Directed Search on a Platform: Meet Fewer to Match More?*

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Abstract

This paper studies a directed search equilibrium in a platform setting with homogeneous buyers and sellers.We show that a *meeting* technology, typically controlled by intermediaries, (e.g., advertisement, interview scheduling, or online search protocol) determines the *matching* outcome as follows. First, a meeting technology that provides full information to market participants is not necessarily efficient. Second, the seller- and buyer-optimal meeting technologies do not require full market transparency either; rather, the latter may be achieved even with the minimum information. Finally, the efficient matching outcome can be decentralized by a profit-maximizing platform who adopts a simple fee-setting policy for its intermediation service.

Keywords: meeting technology, directed search, platform, intermediation

JEL Classification: D83, J64, M37.

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1 Introduction

It is widely recognized that an increasing market transparency leads to better allocations in the absence of search frictions because it helps approximate a competitive market. However, significant search friction exists even in the internet markets which are supposed to exhibit much lower search costs than in the offline markets. This paper aims to study the role of market transparency on efficiency and surplus division in an environment characterized by search frictions. Does allowing one side of the market to observe more options on the other side always help generate matches in a frictional market? Why do some intermediaries deliberately restrict users' access to potential trading partners? If intermediaries do so, will it lead to inefficiency? We provide one possible answer to these questions by studying how meeting technologies adopted by platforms affect matching efficiency, seller profit, and buyer surplus.

In many search/matching markets, a successful transaction between a buyer and a seller takes several steps to realize. First, the buyer needs to be aware of the seller, possibly together with the price and the product features. We refer to this step as *meeting*.[1] Second, if the buyer meets multiple sellers, she needs to decide which seller to visit and buy the product. This is the step of *search*. Finally, if a seller encounters multiple buyers but only has limited capacity, he can only select a subset of buyers to trade with. This step is the *matching* between buyers and sellers. We study how the information transparency implied by the meeting technology affects buyers' search behavior and eventually the matching outcome in a directed search market.

Consider an online platform for service workers such as plumbers, carpenters, or cleaners. A customer, who wishes to find such a service worker, lodge in a request on the platform, and then the platform forwards the request to a limited number of workers. It is often the case that the platform has a large database of qualified workers but only allows two or three to contact the customer. Each worker only has limited availability in terms of working hours and needs to decide which customer to contact. With a lack of coordination among workers over which customer to contact, several workers might contact the same customer, in which case they need to compete for the customer's

[1] The terminology "meeting" has been used in the literature of directed search (e.g. Eeckhout and Kircher, 2010). See Section 6.1 for a discussion of the relation between our meeting technology to the ones adopted in Eeckhout and Kircher, 2010).

request. This leads to the coexistence of unmatched customers and unmatched workers. Our model captures all the key elements of this example and explores the impact of limiting the number of workers that a customer can contact.

In particular, we assume that there exists a continuum of product categories on a monopoly platform and in each category two homogeneous sellers (i.e. customers in the example), each holding one unit of the product, compete by setting reserve prices. A meeting technology specifies the number of buyers (i.e. workers in the example) a seller can meet. A buyer can visit a seller only if she observes this seller, and if more than one buyer visit the same seller then the seller auctions off the product.

In each product category, we show that there will be two matches, which is the most efficient outcome, if information spreads appropriately, i.e., if there are two partially informed buyers, with one buyer only observing seller 1 and the other buyer only observing seller 2. An unmatched seller exists if: (i) all buyers are fully informed; (ii) all the fully informed buyers select the same seller. These two conditions jointly create the matching friction in this market. We refer to the probability of (i) as the *extensive margin* of frictions and the probability of (ii) as the *intensive margin* of frictions.

An increase in the meeting transparency, measured by the number of buyers a seller can meet, can have a non-monotonic effect on matching efficiency. On the one hand, a higher degree of transparency always decreases the intensive margin of frictions, thereby enhancing the match efficiency. On the other hand, it decreases the extensive margin only if the initial meeting transparency is low. If the initial meeting transparency is not too low, a further increase in the transparency will increase the extensive margin. This is because in the latter situation, there are only few remaining uninformed buyers and so the additional meeting requests are likely to be sent to the same buyer, thereby increasing the total number of fully informed buyers. When both these effects are at work, the expected number of total matches is maximized not at the full transparency level but at an intermediate level of transparency.

We also characterize the optimal degree of meeting transparency for sellers and buyers respectively. Individual seller's profit is also maximized by an intermediate degree of transparency. Compared to the full transparency, an intermediate degree of transparency gives a higher probability of having partially informed buyers, which softens the competition between sellers and allows for a higher reserve price. Compared to the minimum degree of transparency, the probability of making sales under an intermediate degree of transparency is a lot higher. To the contrary, buyers' expected utilities are maximized at the minimum degree of meeting transparency. Although the price they expect to pay is very high in this situation, buyers can successfully trade for sure conditional on observing at least one seller.

Finally, we endogenize the meeting transparency. We show that a profit-maximizing platform implements the efficient meeting transparency. In each product category, the total volume of transactions is inelastic in fees, provided that agents are still willing to trade on the platform. Consequently, using a simple transaction fee, the platform will chooses the meeting technology that maximizes the total number of matches, which yields the efficient outcome. Our theory thus provides an explanation of why platforms often limit buyers' available options in search/matching markets.

Our paper is related to several strands of literature. First, a small stream in the directed search literature considers the implication of buyers' information acquisition. Peters (1984) considers costless advertising in a directed search framework. He studies the effect of a prohibition on sellers' advertisement on product-prices and finds that this type of restrictions may benefit high-cost sellers. Lester (2011) introduces buyers' heterogeneous search costs–some buyers with low search cost can observe all posted prices and other buyers with high search cost can only observe one price. He shows that an increase in the number of consumers with low search cost does not necessarily lower the equilibrium price. Gomis-Porqueras, Julien and Wang (2017) study costly advertisement in a directed-search framework. Due to the probabilistic nature of advertising, consumers become heterogeneous in terms of the number of prices they observe, similar as in Lester (2011). All these models do not explore the implication of the exact number of ads each seller sends, or the exact number of meetings each seller can schedule, which is the main focus of the current paper.

Second, fee-setting platforms have been systematically studied in the literature of two-sided markets (Armstrong, 2006, Caillaud and Jullien, 2003, and Rochet and Tirole, 2003, 2006 are the classics). It is widely recognized that platforms facilitate transactions between buyers and sellers. Frictions are, however, still present in many platforms and buyers need to search sellers and/or products. The sequential search framework with differentiated products developed by Wolinsky (1986) and Anderson and Renault (1999) has been imbedded into platform settings to investigate various issues such as search pool quality (Eliaz and Spiegler, 2011), platform targeting (de Corniere, 2016), search cost reduction (Wang and Wright, 2016), price parity clauses (Wang and Wright, 2020), and platform steering (Teh and Wright, 2020). Unlike these studies, we use the directed search approach that applies more appropriately to a different set of markets such as labor and real estate markets. Kennes and Schiff (2007) and Gautier, Hu and Watanabe (2019) also consider intermediaries' role in certifying and market-making in a directed search environment. Instead of addressing these issues, we introduce meeting technologies to the directed search environment on a platform and investigate the platform's optimal choice of the degree of meeting transparencies.

Third, our work is part of an emerging literature on information design by platforms. Johnson, Rhodes and Wildenbeest (2020) theoretically and experimentally study a policy of discriminating sellers on a platform in terms of sales promotion, and its effect on competition and the platform's profit. They show that this simple policy can be pro-competitive. Armstrong and Zhou (2020) introduce a private signal of consumers' preferences over products, and derive the optimal information structures, which can be chosen by a platform. Teh (2020) shows that whether the platform's information design is distorted towards insufficient or excessive seller competition, depends on the platform's fee structure. We study information design problems in a directed-search environment and show that the full information transparency is often suboptimal in forming matches. Our finding that the optimal degree of market transparency in a buyers-sellers equilibrium can be decentralized by a profit-maximizing platform is also new to this literature.

Fourthly, several papers, though studying very different mechanisms, find that restricting participants' choices can either improve matching efficiency or maximize a platform's profit. In the context of job network formation, Calvó-Armengo and Zenou (2005) show that having more contacts on average does not necessarily increase the job matching rate. When the network size grows large enough, the congestion effect will be sufficiently strong so that it becomes harder for workers to land a job. Instead of looking into network formation, we derive a matching function endogenously that is very different from the one used in Calvó-Armengo and Zenou (2005) by explicitly analyzing participants' strategic interactions in a finite market. In the industrial organization literature on platforms, Casadesus-Masanell and Halaburda (2014) study an application platform where users can not only enjoy application variety but also benefit from consumption complementarities. By limiting users' choices, the platform enables users to better coordinate consumption and therefore increase the platform value. Halaburda, Piskorski and Yildirim (2018) consider competing dating platforms. Users with low outside options have higher willingness to pay for a platform who restricts their choices because it allows them to match more quickly. Our model also exhibits within-side network effects but these externalities are results of a directed-search setup with transferrable utilities.

Finally, there are many other possibilities of how and why users' choices on a platform are limited. A lot of discussions in this direction can be found in the literature of operation research, information system or marketing. Kanoria and Saban (2017) considered a dynamic search market in which agents engage in costly search and the match values are pair-specific. In their models, both sides can screen players on the other side and make proposal to match. The authors found that the platform can mitigate wasteful competition in partner search via restricting what agents can see/do. Romanyuk (2017) also studied a dynamic search model in which the platform controls the information the sellers observe about the buyers before forming a match. The paper shows that full information disclosure is inefficient because of excessive rejections by sellers. Li and Netessine (2019) empirically studied a related problem but from a broader perspective—whether a higher market thickness increases the matching rate. By using data from an online peer-to-peer holiday property rental platform, they showed that doubling market size leads to a 5.6% reduction of matches.

The rest of the paper is organized as follows. Section 2 introduces the model and sets up a benchmark case with full meeting transparency. In Section 3, we first use a numerical example to illustrates the main idea and provide the full characterization of the equilibrium. Section 4 identifies the efficient degree of meeting transparency and the optimal degrees of meeting transparency for buyers and sellers respectively. We provide discussions on meeting technologies and an extension to more than two sellers in Section 6. Section 7 concludes. All the proofs, and some omitted details, are in the Appendix.

2 Basic setup and benchmark

We consider an economy with a mass of product categories. Each seller has only one product category. For simplicity, we assume that there are two sellers in each product category and categories are independent. The independence between categories captures the fact that, although platforms usually list many items (a continuum in our model), there is competition between only a few of them (see Karle, Peitz and Reisinger, 2020). Hence, we focus attention on a representative product category with two homogeneous sellers, indexed by $i = 1, 2$, and $B \geq 2 \in \mathbb{N}$ homogeneous buyers. Each seller has one unit of that product with zero production/inventory costs. The products's consumption value to sellers is normalized to one. The market is operated by a platform. The platform offers a meeting technology which facilitates the trades between buyers and sellers. The meeting technology determines a subset of buyers who can observe and trade with each individual seller (see below). For simplicity, we assume that buyers and sellers cannot trade without using the platform. We shall refer to a buyer who observes both sellers as a *fully informed* buyer, a buyer who observes only one seller as a *partially informed buyer*, and a buyer who does not observe any seller an *uninformed buyer*.

