Estimating Cost Functions in Differentiated Product Oligopoly Models without Instruments

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An Endogeneity Problem in Empirical Cost Functions

- A broad number of existing studies estimate cost functions without considering an endogeneity problem even when the differentiated products exist in a market.
- Their estimated cost function parameters are possibly biased.
- Theoretically, firms decide the quantity of outputs q_i which maximize their profits, considering the qualities of their differentiated products.
- In their empirical cost functions, an error term u_i that includes a cost shock could correlate with the quantity of output q_i.

$$\ln C_i = \alpha + \beta \ln q_i + \gamma \ln w_i + u_i, \quad plim\hat{\beta} = \beta + \underbrace{\frac{Cov(q_i, u_i)^{\neq 0}}{Var(q_i)}}_{\text{Endogeneity bias}} \neq \beta$$

- As a cost shock increases, a profit-maximizing firm reduces the quantity of output. This correlation causes a negative sign of the estimated coefficient of the quantity of output.

How to Address an Endogeneity Problem (1)

- The most common method: Using instrumental variables (IV)
  (An Instrumental Variable (IV) method or a 2 Step Least Square (2SLS) method)
  - An instrumental variable may not be available. It is hard to find a valid instrumental variable that is exogenous and relevant to an endogenous explanatory variable.
  - Two possible instrumental variables are consumers' income and the number of family members.
    - Both could affect demand for products (relevant), while both could not affect a production cost directly.
    - However, both could correlate with a cost shock through unobserved public amenities or environment (not exogenous).

$$\ln C_i = \alpha + \beta \ln q_i + \gamma \ln w_i + u_i. \longleftarrow$$ u_i includes a cost shock.

IVs: Consumers' income; and the number of family members

How to Address an Endogeneity Problem (2)

- Our method: A kind of structural estimation
  - Our idea is to eliminate the cause of an endogeneity problem from estimated models by considering market structure.
  - Based on the same idea, Byrne et al. [2021] proposes a new method to deal with an endogeneity problem without any instrumental variable in the empirical demand functions.
  - Imai et al. [2019] summarizes Byrne et al. [2021].

Highlight: Our Method to Address an Endogeneity Problem

- A total cost C is defined as a function of the number of outputs q, input factor price w, observable qualities of product x, and cost shock v as follows;

$$C(q,w,x,v) = \tilde{C}(q,w) * \exp(\varphi(x,v)).$$

A cost shock causes an endogeneity problem.

- A marginal cost derived from a total cost necessarily includes the same cost shock as a total cost.

$$MC(q,w,x,v) = \widetilde{MC}(q,w) * \exp(\varphi(x,v))$$

- Therefore, a total cost is expressed as a function, where all explanatory variables are observables, using the first-order condition for profit maximization (F.O.C.).

$$C = C * \frac{MR}{MC} = \tilde{C} * \exp(\varphi(x,v)) * \frac{MR}{\widetilde{MC} * \exp(\varphi(x,v))} = \frac{\tilde{C}(q,w)}{\widetilde{MC}(q,w)} * MR$$

$\frac{MR}{MC} = 1$

Highlight: Monte-Carlo Experiments

- A total cost C is a function of a product price p, the number of market sales q, market share of the products of the firm s, input factor price w, observable product quality X.

$$C(q, w, x, v) = \frac{\tilde{C}(q, w)}{\widetilde{MC}(q, w)} * MR(p, s, X).$$

- The estimated model is obtained by specifying $\tilde{C}$, $\widetilde{MC}$, and MR.
  - A total cost $\tilde{C}$ and a marginal cost $\widetilde{MC}$ is driven from either a Cobb=Douglas production function or a translog production function.
  - A marginal revenue MR is specified with either a logit model (Berry [1994]) or a random coefficient logit model (Berry et al. [1995]).
  - A consideration of measurement error is also needed.
- Our results of the Monte-Carlo experiments show that we can obtain consistent estimators of a cost function by using our method.

When Can Our Method Work? (1)

When the following assumptions hold, our method can work.

Assumption 1 Data requirements

Supply side: A total cost; the number of outputs; and input factor price
demand for input factors (not necessarily needed)
Demand side: market share; and observed product characteristics

Assumption 2 Markets are isolated.

Assumption 3 Differentiated-products oligopoly model: A logit model (Berry [1994]) or a random coefficient logit model (Berry et al. [1995]) is employed for a demand side.

Assumption 4 Bertrand-Nash equilibrium holds in each market. Then, $MC = MR$ holds.

Assumption 6 Following Gandhi et al. [2020], a total cost C can be expressed as follows;

$$C = C^* + u = \tilde{C}^*(q, w;\ \theta_c) * \exp(\varphi(x, v)) + u,$$

where φ is the cause of bias which is independent from a true cost $\widetilde{C}^*$,

θ_c is a parameter vector of a cost function.
To apply Shepard's lemma to a cost function, $\tilde{C}^*(q, w, x;\ \theta_c) * \exp(v) * \exp(u)$ is avoided.

When Can Our Method Work? (2)

Even when data on demand for input factors is not available, we can obtain consistent estimators of a cost function by adding the orthogonality conditions (Assumption 5).

Assumption 1 Data requirements

Supply side: **A total cost; the number of output; input factor price;** ~~**demand for input factors**~~

Demand side: **market share; observed product characteristics**

Assumption 5 The orthogonality conditions

The Advantages and Disadvantages of Our Method: A Comparison with an IV method and 2SLS

Advantages:

- Our method could cover the disadvantage of using instrumental variables (IV).
    - No statistical test can examine that an IV satisfies the exogeneity requirement. The Generalized Method of Moments (GMM) overidentification test can verify the null hypothesis that any statistical difference exists among the IV estimators. When the null hypothesis is accepted, there is no statistical way to distinguish the valid instruments from others. Even if the hypothesis is rejected, all IVs may not exogenous. (Wooldrige [2016])
    - So far, no instrumental variable can deal with the selection bias when the potential entrants are unobservable.

