Buck-passing Dumping in a Pure Exchange Game of Bads

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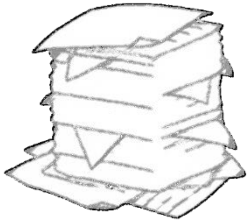
Bads?

A “bad” is a commodity that causes **disutility** to its owner.

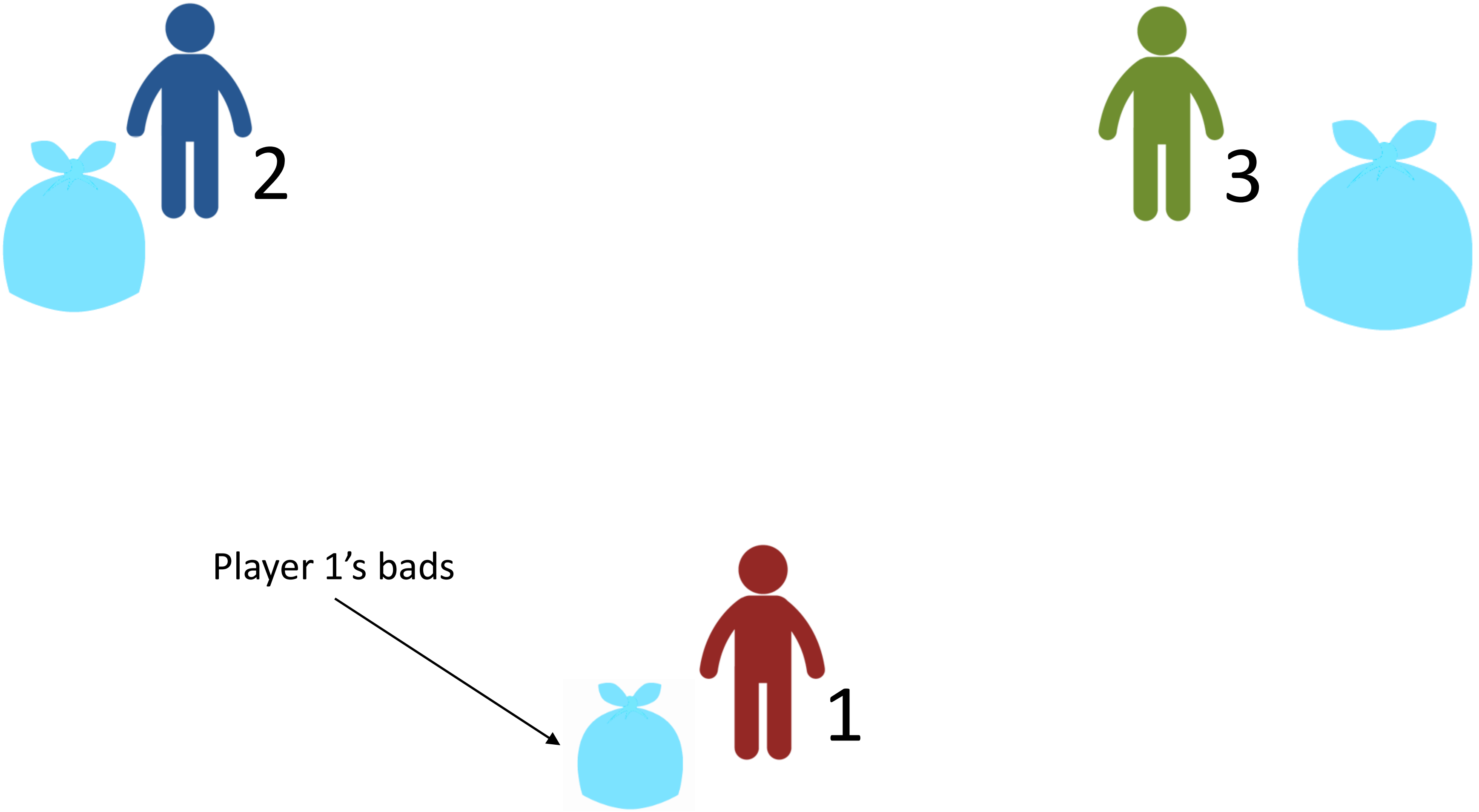
Garbage



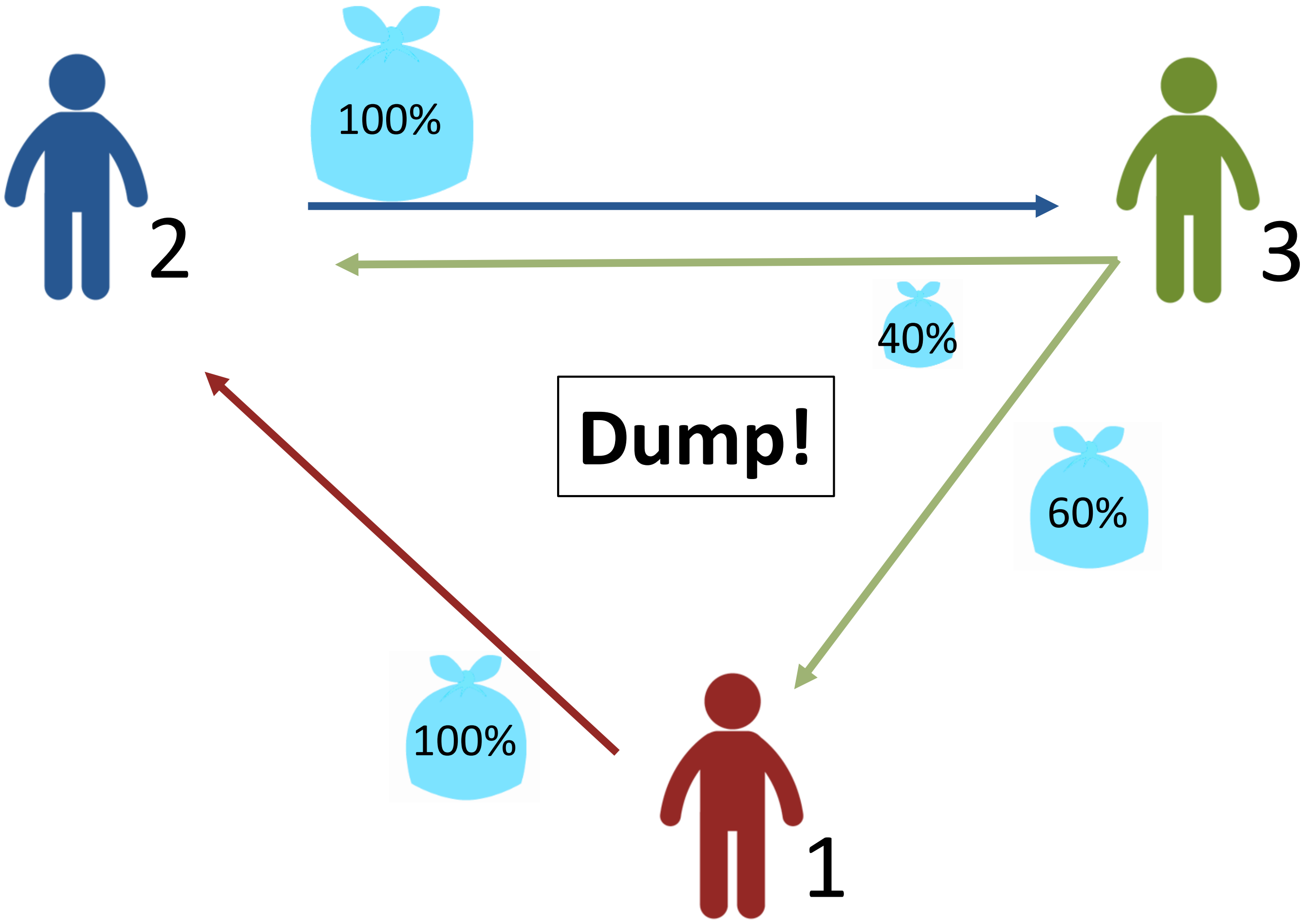
Unimportant tasks

Goods: Utility ***increases*** as its quantity increases.
Bads: Utility ***decreases*** as its quantity increases.

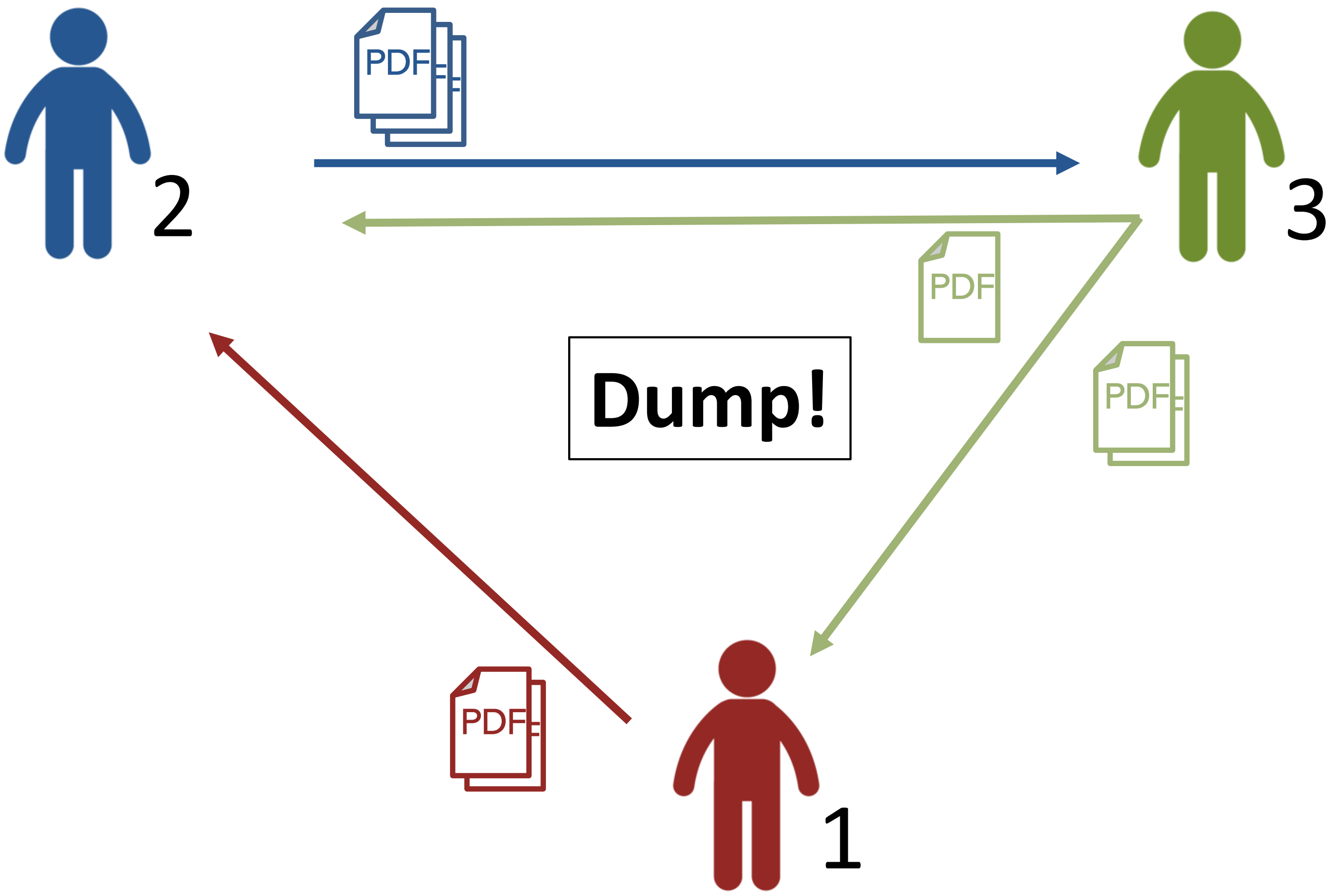
Dumping bads



Dumping bads



Dumping bads



Shapley-Shubik (1969) & Hirai et al. (2006)