□ **Trading protocols**. Individual sellers offer a first-price auction to their buyers.[2] Each seller i posts a reserve price, denoted by r_i, $i = 1, 2$. Once observing those prices, buyers decide which seller to visit without coordination. Each buyer can visit only one seller. A buyer can visit a seller only if she observes this seller (see below). The reserve price r_i is honoured only when one buyer participates in the seller i's auction. If more than one buyer participate, buyers bid for trade. If multiple buyers submit the same bid then each of them obtain the product with equal probability. Modeling trading protocols using auction captures the idea that sellers only have limited commitment power with respect to the posted price.

When attending an auction, a buyer's bidding strategy depends on the posted reserve price, r_i, and the observed number of participants, denoted by n_i, $i = 1, 2$. Bertrand

[2] A second-price auction will yield the same outcome in this environment.

type of reasoning yields the optimal bidding strategy,

$$b(r_i, n_i) = \begin{cases} r_i & \text{if} \quad n_i = 1; \\ 1 & \text{if} \quad n_i > 1. \end{cases}$$

Seller i's realized profit is given by

$$\Pi_i(r_i, n_i) = \begin{cases} 0 & \text{if} \quad n_i = 0; \\ r_i & \text{if} \quad n_i = 1; \\ 1 & \text{if} \quad n_i > 1. \end{cases}$$

□ **Meeting technology/Transparency**. The meeting process between buyers and sellers is determined by the platform's meeting technologies. The key function of the meeting technology is to control market *transparency*, which is summarized by the number of buyers who observe an individual seller, denoted by N. We can also interpret N as the number of meetings a seller can schedule or the number of ads a seller can send. Obviously, $1 \leq N \leq B$. Both buyers and sellers take N as given. For the moment, we treat N as an exogenous parameter, but later we will allow the platform to determine N optimally. We assume the following properties of meeting technologies.

- **Symmetry:** Any two sellers are treated equally in reaching buyers.
- **Anonymity:** Any two buyers observe the same seller with equal probability.
- **No waste:** The N buyers who observe a seller are all distinct buyers.

Note that the "no waste" assumption excludes the advertising technology proposed in Butters (1977) since Butters allows a buyer to receive multiple ads from a single seller. One meeting technology we will repeatedly use throughout the paper can be motivated by the following example.

Example 1 (Scheduling job interviews)**.** *Consider two employers, each has one vacancy, and B job seekers. Each employer can schedule $N = \{1, 2, ..., B\}$ interviews with job seekers. The selection of job seekers to interview is at random. Each job seeker is scheduled at most one interview with each employer.*

With this meeting technology, each job seeker (buyer) receives an interview request from an employer (seller) with probability $\frac{N}{B}$. This meeting technology is the reminiscence of the Non-Frictional Matching technology where the short side of the matching market is always cleared (Stevens, 2007). We therefore refer to it as a **Non-Frictional Meeting (NFM) technology**.

Denote by $\Gamma(k|N,B)$ the probability of having $k = 0,...,N$ fully informed buyers (i.e., the probability that k buyers observe both sellers) when each individual seller is observed by N random buyers out of B buyers (i.e., when each individual seller is introduced to $N \leq B$ buyers). As we will see later, this probability function plays the key role in our analysis. We now compute $\Gamma(k \mid N, B)$ with the NFM technology. First, consider $\Gamma(N \mid N, B)$. To introduce seller 1 to N random buyers, there are in total $C_B^N = \frac{B!}{N!(B-N)!}$ cases. Similarly, to introduce seller 2 to N random buyers, there are in total C_B^N cases. On the other hand, to introduce both sellers 1 and 2 to N random buyers, there are in total C_B^N cases. Hence, the probability that both sellers 1 and 2 are introduced to N random buyers (i.e., the probability of having N fully informed buyers) is $\frac{C_B^N}{(C_B^N)^2} = \frac{1}{C_B^N}$, i.e.,

$$\Gamma(N|N,B) = \frac{1}{C_B^N}. \tag{1}$$

Next, we compute $\Gamma(N-1 \mid N, B)$. To have $N-1$ fully informed buyers, we must introduce both sellers to $N-1$ buyers, and simultaneously have one random buyer who observes seller 1 but not seller 2, and another random buyer who observes seller 2 but not seller 1. There are in total $C_B^{N-1} C_{B-N+1}^1 C_{B-N}^1$ such cases. Hence, the probability of having $N-1$ fully informed buyers is given by $\frac{C_B^{N-1} C_{B-N+1}^1 C_{B-N}^1}{(C_B^N)^2}$, which, by using $C_B^{N-1} = \frac{N}{B-N+1} C_B^N$, can be written as

$$\Gamma(N-1|N,B) = \frac{N(B-N)}{C_B^N}. \tag{2}$$

The following is another example that is consistent with our assumptions on meeting technologies.

Example 2 (Jointly displayed ads). *Suppose that the two sellers' ads are jointly displayed to randomly selected N out of B buyers. This is as if using a NFM technology but with sellers 1's and seller 2's ads as a bundle (i.e. the two ads are always shown together on the same page).*

We shall refer to this meeting technology a **Bundled Non-Frictional Meeting (BNFM)** technology. Under the BNFM technology, $\Gamma(N|N,B)=1$ and $\Gamma(k|N,B)=0$ for any $k<N$.

□ **Timing of the game**. The timing of the game is as follows.

- Stage 1: The platform sets fee(s) and a meeting technology summarized by N.
- Stage 2: Sellers and buyers decide whether to join the platform. Participating sellers set a reserve price.
- Stage 3: Participating buyers' information sets are realized. The buyers choose a seller to visit (if not uninformed).
- Stage 4: The chosen sellers and the informed buyers trade through auctions.

The equilibrium concept we use is Subgame Perfect Nash Equilibrium (SPNE).

□ **A benchmark with full transparency** $N=B$. We first establish a benchmark case with $N=B$, i.e., all buyers are fully informed, which corresponds to the setting in Julien, Kennes and King (2000, hereafter JKK). We use this benchmark case to explain the basic mechanics of buyers' directed search and illustrate what when only one type of friction (search friction) is present but the other friction (i.e. information friction rising from that not every buyer is fully informed) is absent.

We work backwards and start with buyers' directed search. Having observed the posted reserve prices, buyers decide simultaneously which seller to visit. Denote by σ_i the symmetric probability that a buyer selects seller i. Because a buyer can get a strictly positive payoff only when no other buyers visit the same seller, the expected payoff of visiting seller 1 is

$$u_1(r_1,r_2)=(1-r_1)(1-\sigma_1)^{B-1},$$

where $(1-\sigma_1)^{B-1}$ $(=\sigma_2^{B-1})$ is the probability that all other $B-1$ buyers visit seller 2 rather than seller 1. Similarly, the expected payoff of visiting seller 2 is

$$u_2(r_1,r_2)=(1-r_2){\sigma_1}^{B-1},$$

where σ_1^{B-1} $(=(1-\sigma_2)^{B-1})$ is the probability that all other $B-1$ buyers visit seller 1 rather than seller 2. The lack of coordination among buyers implies that they must use symmetric strategies in equilibrium. It has been shown in JKK that there is unique symmetric equilibrium strategy. We can pin down the equilibrium strategy $\sigma_1 = \sigma_1(r_1, r_2)$ using the indifference condition, $u_1(r_1, r_2) = u_2(r_1, r_2)$, which leads to

$$\sigma_1(r_1, r_2) = \frac{1}{1 + \left(\frac{1-r_2}{1-r_1}\right)^{\frac{1}{B-1}}}.$$

The visiting probability $\sigma_1(r_1, r_2)$ is decreasing in the reserve price r_1.

Given the buyers' directed search described above, we next study the sellers' problem. Seller 1's profit equals to zero if no buyer selects him, which occurs with probability $(1-\sigma_1)^B$, r_1 if only one buyer selects him, which occurs with probability $B\sigma_1(1-\sigma_1)^{B-1}$, and 1 otherwise. Thus, seller 1 solves

$$\max_{r_1} r_1 B\sigma_1(1-\sigma_1)^{B-1} + 1 - (1-\sigma_1)^B - B\sigma_1(1-\sigma_1)^{B-1},$$

where $\sigma_1 = \sigma_1(r_1, r_2)$ is decreasing in r_1 as shown above. Applying the first-order condition, we obtain the symmetric equilibrium reserve price, denoted by $r_{N=B}$ $(= r_1 = r_2)$,

$$r_{N=B} = \frac{B-1}{B}. \tag{3}$$

In what follows, we use the subscript $N = 1, .., B$ for equilibrium variables to index the meeting technology. The equilibrium matching rate for each individual seller is $1 - \left(\frac{1}{2}\right)^B$, because $\sigma_i = \frac{1}{2}$, $i = 1, 2$, in equilibrium. Each seller's equilibrium expected profit, denoted by $\pi_{N=B}$, is

$$\pi_{N=B} = 1 - \left(\frac{1}{2}\right)^{B-1}. \tag{4}$$

Let n_i be the number of buyers who visit seller i. Then, the equilibrium total number of matches, denoted by $T_{N=B}$, is

$$\begin{aligned} T_{N=B} &= \Pr.[n_1 \geq 1]\Pr.[n_2 = 0] + \Pr.[n_1 = 0]\Pr.[n_2 \geq 1] + 2 \cdot \Pr.[n_1 \geq 1]\Pr.[n_2 \geq 1] \\ &= 2\left(1 - \left(\frac{1}{2}\right)^B\right)\left(\frac{1}{2}\right)^B + 2\left(1 - \left(\frac{1}{2}\right)^B\right)^2 \\ &= 2\left(1 - \left(\frac{1}{2}\right)^B\right). \end{aligned} \tag{5}$$

A buyer's equilibrium expected payoff, denoted by $u_{N=B}$, is

$$u_{N=B} = (1 - r_{N=B})\left(\frac{1}{2}\right)^{B-1} = \frac{1}{B}\left(\frac{1}{2}\right)^{B-1}. \tag{6}$$

3 Equilibrium characterization

In this section, we allow for $N < B$ and examine the effect of imperfect transparency on the equilibrium. In Section 3.1, we use a numerical example with $B = 3$ to illustrate the basic intuition. It turns our that the equilibrium involves sellers' using symmetric mixed strategy when $N = 1$. We generalize our intuition in the main analysis, allowing for an arbitrary number of buyers and consider the case $N \geq 2$ in Section 3.2 and the case $N = 1$ in Section 3.3.

3.1 A numerical example

Consider a numerical example with $B = 3$ and the NFM technology (see Example 1). First consider the case $N = 3$ (just like in JKK). Then, using (3) to (6), we have $r_{N=3} = \frac{2}{3}$, $\pi_{N=3} = \frac{3}{4}$, $T_{N=3} = \frac{7}{4}$ and $u_{N=3} = \frac{1}{12} \approx 0.083$.

Consider next the case $N = 2$. With imperfect transparency, i.e., $N < B$, note that buyers' information is dispersed: potentially, there exist buyers who observe no seller, one seller, and two sellers. If a buyer observe no seller, then she has no one to visit and her payoff is zero. If a buyer observes one seller, she is a partially informed buyer and she visits the seller with probability one. Her payoff depends on whether the seller in question receives visits from other buyers (see below).

