots, T^* - 1\}$ “indecisive” periods:

$$U_{A|T^*}(ind^T) \equiv \sum_{\theta \in \{d, a\}} \Pr(\theta|ind^T) U_{A|T^*}^\theta(ind^T),$$

where $U_{A|T^*}^\theta(ind^T)$ is A 's continuation payoff from fighting in θ after T times of ind if D surrenders in $T^* + 1$:¹⁰

$$\begin{aligned} U_{A|T^*}^\theta(ind^T) &\equiv \left(\sum_{t=T+1}^{T^*} \Pr(ind|\theta)^{t-T-1} u_A^\theta + \Pr(ind|\theta)^{T^*-T} \right) W_A \\ &= \left((1 - (\delta^\theta)^{T^*-T}) p_A^\theta + (\delta^\theta)^{T^*-T} \right) W_A - \frac{1 - (\delta^\theta)^{T^*-T}}{1 - (\delta^\theta)} c_A. \end{aligned}$$

In particular, for the first period, $U_{A|T^*} \equiv U_{A|T^*}(ind^0)$ and $U_{A|T^*}^\theta \equiv U_{A|T^*}^\theta(ind^0)$.

As the war evolves (with T rising), two factors influence A 's payoff from fighting: the timing of D 's surrender approaches ($U_{A|T^*}^\theta(ind^T)$ increases for each $\theta \in \{d, a\}$); and it becomes more likely that A is stronger ($\Pr(a|ind^T)$ increases). The assumption below guarantees these two factors surely raise A 's payoff from fighting over time:¹¹

Assumption 3 *It is in A 's interest to be stronger, or $(1 - \delta^d) p_A^d < (1 - \delta^a) p_A^a$.*

¹⁰To make a distinction, small letters (u and later-shown v) are assigned to per-period payoffs, and large letters (U and V) to continuation payoffs throughout the article.

¹¹While the former factor raises A 's payoff for sure, the latter has an ambiguous effect without Assumption 3, because the relative size between u_A^a and u_A^d is indeterminate—in state a , although the victory is more likely ($p_A^a > p_A^d$), the war tends to last longer ($\delta^a > \delta^d$). Assumption 3 suffices that $U_{A|T^*}^a(ind^T) > U_{A|T^*}^d(ind^T)$.

With Assumption 3, A never surrenders once it starts fighting, because $U_{A|T^*}(\text{ind}^T)$ increases with T . As the war persists, while D 's incentive to fight diminishes, A 's incentive strengthens. Given these contrasting incentives, their decision problems can be interpreted that D chooses when to surrender, but A determines whether or not to fight in the first period.¹² The key condition that generates these contrasting incentives and characterizes the equilibrium behavior is the difference in the probabilities of indecisive outcomes between the two states $\Pr(\text{ind}|d) < \Pr(\text{ind}|a)$ (Assumption 1).

Proposition 1 (i) If $U_{A|T^*} > 0$, the baseline model holds a unique subgame perfect Nash equilibrium, where D fights for T^* periods and surrenders in $T^* + 1$ and all subsequent periods, while A fights forever.¹³ (ii) If $U_{A|T^*} \leq 0$, A immediately surrenders in equilibrium.

Proof. (i) If $U_{A|T^*} > 0$, A fights in the first period regardless of D 's strategy. It continues to fight in all the subsequent periods, or $U_{A|T^*}(\text{ind}^T)$ increases with T , because $U_{A|T^*}^\theta(\text{ind}^T)$ increases for each $\theta \in \{d, a\}$, $\Pr(a|\text{ind}^T)$ increases (Assumption 1), and $U_{A|T^*}^a(\text{ind}^T) > U_{A|T^*}^d(\text{ind}^T)$ for any T (Claim 1 in Appendix B). Against A 's fight forever, D 's sequentially-rational strategy adopts T^* (Lemma 1). Even off the equilibrium path ($t \geq T^* + 2$), sequential rationality mandates D to surrender and A to fight. (ii) It is straightforward that A surrenders in the first period if $U_{A|T^*} \leq 0$. ■

This model is further augmented to examine how domestic regime types matter for war by incorporating democratic citizens and soldiers.

¹²To put it in another way, D 's marginal benefit from fighting decreases over time, but A 's one increases. A more detailed discussion appears in Appendix A.

¹³Note that unlike games with private information, this game contains only one information set in each period. It has no distinction between separating or pooling equilibria. No separation of the two states is possible. Any equilibrium must pool the two states.

4 Model I: Decreasing Audience Costs

The second model, Model I, aims to capture the Tocqueville-Iklé proposition that democracies face political difficulties with producing the decision to end a war. It highlights democratic citizens, who collectively influence their leader’s behavior by inflicting domestic political costs on him.

4.1 Public Opinion

Into the baseline model, Model I incorporates a number of democratic citizens C , whose population size is normalized to be one. Citizens have the same preference as their leader D except that they have diverse evaluations of the war $W_C \in (0, \infty)$, which follows a cumulative distribution function $F_C(\cdot)$ with its density $f_C(\cdot)$. A more hawkish citizen tends to have a larger W_C . As in the baseline model, all D , A , and C are presumed to share the common priors $\Pr(d)$ and $\Pr(a)$. All Assumptions 1, 2, and 3 set for the baseline model are kept in Model I.

Citizen C ’s per-period payoff from fighting for W'_C after T periods of “indecisive” battle outcomes is:

$$u_C(W'_C | ind^T) \equiv \sum_{\theta \in \{d, a\}} \Pr(\theta | ind^T) ((1 - \delta^\theta) p_D^\theta W'_C - c_D).$$

This citizen supports the war if $u_C(W'_C | ind^T) > 0$ and opposes it if $u_C(W'_C | ind^T) < 0$.¹⁴ Define $\bar{W}_C(T)$ with which C is indifferent between “fight” and “surrender” given ind^T , or $u_C(\bar{W}_C(T) | ind^T) = 0$, so that

$$\bar{W}_C(T) \equiv \frac{c_D}{\sum_{\theta \in \{d, a\}} \Pr(\theta | ind^T) (1 - \delta^\theta) p_D^\theta}. \quad (2)$$

Then, citizens with $W_C > \bar{W}_C(T)$ are those supporting the war, and they amount to:

$$\#cit(T) \equiv 1 - F_C(\bar{W}_C(T)),$$

which will determine the civilian influence on the democratic leader.

