

# (In)Efficient Separations, Firing Costs and Temporary Contracts

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*The views and opinions expressed herein are those of the authors and do not necessarily reflect the position of the Bank of Italy.*

## Motivation

- **Labour market rigidities** are often blamed for the relatively poor performance of the European labor market compared to US
- During the '90s and the '00s many European countries have implemented labor market reforms **mixed success**:
  - ▶ Spain and Italy: reforms and counter-reforms to liberalize the use of fixed-term contracts and lower dismissal costs
  - ▶ Success of **broad reform packages**: Hartz reforms in Germany (2004-2007), Portugal's Memorandum (2011-2014)

## This paper

- We investigate how **employment protection** affects social efficiency and unemployment.
  - ▶ Employment Protection on Regular contracts (EPR)
  - ▶ Introduction of fixed-term, Temporary contracts (Dual mkt)
- Crucially, the answer depends on the **interaction** with other labor market institutions (LMIs), such as generosity of unemployment benefits and wage setting protocol.
- What is the *constrained optimal* level of employment protection?
  - ▶ Analytical results
  - ▶ Quantitative application on Italy
- We adopt a **general equilibrium** approach to take into account externalities.

## Related literature

- ***Macro effects of labor market institutions***

- ▶ *Empirics*: Nickell and Layard (1999), Blanchard and Wolfers (2000), Bassanini and Duval (2009), Gnocchi, Lagerborg, and Pappa (2015), Boeri and Jimeno (2016)
- ▶ *Theory*: Mortensen and Pissarides (1999), Blanchard and Giavazzi (2003), Garibaldi and Violante (2005), Zanetti (2011), Cacciatore and Fiori (2016), Murtin and Robin (2018)

- ***Effects of employment protection***

- ▶ *EPR*: Blanchard and Tirole (2008), Karabay and McLaren (2011), Boeri, Garibaldi, and Moen (2017), Lalé (2019)
- ▶ *Dual mkt*: Bentolila and Saint-Paul (1992), Blanchard and Landier (2002), Sala, Silva, and Toledo (2012), Berton and Garibaldi (2012), Cahuc, Charlot, and Malherbet (2016), Veracierto (2007), Alonso-Borrego et al. (2005)

## Preview of the results

- The **optimal level of EPR** can be positive and it increases with the degree of rigidity introduced by **other LMIs**.
  - ▶ In Italy the optimal level of firing costs during the period 1985-1995 was estimated at 2.5 monthly wages
- **EPR is welfare improving** as long as it reduces job destruction but there is an upper bound to this effect. However, too high firing costs generate sizable welfare losses.
  - ▶ In Italy too high firing costs during the period 1985-1995 (3.5 monthly wages) generated a consumption loss of 1.7% compared to the second best allocation
- **The introduction of temporary contracts is welfare improving** when EPR is *too high*, but generates high labor turnover, with potential side effects.
  - ▶ The flexibility introduced in Italy during the 2000s closed around 1/4 of the gap between the inefficient pre-reform allocation and the single-contract second best allocation

## Outline

### 1. Introduction

### 2. Single contract

The setup

LMIs and social efficiency

Constrained optimal level of EPR

### 3. Dual market

Introduce temporary contracts

Quantitative analysis

### 4. Conclusions

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## Overview of the model

- The **representative household** consumes and invests in bonds. Labor supply is fixed **Households**
- The **government** gets fiscal revenues from layoff taxes and from a lump-sum component, which adjusts to finance unemployment benefits (balanced budget) **Gov't**
- The labor market is characterized by **labor market frictions**: unemployed workers ( $u_{t-1}$ ) search for jobs and firms post vacancies ( $v_t$ ). Hires are determined by an aggregate matching function  $m(v_t, u_{t-1})$  **Laws of motion**
- Each firm-worker match **produces** using linear technology s.t. aggregate & idiosyncratic productivity shocks:  $y_{it} = A_t + z_{it}$  **Resource const.**



## Labor market institutions

We consider these labor market institutions (LMIs):

1.  $\gamma$ : level of **firing costs** (EPR: Employment Protection on Regular contracts) paid by the firm upon layoff
2.  $b$ : **unemployment benefits** paid to unemployed individuals and financed by the government through lump-sum taxes
3.  $\Theta_w$ : **trade union density**, which affects wage rigidity (the higher  $\Theta_w$ , the less firms can adjust wages to idiosyncratic shocks)