We now describe the visiting strategy of a fully informed buyer who observes two sellers. Let $\sigma_1 \in (0, 1)$ be the symmetric equilibrium probability that a fully informed buyer attends seller 1's auction. She obtains a positive payoff from seller 1 only if she is the only one to choose him. This occurs only when there is another fully informed buyer (otherwise, other two buyers are both partially informed, implying either of them should select seller 1 for sure) and that buyer does not select seller 1. Hence, given the buyer in question is fully informed, there is another fully informed buyer with probability $\frac{1}{2}$ (where one buyer observes both sellers and the other buyer observes none), and so her expected payoff from attending seller 1's auction is

$$u_1(r_1, r_2) = (1 - r_1)\frac{1}{2}(1 - \sigma_1),$$

where we note that the other fully informed buyer chooses seller 1 with probability σ_1.

Similarly, her expected payoff from attending seller 2's auction is

$$u_2(r_1, r_2) = (1 - r_2)\frac{1}{2}\sigma_1.$$

Given the directed search described above, seller 1 obtains profit r_1 if he meets only one buyer and profit 1 if he meets more than one buyer. He meets no buyer if there are two fully informed buyers and neither select him (remember that each seller can meet two buyers at most when $N = 2$). Hence, the seller 1's probability of meeting no buyer is $\Pr.[n_1 = 0] = \frac{1}{3}(1 - \sigma_1)^2$. The seller 1's probability of meeting one buyer is $\Pr.[n_1 = 1] = \frac{2}{3}(1 - \sigma_1^2)$, because he meets one buyer if either (i) there are two fully informed buyers and only one of them selects seller 1 (which occurs with probability $\frac{2}{3}\sigma_1(1 - \sigma_1)$), or (ii) there is only one fully buyer informed and she does not select seller 1 (which occurs with probability $\frac{2}{3}(1 - \sigma_1)$). So, seller 1's expected profit becomes

$$r_1\frac{2}{3}(1 - \sigma_1^2) + 1 - \frac{1}{3}(1 - \sigma_1)^2 - \frac{2}{3}(1 - \sigma_1^2),$$

where $\sigma_1 = \sigma_1(r_1, r_2)$ is determined by $u_1 = u_2$, which leads to $\frac{d\sigma_1}{dr_1} = \frac{-(1-\sigma_1)^2}{1-r_2}$. Taking the first-order conditions, we obtain the equilibrium prices which are symmetric,

$$r_{N=2} = \frac{3}{4}.$$

Each seller's equilibrium profit[3] is

$$\pi_{N=2} = \frac{19}{24}.$$

The total number of matches in equilibrium is

$$\begin{aligned} T_{N=2} &= \Pr.[n_1 \geq 1]\Pr.[n_2 = 0] + \Pr.[n_1 = 0]\Pr.[n_2 \geq 1] + 2 \cdot \Pr.[n_1 \geq 1]\Pr.[n_2 \geq 1] \\ &= 2\left(1 - \frac{1}{3}\left(\frac{1}{2}\right)^2\right)\frac{1}{3}\left(\frac{1}{2}\right)^2 + 2\left(1 - \frac{1}{3}\left(\frac{1}{2}\right)^2\right)^2 \\ &= 2\left(1 - \frac{1}{3}\left(\frac{1}{2}\right)^2\right) = \frac{11}{6}. \end{aligned}$$

[3] It is worth noting that a seller i does not want to deviate to set $r_i = 1$ to exclusively sell to partially informed consumers. When r_i is sufficiently higher than $\frac{3}{4}$, only those consumers who only observe seller i will buy from seller i. Given that the demand is completely inelastic for this range of r_i, the optimal deviating price is $r_i = 1$. When $N = 2$ and $B = 3$, the probability of having such a partially informed buyer is $\frac{2}{3}$. The corresponding deviating profit is therefore $\frac{2}{3}$, which is strictly lower than the equilibrium profit $\frac{19}{24}$.

We now compute the buyers' expected utility. As mentioned before, a buyer, no matter whether she observes only one seller or both sellers, can get a strictly positive payoff only if there exists a fully informed buyer who chooses to visit a seller different than the one she chooses. If a buyer already observes both sellers which happens with probability $\left(\frac{2}{3}\right)^2$), the probability of having another fully informed buyer is $\frac{2}{(C_2^1)^2} = \frac{1}{2}$. If a buyer observes only one seller (which happens with probability $2 \times \frac{2}{3} \times \left(1 - \frac{2}{3}\right)$), say seller 1, one opportunity to meet seller 1 and two opportunities of meeting seller 2 are randomly allocated between the other two buyers subject to the no-waste constraint. In this case, the probability of having another fully informed buyer is 1. To sum up, each buyer's equilibrium expected payoff is

$$u_{N=2} = (1 - r_{N=2}) \left[\left(\frac{2}{3}\right)^2 \times \frac{1}{2} + 2 \times \frac{2}{3} \times \left(1 - \frac{2}{3}\right) \times 1 \right] \frac{1}{2} = \frac{1}{12} \approx 0.083.$$

Finally, consider the case $N = 1$. There are only two possibilities, either one fully informed buyer exists (which happens with probability $\frac{1}{3}$) or two partially informed buyers exist (which happens with probability $\frac{2}{3}$). In the former case, sellers have incentive to reduce reserve prices to compete for, while in the latter case, sellers want to raise reserve price to exploit. As we show below, the symmetric equilibrium should involve mixed strategy. Denote by the symmetric equilibrium mixed strategy a distribution function $F(r)$ on $[\underline{r}, \overline{r}]$. By the standard argument given in Varian (1980), there is no gap in the support of $F(r)$ and no mass point. Note that all reserve prices in $[\underline{r}, \overline{r}]$ should yield the same profit, denoted by $\pi(r)$. Suppose $\overline{r} < 1$. Then at $\overline{r}$, only a partially informed consumer will buy. However, seller i can instead set $r = 1$ without losing demand, and hence can make a strictly higher profit. So we must have $\overline{r} = 1$. By the definition of mixed strategies, for any $r \in [\underline{r}, \overline{r}]$,

$$\pi(1) = \frac{2}{3} = r \left(\frac{1 - F(r)}{3} + \frac{2}{3} \right) = \pi(r).$$

So $F(r) = 3 - \frac{2}{r}$. Finally, from $F(\underline{r}) = 0$, $\underline{r} = \frac{2}{3}$. Therefore, the symmetric mixed-strategy equilibrium is characterized by the distribution function $F(r) = 3 - \frac{2}{r}$ with $r \in [\frac{2}{3}, 1]$. The expected equilibrium reserve price is $\mathbb{E}[r_{N=1}] = \ln(\frac{9}{4})$. Moreover, the total number of matches in equilibrium is given by

$$
\begin{aligned}
T_{N=1} &= \Pr.[n_1 = 1]\Pr.[n_2 = 0] + \Pr.[n_1 = 0]\Pr.[n_2 \geq 1] + 2 \times \Pr.[n_1 = 1]\Pr.[n_2 = 1] \\
&= 2\left(1 - \frac{1}{3} \times \frac{1}{2}\right)\frac{1}{3} \times \frac{1}{2} + 2\left(1 - \frac{1}{3} \times \frac{1}{2}\right)^2 \\
&= 2\left(1 - \frac{1}{3} \times \frac{1}{2}\right) = \frac{5}{3}.
\end{aligned}
$$

The equilibrium expected profit is $\pi_{N=1} = \frac{2}{3}$ and each buyer's equilibrium expected payoff is $u_{N=1} = (1 - \mathbb{E}[r_{N=1}])[1 - (\frac{2}{3})^2] = [1 - \ln(\frac{9}{4})]\frac{5}{9} \approx 0.105$.

To summarize the above analysis, we have:

$$
\begin{aligned}
&\pi_{N=2} > \pi_{N=3} > \pi_{N=1}, \\
&T_{N=2} > T_{N=3} > T_{N=1}, \\
&u_{N=1} > u_{N=2} = u_{N=3}.
\end{aligned}
$$

Given $B = 3$, the expected total number of matches is maximized when the information transparency level is moderate, $N = 2$. For all values of N, the total number of matches in equilibrium is decreasing in the probability that a seller receives no buyer. Recall that $\Pr.[n_i = 0] = \left(\frac{1}{2}\right)^3$ for $N = 3$, $\Pr.[n_i = 0] = \frac{1}{3}\left(\frac{1}{2}\right)^2$ for $N = 2$, and $\Pr.[n_i = 0] = \frac{1}{3}\left(\frac{1}{2}\right)$ for $N = 1$. This probability is clearly minimized at $N = 2$. Note also that this probability is the product between the probability of having N fully informed buyers and the probability that the N buyers select the same seller. The former probability is a measure of the *extensive margin* of market frictions, while the latter is a measure of the *intensive margin* of market frictions. When $N = 3$, the extensive margin is too high–actually, every buyer is fully informed so that the former probability equals one. When $N = 1$, the intensive margin is too high–actually, the fully informed buyer can only select one seller so that the latter probability is the highest, equals one half. The moderate transparency level, $N = 2$, achieves the balance in minimizing these two margins and achieves the highest number of matches.

Similarly, an individual seller's expected payoff is maximized when $N = 2$. Compared to the case with full transparency ($N = 3$), with the imperfect transparency ($N = 2$) the competition between sellers is softened and therefore $r_{N=2} > r_{N=3}$. Moreover, from the seller's point of view, the probability of trading is higher under $N = 2$ than under $N = 3$. Compared to the minimum transparency ($N = 1$), while the expected reserve price is lower ($\mathbb{E}(r_{N=1}) \approx 0.811 > 0.75 = r_{N=2}$), the trading probability is much higher under $N = 2$ ($\frac{11}{12} > \frac{5}{9}$), giving a good balance between trade efficiency and price.

Finally, a buyer's ex-ante payoff is maximized at the minimum level of information, i.e. $N = 1$. Notice that there is essentially no competition among buyers when $N = 1$, because there is either one fully informed buyer, or two partially informed buyers who will visit different sellers. Thus, whenever a buyer trades, she only pays the reserve price. Because of this, the probability of a buyer getting positive payoff under $N = 1$ is extremely high. Therefore, despite reaching the maximum expected reserve price, $N = 1$ is most favored by buyers.

3.2 Imperfect transparency: $N \geq 2$

We generalize the above intuition using the same setup with an arbitrary number of buyers, $B \geq 2$. Let us start with the case $N \geq 2$. Consider the problem of a fully informed buyer. As before, in the presence of another buyer who is a partially informed and hence has no other choice than attending a seller's auction, the *ex post* competition would shift all buyer surplus to the seller. Hence, a fully informed buyer gets positive surplus from selecting seller 1 only if all other $N - 1$ buyers are fully informed, which happens with probability $\Gamma(N - 1 \mid N - 1, B - 1)$, and none of them select seller 1, which happens with probability $(1 - \sigma_1)^{N-1}$. Her expected payoff of selecting seller 1 is

$$u_1(r_1, r_2) = (1 - r_1)\Gamma(N - 1|N - 1, B - 1)(1 - \sigma_1)^{N-1}.$$

Her expected payoff of selecting seller 2 can be similarly derived. The equilibrium selecting strategy σ_1 is implicitly determined by $u_1 = u_2$.

