Experimentation and Entry with Common Values*

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Abstract

We model experimentation and entry decisions of rival firms into a new market with uncertain common entry costs and potential product market competition. We show that a herding equilibrium where firms enter immediately when they learn that the cost is low and are immediately followed by their rival always exist. We also show the existence of equilibria that avoid herding. In these equilibria, uninformed firms coordinate to enter at specific entry dates with positive probability and firms that learned that the cost is low before those dates delay their entry to hide under the cover of the uninformed firms. We show that no-herding equilibria are more likely to exist with an early than a late entry date, are unique given a fixed entry date and that, in them, the probability of uninformed entry is decreasing over time. We generalize our results to no-herding equilibria with multiple entry dates for uninformed firms.

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1 Introduction

Firms spend considerable time and resources collecting information about the costs and profitability of entering new markets.[1] For a monopolistic firm, the decision to enter a new market results from a standard optimal stopping problem. The firm collects information over time and has to decide at which point to enter and stop collecting additional information. When firms compete, and receive covertly information about the profitability and costs of entry on the new market, the problem is markedly more complex. Firms anticipate that, by entering, they might reveal information to the other firm, and take into account these informational leakages in the timing of their entry decision. We study these experimentation and entry decisions making use of two frequent observations. First, firms often enter markets or introduce technologies before they have learned about their economic and technical viability, pre-empting their rivals' entry. Second, in many industries, much of the entry or the introduction of new technologies occurs at well-defined dates such as trade fairs or annual product launch events; after which it often takes considerable time for further competitors, who were working in the same product space, to follow suit with their entry.[2]

Building on these two observations, we investigate equilibria in an entry game between two rival firms which gradually acquire information about the common entry cost into a new market. The two firms decide at any point in time whether to enter the market. By entering the market, they incur the fixed entry cost, and start collecting profits. Because duopoly profits are smaller than monopoly profits, the expected payoff of a firm depends on the entry decision of the rival firm.

We characterize the equilibria of this entry game and show that a *herding equi-*

[1]For new geographical markets, firms commission market studies, investigate distribution channels and regulatory constraints before deciding whether or not to enter. For markets for new products, firms engage in research and development, build prototypes, analyze production processes and costs before launching operations at a full scale.

[2]In the mobile electronics industry, for example, new hardware and new services are often introduced at events, such as the Barcelona Mobile World Congress or the Berlin Internationale Funkausstellung. These events see the introduction of both, successes and failures. In 2019, for instance, both Huawei and Samsung launched foldable designs for their smart phones at Mobile World Congress. Both designs have since been widely considered as premature, but their launch gave rivals pause to reconsider their own entries. Other competitors, such as Motorola, introduced their foldable designs only after a considerable delay.

librium – where uninformed firms never enter, firms that learn that the cost is low enter and are immediately followed by the other firm – always exist. In the herding equilibrium, firms collect duopoly profits and start collecting it as soon as one of the two firms learns that the cost is low.

Under some restrictions on the parameters, we also prove existence of other, more complex equilibria, which *avoid herding*. In these equilibria, uninformed firms coordinate on specific dates at which they enter with positive probability. For this entry of an uninformed firm to be profitable, it must be that it is not immediately followed by the other firm, and hence results with positive probability in a period in which the firm collects monopoly profits. These monopoly profits, in turn, give firms that learn the entry cost is low before this entry date an incentive to delay entry and hide under the cover of the entry of the uninformed firms in order to avoid herding. As waiting entails a cost, there is a maximum delay that an informed firm is willing to incur before entering. A single-entry equilibrium at date τ is thus characterized both by an entry probability and a delay for the informed firm.

