Diagnostic Expectations and Macroeconomic Volatility[*]

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Abstract

Diagnostic expectations have emerged as an important departure from rational expectations in macroeconomics and finance. We present a first treatment of diagnostic expectations in linear macroeconomic models. To this end, we establish a strong additivity property for diagnostic expectations. The solution method and stability properties are discussed in full generality. Under some conditions, diagnostic expectations generate higher volatility than rational expectations. We show that this is true in standard New Keynesian models, as in medium-scale DSGE models; in real business cycle models output and investment are characterized by dampening, instead. Finally, we discuss how the combination of diagnosticity with imperfect information can rationalize under- and over-reaction in macroeconomics.

Keywords: Heuristics, representativeness, shocks, endogenous cycles.

JEL codes: E12, E32, E71.

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1 Introduction

Diagnostic expectations have emerged as an important departure from rational expectations in macroeconomics and finance. The appeal of the use of diagnostic expectations is that it leads to a psychologically microfounded approach based on the pioneering work by Kahneman and Tversky (1972). Moreover, it delivers a great deal of tractability, as recent efforts in economics and finance have demonstrated (Bordalo, Gennaioli, and Shleifer 2018; Bordalo, Gennaioli, La Porta, and Shleifer 2019; Bordalo, Gennaioli, Ma, and Shleifer 2020).

In this paper, we seek to improve the portability of diagnostic expectations towards macroeconomics. We extend the diagnosticity and representativeness approach to the general class of linear models, which forms a cornerstone of macroeconomics. As a result, we obtain a number of methodological and substantive results.

Specifically, we present five set of results.

First, we provide a general treatment of diagnostic expectations in linear recursive models. To this end, we establish a strong additivity result for linear-Gaussian setups. We use this result to obtain the solution to linear models with diagnostic expectations. Our strategy consists in showing that one can obtain a rational expectations representation of the diagnostic expectations model. Once this is achieved, the model can be solved using standard linear techniques. Importantly, we show that the stability properties are identical to those obtained under rational expectations. To our knowledge, all of this has not been studied before in the literature.

Second, we point out that it is straightforward to look at the structure of the solution to linear models with diagnostic expectations and obtain conditions under which this type of expectations lead to extra volatility of the endogenous variables over the rational expectations benchmark. We show that the conditions for extra volatility are typically verified in demand-determined economies, whereas these conditions typically fail in supply-determined economies. We present a range of applications to quantitatively assess the degree of extra volatility case by case.

Third, we show how to loglinearize macroeconomic models under diagnostic expectations. Different from many other departures from the full-information rational expectations case, as for example the introduction of imperfect information (Woodford 2002), rational inattention (Mackowiak and Wiederholt 2009), or other behavioral models (Garcia-Schmidt and Woodford 2019), it turns out that diagnostic expectations change the structure of equilibrium conditions of the loglinear model, leading to novel implications in forward looking equations. For example, under diagnostic expectations, we obtain the following Fisher equation, linking the real interest rate to the nominal interest rates and the inflation rate:

$$\hat{r}_t = \hat{i}_t - \mathbb{E}_t^\theta[\pi_{t+1}] - \theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])$$

So, the computation of real rate $\hat{r}_t$ not only takes expected inflation into consideration, but also the *revision* in inflation expectations, up to a factor equal to the diagnosticity parameter θ. (See Section 3.)

Fourth, we show that in the simple New Keynesian model, diagnostic expectations lead to contractions following a negative TFP shock, a feature that fits the evidence regarding the Covid-19 recession. It is the subject of a growing literature discussed in Section 3 (whereas the rational expectations benchmark fails in this regard.)

Fifth, we combine diagnosticity with imperfect information and attempt to contribute to the recent literature on under/overreaction of macroeconomic expectations[1] Our results illustrate two interesting cases: First, a case of aggregate overreaction even in the presence of noisy information, and second, a case of a gradual buildup of overreaction through a learning channel.

Related Literature. The paper is related to the emerging literature on diagnostic expectations. See Gennaioli and Shleifer (2010), Bordalo, Gennaioli, and Shleifer (2018), Bordalo, Gennaioli, La Porta, and Shleifer (2019), and Bordalo, Gennaioli, Ma, and Shleifer (2020) for example, and Gennaioli and Shleifer (2020) for a review. These are forward looking models of extrapolative expectations where agents over-weight future states that are representative of recent news. Bordalo, Gennaioli, Shleifer, and Terry (2020), and Maxted (2020) incorporate diagnostic expectations in macroeconomic frameworks. Our contribution is to provide a general treatment of diagnostic expectations in linear macroeconomic models. To our knowledge, our paper is the first to evaluate diagnostic expectations for endogenous variables in a closed-economy general equilibrium setup.

More broadly, our paper fits into the macroeconomics literature that models departure from rational expectations with various behavioral assumptions. Some references for this literature include Howitt (1992), Evans and Honkapohja (2001), Woodford (2002), Sims (2003), Mackowiak and Wiederholt (2009), Angeletos and La'O (2013),

[1] See Coibion and Gorodnichenko (2012), Bordalo et al. (2020), Kohlhas and Walther (2020), among others.

Woodford (2013), and Eusepi and Preston (2018).

Some of the recent applications have focused on resolving puzzles in New Keynesian models with behavioral assumptions. Angeletos and Lian (2018), Farhi and Werning (2019), Gabaix (2020), and Garcia-Schmidt and Woodford (2019) are some of the papers that propose departures from rational expectations to attenuate the strength of forward guidance. Iovino and Sergeyev (2020) study the effectiveness of central bank balance sheet policies with level-k thinking. Farhi and Werning (2020) study the role of monetary policy as a macro-prudential tool when agents form extrapolative expectations. Malmendier and Nagel (2016) propose and test a framework where agents overweight inflation experiences in their lifetimes. Fuster, Laibson, and Mendel (2010) propose a framework where agents extrapolate from recent changes, which creates an over-reaction to news, and excess volatility in consumption.

An active literature has consistently rejected the full-information rational expectations (FIRE) hypothesis in the data. Mankiw and Reis (2002) resolve the delayed response of inflation to monetary policy shocks via a sticky-information model of nominal rigidities. Coibion and Gorodnichenko (2012) find that response of forecast errors to structural shocks, in survey data, is inconsistent with FIRE models, but consistent with models of informational rigidities. Using survey forecasts, Coibion and Gorodnichenko (2015) find evidence for a general property of models with informational rigidities, namely revisions in forecasts systematically predict future forecast errors. The correlation between future forecast errors and forecast revisions is found to be positive in Coibion and Gorodnichenko (2015) when looking at consensus forecasts. This result is referred to as under-reaction of forecasts relative to the FIRE benchmark. Bordalo, Gennaioli, La Porta, and Shleifer (2019), among others, find evidence of a negative correlation between individual forecast errors and forecast revisions. Bordalo, Gennaioli, Ma, and Shleifer (2020) propose a model of diagnostic expectations with dispersed information to reconcile why there may be an over-reaction in forecasts at the individual level, but under-reaction at the aggregate level. Angeletos, Huo, and Sastry (2020) document that expectations of inflation and unemployment under-react initially, and over-shoot gradually in response to business cycle shocks. In Section 4, we propose a model of diagnostic expectations with learning that can generate under-reaction, over-reaction or delayed over-reaction to shocks.

Because we explore the implications of combining diagnostic expectations with imperfect information (as Bordalo, Gennaioli, Ma, and Shleifer 2020 do), our paper is related to the seminal work of Lorenzoni (2009). Blanchard, L'Huillier, and Lorenzoni

(2013) extend this work empirically and study how to fit a medium-scale DSGE to U.S. business cycle data. Cao and L'Huillier (2018) apply a similar methodology to crisis episodes. Melosi (2014) presents an important empirical application under dispersed information.

Paper Organization. The paper is organized as follows. Section 2 presents our solution procedure and general conditions for extra volatility. Section 3 presents the applications. Section 4 introduces imperfect information with the objective of discussing related literature. Section 5 concludes. The Appendix provides supplementary materials and collects all the proofs.

2 General Formulation, Solution Procedure, and General Properties

We present a general linear model where agents use diagnostic expectations to form beliefs about the future evolution of the exogenous drivers buffeting the economy and all endogenous variables. We then show how one can obtain the solution to this class of models. Our strategy consists in showing that one can obtain a rational expectations (RE) representation of the diagnostic expectations (DE) model. Once this is achieved, the model can be solved using standard techniques.[2]

The main technical difficulty in the context of DSGE models is the presence of predetermined variables. We show how to obtain additivity in this case, an issue that has not been addressed in previous literature. A key step is to appropriately specify the reference distribution. To this end, we extend the "no-news" approach introduced by Bordalo, Gennaioli, and Shleifer (2018) to predetermined variables. This delivers tractability.

[2] For clarity, the model is set up in a perfect information context. It is straightforward to extend this to an imperfect information environment by exploiting the relation between the diagnostic Kalman filter (Bordalo, Gennaioli, Ma, and Shleifer 2020) and the rational or true Kalman filter. This is what we use in the imperfect information extension presented in Section 4.

2.1 Preliminary Considerations: Linearity Results for the Diagnostic Expectation Operator

Define the following two AR(1) processes for random variables x_t and y_t:

$$x_t = \rho_x x_{t-1} + \varepsilon_t$$

$$y_t = \rho_y y_{t-1} + \eta_t$$

where ε_t and η_t are Gaussian and orthogonal exogenous shocks:

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2), \qquad \eta_t \sim N(0, \sigma_\eta^2)$$

ρ_x and ρ_y are persistence parameters satisfying $\rho_x, \rho_y \in [0, 1)$, and σ_ε^2 and σ_η^2 are the shocks' variances.

We now define the diagnostic expectation operator that agents in the model will use to form beliefs about future realizations of these random variables. For the sake of the argument, we first focus on x_t. In an environment where agents have diagnostic beliefs, agents will form these expectations based on a probability distribution function (pdf) distorted by the representativeness heuristic (Kahneman and Tversky 1972; Kahneman, Slovic, and Tversky 1982), which following the language of Bordalo, Gennaioli, and Shleifer (2018) (and earlier related work as Gennaioli and Shleifer 2010), leads to 'diagnosticity' in beliefs.

The true (or non-distorted) pdf of x_{t+1} is

$$f(x_{t+1}|x_t) \propto \varphi\left(\frac{x_{t+1} - \rho_x x_t}{\sigma_\varepsilon}\right)$$

where $\varphi(x)$ is the density of a standard normal distribution:

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

We follow the formulation by Bordalo et al. (2018) closely and make assumptions such that, due to the representativeness, agents overweight the last realization of the x_t when forming beliefs about the future realization of x_{t+1}.[3] The diagnostic distribution

[3] A related, more complex approach, specifies this overweighting as a function not only on the last realization, but also a number of previous ones as well. This is considered in the appendix of Bordalo et al. (2018) and also in Maxted (2020) and Bordalo, Gennaioli, Ma, and Shleifer (2020). In the context of medium-scale DSGEs, extending

is defined as

$$f_t^{\theta}(x_{t+1}) = f(x_{t+1}|G) \cdot \left[\frac{f(x_{t+1}|G)}{f(x_{t+1}|-G)}\right]^{\theta} \cdot C$$

where G and $-G$ are conditioning events. G encodes current conditions, and $-G$ encodes a reference group (i.e. a reference set of events). Accordingly, $f(x_{t+1}|-G)$ is named the "reference distribution". $\theta \geq 0$ is the diagnosticity parameter, and C is a constant ensuring that $f_t^{\theta}(x_{t+1})$ integrates to 1. Notice the role played by the likelihood ratio $f(x_{t+1}|G)/f(x_{t+1}|-G)$ in distorting beliefs to a degree parametrized by θ.

In this dynamic setting, current conditions are the state at t, thus $G \equiv \{x_t = \bar{x}_t\}$.[4] Following Bordalo, Gennaioli, and Shleifer (2018), we shall impose that the event $-G$ carries "no news" about exogenous variables at time t (henceforth no-news assumption or NNA). In this dynamic setting, this is interpreted as if agents did not learn anything new at time t.

Assumption 1 (Univariate No-News Assumption)

$$f(x_{t+1}|-G) = f(x_{t+1}|x_t = \rho_x \bar{x}_{t-1})$$

This assumption imposes that beliefs about future x_{t+1} are formed conditional on the event that the random variable x_t, conditional on the past realization $\bar{x}_{t-1}$, is what it was expected to be, so $\varepsilon_t = E[\varepsilon_t] = 0$, which is equivalent to $x_t = \rho_x \bar{x}_{t-1}$. As we shall see, this assumption ensures tractability. We make this assumption throughout the paper.

Under the NNA, the diagnostic distribution is then written

$$f_t^{\theta}(x_{t+1}) = f(x_{t+1}|x_t = \bar{x}_t) \cdot \left[\frac{f(x_{t+1}|x_t = \bar{x}_t)}{f(x_{t+1}|x_t = \rho_x \bar{x}_{t-1})}\right]^{\theta} \cdot C \tag{1}$$

Notice that the distribution (1) is conditional on two elements: first, it is conditional on the current realization of x_t, because this enters the true distribution of x_{t+1}; second, it is conditional on the reference event $-G \equiv \{x_t = \rho_x \bar{x}_{t-1}\}$, which defines the reference

the number of past realizations that are considered beyond the last presumably is not crucial given the complex, sluggish, dynamics already embedded in this type of models via the common 'bells and whistles', such as habit formation, capital adjustment costs, and so on. Therefore, for simplicity, we only consider diagnosticity induced by the only last realization.

[4]Throughout this subsection it will be needed to differentiate, in our notation, between a random variable x_t, and a given realization, $\bar{x}_t$.

distribution. In this dynamic setting, the reference event happens "between $t-1$ and t".

Following previous literature, we denote the diagnostic expectation operator at time t by $\mathbb{E}_t^\theta[\,\cdot\,]$. The diagnostic expectation is formally defined as

$$\mathbb{E}_t^\theta[x_{t+1}] = \int_{-\infty}^{\infty} x f_t^\theta(x) dx$$

Thanks to the NNA (Assumption 1), the following result by Bordalo et al. (2018, Proposition 1) follows, establishing a linear expression for the diagnostic expectation in terms of the current and lagged true (or 'rational') expectations.

Lemma 1 (Univariate Rational Expectations Representation)

$$\mathbb{E}_t^\theta[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta(\mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}])$$

For completeness, a proof that closely follows Bordalo et al. (2018) is presented in the appendix. In this formula, the lagged expectation $\mathbb{E}_{t-1}[x_{t+1}]$ is the expectation conditional on information available at $t-1$, that is, conditional on $\bar{x}_{t-1}$. Thus, $\mathbb{E}_t[x_{t+1}] = \rho_x \bar{x}_t$ and $\mathbb{E}_{t-1}[x_{t+1}] = \rho_x^2 \bar{x}_{t-1}$. For the processes above and a given realized $\bar{\varepsilon}_t$, this implies the following:

$$\mathbb{E}_t^\theta[x_{t+1}] = E_t[x_{t+1}] + \theta \rho_x \bar{\varepsilon}_t$$

such that $\mathbb{E}_t^\theta[x_{t+1}] > \mathbb{E}_t[x_{t+1}]$ if and only if $\bar{\varepsilon}_t > 0$, that is diagnostic expectations indeed extrapolate the past shock into future beliefs.

It is easy to extend this result to the sum $x_{t+1} + y_{t+1}$. (See Appendix A.) In DSGE models, a case of interest is the one of predetermined variables, i.e. when one of the variables has been realized at t (and therefore it is degenerate.) Let us suppose, momentarily, that this is y_t. The question we are interested in is: What properties does the diagnostic expectation of $x_{t+1} + y_t$ obey?

In order to answer this question, we recur to the Dirac delta function, defined as follows.[5] Suppose that $\bar{y}_t$ is the realization of y_t. Since y_t is degenerate, it can be

[5]This is also pointed out in Bordalo et al. (2018, appendix).

represented by a cumulative distribution function (cdf) with vanishing uncertainty:

$$Pr(y_t \leq \bar{y}|y_t = \bar{y}_t) = \lim_{\sigma_\eta \to 0^+} \frac{1}{\sigma_\eta}\Phi\left(\frac{\bar{y} - \bar{y}_t}{\sigma_\eta}\right)$$

This is the probability that y_t is below any given value $\bar{y}$, where $\Phi(x)$ is the cumulative distribution function (cdf) of a standard normal random variable:

$$\Phi\left(\bar{x}\right) = \int_{-\infty}^{\bar{x}} \varphi(x)dx$$

This implies that $Pr(y_t = \bar{y}_t) = 1$ and $Pr(y_t \neq \bar{y}_t) = 0$, also denoted using the Dirac delta function $\delta(x)$:

$$\delta(x) = \lim_{a \to 0^+} \frac{1}{a}\varphi\left(\frac{x}{a}\right)$$

with the requirement that $\delta(x)$ is a pdf. Using this notation, $\delta(y_t - \bar{y}_t)$ is the pdf of y_t, and thus

$$Pr(y_t \leq \bar{y}|y_t = \bar{y}_t) = \int_{-\infty}^{\bar{y}} \delta(y - \bar{y}_t)dy$$

is equal to 1 for $\bar{y} \geq \bar{y}_t$ and equal to 0 otherwise.

Armed with these definitions, we discuss how to evaluate the time-t diagnostic expectation of y_t under the NNA. In this case, the reference distribution of y_t is degenerate, with expectation $\rho_y \bar{y}_{t-1}$, where $\bar{y}_{t-1}$ is the past realization. We represent this by a cdf with vanishing uncertainty, as follows

$$Pr(y_t \leq \bar{y}|y_t = \rho_y \bar{y}_{t-1}) = \lim_{\sigma_\eta \to 0^+} \frac{1}{\sigma_\eta}\Phi\left(\frac{\bar{y} - \rho_y \bar{y}_{t-1}}{\sigma_\eta}\right) \tag{2}$$

In this case the following lemma obtains.

Lemma 2

$$\mathbb{E}_t^\theta[y_t] = \bar{y}_t + \theta(\bar{y}_t - \rho_y \bar{y}_{t-1})$$

This lemma generalizes the diagnostic expectation representation obtained in Lemma 1 to degenerate variables. We highlight that the NNA is crucial for this result. As we explain in Appendix A, alternative conditioning sets deliver a different result (a case in which one loses tractability of DSGE models with predetermined variables.)

This appendix also derives the distribution of $x_{t+1} + y_t$ (Lemma 4), and computes the diagnostic expectation of this expression (Proposition 5). These results are used to generalize the following additivity result by Bordalo et al. (2018, appendix p. 5):

$$\mathbb{E}_t^\theta[x_{t+r} + y_{t+s}] = \mathbb{E}_t^\theta[x_{t+r}] + \mathbb{E}_t^\theta[y_{t+s}], \quad r, s \geq 1 \tag{3}$$

Indeed, by extending the NNA to degenerate variables, we are able to establish a stronger version of additivity, as follows.

Proposition 1 (Strong Additivity of the Diagnostic Expectation)

$$\mathbb{E}_t^\theta[x_{t+r} + y_{t+s}] = \mathbb{E}_t^\theta[x_{t+r}] + \mathbb{E}_t^\theta[y_{t+s}], \quad r, s \geq 0$$

The proof is in the appendix. This stronger result, then, establishes additivity for the cases in which either x_t or y_t are degenerate (r or s equal to 0.) This turns out to be crucial in the application to linear DSGEs, since these models typically feature predetermined variables.[6] The next section will use this result to find the solution to these models.

2.2 Linear Diagnostic Expectations Model

2.2.1 Exogenous Processes.

We start by specifying the exogenous drivers of the economy. Exogenous variables are staked in a $(n \times 1)$ vector $\mathbf{x}_t$ that is assumed to follow the AR(1) stochastic process

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{v}_t \tag{4}$$

where $\mathbf{v}_t$ is a $(k \times 1)$ vector of Gaussian and orthogonal exogenous shocks:

$$\mathbf{v}_t \sim N(0, \Sigma_{\mathbf{v}})$$

and A is a diagonal matrix of persistence parameters.

Following Bordalo et al. (2018), we make a no-news assumption for this multivariate setup.

Assumption 2 (Multivariate No-News Assumption)

$$f(\mathbf{x}_{t+1}| - G) = f(\mathbf{x}_{t+1}|\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1})$$

[6] Examples include the capital stock, or past consumption (in the case of habit formation).

2.2.2 Stochastic Difference Equation

The class of forward-looking models we analyze are written in form of a stochastic difference equation. To this end, let $\mathbf{y}_t$ denote a $(m \times 1)$ vector of endogenous variables (including jump variables and states) and $\mathbf{x}_t$, as above, denote the $(n \times 1)$ vector of exogenous states. The model is:

$$\mathbb{E}_t^\theta[\mathbf{F}\mathbf{y}_{t+1} + \mathbf{G_1}\mathbf{y}_t + \mathbf{M}\mathbf{x}_{t+1} + \mathbf{N_1}\mathbf{x}_t] + \mathbf{G_2}\mathbf{y}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{N_2}\mathbf{x}_t = 0 \tag{5}$$

where $\mathbf{F}$, $\mathbf{G_1}$, $\mathbf{G_2}$, $\mathbf{M}$, $\mathbf{N_1}$, $\mathbf{N_2}$, and $\mathbf{H}$, are matrices of parameters. $\mathbf{F}$, $\mathbf{G_1}$, $\mathbf{G_2}$, and $\mathbf{H}$ are $(m \times m)$ matrices, $\mathbf{N_1}$ and $\mathbf{N_2}$ are $(m \times n)$ matrices. $\mathbb{E}_t^\theta[\,\cdot\,]$, as above, denotes the diagnostic expectation operator, which is now taken over *endogenous and exogenous* variables.

