Experimentation and Entry with Common Values

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Motivation

- Consider market entry (new geographical market, new product) when the cost of entry is unknown.
- Firms conduct market research (or R&D) to learn about the viability of the product (or the entry cost).
- Suppose that two rival firms experiment to learn the value of the cost.
- How will this result in market entry?

Main insights

- A firm that learns that the market is profitable, if it enters, will immediately be followed.
- Firms have an incentive to wait and hide under the cover of entry of uninformed firms.
- There will be equilibria where uninformed firms enter at a fixed date and will not be followed.
  - Can be uncoordinated.
  - Can be coordinated.
- We characterize these "uninformed entry equilibria".
- They correspond, e.g., to trade fairs or regular unilateral product release events at which firms present new products, even at very early stages before the market is known to be viable.

Related Literature

- Optimal stopping games with learning: Décamps Mariotti (2004), Rosenberg, Solan Vieille (2007), Murto Valimaki (2011), Wagner (2018), Klein and Wagner (2019).
- R&D races with learning: Choi (1991), Malueg Tsutsui (1997), Moscarini Squintani (2010), Akcigit Liu (2016), Halac, Kartic, Liu (2017).
- Entry timing with learning: Bloch, Fabrizi Lippert (2015), Kolb (2015).

The Model

The Model

- Two firms learn about a common value binary entry cost $c \in \{0, \theta\}$.
- Time is discrete $t = 0, 1, \ldots$.
- At each time t, with probability μ each firm learns the true value of the cost, with probability $1 - \mu$, the firm does not learn.
- The two firms have common discount factor δ.
- The discounted value of duopoly is Π_d.
- The discounted value of monopoly is $\Pi_m = \Pi_d + \Delta$.

Assumptions

- The prior probability (shared by the two firms) is that the two costs are equiprobable.

Assumption

$$0 < \Pi_d < \frac{\theta}{2} < \Pi_m < \theta.$$

Continuous time limit

- Let Λ be a time interval (taken to 0).
- Let $\tau = t\Lambda$.
- Let $a = d\Lambda$.
- Let $\mu = \lambda\Lambda$.
- Let $\delta = e^{-r\Lambda}$.

Strategies

- At each time t, firms decide whether to enter or not
- If the other firm has entered at t, the firm can choose to follow suit immediately.
- So the actions are:
  - The entry decision e_i^t when firm i is uninformed, denoted p_i^t
  - The entry decision e_i^t when firm i has learned that the cost is low at some period $s \leq t$ denoted $q_i^{t,s}$
  - The follow-up decision d_i^t when firm i is uninformed, denoted u_i^t.

Histories and Equilibrium

- A history h^t records the past entry decisions of the firms, $h^t = ((e_i^s, e_j^s, d_i^s, d_j^s)_{s=1}^{t-1})$.
- A strategy for firm i is a mapping from any history h^t to probabilities $p_i^t(h^t), q_i^{t,s}(h^t)$ for any $s \leq t$ and $u_i^t(h^t)$ in $[0, 1]$.
- We also let $\chi_i^t(h^t)$ denote firm i's belief that the cost is high after history h^t.
- *The only history at which nontrivial decisions are made is when no firm has entered yet.*
- The solution concept we consider is a *symmetric Bayesian perfect equilibrium.*

Preliminaries

Monopoly solution

- Entering immediately before learning the cost gives
$$\Pi_E = \Pi_d + \Delta - \frac{\theta}{2}.$$
- Experimenting (using the facilities of both firms) and entering when it learns that the cost is low gives
$$\Pi_W = \frac{\lambda}{2\lambda + r}(\Pi_d + \Delta).$$
- A monopolist operating both firms has no incentive to wait if and only if $\Pi_E \geq \Pi_W$, which in the continuous time limit is satisfied whenever the following condition on the parameters holds:
$$\frac{\theta}{2} \leq \frac{\lambda + r}{2\lambda + r}(\Pi_d + \Delta).$$

Herding equilibrium

1. $p^t = 0$ for all t.
2. $q^{s,s} = 1$.
3. $u^t = 1$ for all t.