¹⁴Model I presumes that citizens condition their behavior on their beliefs as to the relative strength ($\Pr(d | ind^T)$). In contrast, Mueller (1973) found that the public support for war depends on accumulated casualties (cT). Our modeling approach would reconcile Mueller’s finding if citizens infer the relative strength from accumulated casualties.

For citizens to influence the leader's decision, some restrictions are put on $F_C(\cdot)$ and $f_C(\cdot)$.

Assumption 4 $F_C(\cdot)$ and $f_C(\cdot)$ satisfy the following three inequalities:

$$\begin{aligned}
(i) \quad & F_C \left(\frac{c_D}{\sum_{\theta \in \{d,a\}} \Pr(\theta) (1 - \delta^\theta) p_D^\theta} \right) < 1/2 \\
(ii) \quad & F_C \left(\frac{c_D}{(1 - \delta^a) p_D^a} \right) > 1/2 \\
(iii) \quad & f_C(W_C) > 0 \text{ for } W_C \in \left[\frac{c_D}{\sum_{\theta \in \{d,a\}} \Pr(\theta) (1 - \delta^\theta) p_D^\theta}, \frac{c_D}{(1 - \delta^a) p_D^a} \right).
\end{aligned}$$

By Assumption 4-(i), a majority of citizens support the war at the onset. By (ii), they oppose it in state a . By (iii), citizens have diverse evaluations on the war.

4.2 Domestic Influence on War

To delineate the civilian influence, suppose that by maintaining the war, the leader receives a per-period political payoff $ac(\#cit(T))$, which varies with the public approval for fighting $\#cit(T)$. Then D chooses to fight after T periods of ind if and only if $u_D(ind^T) + ac(\#cit(T)) > 0$, or equivalently

$$u_D(ind^T) > -ac(\#cit(T)),$$

from which $ac(\#cit(T))$ can also be interpreted as “audience costs” that D incurs when he surrenders despite a fraction $\#cit(T)$ of the opposition to surrendering. That is, the democracy continues to fight until the payoff from fighting falls below the cost of breaking the prewar commitment to the defense of national interests W_D . On $ac(\#cit(T))$, two restrictions are reasonably set: (i) $ac(\#cit(T))$ increases with $\#cit(T)$ (i.e., surrendering is more costly if more citizens support the war); and (ii) $ac(1/2) = 0$ (i.e., the sign of such costs depends on whether the median citizen supports or opposes the war).

As the democracy appears less and less advantageous over time, the civilian support for the war declines, and audience costs shrink.

Lemma 2 $ac(\#cit(T))$ decreases with T .

Proof. As T rises, $\Pr(d|ind^T)$ decreases (Assumption 1), $\overline{W}_C(T)$ increases (to maintain $u_C(\overline{W}_C(T)|ind^T) = 0$), $F_C(\overline{W}_C(T))$ increases (Assumption 4-(iii)), $\#cit(T)$ decreases (by definition of $\#cit(\cdot)$), and then $ac(\#cit(T))$ decreases. ■

The timing of D 's surrender decision could be affected by the public opinion. By taking his political payoff of fighting into account, D surrenders right after period T^I when the payoff from fighting is outweighed by audience costs:

Lemma 3 *There exists a unique $T^I \geq 1$ such that $u_D(ind^{T^I-1}) > -ac(\#cit(T^I - 1))$ and $u_D(ind^{T^I}) \leq -ac(\#cit(T^I))$.*

Proof. By Lemma 1, the left-hand side $u_D(ind^T)$ is originally positive, monotonically decreases with T and converges to u_D^a , which is negative. In contrast, the right-hand side $-ac(\#cit(T))$ is originally negative (Assumptions 4-(i)), monotonically increases with T (Lemma 2) and will turn to be positive (Assumption 4-(ii)). Thus, there must exist a unique time period $T^I \geq 1$ around which the left-hand side becomes lower than the right-hand side. ■

If audience costs persist even after the war's outbreak, a majority of citizens would disallow their government to easily revoke its original war aims. In response, expecting that the democracy is willing to fight tenaciously, the autocracy's decision to initiate the war could also be altered.

Proposition 2 (Tocqueville [1835, 1840] 2004; Iklé 1971) *If a majority of citizens still endorse the war in period T^* ($\#cit(T^*) > \frac{1}{2}$) and their influence is sufficiently large ($ac(\#cit(T^*)) > -u_D(ind^{T^*})$), then the democracy will postpone its surrender decision and fight longer ($T^I > T^*$). In response, the autocracy which would fight forever without audience costs will immediately surrender if W_A is in the range where $U_{A|T^*} > 0$ and $U_{A|T^I} \leq 0$.¹⁵*

Proof. By $\#cit(T^*) > \frac{1}{2}$, $ac(\#cit(T^*)) > 0$. By $ac(\#cit(T^*)) > -u_D(ind^{T^*})$, D is willing to fight at least in period $T^* + 1$, and thus $T^I > T^*$, which suffices $U_{A|T^I} < U_{A|T^*}$. ■

In light of the domestic audience, the democracy would keep on fighting even after its leader no longer deemed the war to be worth fighting without domestic constraints.

¹⁵Conversely, the decision can be made earlier if it is expedited by the majority (Bennett and Stam 1996). Put formally, $T^I < T^*$ if $\#cit(T^* - 1) < \frac{1}{2}$ and $ac(\#cit(T^* - 1)) < -u_D(ind^{T^*-1})$.

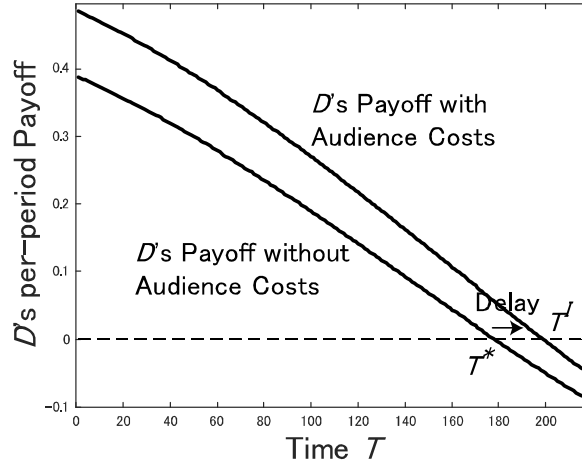


Figure 1: The delay in the democracy's surrendering caused by audience costs

Put differently, democracies are less likely to lose short wars (Bennett and Stam 1998), because of the domestic opposition to compromise in haste. However, as audience costs decrease over time, the political decision to end a war, which was difficult in the short run, can be made more easily in the long run. Decreasing audience costs can thus cause a delay to end wars, as shown in Figure 1, which contrasts D 's per-period payoffs with and without audience costs ($u_D(ind^T)$ and $u_D(ind^T) + ac(\#cit(T))$).¹⁶