Consider next the seller 1's problem to choose r_1. Let n_1 be the realized number of buyers who select seller 1. Then, the seller 1's expected profit is

$$\pi_1(r_1, r_2) = r_1 \Pr(n_1 = 1) + \Pr(n_1 > 1) = 1 - \Pr(n_1 = 0) - \Pr(n_1 = 1)(1 - r_1).$$

Note that $n_1 = 0$ when there are N fully informed buyers and none of them select seller 1. The probability of this event is $\Gamma(N|N, B)(1 - \sigma_1)^N$. Also, $n_1 = 1$ when (i) there are N fully informed buyers but only one of them select seller 1, which happens with probability $\Gamma(N|N, B)N\sigma_1(1-\sigma_1)^{N-1}$; or (ii) there are $N-1$ fully informed buyers (and therefore two partially informed buyers) but none of them select seller 1, which happens with probability $\Gamma(N-1|N,B)(1-\sigma_1)^{N-1}$–note that in this case, it is a partially informed buyer who only observes seller 1 that participates in the seller 1's auction. Then,

$$\begin{aligned}\pi_1(r_1,r_2)=&1-\Gamma(N|N,B)(1-\sigma_1)^N\\&-[\Gamma(N|N,B)N\sigma_1+\Gamma(N-1|N,B)](1-\sigma_1)^{N-1}(1-r_1)\end{aligned}$$

From $u_1=u_2$, we can get

$$\frac{d\sigma_1}{dr_1}=\frac{-(1-\sigma_1)^N}{(N-1)(1-r_2)(\sigma_1)^{N-2}}.$$

Applying the first-order conditions, we obtain the symmetric equilibrium reserve price,

$$r_{N\geq 2}=1-\frac{\Gamma(N|N,B)N}{\Gamma(N|N,B)N^2+2\Gamma(N-1|N,B)(N-1)}. \tag{7}$$

In the Appendix, we show that the first order condition is necessary and sufficient, and (7) is indeed a unique equilibrium. If the meeting technology is NFM, the equilibrium reserve price is given by

$$r_{N\geq 2}=1-\frac{1}{N+2(N-1)(B-N)}. \tag{8}$$

We now compute the equilibrium payoffs and total number of matches. Given $r_{N\geq 2}$, each seller's equilibrium matching rate is $1-\Gamma(N|N,B)(\frac{1}{2})^N$ and their equilibrium profit is

$$\pi_{N\geq 2}=1-\Gamma(N|N,B)\left(\frac{1}{2}\right)^N-\frac{[\Gamma(N|N,B)\frac{N}{2}+\Gamma(N-1|N,B)]\Gamma(N|N,B)N(\frac{1}{2})^{N-1}}{\Gamma(N|N,B)N^2+2\Gamma(N-1|N,B)(N-1)}. \tag{9}$$

If the meeting technology is NFM, applying (1) and (2), we have

$$\pi_{N\geq 2}=1-\frac{1}{C_B^N}\left(\frac{1}{2}\right)^{N-1}\left[1+\frac{1}{\frac{N}{B-N}+2(N-1)}\right]. \tag{10}$$

To write down a buyer's ex ante expected payoff in equilibrium, we need to take into account not only the fully informed case (as already described above) but also the partially informed case. In either case, note that, as before, a buyer can get a positive payoff from a seller only if all other $N-1$ buyers are fully informed and none of them select the seller. With a slight abuse of notation, we use $\Gamma(N-1|(N-1,N),B-1)$

to denote the probability of having $N-1$ fully informed buyers out of $B-1$ buyers when one seller can be observed by N buyers and the other seller can be observed by $N-1$ buyers. We need this adjustment of the probability because if a partially informed buyer obtains a strictly positive payoff from a seller, then the seller should be observed by $N-1$ fully informed buyers who also observe the other seller. Let ϕ_1 and ϕ_2 be the probability of her being partially and fully informed, respectively. Then, a buyer's expected payoff when $N \geq 2$ is

$$u_{N\geq 2} = (1 - r_{N\geq 2})\left[\phi_1 \Gamma(N-1|(N-1,N), B-1) + \phi_2 \Gamma(N-1|N-1, B-1)\right]\left(\frac{1}{2}\right)^{N-1}. \tag{11}$$

Under the NFM technology, $\phi_1 = 2\frac{N}{B}\left(1-\frac{N}{B}\right)$ and $\phi_2 = \left(\frac{N}{B}\right)^2$. Also, (1) implies $\Gamma(N-1|N-1,B-1) = \frac{1}{C_{B-1}^{N-1}}$. To compute $\Gamma(N-1|(N-1,N),B-1)$, suppose that a buyer observes seller 1 but not seller 2. Among the other $B-1$ buyers, $N-1$ buyers should observe seller 1 and N $(\leq B-1)$ buyers should observe seller 2. There are in total $C_{B-1}^{N}C_{B-1}^{N-1}$ such cases. On the other hand, for $N-1$ of them to be fully informed, both of the sellers must be introduced to $N-1$ buyers, which has C_{B-1}^{N-1} cases, and seller 2 should be introduced to one of the $(B-1)-(N-1) = B-N$ remaining buyers, which has $C_{B-N}^{1} = B-N$. To sum up, the probability of having $N-1$ fully informed buyers when there is already a partially informed buyer is $\frac{(B-N)C_{B-1}^{N-1}}{C_{B-1}^{N}C_{B-1}^{N-1}} = \frac{B-N}{C_{B-1}^{N}}$, and so a buyer's expected payoff for $N \leq B-1$ is given by

$$u_{N\geq 2} = (1 - r_{N\geq 2})\left[2\frac{N}{B}\left(1-\frac{N}{B}\right)\frac{B-N}{C_{B-1}^{N}} + \left(\frac{N}{B}\right)^2 \frac{1}{C_{B-1}^{N-1}}\right]\left(\frac{1}{2}\right)^{N-1}. \tag{12}$$

For $N = B$, it is given by (6).

Finally, we compute the expected total number of matches. Whenever there are partially informed buyers, which occurs with probability $1-\Gamma(N|N,B)$, both sellers can make sales and there are two matches. Otherwise, there are N fully informed buyers, and there are two matches with probability strictly less than one: there will be only one match with probability $2(\frac{1}{2})^N = (\frac{1}{2})^{N-1}$; there will be two matches with probability $1-(\frac{1}{2})^{N-1}$. So the expected total number of matches is

$$\begin{aligned} T_{N\geq 2} &= \Gamma(N|N,B)\left[\left(\frac{1}{2}\right)^{N-1}+2\left(1-\left(\frac{1}{2}\right)^{N-1}\right)\right]+2(1-\Gamma(N|N,B)) \\ &= 2-\left[\left(\frac{1}{2}\right)^{N-2}-\left(\frac{1}{2}\right)^{N-1}\right]\Gamma(N|N,B) \\ &= 2\left[1-\left(\frac{1}{2}\right)^{N}\Gamma(N|N,B)\right] \qquad (13) \end{aligned}$$

If the meeting technology is NFM, we have

$$T_{N\geq 2}=2\left[1-\left(\frac{1}{2}\right)^{N}\frac{1}{C_B^N}\right]. \qquad (14)$$

We can summarize the analysis so far as follows.

Theorem 1. *Consider a model of imperfect transparency with the parameter $N\in[2,B]$ for an arbitrarily number of buyers, $B\geq 2$. A directed search equilibrium exists and is unique with the symmetric equilibrium reserve price given by (7). With the NFM meeting technology, the symmetric equilibrium reserve price given by (8).*

3.3 Imperfect transparency: $N=1$

Now we switch to the special case of $N=1$, where it is obvious that the analysis above does not readily extend. Indeed, there is no symmetric pure-strategy equilibrium when $N=1$. If both sellers set some $r_1=r_2>0$, one of them can undercut the reserve price slightly and get the fully informed buyer for sure. But neither seller will set zero reserve price since they can make positive expected profit by setting a positive reserve price and selling to the partially informed buyer with positive probability.

When $N=1$, there are two possible scenarios regarding buyers' information: (i) a single buyer observes both sellers; (ii) one buyer observes only seller 1 and another buyer observes only seller 2. If a buyer is fully informed, she will select the seller with lower reserve price. If a buyer is partially informed, she will select the observed seller provided the reserve price is no greater than 1. Note that the *ex post* bidding never takes place when $N=1$ as each seller can meet at most one buyer.

Let the symmetric mixed strategy equilibrium be denoted by the distribution function $F(r)$ with support $[\underline{r}, \bar{r}]$. By the similar argument as we made in the numerical example, we must have $\bar{r} = 1$. Also, there should be no gap in the support of $F(r)$ and no mass point, just like in Varian (1980).

We now derive an individual seller's (say, seller 1's) expected profit and the lower bound of the equilibrium price distribution, $\underline{r}$. There is a fully informed buyer with probability $\Gamma(1|1, B)$ and, given that seller 2 mixes using the price distribution $F(\cdot)$, seller 2's price is higher than a price r_1 with probability $1 - F(r_1)$, in which case the fully informed buyer will buy from seller 1. On the other hand, with probability $\Gamma(0|1, B)$, there is a partially informed buyer who can only buy from seller 1. To sum up, seller 1's expected profit with a reserve price r_1 is

$$\pi_1(r_1, F(r)) = r_1[\Gamma(1|1, B)(1 - F(r_1)) + \Gamma(0|1, B)].$$

In equilibrium, sellers use a mixed strategy, and so they must be indifferent between any $r \in [\underline{r}, 1)$ and $r = 1$, which yields an expected profit $\Gamma(0|1, B)$. The indifference condition is then

$$r[\Gamma(1|1, B)(1 - F(r)) + \Gamma(0|1, B)] = \Gamma(0|1, B)$$

This condition generates the equilibrium price distribution,

$$F(r) = 1 - \frac{\Gamma(0|1, B)}{\Gamma(1|1, B)} \left(\frac{1}{r} - 1 \right). \tag{15}$$

Further, the equilibrium must satisfy $F(\underline{r}) = 0$, which in turn yields an expected profit $\underline{r} = \Gamma(0|1, B)$. Hence, the reserve price distribution (15) with support $[\Gamma(0|1, B), 1]$ constitutes an equilibrium.

We now compute the equilibrium payoffs and outcomes. The above analysis shows that the equilibrium expected profit of individual sellers is

$$\pi_{N=1} = \Gamma(0|1, B). \tag{16}$$

When $N = 1$, a buyer who observes any seller can trade with probability one since there will be no competitors between buyers. So a buyer's equilibrium expected payoff is

$$u_{N=1} = (1 - \mathbb{E}[r_{N=1}])(\phi_1 + \phi_2) \tag{17}$$

where the equilibrium expected reserve price is

$$\mathbb{E}(r_{N=1}) = \int_{\Gamma(0|1,B)}^{1} r dF(r) = -\frac{\Gamma(0|1,B)}{\Gamma(1|1,B)} \ln(\Gamma(0|1,B)).$$

Finally, when $N = 1$, there is one fully informed buyer (and so one match) with probability $\Gamma(1|1,B)$, and there are two partially informed buyer (and so two matches) with probability $\Gamma(0|1,B)$. Hence, the expected total number of matches is

$$T_{N=1} = \Gamma(1|1,B) + 2\Gamma(0|1,B) = 2\left[1 - \frac{\Gamma(1|1,B)}{2}\right]. \tag{18}$$

When the meeting technology is NFM, the equilibrium is described by the price distribution

$$F(r) = 1 - (B-1)\left(\frac{1}{r} - 1\right) \tag{19}$$

with support $[1 - \frac{1}{B}, 1]$. Further, we have:

$$\pi_{N=1} = 1 - \frac{1}{B}; \tag{20}$$

$$u_{N=1} = (1 - \mathbb{E}[r_{N=1}])\left[1 - \left(1 - \frac{1}{B}\right)^2\right] \tag{21}$$

where

$$\mathbb{E}(r_{N=1}) = (B-1)\ln\left(\frac{B}{B-1}\right);$$

and

$$T_{N=1} = 2 - \frac{1}{B}. \tag{22}$$

Theorem 2. *Consider a model of imperfect transparency with the parameter $N = 1$ for an arbitrarily number of buyers, $B \geq 2$. A directed search equilibrium exists and is unique, characterized by a non-degenerate distribution of reserve prices (15) on $\Gamma(0|1,B), 1$. With the NFM meeting technology, the equilibrium distribution of reserve prices is give by (19) on $[1 - \frac{1}{B}, 1]$.*

4 Optimal meeting transparency

Given the equilibrium results derived above, we study the optimal meeting transparency from the viewpoint of the society (Section 4.1), sellers (Section 4.2) and buyers (Section 4.3). We show that the intuition obtained from the numerical example in Section 3.1 is valid with the more general case with an arbitrary number of buyers.