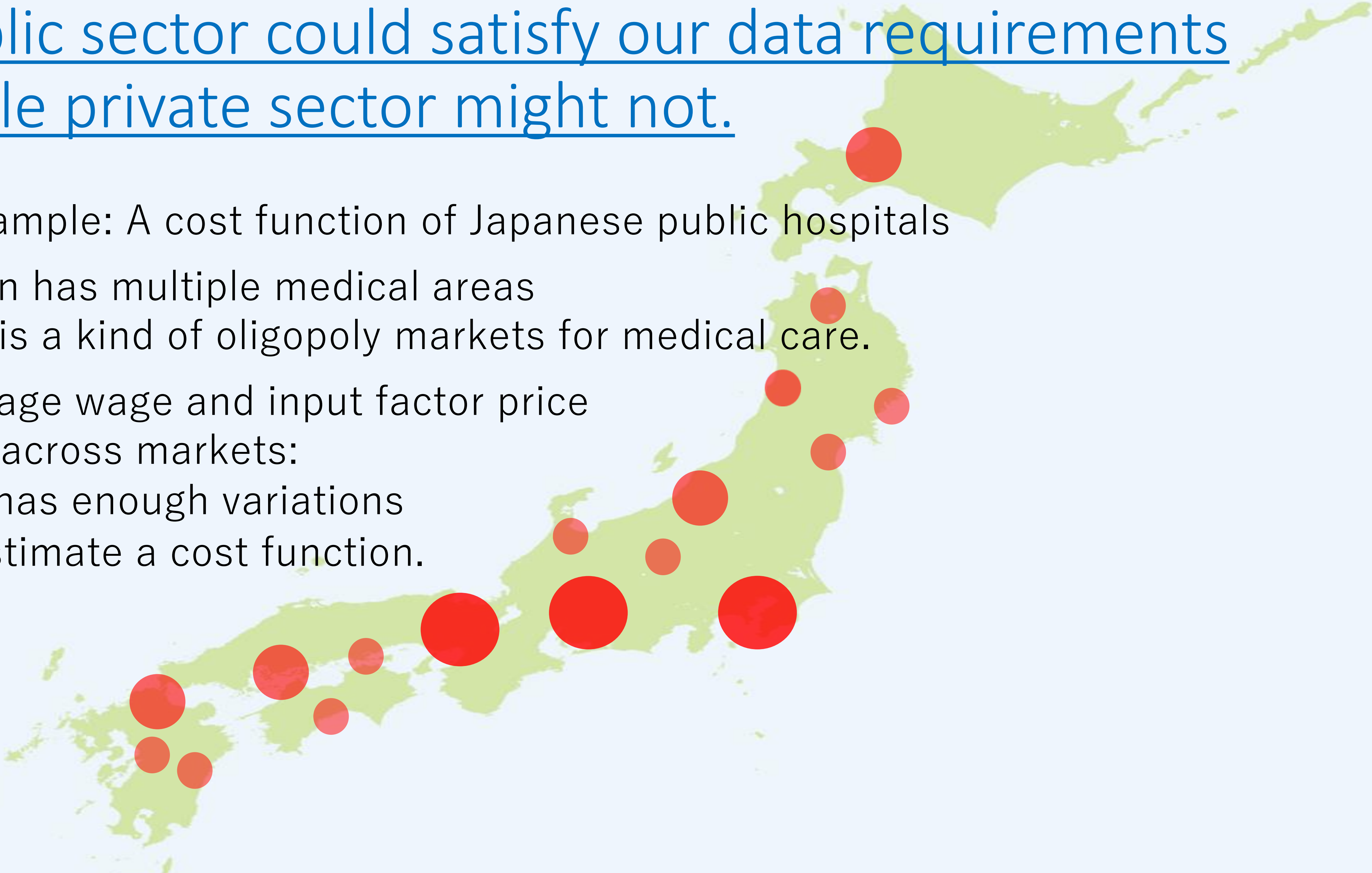
Disadvantages:

- Cost data are required. Our method requires both input factor price and demand for input factors.

Public sector could satisfy our data requirements while private sector might not.

An Example: A cost function of Japanese public hospitals

- Japan has multiple medical areas
  that is a kind of oligopoly markets for medical care.
- Average wage and input factor price
  vary across markets:
  w_{jm} has enough variations
  to estimate a cost function.



Some Examples of the Specific Models

1. A Cobb-Douglas cost function and a logit demand function
2. A translog cost function and a logit demand function
3. A Cobb-Douglas cost function and a random coefficient logit demand function
4. A translog cost function and a random coefficient logit demand function

Supply side	Demand side
A Cobb-Douglas cost function	A logit demand function
A translog cost function	A random coefficient logit demand function

How to Derive A Cobb=Douglas Cost Function (1)

- Let us assume a Cobb=Douglass production function, where a constant return to scale does not hold. Firms change the quantities of outputs q given input factor prices, depending on the qualities of products. Thus, observable product quality x in addition to a cost shock ν affects productivity.

$$q = [B \exp(x\eta + \nu)]^{-(\alpha_c+\beta_c)} L^{\alpha_c} K^{\beta_c}$$

- A Cobb=Douglass cost function is driven from the cost minimization problem as follows;

$$C^*(q, w, r, x, v;\ \theta_c) = \min_{K,L} (rK + wL)$$

$$s.t.\ \ q = [B \exp(x\eta + \nu)]^{-(\alpha_c+\beta_c)} L^{\alpha_c} K^{\beta_c}.$$