Proposition (Existence of the herding equilibrium)

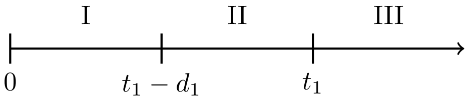
A herding equilibrium exists for all parameter values.

- A herding equilibrium gives expected payoff

$$\Pi_h = \frac{\Pi_d \lambda}{2\lambda + r}.$$

Single-firm, uninformed entry equilibria

Single-firm, one-uninformed-entry-date equilibrium



1. $p_1^t > 0$ for $t = t_1$; $p_1^t = 0$ for all $t \neq t_1$; $p_2^t = 0$ for all t.
2. $q_1^{s,s} = 1$ at $s < t_1 - d_1$ and $s > t_1$ and $q_1^{t_1,s} = 1$ for all $t_1 - d_1 \leq s \leq t_1$; $q_2^{s,s} = 1$.
3. $u_1^s = 1$ for all s; $u_2^s = 1$ for all $s \neq t_1$ and $u_2^{t_1} = 0$.

- A single-firm, one-uninformed-entry-date equilibrium is characterized by a period t_1, an entry probability p_1 and a delay d_1.

Single-firm, one-uninformed-entry-date equilibrium

Entry of an uninformed firm 1

- Profit of entry

$$\Pi_E = (1 - e^{-\tau_1 \lambda})(\Pi_d + \Delta - \theta) + e^{-\tau_1 \lambda}(\Pi_d + \Delta - \frac{\lambda}{2(\lambda + r)}\Delta - \frac{\theta}{2}).$$

- Profit of waiting

$$\Pi_W = e^{-\tau_1 \lambda}\frac{\lambda}{2\lambda + r}\Pi_d.$$

Single-firm, one-uninformed-entry-date equilibrium

Entry of an uninformed firm 1

- Yields a latest uninformed entry date $\overline{\tau}_1$ given by indifference between Π_E and Π_W:

$$\overline{\tau}_1 = \frac{1}{\lambda} \ln \left(\frac{\frac{\lambda}{2\lambda+r}\Pi_d + \frac{\lambda}{2(\lambda+r)}\Delta - \frac{\theta}{2}}{\Pi_d + \Delta - \theta} \right).$$

- Latest uninformed entry date $\overline{\tau}_1 \geq 0$ if

$$\frac{\theta}{2} \leq \frac{\lambda+r}{2\lambda+r}\Pi_d + \frac{\lambda+2r}{2(\lambda+r)}\Delta = \frac{\lambda+r}{2\lambda+r}(\Pi_d+\Delta) + \frac{\lambda r}{2(\lambda+r)(2\lambda+r)}\Delta.$$

- Stronger incentives to enter uninformed than in monopoly.
- Latest entry date is decreasing in λ, r, and θ, and increasing in Π_d and Π_m.

Single-firm, one-uninformed-entry-date equilibrium

Entry of an informed firm 1

- Profit of entry

$$\Pi_E = \Pi_d.$$

- Profit of waiting

$$\Pi_W = (1 - e^{-a_1(\lambda+r)})\frac{\lambda}{\lambda + r}\Pi_d + e^{-a_1(\lambda+r)}(\Pi_d + \Delta - \frac{\lambda}{\lambda + r}\Delta).$$

- Yields a unique $a_1 = -\frac{1}{\lambda+r}\ln\left(\frac{\Pi_d}{\Pi_d+\Delta}\right)$ such that an informed firm 2 prefers to wait between $\tau_1 - a_1$ and τ_1.
- The delay a_1 is decreasing in λ and r, increasing in Π_d, and decreasing in Δ.

Single-firm, one-uninformed-entry-date equilibrium

Herding decision of an uninformed firm 2

- Profit of herding (copying entry) at t_1

$$\Pi_E = \frac{e^{-(\tau_1 - a_1)\lambda} - e^{-\tau_1 \lambda}}{2}\Pi_d + p_1 e^{-\tau_1 \lambda}(\Pi_d - \frac{\theta}{2}).$$

- Profit of not herding (not copying entry) at t_1

$$\Pi_W^1 = \frac{e^{-(\tau_1 - a_1)\lambda} - e^{-\tau_1 \lambda}}{2}\frac{\lambda}{\lambda + r}\Pi_d + p_1 e^{-\tau_1 \lambda}\frac{\lambda}{2(\lambda + r)}\Pi_d.$$

- Yields a minimum entry probability p_1 such that herding is not profitable of

$$p_1 \geq g_1(a_1; \lambda, r, \theta, \Pi_d) = \frac{(e^{a_1 \lambda} - 1)\frac{r}{\lambda + r}\Pi_d}{\theta - \frac{\lambda + 2r}{\lambda + r}\Pi_d}.$$

- g_1 increases in a_1 with $g_1(0; \lambda, r, \theta, \Pi_d) = 0$.