Furthermore, this delay might undermine the autocracy's willingness to fight, because longer wars are likely to inflict larger costs. In this sense, the audience-costs logic of deterrence pertains even to the termination of war as well as the onset (Croco 2011). In contrast to extant audience-costs models of war onset (Fearon 1994; Guisinger and Smith 2002; Slantchev 2006; Tarar and Leventoglu 2009, 2013), which suggest the size of audience costs matters for deterrence, our theory holds that the deterrent effect of audience costs depends not just on the size, but also on how durably such costs persist after a war's outbreak.¹⁷

¹⁶The following parameters and functions are adopted in Figure 1: $\Pr(d) = 8/10$; $\delta^d = 98/101$; $\delta^a = 98/100$; $p_D^d = 8/10$; $p_D^a = 4/10$; $W_D = 50$; $c_D = 1$; $W_C \sim U\left[c_D / \sum_{\theta \in \{d,a\}} (1 - \delta^\theta) p_{D,\theta}^{\theta, c_D} / (1 - \delta^a) p_D^a\right]$; $ac(\#cit(T)) = \#cit(T) / 10$.

¹⁷In our model, key determinants of such durability are the public evaluation of war ($F_C(\cdot)$), their influence ($ac(\cdot)$), and the cost of fighting (c_D).

5 Model II: Declining Battlefield Effectiveness

The next model, Model II, incorporates soldiers instead of citizens in democracy. The soldiers collectively influence war outcomes, as their effectiveness on the battlefield changes over time (Reiter and Stam 1998, 2002).¹⁸ Model II critically differs from the baseline model in that the probability distribution of defeats is endogenously determined.

5.1 Individual Soldiering

In Model II, the democracy has mobilized a number of soldiers S , whose manpower is normalized to be one. They share the same information on $\theta \in \{d, a\}$ with D and A . In every period, each soldier decides to engage in his mission or evade it. If he engages, he incurs a per-period cost $c_S > 0$ and accomplishes the mission with per-period probability $\rho^\theta \in (0, 1)$ in state θ , for which $\rho^a < \rho^d$. In words, the democracy's military missions are more likely to be accomplished in state d than in a . If a soldier evades, he earns nothing. Soldiers are heterogeneous in evaluation of mission accomplishment $W_S \in (0, \infty)$, which distributes according to its c.d.f. $F_S(\cdot)$ and p.d.f. $f_S(\cdot)$ with the following restriction:

Assumption 4' W_S distributes so broadly that

$$f_S(W_S) > 0 \text{ for } W_S \in \left[\frac{c_S}{\sum_{\theta \in \{d, a\}} \Pr(\theta) \rho^\theta}, \frac{c_S}{\rho^a} \right).$$

A soldier's payoff from engaging in a mission with value W'_S after T "indecisive" periods can be shown as:

$$v_S(W'_S | ind^T) \equiv \sum_{\theta \in \{d, a\}} \Pr(\theta | ind^T) \rho^\theta W'_S - c_S.$$

A soldier engages in his mission (and is called "effective") if $v_S(W'_S | ind^T) > 0$ and evades it (being "ineffective") if $v_S(W'_S | ind^T) < 0$. From the condition that

¹⁸In contrast, autocratic soldiers are abstracted away from Model II on the ground that the probability of autocratic victory appears relatively stable over time in past wars (Bennett and Stam 1998).

$v_S(\overline{W}_S(T) | ind^T) = 0$, the threshold $\overline{W}_S(T)$ that demarcates the effective and ineffective soldiers given ind^T can be derived as:

$$\overline{W}_S(T) \equiv \frac{c_S}{\sum_{\theta \in \{d,a\}} \Pr(\theta | ind^T) \rho^\theta}. \quad (3)$$

Because those with $W_S > \overline{W}_S(T)$ are effective, the fraction of effective soldiers after T periods of ind is:

$$\#sol(T) \equiv 1 - F_S(\overline{W}_S(T)),$$

which in turn determines $pp_i^\theta(\#sol(T))$, or the per-period probability of i 's winning in state θ after T periods of ind such that $pp_D^\theta(\#sol(T)) + pp_A^\theta(\#sol(T)) = 1$ for $\theta \in \{d, a\}$ and $T \in \{0, 1, 2, \dots\}$. If more soldiers effectively engage in their missions, the democracy is more likely to prevail in battles; so that $pp_D^\theta(\#sol(T))$ is assumed to increase with $\#sol(T)$ for each $\theta \in \{d, a\}$. Because $\Pr(d | ind^T)$ decreases with T , the soldiers tend to reduce their confidence with mission success, and they undertake less and less missions in the battlefield:

Proposition 3 (Reiter and Stam 1998, 2002) $pp_D^\theta(\#sol(T))$ decreases with T for each $\theta \in \{d, a\}$.

Proof. The proof resembles that of Lemma 2. As T rises, $\Pr(d | ind^T)$ decreases (Assumption 1), $\overline{W}_S(T)$ increases, $F_S(\overline{W}_S(T))$ increases (Assumption 4'), $\#sol(T)$ decreases, and finally $pp_D^\theta(\#sol(T))$ decreases. ■

By Proposition 3, as the democracy becomes less advantageous over time, the autocracy has more chances to prevail. Note that the actual decline of the democracy's battlefield effectiveness ($\sum_{\theta \in \{d,a\}} \Pr(\theta) pp_D^\theta(\#sol(T))$) is different from the decline we observe in raw data ($\sum_{\theta \in \{d,a\}} \Pr(\theta | ind^T) pp_D^\theta(\#sol(T))$), as the latter contains the selection effect—because strong democracies tend to win quickly and decisively, only weak democracies are left in long wars.¹⁹ This effect generates a downward bias in the democracy's observed battlefield effectiveness. Therefore, the actual decline might not be so significant as it appears to be. Furthermore, this bias even widens as the war lengthens (Figure 2).²⁰

¹⁹The effect is caused by the change in the probability distribution of the states $\Pr(\theta | ind^T)$.