How to Derive A Cobb=Douglas Cost Function (2)

- A long-term cost function is derived by solving the cost minimization problem, using a Lagrange multiplier method.

$$C^*(q, w, r, x, v;\ \theta_c) = \left[(\alpha_c + \beta_c)\left(\frac{w}{\alpha_c}\right)^{\frac{\alpha_c}{\alpha_c+\beta_c}}\left(\frac{r}{\beta_c}\right)^{\frac{\beta_c}{\alpha_c+\beta_c}}\right] B \exp(x\eta + v)\, q^{\frac{1}{\alpha_c+\beta_c}}$$

A cost shock

$$\ln C^* = \ln B + \ln(\alpha_c + \beta_c) - \left(\frac{\alpha_c}{\alpha_c + \beta_c}\right)\ln \alpha_c - \left(\frac{\beta_c}{\alpha_c + \beta_c}\right)\ln \beta_c + \left(\frac{\alpha_c}{\alpha_c + \beta_c}\right)\ln w + \left(\frac{\beta_c}{\alpha_c + \beta_c}\right)\ln r + \left(\frac{1}{\alpha_c + \beta_c}\right)\ln q + \underbrace{\eta x + \upsilon}$$

The causes of bias

How to Derive a Marginal Cost Function

- Now, we have a following cost function;

$$C^*_{jm}(q_{jm}, w_{jm}, r_{jm}, x_{jm}, v_{jm};\ \theta_c) = \left[(\alpha_c + \beta_c)\left(\frac{w_{jm}}{\alpha_c}\right)^{\frac{\alpha_c}{\alpha_c+\beta_c}}\left(\frac{r_{jm}}{\beta_c}\right)^{\frac{\beta_c}{\alpha_c+\beta_c}}\right] B\exp(x_{jm}\eta + v_{jm})\, q_{jm}^{\frac{1}{\alpha_c+\beta_c}}.$$

- Let us differentiate a cost function by the number of outputs to derive a marginal cost function.

$$MC^*_{jm}(q_{jm}, w_{jm}, r_{jm}, x_{jm}, v_{jm};\ \theta_c) = \frac{\partial\, C^*(q_{jm}, w_{jm}, r_{jm}, x_{jm}, v_{jm})}{\partial q_{jm}} = \left[\left(\frac{w_{jm}}{\alpha_c}\right)^{\frac{\alpha_c}{\alpha_c+\beta_c}}\left(\frac{r_{jm}}{\beta_c}\right)^{\frac{\beta_c}{\alpha_c+\beta_c}}\right] B\exp(x_{jm}\eta + v_{jm})\, q_{jm}^{\frac{1}{\alpha_c+\beta_c}-1}$$

- A cost shock in a total cost and a cost shock in a marginal cost are canceled each other out, using the first-order condition for profit maximization (F.O.C.)

$$C^*_{jm} = C^*_{jm} * \frac{MR^*_{jm}}{MC^*_{jm}} = (\alpha_c + \beta_c) q_{jm} MR_{jm}$$

$\frac{MR^*_{jm}}{MC^*_{jm}} = 1$

Returns to scale: $(\alpha_c + \beta_c)$

How to Derive a Marginal Revenue Function (1)

- Following Berry (1994), the utility share of product j in market m is specified as follows;

$$s_{jm} = \frac{\exp(X_{jm}\beta - p_{jm}\alpha + \xi_{jm})}{\sum_{j=0}^{J} \exp(X_{jm}\beta - p_{jm}\alpha + \xi_{jm})},$$

  where X_{jm} is observable product quality of rival firms, p_{jm} is a product price, and ξ_{jm} is unobservable product characteristics.

- The logit demand function is driven using the utility that consumer i obtains from product j in market m that is specified as follows;

$$u_{ijm} = X_{jm}\beta - p_{jm}\alpha + \xi_{jm}.$$

How to Derive a Marginal Revenue Function (2)

- The quantity of output q_{jm} can be expressed as the product of the market share s_{jm} times the market size Q_m $(q_{jm} = Q_m s_{jm})$.
- Then, a marginal revenue function is as follows;

$$MR_{jm} = \frac{\partial p_{jm} q_{jm}}{\partial q_{jm}} = \frac{\partial p_{jm} Q_m s_{jm}}{\partial Q_m s_{jm}} = \frac{Q_m \partial p_{jm} s_{jm}}{Q_m \partial s_{jm}}$$

$$= p_{jm} + s_{jm} \left[\frac{\partial s_{jm}(\boldsymbol{p}_m, \boldsymbol{X}_m, \boldsymbol{\xi}_m; \theta_d)}{\partial p_{jm}} \right]^{-1} = p_{jm} - \frac{1}{(1 - s_{jm})\alpha}.$$

How to Derive a Marginal Revenue Function (3)

Product formula is used.

$$\frac{\partial s_{jm}(\boldsymbol{p}_m, \boldsymbol{X}_m, \boldsymbol{\xi}_m; \theta_d)}{\partial p_{jm}} = \frac{\partial \exp\left(X_{jm}\beta - p_{jm}\alpha + \xi_{jm}\right)\left[\sum_{j=0}^{J} \exp(X_{jm}\beta - p_{jm}\alpha + \xi_{jm})\right]^{-1}}{\partial p_{jm}},$$

$$= \exp\left(X_{jm}\beta - p_{jm}\alpha + \xi_{jm}\right)\left[\sum_{j=0}^{J} \exp(X_{jm}\beta - p_{jm}\alpha + \xi_{jm})\right]^{-1} \alpha - \exp\left(X_{jm}\beta - p_{jm}\alpha + \xi_{jm}\right)^{2}\left[\sum_{j=0}^{J} \exp(X_{jm}\beta - p_{jm}\alpha + \xi_{jm})\right]^{-2} \alpha$$

$$= \exp\left(X_{jm}\beta - p_{jm}\alpha + \xi_{jm}\right)\left[\sum_{j=0}^{J} \exp(X_{jm}\beta - p_{jm}\alpha + \xi_{jm})\right]^{-1}\left[1 - \exp\left(X_{jm}\beta - p_{jm}\alpha + \xi_{jm}\right)\left[\sum_{j=0}^{J} \exp\left(X_{jm}\beta - p_{jm}\alpha + \xi_{jm}\right)\right]^{-1}\right]\alpha$$

$$= -s_{jm}\left(1 - s_{jm}\right)\alpha$$

How to Identify Parameters of a Cost Function When Data on K and L is Available (1)