4.1 Match-maximizing meeting

We first consider the degree of transparency N that maximizes the total matches, denoted by N^*. Before turning our focus to the NFM technology, we first present a sufficient condition under which the full meeting transparency is optimal in generating matches.

Proposition 1. *Under the general meeting technology, the total number of matches is maximized at the full transparency, $N^* = B$, if the probability of having N fully informed buyers, $\Gamma(N|N, B)$, is non-increasing in N.*

The proof is trivial from (13) and (18). A meeting technology that satisfies this requirement is the BNFM technology (see Example 2) where it holds that $\Gamma(N|N, B) = 1$ for any N and therefore $\Gamma(N|N, B)$ is non-increasing in N. The result is quite intuitive. Under the BNFM, partially informed buyers never exist and so the extensive margin of market frictions, captured by the probability of having N fully informed buyers, $\Gamma(N|N, B)$, is constant. Therefore, the number of total matches only depends on the intensive margin of market frictions, captured by the probability that N buyers choose the same seller, $\left(\frac{1}{2}\right)^N$. The intensive margin is strictly decreasing in N. Therefore, $N = B$ generates the greatest number of total matches.

In what follows, let us focus on the NFM technology with $B \geq 3$.[4] The next proposition is our first main result which shows that an intermediate level of meeting transparency is optimal in generating matches.

Proposition 2. *With the NFM meeting technology, the total number of matches is maximized at some intermediate level of transparency, $N^* \in (1, B)$.*

As mentioned before, the expected total number of matches $T_{N\geq 2}$ in (13) depends negatively on two probabilities, which in combination lead to the probability of having just one match: one is the probability that all the N fully informed buyers visit the same seller, $(\frac{1}{2})^N$, capturing the intensive margin of frictions, and the other one is the

[4]The case $B = 2$ is somewhat trivial and so we omit it in our presentation.

probability of having N fully informed buyers, $\Gamma(N \mid N, B)$, capturing the extensive margin of frictions. The former probability decreases as N increases since it become harder for fully informed buyers to concentrate on visiting the same seller if there are more of them. This effect improves match efficiency. As for the latter probability, noting $C_B^N = \frac{B+1-N}{N} C_B^{N-1}$, observe that

$$\Gamma(N \mid N, B) = \frac{1}{C_B^N} > \frac{1}{C_B^{N-1}} = \Gamma(N-1 \mid N-1, B) \Longleftrightarrow N > \frac{B+1}{2}.$$

That is, given values of B, the probability of having N fully informed buyers decreases in N first and then increases. Intuitively, when N is relatively small, due to the no waste, it is not likely that additional meeting opportunities with each seller can be concentrated on particular buyers among the remaining $B - N$ buyers (e.g. imagine $N = 2$ versus $B = 100000$). When N is relatively large, it is likely that additional meeting opportunities reach at the targeted buyers (e.g. imagine an increase from $N = B - 1$ to $N = B$).

To sum up, when N is low, both the intensive and extensive margins of frictions decrease in N, improving match efficiency, and so within this region a greater N helps generate more matches. As N increases further, the extensive margin starts to increase in N. In order to generate more matches, we need to tradeoff a higher probability of having N fully informed buyers, which decreases match efficiency, against a lower probability of allocating all the N fully informed buyers to the same seller, which increases match efficiency. This results in an interior optimum, $N^* \in (1, B)$.

4.2 Seller-optimal meeting

We next explore the seller-optimal transparency level, denoted by N^S, that maximizes each individual seller's expected profit. Remember that under the NFM, a seller's expected profit is given by (10) when $N \geq 2$, and is given by (20) when $N = 1$.

Proposition 3. *With the NFM meeting technology, sellers' profits are maximized at some intermediate level of transparency, $N^S \in (1, B)$.*

The logic for the interior optimum, i.e., $N^S \in (0, B)$, is similar to the one for the match-maximizing meeting. A seller cannot sell if there are N fully informed buyers and all of them select the rival seller. When N is relatively large, the probability of having N fully informed buyer increases with N, which makes $N = B$ suboptimal. When N is relatively small, increasing N lowers both the probability of having N fully informed buyer and the probability that all these fully informed buyers choose the rival seller. As a result, $N^S \in (0, B)$. Further, by comparing (10) and (13), it is clear that the only difference between the per seller match rate $\frac{T_{N\geq 2}}{2}$ and the equilibrium profit $\pi_{N\geq 2}$ is the term related to competition,

$$2\left[1 + \frac{1}{\frac{N}{B-N} + 2(N-1)}\right],$$

which is strictly decreasing in N. Hence, we have the optimal meeting transparency required for maximizing each seller's profit is higher than the one required for maximizing the total number of matches.

Corollary 1. *With the NFM meeting technology, $N^S > N^*$.*

4.3 Buyer-optimal meeting

We next consider the buyer-optimal level of transparency, denoted by N^B. The goal is to maximize each individual buyer's expected payoff. Under the NFM technology, a buyer's expected payoff is given by (12) when $N \geq 2$ and by (21) when $N = 1$.

Proposition 4. *With the NFM meeting technology, buyers' expected utility is maximized at the minimum transparency level, $N^B = 1$.*

Proposition 4 implies that buyers prefer the minimum level of meeting transparency, which is somewhat counter-intuitive. The major reason why buyers prefer $N = 1$ is because they do not face any competition once they are informed and therefore can always receive positive payoffs through trading. Hence, in spite of the low probability of becoming informed, buyers prefer $N = 1$ due to the removal of competition on the buyer side.

5 Profit-maximizing platform

So far, we have treated N as an exogenous parameter. In this section, we allow the platform to be profit–maximizing, and to determine the transparency level. The platform could charge a fee to the participating agents and/or to the realized transactions. Just for the sake of illustration, we assume here that the platform can charge each seller a transaction fee denoted by $f \geq 0$. That is, a seller needs to pay the platform f if he makes a sale. As it will be clear shortly, incorporating other types of fees does not affect our results. The platform's objective is to set f and N to maximize its own profit, which is given by $\Pi = f \cdot T_N$, subject to the buyers' and the sellers' participation, where T_N is the total expected number of trades in equilibrium described above conditioned on the value of $N = 1, ..., B$.

Following the platform's choice of f and N, sellers play the same game as before except that their profit margins are now either $1-f$, if the sale price is 1, or $r-f$, if the sale price is the reserve price r. No matter what f is, sellers play the symmetric-strategy equilibrium in the continuation subgames.

With the transaction fee, the fully informed buyers' problem of choosing sellers is the same as before. With $N \geq 2$, seller 1's expected profit is

$$\begin{aligned}\pi_1(r_1, r_2, f) &= (r_1 - f)\Pr(n_1 = 1) + (1-f)(1 - \Pr(n_1 = 0) - \Pr(n_1 = 1)) \\ &= (1-f)(1 - \Pr(n_1 = 0)) - (1 - r_1)\Pr(n_1 = 1).\end{aligned}$$

Seller 2's profit can be similarly derived. Following the same steps as in Section 3.2, we can derive the symmetric equilibrium reserve price for given values of f with the NFM meeting technologies as

$$r(f) = 1 - \frac{1-f}{N + 2(N-1)(B-N+1)}.$$

Observe that $r(f) = 1$ when $f = 1$ which is in fact the highest possible fee the platform can charge without causing sellers and buyers to withdraw.

It is important to note that the fee does not influence the total number of matches in the symmetric equilibrium where $r_1 = r_2$ and fully informed buyers select any seller with probability $\frac{1}{2}$ so that the total number of matches is independent of reserve prices. Hence, the platform can charge a fee without fear of losing trades, and so optimally selects $f^* = 1$ for any given $N \geq 2$. Given $f^* = 1$, as for the selection of N, the platform cares only about the total expected number of matches. We know from Proposition 2 that it is maximized at $N = N^* \in (1, B)$.

Under the solution $f = 1$ and $N^* \in (0, B)$, the platform indeed obtains the maximum profit, $\Pi = T_{N=N^*}$, and implements the full surplus extraction, leaving both the buyers and sellers zero utility/profit. The solution is indeed optimal, i.e., it dominates any other $f < 1$ and $N \neq N^*$ (including $N = 1$). Further, it is worth noting that in general, it holds that $T_N = 2\pi_N + Bu_N$, where π_N (u_N) is the expected profit (utility) derived above conditioned on the value of $N = 1, 2, ..., B$. This implies that the total expected number of trades equals total welfare in equilibrium, and so we can interpret our solution as a welfare maximizing solution.

Proposition 5. *Consider a profit-maximizing platform who is able to determine the transparency level, and charges a transaction fees for its intermediation service. Then, the platform selects $N = N^*$. i.e., it maximizes the expected total number of matches.*

We find this result particularly interesting. Despite the potential complexity introduced by imperfect transparency, the efficient meetings, which maximize the expected total number of matches or surplus, can be implemented by a profit-maximizing platform who adopts a simple fee-setting policy for its intermediation service. In other words, the efficient matching outcome can be decentralized by a profit-maximizing platform who introduces imperfect transparency to its participants.

6 Discussions and extensions

6.1 Meeting technologies

Eeckhout and Kircher (2010) distinguished meeting from matching in the directed search framework. The meeting considered in their model is ex post in the sense that buyers first search submarkets where individual sellers post prices and then meeting takes place in the submarkets according to the meeting technology. In contrast, the meeting technology proposed in our model generates submarkets or a network of contacts between buyers and sellers within which buyers can search individual sellers. As in Eeckhout and Kircher (2010), our meeting technologies, in particular the NFM technologies, can captures various types of rivalry in meeting, including rival, non-rival, and partially rival meetings. Namely, the meeting technology is rival when $N = 1$ where a meeting schedule between a seller and a buyer implies that any other buyers are not allowed to meet the seller. The meeting technology is non-rival when $N = B$ where a meeting between a seller and a buyer does not affect the meeting opportunity between this seller and any other buyers. Finally, the meeting technology is partially rival when $N \in (1, B)$, where a meeting opportunity between a seller and a buyer reduces the opportunity for other buyers to meet this seller but it is not completely eliminated. Unlike in Eeckhout and Kircher (2010), we use auction as the trading mechanism, and explore the optimal meeting technology for the society, sellers and buyers respectively. Further, we endogenize the meeting technology with a profit maximizing platform and show that the social optimum can be decentralized.