We study the **interaction** among them, and whether there exist an **optimal level** of  $\gamma$ , given the levels of  $b$  and  $\Theta_w$

## Firm's decisions

- **Value of a productive match:**

$$J(z_{it}) = y(\cdot) - w(\cdot) + \mathbb{E}_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} [(1 - \phi) \max \{J_{t+1}, V_{t+1} - \gamma\} + \phi V_{t+1}] \right\}$$

For a new match:  $z_{it} = \bar{z}$  (max of the idiosyncratic prod). Workers

- **Job creation:** free-entry by firms until  $V_t = 0$  or:

$$\frac{c}{q_t} = J_{0t}(\bar{z})$$

where  $q(\theta_t)$  is the job filling probability.

- **Job destruction:** exogenous (at rate  $\phi$ ) + endogenous  $\rightarrow$  in any period a new  $z_{it}$  is drawn and the firm decides to continue iff  $z_{it} \geq \tilde{z}_t$ . Otherwise the worker is laid off and the firm pay the firing cost  $\gamma$

$$S_t^F(\tilde{z}_t) = J_t(\tilde{z}_t) - V_t + \gamma = 0$$

## Wage setting schemes

1. **Flexible wages:** wages are renegotiated whenever a shock occurs and set through Nash bargaining. Nash
2. **Rigid wages:** The wage is a weighted average of the efficient Nash-bargained wage –  $w^n(z_{it})$  – and a wage norm –  $w_t^*$ .

$$w(z_{it}) = (1 - \Theta)w^n(z_{it}) + \Theta w_t^*$$

where  $\Theta$  is a proxy for trade union density.

When  $\Theta > 0$  wages cannot perfectly adjust to idiosyncratic productivity shocks  $\rightarrow$  there exist **privately inefficient** separations when wage is too high and firms cease a match with negative *firm* surplus, but positive *total* one.

## Effects of LMIs on social efficiency

**Question:** How do LMIs affect allocative efficiency?

Compare:

- (i) The decentralized competitive allocation [Details](#)
- (ii) Social planner allocation (*first best*): efficient allocation that would be selected by a social planner who maximizes welfare subject *only* to technological constraints and search and matching frictions, abstracting from LMIs [Details](#)

Steps:

1. Study the role of unemployment benefits ( $b$ ) and trade union density ( $\Theta$ ) by setting firing costs to 0 ( $\gamma = 0$ )
2. Re-introduce firing costs as a way to compensate inefficiencies generated by other institutions (taken as given)

## Effects of LMIs on social efficiency: results

### Proposition 1

1. *Positive **unemployment benefits** determine a sub-optimal low level of job creation and a sub-optimal high level of job destruction*
2. ***Wage rigidities** determine a sub-optimal high level of both job creation and job destruction*

Sub-optimal high level of job destruction  $\Rightarrow$  market allocation generates *socially inefficient* separations:

- **unemployment subsidies**: they are distortive while taxes to finance them are lump-sum. So hh's fail to recognize that subsidies cancel out with taxes
- **wage rigidity**: here separations are also *privately inefficient* because they could be neutralized by appropriate side payments between the firm and the worker

## Can firing costs restore efficiency?

*Question:* Can EPR be used to compensate the inefficiencies generated by the other LMIs?

- The effects of EPR is *a priori* ambiguous because it may help reducing excessive job destruction but may also hinder job creation.
- We need to find the optimal level of EPR conditional on the other LMIs (*second best*):
  1. We derive analytically the constrained optimal level of firing costs by solving a suitable Ramsey problem [Details](#)
  2. Application to the Italian economy

## Can firing costs restore efficiency? (1) Analytical results

### Proposition 2

*The optimal level of firing costs which implements the second-best allocation is:*

1. *equal to zero in absence of unemployment benefits and trade union density ( $b = \Theta_w = 0$ )*
2. *monotonically increasing in the amount of unemployment benefits  $b$  ( $\frac{\partial \gamma^*}{\partial b} > 0$ )*
3. *monotonically increasing in trade union density  $\Theta_w$  ( $\frac{\partial \gamma^*}{\partial \Theta_w} > 0$ )*

## Can firing costs restore efficiency? (2) The case of Italy

Calibrate the model to the Italian economy in 1985-1995 (high firing costs); afterwards, several reforms liberalized the labor market by introducing temporary contracts