Notice that in writing model (5), we were careful in allowing both the expectation of time t variables (e.g. $\mathbb{E}_t^\theta[\mathbf{N_1}\mathbf{x}_t]$), and the variables themselves (e.g. $\mathbf{N_2}\mathbf{x}_t$). This is necessary because diagnostic expectations depend on uncertainty between $t-1$ and t via the reference distribution. (Appendix A discusses this technical issue in detail.)

An open question is how to evaluate the expectation

$$\mathbb{E}_t^\theta[\mathbf{F}\mathbf{y}_{t+1} + \mathbf{G_1}\mathbf{y}_t + \mathbf{M}\mathbf{x}_{t+1} + \mathbf{N_1}\mathbf{x}_t]$$

in a manner consistent with a solution of the equation, and whether some form of linearity is maintained with diagnostic expectations, so that equation (5) remains linear after having taken this expectation.[7] Relatedly, we want to know if this equation admits a solution, under which conditions this solution is unique, and how stringent these conditions are when compared to a benchmark of rational expectations.[8]

We tackle all of these issues in the next section.

[7] It is possible that the linear model, in its original form, is written with this expectation broken up into different terms $\mathbb{E}_t^\theta[\mathbf{F}\mathbf{y}_{t+1} + \mathbf{G_1}\mathbf{y}_t + \mathbf{M}\mathbf{x}_{t+1}] + \mathbb{E}_t^\theta[\mathbf{N_1}\mathbf{x}_t]$, say, or with sums of expectations that involve the same variables, i.e. $\mathbb{E}_t^\theta[\mathbf{F_1}\mathbf{y}_{t+1}] + \mathbb{E}_t^\theta[\mathbf{F_2}\mathbf{y}_{t+1}]$, for example. As we will explain below, due to the structure of the solution, the additivity property established earlier (Proposition 1) will render these issues moot.

[8] The seminal paper is by Blanchard and Kahn (1980), who refer to "existence and uniqueness" of a stable rational expectations equilibrium. In linear monetary models these conditions are known as determinacy conditions. Uhlig (1995) uses the language of "stability conditions", who relates the concept to the mathematics of dynamical systems. The idea is that they ensure the solution exists and is unique, ruling out sunspots. The DSGE models used for policy analysis typically satisfy this requirement.

2.3 Solution Procedure

Our procedure to obtain the solution to the model given by equations (4) and (5) consists of first showing how to obtain a representation of the model in terms of rational expectations. To this end, we use the strong additivity result above (Proposition 1) which allows to write equation (5) in the more familiar form[9]

$$\mathbf{F}\mathbb{E}_t^\theta[\mathbf{y}_{t+1}] + \mathbf{G_1}\mathbb{E}_t^\theta[\mathbf{y}_t] + \mathbf{G_2}\mathbf{y}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{M}\mathbb{E}_t^\theta[\mathbf{x}_{t+1}] + \mathbf{N_1}\mathbb{E}_t^\theta[\mathbf{x}_t] + \mathbf{N_2}\mathbf{x}_t = 0 \qquad (6)$$

Once this is achieved, the remaining steps are the following. First, postulate a form for the solution, such that the endogenous variables follow a multivariate normal distribution. Second, as a consequence this property, obtain a rational expectations representation of the model. Third, solve for the model expressed in terms of rational expectations using standard tools (as the method of undetermined coefficients, for instance).

2.3.1 Form of the Solution

We look for a solution of the form

$$\mathbf{y}_t = \mathbf{P}\mathbf{y}_{t-1} + \mathbf{Q}\mathbf{x}_t + \mathbf{R}\mathbf{v}_t \qquad (7)$$

Notice the dependence of endogenous variables *directly* on the shocks $\mathbf{v}_t$ (besides the dependence *through* the exogenous variables $\mathbf{x}_t$ as in RE models.) We make this guess based on the extrapolative nature of DE.

2.3.2 Rational Expectations Representation

Notice that the above guess (7) implies that the endogenous variables of the model, $\mathbf{y}_t$, are normally distributed. We obatin the following result.

Proposition 2 (Multivariate Rational Expectations Representation) *Model (5)*

[9] Together with the property of the diagnostic expectation that for any constant c and random variable Z_{t+1}, $\mathbb{E}_t^\theta[cZ_{t+1}] = c\mathbb{E}_t^\theta[Z_{t+1}]$, which follows from the theorem of the expectation of a monotonic transformation of a random variable.

admits the following rational expectations representation:

$$
\begin{aligned}
&\mathbf{F}\left(\mathbb{E}_t[\mathbf{y}_{t+1}] + \theta\big(\mathbb{E}_t[\mathbf{y}_{t+1}] - \mathbb{E}_{t-1}[\mathbf{y}_{t+1}]\big)\right) + \mathbf{G}\mathbf{y}_t + \mathbf{G}_1\theta\big(\mathbb{E}_t[\mathbf{y}_t] - \mathbb{E}_{t-1}[\mathbf{y}_t]\big) + \mathbf{H}\mathbf{y}_{t-1} \\
&\quad + \mathbf{M}\left(\mathbb{E}_t[\mathbf{x}_{t+1}] + \theta\big(\mathbb{E}_t[\mathbf{x}_{t+1}] - \mathbb{E}_{t-1}[\mathbf{x}_{t+1}]\big)\right) + \mathbf{N}\mathbf{x}_t + \mathbf{N}_1\theta\big(\mathbb{E}_t[\mathbf{x}_t] - \mathbb{E}_{t-1}[\mathbf{x}_t]\big) = 0 \quad (8)
\end{aligned}
$$

where $\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_2$ and $\mathbf{N} = \mathbf{N}_1 + \mathbf{N}_2$.

The proof is omitted, but it follows immediately from Lemma 1 and Lemma 2 provided in Appendix A, which allow to express the diagnostic expectation of all variables, endogenous or exogenous, future and present, in terms of rational expectations.

2.3.3 Solution

Once the representation (8) has been obtained, it is straightforward to proceed by the method of undetermined coefficients, looking for a solution of the form (7). Appendix B shows that this leads to a system of three matrix equations. Specifically, the matrices $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ can be found solving

$$\mathbf{F}\mathbf{P}^2 + \mathbf{G}\mathbf{P} + \mathbf{H} = 0 \quad (9)$$

$$\mathbf{F}\mathbf{P}\mathbf{Q} + \mathbf{F}\mathbf{Q}\mathbf{A} + \mathbf{G}\mathbf{Q} + \mathbf{M}\mathbf{A} + \mathbf{N} = 0 \quad (10)$$

$$\theta\mathbf{F}\mathbf{P}\mathbf{Q} + (1+\theta)\mathbf{F}\mathbf{P}\mathbf{R} + \theta\mathbf{F}\mathbf{Q}\mathbf{A} + \theta\mathbf{G_1}\mathbf{Q} + \mathbf{G}\mathbf{R} + \theta\mathbf{G_1}\mathbf{R} + \theta\mathbf{M}\mathbf{A} + \theta\mathbf{N_1} = 0 \quad (11)$$

We can use the techniques discussed in Uhlig (1995) to solve the quadratic matrix equation (9) in $\mathbf{P}$. The solution of the other two equations is straightforward as they are linear in $\mathbf{Q}$ and $\mathbf{R}$: After vectorization, equation (10) becomes

$$(\mathbf{I}_m \otimes \mathbf{F}\mathbf{P})vec(\mathbf{Q}) + (\mathbf{A}^T \otimes \mathbf{F})vec(\mathbf{Q}) + (\mathbf{I}_m \otimes \mathbf{G})vec(\mathbf{Q}) + vec(\mathbf{M}\mathbf{A}) + vec(\mathbf{N}) = 0$$

such that

$$vec(\mathbf{Q}) = -\Big((\mathbf{I}_m \otimes \mathbf{F}\mathbf{P}) + (\mathbf{A}^T \otimes \mathbf{F}) + (\mathbf{I}_m \otimes \mathbf{G})\Big)^{-1} \times (vec(\mathbf{M}\mathbf{A}) + vec(\mathbf{N}))$$

$\mathbf{R}$ can be found from (11):

$$\mathbf{R} = -\big((1+\theta)\mathbf{F}\mathbf{P} + \mathbf{G} + \theta\mathbf{G_1}\big)^{-1}\big(\theta\mathbf{F}\mathbf{P}\mathbf{Q} + \theta\mathbf{F}\mathbf{Q}\mathbf{A} + \theta\mathbf{G_1}\mathbf{Q} + \theta\mathbf{M}\mathbf{A} + \theta\mathbf{N_1}\big)$$

2.3.4 Stability Conditions

It turns out that the model under DE is subject to the same stability conditions as the model under RE. More precisely, consider the same model above, but under rational expectations:

$$\mathbf{F}\mathbb{E}_t[\mathbf{y}_{t+1}] + \mathbf{G}\mathbf{y}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{M}\mathbb{E}_t[\mathbf{x}_{t+1}] + \mathbf{N}\mathbf{x}_t = 0 \tag{12}$$

where the matrices $\mathbf{F}$, $\mathbf{G}$, $\mathbf{H}$, $\mathbf{M}$ and $\mathbf{N}$ are defined above. The following result holds.

Lemma 3 (Stability) *The stability conditions for the model under diagnostic expectations given by equations* (4) *and* (5) *are identical to the stability conditions for the model under rational expectations given by* (4) *and* (12)*.*

The proof can be found in Appendix B.

2.4 Volatility

In the case of RE, the solution of model takes the form

$$\mathbf{y}_t = \mathbf{P}\mathbf{y}_{t-1} + \mathbf{Q}\mathbf{x}_t \tag{13}$$

Comparing (13) and (7) immediately leads to conjecture that under DE there should be extra volatility due to the presence of the extra term $\mathbf{R}\mathbf{v}_t$. This intuition is correct, however, obviously, whether this is true for a given set of parameters will also depend on the covariance of the matrix $\mathbf{Q}$ with the other matrices of parameters in the solution. This is what the following proposition makes precise.

Proposition 3 (Extra Volatility) *Let y_{it}^{DE} and y_{it}^{RE} respectively denote the i-th component of the vector of endogenous variables $\mathbf{y}_t^{DE}$ and $\mathbf{y}_t^{RE}$ and $Var(y_{it}^{DE})$ and $Var(y_{it}^{RE})$ denote the variance of the variable y_{it}^{DE} and of the variable y_{it}^{RE}. Then, $Var(y_{it}^{DE})$ is larger than $Var(y_{it}^{RE})$ if and only if:*

$$diag(\mathbf{R}\boldsymbol{\Sigma}_{\mathbf{v}}\mathbf{R}' + 2\mathbf{Q}\boldsymbol{\Sigma}_{\mathbf{v}}\mathbf{R}')_i > 0 \tag{14}$$

where $\boldsymbol{\Sigma}_{\mathbf{v}}$ is the variance-covariance matrix of $\mathbf{v}_t$.

The proof can be found in Appendix B. In the next section, we go through a few applications to explore whether the set of conditions (14) is satisfied for a collection of benchmark models.

3 Applications

The aim of this section is to apply the solution method established above to a number of benchmark models. We start by writing down a quantitative medium-scale dynamic stochastic general equilibrium (DSGE) model. The model is quite standard and it is based on the seminar work of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). We then look at the effect of diagnostic expectations by first considering simpler, special cases of this model. At the end of the section we present the results in the context of the full model.

3.1 A Medium-Scale DSGE model

We evaluate the implications for macroeconomic volatility in a quantitative medium-scale dynamic stochastic general equilibrium (DSGE) model. The model follows the exposition in Gust, Herbst, López-Salido, and Smith (2017), henceforth referred to as GHLS. The economy comprises of following agents: a continuum of households supplying differentiated labor, a continuum of firms producing differentiated goods, a perfectly competitive final goods firm, a perfectly competitive labor agency that provides the composite labor input demanded by firms, and a government in charge of fiscal and monetary policy.

3.1.1 Monopolistically competitive producers

Assume there is a continuum of differentiated intermediated good producers that sell the intermediate good Y_{jt}. A perfectly competitive firm aggregates intermediate goods into a final composite good $Y_t = \left[\int_0^1 Y_{jt}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$, where $\varepsilon_p > 1$ is constant elasticity of demand. The iso-elastic demand for intermediate good j is given by: $Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon_p} Y_t$, where P_t is the aggregate price index and P_{jt} is the price of intermediate goods j. Each intermediate good j is produced by a price-setting monopolistically competitive firm using labor L_{jt} and physical capital K_{jt}:

$$Y_{jt} = A_t \left(Z_t L_{jt}\right)^{1-\alpha} \; K_{jt}^{\alpha} \tag{15}$$

where Z_t is the non-stationary aggregate TFP process, and A_t is the stationary aggregate TFP process. The variable Z_t denotes a non-stationary TFP series that evolves according to:

$$\frac{Z_t}{Z_{t-1}} = \left(\frac{Z_{t-1}}{Z_{t-2}}\right)^{\rho_Z} G_Z^{1-\rho_Z} \exp(\epsilon_{Z,t}); \quad \epsilon_{Z,t} \sim iid\ N(0, \sigma_z^2)$$

where ρ_Z is the persistence of the shock process, and $\epsilon_{Z,t}$ is a random disturbance that causes deviations of the TFP growth from its balanced growth rate G_Z. Stationary TFP evolves as follows:

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t}; \quad \epsilon_{A,t} \sim iid\ N(0, \sigma_A^2) \tag{16}$$

Firms choose inputs to minimize total cost each period. Marginal cost, independent of firm-specific variables, is given by $mc_t = \frac{1}{A_t}\frac{1}{Z_t^{1-\alpha}}\left(\frac{R_t^k}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}$, where $\frac{R_t^k}{P_t}$ and $\frac{W_t}{P_t}$ denote aggregate rental rate of capital and real wage. A firm j pays a quadratic adjustment cost in units of final good (Rotemberg 1982) to adjust its price P_{jt}. The cost is given by $\frac{\psi_p}{2}\left(\frac{P_{jt}}{\tilde{\Pi}_{t-1}P_{jt-1}} - 1\right)^2 P_t Y_t$, where $\psi_p \geq 0$ regulates the adjustment costs. Price change is indexed to $\tilde{\Pi}_{t-1} = \bar{\Pi}^{1-\iota_p}\Pi_{t-1}^{\iota_p}$, where ι_p governs indexation between previous period inflation rate Π_{t-1} and steady state inflation rate $\bar{\Pi}$. Firm's per period profits are given by: $\Gamma_{jt} \equiv P_{jt}Y_{jt} - P_t mc_t Y_{jt} - \frac{\psi_p}{2}\left(\frac{P_{jt}}{\tilde{\Pi}_{t-1}P_{jt-1}} - 1\right)^2 P_t Y_t$. Each period, the firm chooses P_{jt} to maximize present discounted value of real profits:

$$\max_{P_{jt}} \left\{ \frac{\Lambda_t \Gamma_{jt}}{P_t} + \mathbb{E}_t^{\theta}\left[\sum_{s=1}^{\infty} \frac{\Lambda_{t+s}\Gamma_{jt+s}}{P_{t+s}}\right] \right\}$$

where Λ_t is the marginal utility of consumption in period t, and $\mathbb{E}_t^{\theta}[\,\cdot\,]$ is the diagnostic expectation operator regulated by parameter θ. Notice that we write dynamic maximization problems by explicitly separating time t choice variables from the expectation of future choice variables. As explained below, this separation is crucial for solving the model with diagnostic expectations.

3.1.2 Households

There is a continuum of monopolistically competitive households, indexed by $i \in [0, 1]$, supplying a differentiated labor input $L_{i,t}$. A perfectly competitive employment agency aggregates various labor types into a composite labor input L_t supplied to firms, in a Dixit-Stiglitz aggregator: $L_t = \left[\int_0^1 L_{i,t}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$, where $\varepsilon_w > 1$ is constant elasticity of demand. The iso-elastic demand for labor input i is given by: $L_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\varepsilon_w} L_t$, where $W_{i,t}$ is household i's wage rate, and W_t is the aggregate wage rate that the household takes as given.

The household i has following lifetime-utility at time t:

$$\left(\log(C_{i,t} - h\tilde{C}_{t-1}) - \frac{\omega}{1+\nu} L_{i,t}^{1+\nu} - \psi_{i,t}^w\right) + \mathbb{E}_t^\theta \left[\Sigma_{s=t+1}^{\infty} \beta^{s-t} \left(\log(C_{i,s} - h\tilde{C}_{s-1}) - \frac{\omega}{1+\nu} L_{i,s}^{1+\nu} - \psi_{i,s}^w\right)\right]$$

where h is the degree of habit formation on external habits over aggregate consumption $\tilde{C}_{t-1}$, which the household takes as given, $\nu > 0$ is inverse of the Frisch elasticity of labor supply, $\omega > 0$ is a parameter that pins down the steady-state level of hours, and the discount factor β satisfies $0 < \beta < 1$. $\psi_{i,t}^w$ is the loss in utility in adjusting wages. We assume a quadratic adjustment cost given by $\psi_{i,t}^w = \frac{\psi_w}{2}\left[\frac{W_{it}}{\tilde{\Pi}_{t-1}^w W_{it-1}}\right]^2$, where $\psi_w \geq 0$ is a parameter, and wage contracts are indexed to productivity and price inflation. We assume $\tilde{\Pi}_{t-1}^w = G_Z \bar{\Pi}^{1-\iota_w} \left(\exp(\epsilon_{Z,t})\Pi_{t-1}\right)^{\iota_w}$ with $0 \leq \iota_w < 1$.

The household's budget constraint in period t is given by

$$P_t C_{i,t} + P_t I_{i,t} + \frac{B_{i,t+1}}{(1+i_t)\eta_t} = B_{i,t} + W_{i,t} L_{i,t} + \Gamma_t + T_t + R_t^K u_{i,t} K_{i,t}^u - P_t a(u_{i,t}) K_{i,t}^u$$

where $I_{i,t}$ is investment, $W_{i,t}L_{i,t}$ is labor income, and $B_{i,t}$ is income from nominal bonds paying nominal interest rate i_t. Households own an equal share of all firms, and thus receive Γ_t dividends from profits. Finally, each household receives a lump-sum government transfer T_t. Following Smets and Wouters (2007) and Christiano, Eichenbaum, and Trabandt (2015), we assume a risk-premium shock process η_t:

$$\log \eta_t = \rho_\eta \log(\eta_{t-1}) + \epsilon_{\eta,t}; \quad \epsilon_{\eta,t} \sim iid\ N(0, \sigma_\eta^2)$$

The households own capital, $K_{i,t}^u$, and choose the utilization rate, $u_{i,t}$. The amount of effective capital, $K_{i,t}$, that the households rent to the firms at nominal rate R_t^K is given by $K_{i,t} = u_{i,t} K_{i,t}^u$. The (nominal) cost of capital utilization is $P_t a(u_{i,t})$ per unit of physical capital. As in the literature, we assume $a(1) = 0$ in the steady state and $a'' > 0$. Following GHLS, we assume investment adjustment costs, $S\left(\frac{I_{i,t}}{G_Z I_{i,t-1}}\right)$, in the production of capital, where G_Z is the steady state growth rate of Z_t. Law of motion for capital is as follows:

$$K_{i,t+1}^u = \mu_t \left[1 - S\left(\frac{I_{i,t}}{G_Z I_{i,t-1}}\right)\right] I_{i,t} + (1-\delta_k) K_{i,t}^u$$

where δ_k denotes depreciation rate, and μ_t is an exogenous disturbance to the marginal efficiency of investment that follows:

$$\log(\mu_t) = \rho_\mu \log(\mu_{t-1}) + \epsilon_{\mu,t}; \quad \epsilon_{\mu,t} \sim iid\ N(0, \sigma_\mu^2)$$

As in the literature, we assume that $S(1) = S'(1) = 0$, and calibrate $S''(1) > 0$.

3.1.3 Government

The central bank follows a Taylor rule in setting the nominal interest rate i_t. It responds to deviations in (gross) inflation rate Π_t from its target rate $\bar{\Pi}$, output gap, and output growth rate.

$$\frac{1+i_t}{1+i_{ss}} = \left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} X_t^{\phi_x} \left(\frac{Y_t}{G_Z Y_{t-1}}\right)^{\phi_{dy}}\right]^{1-\rho_R} \exp(\epsilon_{mp,t}); \quad \epsilon_{mp,t} \sim iid\ N(0, \sigma_{mp}^2) \tag{17}$$

with $0 < \rho_R < 1$, $\phi_\pi \geq 0$, $\phi_x \geq 0$, and $\phi_{dy} \geq 0$. i_{ss} is the steady state nominal interest rate, and $\epsilon_{mp,t}$ is the monetary policy shock. Output gap $X_t \equiv \frac{Y_t}{Y_t^*}$ is measured as deviations of output, Y_t, from output under natural rate allocation, Y_t^*.[10]

We assume government balances budget every period $P_t T_t = P_t G_t$, where G_t is the government spending. G_t is determined exogenously as as a fraction of GDP: $G_t = \left(1 - \frac{1}{\lambda_t^g}\right) Y_t$ where the government spending shock follows the process:

$$\log \lambda_t^g = (1-\rho_g)\log \lambda^g + \rho_g \log \lambda_{t-1}^g + \epsilon_{g,t}; \quad \epsilon_{g,t} \sim N(0, \sigma_g^2)$$

λ^g is the steady state share of government spending in final output.