Single-firm, one-uninformed-entry-date equilibrium

Equilibrium

1. An uninformed firm 1 is indifferent between entering and waiting at $\tau_1 = \overline{\tau}_1$; and strictly prefers to enter at $\tau_1 < \overline{\tau}_1$.
2. An informed firm 1 prefers to enter before $\tau_1 - a_1$ and to wait between $\tau_1 - a_1$ and τ_1.
3. An uninformed firm 2 does not imitate at τ_1 if $p \geq g_1(a_1; \lambda, r, \theta, \Pi_d)$.
4. The delay cannot be larger than the date: $a_1 \leq \tau_1$.

- For $\tau_1 = \overline{\tau}_1$, the expected profits of firms 1 and 2 are decreasing in p_1.
  - For $\tau_1 < \overline{\tau}_1$, $p_1 = 1$, and $a_1 = \min\left\{-\frac{1}{\lambda+r}\ln\left(\frac{\Pi_d}{\Pi_d+\Delta}\right), \tau_1\right\}$.
  - For $\tau_1 = \overline{\tau}_1$, $p_1 = g_1(a_1; \lambda, r, \theta, \Pi_d)$ and $a_1 = \min\left\{-\frac{1}{\lambda+r}\ln\left(\frac{\Pi_d}{\Pi_d+\Delta}\right), \overline{\tau}_1\right\}$.

Single-firm, one-uninformed-entry-date equilibrium

Equilibrium

Proposition (Existence of the single-firm, one-uninformed-entry-date eqm)

If a single-firm, one-uninformed-entry-date equilibrium exists, then

$$\theta \leq \frac{2(\lambda + r)}{2\lambda + r}\Pi_d + \frac{\lambda + 2r}{\lambda + r}\Delta.$$

- Condition is necessary and sufficient for existence of eqm at $\tau = 0$ (immediate single-firm-entry eqm).

Single-firm, one-uninformed-entry-date equilibrium

Entrant's vs. non-entrant's profit at the latest entry date

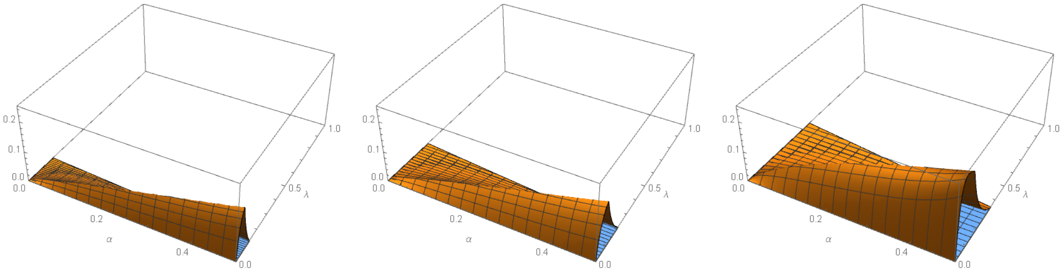


Figure: Probability of entry in the single one-entry equilibrium as a function of the competition in the product market, α, where $\Pi_d = \alpha\Pi_m$, and the speed of learning, λ. Left: $\theta = 1.4\Pi_m$, $r = 0.12$. Center: $\theta = 1.4\Pi_m$, $r = 0.2$. Right: $\theta = 1.2\Pi_m$, $r = 0.12$.