Shapley and Shubik (1969)

Cooperative game with transferable utility

- Each player has a bag of garbage.
- Each player dumps his/her bads in someone's yard.
- Players can form **a coalition S**.
- If S is formed, S dumps bads all to $N \setminus S$, and $N \setminus S$ to S: for any $S \subsetneq N$,

$$v(S) = \sum_{j \in N \setminus S} b^j; \text{ and } v(N) = \sum_{j \in N} b^j.$$

Hirai, Masuzawa, and Nakayama (2006)

Strategic form game with joint strategies

- Each player strategically dumps bads to someone else.
- **Player i's strategy is a distribution of i's bads** over all players.
- Players can form a coalition S and take a joint strategy.
- Scarf's (1971) pure exchange game with goods being replaced by bads.

Motivation and model selection

Our research question is:
> why does **buck-passing dumping behavior** last everywhere?
> why do a small number of individuals or nations dispose of a large quantity of bads?
✓ Explain these cases in terms of α-stability.

We need
- who dumps how many bads to whom.
- what redistribution of bads results.

<u>Shapley and Shubik (1969)</u>
Cooperative game with transferable utility

<u>Hirai, Masuzawa, and Nakayama (2006)</u>
Strategic form game with joint strategies

Figure: Demonstrators hold placards while lying down on the road during a protest at the Canadian embassy in the Philippines.

Photograph: Mark R Cristino/EPA
www.theguardian.com/world/2019/may/23/philippines-threatens-to-dump-rubbish-back-in-canadian-waters-as-row-deepens

Model

- $N = \{1, \dots, n\}$ a player set
- $b^i > 0$ i's initial endowment of bads
  - one type
  - homogeneous
  - divisible

  $b = (b^1, \dots, b^n) \in \mathbb{R}^N_{++}$ $b^n \geq \dots \geq b^1$ without loss of generality

- $x^i = \left(x^{i1}, \dots, x^{in}\right) \in \mathbb{R}^N_+$ with $\sum_{j \in N} x^{ij} = b^i$ i's strategy

  $X^i_b = \left\{x^i \in \mathbb{R}^N_+ \mid \sum_{j \in N} x^{ij} = b^i\right\}$ i's strategy set

  $x = (x^1, \dots, x^n) \in \mathbb{R}^{N \times N}_+$ a strategy profile

- $v^i(x) := u^i\left(\sum_{j \in N} x^{ji}\right)$ u^i is a ***strictly decreasing*** utility function of i

- $G_b = \left(N, \{X^i_b\}_{i \in N}, \{v^i\}_{i \in N}\right)$ A pure exchange game of bads w.r.t. b

Stability

α-Core
Aumann and Peleg (1960)

β-Core
Aumann and Peleg (1960)

Strong Nash Equilibria

γ-Core
Chander and Tulkens (1997)

δ-Core
Currarini and Marini (2004)

Coalition-proof Nash Equilibria

Stability

α-Core Aumann and Peleg (1960)	$\supseteq$	β-Core Aumann and Peleg (1960)	$\supseteq$	Strong Nash Equilibria
		In general		$\cap\vert$
γ-Core Chander and Tulkens (1997)	$\supseteq$	δ-Core Currarini and Marini (2004)		Coalition-proof Nash Equilibria

Stability

α-Core
Aumann and Peleg (1960)

=

β-Core
Aumann and Peleg (1960)

$\supseteq$

Strong Nash Equilibria

||

Coalition-proof Nash Equilibria

In an exchange game of bads

Empty
γ-Core
Chander and Tulkens (1997)

Empty
δ-Core
Currarini and Marini (2004)

Stability

α-Core
Aumann and Peleg
(1960)

$=$

β-Core
Aumann and Peleg
(1960)

$\supseteq$

Strong
Nash Equilibria

$||$ In an exchange game of ***goods*** $||$

Nonempty if quasi-concave

γ-Core
Chander and Tulkens
(1997)

$=$

δ-Core
Currarini and Marini
(2004)

Coalition-proof
Nash Equilibria

Stability

<u>α-Core (Aumann and Peleg, 1960)</u>

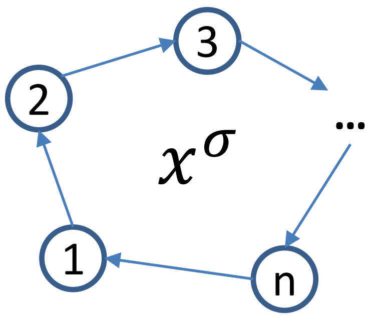
- A coalition $S \subseteq N$ is ***α-effective*** for $x \in X^N$ if there is a strategy profile $y^S \in X^S$ such that for any $z^{N \setminus S} \in X^{N \setminus S}$,
$$v^i\left(y^S, z^{N \setminus S}\right) > v^i(x) \text{ for every } i \in S.$$

- A strategy profile x is ***α-stable***, or is an α-core element, if no coalition is α-effective for $x \in X^N$.

What profile is α-stable?

Facts by Hirai et al. (2006)

σ-cycle dumping



Ordering

- Let σ be an **ordering** of all players: $\sigma(k)$ is the kth player.
- Let $\sigma(1) = 1$.
- Let $\sigma(n+1) = \sigma(1)$ and $\sigma(1-1) = \sigma(n)$

For $\sigma \in \Psi^N$, $\lambda(i)$ is the **predecessor** of i and $\eta(i)$ is the **successor** of i: for some index k with $i = \sigma(k)$,

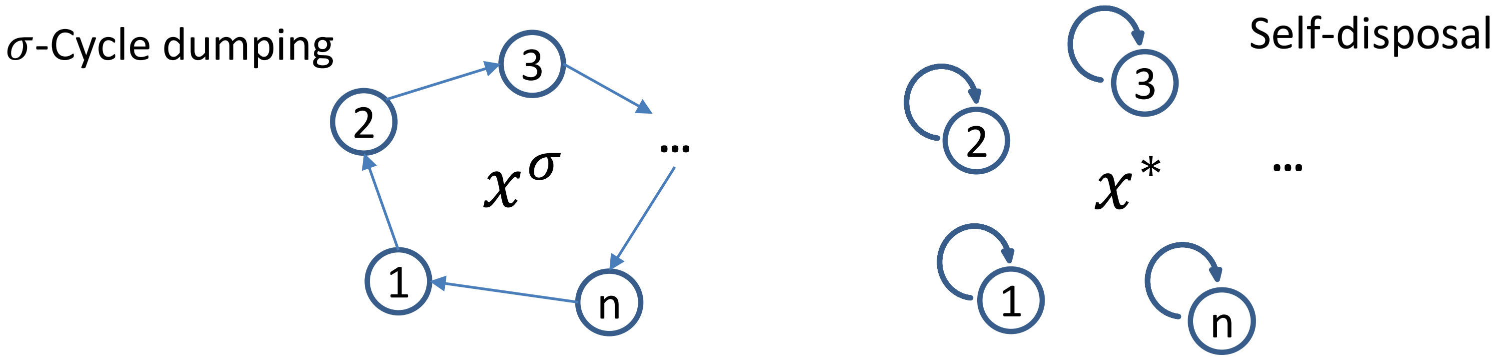
$$\lambda(i) := \sigma(k-1), \eta(i) := \sigma(k+1).$$

σ-Cycle dumping

Let $\sigma \in \Psi^N$. ***σ-Cycle dumping*** x^σ is given as follows: for any $b \in B^N$ and any $i \in N$,

$$x^\sigma(b)^{i\eta(i)} = b^i.$$

Facts by Hirai et al. (2006)



Self-disposal

- Define $x^*(b)$ as follows: for any $b \in B^N$, $x^*(b)^{ii} = b^i$ for all $i \in N$.

Facts by Hirai et al. (2006)

Hirai et al. (2006)

1. For any $b \in B^N$ and any ordering $\sigma \in \Psi^N$, $x^\sigma(b)$ is a strong Nash equilibrium.
2. If $\sum_{j=1}^{k} b^j \geq b^{k+1}$ for all $k = 1, \ldots, n-1$, then $x^*(b)$ is α-stable.

1. Strong Nash equilibrium:

Let $SNE(b)$ be the set of all strong Nash equilibria for b.

Hirai et al. (2006) show that for any $b \in B^N$,

$$SNE(b) \supseteq \{x^\sigma(b) | \forall \sigma \in \Psi^N\}.$$

We show that for any $b \in B^N$,

$$SNE(b) = \{x^\sigma(b) | \forall \sigma \in \Psi^N\}.$$

>> σ-cycle dumping is the only dumping strategy that generates a strong Nash equilibrium.

Therefore, we have $SNE(b) = CPNE(b) = \{x^\sigma(b) | \forall \sigma \in \Psi^N\}$.

2. α-Stability

- Hirai condition requires that there is **no "very big" player** such that $b^{k^*} > \sum_{j=1}^{k^*-1} b^j$. This is a sufficient condition for self-disposal profile to be α-stable.

Our approach

Hirai's proposition shows that
a particular profile becomes α-stable if **a condition for $\boldsymbol{b}$ is satisfied**.