3.1.4 Market clearing

We focus on a symmetric equilibrium where all intermediate goods producing firms and households make the same decisions. Therefore, we can drop subscripts i and j. The aggregate production function, in the symmetric equilibrium, is then given by: $Y_t = A_t (Z_t L_t)^{1-\alpha} K_t^\alpha$, since $K_t = K_{i,t} = K_{jt}$ and $N_t = N_{i,t} = N_{jt}$. The market clearing

[10] We define the natural rate allocation as the one where prices and wages are flexible from today onwards taking as given the evolution of the state-variable.

for the final good, in the symmetric equilibrium, requires that

$$Y_t = C_t + I_t + a(u_t)K_t^u + G_t + \frac{\psi_p}{2}\left[\frac{\Pi_t}{\tilde{\Pi}_{t-1}} - 1\right]^2 Y_t$$

This completes the presentation of the full DSGE model.

3.1.5 Technical Remark

We make a technical remark, important for the loglinearization of the model above (and any DSGE model that uses diagnostic expectations more generally.) In fact, because the diagnostic distribution of a random variable realized at time t, say X_t, depends on uncertainty between $t-1$ and t (which enters through the reference distribution), generally

$$\mathbb{E}_t^\theta[X_t] \neq X_t$$

In other words, one loses the usual property of the expectation conditional on time[11] t:

$$\mathbb{E}_t[X_t] = X_t$$

As a consequence, care needs to be exercised when loglinearizing the model under diagnostic expectations. The naive in approach in which one would proceed to "substitute" the rational expectations operator by the diagnostic expectations operator for the loglinear equations is unfortunately not correct. To see this, consider for instance the Euler equation for the optimal household choice in a standard simple monetary economy:

$$\frac{u'(C_t)}{P_t} = \beta(1+i_t)\mathbb{E}_t^\theta\left[\frac{u'(C_{t+1})}{P_{t+1}}\right]$$

where we follow the notation of the DSGE model above. Because $\mathbb{E}_t^\theta[\,\cdot\,]$ is *not* conditional on t, but on $t-1$, then one cannot proceed by arranging the equation to have all variables on the right-hand side, then introducing all time-t variables into the expectation, with the goal of obtaining an expression for gross inflation at t and then proceed by loglinearization. Appendix C shows which procedure to adopt to handle the diagnostic expectation. It also shows that this changes the resulting loglinear approximation. (This fact is a major complication in the loglinearization of the model

[11] Under DE however, we know that

$$\mathbb{E}_t^\theta[X_{t-1}] = X_{t-1}$$

when the agent uses a reference distribution back to $t-1$.

above, and we present the loglinearized model in Appendix E.)

The reader may wonder at this point whether these issues introduce time inconsistency in agents' choices. It turns out that this is not the case in the loglinear approximation. By the law of iterated expectations (which holds for the diagnostic expectation in linear-Gaussian settings), time $t+1$ policy functions are consistent with agents' expectations (about their time $t+1$ policy functions.)

3.1.6 Calibration

Table 3 in Appendix F displays the calibrated parameters. These are the parameters estimated in the linearized model of GHLS. The only difference with respect to GHLS is that we model external habits, as opposed to internal habits. Subsequently, we calibrate $h = 0.5$ as a standard value in the literature.[12]

In addition, we set the diagnosticity parameter $\theta = 1$, which is broadly consistent with the evidence presented by Bordalo, Gennaioli, Ma, and Shleifer (2020). This choice is also close to what is used by Bordalo, Gennaioli, Shleifer, and Terry (2020). (Using a different value does not qualitatively change our conclusions.)

3.2 Inspecting the Effects of Diagnostic Expectations Through Special Cases

In principle, expectational errors induced by diagnostic expectations have the potential to propagate through model economies and generate novel dynamics of endogenous variables. Intuitively, diagnostic expectations ought to generate extrapolation in beliefs, together with sharp reversals towards rational expectations. One may conclude that diagnostic expectations can generate extra volatility relative to rational expectations framework.

In order to study these questions, we consider two special cases of the full model to illustrate that, even though this intuition is correct, the way in which diagnostic expectations interact with equilibrium models paints a nuanced answer in terms of the exact implications for equilibrium variables such as output, consumption, or employment. Indeed, the errors in expectations interact with the frictions present in benchmark models and have powerful effects, or—on the contrary—might be muted by general equilib-

[12] GHLS estimate five shock processes: monetary policy shocks, risk-premium shocks, investment quality shocks, government spending shocks and TFP non-stationary shocks. We have added a stationary TFP shock to the model above to be used in the special cases below. This shock is calibrated using standard values.

rium forces. Thus, in order to exposit the potential of these errors to propagate in a given economy, we resort to two polar opposites: a prototypical demand-determined economy as the New Keynesian model, and a prototypical supply-determined economy as the Real Business Cycle (RBC) model. In both models, diagnostic expectations will generate volatility in beliefs impacting the intertemporal consumer problem, and lead to higher volatility in consumption compared to a rational expectations benchmark. In the former, diagnostic expectations will rely on nominal frictions to propagate this volatility towards output and other quantities. In the later, general equilibrium adjustment in prices will, instead, lead to a limited propagation, with output volatility almost unchanged. Furthermore, another interesting result we found is that, by the presence of diagnostic expectations in the NK model, the output gap *falls* of the output gap following a contractionary TFP shock.[13]

3.2.1 The Simple New Keynesian Model

We consider a textbook New Keynesian model (Woodford 2003; Galí 2015). This simple New Keynesian (NK) model is the special case of the medium-scale DSGE model described above, when there is no capital ($\alpha = 0$), zero trend inflation ($\bar{\Pi} = 1$), no price or wage indexation ($\iota_p = \iota_w = 0$), zero trend growth rate ($\log(G_Z) = 0$), no habits ($h = 0$), flexible wages ($\psi_w = 0$) without any labor market power, zero government spending ($\lambda_g = 1$), and a few coefficients of the interest rate rule are set to zero ($\rho_R = \phi_{dy} = 0$). To keep our discussion focused, we assume there is one exogenous shock process, namely shocks to stationary TFP (A_t). We present the detailed derivation of this model in Appendix C.

Under diagnostic expectations, the equilibrium conditions of the simple new Keynesian model are as follows:

$$\hat{y}_t = \mathbb{E}_t^\theta[\hat{y}_{t+1}] - (\hat{i}_t - \mathbb{E}_t^\theta[\pi_{t+1}]) + \theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])$$
$$\pi_t = \beta\mathbb{E}_t^\theta[\pi_{t+1}] + \kappa(\hat{y}_t - \hat{a}_t)$$
$$\hat{i}_t = \phi_\pi \pi_t + \phi_x(\hat{y}_t - \hat{a}_t)$$

where β is the discount factor, $\kappa \equiv \frac{\varepsilon_p - 1}{\psi_p}(1+\nu)$, ϵ_p is the elasticity of substitution across intermediate goods' varieties, ψ_p is a Rotemberg price-adjustment cost parameter, and ν is the inverse of Frisch elasticity of labor supply. Variables $\hat{y}_t$, $\hat{i}_t$, π_t, $\hat{a}_t$ denote

[13] There is a growing interest in this type of result in macroeconomic models due to the Covid-19 pandemic. See the discussion below for how it relates to recent works.

log-deviations of output, nominal interest rate, inflation rate, and TFP from their respective steady state values. The shock process is given by (16).

Note that the equations depart from the rational-expectations formulation in that forward-looking terms bear a diagnostic expectations operator instead of a rational-expectations operator, $\mathbb{E}_t[\,\cdot\,]$. Moreover, the Euler-equation carries an additional term that captures the time-t rational forecast error of inflation. In particular, the log-linear Fisher relationship between nominal interest and real interest rate is altered under diagnostic expectations, and it is given by:

$$\hat{r}_t = \hat{i}_t - \mathbb{E}_t^\theta[\pi_{t+1}] - \theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])$$

Time-t *rational* forecast errors about inflation, and not just expected inflation, links the two interest rates with diagnostic expectations. When $\theta = 0$, the diagnostic expectations equilibrium is equivalent to the rational expectations equilibrium.

In the following proposition, we establish some key properties of the diagnostic expectations equilibrium when prices are completely rigid, that is $\psi_p \to \infty$. The details are presented in Appendix D.

Proposition 4 (Output Gap Characterization) *Assume that $\psi_p \to \infty$ and that the diagnosticity parameter is high enough, that is, $\theta > 2(1-\rho_A)(1+\phi_x)/(\phi_x\rho_A)$. Then,*

1. *The output gap positively co-moves with the unanticipated component of TFP. That is, $\frac{\partial \hat{x}_t}{\partial \epsilon_{A,t}} > 0$.*
2. *The output gap under diagnostic expectations is more volatile than that under rational expectations. That is, $Var(\hat{x}_t)_{DE} > Var(\hat{x}_t)_{RE}$.*

The volatility condition in Proposition 4 is a special case of the more general condition presented in Proposition 3.[14] For standard parameter values, this condition is satisfied. For example, when $\rho_A = 0.9$ and $\phi_x = 0.5$, this condition requires that θ be greater than 0.67 for diagnostic expectations to generate extra volatility.

The analytical results noted in the case of completely rigid prices also hold when prices are sticky (not-completely rigid), i.e. $0 < \psi_p < \infty$. To numerically demonstrate this, we use the calibration discussed above in the context of the full model. The only difference is in the calibration of the stationary TFP process, which is calibrated

[14] Let $\mathbf{R} \equiv \frac{\phi_x}{1+\phi_x-\rho_A}$ and $\mathbf{Q} \equiv \frac{\theta\rho\phi_x}{(1+\phi_x)(1+\phi_x-\rho_A)}$. Propositions 3 state that the output gap is more volatile under diagnostic expectations when $\mathbf{R}^2 + 2\mathbf{QR} > 0$, consistent with the condition provided in Proposition 4.

Table 1: Model-Implied Volatilities with Stationary TFP Shocks

(a) Simple New Keynesian Model

Variable	Rational Expectations	Diagnostic Expectations	Percentage Increase
Output	0.0048	0.0085	77%
Consumption	0.0048	0.0085	77%
Investment	–	–	–

(b) Real Business Cycle Model

Variable	Rational Expectations	Diagnostic Expectations	Percentage Increase
Output	0.0064	0.0059	-7%
Consumption	0.0015	0.0030	100%
Investment	0.0533	0.0503	-6%

Notes: The table reports the standard deviations of output growth, consumption growth and investment growth in the simple NK model and the RBC model in Panels (a) and (b) respectively. Final column titled "Percentage Increase" shows the percentage increase in standard deviation under the diagnostic expectations model relative to the rational expectations benchmark. There is one shock process in the two models. Stationary TFP follows an AR(1) process with persistence 0.9 and standard deviation 0.0050, as shown in equation 16.

with persistence 0.90 and standard deviation of 0.0050. Panel a) in Table 1 shows unconditional volatilities of output growth, and consumption growth under diagnostic and rational expectations. Since there is no government spending or investment, output growth and consumption growth are equivalent in the simple NK model. Table 1 shows that the output gap under diagnostic expectations exhibits 77 percent higher standard deviation relative to the output gap under rational expectations.[15]

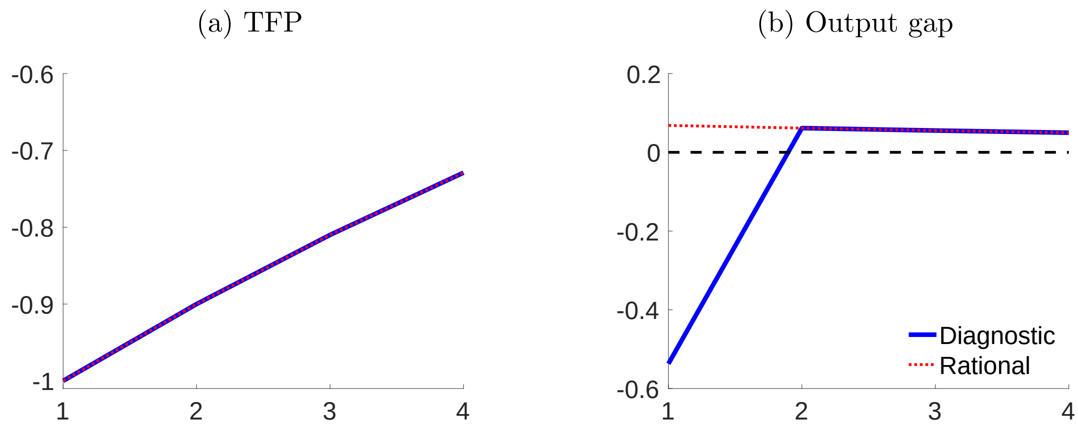
Furthermore, the condition in Proposition 4 also implies that the output gap can be negative with negative unanticipated productivity shocks. A recent literature, following the Covid-19 pandemic, focuses on generating this positive co-movement. Some of the alternate explanations include the use of input-output networks (Baqaee and Farhi 2020), endogenous firm-entry (Bilbiie and Melitz 2020), heterogenous risk-tolerance (Caballero and Simsek 2020), endogenous productivity growth (Fornaro and Wolf 2020), multiple consumption goods (Guerrieri, Lorenzoni, Straub, and Werning 2020), among others.

Figure 1 plots the evolution of TFP and the output gap in the case of the NK model. Following a negative TFP shock, with diagnostic beliefs the economy enters a recession: the output gap and employment falls.

Thus, diagnostic expectations present a behavioral mechanism by which the output gap correlates positively with productivity in the short-run. This result is in contrast to

[15]We also compared results with a textbook calibration (Galí 2015; Woodford 2003): $\beta = 0.99$, $\kappa = 0.025$, $\phi_\pi = 1.50$, $\phi_x = 0.5$, and $\theta = 1$. This calibration implies 68% higher standard deviation under diagnostic expectations relative to rational expectations.

Figure 1: Impulse Responses to a Stationary TFP Shock in the Simple NK Model



Notes: The left and right panels depict the impulse responses of TFP and output gap to a one unit negative shock to TFP. TFP shock process is given by equation 16. The blue solid lines denote impulses responses with diagnostic expectations, whereas the red dotted lines denote responses with rational expectations. The dynamics of employment are exactly the same as the output gap.

the well-known result under rational-expectations going back at least to Blanchard and Quah (1989) and Galí (1999): the fall of productivity, for the same level of aggregate demand, increases the demand for labor; this generates a boom in the labor market, together with a *rise* in the output gap.

Next, we turn to the describe what happens, following the same economic shock, in the RBC model.

3.2.2 The Real Business Cycles Model

The baseline real business cycle (RBC) model is the special case of the full model for flexible prices and wages ($\psi_p = \psi_w = 0$), no indexation, ($\iota_w = \iota_p = 0$), zero capital adjustment costs ($S''(1) = 0$), full capacity utilization, zero trend growth rate ($\log(G_Z) = 0$), no habits ($h = 0$), no product market or labor market power, and zero government spending ($\lambda_g = 1$), The remaining parameters are same as calibrated in the case of full model, except for the TFP process which is calibrated as in the NK model to allow for a clean comparison.

Panel b) in Table 1 shows unconditional volatilities of output growth, consumption growth, and investment growth, both under diagnostic and rational expectations. Consumption growth is twice as volatile under diagnostic expectations than under rational expectations. On the other hand, investment growth and output growth are dampened under diagnostic expectations due to the general equilibrium adjustment of the interest rate.[16] Diagnosticity, therefore, does not always generate extra amplification.

3.3 Results for the Full DSGE Model

We simulate the model based on the five shocks estimated by GHLS: monetary policy shocks, risk-premium shocks, investment quality shocks, government spending shocks and TFP growth rate shocks. We gauge how much extra volatility diagnostic expectations can generate in the quantitative medium-scale DSGE model presented above. (We present the full set of impulse response functions in Appendix F.) The interest of this exercise is that this model contains a set of more realistic ingredients that the previous two simple models did not contain.

Table 2 shows unconditional volatilities of output growth, consumption growth, investment growth, employment, capacity utilization, capital stock growth, inflation, and the nominal interest rate both under diagnostic and rational expectations. Consistent with our analysis of the simple NK, diagnostic expectations generate higher volatility for all these variables. This is because of the presence of nominal rigidities (for prices and wages), which dampen the general equilibrium adjustment of prices, which could offset the extrapolation of beliefs induced by diagnostic expectations. The variable for which the introduction of diagnosticity is most impactful is consumption. Looking at the previous results for the simple models indicates that this is due to the forward looking nature of consumption through the Euler equation. Expectations of future marginal utility are impacted by diagnosticity, leading to high consumption volatility. As a by-product, output volatility is much higher with diagnosticity as well.

Figure 4 in Appendix F looks at the degree of extra output volatility generated by diagnosticity by varying θ from 0 to 1.5. The figure shows that this relation is monotonic.

The overarching conclusion is that the impact of diagnostic expectations in this more realistic model resemble more the NK model than the RBC model. Given that this model is considered a realistic building block for empirical analyses of the business cycle and for policy analysis, this result suggests great promise for the incorporation of representativeness into monetary economics and business cycle analysis in the presence

[16]Indeed, Figure 3 in Appendix F plots the impulse response of the exogenous TFP, consumption, output, investment, capital stock, and real interest rate to a one standard deviation shock to TFP. The greater reduction in real-interest rate under diagnostic expectations attenuates the fall in investment, and explains why there is lower volatility in output and investment with diagnostic beliefs, compared to the corresponding variables under rational expectations.

Table 2: Model-Implied Volatilities in the Medium-Scale DSGE Model

Variable	Rational Expectations	Diagnostic Expectations	Percentage Increase
Output	0.0139	0.0192	38%
Consumption	0.0113	0.0180	59%
Investment	0.0626	0.0751	20%
Employment	0.0558	0.0599	7%
Capacity Utilization	0.0154	0.0165	7%
Capital Stock	0.0101	0.0108	7%
Inflation	0.0032	0.0039	22%
Nominal Interest Rate	0.0089	0.0100	12%

Notes: The table reports the standard deviations of output growth, consumption growth, investment growth, employment, capacity utilization, capital stock growth, inflation, and the nominal interest rate in the medium-scale DSGE model. Final column titled "Percentage Increase" shows the percentage increase in standard deviation under the diagnostic expectations model relative to the rational expectations benchmark. There are five shock processes in the model, as estimated by Gust, Herbst, López-Salido, and Smith (2017): monetary policy shocks, risk-premium shocks, investment quality shocks, government spending shocks and TFP growth rate shocks. The parameters of the shock processes are reported in Table 3.

of frictions. For example, one immediate implication is that diagnosticity could allow fitting the data with a lower size of underlying disturbances. Generally, previous research has strived to find mechanisms to generate amplification,[17] and thus this appears to be a potentially important implication of our analysis. While we have not yet estimated the model to verify this conjecture, we plan to do so in future work.

4 Introducing Imperfect Information: Under- and Overreaction

Whether beliefs as measured by surveys feature under- or overreaction appears to be the subject of an important debate in recent literature. Indeed, Coibion and Gorodnichenko (2012) provide evidence of underreaction of consensus forecasts, whereas Bordalo, Gennaioli, Ma, and Shleifer (2020) provide evidence of overreaction at the level of the individual forecaster. Kohlhas and Walther (2020) claim that there is overreaction, in some cases, even at the aggregate level. In a complementary way, Angeletos, Huo, and Sastry (2020) stress that at the aggregate level one can observe both under- and

[17] See, for instance, the comments by Chari, Kehoe, and McGrattan (2009), who write "Our critique focuses heavily on the dubiously structural shocks. That includes four of the shocks in the New Keynesian Smets-Wouters model: shocks to wage markups, price markups, exogenous spending, and risk premia. As it appears in the Smets-Wouters model, the wage markup shock is highly questionable. This shock is modeled by Smets and Wouters (2007) as arising from fluctuations in the elasticity of substitution across different types of labor. That interpretation makes little sense. When expressed in units of a markup, the shock has a mean of 50 percent and a standard deviation of over 2,500 percent. Clearly, this level of volatility is absurd when it is interpreted as reflecting variations in the elasticity of substitution between workers such as carpenters, plumbers, neurosurgeons, and economists."

overreation. According to them, what matters is the horizon: there is underreaction in the short run, whereas overreaction dominates in the medium run. These authors arrive at this conclusion by looking at inflation and unemployment, but their theoretical argument can be applied more broadly.

The importance of the horizon at which one observes the dynamics of forecasts has also been stressed in an application to stock returns by Bordalo, Gennaioli, La Porta, and Shleifer (2019). The authors stress that the key is to look at the medium-term forecast errors to find evidence of overreaction to news. The explanation is the following. A gradual arrival of news can happen some time after an anticipated event, and a buildup of the overreaction can move forecasts away from the underreaction generated by imperfect information on impact.

Based on the premise by Bordalo et al. (2019), our broad aim in this section is to contribute to this debate by presenting a simple model about long-term beliefs guided by the diagnostic Kalman filter. The key innovation of our setup compared to previous exercises in the literature is that agents form beliefs about a hidden component that features both sizeable *persistence*, and is also *permanent* (in the sense that the underlying process has a unit root.) To model the long-term nature of this hidden object, we calibrate this persistence to a high value, which conceptually connects our exercise to the long-run risks approach (Bansal and Yaron 2004). However, ours is a general equilibrium representative-agent macroeconomic model where consumers are concerned with the long run path of income.

