Gaussian approximations for high-dimensional random fields

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In this talk, we derive Gaussian approximations and develops bootstrap methods for high-dimensional sample means of a random field $\{\boldsymbol{Y}(\boldsymbol{s}) : \boldsymbol{s} \in \mathbb{R}^d\}$ in $\mathbb{R}^p$ which is approximated by a strictly stationary random field and is observed at a finite number of locations in the sampling region R_n with the volume $O(\lambda_n^d)$.

In particular, we give Gaussian and bootstrap approximations for probabilities that the normalized sample means of discretely observed random field of the form $\sqrt{\lambda_n^d}\bar{\boldsymbol{Y}}_n = (n^2\lambda_n^{-d})^{-1/2}\sum_{j=1}^n \boldsymbol{Y}(\boldsymbol{s}_j)$ hit hyperrectangles even if $p = p_n \to \infty$ and $p \gg n$ as $n \to \infty$, where the locations $\boldsymbol{s}_j$'s are specified by a stochastic spatial sampling design with mixed increasing asymptotics ($\lim_{n\to\infty} n\lambda_n^{-d} = \infty$). The proposed bootstrap method is an extension of a wild bootstrap for (irregularly spaced) time series to irregularly spaced spatial data.

A notion of approximately m_n-dependent random field is introduced and we find that our results can be applied to a wide class of random fields such as multivariate continuous autoregressive moving average (CARMA) random fields on $\mathbb{R}^d$ which are multivariate extensions of univariate CARMA random fields developed in Brockwell and Matsuda (2017).

Our results can be applied to (i) the simultaneous inference of the mean vector of a multivariate random field, (ii) the inference of time-varying mean of a univariate spatio-tamporal compound Poisson-driven CARMA random field, (iii) the construction of joint confidence bands for mean functions of a multivariate spatio-temporal model, and (iv) multiple change point tests in temporal domain for uni- and multivariate spatio-temporal models.

References

Brockwell, P.J. and Matsuda, Y. (2017). Continuous auto-regressive moving average random fields on $\mathbb{R}^n$. *J. Roy. Statist. Soc. Ser. B Stat. Methodol.* **79** 833-857.