Step 1: Can we capture all α-stable profiles?
>>> Offer **a necessary and sufficient condition** (in terms of x) for x to be α-stable.

Step 2: What dumping behavior or strategies generate α-stable profiles?
>>> Provide **some dumping strategies** that generate α-stable profiles **without any condition for $\boldsymbol{b}$**.

Necessary and sufficient condition

Notation

For each $i \in N$, we write $r_x^i := \sum_{j \in N} x^{ji}$, the quantity of the bads player i receives in profile x. Let $r_x = (r_x^1, \ldots, r_x^n)$.

Proposition 1

Let $b \in B^N$. A strategy profile $x \in X_b^N$ is α-stable if and only if for any $S \subsetneq N$,

$$\sum_{j \in N \setminus S} b^j \geq r_x^i \text{ for some } i \in S.$$

Implication

Profile r_x is **informative enough** (and is the only information needed) to verify whether x is α-stable.

$$x \in \mathbb{R}_+^{N \times N} \xRightarrow{\text{Reduction}} r_x \in \mathbb{R}_+^N$$

However…

We cannot derive any **strategic behavior** that generates stable profiles from Prop.1.

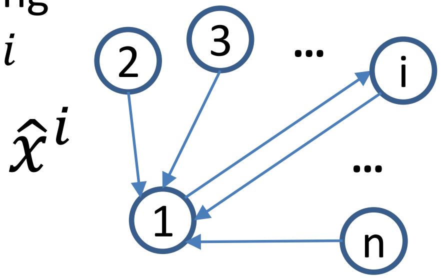
New profiles – Focus dumping

Since $b^n \geq \ \ldots \geq b^1$, we call player 1 **the "smallest" player**.

- Let $i \in N \setminus \{1\}$. ***Focus dumping* on 1 against *i***, $\hat{x}^i$, is given as follows: for any $b \in B^N$ and every $j \in N \setminus \{1\}$,

$$\hat{x}^i(b)^{j1} = b^j \text{ and } \hat{x}^i(b)^{1i} = b^1.$$

Focus dumping on 1 against i



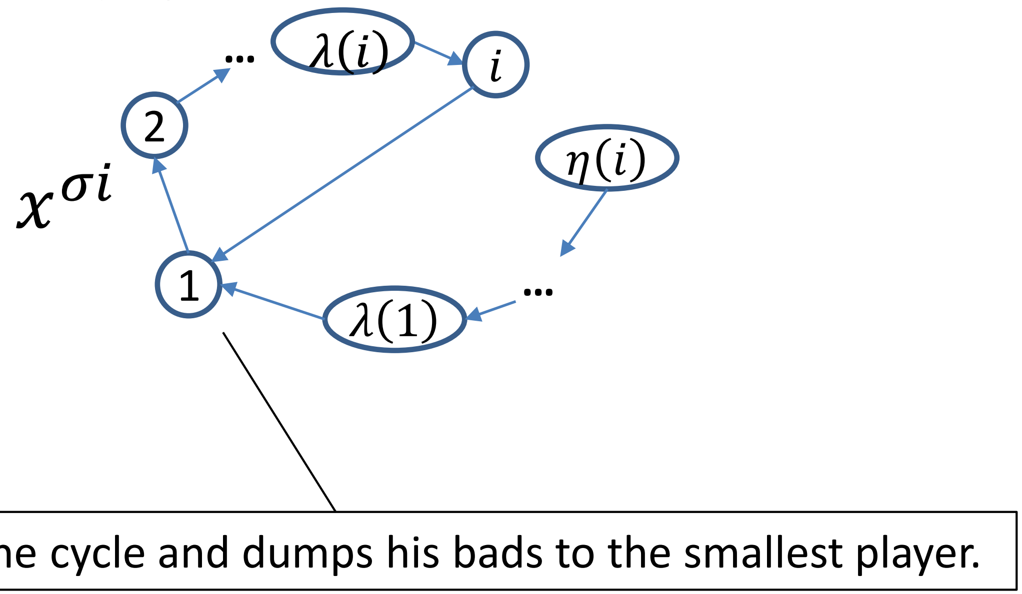
All the players dump their bads onto **the smallest player**.
Player 1 dumps his bads to player i.

New profiles – Incomplete cycle dumping

- Let $\sigma \in \Psi^N$ and $i \in N \setminus \{\lambda(1)\}$. ***i-Incomplete σ-cycle dumping*** $x^{\sigma i}(b)$ is given as follows: for any $b \in B^N$ and every $j \in N \setminus \{i\}$,

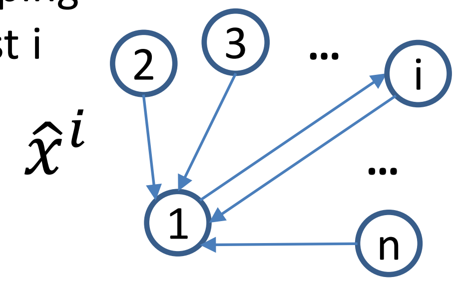
$$x^{\sigma i}(b)^{j\eta(j)} = b^j \text{ and } x^{\sigma i}(b)^{i1} = b^i.$$

i-Incomplete σ-cycle dumping

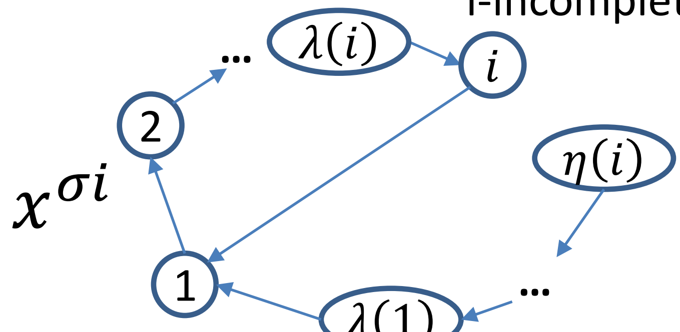


New profiles

Focus dumping
on 1 against i



i-incomplete cycle dumping



Proposition 2
For any b and any σ, all nonnegative convex combinations of the following profiles are α-stable:

$$x^{\sigma}(b), \qquad \hat{x}^{i}(b) \text{ for all } i \in N \setminus \{1\}, \qquad x^{\sigma i}(b) \text{ for all } i \in N \setminus \{\lambda(1)\}$$

Cycle dumping — Focus dumping — Incomplete cycle dumping

$\sigma_{\mathcal{P}}$-Cycle dumping

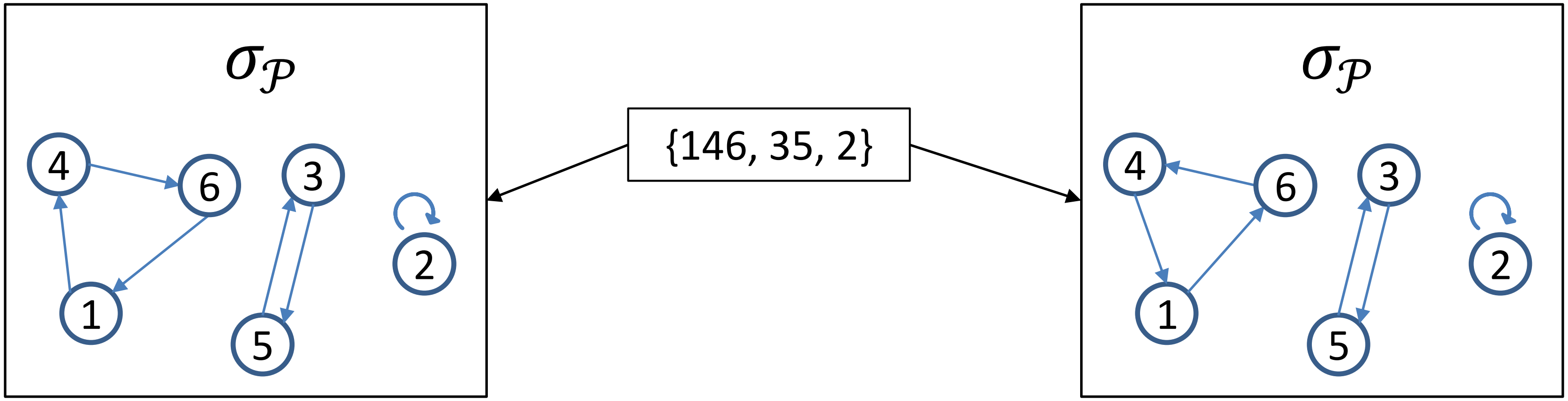
- $\Pi(N)$ Set of all partitions of N, $\Pi^*(N) := \Pi(N) \setminus \{\{N\}\}$

For each $\mathcal{P} \in \Pi(N), \mathcal{P}(i)$ is the coalition to which player i belongs in $\mathcal{P}$.

- Ψ^S Set of all orderings of S

For each $\mathcal{P} \in \Pi^*(N)$, $\Psi^{\mathcal{P}} = \times_{S \in \mathcal{P}} \Psi^S$

- $\sigma_{\mathcal{P}} = \left(\sigma_{S_1}, \ldots, \sigma_{S_{|\mathcal{P}|}}\right) \in \Psi^{\mathcal{P}}$ A partitional ordering