The model is based on Blanchard, L'Huillier, and Lorenzoni (2013). The model features imperfect information. The innovation here is the addition of diagnosticity to this model.

We have two specific goals. First, we show that when the representative agent's signal is precise enough, diagnosticity can dominate imperfect information, leading to aggregate overreaction in the short run. Second, we show that, on the other hand, for an imprecise signal, learning can lead to a gradual buildup of overreaction over the medium term, even though there is short-term underreaction. This second delayed-learning result has been found to be empirically relevant in related study (Cao and L'Huillier 2018), where the learning process is estimated via maximum likelihood for different macroeconomic episodes surrounding boom-bust cycles.

A point that is ex-post straightforward, but perhaps not so obvious at first, is that the diagnostic Kalman filter implies rich dynamics of beliefs. Many patterns can emerge depending on the parameters that govern the underlying hidden process, its exact specification, and the parameters governing the signal-to-noise ratio. As a matter of fact, the model we present (and other models previously employed in the literature) can be thought of as capturing a "horse race" between two competing mechanisms: imperfect information (which pushes towards underreaction compared to the full information rational expectations benchmark, henceforth FIRE), and diagnosticity (which pushes towards overreaction both compared to FIRE and the true, non-diagnostic, Kalman filter).

We start by presenting the model, and then plot the responses of output (beliefs) for selected calibrations.

4.1 The Model

The bare-bones model we use (absent the information structure) is equivalent to the simple New Keynesian model in which prices are completely rigid, as in Appendix D. Consumption is pinned down solely by beliefs about long-run income.[18]

The information structure is as follows. As in the DSGE model above, TFP has both a permanent component (Z_t) and a temporary component (A_t).[19] Agents do not observe these components separately. Instead, they observe realized TFP and a noisy signal s_t about the permanent component.[20] This is written (in logs):

$$s_t = \log Z_t + \epsilon_{S,t}$$

where $\epsilon_{S,t} \sim iid\ N(0, \sigma_S^2)$, and form beliefs using the diagnostic Kalman filter introduced by Bordalo et al. (2020).[21]

The model is calibrated as follows. In order to capture the idea that the agent is forming beliefs about a very long-run object, we calibrate the persistence of the permanent component to a high value, $\rho_Z = 0.98$. This is also broadly consistent with estimates obtained in Blanchard et al. (2013), Cao and L'Huillier (2018), and Flemming, L'Huillier, and Piguillem (2019), which use a similar model and estimate it

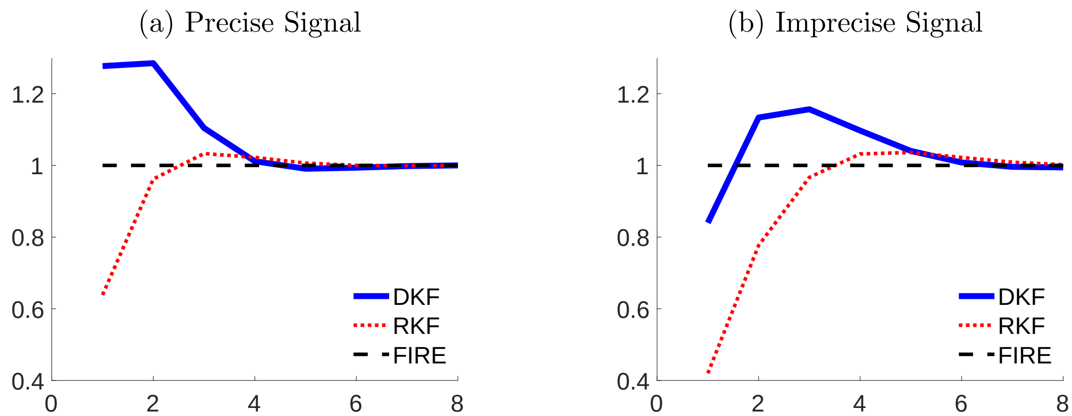
[18] We take the limit $\phi_x \to 0$ and $\psi_p \to \infty$. For brevity we do not write down the equations more explicitly, but this conclusion can be reached by iterating forward the Euler equation. Forward iteration is possible because in linear-Gaussian contexts as ours the law of iterated expectations for DE holds.

[19] In order for the specification in the full model to be equivalent to the one in Blanchard et al. (2013), the temporary component needs to be scaled by $1 - \alpha$.

[20] Even though the model does not explicitly have dispersed information as in Coibion and Gorodnichenko (2012), we follow Lorenzoni (2009) by using a simple representative agent model with aggregate noisy signals.

[21] The filter needs to be adapted to the particular information structure here, but the ideas are the same.

Figure 2: Impulse Responses: Beliefs About the Long-Run



Notes: The panels depict the impulse responses of beliefs about long-run productivity to a one unit positive shock to the permanent component of TFP. The left-hand side panel presents the case of a precise signal ($\sigma_S = 0.01$ and $\theta = 1.0$); the right-hand side panel with the case of an imprecise signal ($\sigma_S = 0.03$ and $\theta = 1.0$).

by maximum likelihood.[22] We normalize the standard deviation of TFP to 1, implying a standard deviation permanent shocks of 0.02.[23] We consider two values of the standard deviation of the signal: a relatively precise signal (of standard deviation 0.01), or a relatively imprecise signal (of standard deviation 0.03). Our baseline diagnosticity parameter θ is fixed at 1 (as in the previous section).

4.2 Results

Figure 2 presents the dynamics for beliefs about long-run productivity in response to a one standard deviation permanent shock. We show three cases: FIRE, the diagnostic Kalman filter (DKF), and the rational Kalman filter (RKF). Figure 2a presents the case of a precise signal, and Figure 2b presents the case of an imprecise signal.

Under FIRE, long-run beliefs jump to 1 on impact and stay there. This is because the standard deviation of TFP innovations has been normalized to 1, and beliefs immediately adjust to the long-run value of TFP after the shock. In the case of a precise signal (Figure 2a), beliefs under the RKF underreact on impact, starting off at 0.70. As learning happens over time, these beliefs rise, gradually converging to 1 in the long run.[24] Instead, beliefs under the DKF strongly overreact on impact. This because the

[22]Following previous work, we set the persistence of the transitory component to the same value, implying a random walk process for TFP.

[23]See Cao and L'Huillier (2018), p. 99, for how to compute this value.

[24]There is a light overreaction in period 3 even in the case of the RKF. This is simply a mechanical implication of the persistence of beliefs inherited from the highly persistent permanent component.

signal is so precise that diagnosticity overwhelms imperfect information.

Turning to the case of an imprecise signal (Figure 2b), beliefs under the RKF underreact significantly, starting off at 0.41. Given that now imperfect information is more severe, DKF beliefs also slightly underreact on impact, starting off at 0.84. However, because agents receive a new signal every period, there is gradual learning. Therefore, as they gather more information, DKF implies a sizeable overreaction over periods 2 to 6, with a peak at 1.16. Notice, the RKF also slightly overreacts around period 5. This is due to a mechanical effect induced by the persistence of beliefs. However, diagnosticity induces overreaction above and beyond this mechanical effect.

We conclude this brief section by noting that we reported results only varying the precision of the signal. By varying the degree of diagnosticity one modifies the degree of overreaction independently. For instance, increasing θ to 1.5 (which is within the range of estimates reported by Bordalo, Gennaioli, Ma, and Shleifer 2020) can generate a slight overreaction in the short run and a stronger overreaction in the medium run, leading to a hump-shaped pattern of beliefs.

5 Conclusion

In this paper, we presented a general treatment of diagnostic expectations in linear recursive models. To our knowledge, this has not been studied before. Using our characterization, we provide conditions under which diagnostic expectations imply extra volatility for variables relative to the rational expectations benchmark. We assess the validity of this claim in the simple new Keynesian model, the real business cycle model, and a quantitative medium-scale DSGE model. Analytically, we show that output gap co-moves positively with unanticipated productivity shocks under diagnostic expectations in the simple new Keynesian model; in contrast to the conventional result obtained with rational expectations. Finally, we show that variations in precision of signal can generate under-reaction, over-reaction or delayed over-reaction when diagnostic expectations are modeled along with imperfect information.

Our focus in this paper has been primarily to provide a general framework to incorporate diagnostic expectations in linear models. A number of important applications and extensions have not been studied in this paper. These include characterizing optimal monetary policy when agents form expectations with diagnosticity, modeling occasionally binding constraints such as the zero lower bound on short-term nominal interest, and incorporating diagnostic expectations in heterogeneous agent frameworks, among others. We hope to pursue some of these directions in future work.

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A Supplementary Material to Linearity Results, Section 2.1

This appendix significantly expands Section 2.1 by providing all missing proofs, by writing out some of the expressions in detail, and discussing some of the intuition for the results. In particular, we present the proof of Lemma 2, and describe in detail how it relates to the comments in the appendix of Bordalo, Gennaioli, and Shleifer (2018).

Explicit Expression for Diagnostic Distribution Under the NNA, and Tractability Intuition. Given (realized) states $\bar{x}_t$ and $\bar{x}_{t-1}$, the diagnostic probability distribution function of x_{t+1} is

$$f_t^\theta(x_{t+1}) = f(x_{t+1}|x_t = \bar{x}_t) \cdot \left[\frac{f(x_{t+1}|x_t = \bar{x}_t)}{f(x_{t+1}|x_t = \rho_x \bar{x}_{t-1})}\right]^\theta \cdot C \tag{18}$$

When looking at equation (18), it is important to notice that, generically, $\bar{x}_t \neq \rho_x \bar{x}_{t-1}$ (due to the realization of the shock ε_t.) However, since ε_t is fixed at 0 *in expectation*, then, the NNA implies that

$$f(x_{t+1}|x_t = \rho_x \bar{x}_{t-1}) \propto \varphi\left(\frac{x_{t+1} - \rho_x^2 \bar{x}_{t-1}}{\sigma_\varepsilon}\right)$$

Thanks to the NNA, the variance of this pdf is σ_ε^2, which is the same as the variance of the true pdf of x_{t+1}. Thus, the true and the reference distributions have the same variance. This allows for tractability, implying that the diagnostic distribution is normally distributed.

We now prove that the diagnostic expectation of a univariate variable can be expressed in terms of rational expectations.

Proof (Lemma 1). The diagnostic expectation of x_{t+1} is given by

$$\mathbb{E}_t^\theta[x_{t+1}] = \int_{-\infty}^{\infty} x f_t^\theta(x) dx$$

The diagnostic pdf is given by

$$f_t^\theta(x) = \frac{\left[\frac{1}{\sigma_\varepsilon}\varphi\left(\frac{x-\rho_x\bar{x}_t}{\sigma_\varepsilon}\right)\right]^{1+\theta}}{\left[\frac{1}{\sigma_\varepsilon}\varphi\left(\frac{x-\rho_x^2\bar{x}_{t-1}}{\sigma_\varepsilon}\right)\right]^{\theta}}C$$

where C is a normalizing constant given by

$$\exp\left\{-\frac{1}{2}\left(\frac{\theta(1+\theta)\rho_x^2\bar{x}_t^2 + \theta(\theta+1)\rho_x^4\bar{x}_{t-1}^2 - 2(1+\theta)\theta\rho_x^3\bar{x}_t\bar{x}_{t-1}}{\sigma_\varepsilon^2}\right)\right\}$$

in which case

$$\begin{aligned}\mathbb{E}_t^\theta[x_{t+1}] &= \int_{-\infty}^{\infty} x f_t^\theta(x)dx \\ &= \int_{-\infty}^{\infty} x\frac{1}{\sigma_\varepsilon}\varphi\left(\frac{x-(\rho_x\bar{x}_t+\theta(\rho_x\bar{x}_t-\rho_x^2\bar{x}_{t-1}))}{\sigma_\varepsilon}\right)dx\end{aligned}$$

Thus, the diagnostic distribution $f_t^\theta(x_{t+1})$ is normal with variance σ_ε^2 and mean

$$\mathbb{E}_t^\theta[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta(\mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}])$$

■

More on Obtaining the Additivity Result. The following is a corollary of the previous lemma, which follows from the fact that the sum $x_{t+1}+y_{t+1}$ is a normal random variable.

Corollary 1

$$\mathbb{E}_t^\theta[x_{t+1}+y_{t+1}] = \mathbb{E}_t[x_{t+1}+y_{t+1}] + \theta(\mathbb{E}_t[x_{t+1}+y_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}+y_{t+1}])$$

It is useful to first record the following lemma, showing that the sum $x_{t+1}+y_t$ follows a normal distribution.

Lemma 4

$$x_{t+1}+y_t \sim N(\rho_x\bar{x}_t+\bar{y}_t, \sigma_\varepsilon^2)$$

Proof. We know that

$$x_{t+1} \sim N(\rho_x \bar{x}_t, \sigma_\varepsilon^2)$$

To derive the pdf of $z_{t+1} \equiv x_{t+1} + y_t$, we evaluate the convolution

$$f_{z_{t+1}}(z) = \int_{-\infty}^{\infty} f_{x_{t+1}}(x) f_{y_t}(z - x) dx = \int_{-\infty}^{\infty} \frac{1}{\sigma_\varepsilon} \varphi\left(\frac{x - \rho_x \bar{x}_t}{\sigma_\varepsilon}\right) \delta(z - x - \bar{y}_t) dx$$

where $f_{z_{t+1}}$ is the pdf of z_{t+1}, $f_{x_{t+1}}$ is the pdf of x_{t+1}, and f_{y_t} is the pdf of y_t, and the second equality follows from the fact that x_{t+1} is normally distributed and y_t follows a Dirac delta distribution centered at $\bar{y}_t$.

By the symmetry of the Dirac delta function,

$$f_{z_{t+1}}(z) = \int_{-\infty}^{\infty} \frac{1}{\sigma_\varepsilon} \varphi\left(\frac{x - \rho_x \bar{x}_t}{\sigma_\varepsilon}\right) \delta(x - z + \bar{y}_t) dx$$

and by the sifting property of the Dirac delta function:[25]

$$f_{z_{t+1}}(z) = \frac{1}{\sigma_\varepsilon} \varphi\left(\frac{z - \bar{y}_t - \rho_x \bar{x}_t}{\sigma_\varepsilon}\right)$$

which is what we wanted to show. ■

Using the previous result Lemma 4, it is easy to obtain the following representation, contained in the appendix of Bordalo, Gennaioli, and Shleifer (2018).

Proposition 5

$$\mathbb{E}_t^\theta[x_{t+1} + y_t] = \mathbb{E}_t[x_{t+1} + y_t] + \theta(\mathbb{E}_t[x_{t+1} + y_t] - \mathbb{E}_{t-1}[x_{t+1} + y_t])$$

Proof. First, we need the reference distribution of $x_{t+1} + y_t$. Under no news, $\varepsilon_t = \eta_t = 0$ and so,

$$x_{t+1} + y_t = \rho_x^2 x_{t-1} + \rho_y y_{t-1} + \varepsilon_{t+1}$$

Then, by an easy extension of Lemma 4,

$$(x_{t+1} + y_t) | \varepsilon_t = \eta_t = 0 \ \sim \ N(\rho_x^2 \bar{x}_{t-1} + \rho_y \bar{y}_{t-1}, \sigma_\varepsilon^2)$$

[25] The Dirac delta function's sifting property is the following. For a continuous function $f(x)$ over $(-\infty, \infty)$,

$$\int_{-\infty}^{\infty} f(x) \delta(x - x_0) dx = f(x_0)$$

It follows that both the reference and representative distributions are normal and have variance σ_ε^2. We then conclude that

$$\mathbb{E}_t^\theta[x_{t+1}+y_t] = \mathbb{E}_t[x_{t+1}+y_t] + \theta(\mathbb{E}_t[x_{t+1}+y_t] - \mathbb{E}_{t-1}[x_{t+1}+y_t])$$

■

We can in fact use this last proposition to compute the expectation of the linear combination for the processes presented in the body. The calculation is as follows:

$$\begin{aligned}
\mathbb{E}_t^\theta[x_{t+1}+y_t] &= \mathbb{E}_t[x_{t+1}+y_t] + \theta(\mathbb{E}_t[x_{t+1}+y_t] - \mathbb{E}_{t-1}[x_{t+1}+y_t]) \\
&= \rho\bar{x}_t + \bar{y}_t + \theta(\rho_x\bar{x}_t + \bar{y}_t - \rho_x^2\bar{x}_{t-1} - \rho_y\bar{y}_{t-1}) \\
&= \rho\bar{x}_t + \bar{y}_t + \theta\rho_x\varepsilon_t + \theta\eta_t \\
&= \rho\bar{x}_t + \bar{y}_t + \theta(\rho_x\varepsilon_t + \eta_t)
\end{aligned}$$

Proof (Lemma 2). The diagnostic expectation of y_t is given by

$$\mathbb{E}_t^\theta[y_t] = \int_{-\infty}^{\infty} y f_t^\theta(y)dy$$

In order to get the diagnostic pdf of y_t, we start by looking at the diagnostic cdf, which by virtue of the NNA is

$$Pr_t^\theta(y_t \leq \bar{y}) = \lim_{\sigma_\eta \to 0^+} \int_{-\infty}^{\bar{y}} \frac{\left[\frac{1}{\sigma_\eta}\varphi\left(\frac{y-\bar{y}_t}{\sigma_\eta}\right)\right]^{1+\theta}}{\left[\frac{1}{\sigma_\eta}\varphi\left(\frac{y-\rho_y\bar{y}_{t-1}}{\sigma_\eta}\right)\right]^{\theta}} \, C \, dy$$

First, note that

$$\begin{aligned}
\frac{\left[\frac{1}{\sigma_\eta}\varphi\left(\frac{y-\bar{y}_t}{\sigma_\eta}\right)\right]^{1+\theta}}{\left[\frac{1}{\sigma_\eta}\varphi\left(\frac{y-\rho_y\bar{y}_{t-1}}{\sigma_\eta}\right)\right]^{\theta}} &= \frac{1}{\sqrt{2\pi}\sigma_\eta}\exp\left\{-\frac{1}{2}\left[(1+\theta)\left(\frac{y-\bar{y}_t}{\sigma_\eta}\right)^2 - \theta\left(\frac{y-\rho_y\bar{y}_{t-1}}{\sigma_\eta}\right)^2\right]\right\} \\
&= \frac{1}{\sqrt{2\pi}\sigma_\eta}\exp\left\{-\frac{1}{2}\frac{\left[y-\left((1+\theta)\bar{y}_t - \theta\rho_t\bar{y}_{t-1}\right)\right]^2}{\sigma_\eta^2}\right\} \times \frac{1}{C}
\end{aligned}$$

where the value of C must be

$$C = \exp\left\{-\frac{1}{2}\left[\frac{\theta(1+\theta)\bar{y}_t^2 + \theta(1+\theta)\rho_y^2\bar{y}_{t-1}^2 - 2\theta(1+\theta)\rho_y\bar{y}_t\bar{y}_{t-1}}{\sigma_\eta^2}\right]\right\}$$

Hence, we can write

$$\begin{aligned}
\mathbb{E}_t^\theta[y_t] &= \lim_{\sigma_\eta\to 0^+}\lim_{u\to\infty}\int_{-\infty}^{u} y\ \frac{\left[\frac{1}{\sigma_\eta}\varphi\left(\frac{y-\bar{y}_t}{\sigma_\eta}\right)\right]^{1+\theta}}{\left[\frac{1}{\sigma_\eta}\varphi\left(\frac{y-\rho_t\bar{y}_{t-1}}{\sigma_\eta}\right)\right]^{\theta}}\ Cdy \\
&= \lim_{\sigma_\eta\to 0^+}\lim_{u\to\infty}\int_{-\infty}^{u} y\ \frac{1}{\sigma_\eta}\varphi\left(\frac{y-\big((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1}\big)}{\sigma_\eta}\right)dy \\
&= \lim_{\sigma_\eta\to 0^+}\lim_{u\to\infty}\left\{\int_{-\infty}^{u}\frac{y-\big((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1}\big)}{\sigma_\eta}\varphi\left(\frac{y-\big((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1}\big)}{\sigma_\eta}\right)dy\right. \\
&\left.+\big((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1}\big)\int_{-\infty}^{u}\frac{1}{\sigma_\eta}\varphi\left(\frac{y-\big((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1}\big)}{\sigma_\eta}\right)dy\right\}
\end{aligned}$$

We will evaluate the integral by change of variables. To this end, define $z \equiv \frac{y-((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1})}{\sigma_\eta}$ such that

$$\mathbb{E}_t^\theta[y_t] = \lim_{\sigma_\eta\to 0^+}\lim_{u\to\infty}\left\{\sigma_\eta\int_{-\infty}^{\frac{u-((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1})}{\sigma_\eta}} z\varphi(z)dz + ((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1})\int_{-\infty}^{\frac{u-((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1})}{\sigma_\eta}}\varphi(z)dz\right\}$$

Since $\lim_{\sigma_\eta\to 0^+}\frac{u-((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1})}{\sigma_\eta} = +\infty$ when $u > (1+\theta)\bar{y}_t - \theta\rho_y\bar{y}_{t-1}$, we have

$$\lim_{\sigma_\eta\to 0^+}\int_{-\infty}^{\frac{u-((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1})}{\sigma_\eta}} z\varphi(z)dz = 0 \quad\text{and}\quad \lim_{\sigma_\eta\to 0^+}\int_{-\infty}^{\frac{u-((1+\theta)\bar{y}_t-\theta\rho_y\bar{y}_{t-1})}{\sigma_\eta}}\varphi(z)dz = 1$$

and

$$Pr_t^\theta(y_t \le \bar{y}) = \lim_{\sigma_\eta\to 0^+}\frac{1}{\sigma_\eta}\Phi\left(\frac{\bar{y}-(\bar{y}_t+\theta(\bar{y}_t-\rho_y\bar{y}_{t-1}))}{\sigma_\eta}\right)$$

Thus,

$$f_t^\theta(y_t) = \delta(y_t - (\bar{y}_t + \theta(\bar{y}_t - \rho_y\bar{y}_{t-1})))$$

and

$$\mathbb{E}_t^\theta[y_t] = \bar{y}_t + \theta(\bar{y}_t - \rho_y\bar{y}_{t-1})$$

■

Proof (Proposition 1). There are two cases:

- The case $s = r = 1$ follows from the fact that both x_{t+1} and y_{t+1} are normal and therefore Lemma 1 and Corollary 1 apply.
- The case of $s = 0$ or $r = 0$ follows from Lemma 2. (Equivalently, when only one of s or r is not 0, this follows from Proposition 5.)