- Let us substitute a marginal revenue function for a cost function as follows;

$$C_{jm}^{*} = (\alpha_c + \beta_c) q_{jm} * MR_{jm}^{*} = (\alpha_c + \beta_c) q_{jm} * \left[p_{jm} - \frac{1}{(1 - s_{jm})\alpha} \right].$$

- The measurement error is added to a cost function as follows;

$$C_{jm} = C_{jm}^{*} + u_{jm} = (\alpha_c + \beta_c) q_{jm} * \left[p_{jm} - \frac{1}{(1 - s_{jm})\alpha} \right] + u_{jm}.$$

- Then, $\widehat{\alpha_c + \beta_c}$ and $\hat{\alpha}$ can be estimated, using an Ordinary Least Square method (OLS) or a Maximum Likelihood method (ML).

How to Identify Parameters of a Cost Function When Data on K and L is Available (2)

- Let us take a natural logarithm to obtain a liner cost function as follows;

$$\ln C_{jm}^{*} = \ln B + \ln(\alpha_c + \beta_c) - \left(\frac{\alpha_c}{\alpha_c+\beta_c}\right)\ln\alpha_c - \left(\frac{\beta_c}{\alpha_c+\beta_c}\right)\ln\beta_c + \left(\frac{\alpha_c}{\alpha_c+\beta_c}\right)\ln w_{jm} + \left(\frac{\beta_c}{\alpha_c+\beta_c}\right)\ln r_{jm} + \left(\frac{1}{\alpha_c+\beta_c}\right)\ln q_{jm} + \eta x_{jm} + \nu_{jm}\ .$$

$$\frac{\partial \ln C_{jm}^{*}}{\partial \ln w_{jm}} = \frac{\alpha_c}{\alpha_c+\beta_c}.$$

- $\frac{\partial C_{jm}^{*}}{\partial w_{jm}} = L_{jm}$ holds from Shepard's lemma. Then,

$$C_{jm}^{*} = w_{jm}L_{jm} + r_{jm}K_{jm}$$

$$\frac{\partial \ln C_{jm}^{*}}{\partial \ln w_{jm}} = \frac{\partial \ln C_{jm}^{*}}{\partial w_{jm}}\cdot\frac{\partial w_{jm}}{\partial \ln w_{jm}} = \left(\frac{\partial \ln C_{jm}^{*}}{\partial C_{jm}^{*}}\cdot\frac{\partial C_{jm}^{*}}{\partial w_{jm}}\right)\cdot\left(\frac{\partial \ln w_{jm}}{\partial w_{jm}}\right)^{-1} = \left(\frac{1}{C_{jm}^{*}}\cdot L_{jm}\right)\left(\frac{1}{w_{jm}}\right)^{-1} = \frac{w_{jm}L_{jm}}{C_{jm}^{*}},$$

- Therefore, $\frac{\partial \ln C_{jm}^{*}}{\partial \ln w_{jm}} = \frac{w_{jm}L_{jm}}{C_{jm}^{*}} = \frac{\alpha_c}{\alpha_c+\beta_c}$ holds. $\widehat{\alpha_C}$ and $\widehat{\beta_C}$ are identified as follows;

$$\widehat{\alpha_C} = \left(\widehat{\alpha_c+\beta_c}\right)\frac{w_{jm}L_{jm}}{C_{jm}^{*}} = \left(\widehat{\alpha_c+\beta_c}\right)\frac{w_{jm}L_{jm}}{C_{jm}-u_{jm}},\quad \widehat{\beta_C} = \left(\widehat{\alpha_c+\beta_c}\right) - \widehat{\alpha_C}.$$

How to Identify Parameters of a Cost Function When Data on K and L is Not Available (1)

- When data on demand for inputs are not available, the estimated equation to identify α_c and β_c is as follows;

A cost shock ↓

$$\ln C_{jm}^* = \ln B + \ln(\alpha_c + \beta_c) - \left(\frac{\alpha_c}{\alpha_c+\beta_c}\right)\ln \alpha_c - \left(\frac{\beta_c}{\alpha_c+\beta_c}\right)\ln \beta_c + \left(\frac{\alpha_c}{\alpha_c+\beta_c}\right)\ln w_{jm} + \left(\frac{\beta_c}{\alpha_c+\beta_c}\right)\ln r_{jm} + \left(\frac{1}{\alpha_c+\beta_c}\right)\ln q_{jm} + \eta x_{jm} + \nu_{jm},$$

$$\ln C_{jm}^* - \left(\frac{1}{\alpha_c+\beta_c}\right)\ln q_{jm} = (Constant) + \left(\frac{\alpha_c}{\alpha_c+\beta_c}\right)\ln w_{jm} + \left(\frac{\beta_c}{\alpha_c+\beta_c}\right)\ln r_{jm} + \eta x_{jm} + \nu_{jm},$$

$$\ln C_{jm} - \widehat{u_{jm}} - \left(\frac{1}{\widehat{\alpha_c+\beta_c}}\right)\ln q_{jm} = (Constant) + \alpha_c\left(\frac{1}{\widehat{\alpha_c+\beta_c}}\right)\ln w_{jm} + \beta_c\left(\frac{1}{\widehat{\alpha_c+\beta_c}}\right)\ln r_{jm} + \eta x_{jm} + \nu_{jm}.$$