Single-firm, one-uninformed-entry-date equilibrium

Entrant's profit at the latest entry date vs. herding eqm profit

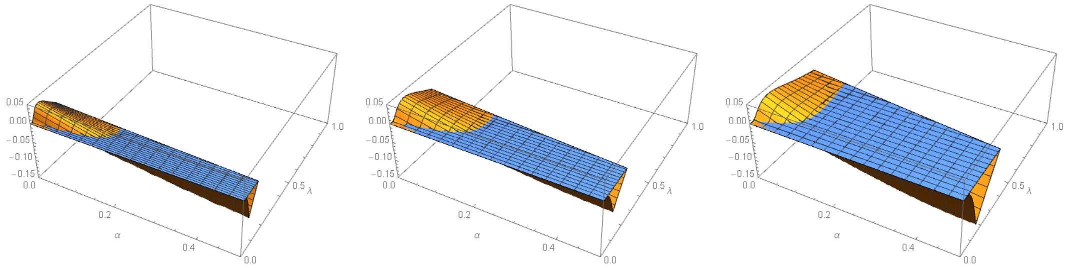


Figure: Difference of the expected profit of the entrant in the single one-entry equilibrium and the expected profit in the herding equilibrium as a function of the competition in the product market, α, where $\Pi_d = \alpha\Pi_m$, and the speed of learning, λ. Left: $\theta = 1.4\Pi_m$, $r = 0.12$. Center: $\theta = 1.4\Pi_m$, $r = 0.2$. Right: $\theta = 1.2\Pi_m$, $r = 0.12$.

Single-firm, one-uninformed-entry-date equilibrium

Non-entrant's profit at the latest entry date vs. herding eqm profit

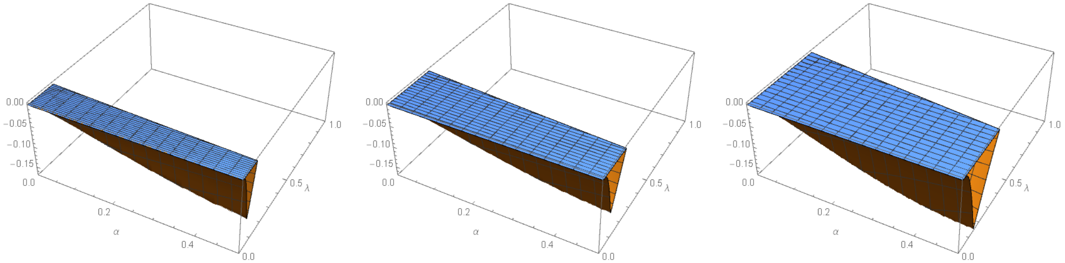


Figure: Difference of the expected profit of the non-entrant in the single one-entry equilibrium and the expected profit in the herding equilibrium as a function of the competition in the product market, α, where $\Pi_d = \alpha\Pi_m$, and the speed of learning, λ. Left: $\theta = 1.4\Pi_m$, $r = 0.12$. Center: $\theta = 1.4\Pi_m$, $r = 0.2$. Right: $\theta = 1.2\Pi_m$, $r = 0.12$.

Single-firm, one-uninformed-entry-date equilibrium

Entrant's vs. non-entrant's profit at the latest entry date

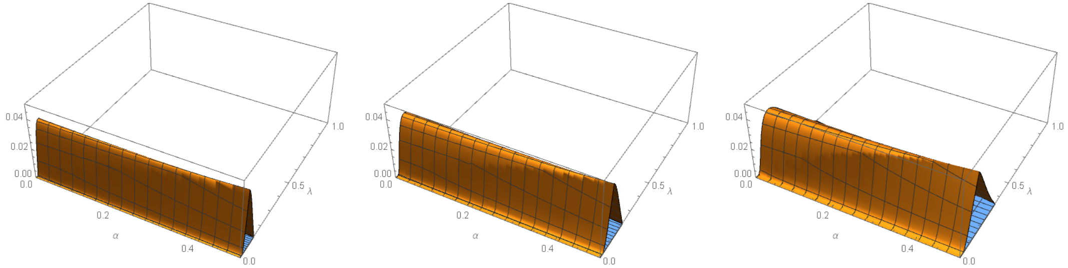
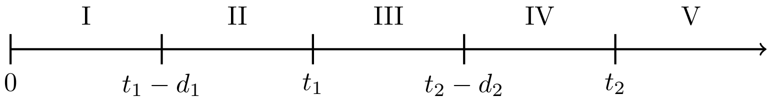


Figure: Profit difference in the single one-entry equilibrium as a function of the competition in the product market, α, where $\Pi_d = \alpha \Pi_m$, and the speed of learning, λ. Left: $\theta = 1.4\Pi_m$, $r = 0.12$. Center: $\theta = 1.4\Pi_m$, $r = 0.2$. Right: $\theta = 1.2\Pi_m$, $r = 0.12$.