We now discuss other assumptions of our meeting technologies. The assumption of "anonymity" rules out the trivial case that the buyers' probability of receives meeting requests from each seller can depend on their identity (which opens a room for removing search frictions in the first place). If the assumption of "symmetry" is violated, sellers could fully extract all the surplus and the platform can transfer that surplus to itself by setting a listing fee. Consider, for example, a case of two sellers and two buyers. Allow seller 1 to send one meeting request but seller 2 to send two. Then, seller 2 knows that there exists one buyer who only receives request from him so he can set $r_2 = 1$ to fully extract surplus of that buyer. Knowing this, seller 1 can also set $r_1 = 1$. The buyer who observes both sellers is indifferent between which seller to visit. So we assume she selects seller 1. Then, seller 1 also obtains a profit equal to 1. The platform can set each seller a listing fee equal to 1 and extract the whole surplus.

The "no waste" assumption is important for our analysis. If this property is violated, the analysis becomes less straightforward. In particular, there can be an unmatched seller even when there exists a partially informed buyer. Consider, for example, a case of three buyers and $N = 3$. Then, it is possible that buyer 1 receives only one meeting request from seller 1, buyer 2 receives two meeting requests from seller 2 and one meeting request of seller 1, and buyer 3 receives one meeting request from each seller. In this case, it is possible that all buyers end up selecting seller 1, leaving seller 2 unmatched. So, when the "no waste" assumption does not hold, the total number of matches does not take the simple form as in (13). A related analysis of advertising in the directed search environment which violates the "no waste" assumption can be found in Gomis-Porqueras, Julien and Wang (2017).

6.2 More than two sellers

Our analysis so far with two sellers in each product category was mainly because of analytical tractability. In this section we shall illustrate the difficulty associated with a more than two sellers setup. The main modification would be that partially informed buyers, who have no choice other than visiting the single observed seller with the two sellers setup, may now have a choice of which seller visit. As seen below, this additional decision would complicate the equilibrium analysis significantly. However, we should emphasize that even with this extension, our main insight that the full transparency is not necessarily optimal would be still valid.

Consider the same setup as before with three sellers and thee buyers. When $N = 1$, the situation is very similar to the one in Burdett and Judd (1983). There exist, in each case with positive probability, fully informed buyers, partially informed buyers who observe two sellers, and partially informed buyers who observe only one seller. The key trade-off we described in Section 3.3 is still valid here, except that sellers need to compete not only for the fully informed buyers but also those partially informed buyers who observe two sellers. As in Section 3.3, the equilibrium exhibits dispersion in reserve prices.

Analysis changes significantly when $N = 2$. Except those observing only one seller, fully informed and partially informed buyers need to estimate the possible information sets and the associated visiting strategies of other buyers. For example, a fully informed buyer knows that there are only two possible scenarios she could be in: (i) there exists another fully informed buyer and an uninformed buyer, or (ii) there is a partially informed buyer who observes two sellers and another partially informed buyer who only observes one seller. The fully informed buyer needs to trade-off the reserve price and the possibility of overlapping with each type of the buyers. In addition, a partially informed buyer who observes two sellers faces a similar trade-off and needs to determine her optimal visiting strategy. Given all these possibilities, each seller needs to assign probabilities to each possible scenario regarding the distribution of buyers' information sets. We solve for the equilibrium with symmetric reserve prices.

Comparing the three specifications of N (see the Appendix for the detailed derivation), we have

$$\pi_{N=1} > \pi_{N=3} > \pi_{N=2},$$

$$T_{N=1} = T_{N=3} > T_{N=2},$$

$$u_{N=3} > u_{N=1} > u_{N=2}.$$

Compared to the case with two sellers, the efficient number of meetings is further restricted and the total number of matches is maximized at $N = 1$ when there are three sellers. With more sellers, the probability that an additional meeting request from each seller overlaps at the same buyer can be high even when the initial meeting transparency is low. So, in order to maximize the total number of matches, the meeting transparency needs to stay at the low level. An individual seller's expected profit is also maximized at $N = 1$ as the competition among sellers is greatly softened under $N = 1$. In addition, compared to the two-seller case, the probability of sales under $N = 2$ is no longer significantly higher than when $N = 1$. Finally, consumers will prefer $N = 3$. This is in stark contrast to the two-seller case where the buyer-optimal meeting requires $N = 1$. With more sellers, the probability of overlapping at the same seller is mitigated while a lower reserve price matters more for buyers.

7 Conclusion

This paper studied how meeting technologies, typically controlled by intermediaries, can affect the matching efficiency, seller profits and buyer surplus in a directed search equilibrium. In a market with a continuum of duopoly product categories, we are able to identify the optimal meeting technology. In particular, we show that the full-information meeting technology does not necessarily lead to the best outcome. We further consider a profit-maximizing platform that can choose the degree of meeting transparency and charge with fees for its intermediation service. We show that this profit-maximizing platform can implement the efficient allocation.

Various market characteristics, such as sellers' entry and exit, ex ante agent heterogeneity, or idiosyncratic match values, are not included in our model. The main purpose of this paper is, by using a stylized model with a fixed number of homogeneous participants, to show that there is a straightforward rationale of why platform may want to restrict participants' meeting choices. As a future extension, the additional market characteristics mentioned above could be incorporated. In particular, it would be interesting to study the optimal fee structure and market transparency when the buyer side of the market is featured by heterogeneous outside options and free entry. In such an extension, a change of meeting technology affects not only the existing buyers' information sets and selection strategies, but also the total number of buyers who participate in the market. We expect an imperfect meeting technology continues to dominate the full transparency in generating matches. However, a profit-maximizing platform is unlikely to be able to achieve the full efficiency with a simple transaction fee and the decentralization result may require more complicated fee schemes.

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Appendix.

Proof of Theorem 1. We have shown in the main text the equilibrium reserve prices in (7) satisfies the first-order condition. We next show that the second-order condition is satisfied. The second-order derivative of firm 1's profit at the symmetric equilibrium price r^* is equal to

$$-2^{-1-N}\frac{[N^2\Gamma(NIN,B)+(2N-3)\Gamma(N-1|N,B)][N^2\Gamma(N|N,B)+(2N-2)\Gamma(N-1|N,B)]}{N(N-1)\Gamma(N|N,B)},$$

which is negative given that $N \geq 2$. Finally, we check that a unilateral deviation to setting $r=1$ and only selling to partially informed consumers is not profitable. It is a matter to compare (9) with $\pi^d = 1 \times (1 - \Gamma(N|N,B))$. After simplification, choosing r^* is better if the following condition holds

$$\left(2^N - 1\right) \frac{N^2\Gamma(N|N,B) + 2\Gamma(N-1|N,B)(N-1)}{N^2\Gamma(N|N,B) + 2\Gamma(N-1|N,B)N} \geq 1.$$

But this inequality always holds for $N \geq 2$. Since the second-order derivative of firm 1's profit is strictly negative, r^* is a local maximizer. Moreover, it yields a profit greater than when choosing $r = 1$ and therefore is also a global maximizer. ■

Proof of Theorem 2.

In text. ■

Proof of Proposition 1.

In text. ■

Proof of Proposition 2. First, to compare $T_{N=1} = 2 - \frac{1}{B}$ and $T_{N=B} = 2 - \left(\frac{1}{2}\right)^{B-1}$, we can instead compare $\ln\left(\frac{1}{B}\right)$ and $\ln\left(\left(\frac{1}{2}\right)^{B-1}\right)$ since the log function is a monotone transformation. The difference of the two terms is $-\ln(B) + (B-1)\ln(2)$, which is strictly positive when $B = 3$. The derivative of the difference is $\ln(2) - \frac{1}{B} > 0$ for any $B \geq 3$. We can conclude that $\frac{1}{B} > \left(\frac{1}{2}\right)^{B-1}$ and so $T_{N=1} < T_{N=B}$ for any $B \geq 3$.

Second, for any $B \geq 3$, observe that

$$\begin{aligned} T_{N=B} - T_{N=B-1} &= -\left(\frac{1}{2}\right)^{B-1} \left[\Gamma(B \mid B, B) - 2\Gamma(B-1 \mid B-1, B)\right] \\ &= -\left(\frac{1}{2}\right)^{B-1} \left(1 - \frac{2}{B}\right) < 0. \end{aligned}$$

Therefore, $N = B$ cannot be optimal, and so we must have $N^* \in (1, B)$. ■

Proof of Proposition 3. First, $\pi_{N=1} = 1 - \frac{1}{B}$ and $\pi_{N=B} = 1 - \left(\frac{1}{2}\right)^{B-1}$. By using the logarithm transformation, it is easy to show that $\pi_{N=1} < \pi_{N=B}$, and so $N = 1$ is never optimal. Second, observe that

$$\begin{aligned} \pi_{N=B} - \pi_{N=B-1} &= -\left(\frac{1}{2}\right)^{B-1} + \frac{1}{B}\left(\frac{1}{2}\right)^{B-2} \left[1 + \frac{1}{B-1+2(B-2)}\right] \\ &= -\left(\frac{1}{2}\right)^{B-1} \left[1 - \frac{2}{B}\left(1 + \frac{1}{B-1+2(B-2)}\right)\right] \\ &= -\left(\frac{1}{2}\right)^{B-1} \frac{3B^2 - 11B + 8}{B(3B-5)} < 0 \end{aligned}$$

for any $B \geq 3$. Therefore, $N = B$ cannot be optimal, and so we must have $N^S \in (1, B)$. ■

Proof of Corollary 1. In text. ■

Proof of Proposition 4. Our numerical example shows $u_{N=1} > u_{N\geq 2}$ when $B = 3$. So we can focus on the case when $B \geq 4$. The proof proceeds case by case: we show the claim $u_{N=1} > u_{N\geq 2}$ in separation for (i) $N = 2$; (ii) $N = 3, 4, ..., B - 1$; (iii) $N = B$. Since these cases altogether cover all the possible values of $N = 2, 3, ..., B$ for all $B \geq 4$, we prove the claim in the proposition.

(i) Case 1 ($N = 2$). Applying $N = 2$ to (12), we get $u_{N=2} = \frac{2B-3}{B^2(B-1)^2}$. Then,

$$u_{N=1} - u_{N=2} = \frac{1}{B^2(B-1)^2}\left[(2B-1)\left[B(B-2) - (B-1)^3 \ln \frac{B}{B-1}\right] + 2\right].$$

In what follows, we show that the term

$$B(B-2) - (B-1)^3 \ln \frac{B}{B-1}$$

is positive. Since this term can be re-written as

$$(B-1)^2 \left(1 - (B-1) \ln \frac{B}{B-1}\right) - 1,$$

it is sufficient to show that $1 - (B-1) \ln \left(\frac{B}{B-1}\right) > \frac{1}{(B-1^2)}$, or using $\frac{1}{2B} > \frac{1}{(B-1)^2}$,

$$\Upsilon(B) \equiv \frac{2B-1}{2B} - (B-1) \ln \left(\frac{B}{B-1}\right) > 0.$$

Observe that: $\Upsilon(4) \approx 0.012 > 0$;

$$\Upsilon'(B) = \frac{1}{2B^2} + \frac{1}{B} - \ln \left(\frac{B}{B-1}\right).$$

Note that $\Upsilon'(4) = -0.006$ and $\lim_{B\to\infty} \Upsilon'(B) = 0$. Also, $\Upsilon''(B) = \frac{1}{B^3(B-1)} > 0$. Therefore, $\Upsilon'(B) < 0$ for all $B \geq 4$. Since $\Upsilon(4) > 0$ and $\lim_{B\to\infty} \Upsilon = 0$, we then can conclude $\Upsilon(B) > 0$ for all $B \geq 4$. This completes the proof of the case $N = 2$.