We first prove that a single-entry equilibrium exists if and only if there exists an equilibrium in which the two firms enter immediately with positive probability. Next, we show that, if the equilibrium exists, it is unique: It is characterized by a unique pair of entry probability and delay. We then analyze the comparative statics effects of changes in the parameters of the model of the equilibrium entry probability of uninformed and the entry delay of informed firms. Interestingly, as the entry date increases and firms become more pessimistic, the equilibrium entry probability decreases and the equilibrium delay increases. But for the equilibrium to exist, it must be that entry is not followed by the rival's entry at the entry date, τ, which implies that the fraction of uninformed firms in the pool of firms entering at τ must be sufficiently large. Hence, for an equilibrium to exist, the entry probability of uninformed firms must be sufficiently high. As this probability is decreasing over time, there exists a last entry date for a single-entry equilibrium. We characterize that last entry date as a solution to a system of three equations.

We next turn our attention to no-herding equilibria with multiple entry dates, where uninformed firms enter at a finite number of different dates. We first show that if a multiple-entry equilibrium exists, a single-entry equilibrium must exist as well. We further observe that, in a multiple-entry equilibrium, the entry probability at any date τ_m depends on the entry probabilities at all other entry dates; that it is decreasing in the entry probability at earlier dates, $\tau_1, \ldots, \tau_{m-1}$; and that an increase in the entry probability at date τ_{m+1} increases the entry probability that makes the firm indifferent between entering and waiting at date τ_m. These comparative statics effects of changes in $p_1, \ldots, p_{m-1}$ and p_{m+1} on p_m are sufficient to prove that, if an equilibrium exists, it must be unique. We finally investigate the relation between entry probabilities at different dates for a given equilibrium. Assuming that the delay between two successive entry dates is small, we show that the equilibrium entry probabilities are decreasing over time: firms enter with lower probability as time passes.

The paper builds on the literature on strategic experimentation and social learning initiated by Chamley and Gale (1994) and Bolton and Harris (1999). The exponential bandit model we use was developed by Keller et al. (2005) and extended to social learning by Keller and Rady (2010, 2015). As we model the firms' entry decisions as irreversible, our paper is most closely related to the literature on stopping games with social learning. Décamps and Mariotti (2004) study a continuous-time model where two firms choose whether to invest in a project after observing private signals on the common value of the project and the experience of the rival. They characterize equilibrium as an extension of the equilibrium of an attrition war. In Rosenberg et al. (2007), two players receive a sequence of private signals and choose when to stop depending on their history of private signals and the publicly observed stopping decision of their rival. They characterize equilibrium in the discrete time game as cut-off equilibria based on the private beliefs and study the qualitative properties of equilibrium strategies. Murto and Välimäki (2011) adopt a framework where players receive binary signals about the profitability of an investment. They give a full characterization of the symmetric Bayesian perfect equilibrium in the discrete time game where players observe their private signals and the stopping decisions of other players. They analyze information aggregation in the limiting equilibrium when the number of players grows large, and show that equilibrium displays “exit waves” with herding. Murto and Välimäki (2013) generalizes the analysis to a larger class of environments. In two recent papers, Wagner (2018) and Klein and Wagner (2018) analyze the effect of information exchange, transparency and disclosure in a model with private signals and irreversible investments.

The paper is also related to the literature on learning in R&D races, where firms gradually learn the rate of success of innovations, and decide whether to drop out of the patent race. The first papers in the literature, by Choi (1991) and Malueg and Tsutsui (1997) analyze patent races where players have common priors, and successes and failures are perfectly observed. Moscarini and Squintani (2010) suppose that the firms initially receive different signals about the common success rate, and choose their optimal exit time based on their private signals. The equilibrium displays a “survivor’s curse” as the firm which remains longest may regret not having exited sooner. Akcigit and Liu (2016) study the incentives to disclose information in an R&D race with private information about successes and failures of competing research paths. Halac et al. (2017) analyze optimal contests with experimentation, where the designer chooses how to allocate the value to different participants and how much information to disclose. Hence, as opposed to an R&D race where all the value accrues to the winner, the designer may choose to allocate some value to the other participants. The analysis shows that experimentation affects the optimal contest design, and that the optimal contest may either take the form of a winner-takes-all contest with information leakage or of an equal sharing contest with no disclosure.