	Parameter	Value	Source
<i>Targeted moments</i>			
Interest rate	-	11.9%	avg. ann. int. rate (1985-1995)
Unemployment rate	$u$	9.4%	avg. unemp. rate (1985-1995)
Job finding rate	$f(\theta)$	0.4	D'Amuri et al. (2021)
Quits out of total separations	$\bar{s} = \phi/\delta$	60%	Comunicazioni Obbligatorie
Hiring costs as % of GDP	$\bar{h} = cn_0/Y$	1%	Blanchard and Galí (2010)
<i>Institutional parameters</i>			
Benefits over labor income	$\frac{bu}{w(1-u)}$	0.04	Luksic (2020)
Trade union density	$\Theta_w$	40%	OECD
Firing cost as fraction of avg. perm wage	$\gamma/w$	<b>1.2</b>	Garibaldi and Violante (2005)



Figure 1: Long-run effects of firing costs

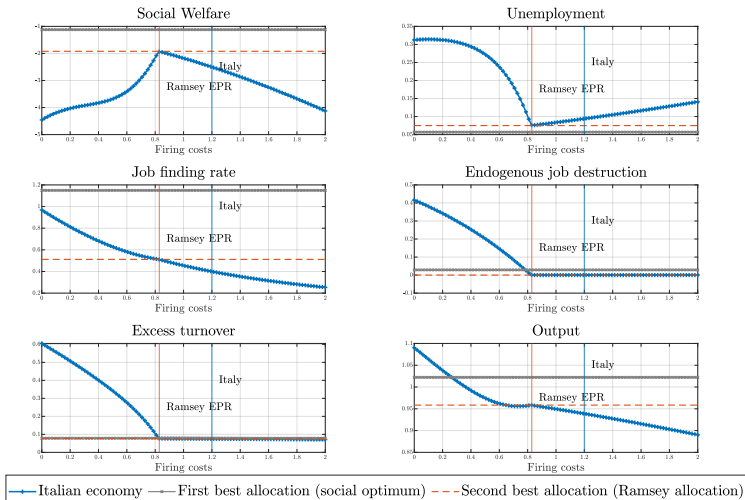
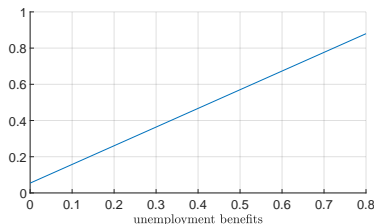
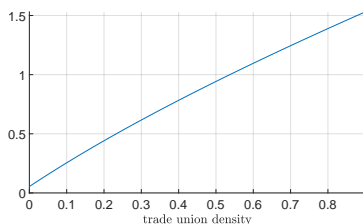


Figure 2: Optimal firing costs  $\gamma^*$ (a)  $\gamma^*$  wrt Unemployment benefits(b)  $\gamma^*$  wrt Trade union density

In panel a) the wage rigidity parameter is set at its benchmark value ( $\Theta_w = 0.4$ ); in panel b) unemployment benefits are at their benchmark level ( $b = 0.37$ ).

## LMIs and efficiency in Italy: summary of results

- The downward wage rigidity determined by unemployment benefits and trade union density generate inefficient job separations
- In this context, a moderate amount of firing costs is welfare-improving: the Ramsey planner would choose a level of firing costs roughly equal to 2.5 monthly wages (0.83 at quarterly frequency)
- Beyond optimal EPR, the negative impact on job creation prevails
- The stricter firing restrictions faced by Italy (3.5 monthly wages) generated a sizeable consumption loss (1.7%) compared to Ramsey
- Practical solution: **flexibility at the margin** (introduction of temporary contracts). Welfare improving?

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## Research questions

- Can temporary contracts restore efficiency when EPR is too high?
- Can temporary contracts substitute for excessive job security in regular contracts?
- What is the constrained optimal level of duality?