(ii) Case 2 ($N \in [3, B-1]$). Using (8) and $C^N_{B-1} = \frac{B-N}{N} C^{N-1}_{B-1}$, we can re-write (12) as

$$u_{N\geq 2} = \left[\frac{1}{N + 2(N-1)(B-N)}\right] \left[\left(\frac{N}{B}\right)^2 \frac{1 + 2(B-N)}{C^{N-1}_{B-1}} \left(\frac{1}{2}\right)^{N-1}\right]. \quad (A1)$$

We prove the claim case by case in separation: $B = 4$ and $B \geq 5$.

- **Case 2-1 ($B = 4$).** Note that when $B = 4$, the only admissible parameter value is $N = 3$. Applying these values to (21) and (12), we get: $u_{N=1} = \left(1 - 3\ln\frac{4}{3}\right)\left(1 - \left(\frac{3}{4}\right)^2\right) \approx 0.0599$; $u_3 = \frac{1}{7}\left(\frac{3}{4}\right)^2 \frac{3}{C_3^2}\left(\frac{1}{2}\right)^2 \approx 0.0201$. Hence, $u_{N=1} > u_{N=3}$.

- **Case 2-2 ($B \geq 5$).** Decomposing (A1) into two terms, we will compare the first term in the expression of $u_{N=1}$ in (21) with the first term in (A1) and the second term in the expression of $u_{N=1}$ in (21) with the second term in (A1). Below, we show that both terms in the expression of $u_{N=1}$ are greater in (21) for $N = 1$ than in (A1) for $N \in [3, B-1]$. Note that the term $N + 2(N-1)(B-N)$ takes the minimum when $N = B - 1$, which equals $3B - 5$ ($> 2B$) for all $B \geq 5$. Hence, the first term in (A1) is strictly smaller than $\frac{1}{2B}$, and so to show $1 - (B-1)\ln\left(\frac{B}{B-1}\right) > \frac{1}{N+2(N-1)(B-N)}$, it is sufficient to show $1 - (B-1)\ln\left(\frac{B}{B-1}\right) > \frac{1}{2B}$. But we have already shown it above, i.e., $\Upsilon(B) > 0$.

We next show that $1 - \left(\frac{B-1}{B}\right)^2 > \left(\frac{N}{B}\right)^2 \frac{1+2(B-N)}{C_{B-1}^{N-1}}\left(\frac{1}{2}\right)^{N-1}$. Given that $C_{B-1}^{N-1} = C_B^N \frac{N}{B}$, this inequality can be written as

$$\frac{2B-1}{1+2(B-N)} > \frac{BN}{C_B^N}\left(\frac{1}{2}\right)^{N-1}.$$

Note further that $C_B^N > \left(\frac{B}{N}\right)^N$.[5] Thus, to show the inequality in question, it is sufficient to show the following inequality:

$$\Psi(B) \equiv \frac{2B-1}{1+2(B-N)} - \frac{BN}{\left(\frac{B}{N}\right)^N}\left(\frac{1}{2}\right)^{N-1} = \frac{2B-1}{1+2(B-N)} - N^2\left(\frac{N}{2B}\right)^{N-1} > 0.$$

Observe that: $\Psi(5) = \frac{9}{11-2N} - N^2\left(\frac{N}{10}\right)^{N-1}$: when $N = 4$, $\Psi(5) = \frac{9}{3} - 4^2\left(\frac{4}{10}\right)^3 = 3 - 1.024 = 1.976$; when $N = 3$, $\Psi(5) = \frac{9}{5} - 3^2\left(\frac{3}{10}\right)^2 = 1.8 - 0.81 = 0.99$; $\lim_{B\to\infty}\Psi(B) = 1$;

$$\Psi'(B) = -\frac{4(N-1)}{[1+2(B-N)]^2} + \frac{N^2(N-1)}{B}\left(\frac{N}{2B}\right)^{N-1}.$$

Since $\min\{\Psi(5), \Psi(\infty)\} > 0$, if $\Psi(B)$ is monotone in all $B \geq 5$ then $\Psi(B) > 0$ for all $B \geq 5$. Suppose otherwise, i.e., $\Psi(B)$ is non-monotone in B. Then, there is a possibility of the existence of a minimum at some interior $\tilde{B} \in (5, \infty)$, satisfying $\Psi'(\tilde{B}) = 0$. We will

[5] To show $C_B^N \geq \left(\frac{B}{N}\right)^N$, it is sufficient to observe that $C_B^N = \frac{B}{N}\frac{B-1}{N-1}\cdots\frac{B-(N-1)}{1}$, where each of these N terms is no less than $\frac{B}{N}$.

check whether the possible minimum attains a positive or negative value. The marginal condition $\Psi'(\tilde{B}) = 0$ gives $N^2\left(\frac{N}{2\tilde{B}}\right)^{N-1} = \frac{4\tilde{B}}{\left[1+2(\tilde{B}-N)\right]^2}$. Plugging this into $\Psi(B)$, we have

$$\Psi(\tilde{B}) = \frac{2\tilde{B}-1}{1+2(\tilde{B}-N)} - \frac{4\tilde{B}}{\left[1+2(\tilde{B}-N)\right]^2} = \frac{4\tilde{B}^2 - 4(N+1)\tilde{B} + 2N - 1}{\left[1+2(\tilde{B}-N)\right]^2} > 0,$$

where the last inequality follows from the fact that the numerator takes the minimum at $N = B-1$, which equals $2B-3 > 0$. Hence, we can conclude that $\Psi(B) > 0$ for all $B \geq 5$. This completes the proof of the case $N \in [3, B-1]$.

(iii) Case 3 ($N = B$). From (6), $u_{N=B} = \frac{1}{B}\left(\frac{1}{2}\right)^{B-1} = \frac{1}{2B}\left(\frac{1}{2}\right)^{B-2}$. To compare it with $u_{N=1} = \left(1 - (B-1)\ln\frac{B}{B-1}\right)\left(1 - \left(\frac{B-1}{B}\right)^2\right)$, note first that $1 - (B-1)\ln\left(\frac{B}{B-1}\right) > \frac{1}{2B}$, as already shown above, i.e., $\Upsilon(B) > 0$. Hence, what remains here is to show that $1 - \left(\frac{B-1}{B}\right)^2 > \left(\frac{1}{2}\right)^{B-2}$ or

$$\Phi(B) \equiv 2B - 1 - B^2\left(\frac{1}{2}\right)^{B-2} > 0.$$

This can be show by observing that: $\Phi(4) = 7 - 4^2\left(\frac{1}{2}\right)^2 = 3$; $\Phi'(B) = 2 - 2B\left(\frac{1}{2}\right)^{B-2} + B^2\left(\frac{1}{2}\right)^{B-2}\ln 2 > 0$ where the last inequality follows from $B\left(\frac{1}{2}\right)^{B-2} < 4\left(\frac{1}{2}\right)^2 = 1$. This completes the proof of the case $N = B$. ■

Proof of Proposition 5.

In text. ■

Omitted details in Section 6.2.

• $N = 3$. This is the JKK model with three sellers and three buyers. The symmetric equilibrium reserve price is $r_{N=3} = \frac{1}{3}$. In equilibrium, each buyer visits a particular seller with probability $\frac{1}{3}$. A seller cannot make a sale if no buyer selects him, which happens with probability $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$, makes a sale at price r if only one buyer selects him, which happens with probability $C_3^1\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 = \frac{12}{27}$, and makes a sale at a price equal to 1 if at least two buyers select him, which happens with probability $1 - \left(\frac{8}{27}\right) - \left(\frac{12}{27}\right) = \frac{7}{27}$. Hence, a seller's expected profit is $\pi_{N=3} = \left(\frac{1}{3}\right)\cdot\left(\frac{12}{27}\right) + 1\cdot\left(\frac{7}{27}\right) = \frac{11}{27}$.

A buyer can get positive payoff by selecting a seller only if the other two buyers select a different seller, which happens with probability $(2/3)^2 = \frac{4}{9}$. So a buyer's expected payoff is $u_{N=3} = \left(1 - \left(\frac{1}{3}\right)\right)\cdot\left(\frac{4}{9}\right) = \frac{8}{27}$.

Finally, there is only one match if all buyers select the same seller. This happens with probability $3\left(\frac{1}{3}\right)^3 = \frac{1}{9}$. There are three matches if each buyer selects a different seller. This happens with probability $P_3^3\left(\frac{1}{3}\right)^3 = \frac{2}{9}$. There are two matches with the reaming probability $1 - \frac{1}{9} - \frac{2}{9} = \frac{2}{3}$. The expected total number of matches is then given by $T_{N=3} = 1 \cdot \left(\frac{1}{9}\right) + 2 \cdot \left(\frac{2}{3}\right) + 3 \cdot \left(\frac{2}{9}\right) = \frac{19}{9}$.

• <u>$N = 1$.</u> With probability $6\left(\frac{1}{3}\right)^2 = \frac{2}{9}$, each buyer observes a distinct seller and selects the seller she observes. With probability $3\left(\frac{1}{3}\right)^3 = \frac{1}{9}$, a buyer observes all sellers and the other two buyers observe nothing. In this case, the fully informed buyer selects the seller with the lowest reserve price. Finally, with probability $2 \cdot 3 \cdot 3 \cdot \left(\frac{1}{3}\right)^3 = \frac{2}{3}$, one buyer observes two sellers while another buyer observes one seller. In this case, the buyer who observes two sellers goes to the one with lower reserve price. As in the two-sellers case, the co-existence of buyers who observe only one seller and buyers who observe more than one seller causes sellers to play a mixed strategy in equilibrium. Let $F(r)$ on $[\underline{r}, \overline{r}]$ be the symmetric equilibrium price distribution. By the same argument as before, the upper bound of the support is $\overline{r} = 1$. Suppose seller 1 sets a reserve price r. If there is a fully informed buyer, the buyer will buy from seller 1 if r is the lowest reserve price. This happens with probability $(1 - F(r))^2$. If there are three buyers, each observes a distinct seller, seller 1 can get a profit equal to r with probability 1. If there is one buyer who observes both seller 1 and another seller, seller 1 can sell with probability $1 - F(r)$. In sum, seller 1's expected profit with a reserve price r is

$$r\left[\frac{1}{9}(1 - F(r))^2 + \left(\frac{2}{9} + \frac{2}{9}\right)(1 - F(r)) + \left(\frac{2}{9} + \frac{2}{9}\right)\right].$$

By the definition of mixed strategy, seller 1 will be indifferent between any r in $[\underline{r}, 1]$ and $r = 1$. If $r = 1$, seller 1 can only sell to the buyers who only observe seller 1. There is a buyer who only observes seller 1 with probability $\frac{4}{9}$. The following indifferent condition must hold

$$r\left[\frac{1}{9}(1 - F(r))^2 + \left(\frac{2}{9} + \frac{2}{9}\right)(1 - F(r)) + \left(\frac{2}{9} + \frac{2}{9}\right)\right] = \frac{4}{9}.$$

The equilibrium price distribution is

$$F(r) = 3 - \frac{2}{\sqrt{r}}.$$

Moreover, $F(\underline{r}) = 0$ implies $\underline{r} = \frac{4}{9}$. Hence, the equilibrium reserve price distribution is described by $F(r) = 3 - \frac{2}{\sqrt{r}}$ on $\left[\frac{4}{9}, 1\right]$. The expected equilibrium reserve price is given by

$$\mathbb{E}(r_{N=1}) = \int_{\frac{4}{9}}^{1} r dF(r) = \frac{2}{3}.$$

Each seller's expected profit is $\pi_{N=1} = \frac{4}{9}$.