More than one entry date

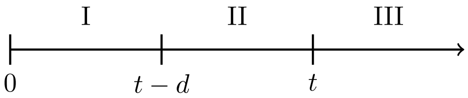
- Can extend to more than one uninformed entry.



- In the case with two entry dates,
  - Latest entry date, delay at latest entry date, and minimum entry probability at latest entry date is the same $\overline{\tau}_1$.
  - First entry date depends on second entry probability; longest delay and minimum entry probability at first entry date are the same as at latest entry date.
- To be finished.

Two-firm, uninformed entry equilibria

Two-firm, one-uninformed-entry-date equilibrium



1. $p^t > 0$, $p^s = 0$ for all $s \neq t$
2. $q^{s,s} = 1$ for all $s < t - d$ and $s > t$ and $q^{t,s} = 1$ for all $t - d \leq s \leq t$
3. $u^s = 1$ for all $s \neq t$ and $u^t = 0$

- A single-entry equilibrium is characterized by a period t, an entry probability p and a delay d.

Two-firm, one-uninformed-entry-date equilibrium

Entry of an uninformed firm

- Profit of entry

$$\Pi_E = \frac{\left(e^{-\lambda(\tau-a)} - e^{-\tau\lambda}\right)}{2}\Pi_d + \frac{1-e^{-\tau\lambda}}{2}(\Pi_d + \Delta - \theta) + e^{-\tau\lambda}\left(p(\Pi_d - \frac{\theta}{2}) + (1-p)(\Pi_d + \Delta - \frac{\lambda}{2(\lambda+r)}\Delta - \frac{\theta}{2})\right)$$

- Profit of waiting

$$\Pi_W = \frac{\left(e^{-\lambda(\tau-a)} - e^{-\tau\lambda}\right)}{2}\frac{\lambda}{\lambda+r}\Pi_d + e^{-\tau\lambda}\left(p\frac{\lambda}{2(\lambda+r)}\Pi_d + (1-p)\frac{\lambda}{2\lambda+r}\Pi_d\right)$$

Two-firm, one-uninformed-entry-date equilibrium

Entry of an uninformed firm

- Yields a probability of entry given by indifference between Π_E and Π_W:

$$p = f(a) = 1 + \frac{\frac{1}{2}\left(e^{a\lambda}-1\right)\frac{r}{\lambda+r}\Pi_d + \frac{1}{2}\left(e^{\lambda\tau}-1\right)\left(\Delta-\theta+\Pi_d\right) + \left(\frac{(\lambda+2r)\Pi_d}{2(\lambda+r)} - \frac{\theta}{2}\right)}{\frac{\lambda\Pi_d}{2(\lambda+r)} - \frac{\lambda\Pi_d}{2\lambda+r} + \frac{\Delta(\lambda+2r)}{2(\lambda+r)}}.$$

Two-firm, one-uninformed-entry-date equilibrium

Entry of an informed firm

- Profit of entry

$$\Pi_E = \Pi_d.$$

- Profit of waiting

$$\Pi_W = (1 - e^{-\lambda a})e^{-ra}\Pi_d + e^{-\lambda a}e^{-ra}\left(p\Pi_d + (1-p)(\Pi_d + \Delta - \frac{\lambda}{\lambda + r}\Delta\right)$$

- Yields a maximum probability of entry as a function of the delay a such that informed firms delay their entry between $\tau - a$ and τ:

$$p \leq k(a) = 1 - \frac{(1 - e^{-ra})\,\Pi_d}{e^{-(\lambda+r)a}\frac{r}{\lambda+r}\Delta}.$$

Two-firm, one-uninformed-entry-date equilibrium

Herding decision

- Profit of entry

$$\Pi_E = \frac{e^{-\lambda(\tau-a)} - e^{-\lambda\tau}}{2}\Pi_d + pe^{-\lambda\tau}(\Pi_d - \frac{\theta}{2}).$$