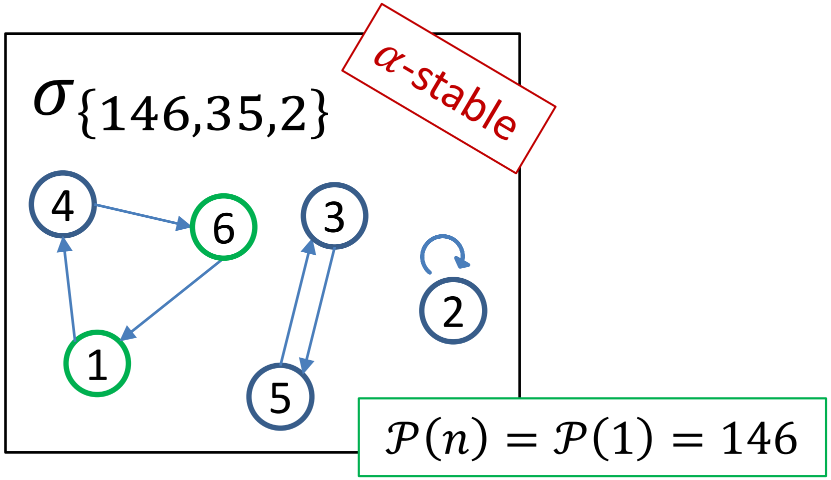
$\sigma_{\mathcal{P}}$-Cycle dumping

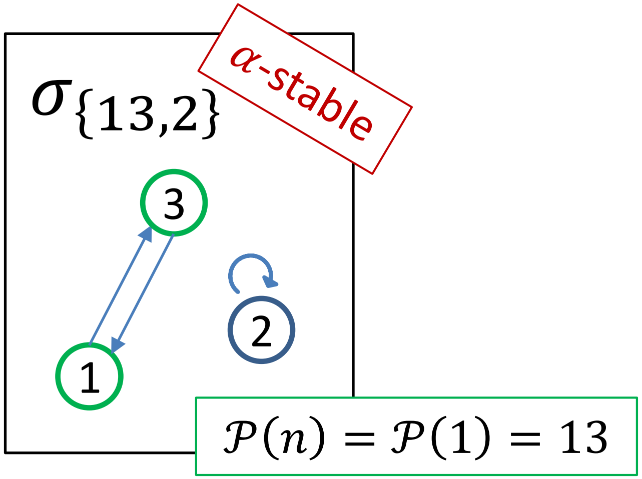
- $\sigma_{\mathcal{P}}$-cycle dumping

Let $\mathcal{P} \in \Pi^*(N)$ and $\sigma_{\mathcal{P}} \in \Psi^{\mathcal{P}}$. ***$\sigma_{\mathcal{P}}$-Cycle dumping*** $x^{\sigma_{\mathcal{P}}}$ is given as follows: for any $b \in B^N$ and every $j \in N$,

$$x^{\sigma_{\mathcal{P}}}(b)^{j\,\eta(j)} = b^j.$$

<u>Lemma</u>
Let $\mathcal{P} \in \Pi^*(N)$ with $\mathcal{P}(n) = \mathcal{P}(1)$. For any $b \in B^N$ and any $\sigma_{\mathcal{P}} \in \Psi^{\mathcal{P}}$, strategy profile $x^{\sigma_{\mathcal{P}}}(b)$ is α-stable.





$\sigma_{\mathcal{P}}$-Cycle dumping with focus

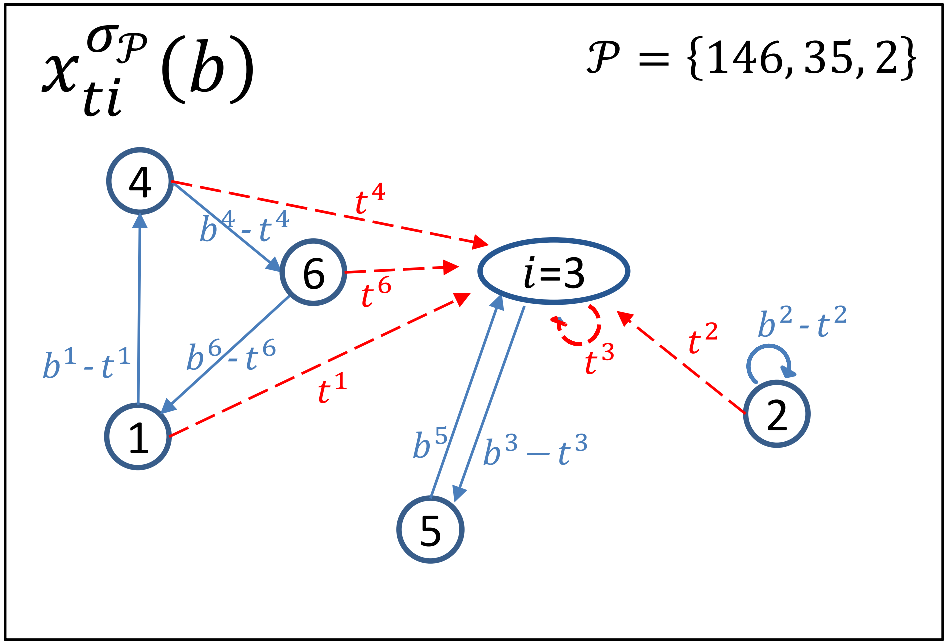
For any $\mathcal{P} \in \Pi^*(N)$ with $\mathcal{P}(n) = \mathcal{P}(1)$, write $T_{\mathcal{P}} \coloneqq \mathcal{P}(n) = \mathcal{P}(1)$.

Let $t \in \mathbb{R}_+$ and $i \in N \setminus T_{\mathcal{P}}$.

- $\sigma_{\mathcal{P}}$-cycle dumping with t-focus on i

$\sigma_{\mathcal{P}}$-Cycle dumping with t-focus on i, $x_{ti}^{\sigma_{\mathcal{P}}}(b)$, is given as follows: for any $b \in B^N$, there is $(t^1, \dots, t^n) \in \mathbb{R}_+^N$ such that

- for every $j \in N$, $0 \leq t^j \leq b^j$, and $t^{\lambda(i)} = 0$,
- $\sum_{j \in N} t^j = t$,
- for every $j \in N$, $x_{ti}^{\sigma_{\mathcal{P}}}(b)^{j\eta(j)} = b^j - t^j$,
- for every $j \in N$, $x_{ti}^{\sigma_{\mathcal{P}}}(b)^{ji} = t^j$.

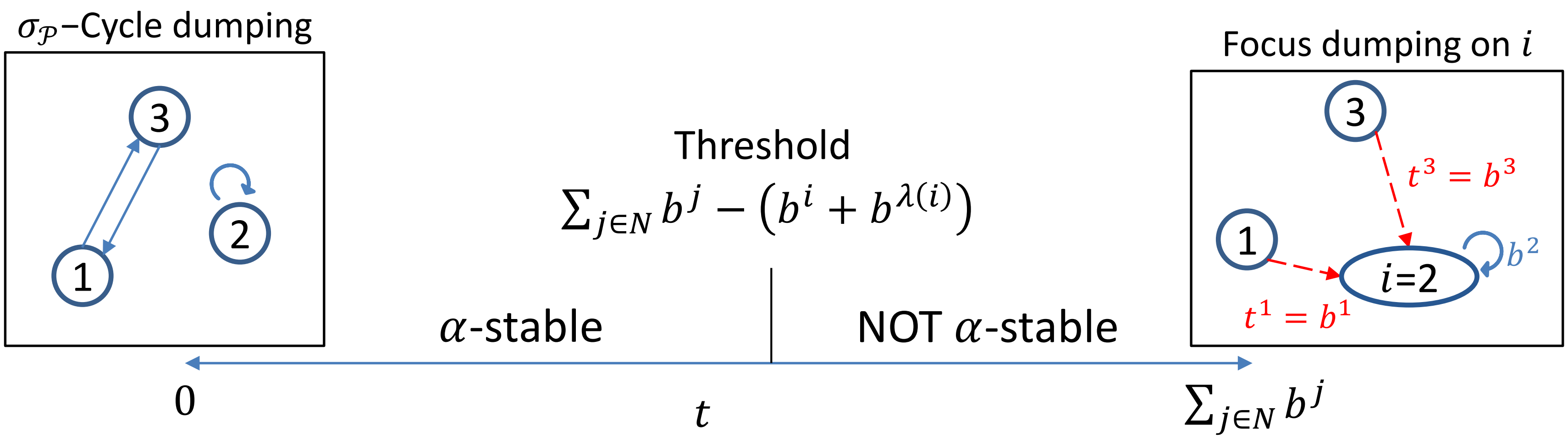


$\sigma_{\mathcal{P}}$-Cycle dumping with focus

Proposition 3

Let $\mathcal{P} \in \Pi^*(N)$ with $\mathcal{P}(n) = \mathcal{P}(1)$, $\sigma_{\mathcal{P}} \in \Psi^{\mathcal{P}}$, $t \in \mathbb{R}_+$, and $i \in N \setminus T_{\mathcal{P}}$.
For $b \in B^N$, the following two statements are equivalent:

- $x_{ti}^{\sigma_{\mathcal{P}}}(b)$ is α-stable,
- $t \leq \sum_{j \in N} b^j - \left(b^i + b^{\lambda(i)}\right)$.



Summary

Proposition 1

We need only r_x to verify whether x is α-stable.

Proposition 2

Cycle dumping, **incomplete cycle** dumping, **focus** dumping, and **their combination** generate an α-stable profile for any $b \in B^N$.

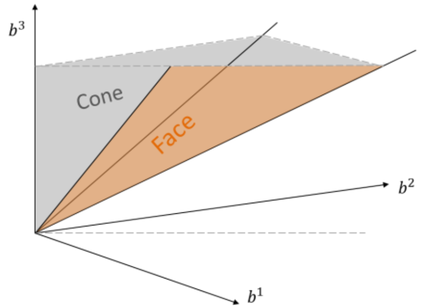
Proposition 3

A profile in which bads are dumped between two disjoint cycles becomes α-stable if and only if the total amount of the bads dumped, namely t, is $\leq$ **the threshold**.

Self-disposal profile

Hirai et al. (2006) show that if b satisfies the condition, then self-disposal profile is α-stable.