- A cost shock ν_{jm} is treated as an error term and assumes that a cost shock follows a normal distribution.
- $\widehat{\alpha_c}$, $\widehat{\beta_c}$, and $\hat{\eta}$ can be identified by regressing $\left(\ln C_{jm} - \widehat{u_{jm}} - \left(\frac{1}{\widehat{\alpha_c+\beta_c}}\right)\ln q_{jm}\right)$ on $\left(\frac{1}{\widehat{\alpha_c+\beta_c}}\right)\ln w_{jm}$, $\left(\frac{1}{\widehat{\alpha_c+\beta_c}}\right)\ln r_{jm}$, and x_{jm}.

How to Identify Parameters of a Cost Function When Data on K and L is Not Available (2)

- If all explanatory variables are not correlated with an error term which is a cost shock here, OLS estimators $\widehat{\alpha_c}$, $\widehat{\beta_c}$, and $\hat{\eta}$ are not biased.

A cost shock

An error term

$$\ln C_{jm} - \widehat{u_{jm}} - \left(\frac{1}{\widehat{\alpha_c+\beta_c}}\right)\ln q_{jm} = (Constant) + \alpha_c \left(\frac{1}{\widehat{\alpha_c+\beta_c}}\right)\ln w_{jm} + \beta_c \left(\frac{1}{\widehat{\alpha_c+\beta_c}}\right)\ln r_{jm} + \eta x_{jm} + v_{jm}.$$

No correlations

- Here, assumption 5 (the orthogonal conditions) needs to be satisfied to obtain consistent estimators of α, β, and η.

$$Cov(w_{jm}, v_{jm}) = 0,\ Cov(r_{jm}, v_{jm}) = 0,\ Cov(x_{jm}, v_{jm}) = 0$$

An Attempt to Obtain an Estimate of $\hat{\beta}$:

- We do not have $\hat{\beta}$ still now. If we can obtain $\hat{\beta}$, we can conduct a policy simulation using an estimated demand function.
- To obtain a consistent estimator of $\hat{\beta}$, the following equation is estimated using an OLS method without addressing the endogeneity problem in an empirical demand function.
  - A logit model (Berry (1994)) is used to identify $\hat{\beta}$.

Utility level of no purchase equals zero.

$$s_{jm}/s_{0m} = \frac{\exp(X_{jm}\beta - p_{jm}\alpha + \xi_{jm})}{\sum_{j=0}^{J}\exp(X_{jm}\beta - p_{jm}\alpha + \xi_{jm})} \Big/ \frac{\exp(0)}{\sum_{j=0}^{J}\exp(X_{jm}\beta - \alpha + \xi_{jm})} = \exp\left(X_{jm}\beta - p_{jm}\alpha + \xi_{jm}\right).$$

$$\ln s_{jm} - \ln s_{0m} = X_{jm}\beta - p_{jm}\alpha + \xi_{jm}$$

$$\ln s_{jm} - \ln s_{0m} + p_{jm}\alpha = X_{jm}\beta + \xi_{jm}$$

- The market share of the 0-th goods
- Purchase the 0-th goods
  = Purchase nothing in market m

  - $\hat{\beta}$ can be obtained by regressing $\left(\ln s_{jm} - \ln s_{0m} + p_{jm}\hat{\alpha}\right)$ on X_{jm}.
  - If X_{jm} is correlated with ξ_{jm}, an OLS estimator $\hat{\beta}$ is biased.

A Comparison of Our Parametric Approach and a Non-parametric Approach in Byrne et al. (2021)

- Byrne et al. (2021) employs a non-parametric approach and estimates a more general function.

Our Parametric Approach	A Non-parametric Approach in Byrne et al. (2021)
$C_{jm}^{*} = \frac{\widetilde{C_{jm}}(q_{jm}, w_{jm}, x_{jm}; \theta_c)}{\widetilde{MC_{jm}}(q_{jm}, w_{jm}, x_{jm}; \theta_c)} * MR_j(p_m, s_m, X_m; \theta_d)$	$C_{jm}^{*} = \psi\left(q_{jm}, w_{jm}, x_{jm}, MR(p_m, s_m, X_m;\ \theta_d)\right)$
When a Cobb-Douglas cost function and a logit market share function are employed; $C_{jm}^{*} = (\alpha_c + \beta_c) q_{jm} \left[p_{jm} - \frac{1}{(1 - s_{jm})\alpha}\right] = \psi\left(q_{jm}, w_{jm}, x_{jm}, MR(p_m, s_m, X_m;\ \theta_d)\right)$	

- While a non-parametric approach in Byrne et al. (2021) needs to estimate ψ to obtain parameter estimates $\widehat{\alpha_c + \beta_c}$ and $\hat{\alpha}$, we directly estimate $\alpha_c + \beta_c$ and α. Our direct estimation is more efficient.