## Introduce temporary contracts (1)

- Temporary contracts (TCs) are characterized by:
  - ▶ No firing costs ( $\gamma^T = 0$ )
  - ▶ No wage rigidity ( $\Theta_w^T = 0$ )
  - ▶ Expiration rate  $\iota$ : if expired, can be converted to permanent; if not, can be renewed a limited number of times
  - ▶ Cost of posting a vacancy  $c^T$
- In this setup  $c^T$  can serve as proxy for legal impediments to the introduction of TCs
- By setting  $c^T$  to a very high number, we are indeed able to replicate the single-contract economy

## Introduce temporary contracts (2)

The introduction of TCs implies a modification of the single contract model by:

- **new budget constraint** Budget constraint
- **new laws of motions** Law of motions
- **new firms' value functions for TCs** Firms' value functions for TCs
- **new workers' value functions for TCs** Workers' value functions for TCs
- **new Nash and effective wages for TCs** Wages for TCs
- **new JC and JD**: see next slides

## Job creation

- **One-sided directed search:**
  - ▶ Firms can post vacancies in either the temporary or permanent submarket
  - ▶ In both submarkets they face the same pool of (unemployed) workers
- **Firms trade off** the *ex-ante* benefits of a quick search with the *ex-post* costs of EPR.
- **Spillovers** from the market of regular jobs to the one of TCs:

$$(\text{job finding rate})^T = f_t^T (1 - f_t^P)$$

$$(\text{job filling rate})^T = q_t^T (1 - f_t^P)$$

- **Free-entry** in both markets
- **Co-existence** of temporary and permanent contracts in the flow of new matches



## The reduced TCs' model

By following (and doubling) the same steps as in the single-contract economy we derive the reduced equilibrium form of the dual market model as:

$$S_t^F(\tilde{z}_t^C) = J_{0t}^P(z_{it}) - V_t = 0$$

$$S_t^F(\tilde{z}_t^T) = J_t^T(z_{it}) - V_t + \gamma^T = 0$$

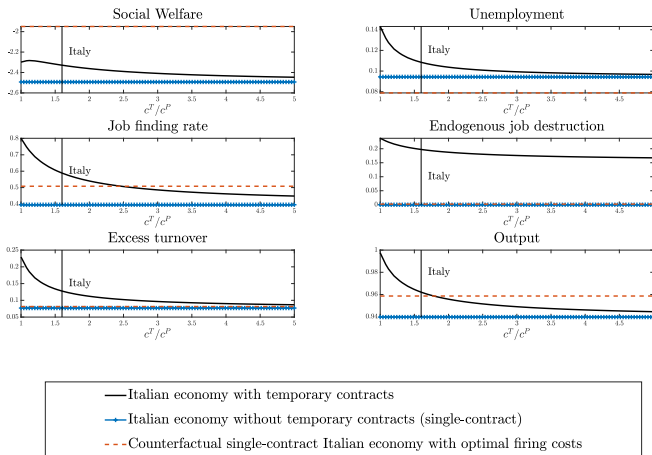
$$\tilde{z}_t^C = \tilde{z}_t^P + \frac{\gamma^P}{[1 - \eta(1 - \Theta_w^P)]}$$

$$\frac{c^T}{\xi_t^T} = [1 - \eta(1 - \Theta_w^T)] (\bar{z} - \tilde{z}_t^T) - \gamma^T$$

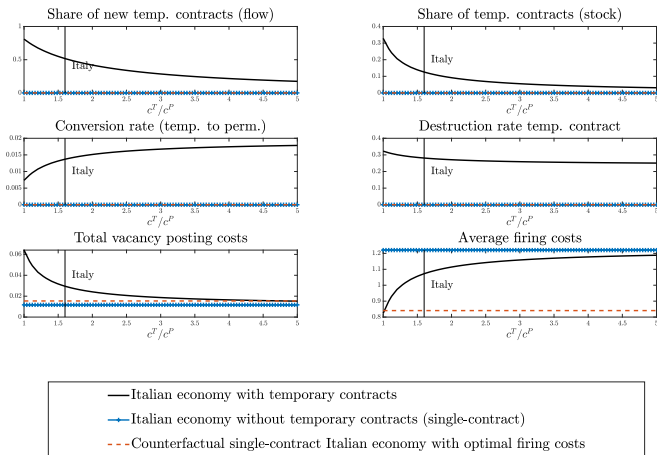
## The experiments

- We calibrate  $c^T$  and  $\iota$  to match Italian data in the period 2003-2012:
  - ▶ average share of TCs must equal 12.6% ( $\rightarrow c^T$  is 60% higher than  $c^P$ )
  - ▶ transition rate from a temporary job to unemployment equal to 0.29 ( $\rightarrow \iota = 0.083$  corresponding to a maximum duration of 12 quarters)
- We then perform several experiments:
  - ▶ lowering  $c^T$  to simulate the gradual liberalization of TCs
  - ▶ lowering  $\iota$  to simulate extending their duration
  - ▶ increasing  $\gamma^T$  to simulate increasing employment protection
  - ▶ increasing  $\Theta_w^T$  to simulate increasing wage stickiness