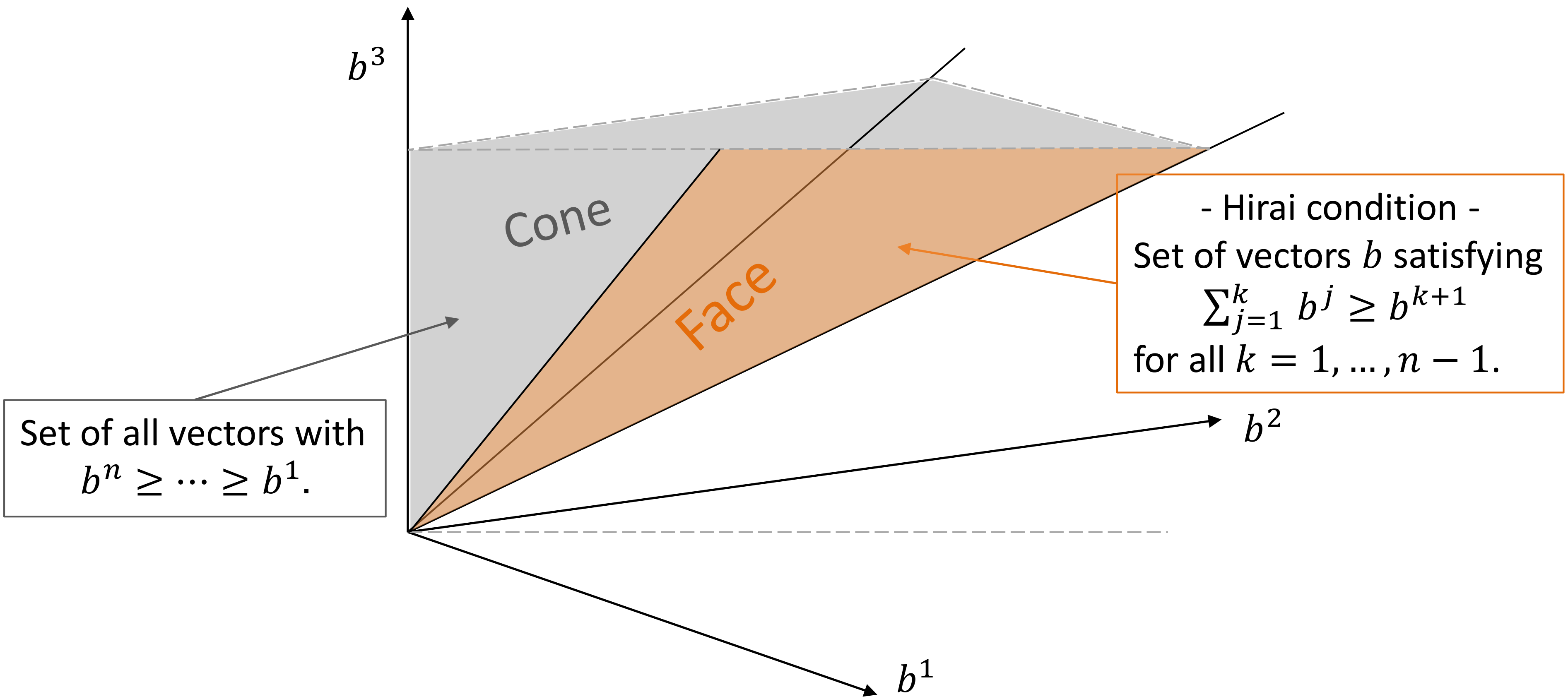
However, Hirai class is not large.



Question:
Can we change the structure of the exchange game to **make self-disposal profile stable** for **any** initial endowments?

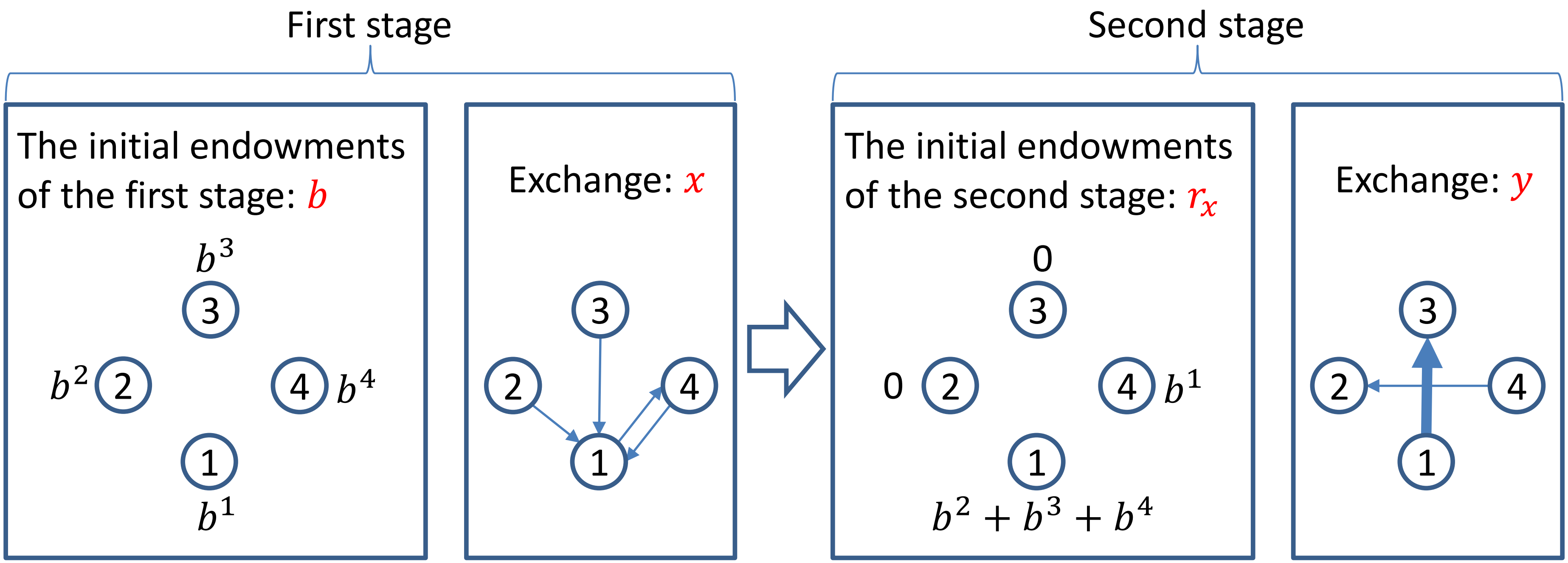
Introduce the second stage.

Hirai condition is restrictive.



Self-disposal profile

Note
$r_x^i := \sum_{j \in N} x^{ji}$
$r_x = (r_x^1, \dots, r_x^n)$



Self-disposal profile

Subgame-perfect equilibrium?

>>> the concept of SPE might be not a good approach because:

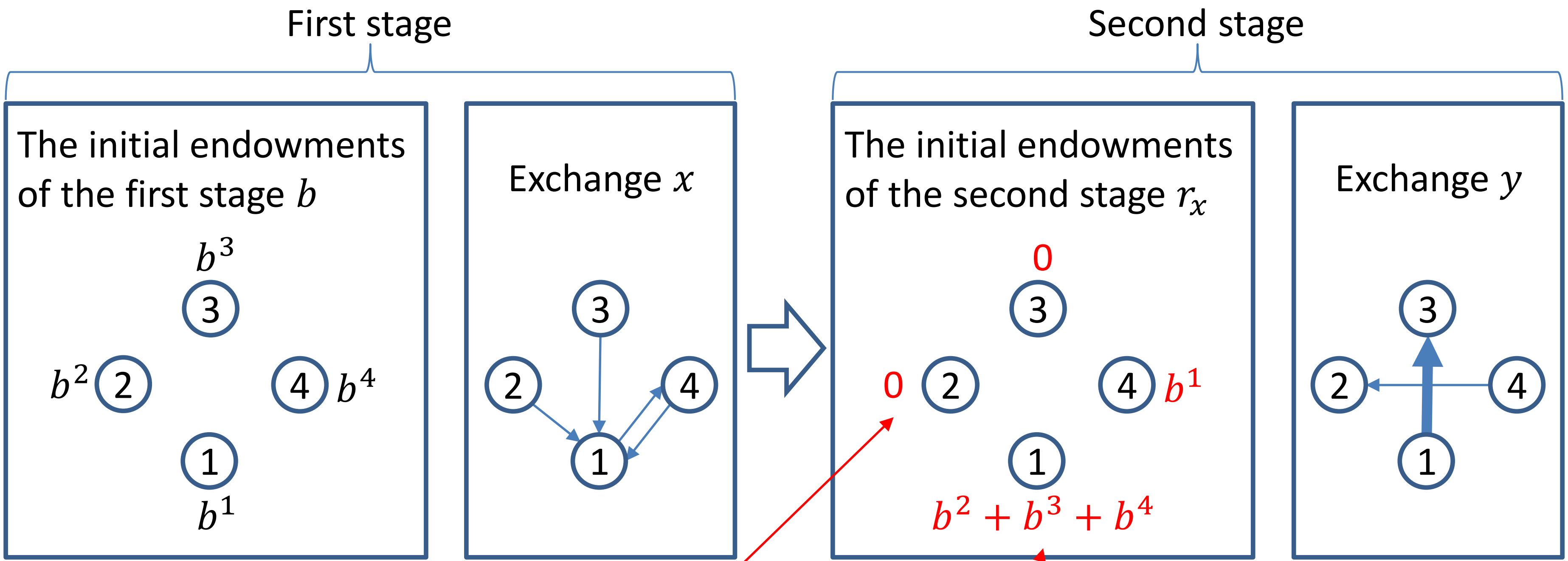
- a SPE captures **individual actions** (not coalitional actions).
- there are **infinitely many SPEs** (because of too many dominant strategies).

⇩

Introducing **another stability notion**
to incorporate *coalitional actions* and *the second stage*.

Self-disposal profile

- How does the second stage influence the players?



$$X_x^2 := X_{r_x}^2 = \{(0,0,0,0)\}$$

$$X_x^1 := X_{r_x}^1 = \left\{(y^{11}, y^{12}, y^{13}, y^{14}) \,\middle|\, \sum_{j \in N} y^{1j} = b^2 + b^3 + b^4\right\}$$

The action sets change depending on x.

m-Stability

Notation

- For any $x \in X_b^N$, let X_x^i denote **the set of actions player *i* can take in the second stage** when x is played in the first stage.

- For every $i \in N$ and $x \in X_b^N$,

$$m^i(x) := \max_{y^i \in X_x^i} \min_{y^{-i} \in X_x^{-i}} v^i\left(y^i, y^{-i}\right),$$

namely, the **maximin payoff** player i guarantees in the second stage when x is played in the first stage.

Definition

- Let $S \subseteq N$. Coalition S ***m-deviates*** from $x \in X_b^N$ if there is $y^S \in X_b^S$ such that for every $i \in S$, $m^i\left(y^S, x^{N \setminus S}\right) > m^i(x)$.

- Profile $x \in X_b^N$ is ***m-stable*** if no coalition m-deviates.

m-Stability

Definition (again)

- Let $S \subseteq N$. Coalition S m-deviates from $x \in X_b^N$ if there is $y^S \in X_b^S$ such that for every $i \in S, m^i\left(y^S, x^{N \setminus S}\right) > m^i(x)$.
- Profile $x \in X_b^N$ is m-stable if no coalition m-deviates.

What is the point of m-stability?

✓ When a coalition S m-deviates from x, the members of S have a joint action y^S by which **all members improve their guaranteed minimum payoffs** in the second stage.

✓ **The cooperation among the members of S is not assumed in the second stage**: the members of S ***agree*** that playing y^S in the first stage gives them higher maximin payoffs than playing x^S and ***not necessarily agree*** that they cooperate with each other again in the second stage.

✓ m-Stability is defined for profiles in the first stage: if a profile is m-stable, the profile is **stationary** in the sense that no player changes his action.

Self-disposal profile

Proposition 4
For any $b \in B^N$, the self-disposal profile $x^*(b)$ is the only profile that is m-stable.

Implication
Counterattacks may block outside dumping: **keeping bads weakens future counterattacks**.