Some Examples of the Specific Models

1. A Cobb-Douglas cost function and a logit demand function
2. A translog cost function and a logit demand function
3. A Cobb-Douglas cost function and a random coefficient logit demand function
4. A translog cost function and a random coefficient logit demand function

Supply side	Demand side
A Cobb-Douglas cost function	A logit demand function
A translog cost function	A random coefficient logit demand function

A Translog Cost Function

- A translog production function is specified as follows;

$$\ln q_{jm} = b_0 + \sum_{k=1}^{K} b_k \ln L_{km} + \frac{1}{2}\sum_{k=1}^{K}\sum_{k'=1}^{K} b_{kk'} \ln L_{km} \ln L_{k'm} - (\eta x_{qjm} + \upsilon_{qjm}).$$

- Then, the corresponding translog cost function is as follows;

$$\ln C^{*}{}_{jm} = \gamma_0 + \gamma_q \ln q_{jm} + \frac{1}{2}\gamma_{qq}(\ln q_{jm})^2 + \sum_{k=1}^{K} \gamma_{kq} \ln w_{kjm} \ln q_{jm}$$

$$+ \sum_{k=1}^{K} \gamma_k \ln w_{kjm} + \frac{1}{2}\sum_{k=1}^{K}\sum_{k'=1}^{K} \gamma_{kk'} \ln w_{kjm} \ln w_{k'jm} + \eta x_{jm} + \upsilon_{jm}.$$

A cost shock (υ_{jm})

The causes of bias ($\eta x_{jm} + \upsilon_{jm}$)

- A restrictions on the homogeneity of degree are as follows:

$$\sum_{k=1}^{K} \gamma_k = 1,\ \sum_{k=1}^{K} \gamma'_{kk} = 0,\ \sum_{k'=1}^{K} \gamma'_{kk} = 0,\ \sum_{k=1}^{K} \gamma_{kq} = 0.$$

Estimated Equations (1)

- A total cost can be rewritten by using the first order condition for profit maximization (F.O.C.) as follows;

$$C^*_{jm} = C^*_{jm} \cdot \underset{\overset{\|}{1}}{\frac{MR^*_{jm}}{MC^*_{jm}}} = \frac{C^*_{jm}}{MC^*_{jm}} \cdot MR^*_{jm}.$$

$$\frac{C^*_{jm}}{MC^*_{jm}} = \underset{MC^*_{jm}}{\left(\frac{\partial C^*_{jm}}{\partial q_{jm}}\right)^{-1}} \left(\frac{1}{C^*_{jm}}\right)^{-1} = \left(\frac{\cancel{\partial C^*_{jm}}}{\partial q_{jm}}\right)^{-1} \left(\frac{\partial \ln C_{jm}}{\cancel{\partial C^*_{jm}}}\right)^{-1} = \left(\frac{1}{q_{jm}}\right)^{-1} \left[\frac{\partial \ln C_{jm}}{\partial \ln q_{jm}}\right]^{-1} = q_{jm}\left[\gamma_q + \gamma_{qq} \ln q_{jm} + \sum_{k=1}^{K} \gamma_{kq} \ln w_{kjm}\right]^{-1}.$$

Insert $\partial \ln q_{jm}$ instead of ∂C^*_{jm}

- Therefore, an estimated translog cost function is as follows;

$$C_{jm} = C^*_{jm} + u_{jm} = q_{jm}\left[\gamma_q + \gamma_{qq} \ln q_{jm} + \sum_{k=1}^{K} \gamma_{kq} \ln w_{kjm}\right]^{-1}\left[p_{jm} - \frac{1}{(1 - s_{jm})\alpha}\right] + u_{jm}.$$

- Then, $\widehat{\gamma_q}$, $\widehat{\gamma_{qq}}$, $\widehat{\gamma_{kq}}$, and $\hat{\alpha}$ can be obtained as OLS estimators or ML estimators.

Estimated Equations (2)

- Now, $\widehat{\gamma_q}$, $\widehat{\gamma_{qq}}$, and $\widehat{\gamma_{kq}}$ can be obtained while $\widehat{\gamma_k}$ and $\widehat{\gamma_{kk'}}$ are still not obtained.

$$\ln C^*{}_{jm} = \alpha_0 + \gamma_q \ln q_{jm} + \frac{1}{2}\gamma_{qq}(\ln q_{jm})^2 + \sum_{k=1}^{K} \gamma_{kq} \ln w_{kjm} \ln q_{jm} + \sum_{k=1}^{K} \gamma_k \ln w_{kjm} + \frac{1}{2}\sum_{k=1}^{K}\sum_{k'=1}^{K} \gamma_{kk'} \ln w_{kjm} \ln w_{k'jm} + v_{jm}$$

- In order to obtain estimates of the rest parameters, $\widehat{\gamma_k}$ and $\widehat{\gamma_{kk'}}$, the equation which is driven from the Shephard's Lemma is estimated.

- $\frac{\partial C^*_{jm}}{\partial w_{jm}} = L_{jm}$ holds from Shepard's lemma. Similar to a case of a Cobb=Douglas cost function, the estimated equation can be obtained as follows;.

$$\frac{\partial \ln C^*(q_{jm}, w_{jm}, x_{jm}, v_{jm};\ \theta_c)}{\partial \ln w_{kjm}} = \frac{w_{kjm}L_{kjm}}{C^*_{jm}} = \gamma_{kq} \ln q + \gamma_k + \sum_{k'=1}^{K} \gamma_{kk'} \ln w_{k'jm},$$

$$\frac{\partial \ln C\ (q_{jm}, w_{jm}, x_{jm}, v_{jm};\ \theta_c)}{\partial \ln w_{kjm}} = \gamma_{kq} \ln q + \gamma_k + \sum_{k'=1}^{K} \gamma_{kk'} \ln w_{k'jm} + e_{jm}.$$

The Monte-Carlo Experiments

- Table 1 shows how to set the parameter values for the Monte-Carlo experiments to examine whether we can obtain consistent parameters using our method.
  - Only input factor price w is exogenous, while other explanatory variables are endogenous.
  - A multiple-inputs-and-a-single-output production model is assumed for each firm.
  - Four firms exist in each oligopoly market.
  - Sample size: $4\, products \times (the\ number\ of\ markets)$

Consumers' Five Choices in Oligopoly Market m

$j = 0$	Not purchase
$j = 1$	Products of firm 1
$j = 2$	Products of firm 2
$j = 3$	Products of firm 3
$j = 4$	Products of firm 4