■

Alternative Assumption for Degenerate Variables. Instead of the NNA (Assumption 1), let us suppose that the reference distribution of y_t is the (non-degenerate) normal distribution:

$$f(y_t|y_{t-1} = \bar{y}_{t-1}) \propto \varphi\left(\frac{y_t - \rho_y \bar{y}_{t-1}}{\sigma_\eta}\right)$$

which corresponds to replacing the NNA by the assumption that the conditioning set is $\{y_{t-1} = \bar{y}_{t-1}\}$. This is the alternative discussed in Bordalo, Gennaioli, and Shleifer (2018), footnote 8. We highlight that this is an assumption about past y_{t-1} instead of current y_t. Indeed, the NNA embeds an assumption about the shock ε_t, on top of the conditioning on the realization $\bar{y}_{t-1}$, resulting in the reference cdf (2) above. In this alternative case, the following lemma obtains.

Lemma 5 *Replace Assumption 1 by* $\{y_{t-1} = \bar{y}_{t-1}\}$. *Then,*

$$\mathbb{E}_t^\theta[y_t] = \bar{y}_t$$

Proof. The diagnostic expectation of y_t is given by

$$\mathbb{E}_t^\theta[y_t] = \int y f_t^\theta(y|y_t = \bar{y}_t, y_{t-1} = \bar{y}_{t-1})dy$$

Notice that in this notation, since we are not using the NNA, we explicitly write the two conditioning events $G = \{y_t = \bar{y}_t\}$ and $-G = \{y_{t-1} = \bar{y}_{t-1}\}$. In order to get the diagnostic pdf of y_t, we start by looking at the diagnostic cdf:

$$Pr_t^\theta(y_t \leq \bar{y}|y_t = \bar{y}_t, y_{t-1} = \bar{y}_{t-1}) = \lim_{a \to 0^+} \int_{-\infty}^{\bar{y}} \frac{\left[\frac{1}{a}\varphi\left(\frac{y-\bar{y}_t}{a}\right)\right]^{1+\theta}}{\left[\frac{1}{\sigma_\eta}\varphi\left(\frac{y-\rho_y\bar{y}_{t-1}}{\sigma_\eta}\right)\right]^\theta} C \; dy$$

Notice that this time it is only the uncertainty in the numerator that vanishes. First, note that

$$\frac{\left[\frac{1}{a}\varphi\left(\frac{y-\bar{y}_t}{a}\right)\right]^{1+\theta}}{\left[\frac{1}{\sigma_\eta}\varphi\left(\frac{y-\rho_y\bar{y}_{t-1}}{\sigma_\eta}\right)\right]^{\theta}} = \frac{1}{\sqrt{2\pi}\frac{a^{1+\theta}}{\sigma_\eta^\theta}} \exp\left\{-\frac{1}{2}\left[(1+\theta)\left(\frac{y-\bar{y}_t}{a}\right)^2 - \theta\left(\frac{y-\rho_y\bar{y}_{t-1}}{\sigma_\eta}\right)^2\right]\right\}$$

$$= \frac{1}{\sqrt{2\pi}\sigma_a} \exp\left\{-\frac{1}{2}\frac{(y-\mu_a)^2}{\sigma_a^2}\right\} \times \frac{1}{C}$$

where

$$\mu_a = \frac{\sigma_\eta^2(1+\theta)\bar{y}_t - a^2\theta\rho_y\bar{y}_{t-1}}{\sigma_\eta^2(1+\theta) - a^2\theta}, \quad \sigma_a^2 = \frac{a^2\sigma_\eta^2}{\sigma_\eta^2(1+\theta) - a^2\theta}$$

and the value of C must be

$$C = \exp\left\{-\frac{1}{2}\left(\frac{\mu_a^2 - k_a}{\sigma_a^2}\right)\right\}\frac{a^{1+\theta}}{\sigma_a\sigma_\eta^\theta}$$

where

$$k_a = \frac{\sigma_\eta^2(1+\theta)\bar{y}_t^2 - a^2\theta\rho_y^2\bar{y}_{t-1}^2}{\sigma_\eta^2(1+\theta) - a^2\theta}$$

Hence, we can write

$$\mathbb{E}_t^\theta[y_t] = \lim_{a\to 0^+}\lim_{u\to\infty}\int_{-\infty}^{u} y \,\frac{\left[\frac{1}{a}\varphi\left(\frac{y-\bar{y}_t}{a}\right)\right]^{1+\theta}}{\left[\frac{1}{\sigma_\eta}\varphi\left(\frac{y-\rho_y\bar{y}_{t-1}}{\sigma_\eta}\right)\right]^{\theta}}\, C\, dy$$

$$= \lim_{a\to 0^+}\lim_{u\to\infty}\int_{-\infty}^{u} y\,\frac{1}{\sigma_a}\varphi\left(\frac{y-\mu_a}{\sigma_a}\right)dy$$

$$= \lim_{a\to 0^+}\lim_{u\to\infty}\left\{\int_{-\infty}^{u}\frac{y-\mu_a}{\sigma_a}\varphi\left(\frac{y-\mu_a}{\sigma_a}\right)dy + \mu_a\int_{-\infty}^{x}\frac{1}{\sigma_a}\varphi\left(\frac{y-\mu_a}{\sigma_a}\right)dy\right\}$$

We will evaluate the integral by change of variables. To this end, define $z \equiv \frac{y-\mu_a}{\sigma_a}$ such that

$$\mathbb{E}_t^\theta[y_t] = \lim_{a\to 0^+}\lim_{u\to\infty}\left\{\sigma_a\int_{-\infty}^{\frac{u-\mu_a}{\sigma_a}} z\varphi(z)dz + \mu_a\int_{-\infty}^{\frac{u-\mu_a}{\sigma_a}}\varphi(z)dz\right\}$$

Notice that

$$\lim_{a\to 0^+}\mu_a = \bar{y}_t$$

and

$$\lim_{a \to 0^+} \sigma_a = 0$$

Since $\lim_{a \to 0^+} \frac{u-\mu_a}{\sigma_a} = +\infty$ when $u > \mu_a$, we have

$$\lim_{a \to 0^+} \int_{-\infty}^{\frac{u-\mu_a}{\sigma_a}} z\varphi(z)dz = 0 \quad \text{and} \quad \lim_{a \to 0^+} \int_{-\infty}^{\frac{u-\mu_a}{\sigma_a}} \varphi(z)dz = 1$$

and

$$Pr_t^\theta(y_t \leq \bar{y} | y_t = \bar{y}_t, y_{t-1} = \bar{y}_{t-1}) = \lim_{a \to 0^+} \frac{1}{a} \Phi\left(\frac{\bar{y} - \bar{y}_t}{a}\right)$$

Thus,

$$f_t^\theta(y) = \delta(y_t - \bar{y}_t)$$

As a consequence,

$$\mathbb{E}_t^\theta[y_t] = \bar{y}_t$$

as we wanted to show. ■

B Supplementary Materials and Proofs for Results in Sections 2.2, 2.3, and 2.4

This appendix significantly expands Sections 2.2, 2.3, and 2.4 by providing missing proofs and by writing out some of the expressions in detail.

Detailed Solution Procedure. We solve for the recursive equilibrium law of motion of a linear diagnostic-expectations DSGE model using the method of undetermined coefficients.

With the strong additivity result from Proposition 1, the class of forward-looking models of our interest is written in the following form:

$$\mathbf{F}\mathbb{E}_t^\theta[\mathbf{y}_{t+1}] + \mathbf{G_1}\mathbb{E}_t^\theta[\mathbf{y}_t] + \mathbf{G_2}\mathbf{y}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{M}\mathbb{E}_t^\theta[\mathbf{x}_{t+1}] + \mathbf{N_1}\mathbb{E}_t^\theta[\mathbf{x}_t] + \mathbf{N_2}\mathbf{x}_t = 0$$

Suppose that there is a unique stable solution of the model:

$$\mathbf{y}_t = \mathbf{P}\mathbf{y}_{t-1} + \mathbf{Q}\mathbf{x}_t + \mathbf{R}\mathbf{v}_t \tag{19}$$

we can rewrite the above stochastic difference equation as follows:

$$\begin{gathered}\mathbf{F}\mathbb{E}_t^\theta\big[\mathbf{P}\mathbf{y}_t + \mathbf{Q}\mathbf{x}_{t+1} + \mathbf{R}\mathbf{v}_{t+1}\big] + \mathbf{G_1}\mathbb{E}_t^\theta\big[\mathbf{P}\mathbf{y}_{t-1} + \mathbf{Q}\mathbf{x}_t + \mathbf{R}\mathbf{v}_t\big] + \mathbf{G_2}\mathbf{P}\mathbf{y}_{t-1} \\ +\mathbf{G_2}\mathbf{Q}\mathbf{x}_t + \mathbf{G_2}\mathbf{R}\mathbf{v}_t + \mathbf{M}\mathbb{E}_t^\theta\big[\mathbf{A}\mathbf{x}_t + \mathbf{v}_{t+1}\big] + \mathbf{N_1}\mathbb{E}_t^\theta\big[\mathbf{x}_t\big] + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{N_2}\mathbf{x}_t = 0\end{gathered}$$

Applying the strong additivity property, diagnostic expectations can be represented as a linear combination of the rational expectations held at t and $t-1$:

$$\begin{aligned}&\mathbf{F}\mathbb{E}_t^\theta\big[\mathbf{P}\mathbf{y}_t + \mathbf{Q}\mathbf{x}_{t+1} + \mathbf{R}\mathbf{v}_{t+1}\big] = (1+\theta)\mathbf{F}\mathbb{E}_t\Big[\mathbf{P}^2\mathbf{y}_{t-1} + \mathbf{P}\mathbf{Q}\mathbf{x}_t + \mathbf{P}\mathbf{R}\mathbf{v}_t + \mathbf{Q}\mathbf{A}\mathbf{x}_t + \mathbf{Q}\mathbf{v}_{t+1} + \mathbf{R}\mathbf{v}_{t+1}\Big] \\ &- \theta\mathbf{F}\mathbb{E}_{t-1}\Big[\mathbf{P}^2\mathbf{y}_{t-1} + \mathbf{P}\mathbf{Q}\mathbf{A}\mathbf{x}_{t-1} + \mathbf{P}\mathbf{Q}\mathbf{v}_t + \mathbf{P}\mathbf{R}\mathbf{v}_t + \mathbf{Q}\mathbf{A}^2\mathbf{x}_{t-1} + \mathbf{Q}\mathbf{A}\mathbf{v}_t + \mathbf{Q}\mathbf{v}_{t+1} + \mathbf{R}\mathbf{v}_{t+1}\Big] \\ &= \mathbf{F}\mathbf{P}^2\mathbf{y}_{t-1} + \mathbf{F}\mathbf{P}\mathbf{Q}\mathbf{x}_t + \theta\mathbf{F}\mathbf{P}\mathbf{Q}\mathbf{v}_t + (1+\theta)\mathbf{F}\mathbf{P}\mathbf{R}\mathbf{v}_t + \mathbf{F}\mathbf{Q}\mathbf{A}\mathbf{x}_t + \theta\mathbf{F}\mathbf{Q}\mathbf{A}\mathbf{v}_t\end{aligned}$$

$$\begin{aligned}\mathbf{G_1}\mathbb{E}_t^\theta\big[\mathbf{P}\mathbf{y}_{t-1} + \mathbf{Q}\mathbf{x}_t + \mathbf{R}\mathbf{v}_t\big] &= (1+\theta)\mathbf{G_1}\mathbb{E}_t[\mathbf{P}\mathbf{y}_{t-1} + \mathbf{Q}\mathbf{x}_t + \mathbf{R}\mathbf{v}_t] - \theta\mathbf{G_1}\mathbb{E}_{t-1}[\mathbf{P}\mathbf{y}_{t-1} + \mathbf{Q}\mathbf{x}_t + \mathbf{R}\mathbf{v}_t] \\ &= \mathbf{G_1}\mathbf{P}\mathbf{y}_{t-1} + \mathbf{G_1}\mathbf{Q}\mathbf{x}_t + \theta\mathbf{G_1}\mathbf{Q}\mathbf{v}_t + (1+\theta)\mathbf{G_1}\mathbf{R}\mathbf{v}_t\end{aligned}$$

$$\mathbf{M}\mathbb{E}_t^{\theta}\big[\mathbf{A}\mathbf{x}_t + \mathbf{v}_{t+1}\big] = (1+\theta)\mathbf{M}\mathbb{E}_t[\mathbf{A}\mathbf{x}_t + \mathbf{v}_{t+1}] - \theta\mathbf{M}\mathbb{E}_{t-1}[\mathbf{A}\mathbf{x}_t + \mathbf{v}_{t+1}]$$
$$= \mathbf{M}\mathbf{A}\mathbf{x}_t + \theta\mathbf{M}\mathbf{A}\mathbf{v}_t$$

$$\mathbf{N_1}\mathbb{E}_t^{\theta}\big[\mathbf{x}_t\big] = (1+\theta)\mathbf{N_1}\mathbb{E}_t[\mathbf{x}_t] - \theta\mathbf{N_1}\mathbb{E}_{t-1}[\mathbf{x}_t] = \mathbf{N_1}\mathbf{x}_t + \theta\mathbf{N_1}\mathbf{v}_t$$

We write the model in the rational expectations representations as

$$0 = \mathbf{F}\mathbf{P}^2\mathbf{y}_{t-1} + \mathbf{F}\mathbf{P}\mathbf{Q}\mathbf{x}_t + \theta\mathbf{F}\mathbf{P}\mathbf{Q}\mathbf{v}_t + (1+\theta)\mathbf{F}\mathbf{P}\mathbf{R}\mathbf{v}_t + \mathbf{F}\mathbf{Q}\mathbf{A}\mathbf{x}_t + \theta\mathbf{F}\mathbf{Q}\mathbf{A}\mathbf{v}_t + \mathbf{G_1}\mathbf{P}\mathbf{y}_{t-1} + ...$$
$$+ \mathbf{G_1}\mathbf{Q}\mathbf{x}_t + \theta\mathbf{G_1}\mathbf{Q}\mathbf{v}_t + (1+\theta)\mathbf{G_1}\mathbf{R}\mathbf{v}_t + \mathbf{G_2}\mathbf{P}\mathbf{y}_{t-1} + \mathbf{G_2}\mathbf{Q}\mathbf{x}_t + \mathbf{G_2}\mathbf{R}\mathbf{v}_t + \mathbf{M}\mathbf{A}\mathbf{x}_t + ...$$
$$+ \theta\mathbf{M}\mathbf{A}\mathbf{v}_t + \mathbf{N_1}\mathbf{x}_t + \theta\mathbf{N_1}\mathbf{v}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{N}_2\mathbf{x}_t$$

It is now straightforward to proceed by the method of undetermined coefficients to find a solution of the form (19), and the matrices $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ can be found solving the following matrix equations.

$$\mathbf{F}\mathbf{P}^2 + \mathbf{G}\mathbf{P} + \mathbf{H} = 0 \tag{20}$$

$$\mathbf{F}\mathbf{P}\mathbf{Q} + \mathbf{F}\mathbf{Q}\mathbf{A} + \mathbf{G}\mathbf{Q} + \mathbf{M}\mathbf{A} + \mathbf{N} = 0$$

$$\theta\mathbf{F}\mathbf{P}\mathbf{Q} + (1+\theta)\mathbf{F}\mathbf{P}\mathbf{R} + \theta\mathbf{F}\mathbf{Q}\mathbf{A} + \theta\mathbf{G_1}\mathbf{Q} + \mathbf{G}\mathbf{R} + \theta\mathbf{G_1}\mathbf{R} + \theta\mathbf{M}\mathbf{A} + \theta\mathbf{N_1} = 0$$

where $\mathbf{G} = \mathbf{G_1} + \mathbf{G_2}$ and $\mathbf{N} = \mathbf{N_1} + \mathbf{N_2}$.

See Section 2.3.3 for the detailed procedure to obtain solution matrices.

The Solution under Rational Expectations. Consider the model under rational expectations:

$$\mathbf{F}\mathbb{E}_t[\mathbf{y}_{t+1}] + \mathbf{G}\mathbf{y}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{M}\mathbb{E}_t[\mathbf{x}_{t+1}] + \mathbf{N}\mathbf{x}_t = 0 \tag{21}$$

where $\mathbf{G} = \mathbf{G_1} + \mathbf{G_2}$ and $\mathbf{N} = \mathbf{N_1} + \mathbf{N_2}$ and, as above, $\mathbf{y}_t$ and $\mathbf{x}_t$ denote vectors of endogenous variables (including controls and states) $(m \times 1)$ and of exogenous states $(n \times 1)$. $\mathbb{E}_t$ denotes the rational expectation operator, and the exogenous process is given by (4).

Suppose that there is a unique stable solution of the model:

$$\mathbf{y}_t = \widetilde{\mathbf{P}}\mathbf{y}_{t-1} + \widetilde{\mathbf{Q}}\mathbf{x}_t$$

then, we can rewrite the stochastic difference equation (21) as follows:

$$\mathbf{F}\mathbb{E}_t\left[\widetilde{\mathbf{P}}\mathbf{y}_t + \widetilde{\mathbf{Q}}\mathbf{x}_{t+1}\right] + \mathbf{G}\widetilde{\mathbf{P}}\mathbf{y}_{t-1} + \mathbf{G}\widetilde{\mathbf{Q}}\mathbf{x}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{M}\mathbf{A}\mathbf{x}_t + \mathbf{N}\mathbf{x}_t = 0$$

We can simplify the above equation to

$$\mathbf{F}\widetilde{\mathbf{P}}^2\mathbf{y}_{t-1} + \mathbf{F}\widetilde{\mathbf{P}}\widetilde{\mathbf{Q}}\mathbf{x}_t + \mathbf{F}\widetilde{\mathbf{Q}}\mathbf{A}\mathbf{x}_t + \mathbf{G}\widetilde{\mathbf{P}}\mathbf{y}_{t-1} + \mathbf{G}\widetilde{\mathbf{Q}}\mathbf{x}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{M}\mathbf{A}\mathbf{x}_t + \mathbf{N}\mathbf{x}_t = 0$$

and can solve similarly for the recursive equilibrium law of motion via the method of undetermined coefficients. Specifically, the matrices $\widetilde{\mathbf{P}}$ and $\widetilde{\mathbf{Q}}$ can be found solving the following matrix equations.

$$\mathbf{F}\widetilde{\mathbf{P}}^2 + \mathbf{G}\widetilde{\mathbf{P}} + \mathbf{H} = 0$$

$$\mathbf{F}\widetilde{\mathbf{P}}\widetilde{\mathbf{Q}} + \mathbf{F}\widetilde{\mathbf{Q}}\mathbf{A} + \mathbf{G}\widetilde{\mathbf{Q}} + \mathbf{M}\mathbf{A} + \mathbf{N} = 0$$

Comparison of these equations with their counterpart under DE immediately shows that $\mathbf{P} = \widetilde{\mathbf{P}}$ and $\mathbf{Q} = \widetilde{\mathbf{Q}}$.

Stability Conditions. Given the quadratic matrix equation (20)

$$\mathbf{F}\mathbf{P}^2 + \mathbf{G}\mathbf{P} + \mathbf{H} = \mathbf{0}$$

for the $m \times m$ matrix $\mathbf{P}$ and $m \times m$ matrices $\mathbf{G}$ and $\mathbf{H}$, define the $2m \times 2m$ matrices $\mathbf{\Xi}$ and $\mathbf{\Delta}$:

$$\mathbf{\Xi} = \begin{bmatrix} -\mathbf{G} & -\mathbf{H} \\ \mathbf{I}_m & \mathbf{0}_m \end{bmatrix}$$

and

$$\mathbf{\Delta} = \begin{bmatrix} -\mathbf{F} & \mathbf{0}_m \\ \mathbf{0}_m & \mathbf{I}_m \end{bmatrix}$$

where $\mathbf{I}_m$ is the identity matrix of size m and $\mathbf{0}_m$ is the $m \times m$ matrix with only zero entries.