²⁰For Figure 2, parameters and functions are set as: $\rho^d = 8/10$; $\rho^a = 4/10$; $c_S = 1$; $W_S \sim U[c_S / \sum_{\theta \in \{d,a\}} \Pr(\theta) \rho^\theta, c_S / \rho^a]$; $pp_D^d(\#sol(T)) = 9\#sol(T) / 10$; $pp_D^a(\#sol(T)) = 9\#sol(T) / 20$.

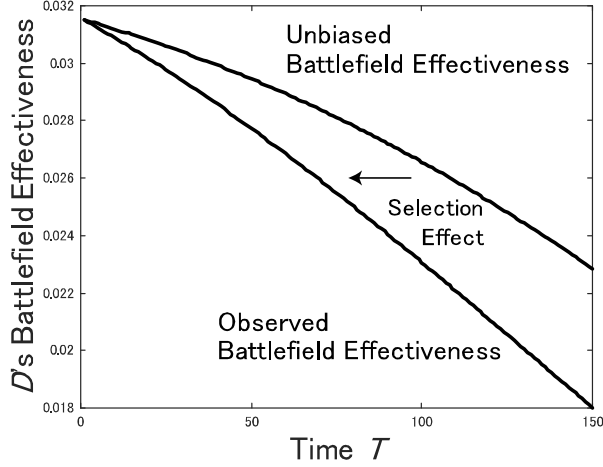


Figure 2: The downward bias in the democracy’s battlefield effectiveness caused by the selection effect

5.2 Military Influence on Political Decisions

In light of changing battlefield effectiveness, the belligerents’ payoffs are slightly modified— p_i^θ is replaced with $pp_i^\theta(\#sol(T))$ in their payoffs (Appendix C). Let $v_D(ind^T)$ denote D ’s per-period payoff from fighting after T “indecisive” periods, and in particular $v_D \equiv v_D(ind^0)$. The assumption below replaces Assumption 2 of the baseline model, holding that D is willing to fight at least once ($v_D > 0$) and unwilling to fight in the long run ($\lim_{T \rightarrow \infty} v_D(ind^T) < 0$).

Assumption 2’ W_D is in the range where:

$$\frac{c_D}{\sum_{\theta \in \{d,a\}} (1 - \delta^\theta) pp_D^\theta(\#sol(0))} < W_D < \lim_{T \rightarrow \infty} \frac{c_D}{(1 - \delta^a) pp_D^a(\#sol(T))}.$$

Assumption 2’ guarantees that the timing of D ’s surrender is uniquely determined:

Lemma 4 *There exists a unique $T^{II} \geq 1$ such that $v_D(ind^{T^{II}-1}) > 0$ and $v_D(ind^{T^{II}}) \leq 0$.*

Proof. Because $pp_D^\theta(\#sol(T))$ decreases with T (Proposition 3), D ’s per-period payoff from fighting in each θ ($v_D^\theta(ind^T)$ in Appendix C) also decreases. Therefore, the monotonicity of $v_D(ind^T)$ holds— $v_D(ind^T)$ decreases with T , because $\Pr(d|ind^T)$ decreases (Assumption 1). By Assumption 2’, T^{II} is greater than or equal to one but is finite. ■

Given D 's strategy of T^{II} , A 's continuation from fighting after T “indecisive” periods can also be specified ($V_{A|T^{II}}(ind^T)$ in Appendix C and $V_{A|T^{II}} \equiv V_{A|T^{II}}(ind^0)$). Corresponding to Assumption 3 of the baseline model, the following restriction suffices that A prefers state a to d .

Assumption 3' *Regardless of T , $(1 - \delta^d) pp_A^d(\#sol(T)) < (1 - \delta^a) pp_A^a(\#sol(T))$.*

Unlike the audience-costs argument of Model I, the inclusion of soldiers in Model II affects not only the timing of surrendering (T^{II}) but also the probabilities of defeats ($pp_i^\theta(\#sol(T))$ for any $T \leq T^{II}$). In other words, soldiers influence the war at both the strategic and tactical levels. At the strategic level, when the democracy surrenders depends on how rapidly its battlefield effectiveness declines. At the tactical level, the democracy is more likely to win short wars, because higher military performance is expected in the early stage. In these senses, the negative association between the probability of democratic victory and the duration of war should be addressed in terms of changing battlefield effectiveness (Reiter and Stam 1998) and also of the shifting incentives to surrender (Bennett and Stam 1998).

Proposition 4 *If the fraction of effective soldiers $\#sol(T^*)$ is so large in period T^* that $v_D(ind^{T^*}) > 0$, then D will fight longer in Model II than in the baseline model ($T^{II} > T^*$). In response, the autocracy which would fight forever without endogenous battlefield effectiveness will immediately surrender if W_A is in the range where $U_{A|T^*} > 0$ and $V_{A|T^{II}} \leq 0$.*

Proof. While $u_D(ind^{T^*}) \leq 0$ (Lemma 1), a sufficiently large $\#sol(T^*)$ guarantees that $v_D(ind^{T^*}) > 0$. Given ind^{T^*} , D fights at least one more period, and thus $T^{II} > T^*$. Because A prefers state a to d (Assumption 3'), and because $pp_A^\theta(\#sol(T))$ increases with T in each θ (Proposition 3), A 's per-period payoff $v_A(ind^T)$ increases with T , and thus A fights forever if $V_{A|T^{II}} > 0$ (Appendix A). Moreover, because $pp_D^\theta(\#sol(T))$ decreases with T in each θ , $v_D(ind^T) > u_D(ind^T)$ for $T \leq T^*$. Because the sum of D and A 's per-period payoffs is constant, this inequality is equivalent to $v_A(ind^T) < u_A(ind^T)$ for $T \leq T^*$, which suffices $V_{A|T^{II}} < U_{A|T^*}$ for $T^{II} > T^*$. ■

The democracy's short-run advantage and long-run disadvantage in the battlefield can affect when it surrenders (T^{II}). Moreover, its endogenous military effectiveness has marked effects on deterrence—whether the autocracy fights or not in the first

place. In Model I, a delay in the democracy's surrendering caused by audience costs magnifies the deterrent effect—if $T^I > T^*$, then $U_{A|T^I} < U_{A|T^*}$. Model II also stipulates the corresponding effect if $T^{II} > T^*$. Moreover, because the democracy's declining battlefield effectiveness means that the autocracy is less advantageous in earlier battles, even greater deterrability can be expected in Model II. In other words, a democracy's short-run military advantage can serve to enhance its ability to deter opponents.