Some Examples of the Specific Models

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Supply side	Demand side
A Cobb-Douglas cost function	A logit demand function
A translog cost function	A random coefficient logit demand function

Table 1: Parameter values of Monte-Calro experiments

Supply side		Demand side	
Cobb-Douglas Production Function $q_{jm} = \left[B exp\left(x_{jm}\eta + v_{jm}\right)\right]^{-(\alpha_c+\beta_c)} L_{jm}^{\alpha_c} K_{jm}^{\beta_c}$		Random Coefficient Logit Demand Function $s_{jm} = \int_\alpha \int_\beta \frac{\exp(X_{jm}\beta - p_{jm}\alpha + \xi_{jm})}{\sum_{k=0}^{J_m} \exp(X_{jm}\beta - p_{jm}\alpha + \xi_{jm})} dF_\beta(\beta;\theta_\beta) dF_\alpha(\alpha;\theta_\alpha)$	
Parameter	Value	Parameter	Value
α_c	0.5	μ_α	2.0
β_c	0.3	σ_α	0.5
μ_w	1.0	μ_β	1.0
σ_w	0.2	σ_β	0.2
μ_r	1.0	μ_X	3.0
σ_r	0.2	σ_X	1.0
μ_v	-5.0	μ_ξ	2.0
σ_v	0.1	σ_ξ	0.5
B	1.0	Lower bound on market size	5.0
The standard deviation of a measurement error for a total cost	0.4	Upper bound on market size	10.0
		J	4.0

Here, a variable returns to scale ($\alpha_c + \beta_c \neq 1$) is assumed.

Table 1: Continued

Endogenities	
Correlation coefficient	Value
$Corr(\xi_{jm}, x_{jm})$	0 or 0.833
$Corr(\xi_{jm}, X_{jm})$	0.833
$Corr(\xi_{jm}, w_m)$	0.833
$Corr(\xi_{jm}, r_m)$	0.833
$Corr(\xi_{jm}, \nu_{jm})$	0.833
$Corr(\xi_{jm}, Q_m)$	0.833
$Corr(\upsilon_{jm}, Q_m)$	0.833

Table 2: Estimated Results when data on K and L is available ($Corr(\xi_{jm}, x_{jm})=0$)

(a) Demand side parameters

Markets	Sample Size	$\hat{\mu}_\alpha$ Mean	$\hat{\mu}_\alpha$ Std. Dev.	$\hat{\mu}_\alpha$ RMSE	$\hat{\sigma}_\alpha$ Mean	$\hat{\sigma}_\alpha$ Std. Dev.	$\hat{\sigma}_\alpha$ RMSE
50	200	2.031	0.3687	0.3682	0.5102	0.1260	0.1258
100	400	1.999	0.2740	0.2726	0.4982	0.1056	0.1051
200	800	1.982	0.1799	0.1798	0.4908	0.0751	0.0753
400	1600	2.001	0.1556	0.1548	0.4965	0.0600	0.0598
True Value		2.0			0.5		

(a) Demand side parameters

Markets	Sample Size	$\hat{\mu}_\beta$ Mean	$\hat{\mu}_\beta$ Std. Dev.	$\hat{\mu}_\beta$ RMSE	$\hat{\sigma}_\beta$ Mean	$\hat{\sigma}_\beta$ Std. Dev.	$\hat{\sigma}_\beta$ RMSE
50	200	0.9913	0.2412	0.2401	0.2167	0.1155	0.1161
100	400	0.9859	0.1880	0.1876	0.2160	0.0973	0.0981
200	800	0.9896	0.1289	0.1287	0.2048	0.0595	0.0594
400	1600	1.0016	0.1034	0.1029	0.2027	0.0404	0.0402
True Value		1.0			0.2		

(b) Production function parameters

Markets	Sample Size	$\hat{\alpha}_c$ Mean	$\hat{\alpha}_c$ Std. Dev.	$\hat{\alpha}_c$ RMSE	$\hat{\beta}_c$ Mean	$\hat{\beta}_c$ Std. Dev.	$\hat{\beta}_c$ RMSE
50	200	0.5233	0.0679	0.0715	0.3111	0.0412	0.0425
100	400	0.5144	0.0497	0.0516	0.3057	0.0316	0.0320
200	800	0.5071	0.0419	0.0423	0.3051	0.0281	0.0284
400	1600	0.5034	0.0295	0.0296	0.3014	0.0180	0.0180
True Value		0.5			0.3		

Markets	Sample Size	$\hat{\eta}$ Mean	$\hat{\eta}$ Std. Dev.	$\hat{\eta}$ RMSE
50	200	0.1998	0.0130	0.0129
100	400	0.2002	0.0083	0.0082
200	800	0.2007	0.0057	0.0057
400	1600	0.1995	0.0048	0.0040
True Value		0.2		

A consistent estimator of each parameter

Table 3: Estimated Results when data on K and L is available $(Corr(\xi_{jm}, x_{jm})>0, Corr(v_{jm}, x_{jm})>0)$

(a) Demand side parameters

Markets	Sample Size	$\hat{\mu}_\alpha$ Mean	$\hat{\mu}_\alpha$ Std. Dev.	$\hat{\mu}_\alpha$ RMSE	$\hat{\sigma}_\alpha$ Mean	$\hat{\sigma}_\alpha$ Std. Dev.	$\hat{\sigma}_\alpha$ RMSE
50	200	2.031	0.3687	0.3682	0.5102	0.1260	0.1258
100	400	1.999	0.2740	0.2726	0.4982	0.1056	0.1051
200	800	1.982	0.1799	0.1798	0.4908	0.0751	0.0753
400	1600	2.001	0.1556	0.1548	0.4965	0.0600	0.0598
True Value		2.0			0.5		