Uhlig (1995) shows that if (a) s is a generalized eigenvector and λ is the corresponding generalized eigenvalue of $\mathbf{\Xi}$ with respect to $\mathbf{\Delta}$, then s can be written as $s' = \left[\lambda x', x'\right]$ for some $x \in \mathsf{R}^m$, and (b) there are m generalized eigenvalues $\lambda_1, ..., \lambda_m$ together with generalized eigenvectors $s_1, ..., s_m$ of $\mathbf{\Xi}$ with respect to $\mathbf{\Delta}$, written as

$s_i^{'} = [\lambda_i x_i^{'}, x_i^{'}]$ for some $x_i \in \mathsf{R}^m$, and if $(x_1, ..., x_m)$ is linearly dependent, then

$$\mathbf{P} = \mathbf{\Omega \Lambda \Omega}^{'}$$

is a solution to the matrix quadratic equation, where $\mathbf{\Omega} = [x_1, ..., x_m]$ and $\mathbf{\Lambda} = diag(\lambda_1, ..., \lambda_m)$.

The stability conditions are given as follows.[26]

Theorem 1 *The solution* $\mathbf{P}$ *is stable if* $|\lambda_i| < 1$ *for all* $i = 1, ..., m$.

Thus, we can easily show that the stability conditions for both models are the same.

Proof (Lemma 3). The solutions $\mathbf{P}$ and $\widetilde{\mathbf{P}}$ are the same since they involve identical matrices $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$. Thus, the stability conditions stated in Theorem 1 are the same for both solutions. ■

Volatility. We show the condition under which a model with DE delivers a larger volatility than its counterpart with RE.

Proof (Proposition 3). We have already shown that $\mathbf{P}$ and $\widetilde{\mathbf{P}}$ are the same and that $\mathbf{Q}$ and $\widetilde{\mathbf{Q}}$ are the same.

Thus, given the exogenous process $\mathbf{x}_t$, the solution for the model with diagnostic expectations and for the model with rational expectations can be formulated as

$$\begin{aligned}\mathbf{y}_t^{DE} &= \mathbf{P}\mathbf{y}_{t-1} + \mathbf{Q}\mathbf{x}_t + \mathbf{R}\mathbf{v}_t \\ \mathbf{y}_t^{RE} &= \mathbf{P}\mathbf{y}_{t-1} + \mathbf{Q}\mathbf{x}_t\end{aligned}$$

such that the variance of the vector of endogenous variables under diagnostic expectations, $\mathbf{y}_t^{DE}$, is given by

$$\begin{aligned}Var(\mathbf{y}_t^{DE}) &= Var(\mathbf{P}\mathbf{y}_{t-1}) + Var(\mathbf{Q}\mathbf{x}_t) + Var(\mathbf{R}\mathbf{v}_t) \\ &\quad + 2\ Cov(\mathbf{P}\mathbf{y}_{t-1}, \mathbf{Q}\mathbf{x}_t) + 2\ Cov(\mathbf{P}\mathbf{y}_{t-1}, \mathbf{R}\mathbf{v}_t) + 2\ Cov(\mathbf{Q}\mathbf{x}_t, \mathbf{R}\mathbf{v}_t) \qquad (22)\end{aligned}$$

Similarly, the variance of the vector of endogenous variables under rational expec-

[26] See Section 6.3 of Uhlig (1995) for a detailed discussion.

tations, $\mathbf{y}_t^{RE}$ is given by

$$Var(\mathbf{y}_t^{RE}) = Var(\mathbf{P}\mathbf{y}_{t-1}) + Var(\mathbf{Q}\mathbf{x}_t) + 2\ Cov(\mathbf{P}\mathbf{y}_{t-1}, \mathbf{Q}\mathbf{x}_t)$$

Since $cov(\mathbf{P}\mathbf{y}_{t-1}, \mathbf{R}\mathbf{v}_t) = 0$, (22) is simplified to

$$Var(\mathbf{y}_t^{DE}) = Var(\mathbf{P}\mathbf{y}_{t-1}) + Var(\mathbf{Q}\mathbf{x}_t) + Var(\mathbf{R}\mathbf{v}_t) + 2\ Cov(\mathbf{P}\mathbf{y}_{t-1}, \mathbf{Q}\mathbf{x}_t) + 2\ Cov(\mathbf{Q}\mathbf{x}_t, \mathbf{R}\mathbf{v}_t)$$

such that by taking the difference of the two variances, we have

$$\begin{aligned} Var(\mathbf{y}_t^{DE}) - Var(\mathbf{y}_t^{RE}) &= Var(\mathbf{R}\mathbf{v}_t) + 2\ Cov(\mathbf{Q}\mathbf{x}_t, \mathbf{R}\mathbf{v}_t) \\ &= Var(\mathbf{R}\mathbf{v}_t) + 2\ Cov(\mathbf{Q}\mathbf{A}\mathbf{x}_{t-1} + \mathbf{Q}\mathbf{v}_t, \mathbf{R}\mathbf{v}_t) \\ &= \mathbf{R}\mathbf{\Sigma}_{\mathbf{v}}\mathbf{R}' + 2\mathbf{Q}\mathbf{\Sigma}_{\mathbf{v}}\mathbf{R}' \end{aligned}$$

Thus, for an endogenous variable y_{it} to have extra volatility with diagnostic expectations, the i-th diagonal component of the matrix $\mathbf{R}\mathbf{\Sigma}_{\mathbf{v}}\mathbf{R}' + 2\mathbf{Q}\mathbf{\Sigma}_{\mathbf{v}}\mathbf{R}'$ must be greater than zero. ■

C New Keynesian Model with Diagnostic Expectations: Detailed Derivation

There are three sets of agents in the economy: households, firms and government. Total output produced is equal to consumption expenditure made by the households and adjustment costs spent in adjusting prices. There is no government spending.

C.1 Households

Households have the following lifetime utility

$$\log C_t - \omega \frac{L_t^{1+\nu}}{1+\nu} + \mathbb{E}_t^\theta \left[\Sigma_{s=t+1}^{\infty} \beta^{s-t} \left[\log(C_s) - \frac{\omega}{1+\nu} L_s^{1+\nu} \right] \right]$$

subject to budget constraint:

$$P_t C_t + \frac{B_{t+1}}{(1+i_t)} = B_t + W_t L_t + \Gamma_t + T_t \, ,$$

$P_t C_t$ is nominal expenditure on final consumption good, B_{t+1} denotes purchase of nominal bonds that pay off $1 + i_t$ interest rate in the following period, $W_t L_t$ denotes labor income, Γ_t and T_t denote dividends from firm-ownership and lumpsum government transfers respectively. $\mathbb{E}_t^\theta$ is the diagnostic expectations operator with diagnosticity parameter θ.

Let $\log C_t \equiv u(C_t)$. The consumption Euler equation is given by:

$$\frac{u'(C_t)}{P_t} = \beta(1+i_t)\mathbb{E}_t^\theta \left[\frac{u'(C_{t+1})}{P_{t+1}} \right]$$

Multiplying with P_{t-1} on both sides:

$$\frac{u'(C_t) P_{t-1}}{P_t} = \beta(1+i_t)\mathbb{E}_t^\theta \left[\frac{u'(C_{t+1}) P_{t-1}}{P_{t+1}} \right]$$

Let $\Pi_t = \frac{P_t}{P_{t-1}}$ be the gross inflation rate. We can rewrite the Euler equation as:

$$\frac{u'(C_t)}{\Pi_t} = \beta(1+i_t)\mathbb{E}_t^\theta \left[\frac{u'(C_{t+1})}{\Pi_t \; \Pi_{t+1}} \right]$$

substitute the functional form for $u(C_t)$ and log-linearize the equation around the deterministic steady state ($\Pi = 1$, $\beta(1+i) = 1$). Hat-variables in small-cases denote log-deviation from steady state, except for inflation where we denote log-deviations with π_t

$$-\pi_t - \hat{c}_t = \hat{i}_t + \mathbb{E}_t^\theta \left[-\hat{c}_{t+1} - \pi_t - \pi_{t+1} \right]$$

Use the resource constraint: $\hat{y}_t = \hat{c}_t$, to get

$$\hat{y}_t = \mathbb{E}_t^\theta \left[\hat{y}_{t+1} + \pi_{t+1} + \pi_t \right] - \pi_t - \hat{i}_t$$

Using additivity, rearranging, and using the fact that

$$\mathbb{E}_t^\theta[\pi_t] = E_t[\pi_t] + \theta(E_t[\pi_t] - E_{t-1}[\pi_t]) = \pi_t + \theta(\pi_t - E_{t-1}[\pi_t])$$

which follows from Lemma 2, we obtain the equation in the body:

$$\hat{y}_t = \mathbb{E}_t^\theta \left[\hat{y}_{t+1} \right] - (\hat{i}_t - \mathbb{E}_t^\theta[\pi_{t+1}]) + \theta(\pi_t - E_{t-1}[\pi_t])$$

C.2 Firms

Monopolistically competitive firms, indexed by $j \in [0, 1]$, produce a differentiated good, $Y_t(j)$. We assume a Dixit-Stiglitz aggregator that aggregates intermediate goods into a final good, Y_t. Intermediate goods demand given by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_p} Y_t$$

where $\varepsilon_p > 1$ is the elasticity of substitution across intermediate goods' varieties, $P_t(j)$ is price of intermediate good j, and P_t is the price of final good Y_t. Each intermediate good is produced using the technology:

$$Y_t(j) = A_t L_t(j)$$

where $\log(A_t)$ is an aggregate TFP process that follows an AR(1) process with persistence coefficient ρ_A:

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t}$$

where $\epsilon_{A,t} \sim iid\ N(0, \sigma_A^2)$. Firm pays a quadratic adjustment cost in units of final good (Rotemberg 1982) to adjust prices:

$$\frac{\psi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t$$

Firm's per period profits are given by:

$$\Gamma_t \equiv P_t(j)Y_t(j) - W_t L_t(j) - \frac{\psi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t$$

Firm's profit maximization problem

$$\max_{P_t(j)} \left\{ P_t(j)Y_t(j) - W_t L_t(j) - \frac{\psi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t + \mathbb{E}_t^\theta \left[\sum_{s=1}^{\infty} \beta^s Q_{t,t+s}\Gamma_{t+s}\right]\right\}$$

where $Q_{t,t+s}$ is the nominal stochastic discount factor of the household. Substitute in the demand for intermediate goods to get:

$$\max_{P_t(j)} \left\{ P_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_p} Y_t - \frac{W_t}{A_t}\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_p} Y_t - \frac{\psi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t + \mathbb{E}_t^\theta \left[\sum_{s=1}^{\infty} \beta^s Q_{t,t+s}\Gamma_{t+s}\right]\right\}$$

Notice that $P_t(j)$ appears in period t profits and period $t+1$ adjustment costs. It doesn't appear anywhere else in the problem. So we can "ignore" the remaining terms as we take the first-order condition. The monopolistically competitive firm solves the following problem:

$$\max_{P_t(j)} \left\{ P_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_p} Y_t - \frac{W_t}{A_t}\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_p} Y_t - \frac{\psi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t - \mathbb{E}_t^\theta \left[\beta Q_{t,t+1}\frac{\psi_p}{2}\left(\frac{P_{t+1}(j)}{P_t(j)} - 1\right)^2 P_{t+1}Y_{t+1}\right]\right\}$$
$$+ \text{ other terms}$$

First order condition:

$$(1-\varepsilon_p)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_p} Y_t + \varepsilon_p \frac{W_t}{A_t P_t}\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_p - 1} Y_t - \psi_p\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)\frac{P_t}{P_{t-1}(j)} Y_t$$
$$-\psi_p \beta \mathbb{E}_t^\theta \left[\frac{u'(C_{t+1}}{u'(C_t)}\left(\frac{P_{t+1}(j)}{P_t(j)} - 1\right)\frac{P_{t+1}(j)}{P_t(j)}\frac{P_t}{P_t(j)} Y_{t+1}\right] = 0$$

Symmetry across all firms implies that reset price equals the aggregate price level.

Define $\Pi_t = \frac{P_t}{P_{t-1}}$

$$(1-\varepsilon_p)Y_t + \varepsilon_p \frac{W_t}{A_t P_t} Y_t - \psi_p(\Pi_t - 1)\Pi_t Y_t + \psi_p \beta \mathbb{E}_t^\theta \left[\frac{u'(C_{t+1})}{u'(C_t)} (\Pi_{t+1} - 1)\Pi_{t+1} Y_{t+1} \right] = 0$$

Divide by Y_t:

$$(1-\varepsilon_p) + \varepsilon_p \frac{W_t}{A_t P_t} - \psi_p(\Pi_t - 1)\Pi_t + \frac{\psi_p}{Y_t} \beta \mathbb{E}_t^\theta \left[\frac{u'(C_{t+1})}{u'(C_t)} (\Pi_{t+1} - 1)\Pi_{t+1} Y_{t+1} \right] = 0$$

Log-linearize around the deterministic steady state such that $A = 1$, $w = \frac{W}{P} = \omega C Y^\nu = \frac{\varepsilon_p - 1}{\varepsilon_p}$, $\Pi = 1$, and $Y_t = Y$. Let $w_t = \frac{W_t}{P_t}$

$$\varepsilon_p w(\hat{w}_t - \hat{a}_t) - \psi_p \pi_t + \psi_p \beta \mathbb{E}_t^\theta \pi_{t+1} = 0$$

Re arrange to get

$$\pi_t = \beta \mathbb{E}_t^\theta [\pi_{t+1}] + \frac{\varepsilon_p \ w}{\psi_p} (\hat{w}_t - \hat{a}_t)$$

From the intra-temporal labor supply first order condition, we have:

$$\hat{w}_t = \hat{c}_t + \nu(\hat{y}_t - \hat{a}_t)$$

Use the resource constraint $\hat{c}_t = \hat{y}_t$, to rewrite the new Keynesian Phillips Curve (NKPC):

$$\pi_t = \beta \mathbb{E}_t^\theta [\pi_{t+1}] + \frac{\varepsilon_p \ w}{\psi_p} (1+\nu)\hat{y}_t$$

Note that $\frac{\varepsilon_p \ w}{\psi_p} = \frac{\varepsilon_p - 1}{\psi_p}$. Then, the NKPC is given by

$$\pi_t = \beta \mathbb{E}_t^\theta [\pi_{t+1}] + \kappa(\hat{y}_t - \hat{a}_t)$$

where $\kappa \equiv \frac{\varepsilon_p - 1}{\psi_p}(1+\nu)$.

C.3 Policy Rule

The government sets nominal interest rate with the following rule:

$$\frac{1+i_t}{1+i_{ss}} = \Pi_t^{\phi_\pi} \left(\frac{Y_t}{Y_t^*} \right)^{\phi_x}$$

where $Y_t^* = A_t$ is the natural rate allocation, $i_{ss} = \frac{1}{\beta} - 1$ is the steady state nominal interest rate, $\phi_\pi \geq 0$, $\phi_x \geq 0$, and steady state inflation $\Pi = 1$. Log-linearized policy rule is given by:

$$\hat{i}_t = \phi_\pi \pi_t + \phi_x(\hat{y}_t - \hat{a}_t)$$

There is no government spending, and nominal bonds are in net zero supply.

C.4 Equilibrium

The log-linearized equilibrium in the simple new Keynesian model with diagnostic expectations is given by following three equations in three unknowns $\{\hat{y}_t, \pi_t, \hat{i}_t\}$ for a given shock process $\{\hat{a}_t\}$.

$$\hat{y}_t = \mathbb{E}_t^\theta\left[\hat{y}_{t+1}\right] - (\hat{i}_t - \mathbb{E}_t^\theta[\pi_{t+1}]) + \theta(\pi_t - E_{t-1}[\pi_t]) \tag{23}$$

$$\pi_t = \beta \mathbb{E}_t^\theta[\pi_{t+1}] + \kappa(\hat{y}_t - \hat{a}_t) \tag{24}$$

$$\hat{i}_t = \phi_\pi \pi_t + \phi_x(\hat{y}_t - \hat{a}_t) \tag{25}$$

where $\kappa \equiv \frac{\varepsilon_p - 1}{\psi_p}(1 + \nu)$, and the shock process is given by:

$$\hat{a}_t = \rho_A \hat{a}_{t-1} + \epsilon_{A,t}$$

where $\epsilon_{A,t} \sim iid\ N(0, \sigma_A^2)$.

D Proof of Proposition 4

The equilibrium with completely rigid prices, i.e. $\psi_p \to \infty$, given by:

$$\hat{y}_t = \mathbb{E}_t^\theta\left[\hat{y}_{t+1}\right] - \hat{i}_t \tag{26}$$

$$\hat{i}_t = \phi_x(\hat{y}_t - \hat{a}_t) \tag{27}$$

where $\hat{a}_t = \rho_A \hat{a}_{t-1} + \epsilon_t$, $\rho_A \in [0,1)$, and $\epsilon_{A,t} \sim iid\ N(0, \sigma_A^2)$. Substituting the policy rule into the Euler equation, we get:

$$\hat{y}_t = \frac{1}{1+\phi_x}\mathbb{E}_t^\theta\left[\hat{y}_{t+1}\right] + \frac{\phi_x}{1+\phi_x}\hat{a}_t$$

By forward iteration, and using the law of iterated expectations under the no-news assumption,

$$\hat{y}_t = \lim_{T\to\infty}\frac{\mathbb{E}_t^\theta\left[\hat{y}_{T+1}\right]}{(1+\phi_x)^{T+1}} + \sum_{i=1}^{\infty}\frac{\phi_x \mathbb{E}_t^\theta\left[\hat{a}_{t+i}\right]}{(1+\phi_x)^{i+1}} + \frac{\phi_x}{1+\phi_x}\hat{a}_t$$

The system is locally determinate if and only if $\phi_x > 0$. Let $\phi_x > 0$. Then,

$$\hat{y}_t = \sum_{i=1}^{\infty}\frac{\phi_x \mathbb{E}_t^\theta\left[\hat{a}_{t+i}\right]}{(1+\phi_x)^{i+1}} + \frac{\phi_x}{1+\phi_x}\hat{a}_t$$

From the definition of the shock process, we know that, $\forall\ i > 0$

$$\mathbb{E}_t^\theta\left[\hat{a}_{t+i}\right] = \rho_A^i(1+\theta)\hat{a}_t - \theta\rho_A^{i+1}\hat{a}_{t-1} = \rho_A^i\left((1+\theta)\hat{a}_t - \theta\rho_A\hat{a}_{t-1}\right)$$

We can then derive the solution for output:

$$\hat{y}_t = \frac{\phi_x\rho_A(1+\theta) + \phi_x(1+\phi_x-\rho_A)}{(1+\phi_x)(1+\phi_x-\rho_A)}\hat{a}_t - \frac{\phi_x\theta\rho_A^2}{(1+\phi_x)(1+\phi_x-\rho_A)}\hat{a}_{t-1}$$

The solution for output gap $\hat{x}_t \equiv \hat{y}_t - \hat{a}_t$ is given by:

$$\hat{x}_t = \frac{-\rho_A(1-\rho_A)(1+\phi_x)}{(1+\phi_x)(1+\phi_x-\rho_A)}\hat{a}_{t-1} + \frac{\theta\phi_x\rho_A - (1-\rho_A)(1+\phi_x)}{(1+\phi_x)(1+\phi_x-\rho_A)}\epsilon_t$$

In response to an unanticipated improvement in productivity, output gap can be positive on impact if and only

$$\theta\phi_x\rho_A - (1-\rho_A)(1+\phi_x) > 0$$

When $\theta = 0$, that is rational expectations, output gap negatively co-moves with productivity shock. Under diagnostic expectations, productivity improvements can be expansionary on impact.

Volatility of output gap is given by:

$$Var(\hat{x}_t) = \left(\frac{\rho_A(1-\rho_A)(1+\phi_x)}{(1+\phi_x)(1+\phi_x-\rho_A)}\right)^2 Var(\hat{a}_{t-1}) + \left(\frac{\theta\phi_x\rho_A - (1-\rho_A)(1+\phi_x)}{(1+\phi_x)(1+\phi_x-\rho_A)}\right)^2 \sigma^2$$

The first coefficient is same under rational and diagnostic expectations. Volatility is higher under diagnostic expectations relative to rational expectations if and only if

$$\begin{aligned}
&(\theta\phi_x\rho_A - (1-\rho_A)(1+\phi_x))^2 > (1-\rho_A)^2(1+\phi_x)^2 \\
\iff &(\theta\phi_x\rho_A)^2 + (1-\rho_A)^2(1+\phi_x)^2 - 2\theta\phi_x\rho_A(1-\rho_A)(1+\phi_x) > (1-\rho_A)^2(1+\phi_x)^2 \\
\iff &(\theta\phi_x\rho_A)^2 > 2\theta\phi_x\rho_A(1-\rho_A)(1+\phi_x) \\
\iff &\theta\phi_x\rho_A > 2(1-\rho_A)(1+\phi_x) \\
\iff &\theta > \frac{2(1-\rho_A)(1+\phi_x)}{\phi_x\rho_A}
\end{aligned}$$

■

In this model, the condition for amplification also implies that output gap is positive on impact from an unanticipated productivity improvement. When $\rho_A = 0.9$, and $\phi_x = 0.5$, the RHS is equal to 0.67. That is, for values of $\theta > 0.67$, the completely-rigid prices model yields amplification under diagnostic expectations.

E Equilibrium Conditions for the Medium-Scale DSGE Model

We summarize the equilibrium conditions for the medium-scale DSGE model presented in Section 3.1.