- Mathematically… Without the second stage, each player's payoff is independent of his own action as long as $x^{ii} = 0$.
  >>> The second stage makes their payoffs dependent on their actions via strategy sets.

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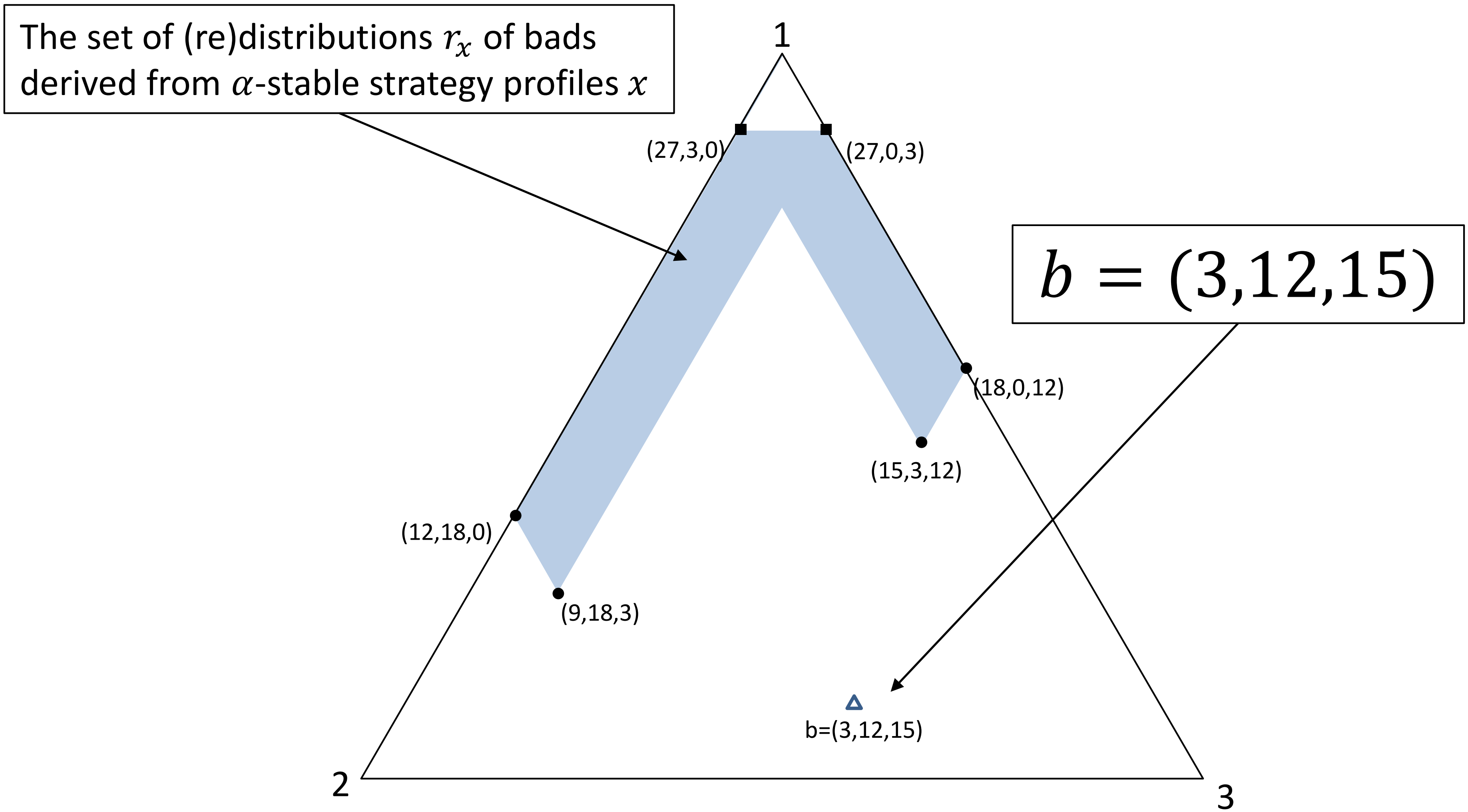
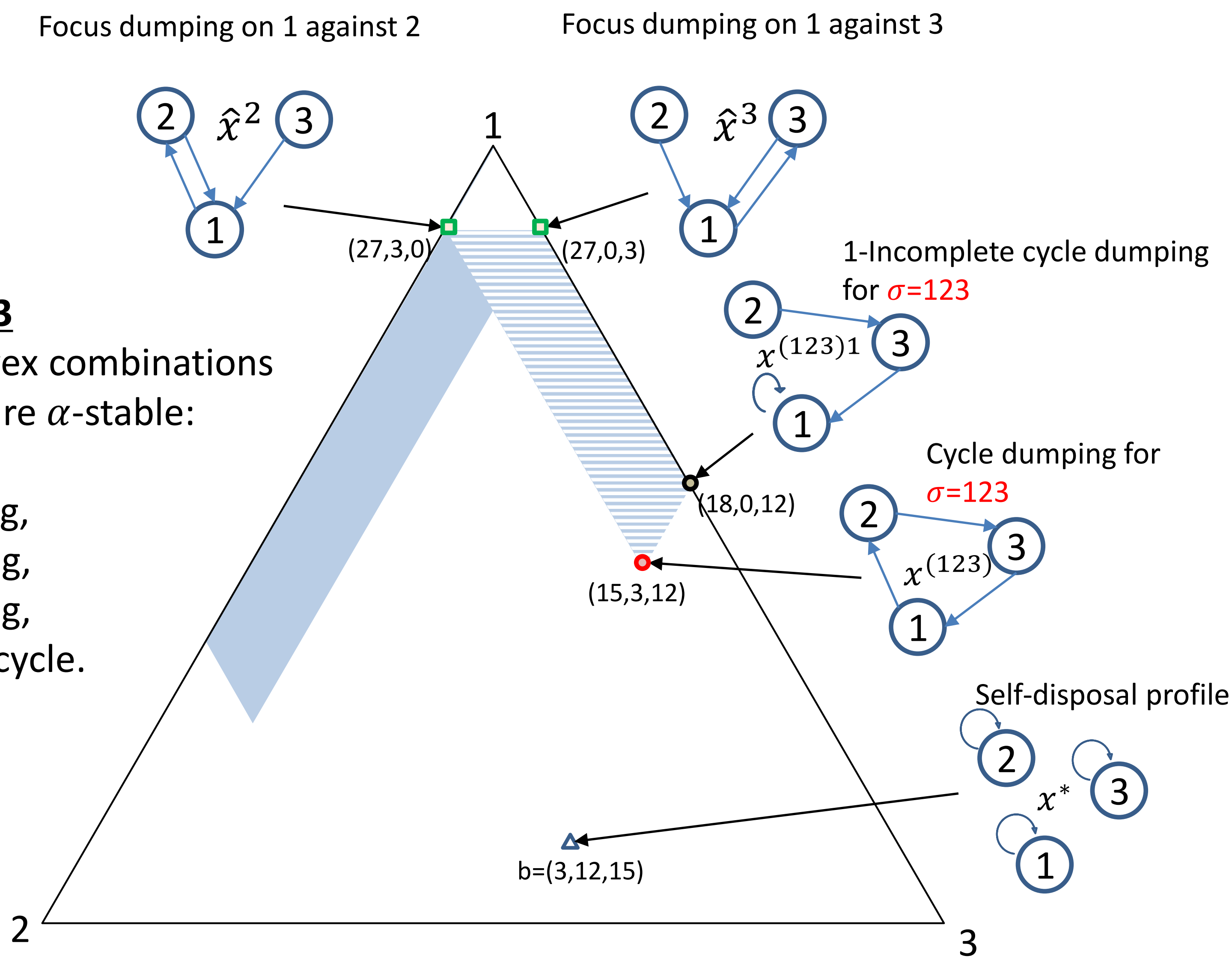
Appendix

Proposition 1



Proposition 2 [$\sigma = 123$]



<u>Proposition 2 for n=3</u>

All nonnegative convex combinations of **the four profiles** are α-stable: for $\boldsymbol{\sigma = 123}$,

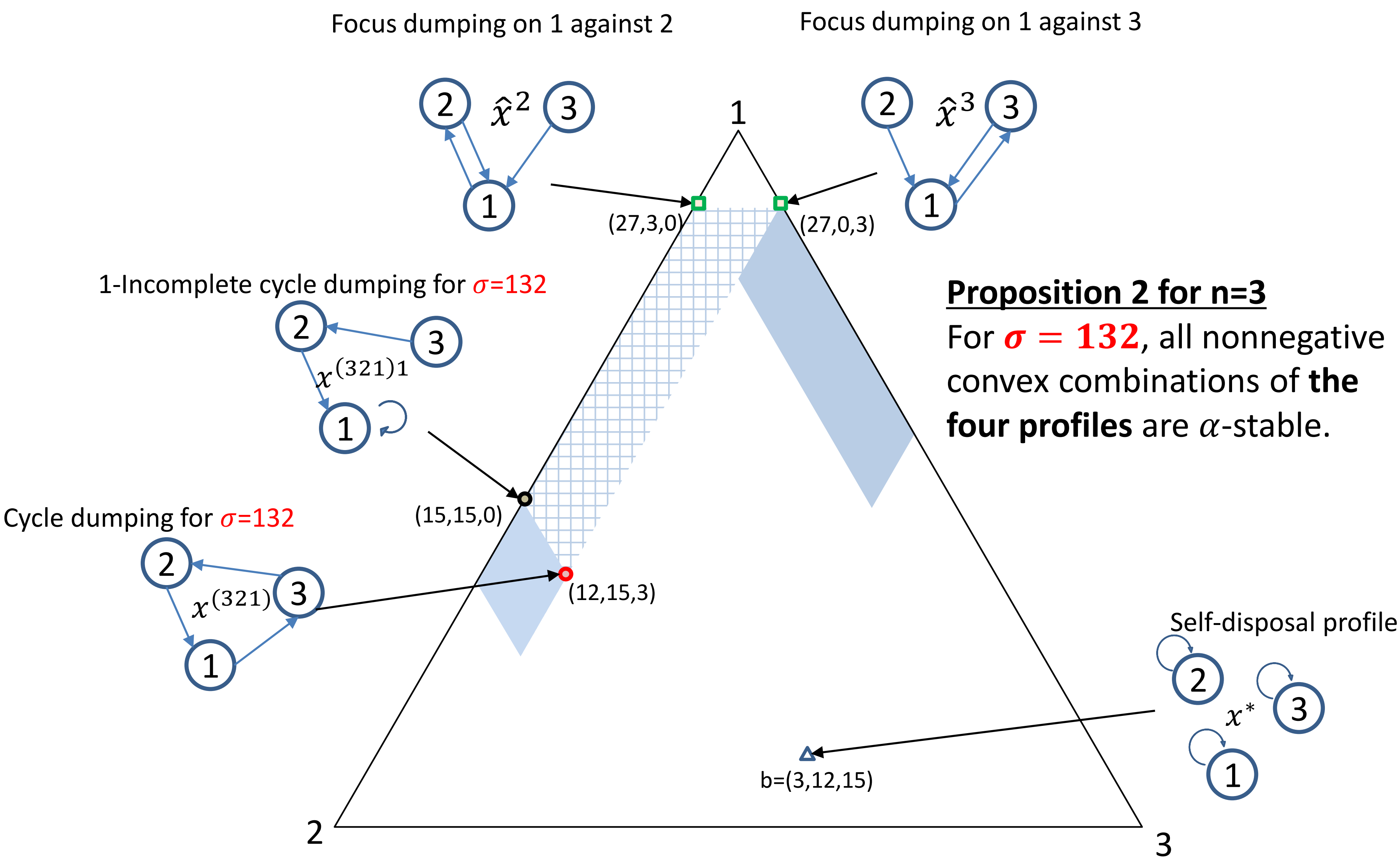
$x^{\sigma}(b)$: cycle dumping,

$\hat{x}^2(b)$: focus dumping,

$\hat{x}^3(b)$: focus dumping,

$x^{\sigma 1}(b)$: incomplete cycle.