- Profit of waiting

$$\Pi_W = \frac{e^{-\lambda(\tau-a)} - e^{-\lambda\tau}}{2}\frac{\lambda}{\lambda+r}\Pi_d + pe^{-\lambda\tau}\frac{\lambda}{2(\lambda+r)}\Pi_d$$

- Yields a minimum entry probability p such that herding is not profitable of

$$p \geq g(a;\lambda,r,\theta,\Pi_d) = \frac{(e^{a\lambda}-1)\frac{r}{\lambda+r}\Pi_d}{\theta - \frac{\lambda+2r}{\lambda+r}\Pi_d}.$$

Two-firm, one-uninformed-entry-date equilibrium

Equilibrium

1. An uninformed firm is indifferent between entering and waiting at τ, $f(a^*) = p^*$
2. An informed firm prefers to enter before $\tau - a^*$ and to wait between $\tau - a^*$ and τ, $k(a^*) = p^*$
3. An uninformed firm does not imitate at τ: $p^* \geq g(a^*)$
4. The delay cannot be larger than the date: $a^* \leq \tau$.

- Two situations may arise:
    1. Either the equilibrium is defined by a pair (a^*, p^*) which satisfies the three conditions $f(a^*) = p^*, k(a^*) = p^*$ and $p^* \geq g(a^*)$
    2. Or the equilibrium is given by (τ, p^*) where $f(\tau) = p^*$, $k(\tau) > p^*$ and $p^* \geq g(\tau)$.

Two-firm, one-uninformed-entry-date equilibrium

Existence and uniqueness of equilibrium

Proposition (Existence)

If a two-firm, one-uninformed-entry-date equilibrium exists, the following condition holds:

$$\frac{\theta}{2} \leq \frac{\lambda + r}{2\lambda + r}\Pi_d + \frac{\lambda + 2r}{2(\lambda + r)}\Delta.$$

- Condition is necessary and sufficient for existence of eqm at $\tau = 0$ (immediate two-firm-entry eqm).

Proposition (Uniqueness)

Suppose that an equilibrium with single entry at date τ exists. Then the equilibrium (p^*, a^*) is unique.

Two-firm, one-uninformed-entry-date equilibrium

Comparative statics

Proposition

Suppose that an equilibrium with single-entry at date τ exists. The following comparative statics hold : (i) $\partial a^*/\partial\theta \geq 0$, (ii) $\partial p^*/\partial\theta < 0$, (iii) $\partial a^*/\partial\Pi_d \leq 0$, (iv) $\partial p^*/\partial\Pi_d \gtreqless 0$, (v) $\partial a^*/\partial\Delta \gtreqless 0$, (vi) $\partial p^*/\partial\Delta > 0$.

Proposition (Comparison of equilibria across dates)

Let (p_1^*, a_1^*) and (p_2^*, a_2^*) be two two-firm, one-uninformed-entry-date equilibrium at dates $\tau_1 < \tau_2$. Then $p_1^* > p_2^*$.

Two-firm, one-uninformed-entry-date equilibrium

Latest entry date

- Consider the system of three equations in three unknowns given by:

$$\begin{aligned} f(\overline{a}, \overline{\tau}) &= \overline{p}, \\ g(\overline{a}) &= \overline{p}, \\ k(\overline{a}) &= \overline{p}. \end{aligned}$$

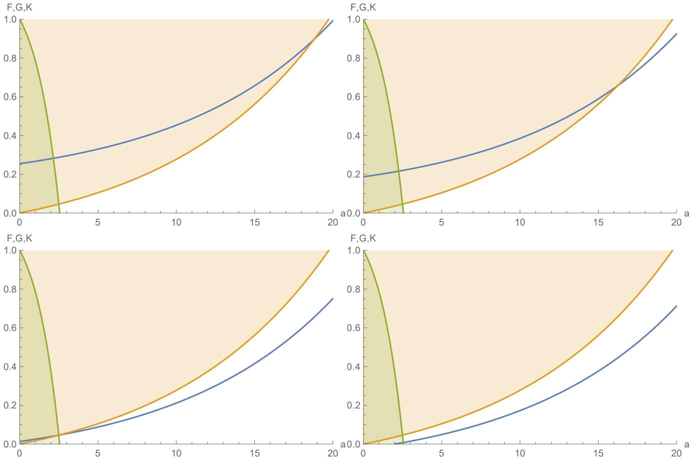
Proposition (Latest entry date)

There is no single-entry equilibrium at $\tau \geq \overline{\tau}$.