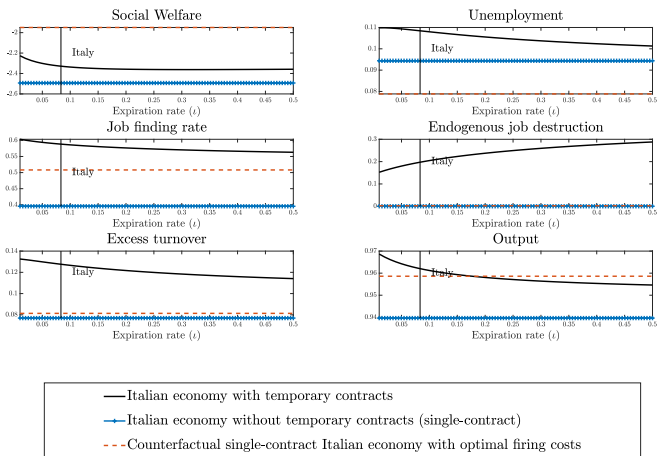
## LR effects of cost of creating a TC (1)



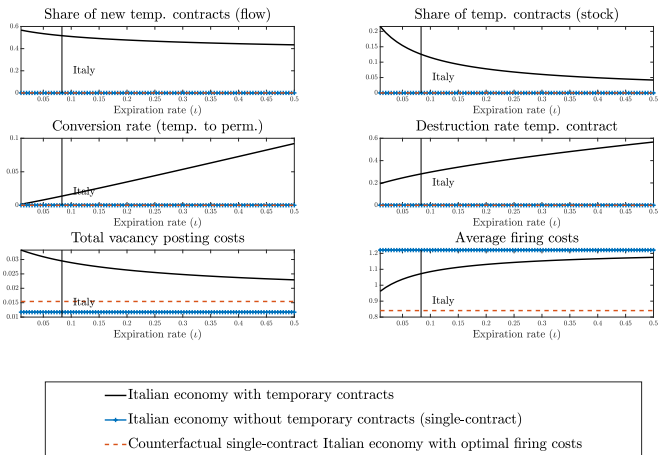
## LR effects of cost of creating a TC (2)



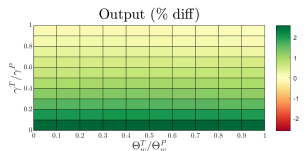
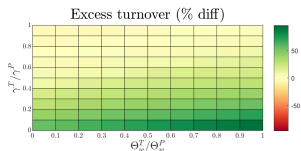
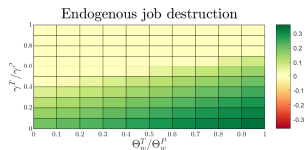
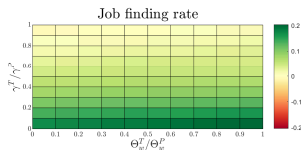
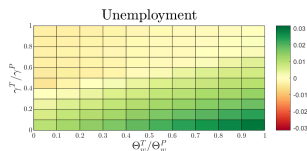
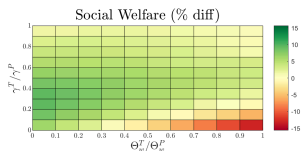
## LR effects of TC's expiration rate (1)



## LR effects of TC's expiration rate (2)



# LR effects of TC's employment protection and wage stickiness (w.r.t. the single-contract economy)



## Answers to the research questions

- The introduction of TCs increases social welfare and closes around **one fourth of the gap** between the decentralized single-contract economy (blue line) and the optimal EPR policy (orange line)
- **Social welfare is maximized when TCs are almost fully liberalized**, hence when there are no limits to their duration and when the cost of posting a vacancy for a temporary job is only slightly higher than the cost of creating a regular job
- Overall, starting from the single-contract scenario, the most dangerous policy is introducing temporary contracts with no firing costs but still rather inflexible wages: ideally, **one would have TCs with fully flexible wages and a moderate degree of firing restrictions**