A buyer who observes at least one seller can trade by paying $r_{N=1}$ for sure since she will face no competition from other buyers. A buyer's expected payoff is therefore $(1 - \mathbb{E}(r_{N=1})) \left[1 - (\frac{2}{3})^3\right] = \frac{19}{81}$.

With probability $\frac{1}{9}$, there is a fully informed buyer, which results in only one match. With probability $\frac{2}{9}$, each buyer observes a distinct seller and there are three matches. With the remaining probability $1 - \left(\frac{1}{9}\right) - \left(\frac{2}{9}\right) = \frac{2}{3}$, there are two matches. Summing them up all, we have $T_{N=1} = 3(\frac{2}{9}) + 2(\frac{2}{3}) + 1(\frac{1}{9}) = \frac{19}{9}$.

• <u>$N = 2$.</u> There are in total $(C_3^2)^3 = 27$ scenarios. Three of them are such that two buyers are fully informed. This means that the probability of having two fully informed buyers is $\frac{3}{27} = \frac{1}{9}$. Another possibility is that each buyer observes two sellers. There are in total 6 such cases, and so the probability of this scenario is $\frac{6}{27} = \frac{2}{9}$. Also, it could be that one buyer is fully informed, another buyer observes two sellers, and the third buyer only observes one seller. There are in total 18 such cases, and so the probability of this scenario is $\frac{18}{27} = \frac{2}{3}$.

Suppose a buyer is fully informed. There can be only two situations: (i) there exists another fully informed buyers, or (ii) there exists another buyer who observes two sellers i and j and a third buyer who observes seller $k \neq i, j$. Hence, conditional on the buyer in question being fully informed, the probability that case (i) happens is $\frac{2}{2+6} = \frac{1}{4}$, and the probability that case (ii) happens is $\frac{6}{2+6} = \frac{3}{4}$. Now, consider a situation in which seller 1 deviates to a price r_1 while the other two sellers set an equilibrium price r. Denote by $\mu(z)$ the probability that a buyer who observes $z = 1, 2, 3$ sellers chooses to visit seller 1. Clearly, $\mu(1) = 1$. If this fully informed buyer selects seller 1, then her expected payoff, denoted by $u_1(3)$, is

$$u_1(3) = (1 - r_1) \left[\frac{1}{4} \cdot (1 - \mu(3)) + \frac{3}{4} \cdot \frac{2}{3} \cdot (1 - \mu(2))\right].$$

If this buyer is the only visitor to seller 1, she obtains $1 - r_1$. With probability $\frac{1}{4}$, there is another fully informed buyer, and with probability $1 - \mu(3)$, this buyer does not select seller 1. With probability $\frac{3}{4}$, there are two partially informed buyers. In this event, one possibility is that seller 1 is in the information set of the buyer who observes two sellers (which happens with probability $\frac{2}{3}$). This buyer will not choose seller 1 with probability $1-\mu(2)$. Another possibility is that seller 1 is in the information set of the buyer who only observes one seller (which happens with probability $\frac{1}{3}$). In this case the buyer will select seller 1 for sure.

If the fully informed buyer instead chooses a non-deviating seller (who sets r), her expected payoff, denoted by $u(3)$, is

$$u(3) = (1-r)\left[\frac{1}{4}\cdot\left(1-\frac{1-\mu(3)}{2}\right)+\frac{3}{4}\left(\frac{1}{3}\cdot\frac{1}{2}+\frac{1}{3}\cdot\mu(2)\right)\right]$$

If there is another fully informed buyer (which happens with probability $\frac{1}{4}$), she will choose to visit the same non-deviating seller with probability $\frac{1-\mu(3)}{2}$. If the other two buyers are partially informed (one observes two sellers and one observes one seller), there are three scenarios. First, seller 1 is observed by the buyer who only observes one seller. This happens with probability $\frac{1}{3}$. In this case, the other buyer visits the two non-deviating sellers with equal probability. Second, seller 1 and the non-deviating seller in question are observed by the same buyer. This happens with probability $\frac{1}{3}$. In this case, the buyer will visit seller 1 with probability $\mu(2)$. Finally, the non-deviating seller in question is observed by the other buyer who only observes one seller. In this case, selecting this seller yields zero payoff.

We next describe the decision problem of a partially informed buyer who observes two sellers. If a buyer only observes the two non-deviating sellers, she will choose each of them with probability $\frac{1}{2}$. If a buyer observes seller 1 and a non-deviating seller, which we name seller 2, her strategy depends on both r_1 and r. Conditional on this event, there are two possible scenarios: (i) there is a fully informed buyer and another buyer who only observes the other non-deviating seller, seller 3, which happens with probability $\frac{1}{2}$, or (ii) two other partially-informed buyers, with one observing sellers 1 and 3 and the other observing sellers 2 ad 3, which happens with probability $\frac{1}{2}$. If the buyer in question (who observes sellers 1 and 2) decides to select seller 1, her expected payoff, denoted by $u_1(2)$, is

$$u_1(2) = (1-r_1)\left[\frac{1}{2}(1-\mu(3))+\frac{1}{2}(1-\mu(2))\right]$$

In scenario (i), seller 1 must be in the information set of the fully informed buyer. This fully informed buyer will not choose seller 1 with probability $1-\mu(3)$. In scenario (ii), firm 1 is in the information set of a buyer who observes two sellers. Then this buyer will not select seller 1 with probability $1-\mu(2)$.

If the buyer in question decides to select seller 2, her expected payoff, denoted by $u(2)$ is

$$u(2)=(1-r)\left[\frac{1}{2}\left(1-\frac{1-\mu(3)}{2}\right)+\frac{1}{2}\cdot\frac{1}{2}\right].$$

In scenario (i), seller 2 must be in the information set of the fully informed buyer. This fully informed buy will not select seller 2 with probability $1-\frac{1-\mu(3)}{2}$. In scenario (ii), seller 2 must be in the information set of another buyer who observes two non-deviating sellers, i.e., seller 2 and 3. This buyer will not select the same non-deviating seller with probability $\frac{1}{2}$.

Using the indifference condition $u_1(3)=u(3)$ and $u_1(2)=u(2)$, we obtain the buyers' equilibrium selection strategies,

$$\mu(2)=\frac{r_1-1}{r+r_1-2} \quad \text{and} \quad \mu(3)=\frac{2\left(-r^2+rr_1+r+r_1{}^2-3r_1+1\right)}{(r+r_1-2)(r+2r_1-3)}.$$

We now turn to seller 1's problem. In the case of two fully informed buyers, the probability of getting zero visitor is $(1-\mu(3))^2$, of getting one visitor is $2\mu(3)(1-\mu(3))$, and of getting two visitors is $(\mu(3))^2$. In the case of having one fully informed buyer and two partially informed buyers, the probability of having zero visitor is $(1-\mu(3))(1-\mu(2))\frac{2}{3}+(1-\mu(3))\cdot 0\cdot\frac{1}{3}=(1-\mu(3))(1-\mu(2))\frac{2}{3}$, of having one visitor is $[\mu(3)(1-\mu(2))+\mu(2)(1-\mu(3))]\frac{2}{3}+(1-\mu(3))(1-\mu(2))\frac{1}{3}$, of having two buyers is $\frac{1}{3}(\mu(2)\mu(3)+\mu(2)+\mu(3))$. In the case with three partially informed buyers, the probability of having zero visitor is $(1-\mu(2))^2$, of having one visitor is $2\mu(2)(1-\mu(2))$, and having two visitor is $(\mu(2))^2$. Aggregating the probabilities over the three scenarios, we get the followings: the probability of having zero visitor,

$$\Pr[n_1=0]=\frac{1}{9}(1-\mu(3))^2+\frac{2}{3}(1-\mu(3))(1-\mu(2))\frac{2}{3}+\frac{2}{9}(1-\mu(2))^2,$$

the probability of having one visitor,

$$\Pr[n_1=1]=\frac{1}{9}[2\mu(3)(1-\mu(3))]+\frac{2}{3}\left[[\mu(3)(1-\mu(2))+\mu(2)(1-\mu(3))]\frac{2}{3}+(1-\mu(3))\frac{1}{3}\right]$$
$$+\frac{2}{9}[2\mu(2)(1-\mu(2))],$$

the probability of having two visitors,

$$\Pr[n_1=2]=\frac{1}{9}(\mu(3))^2+\frac{2}{3}\left[\frac{2}{3}\mu(3)\mu(2)+\frac{1}{3}\mu(3)\right]+\frac{2}{9}(\mu(2))^2.$$

Holding r fixed, seller 1 solves

$$\pi_1 = \Pr[n_1 = 1]r_1 + \Pr[n_1 = 2],$$

subject to $u_1(3) = u(3)$ and $u_1(2) = u(2)$. Using the first-order conditions, we obtain the symmetric equilibrium reserve price,

$$r_{N=2} = \frac{59}{232}.$$

Each seller's expected profit is

$$\pi_{N=2} = \frac{733}{2088} \approx 0.351.$$

A fully informed buyer's expected payoff is $u(3) = \frac{865}{2784}$, a partially informed buyer with two observations has an expected payoff $u(2) = \frac{1211}{2784}$, and a partially informed buyer with one observation has an expected payoff $u(1) = (1 - 1/3)(1/2)(1 - r) = \frac{173}{696}$. A buyer is fully informed with probability $\phi_3 = (1/3)^3 = 1/27$, partially informed with two observations with probability $\phi_2 = 3(1/3)^2(2/3) = 2/9$, and partially informed with one observation with probability $\phi_1 = 3(1/3)(2/3)^2 = 4/9$. So a buyer's expected payoff when $N = 2$ is

$$u_{N=2} = \sum_i \phi_i u(i) = \frac{16435}{75168} \approx 0.219.$$

Each seller's match probability is $1 - \Pr(n = 0) = \frac{112}{152}$. So the total number of matches is

$$T_{N=2} = 3 \cdot \frac{112}{162} \approx 2.07.$$

• **Comparison.** Comparing the above equilibrium outcome/payoffs for the three-by-three case, we have

$$\pi_{N=1} = 0.444 > \pi_{N=3} = 0.407 > \pi_{N=2} = 0.351,$$

$$T_{N=1} = T_{N=3} = 2.111 > T_{N=2} = 2.07,$$

$$u_{N=3} = 0.296 > u_{N=1} = 0.235 > u_{N=2} = 0.219.$$