(a) Demand side parameters

Markets	Sample Size	$\hat{\mu}_\beta$ Mean	$\hat{\mu}_\beta$ Std. Dev.	$\hat{\mu}_\beta$ RMSE	$\hat{\sigma}_\beta$ Mean	$\hat{\sigma}_\beta$ Std. Dev.	$\hat{\sigma}_\beta$ RMSE
50	200	0.9913	0.2412	0.2401	0.2167	0.1155	0.1161
100	400	0.9859	0.1880	0.1876	0.2160	0.0973	0.0981
200	800	0.9896	0.1289	0.1287	0.2048	0.0595	0.0594
400	1600	1.0016	0.1034	0.1029	0.2027	0.0404	0.0402
True Value		1.0			0.2		

(b) Production function parameters

Markets	Sample Size	$\hat{\alpha}_c$ Mean	$\hat{\alpha}_c$ Std. Dev.	$\hat{\alpha}_c$ RMSE	$\hat{\beta}_c$ Mean	$\hat{\beta}_c$ Std. Dev.	$\hat{\beta}_c$ RMSE
50	200	0.5233	0.0679	0.0715	0.3111	0.0412	0.0425
100	400	0.5144	0.0497	0.0518	0.3057	0.0316	0.0320
200	800	0.5071	0.0419	0.0423	0.3051	0.0281	0.0284
400	1600	0.5034	0.0295	0.0296	0.3014	0.0180	0.0180
True Value		0.5			0.3		

Markets	Sample Size	$\hat{\eta}$ Mean	$\hat{\eta}$ Std. Dev.	$\hat{\eta}$ RMSE
50	200	0.1998	0.0130	0.0129
100	400	0.2002	0.0083	0.0082
200	800	0.2007	0.0057	0.0057
400	1600	0.1995	0.0048	0.0040
True Value		0.2		

A consistent estimator of each parameter

Not consistent

Table 4: Estimated Results when data on K and L is not available $(Corr(\xi_{jm}, x_{jm}) = 0)$

(a) Demand side parameters

Markets	Sample Size	$\hat{\mu}_\alpha$ Mean	$\hat{\mu}_\alpha$ Std. Dev.	$\hat{\mu}_\alpha$ RMSE	$\hat{\sigma}_\alpha$ Mean	$\hat{\sigma}_\alpha$ Std. Dev.	$\hat{\sigma}_\alpha$ RMSE
50	200	2.698	2.787	2.860	0.4935	0.2994	0.2990
100	400	2.115	0.5304	0.5403	0.4540	0.1877	0.1923
200	800	2.066	0.3504	0.3546	0.4593	0.1387	0.1439
400	1600	2.035	0.2242	0.2258	0.4837	0.1141	0.1147
True Value		2.0			0.5		

(a) Demand side parameters

Markets	Sample Size	$\hat{\mu}_\beta$ Mean	$\hat{\mu}_\beta$ Std. Dev.	$\hat{\mu}_\beta$ RMSE	$\hat{\sigma}_\beta$ Mean	$\hat{\sigma}_\beta$ Std. Dev.	$\hat{\sigma}_\beta$ RMSE
50	200	1.3284	1.5293	1.5567	0.3348	0.4509	0.4684
100	400	1.0734	0.4003	0.4050	0.2235	0.1437	0.1449
200	800	1.1057	0.2603	0.2652	0.2102	0.0978	0.0979
400	1600	1.1027	0.1593	0.1608	0.2027	0.0755	0.0752
True Value		1.0			0.2		

(b) Production function parameters

Markets	Sample Size	$\hat{\alpha}_c$ Mean	$\hat{\alpha}_c$ Std. Dev.	$\hat{\alpha}_c$ RMSE	$\hat{\beta}_c$ Mean	$\hat{\beta}_c$ Std. Dev.	$\hat{\beta}_c$ RMSE
50	200	0.5216	0.0733	0.0761	0.3048	0.0917	0.0914
100	400	0.5101	0.0610	0.0615	0.3101	0.0724	0.0728
200	800	0.4997	0.0443	0.0441	0.2983	0.0530	0.0528
400	1600	0.4992	0.0342	0.0340	0.3024	0.0379	0.0378
True Value		0.5			0.3		

Markets	Sample Size	$\hat{\eta}$ Mean	$\hat{\eta}$ Std. Dev.	$\hat{\eta}$ RMSE
50	200	0.1968	0.0168	0.0170
100	400	0.1998	0.0106	0.0105
200	800	0.2007	0.0068	0.0068
400	1600	0.1996	0.0060	0.0059
True Value		0.2		

A consistent estimator of each parameter

The estimators in Table 4 are less consistent than those in Table 2. It is probably because a cost shock is treated as an error term.

Remaining Problems and the Future Research (1)

- To compare our method with a method using instrumental variables (IVs), using the Monte-Carlo experiments
  - Our method considers a measurement error differently from a method using IVs. A comparison of our method and an IV method is needed.
    - Our Method: $\tilde{C}^*(q, w, x;\ \theta_c) * \exp(v) + u$
    - An IV Method: $\tilde{C}^*(q, w, x;\ \theta_c) * \exp(v) * \exp(u)$
- To improve the consistency of $\hat{\beta}$ when ξ_{jm} is correlated with x_{jm}
- To extend the current single output model to a multiple output model

Remaining Problems and the Future Research (2)

- A theory of a cost function is based on the assumption that all product quality is homogenous. However, our method assumes a homogenous production function for all product differentiated firms. We need to discuss this more carefully.

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- Thank you for listening.
- I am determined to continue the discussion on this research and would be grateful for all comments and suggestions.

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Note that ‘u’ is not included here.