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## Wrap up

- Employment protection on regular contracts has non-linear effects
- The *optimal* level of employment protection increases with downward real wage rigidities generated by other LMIs
- Flexible contracts can mitigate the effects of excessive firing costs but exacerbate turnover
- Policy implication: need of unified design of labor market policies

THANKS FOR YOUR ATTENTION

Households [← Back](#)

The **representative household** consumes and invests in bonds. Labor supply is fixed and equal to 1.

$$W_t^H = \max_{C_t, B_t} [\log C_t + \mathbb{E}_t \beta W_{t+1}^H]$$

$$\text{s.t. } C_t + B_t = R_{t-1} B_{t-1} + w_{0t} n_{0t} + w_t n_t + bu_t + \Pi_t^F - \bar{T}_t$$

$$\Pi_t^F = Y_t - w_{0t} n_{0t} - w_t n_t - cv_t - (1 - \phi) G(\tilde{z}_t) (n_{0t} + n_t) \gamma$$

$$\bar{T}_t = T_t + (1 - \phi) G(\tilde{z}_t) (n_{0t} + n_t) \gamma$$

where

- We distinguish between **newly hired** ( $n_{0t}$ ) and **incumbent workers** ( $n_t$ ) because they earn a different wage ( $w_{0t}$  and  $w_t$ )
- $\Pi_t^F$ : **firm's profits**  $\Rightarrow$  production ( $Y$ ) - labor income - vacancy costs ( $cv_t$ ) - firing costs  $((1 - \phi) G(\tilde{z}_t) (n_{0t} + n_t) \gamma)$
- $\bar{T}_t$ : **lump-sum taxes**

## The government [◀ Back](#)

$$\bar{T}_t = T_t + (1 - \phi) G(\tilde{z}_t) (n_{0t} + n_t) \gamma$$
$$bu_t = T_t$$

Notice that:

- Total taxes  $\bar{T}_t$  are made of two components:  $T_t$ , collected lump-sum but adjusted each period to finance unemployment benefits, plus layoff taxes, collected via dismissals [Albertini and Fairise (2013)]
- **Unemployment subsidy** ( $b$ ) does not have any proper insurance motive (pooled consumption within the household): however, it affects both labor supply and demand, interacting with other LMIs
- **Firing costs** ( $\gamma$ ) do not represent a pure waste because they finance the government and reduce the need to levy additional taxes

## Laws of motion

$$1 = n_{0t} + n_t + u_t$$

$$n_{0t} = f_t u_{t-1}$$

$$n_t = (1 - \delta_t)(n_{0t-1} + n_{t-1})$$

where

- $f_t$  is the job finding rate
- $\delta_t$  is the overall separation rate (exogenous + endogenous separations)

◀ back

## Aggregate resource constraint

- Total production

$$Y_t = n_{0t} (A_t + \bar{z}) + n_t \left( A_t + \int_{\bar{z}_t}^{\bar{z}} z' dG(z') \right)$$

- Aggregate resource constraint

$$Y_t = C_t + cv_t$$

◀ back

## Worker's value functions

- Value of unemployment:

$$U_t = b + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} [f_{t+1} W_{0t+1}(\bar{z}) + (1 - f_{t+1}) U_{t+1}]$$

where  $f_t$  is the job finding rate.

- Value function for a worker:

$$W_t(z_{it}) = w_t(z_{it}) + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \phi) \left[ \int_{\bar{z}_{t+1}}^{\bar{z}} W_{t+1}(z') dG(z') + G(\bar{z}_{t+1}) U_{t+1} \right] \\ + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \phi U_{t+1}$$

◀ back



## Nash bargaining

Wage setting:

$$(1 - \eta) (W_t(z_{it}) - U_t) = \eta (J_t(z_{it}) - V_t + \gamma)$$

$$(1 - \eta) (W_{0t}(\bar{z}) - U_t) = \eta (J_{0t}(\bar{z}) - V_t + \gamma)$$

After some algebra we obtain:

$$w_t^n(z_{it}) = (1 - \eta)b + \eta \left\{ y_t(A_t, z_{it}) + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} c \theta_{t+1} + \gamma \left[ 1 - \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \phi - f_{t+1}) \right] \right\}$$
$$w_{0t}^n(\bar{z}) = w_t^n(\bar{z})$$