Two-firm, one-uninformed-entry-date equilibrium

Latest entry date

- Blue: $f(a)$ entry probability;
- Green: $k(a)$ waiting constraint;
- Orange: $g(a)$ no-copy constraint.
- Parameters: $\alpha = 0.2$, $\beta = 1.4$, $r = 0.5$, $\lambda = 0.1$. Top left: $\tau = 1$, top right: $\tau = 3$, bottom left: $\tau = 6.81604$, bottom right: $\tau = 7.5$



Two-firm, multiple-uninformed-entry-date equilibrium

- Let $t_1, \dots, t_M$ be entry dates
- We associate $p_1, \dots, p_M$ entry probabilities
- And $d_1, \dots, d_M$ entry delays
- When do multiple entry equilibria exist?

Two-firm, multiple-uninformed-entry-date equilibrium

Existence and uniqueness

Proposition (Existence)

If a multi-entry equilibrium with last entry date τ_M exists, then there exists a single-entry equilibrium at date $\tau_M - \tau_{M-1}$.

- Suppose that

$$\frac{2r+\lambda}{\lambda+r}\Delta < \theta.$$

Proposition (Uniqueness)

Suppose that an equilibrium with multiple entry at dates $\tau_1, \ldots, \tau_M$ exists. Then the equilibrium entry probabilities and delays $(p_1^*, a_1^*, \ldots, p_M^*, a_M^*)$ are unique.

Two-firm, multiple-uninformed-entry-date equilibrium

Comparison of probabilities

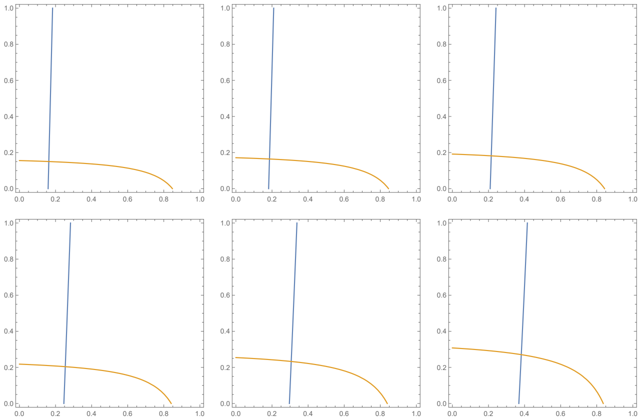
Proposition

There exists an upper bound $\overline{\Delta\tau}$ *such that, if* $\Delta\tau < \overline{\Delta\tau}$*, the equilibrium continuation values are increasing over time,* $W_{m+1} > W_m$*, and the equilibrium entry probabilities are decreasing over time,* $p_m > p_{m+1}$*.*

Two-firm, multiple-uninformed-entry-date equilibrium

A two-entry equilibrium illustration

- Indifference of type 1 firms at $t = 0$ and $t = 1$ in $p_0 - p_1$ space.
- Horizontal axis: p_0, vertical axis: p_1.
- Blue: $\Pi^E(1,0) = \Pi^W(1,0)$.
- Yellow: $\Pi^E(1,1) = \Pi^W(1,1)$.
- Parameters: $\lambda = .1$, $\delta = .75$, $\beta = 1.4$. Top left to bottom right, $\alpha \in \{0, .1, .2, .3, .4, .5\}$.



Conclusion

Summary

- We study entry with learning about a common value cost
- A no-entry equilibrium always exists
- There also exist equilibria where uninformed firms coordinate entry at specific dates
- There is a last entry date for these equilibria
- Equilibrium is unique given fixed entry dates, and are characterized by entry probabilities and entry delays
- Multiple entry equilibria are harder to support than single-entry equilibria

Extensions

- Multiple values of the costs
- Imperfect signals
- Private values vs. common values
- Endogenous signal acquisition