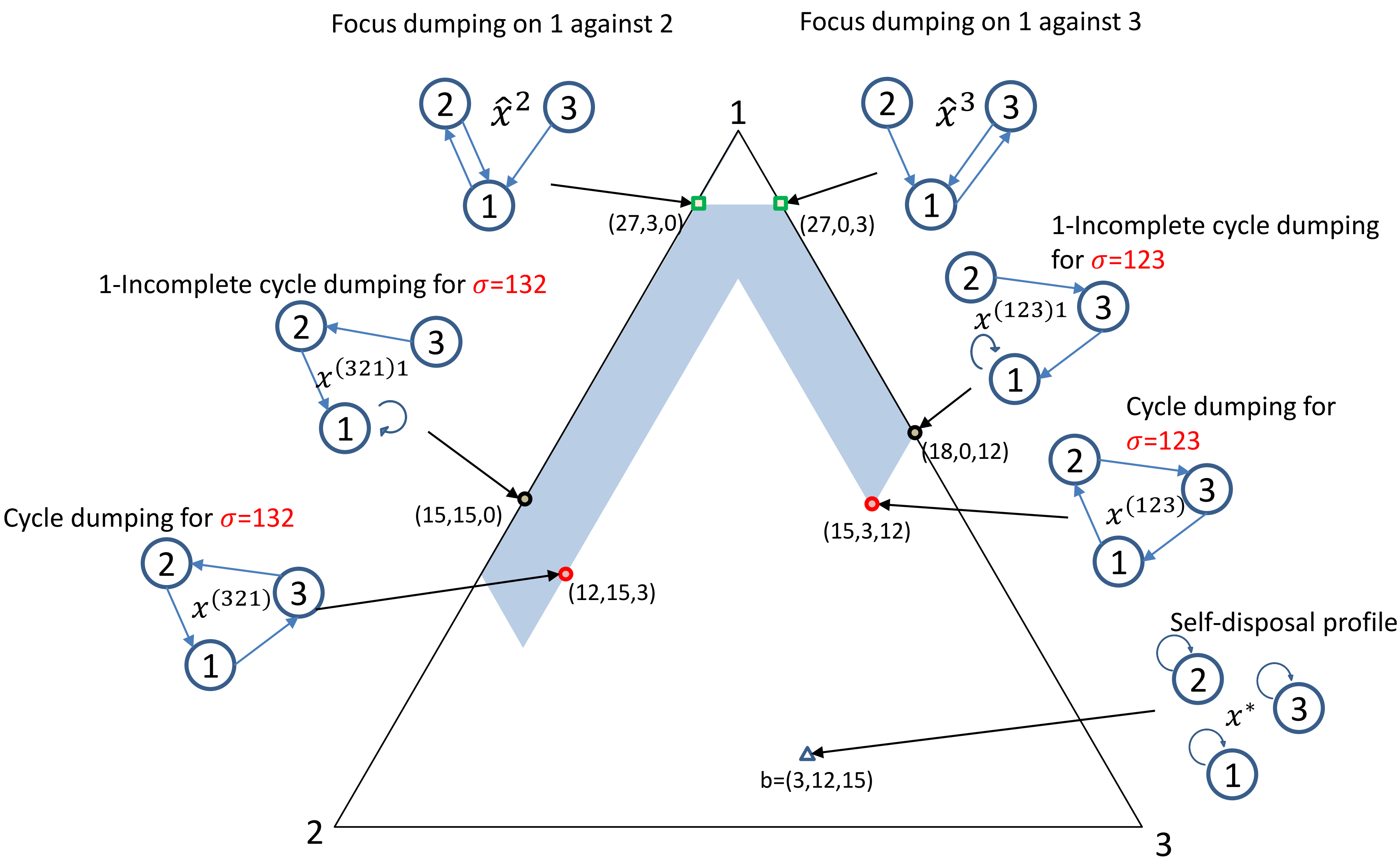
Proposition 2 [$\sigma = 132$]



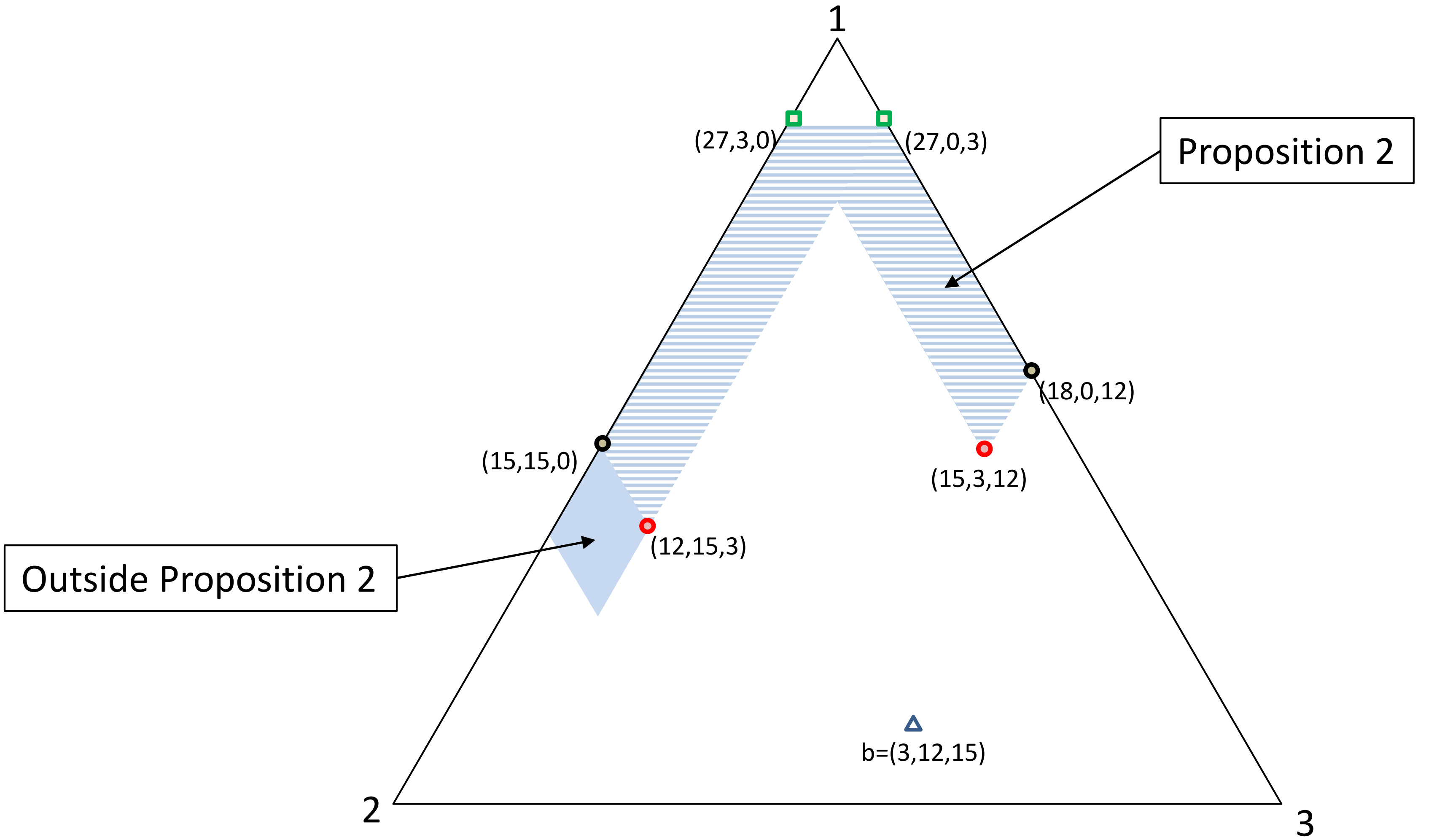
Proposition 2 for n=3

For $\boldsymbol{\sigma = 132}$, all nonnegative convex combinations of **the four profiles** are α-stable.