◀ back

## Decentralized allocation

We collapse the main equations of the model to obtain two conditions for the labor market tightness ( $\theta_t$ ) and the job destruction threshold ( $\tilde{z}_t$ ) which characterize the decentralized allocation and evaluate them in steady state:

$$\begin{aligned} \frac{c}{q(\theta^{dec})} &= [1 - \eta(1 - \Theta_w)] (A + \bar{z}) + [1 - \eta(1 - \Theta_w)] \beta(1 - \delta) [H(\tilde{z}^{dec}) - \tilde{z}^{dec}] \\ &\quad - \eta(1 - \Theta_w)\beta c\theta^{dec} - (1 - \Theta_w)(1 - \eta)b - \Theta_w w^* \\ &\quad - \eta(1 - \Theta_w) [1 - \beta(1 - \phi)] \gamma - \beta(1 - \phi)\gamma - \eta(1 - \Theta_w)\beta f(\theta^{dec})\gamma \\ \frac{c}{q(\theta^{dec})} &= [1 - \eta(1 - \Theta_w)] (\bar{z} - \tilde{z}^{dec}) - \gamma \end{aligned}$$

◀ back

## Social planner allocation (first best)

The social planner chooses labor market tightness, unemployment and the job destruction threshold to solve the following problem:

$$\begin{aligned} \max_{\{\theta_t, u_t, \bar{z}_t\}_{t=0}^{\infty}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \\ \text{s.t.} \quad & C_t = n_{0t} (A_t + \bar{z}) + n_t \left[ A_t + \frac{1}{1 - G(\bar{z}_t)} \int_{\bar{z}_t}^{\bar{z}} xg(x)dx \right] - c\theta_t u_{t-1} \\ & n_{0t} = f(\theta_t)u_{t-1} \\ & n_t = (1 - \phi) [1 - G(\bar{z}_t)] (n_{0t-1} + n_{t-1}) \\ & 1 = u_t + n_{0t} + n_t \\ & f(\theta_t) = \Phi\theta_t^\varepsilon \end{aligned}$$

## Social planner allocation: solution

By computing the planner's FOCs and simplifying we obtain two conditions characterizing labor market tightness and the job destruction threshold for the optimal allocation in steady state:

$$\frac{c}{q(\theta^{opt})} = \varepsilon(A + \bar{z}) + \varepsilon\beta(1 - \delta) [H(\tilde{z}^{opt}) - \tilde{z}^{opt}] - (1 - \varepsilon)\beta c\theta^{opt}$$
$$\frac{c}{q(\theta^{opt})} = \varepsilon(\bar{z} - \tilde{z}^{opt})$$

[◀ back](#)

## Ramsey allocation (second best) [← back](#)

We solve the Ramsey problem of a social planner who takes into account the constraints due to agents' choices in the decentralized economy, which are affected by LMIs. The planner takes as given  $b$  and  $\Theta_w$  and choose the best possible allocation that can be attained by changing  $\gamma$ .

$$\begin{aligned} & \max_{\{\theta_t, u_t, \bar{z}_t, \gamma\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \\ \text{s.t.} \quad & C_t = n_{0t} (A_t + \bar{z}) + n_t \left[ A_t + \frac{1}{1 - G(\bar{z}_t)} \int_{\bar{z}_t}^{\bar{z}} xg(x)dx \right] - c\theta_t u_{t-1} \\ & n_{0t} = f(\theta_t)u_{t-1} \\ & n_t = (1 - \phi) [1 - G(\bar{z}_t)] (n_{0t-1} + n_{t-1}) \\ & 1 = u_t + n_{0t} + n_t \\ & f(\theta_t) = \Phi\theta_t^{\xi} \\ & \frac{c}{q(\theta_t)} = [1 - \eta(1 - \Theta_w)] (\bar{z} - \bar{z}_t) - \gamma \\ & J_t(\bar{z}_t) - V_t + \gamma = 0 \end{aligned}$$

## The dual labor market

HHs' budget constraint in the dual labor market model:

$$C_t + B_t = R_{t-1}B_{t-1} + w_{0t}^P n_{0t}^P + w_t^P n_t^P + w_{0t}^T n_{0t}^T + w_t^T n_t^T + bu_t + \Pi_t^F - \bar{T}_t$$