E.1 Stationary allocation

We normalize the following variables :

$$y_t = Y_t/Z_t\,,$$

$$c_t = C_t/Z_t\,,$$

$$k_t = K_t/Z_t\,,$$

$$k_t^u = K_t^u/Z_{t-1}\,,$$

$$\mathbb{I}_t = I_t/Z_t\,, \quad \text{capital investment}\,,$$

$$w_t = W_t/(Z_t P_t)\,,$$

$$r_t^k = R_t^k/P_t\,,$$

$$\lambda_t = \Lambda_t Z_t\,,$$

Definition 1 (Normalized equilibrium) *17 endogenous variables* $\{\lambda_t,\, i_t,\, c_t,\, y_t,\, \Pi_t,\, mc_t,\, \tilde{\Pi}_{t-1},\, \Pi_t^w,\, \tilde{\Pi}_{t-1}^w,\, w_t,\, L_t,\, k_{t+1}^u,\, r_t^K,\, \mathbb{I}_t,\, q_t,\, u_t,\, k_t\}$*, 5 endogenous shock processes* $\{G_{Z,t},\, A_t,\, \lambda_t^g,\, \mu_t,\, \eta_t\}$*, 6 exogenous shocks* $\{\epsilon_{g,t}, \epsilon_{A,t}, \epsilon_{mp,t}, \epsilon_{Z,t}, \epsilon_{\mu,t}, \epsilon_{\eta,t}\}$ *given initial values of* k_{t-1}^u*, and natural rate allocation* $\{y_t^*\}$*.*

Consumption Euler equation

$$\frac{\lambda_t}{G_{Z,t}\Pi_t} = \beta(1+i_t)\eta_t \mathbb{E}_t^\theta \left[\frac{\lambda_{t+1}}{G_{Z,t}G_{Z,t+1}} \frac{1}{\Pi_t \Pi_{t+1}}\right]\,, \tag{28}$$

$$\lambda_t = \frac{1}{c_t - \frac{hc_{t-1}}{G_{Z,t}}}\,, \tag{29}$$

Price-setting

$$(1-\varepsilon_p)+\varepsilon_p\ mc_t-\psi_p\left(\frac{\Pi_t}{\tilde{\Pi}_{t-1}}-1\right)\frac{\Pi_t}{\tilde{\Pi}_{t-1}}+\psi_p\frac{\beta\Pi_t}{\Lambda_t Y_t}\mathbb{E}_t^\theta\left[\Lambda_{t+1}\left(\frac{\Pi_{t+1}}{\tilde{\Pi}_t}-1\right)\frac{\Pi_{t+1}}{\tilde{\Pi}_t}\frac{Y_{t+1}}{\Pi_t}\right]=0 \tag{30}$$

$$\tilde{\Pi}_{t-1}=\bar{\Pi}^{1-\iota_p}\Pi_{t-1}^{\iota_p} \tag{31}$$

Wage-setting

$$\psi_w\left[\frac{\Pi_t^w}{\tilde{\Pi}_{t-1}^w}-1\right]\frac{\Pi_t^w}{\tilde{\Pi}_{t-1}^w}=\psi_w\beta\mathbb{E}_t^\theta\left[\frac{\Pi_{t+1}^w}{\tilde{\Pi}_t^w}-1\right]\frac{\Pi_{t+1}^w}{\tilde{\Pi}_t^w}+L_t\lambda_t\varepsilon_w\left[\omega\frac{L_t^\nu}{\lambda_t}-\frac{\varepsilon_w-1}{\varepsilon_w}w_t\right] \tag{32}$$

$$\tilde{\Pi}_{t-1}^w=G_Z\bar{\Pi}^{1-\iota_w}\left(\exp(\epsilon_{Z,t})\Pi_{t-1}\right)^{\iota_w} \tag{33}$$

$$\Pi_{W,t}=\frac{w_t}{w_{t-1}}\Pi_t G_{Z,t}\,, \tag{34}$$

Capital investment

$$k_{t+1}^u=\mu_t\left[1-S\left(\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}}\frac{G_{Z,t}}{G_Z}\right)\right]\mathbb{I}_t+(1-\delta_k)\frac{k_t^u}{G_{Z,t}}\,, \tag{35}$$

$$q_t=\frac{\beta G_{Z,t}}{\lambda_t}\mathbb{E}_t^\theta\left[\frac{\lambda_{t+1}}{G_{Z,t}G_{Z,t+1}}\left(r_{t+1}^K u_{t+1}-a(u_{t+1})+q_{t+1}(1-\delta_k)\right)\right]\,, \tag{36}$$

$$\begin{aligned}&q_t\mu_t\left[1-S\left(\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}}\frac{G_{Z,t}}{G_Z}\right)-S'\left(\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}}\frac{G_{Z,t}}{G_Z}\right)\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}}\frac{G_{Z,t}}{G_Z}\right]\\+&\frac{\beta G_{Z,t}}{\lambda_t}\mathbb{E}_t^\theta\left[\mu_{t+1}\frac{\lambda_{t+1}}{G_{Z,t}}q_{t+1}\frac{G_{Z,t+1}}{G_Z}\left(\frac{\mathbb{I}_{t+1}}{\mathbb{I}_t}\right)^2S'\left(\frac{\mathbb{I}_{t+1}}{\mathbb{I}_t}\frac{G_{Z,t+1}}{G_Z}\right)\right]=1\end{aligned} \tag{37}$$

Capital utilization rate

$$k_t=u_t\frac{k_t^u}{G_{Z,t}}\,, \tag{38}$$

$$r_t^K=a'(u_t)\,, \tag{39}$$

Production technologies

$$y_t=k_t^\alpha(A_tL_t)^{1-\alpha}\,, \tag{40}$$

$$\frac{K_t}{L_t}=\frac{w_t}{r_t^k}\frac{\alpha}{1-\alpha}\,, \tag{41}$$

$$mc_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(r_t^k)^\alpha w_t^{1-\alpha}}{A_t^{1-\alpha}}\,, \tag{42}$$

Government

$$\frac{1+i_t}{1+i_{ss}} = \left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{y_t}{y_t^*}\right)^{\phi_x} \left(\frac{y_t G_{Z,t}}{G_Z y_{t-1}}\right)^{\phi_{dy}}\right]^{1-\rho_R} \exp(\epsilon_{mp,t})\,, \tag{43}$$

Market clearing

$$y_t = c_t + \mathbb{I}_t + a(u_t)\frac{k_t^u}{G_{Z,t}} + \left(1 - \frac{1}{\lambda_t^g}\right) y_t\,, \tag{44}$$

Law of motion of Shocks

$$\log \lambda_t^g = (1-\rho_g)\log \lambda^g + \rho_g \log \lambda_{t-1}^g + \epsilon_{g,t}, \tag{45}$$

$$\log \eta_t = \rho_\eta \log(\eta_{t-1}) + \epsilon_{\eta,t}, \tag{46}$$

$$G_{Z,t} = Z_t / Z_{t-1} = G_Z \exp(\epsilon_{Z,t}), \tag{47}$$

$$\log \mu_t = \rho_\mu \log(\mu_{t-1}) + \epsilon_{\mu,t}, \tag{48}$$

Disturbances

$$\text{Monetary Policy} \quad \epsilon_{mp,t} \sim iid\ N(0, \sigma_{mp}^2) \tag{49}$$

$$\text{TFP growth} \quad \epsilon_{Z,t} \sim iid\ N(0, \sigma_Z^2), \tag{50}$$

$$\text{Stationary TFP} \quad \epsilon_{A,t} \sim iid\ N(0, \sigma_A^2), \tag{51}$$

$$\text{Risk Premium} \quad \epsilon_{\eta,t} \sim iid\ N(0, \sigma_\eta^2) \tag{52}$$

$$\text{MEI shock} \quad \epsilon_{\mu,t} \sim iid\ N(0, \sigma_\mu^2) \tag{53}$$

$$\text{Govt Spending} \quad \epsilon_{g,t} \sim iid\ N(0, \sigma_g^2) \tag{54}$$

E.2 Steady state

$$1 = \beta \frac{1}{G_Z} \frac{1+i}{\Pi}\,,$$

$$\lambda = \frac{G_Z}{c(G_Z - h)}\,,$$

$$mc = \frac{\varepsilon_p}{\varepsilon_p - 1}\,,$$

$$\frac{\omega L^{\nu}}{\lambda} = \frac{\varepsilon_w - 1}{\varepsilon_w} w\,,$$

$$\Pi^w = \Pi G_Z\,,$$

$$\Pi = \bar{\Pi}$$

$$q = 1\,,$$

$$u = 1\,,$$

$$(1 - \frac{1-\delta_k}{G_Z})k^u = \mathbb{I}\,,$$

$$1 = \beta \left[\frac{1}{G_Z} \left(r^K + (1-\delta_k) \right) \right]\,,$$

$$k = \frac{k^u}{G_Z}\,,$$

$$r^K = a'(1)\,,$$

$$y = k^{\alpha} L^{1-\alpha}\,,$$

$$r^k = \frac{\varepsilon_p}{\varepsilon_p - 1} \alpha \frac{y}{k}\,,$$

$$w = \frac{\varepsilon_p}{\varepsilon_p - 1} (1-\alpha) \frac{y}{L}\,,$$

$$y = c + \mathbb{I} + \left(1 - \frac{1}{\lambda^g} \right) y\,,$$

$$S(1) = S'(1) = 0; S" > 0$$

$A = 1.$

E.3 Log-linearized model

Consumption Euler equation

$$\hat{\lambda}_t - \hat{G}_{Z,t} - \pi_t = \hat{i}_t + \hat{\eta}_t + \mathbb{E}_t^{\theta} \left[\hat{\lambda}_{t+1} - \hat{G}_{Z,t} - \hat{G}_{Z,t+1} - \pi_t - \pi_{t+1} \right] \tag{55}$$

$$\hat{\lambda}_t + \frac{G_Z}{G_z - h}\hat{c}_t - \frac{h}{G_z - h}\left(\hat{c}_{t-1} - \hat{G}_{Z,t}\right) = 0 \tag{56}$$

Price-setting

$$\pi_t = \beta \mathbb{E}_t^\theta\left[\pi_{t+1}\right] - \iota_p \beta\left[\mathbb{E}_t^\theta \pi_t\right] + \iota_p \pi_{t-1} + \frac{\varepsilon - 1}{\psi_p}\hat{mc}_t \tag{57}$$

Wage-setting

$$\pi_t^w = \beta \mathbb{E}_t^\theta\left[\pi_{t+1}^w\right] - \iota_w \beta \mathbb{E}_t^\theta\left[\pi_t\right] - \iota_w \beta \mathbb{E}_t^\theta\left[\hat{G}_{Z,t+1}\right] + \iota_w \pi_{t-1} + \iota_w \hat{G}_{Z,t} + \frac{\varepsilon_w \omega L^{1+\nu}}{\psi_w}\left[\nu \hat{L}_t - \hat{w}_t - \hat{\lambda}_t\right] \tag{58}$$

$$\pi_t^w = \hat{w}_t - \hat{w}_{t-1} + \pi_t + \hat{G}_{Z,t} \tag{59}$$

Capital investment

$$\hat{k}_{t+1}^u = \frac{\mathbb{I}}{k^u}\left(\hat{I}_t + \hat{\mu}_t\right) + \frac{1 - \delta_k}{G_Z}\left(\hat{k}_t^u - \hat{G}_{Z,t}\right) \tag{60}$$

$$\hat{q}_t - \hat{G}_{Z,t} + \hat{\lambda}_t = \mathbb{E}_t^\theta\left[\hat{\lambda}_{t+1} - \hat{G}_{Z,t} - \hat{G}_{Z,t+1} + \frac{r^K}{r^K + 1 - \delta_k}\hat{r}_{t+1}^K + \frac{1 - \delta_k}{r^K + 1 - \delta_k}\hat{q}_{t+1}\right] \tag{61}$$

$$\hat{q}_t + \hat{\mu}_t - S"(1)\left(\hat{I}_t - \hat{I}_{t-1} + \hat{G}_{Z,t}\right) + \beta S"(1)\mathbb{E}_t^\theta\left[\hat{I}_{t+1} - \hat{I}_t + \hat{G}_{Z,t+1}\right] = 0 \tag{62}$$

Capital utilization rate

$$\hat{k}_t = \hat{u}_t + \hat{k}_t^u - \hat{G}_{Z,t} \tag{63}$$

$$\hat{r}_t^K = \frac{a"(1)}{a'(1)}\hat{u}_t\,, \tag{64}$$

Production technologies

$$\hat{y}_t = \alpha \hat{k}_t + (1 - \alpha)(\hat{A}_t + \hat{L}_t) \tag{65}$$

$$\hat{r}_t^K = \hat{w}_t + \hat{L}_t - \hat{k}_t \tag{66}$$

$$\hat{mc}_t = \alpha \hat{r}_t^K + (1 - \alpha)(\hat{w}_t - \hat{A}_t) \tag{67}$$

Government

$$\hat{i}_t = \rho_R \hat{i}_{t-1} + (1 - \rho_R)\left(\phi_\pi \pi_t + \phi_x(\hat{y}_t - \hat{y}_t^*) + \phi_{dy}(\hat{y}_t - \hat{y}_{t-1} + \hat{G}_{Z,t})\right) + \epsilon_{mp,t}\,, \tag{68}$$

Market clearing

$$\frac{1}{\lambda^g}\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{\mathbb{I}}{y}\hat{I}_t + \frac{a'(1)k}{y}\hat{u}_t + \frac{1}{\lambda^g}\hat{\lambda}_t^g \tag{69}$$

Law of motion of shocks

$$\hat{\lambda}_t^g = \rho_g \hat{\lambda}_{t-1}^g + \epsilon_{g,t}, \tag{70}$$

$$\hat{\eta}_t = \rho_\eta \hat{\eta}_{t-1} + \epsilon_{\eta,t}, \tag{71}$$

$$\hat{G}_{Z,t} = \epsilon_{Z,t}, \tag{72}$$

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \epsilon_{A,t} \tag{73}$$

$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \epsilon_{\mu,t}, \tag{74}$$

Disturbances

$$\text{Monetary Policy} \quad \epsilon_{mp,t} \sim iid\ N(0, \sigma_{mp}^2) \tag{75}$$

$$\text{TFP growth} \quad \epsilon_{Z,t} \sim iid\ N(0, \sigma_Z^2), \tag{76}$$

$$\text{Stationary TFP} \quad \epsilon_{A,t} \sim iid\ N(0, \sigma_A^2), \tag{77}$$

$$\text{Risk Premium} \quad \epsilon_{\eta,t} \sim iid\ N(0, \sigma_\eta^2) \tag{78}$$

$$\text{MEI shock} \quad \epsilon_{\mu,t} \sim iid\ N(0, \sigma_\mu^2) \tag{79}$$

$$\text{Govt Spending} \quad \epsilon_{g,t} \sim iid\ N(0, \sigma_g^2) \tag{80}$$

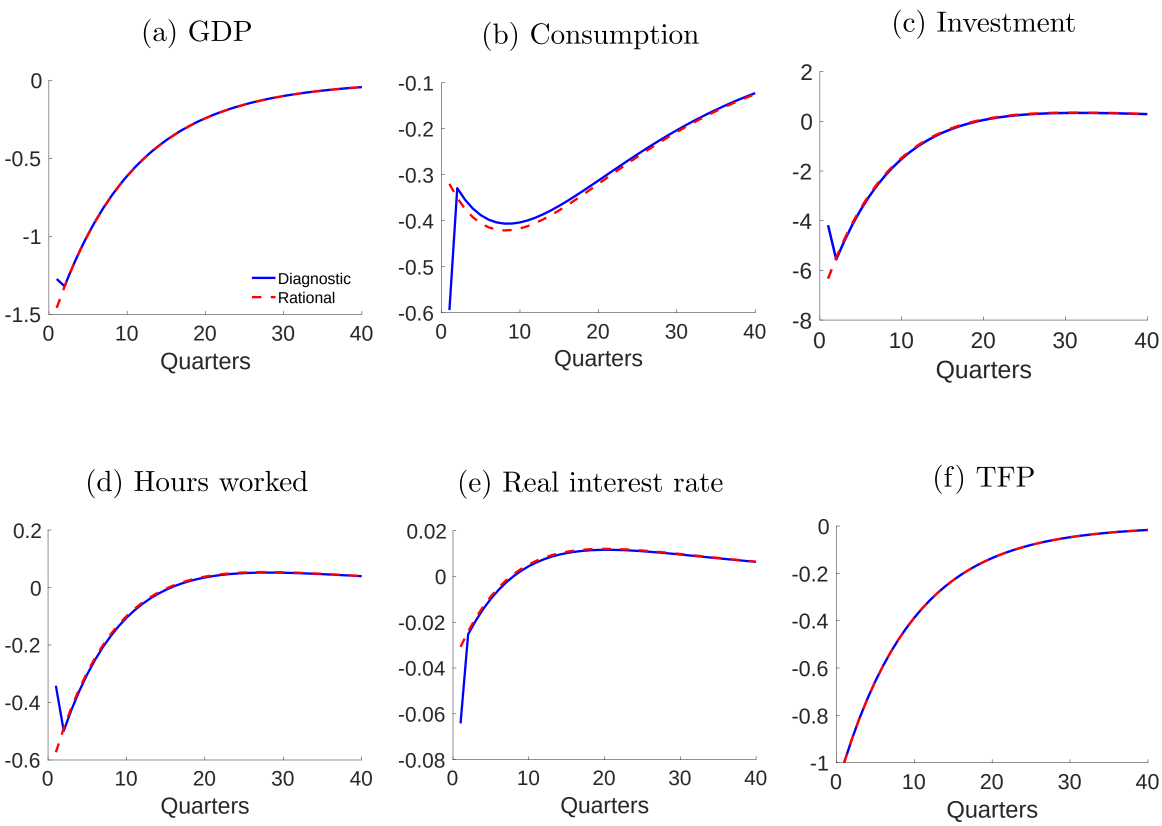
F Extra Tables and Figures

Table 3: Parameters

β	δ_k	α	$100\log(G_Z)$	$1-\frac{1}{\lambda_g}$	ω
Discount factor	Capital depreciation rate	Capital share	Trend growth rate	Government spending share	Labor preference
0.9984	0.025	0.20	0.50	0.20	1
ν	ε_p	ε_w	ψ_p	ψ_w	h
Inverse of Frisch elasticity	Elast of goods demand	Elast of labor demand	Price adjustment	Wage adjustment	(External) habit
2.05	6	6	102.25	5102.38	0.5
$\frac{a''(1)}{a'(1)}$	$S"(1)$	ϕ_π	ϕ_x	ϕ_{dy}	$100(\bar{\Pi}-1)$
Capital utilization cost	Investment adjustment cost	inflation coef Taylor rule	output gap coef Taylor rule	output growth coef Taylor rule	Inflation Target
5.27	3.51	1.78	0.07	0.57	0.64
ι_p	ι_w	θ			
Price indexation	Wage indexation	Diagnosticity parameter			
0.64	0.55	1			
Standard Deviation and Persistence of Shock Processes					
ρ_R	ρ_η	ρ_μ	ρ_g	ρ_Z	ρ_A
Persistence Taylor rule	Persistence η shock	Persistence μ shock	Persistence λ_g shock	Persistence Z shock	Persistence A shock
0.77	0.85	0.73	0.70	0	0.90
$100\sigma_{mp}$	$100\sigma_\eta$	$100\sigma_\mu$	$100\sigma_g$	$100\sigma_Z$	$100\sigma_A$
Std dev MP shock	Std dev η shock	Std dev μ shock	Std dev λ_g shock	Std dev Z shock	Std dev A shock
0.17	0.50	9.12	0.15	0.50	0.5

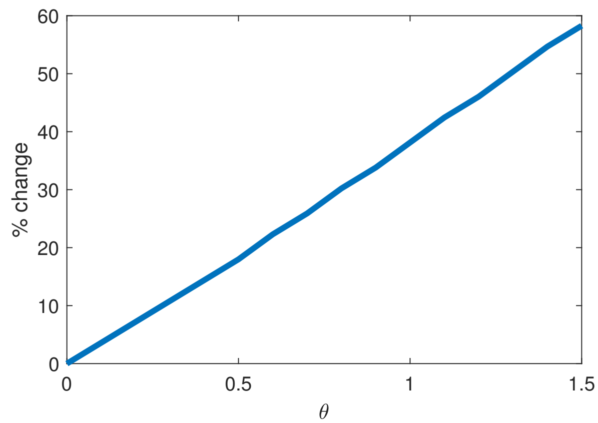
Notes: The table shows the parameter values of the model for the baseline calibration. Most parameters taken from Gust, Herbst, López-Salido, and Smith (2017). See Section 3.1 for details.