6 Regime Types and Military Strategies

The final analysis further extends Models I and II to seek implications toward military strategies. The purposes of the extensions are: (i) to test whether the choices of strategies predicted by these models are consistent with empirical findings (Reiter and Meek 1999); and once the consistency is confirmed, (ii) to deliver the rationales for the adoptions of the predicted strategies.

The military strategies addressed below are those archetypal in the literature (Mearsheimer 1983; Reiter and Stam 2002; Stam 1996). One is a shortening strategy that aims to end war sooner (e.g., blitzkrieg or maneuver), and the other a protracting strategy that tends to make war longer (e.g., attrition). The immediate effects of these strategies are captured by the difference in battle decisiveness in the models.

6.1 Shortening vs. Protracting Strategies

The extensions below apply to both Models I and II. In the extended models, the belligerents simultaneously choose either shortening or protracting strategy at the onset to maximize their own expected payoffs. The combination of their strategies (henceforth, called the “military strategy profile”) determines the probability of the “indecisive” battle outcome in each state θ . To be concrete, $\Pr(ind|\theta)$ equals ε^θ if both D and A choose the shortening strategy, η^θ if only either side chooses it, and δ^θ if neither side chooses it, where $0 < \varepsilon^\theta < \eta^\theta < \delta^\theta < 1$ for each θ . Regarding the protracting strategy as default, a belligerent can make battles more decisive by switching to the shortening strategy. The military strategy profile corresponding to each probability $\sigma^\theta \in \{\varepsilon^\theta, \eta^\theta, \delta^\theta\}$ is denoted in a reduced form $\sigma \in \{\varepsilon, \eta, \delta\}$.

The assumption below ensures that military strategies have no informational effects:

Assumption 1' $\varepsilon^a/\varepsilon^d = \eta^a/\eta^d = \delta^a/\delta^d > 1$.

With Assumption 1', Bayesian learning of the states is unaffected by the choices of strategies. Unless the purpose of choosing a certain strategy is informational, the assumption should make sense and can illuminate other possible effects of the strategies. In addition, the extended models take over Assumptions 2-4 and 2'-4' regardless of $\sigma^\theta \in \{\varepsilon^\theta, \eta^\theta, \delta^\theta\}$.

6.2 Military Strategies in Light of Audience Costs or Endogenous Battlefield Effectiveness

Below we derive the rational military strategies in extended Models I and II. To address the choices of military strategies, it must be taken into account how the choices influence the belligerents' subsequent decisions to fight or surrender.

Lemma 5 *In both extended Models I and II, D can postpone its decision to surrender with the shortening strategy, regardless of A 's military strategy; i.e., $T^I(\delta) \leq T^I(\eta) \leq T^I(\varepsilon)$ and $T^{II}(\delta) \leq T^{II}(\eta) \leq T^{II}(\varepsilon)$, where $T^I(\sigma)$ and $T^{II}(\sigma)$ are the numbers of periods before D surrenders given military strategy profile σ in Models I and II, respectively.*

Proof. The proofs of Lemma 5 above and Proposition 5 below are left in Online Appendix E. ■

By choosing the shortening strategy, the democracy can increase the chance of short-run victory. Moreover, in extended Model I, the increased chance raises the audience costs of surrendering through promoting the public support of the war. In extended Model II, the shortening strategy assists the democracy in defeating its opponent before its battlefield effectiveness diminishes.²¹ These factors altogether induce the democracy to fight longer (with larger $T^I(\sigma)$ and $T^{II}(\sigma)$) although battles are made more decisive (with a smaller σ^θ).

While the democracy benefits from the shortening strategy, the autocracy has rather mixed incentives for shortening and protracting strategies.

Proposition 5 *In both extended Models I and II, D 's optimal military strategy is the shortening strategy regardless of A 's military strategy, while A 's optimal military strategy is indeterminate.*

Even though the two models have distinct theoretical grounds, they share analogous predictions as to the choices of military strategies. In fact, these predictions are consistent with empirical findings that democracies tend to adopt the maneuver strategy, whereas the choices of autocracies are rather ambiguous (Reiter and Meek

²¹If the shortening strategy has some tactical advantage, it might raise ρ^θ (the probability that a soldier in D accomplishes a mission). Extended Model II suggests that even without such advantage, the shortening strategy can benefit D .

	Democracy (D)	Autocracy (A)
Shortening strategy (maneuver)	To reduce cost of fighting	
	To end war before its public support and/or battlefield effectiveness diminish	To reduce cost of fighting
	To postpone its surrendering	
Protracting strategy (attrition)	None	To fight long until D 's public support and/or battlefield effectiveness diminish
		To hasten D 's surrendering

Table 2: Rationales for the choices of military strategies

1999). These results suggest the possibility that both the mechanisms of audience costs and of battlefield effectiveness might be at work in democracies' prosecution of wars. For instance, democratic soldiers outperform autocratic counterparts in the short run if $pp_D^\theta(\#sol(T)) > 1/2$ for $T < T^{II}$ (Reiter and Stam 1998), but democratic citizens refuse futile fighting in the long run if $ac(\#cit(T)) < 0$ for $T > T^I$ (Bennett and Stam 1998). These effects caused by regime types could shape wars fought by democracies.

According to our models, the shortening strategy surely advantages the democracy in having a higher probability of short-run victory, while reducing the risk of fighting long. A lengthy war disadvantages the democracy, because the public opposes fighting long, and also because its forces cannot effectively fight long. In contrast, the effects of the shortening strategy on the autocracy are more complicated. On one hand, it can reduce the risk of fighting long. On the other hand, it can also lower the probability that the democracy ultimately surrenders, because it makes battles more decisive and the war more likely to end before the democracy surrenders, and also because the shortening strategy taken by the autocracy can induce the democracy to fight more durably (Lemma 5). These conflicting effects make the autocracy's optimal military strategy indeterminate. According to our models, it depends mainly on the per-period cost of fighting (c_A) and on how durably the democracy can fight ($T^I(\sigma)$ and $T^{II}(\sigma)$). That means, the protracting strategy would be desirable for the autocracy if the per-period cost of fighting is smaller and/or if the democracy is expected to give in earlier.

At bottom, the logic behind democracies' adoption of the shortening strategy is to have higher chances of short-run victory. With the shortening strategy, a democracy can end a war before the public withdraws their support and before its soldiers reduce their performance on the battlefield. On the other hand, autocracies may benefit from either shortening or protracting strategy. While the shortening strategy helps to reduce the risk of prosecuting a lengthy war, the protracting strategy might also be functional if an autocracy is willing to fight until a democracy's long-run weakness materializes. The rationales for the choices of military strategies are summarized in Table 2.