Law of motions in the dual labor market model:

$$\begin{aligned} 1 &= n_{0t}^T + n_t^T + n_{0t}^P + n_t^P + u_t \\ n_{0t}^T &= \chi_t^T u_{t-1} \\ n_{0t}^P &= f_t^P u_{t-1} + \zeta_t (n_{0t-1}^T + n_{t-1}^T) \\ n_t^T &= (1 - \delta_t^T - \zeta_t)(n_{0t-1}^T + n_{t-1}^T) \\ n_t^P &= (1 - \delta_t^P)(n_{0t-1}^P + n_{t-1}^P) \end{aligned}$$

## The dual labor market

### Firms' value functions for TCs in the dual labor market

$$J_t^T(z_{it}) = y_t(A_t, z_{it}) - w_t^T(A_t, z_{it}) + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1 - \phi) \left[ \iota \int_{\bar{z}_{t+1}^C}^{\bar{z}} J_{t+1}^P(z') dG(z') + (1 - \iota) \left[ \int_{\bar{z}_{t+1}^T}^{\bar{z}} J_{t+1}^T(z') dG(z') - G(\bar{z}_{t+1}^T) \gamma^T \right] \right] \right.$$

$$\left. J_{0t}^T(\bar{z}) = y_{0t}(A_t, \bar{z}) - w_{0t}^T(A_t, \bar{z}) + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1 - \phi) \left[ \iota \int_{\bar{z}_{t+1}^C}^{\bar{z}} J_{t+1}^P(z') dG(z') + (1 - \iota) \left[ \int_{\bar{z}_{t+1}^T}^{\bar{z}} J_{t+1}^T(z') dG(z') - G(\bar{z}_{t+1}^T) \gamma^T \right] \right] \right\}$$

$$V_t^T = -c^T + \xi_t^T J_{0t}^T(\bar{z}) + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \xi_t^T) V_{t+1}^T$$

$$S_t^{FT}(z_{it}) = J_t^T(z_{it}) - V_t^T + \gamma^T$$

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## The dual labor market

### HHs' value functions for TCs in the dual labor market

$$W_t^T(z_{it}) = w_t^T(A_t, z_{it}) + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1 - \phi) \left[ \begin{array}{l} \iota \int_{\bar{z}_{t+1}^C}^{\bar{z}} W_{t+1}^P(z') dG(z') \\ + (1 - \iota) \left[ \int_{\bar{z}_{t+1}^T}^{\bar{z}} W_{t+1}^T(z') dG(z') \right] \\ + \left[ \iota G(\bar{z}_{t+1}^C) + (1 - \iota) G(\bar{z}_{t+1}^T) \right] U_{t+1} \end{array} \right] + \phi U_{t+1} \right\}$$

$$W_{0t}^T(\bar{z}) = w_{0t}^T(A_t, \bar{z}) + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1 - \phi) \left[ \begin{array}{l} \iota \int_{\bar{z}_{t+1}^C}^{\bar{z}} W_{t+1}^P(z') dG(z') \\ + (1 - \iota) \left[ \int_{\bar{z}_{t+1}^T}^{\bar{z}} W_{t+1}^T(z') dG(z') \right] \\ + \left[ \iota G(\bar{z}_{t+1}^C) + (1 - \iota) G(\bar{z}_{t+1}^T) \right] U_{t+1} \end{array} \right] + \phi U_{t+1} \right\}$$

$$U_t = b + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left[ \chi_t^T W_{0t+1}^T(\bar{z}) + f_{t+1}^P W_{0t+1}^P(\bar{z}) + \left( 1 - \chi_t^T - f_{t+1}^P \right) U_{t+1} \right]$$

$$S_t^{WT}(z_{it}) = W_t^T(z_{it}) - U_t$$



## The dual labor market

### Nash and effective wages in the dual labor market

$$(1 - \eta) (W_t^T(z_{it}) - U_t) = \eta (J_t^T(z_{it}) - V_t^T + \gamma^T)$$

$$(1 - \eta) (W_{0t}^T(\bar{z}) - U_t) = \eta (J_{0t}^T(\bar{z}) - V_t^T + \gamma^T)$$

$$w_t^T(z_{it}) = (1 - \Theta_w^T) w_t^{nT}(z_{it}) + \Theta_w^T w_t^{T*}$$

$$w_{0t}^T(\bar{z}) = (1 - \Theta_w^T) w_{0t}^{nT}(\bar{z}) + \Theta_w^T w_t^{T*}$$

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