Figure 3: Impulse responses to a stationary TFP shock in the RBC model



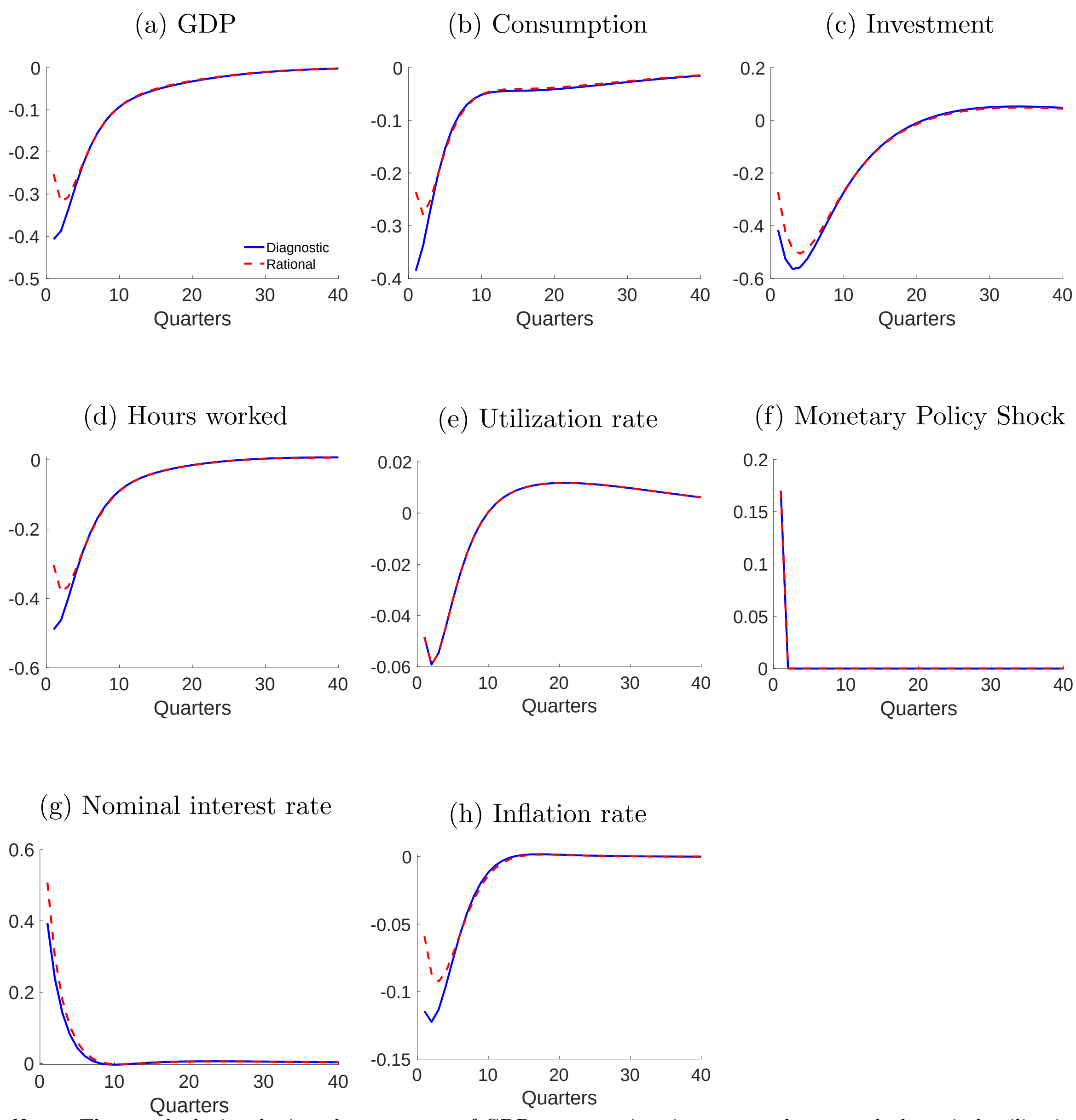
Notes: The panels depict the impulse responses of GDP, consumption, investment, hours worked, real interest rate and TFP shock ($\hat{a}$) to a unit shock to TFP, $\epsilon_{a,t}$. TFP shock process is given by equation 16. The blue solid lines denote impulses responses with diagnostic expectations, whereas the red dashed lines denote responses with rational expectations. See Table 3 for parameters corresponding to the RBC model, presented in Section 3.2.2.

Figure 4: The Degree of Extra Output Volatility and Diagnosticity



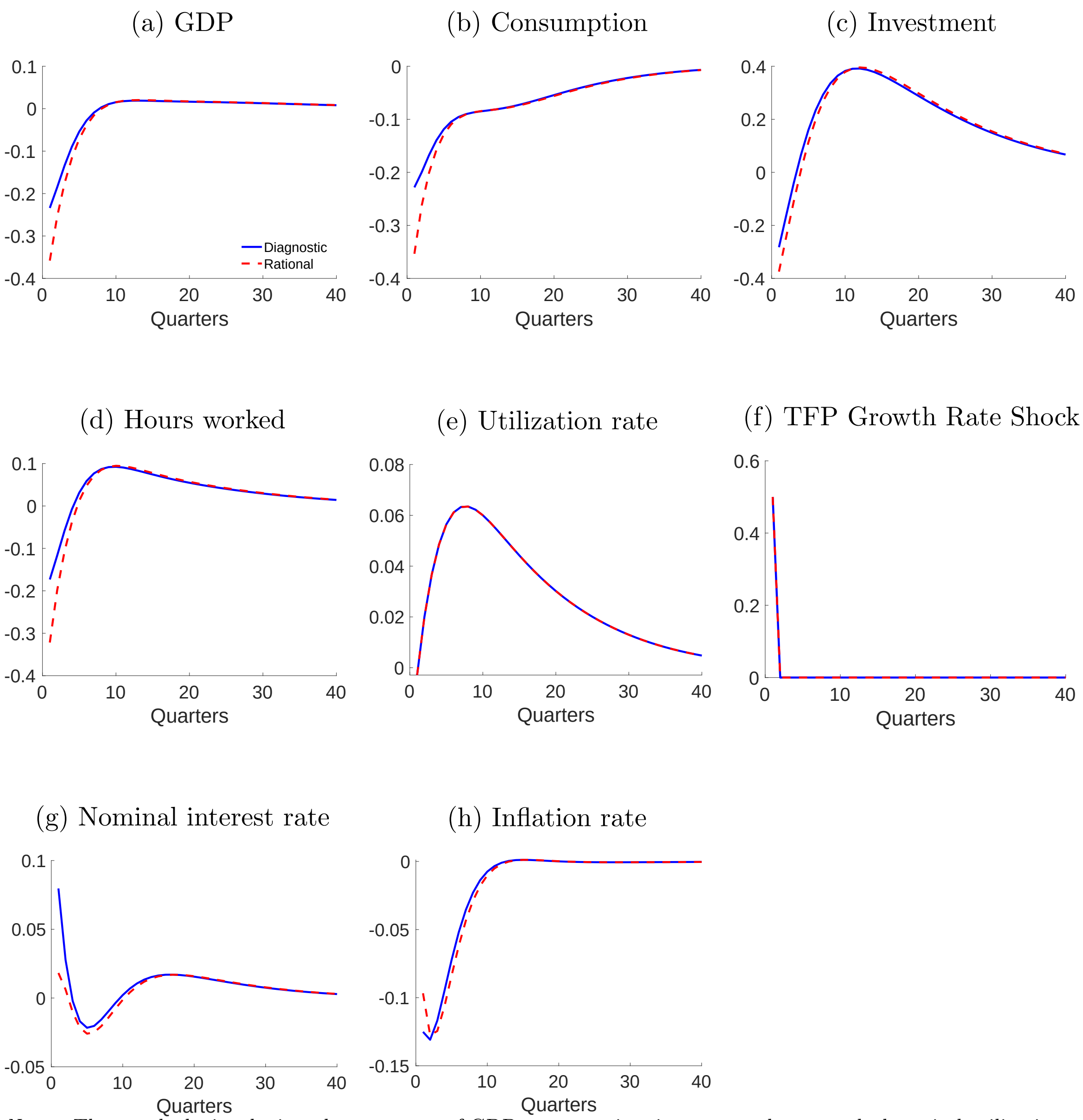
Notes: The panel depicts the relationship between extra output volatility and the diagnostic parameter θ. Extra output volatility is given by a percentage change in the standard deviation of output growth from the rational expectations benchmark to diagnostic expectations with given θ. We use the parameters in Table 3 and vary the parameter value θ from 0 to 1.5.

Figure 5: Impulse responses to a monetary policy shock ($\epsilon_{mp,t}$) in the DSGE model



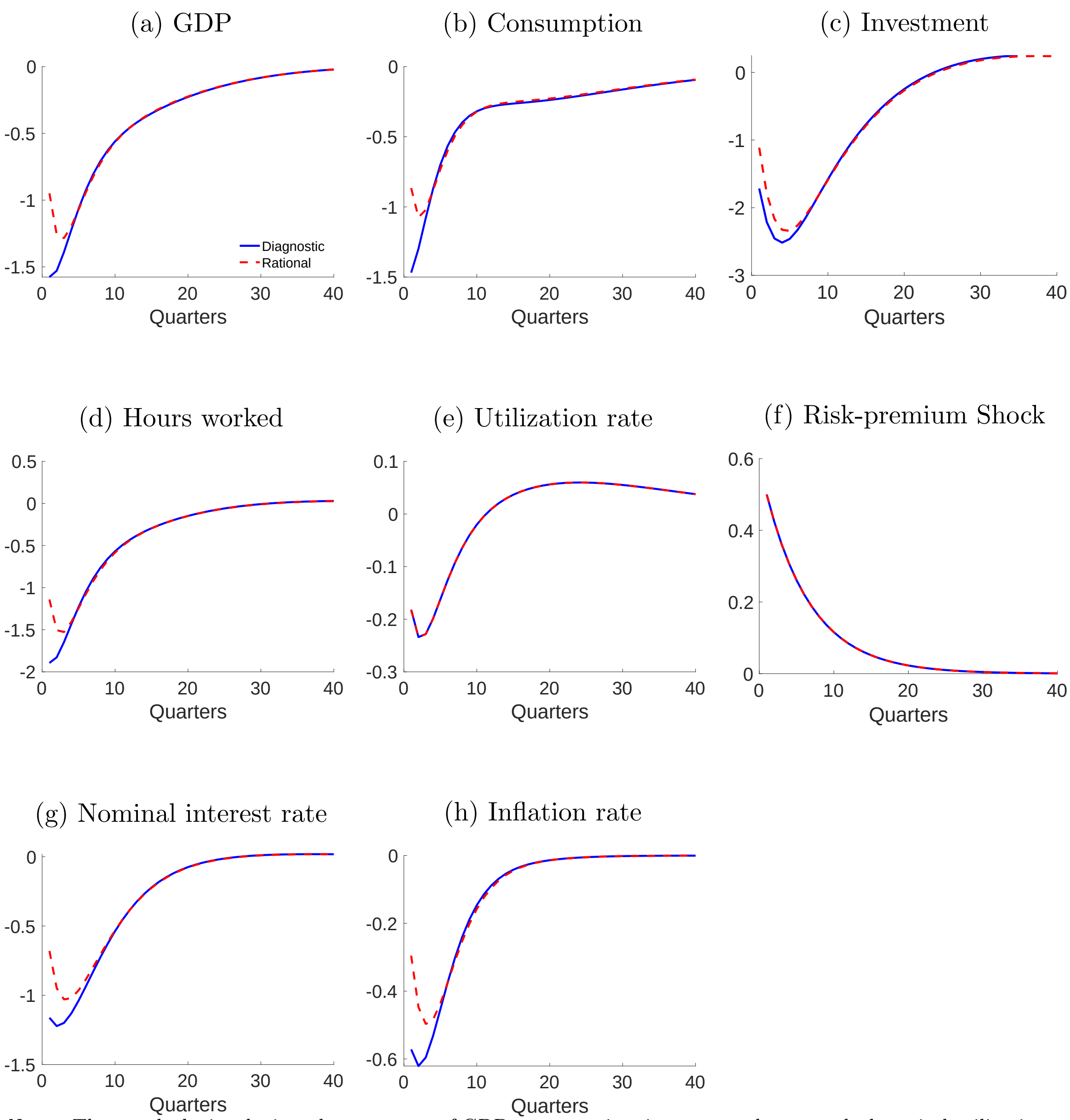
Notes: The panels depict the impulse responses of GDP, consumption, investment, hours worked, capital utilization rate, monetary policy shock ($\epsilon_{mp,t}$), annualized nominal interest rate, and annualized inflation rate to a one standard deviation shock to monetary policy ($\epsilon_{mp,t}$). The blue solid lines denote impulses responses with diagnostic expectations, whereas the red dashed lines denote responses with rational expectations. See Table 3 for parameters. See Section 3.1 for details.

Figure 6: Impulse responses to a TFP growth rate shock ($\hat{G}_{Z,t}$) in the DSGE model



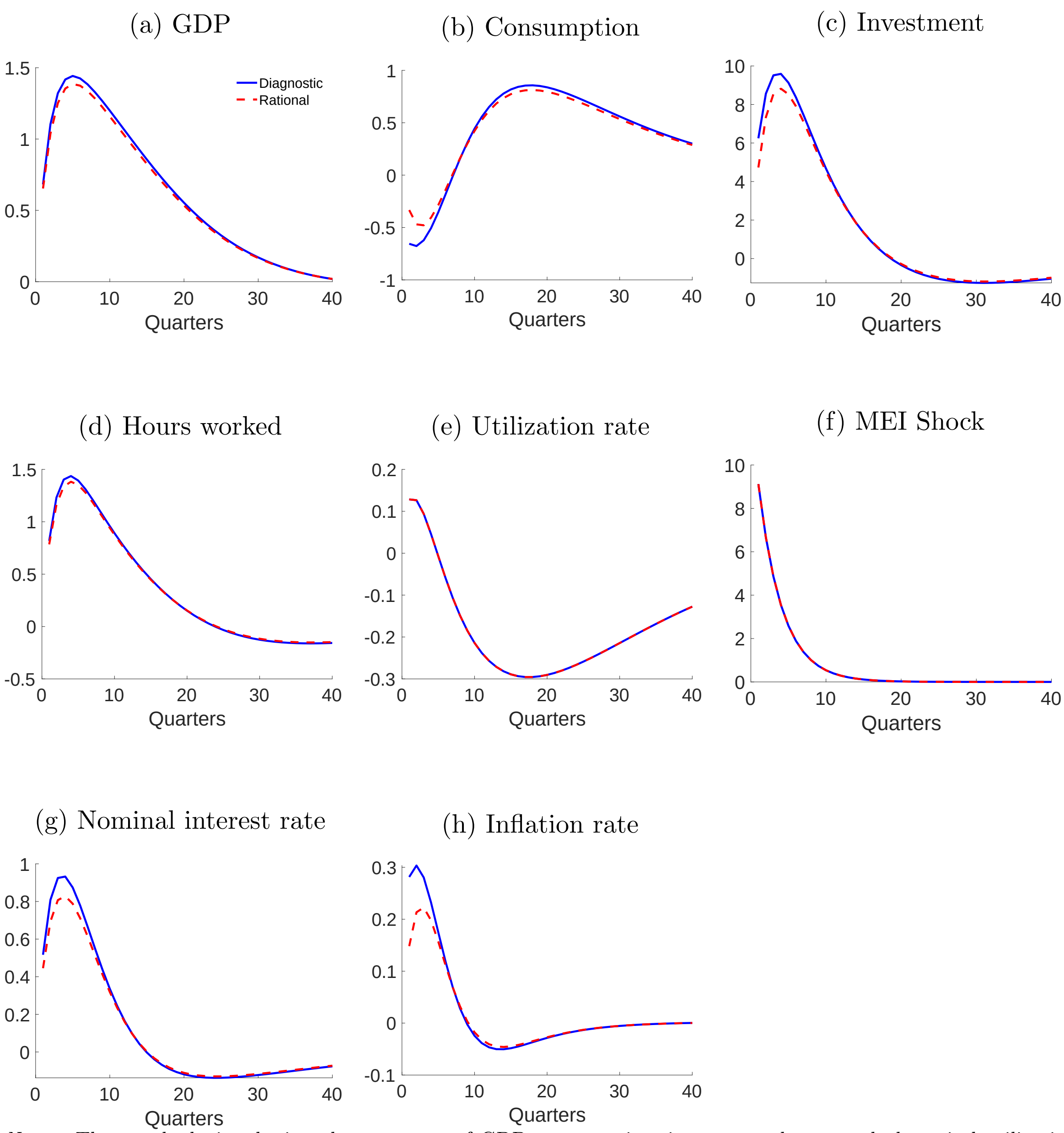
Notes: The panels depict the impulse responses of GDP, consumption, investment, hours worked, capital utilization rate, TFP growth rate shock $\hat{G}_{Z,t}$, annualized nominal interest rate, and annualized inflation rate to a one standard deviation shock to TFP growth rate, $\hat{G}_{Z,t}$. The blue solid lines denote impulses responses with diagnostic expectations, whereas the red dashed lines denote responses with rational expectations. See Table 3 for parameters. See Section 3.1 for details.

Figure 7: Impulse responses to a risk-premium shock ($\hat{\eta}$) in the DSGE model



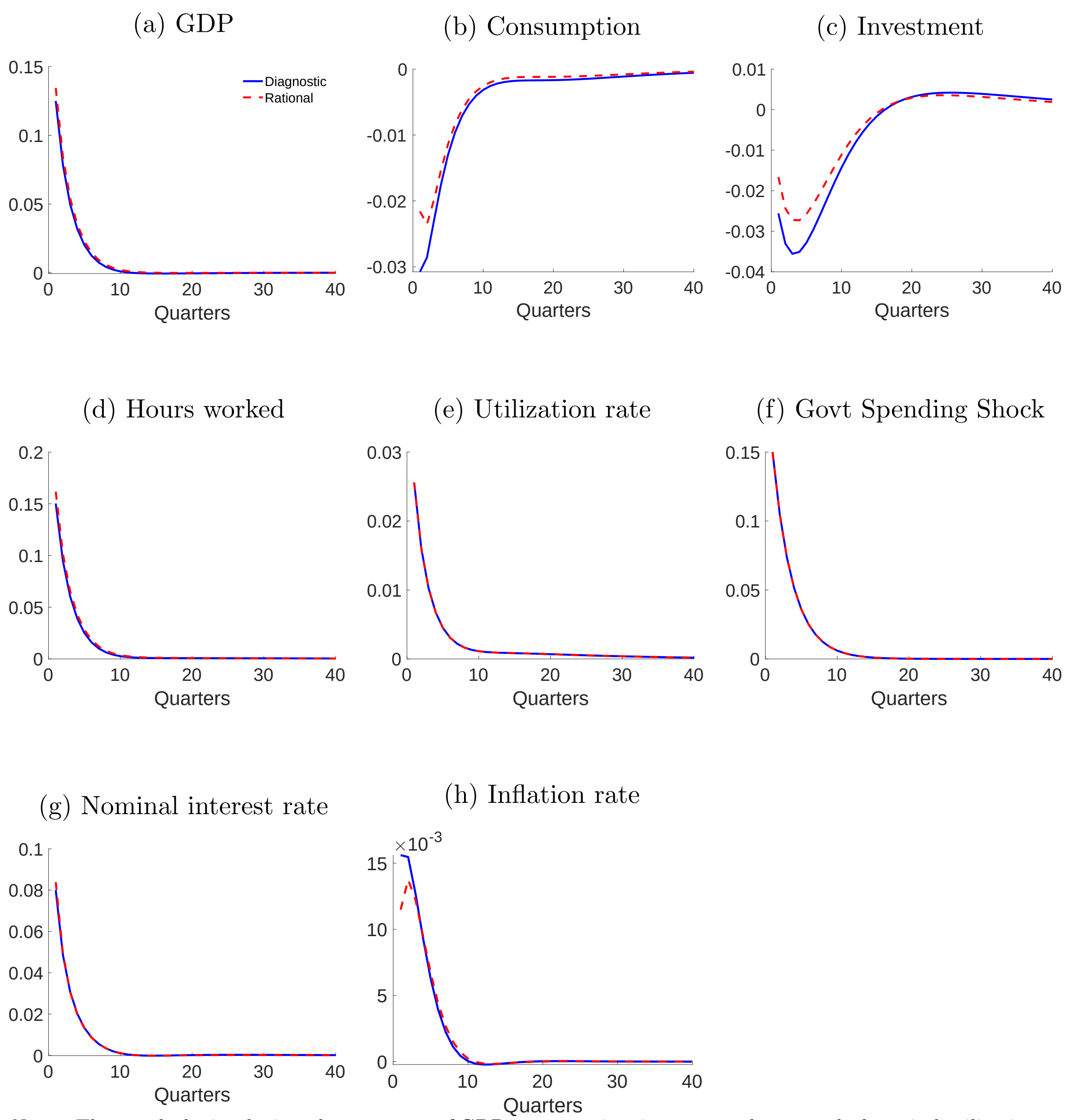
Notes: The panels depict the impulse responses of GDP, consumption, investment, hours worked, capital utilization rate, risk-premium shock $\hat{\eta}$, annualized nominal interest rate, and annualized inflation rate to a one standard deviation shock to risk-premium, $\epsilon_{\eta,t}$. The blue solid lines denote impulses responses with diagnostic expectations, whereas the red dashed lines denote responses with rational expectations. See Table 3 for parameters. See Section 3.1 for details.

Figure 8: Impulse responses to a marginal efficiency of investment (MEI) shock ($\hat{\mu}$) in the DSGE model



Notes: The panels depict the impulse responses of GDP, consumption, investment, hours worked, capital utilization rate, marginal efficiency of investment (MEI) shock ($\hat{\mu}$) , annualized nominal interest rate, and annualized inflation rate to a one standard deviation shock to MEI, $\epsilon_{\mu,t}$. The blue solid lines denote impulses responses with diagnostic expectations, whereas the red dashed lines denote responses with rational expectations. See Table 3 for parameters. See Section 3.1 for details.

Figure 9: Impulse responses to a govt spending shock ($\hat{\lambda}_g$) in the DSGE model



Notes: The panels depict the impulse responses of GDP, consumption, investment, hours worked, capital utilization rate, govt spending shock ($\hat{\lambda}_g$) , annualized nominal interest rate, and annualized inflation rate to a one standard deviation shock to govt spending, $\epsilon_{g,t}$. The blue solid lines denote impulses responses with diagnostic expectations, whereas the red dashed lines denote responses with rational expectations. See Table 3 for parameters. See Section 3.1 for details.