7 Concluding Remark

Conceivably, a straightforward interpretation of the negative association between the probability of democratic victory and the duration of war can be found in the proposition that democracies' battlefield effectiveness declines over time (Reiter and Stam 1998, 2002). This decline seems to disadvantage democracies in long wars. We are concerned that this proposition hinges on the presumption that the duration of war is exogenously given. If democracies' military advantages dissipate in around eighteen months (Bennett and Stam 1998), why do not they stop fighting by then? If they do stop fighting, there should be few lengthy wars fought by democracies, generating a bias in the data set on past wars. If they do not, that is presumably because they cannot. Namely, the domestic opposition prevents democracies from ending wars too hastily. A consequential delay in the process of ending wars is well exemplified by the closing phase of the Vietnam War—the U.S. decision to withdraw troops was postponed by the successive presidents' concerns about their next elections (Ellsberg 1972: 100-107). This is what Tocqueville ([1835, 1840] 2004) foresaw and Iklé (1971) was afraid of.

Indeed, the duration of war is a choice of belligerents. That is, they choose to fight until the condition for peace matures (Fearon 2007; Powell 2004; Slantchev 2003b; Smith and Stam 2004; Wagner 2000). Hence, the seeming decline of democracies' advantages may be a result of the selection effect—as strong democracies win swiftly and decisively, only weak democracies are likely to fight long. In this regard, the selection effect may operate even after a war's outbreak (Koch 2009) as well as before (Bueno de Mesquita et al. 1999; Bueno de Mesquita and Siverson 1995; Clark and Reed 2003; Reed and Clark 2000). It might still hold true that democracies are strong especially at short wars, but the decline might not be so significant in light of the selection effect. In fact, we have demonstrated that observed battlefield effectiveness of democracies would contain a downward bias if the selection effect is not controlled for (Figure 2). It is then worth postulating the causality in reverse; i.e., war outcomes do affect the duration.

We have thus explored the alternative proposition (Tocqueville [1835, 1840] 2004; Iklé 1971)—when democracies are disadvantageous, wars tend to be long—as well as the one by Reiter and Stam (1998, 2002) shown above. In doing so, we adopt the formal-modeling approach that is helpful not only to delineate the strategic inter-

	Model I	Model II
Key variables	Audience costs	Battlefield effectiveness
Proponents	Tocqueville ([1835, 1840] 2004) and Iklé (1971)	Reiter and Stam (1998, 2002)
Pivotal players	Democratic citizens	Democratic soldiers
Influence on battle outcomes	No	Yes $\left(\begin{array}{c} \text{decreasing} \\ \Pr(\text{win}_D \theta) \end{array} \right)$
Influence on surrender decisions	Possible ($T^I \neq T^*$)	Possible ($T^{II} \neq T^*$)
Deterrence effect on autocracy (given $T^I = T^{II}$)	Weaker (larger $U_{A T^I}$)	Stronger (smaller $V_{A T^{II}}$)
Military strategies $\left(\begin{array}{c} \text{maneuver} \\ \text{vs. attrition} \end{array} \right)$	D : maneuver A : indeterminate	D : maneuver A : indeterminate

Table 3: Summary comparison between Models I and II

actions across the key players (e.g., state leaders, citizens, and soldiers) but also to illuminate the causal link between the key variables (the probabilities of outcomes and the duration of war). With a series of models, we have examined these two propositions by testing: (i) whether they possess behavioral rationalist foundation; and (ii) whether they are consistent with empirical findings on the choices of military strategies.²² To our surprise, both the propositions satisfy these two qualifications.²³ That means, our analysis suggests that in explaining the negative association, both decreasing audience costs and declining battlefield effectiveness complement the selection effect (Reed and Clark 2000). Our summary comparison between Models I and II appears in Table 3.

While empirical studies have increasingly emphasized the relevance of domestic politics to war termination, formal studies on this relevance have been limited presumably because of complexity with theoretical analyses. With formal models that

²²Our theory is also consistent with the empirical finding that an opponent adopting the maneuver strategy is more difficult to deter (Mearsheimer 1983), because the expected cost of implementing the strategy is lower.

²³We were originally doubtful that the Tocqueville-Iklé proposition meets the latter qualification, because if war outcomes affect the duration, any attempt to manipulate the duration seems not to change the outcomes. However, our theory holds that although citizens have no influence on the battlefields, they do affect both the outcomes and duration through their influence on the government's decision to fight or surrender.

highlight a democracy's citizens and soldiers, we have attempted to depict the relevance. As the extant theoretical literature on domestic politics and international relations have greatly contributed to our understanding of war outbreak, future theoretical progress will, we hope, assist our understanding of war termination.

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APPENDIX

A Decision Rules to Fight or Surrender

In our models, each player decides whether to fight or surrender in every time period. This decision problem can be redefined as another problem of when to surrender across periods (or to fight forever). Given an information set, a player chooses the best timing of surrendering. This problem is tractable if monotonicity holds for per-period payoffs.

Case 1 *If a player's per-period payoff from fighting decreases over time, and his payoff from surrendering is constant, he compares his current-period payoff from fighting with the payoff from surrendering to choose when to surrender.*

In Case 1, even if fighting is unworthy in future periods, it may be worth fighting in the current period. A player keeps on fighting as long as the current-period payoff from fighting is larger than the payoff from surrendering. He then stops fighting when it becomes unworthy. Case 1 applies to D of all the three models, C of Model I (for the support decision), and S of Model II (for the mission decision).

Case 2 *If a player's per-period payoff from fighting increases over time, he compares his continuation payoff from fighting forever with the payoff from surrendering to decide whether or not to fight in the first period.*

In Case 2, even if short fighting is unworthy, it may still be worth fighting long. In other words, if it is worth fighting in the first period, it must be worth fighting for all the future periods. A player thus takes all possible future payoffs from fighting into account, anticipating some benefits (e.g., his opponent's surrendering). Case 2 applies to A of all the three models.

B Claim and Its Proof for Baseline Model

Claim 1 $U_{A|T^*}^a(ind^T) > U_{A|T^*}^d(ind^T)$ for any $T \in \{0, 1, 2, \dots, T^* - 1\}$.