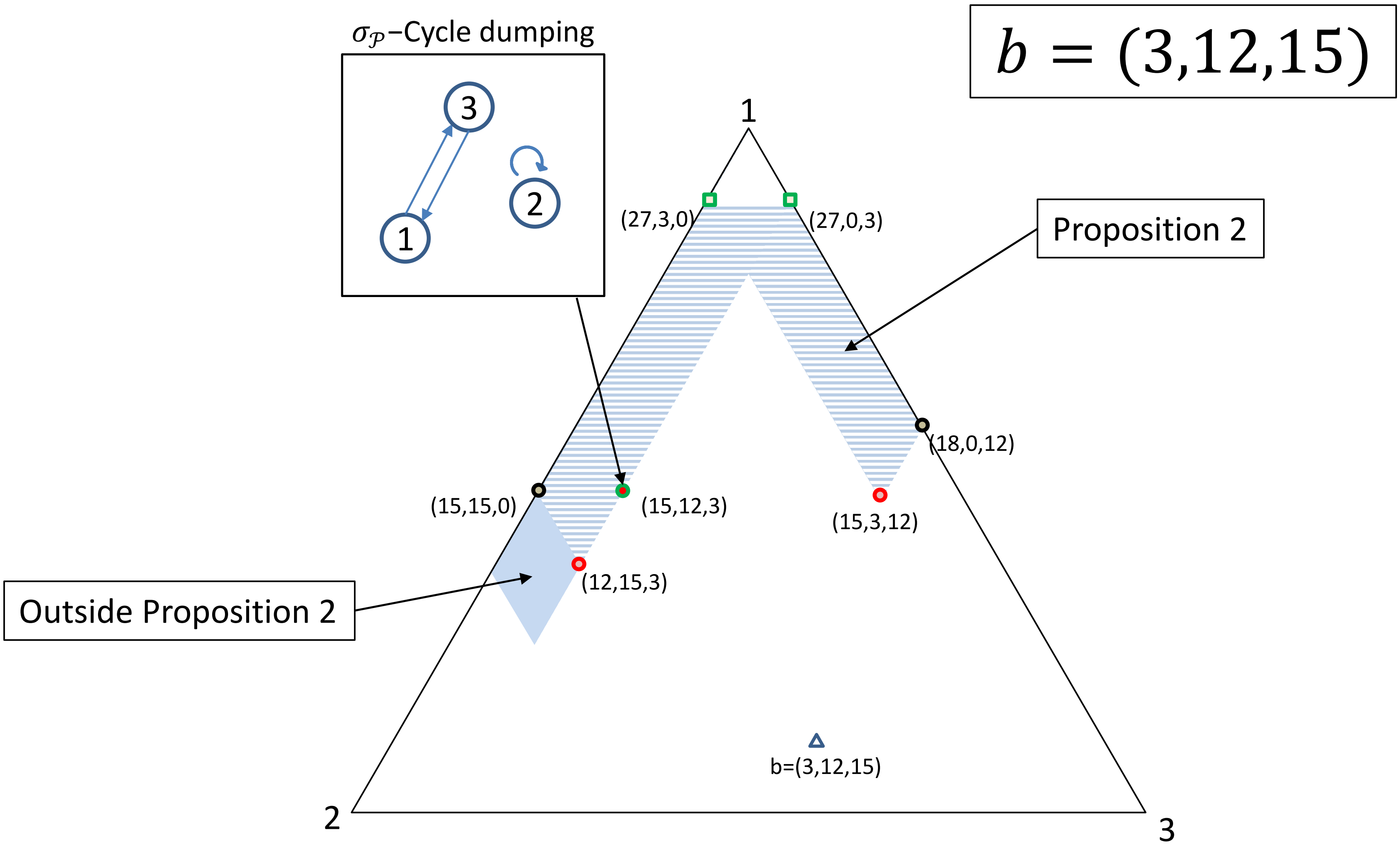
Proposition 2



Proposition 2

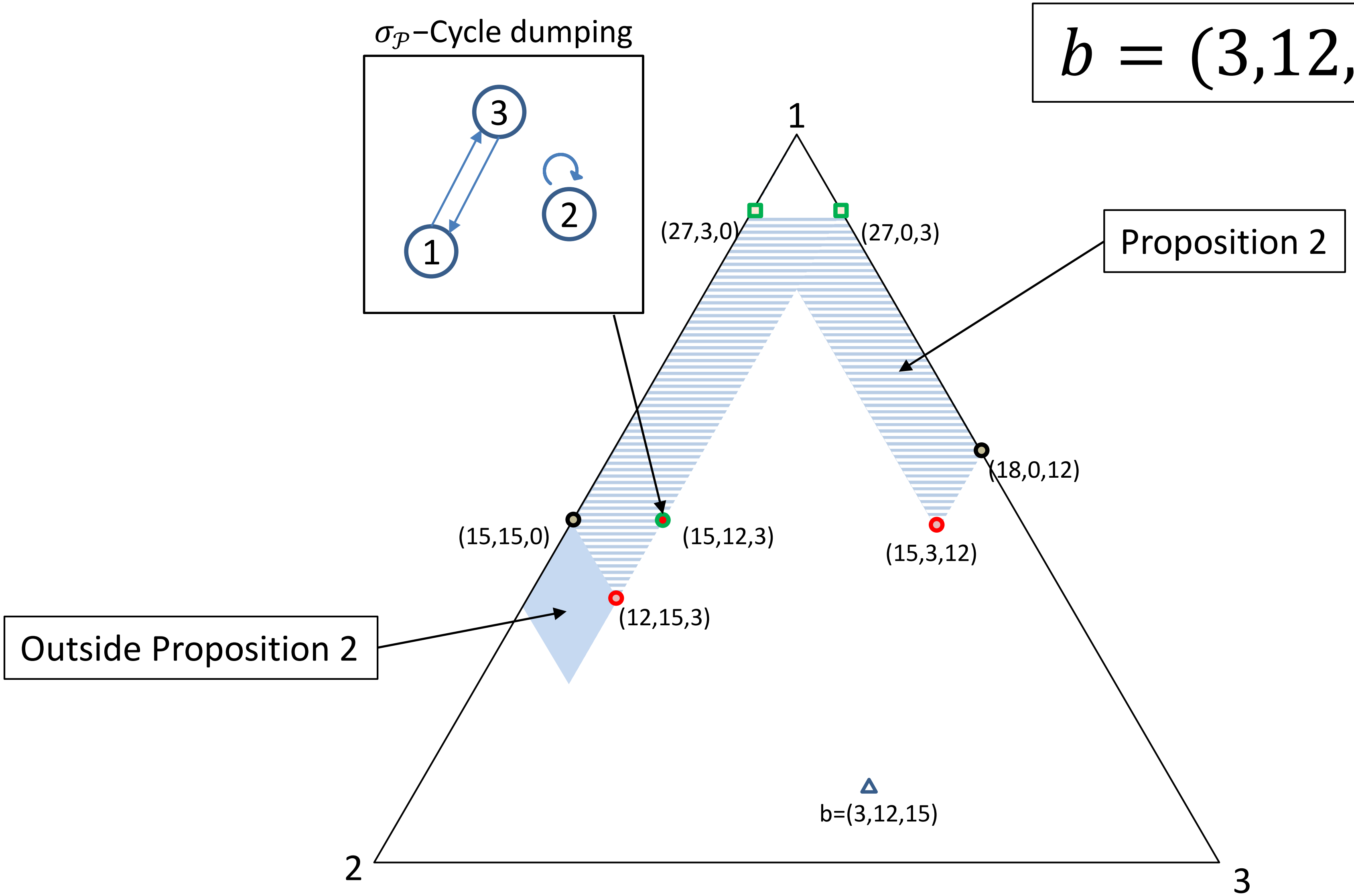


$\sigma_{\mathcal{P}}$-Cycle dumping



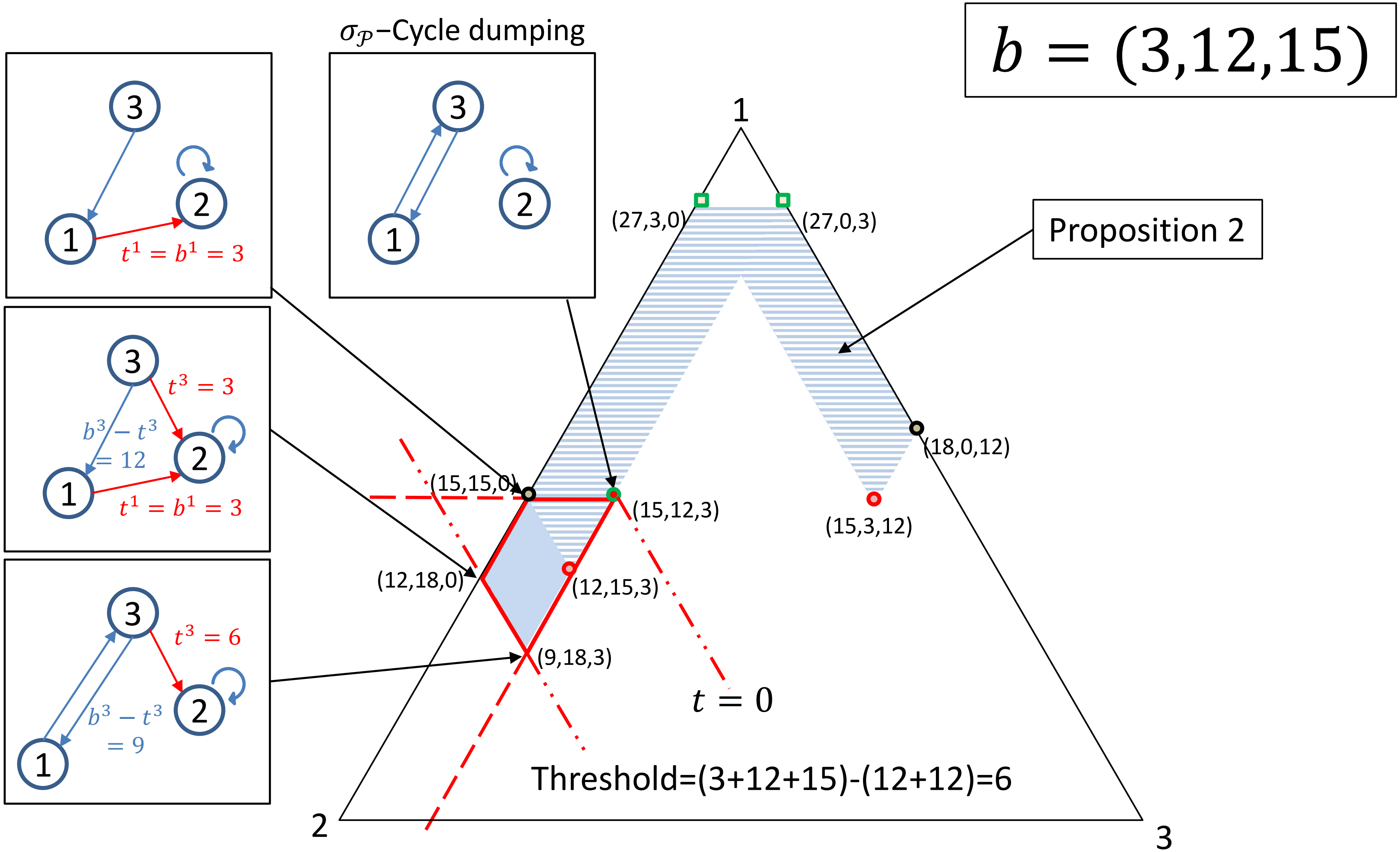
Proposition 3

$$b = (3,12,15)$$



Proposition 3

$b = (3,12,15)$



Proposition 3

$$b = (3,12,15)$$