Proof. By definition, $U_{A|T^*}^\theta(ind^T) = U_{A|T^*-T}^\theta$. Also by definition and some algebra,

$$U_{A|T^*}^\theta = \left(1 - (\delta^\theta)^{T^*}\right) U_A^\theta + (\delta^\theta)^{T^*} W_A.$$

To assess the relative size between $U_{A|T^*-T}^a$ and $U_{A|T^*-T}^d$,

$$\begin{aligned} U_{A|T^*-T}^a - U_{A|T^*-T}^d &= \left(1 - (\delta^a)^{T^*-T}\right) U_A^a + (\delta^a)^{T^*-T} W_A \\ &\quad - \left(\left(1 - (\delta^d)^{T^*-T}\right) U_A^d + (\delta^d)^{T^*-T} W_A\right), \end{aligned}$$

which is positive, because $T^* - T \geq 1$ (or $T \leq T^* - 1$), $\delta^d < \delta^a$ (Assumption 1), and $U_A^d < U_A^a < W_A$ (Assumption 3). ■

C Payoffs from Fighting in Model II

For $i \in \{D, A\}$, i 's per-period payoff from fighting after T “indecisive” periods is:

$$v_i(ind^T) \equiv \sum_{\theta \in \{d, a\}} \Pr(\theta | ind^T) v_i^\theta(ind^T),$$

where

$$v_i^\theta(ind^T) \equiv (1 - \delta^\theta) pp_i^\theta(\#sol(T)) W_i - c_i. \quad (\text{A1})$$

A 's continuation payoff from fighting D with T^{II} after T “indecisive” periods is:

$$V_{A|T^{II}}(ind^T) \equiv \sum_{\theta \in \{d, a\}} \Pr(\theta | ind^T) V_{A|T^{II}}^\theta(ind^T),$$

where

$$V_{A|T^{II}}^\theta(ind^T) \equiv \sum_{t=T+1}^{T^{II}} (\delta^\theta)^{t-T-1} v_A^\theta(ind^{t-1}) + (\delta^\theta)^{T^{II}-T} W_A. \quad (\text{A2})$$

ONLINE APPENDIX

D Wars Fought by Democracies, 1816-2007

All the 33 wars fought by democracies since 1816 through 2007 are listed in Table A1.²⁴

E Proofs for Extended Models I and II

For the proofs below, $\sigma \in \{\varepsilon, \eta, \delta\}$ will be added to the subscript of relevant symbols for denotation.

Proof of Lemma 5. For Model I, the existence of $T^I(\varepsilon)$, $T^I(\eta)$, and $T^I(\delta)$ is guaranteed by Assumptions 1', 2, and 4, as with the proof of Lemmas 3. For Model II, the existence of $T^{II}(\varepsilon)$, $T^{II}(\eta)$, and $T^{II}(\delta)$ is derived from Assumptions 1', 2', and 4' (Lemma 4). Moreover, because military strategies have no influence on $\Pr(\theta|ind^T)$ (Assumption 1'), i 's payoff with a certain military strategy is larger than the payoff with the other strategy if the former is larger in any states. In other words, a comparison of i 's payoffs across i 's military strategies in each θ suffices to determine i 's incentives.

For Model I, with a smaller σ^θ , $u_{D|\sigma}^\theta$ is larger, where $u_{D|\sigma}^\theta \equiv (1 - \sigma^\theta)p_D^\theta W_D - c_D$. Thus, $u_{D|\sigma}(ind^T)$ is also larger. Also, with a smaller σ^θ , $ac(\#cit_\sigma(T))$ is larger, because $\overline{W}_{C|\sigma}(T)$ is smaller (Equation (2)), and thus $\#cit_\sigma(T)$ larger. These two changes tighten the condition for D 's surrender (raising the left-hand side and lowering the right-hand side of $u_{D|\sigma}(ind^T) < -ac(\#cit_\sigma(T))$) and therefore tend to increase $T^I(\sigma)$. With discrete time periods, $T^I(\varepsilon)$ ($T^I(\eta)$) is at least as large as $T^I(\eta)$ ($T^I(\delta)$).

For Model II, with a smaller σ^θ , $v_{D|\sigma}^\theta(ind^T)$ is larger (Equation (A1) with σ in Appendix C). So is $v_{D|\sigma}(ind^T)$. A larger $v_{D|\sigma}(ind^T)$ can hold the condition to fight ($v_{D|\sigma}(ind^T) > 0$) even with a larger T . Therefore, a smaller σ^θ tends to raise $T^{II}(\sigma)$. (Note that $\#sol(T)$ is independent of σ . Unlike $\overline{W}_C(T)$ of Equation (2), $\overline{W}_S(T)$ of Equation (3) does not contain δ^θ .) ■

²⁴The Turco-Cypriot War of 1974 was fought between two democratic states and thus is regarded as an exception and excluded from the list.

War (year)	Democratic belligerent(s)	Duration (months)	Outcome for democracy
Mexican-American (1846-47)	USA	17	Win
Conquest of Egypt (1882)	UK	2	Win
Sino-French (1884-85)	France	12	Win
Greco-Turkish (1897)	Greece	3	Loss
Spanish-American (1898)	USA	4	Win
Boxer Rebellion (1900)	UK, USA, etc.	2	Win
First Balkan (1912-23)	Greece	6	Win
Second Balkan (1913)	Greece	1	Win
World War I (1914-18)	UK, USA, etc.	52	Win
Estonian Liberation (1918-20)	Finland	14	Win
Latvian Liberation (1918-20)	Estonia	14	Win
Russo-Polish (1919-20)	Poland	20	Win
Hungarian Adversaries (1919)	Czechoslovakia	4	Win
Franco-Turkish (1919-21)	France	23	Tie
Lithuanian-Polish (1920)	Poland	5	Win
World War II (1939-45)	UK, USA, etc.	71	Win
Arab-Israeli (1948)	Israel	8	Win
Korean (1950-53)	UK, USA, etc.	37	Stalemate
Sinai War (1956)	Israel, UK, etc.	1	Win
Vietnam War, Phase 2 (1965-75)	Australia, USA	122	Loss
Six Day War (1967)	Israel	Less than 1	Win
Second Laotian, Phase 2 (1968-73)	USA	63	Loss
War of Attrition (1969-70)	Israel	17	Stalemate
Communist Coalition (1970-71)	USA	16	Transformed
Bangladesh (1971)	India	12	Win
Yom Kippur War (1973)	Israel	Less than 1	Win
Falkland Islands (1982)	UK	3	Win
Gulf War (1990-91)	UK, USA, etc.	8	Win
Cenepa Valley (1995)	Ecuador	1	Tie
War for Kosovo (1999)	UK, USA, etc.	3	Win
Kargil War (1999)	India	2	Win
Invasion of Afghanistan (2001)	UK, USA, etc.	2	Transformed
Invasion of Iraq (2003)	UK, USA, etc.	2	Transformed

Table A1: All 33 wars fought by democracies, 1816-2007

Proof of Proposition 5. As in Propositions 2 and 4, D and A 's decisions to fight or surrender are uniquely determined for a given military strategy profile σ in both extended Models I and II, where regardless of σ , D 's per-period payoff from fighting monotonically decreases over time, while A 's payoff from fighting increases (Assumptions 3 and 3').

For Model I, D 's payoff comparison is immediate from the proof of Lemma 5: $U_{D|\delta} < U_{D|\eta} < U_{D|\varepsilon}$, where for $\sigma \in \{\varepsilon, \eta, \delta\}$, $U_{D|\sigma}$ is D 's continuation payoff from fighting given σ

$$U_{D|\sigma} \equiv \sum_{\theta \in \{d,a\}} \Pr(\theta) \sum_{t=1}^{T^I(\sigma)} (\sigma^\theta)^{t-1} (u_{D|\sigma}^\theta + ac(\#cit_\sigma(t-1))).$$

For A , a smaller σ^θ has both positive and negative effects on $U_{A|T^I(\sigma),\sigma}^\theta$. The positive effect is to reduce the duration of the war $((1 - (\sigma^\theta)^{T^I(\sigma)}) / (1 - \sigma^\theta))$ for given $T^I(\sigma)$. The negative effect is to lower the probability of reaching D 's surrendering $((\sigma^\theta)^{T^I(\sigma)})$. The latter effect is even more significant with a larger $T^I(\sigma)$ (Lemma 5). Whether $U_{A|T^I(\sigma),\sigma}$ increases or decreases depends on which of these effects on $U_{A|T^I(\sigma),\sigma}^\theta$ dominates.

For D of Model II, it is immediate from the proof of Lemma 5 that $V_{D|\delta} < V_{D|\eta} < V_{D|\varepsilon}$, where $V_{D|\sigma}$ is D 's continuation payoff from fighting given σ

$$V_{D|\sigma} \equiv \sum_{\theta \in \{d,a\}} \Pr(\theta) \sum_{t=1}^{T^{II}(\sigma)} (\sigma^\theta)^{t-1} v_{D|\sigma}^\theta (ind^{t-1}).$$

For A , the shortening strategy may raise or lower $V_{A|T^{II}(\sigma),\sigma}$, depending on the relative size between its positive and negative effects on $V_{A|T^{II}(\sigma),\sigma}^\theta$ (Equation (A2) with σ and ind^0 in Appendix C). For the positive effect, a smaller σ^θ reduces the duration of war $((1 - (\sigma^\theta)^{T^{II}(\sigma)}) / (1 - \sigma^\theta))$. For the negative effect, a smaller σ^θ makes D 's surrendering less likely to be reached (with smaller $(\sigma^\theta)^{T^{II}(\sigma)}$). It is even less likely with a larger $T^{II}(\sigma)$ (Lemma 5). ■

δ^a	BM		Model I					Model II				
	T^*	$U_{A T^*}$	T^I	U_D	$U_{A T^I}$	P_A^I	Dur^I	T^{II}	V_D	$V_{A T^{II}}$	P_A^{II}	Dur^{II}
0.96	300	6.84	294	16.83	6.84	0.28	21.16	174	18.79	-1.04	0.20	21.15
0.97	178	1.26	198	11.66	1.20	0.2804	26.84	130	12.77	-5.44	0.21	26.63
0.98	23	47.83	78	2.92	3.45	0.36	32.30	56	3.18	7.45	0.36	28.73

Table A2: Results in the numerical example

F Numerical Example

With specific parameters and functions, we confirm our theoretical predictions. From the results summarized in Table A2, the following implications can be read:

- Audience costs influence the timing of D 's surrendering (Proposition 2). In particular, D 's surrendering is delayed ($T^I > T^*$) when $\delta^a = 0.97$ or $\delta^a = 0.98$. Moreover, this delay generates the deterrence effect on A ($U_{A|T^I} < U_{A|T^*}$).
- Endogenous battlefield effectiveness also influences the timing (Proposition 4). D fights longer ($T^{II} > T^*$) for $\delta^a = 0.98$. In Model II, D 's short-run advantage can enhance its ability to deter A ($V_{A|T^{II}} < U_{A|T^*}$ regardless of δ^a).
- With the shortening strategy, D 's surrendering is delayed (Lemma 5). As δ^a decreases, T^I and T^{II} increase.
- With the shortening strategy, D can raise its continuation payoff (Proposition 5). With a smaller δ^a , U_D and V_D tend to be larger.
- With the shortening strategy, A does not necessarily raise its continuation payoff (Proposition 5). As δ^a falls, $U_{A|T^I}$, and $V_{A|T^{II}}$ change non-monotonically. Whether the shortening strategy raises or lowers A 's continuation payoff depends on its effects on the probability of A 's winning (P_A^I and P_A^{II}) and the expected duration (Dur^I and Dur^{II}).

The parameters and functions are set as follows: for the baseline model, $\Pr(d) = \frac{8}{10}$, $\frac{\delta^a}{\delta^d} = \frac{101}{100}$ (Assumption 1'), $p_D^d = \frac{8}{10}$, $p_D^a = \frac{4}{10}$, $W_D = 50$, $c_D = 1$, $W_A = 100$, and $c_A = 1$; in addition to those above, for Model I, $W_C \sim U \left[\frac{c_D}{\sum_{\theta \in \{d,a\}} \Pr(\theta)(1-\delta^\theta)p_D^\theta}, \frac{c_D}{(1-\delta^a)p_D^a} \right]$ and $ac(\#cit(T)) = \frac{1}{10}\#cit(T)$; and for Model II, $W_S \sim U \left[\frac{c_S}{\sum_{\theta \in \{d,a\}} \Pr(\theta)\rho^\theta}, \frac{c_S}{\rho^a} \right]$, $c_S = 1$, $pp_D^d(\#sol(T)) = \frac{9}{10}\#sol(T)$, and $pp_D^a(\#sol(T)) = \frac{9}{20}\#sol(T)$.