Our model differs from the existing literature on social learning in common values environments and R&D contest in two dimensions. On the one hand, we consider a much simpler environment with a binary state variable, perfect signals and discrete time. On the other hand, we consider a game where the value of the project depends on the timing of the entry of the rival firm, as firms collect monopoly payoffs as long as the other firm does not enter, and duopoly payoffs thereafter. This difference in payoffs generates very different equilibrium structures, as firms may have an incentive to delay entry to avoid herding, a phenomenon which does not arise in the existing literature.

Finally, we relate our paper to two recent papers on market entry timing with learning. In a previous paper, Bloch et al. (2015), we analyzed entry decisions when firms learn private values of entry costs. The differences between the private values and common values models are striking. The common values model offers the possibility of a new type of equilibria which do not exist in the private values model. The private values model typically only has one equilibrium with preemption whereas in the common values model, multiple equilibria can exist, parameterized by the entry dates on which uninformed firms coordinate. We discuss in detail the differences between the private values and common value setting in Section 6. Kolb (2015) studies a different dynamic model, where one firm (the incumbent) has known cost whereas the other firm (the entrant) has a privately known cost. Public news about the cost arrives over time following a Brownian process. As in our model, entry can either occur at time zero, or through "interval equilibria" where both types of entrants wait until they have received enough news before entering. This delay allows the weak players to hide under the cover of strong players, making the incumbent concede rather than fight entry. Hence, the structure of equilibrium bears some similarity to the equilibria we construct in this paper, but the underlying mechanisms are different.

The rest of the paper is structured as follows. We describe the model and introduce notations in the next section. Section 3 contains preliminary results on optimal strategies, beliefs and the no-entry equilibrium. Section 4 contains our analysis of single-entry equilibria. We extend the analysis to multiple-entry equilibria in Section 5. We compare equilibria and discuss the robustness of our results in Section 6 and conclude in Section 7. All proofs are relegated to the Appendix.

2 Model

We consider a model where two potentially rival firms choose whether to enter a new market. Time is discrete and runs as $t = 0, 1, 2, \ldots, \infty$. We use the notation d for the difference between two time periods. The two firms share the same discount factor δ. At the beginning of the game, nature chooses the firm's common entry cost, c, which can either be high or low with equal probability. Without lost of generality, we normalize the low cost to 0 and denote the high cost by $\theta > 0$ so that $c \in \{0, \theta\}$. The common entry cost is initially unknown to both firms, which are assumed to be risk neutral. Therefore, each firm's ex-ante expected entry cost is given by $\widetilde{c} = \theta/2$.

Firms can learn their common cost over time. Each firm's belief about the cost depends on the information arriving gradually through the game. This is the result of firms experimenting over time to find out the extent of this cost. We capture this by assuming that, at any time period, each firm receives a potentially informative signal $\zeta \in \{0, 1, 2\}$, as follows: $\Pr(\zeta = 0|c = 0) = \Pr(\zeta = 2|c = \theta) = \frac{\mu}{2}$, $\Pr(\zeta = 1|c = 0) = \Pr(\zeta = 1|c = \theta) = 1 - \mu$ and $\Pr(\zeta = 2|c = 0) = \Pr(\zeta = 0|c = \theta) = 0$ where μ is a commonly known parameter. Hence, with probability $1 - \mu$, the firm does not learn the entry cost at a given experimentation period, and with probability μ, the experimenting firm receives a perfect signal about the entry cost. Signals are independent across periods and across firms and are privately observed by each firm. No payoff is collected by a firm during its experimentation phase.

Each period t is divided into two sub-periods, $(t.1)$ and $(t.2)$. First, at $t.1$, both firms simultaneously make a binary entry decision, $e_i^t \in \{0, 1\}$. If $e_i^t = 1$, firm i enters the market, pays the entry cost c, stops the experimentation phase and starts collecting profits. Subsequently, at $t.2$, if one firm has entered at $t.1$, the other firm makes a binary decision, $d_i^t \in \{0, 1\}$, to follow suit and enter the market immediately. Hence we allow firms to enter the market *immediately after the other firm*, before receiving any additional signal on the entry cost.[3] The profit collected by a firm depends on the entry of the other firm. When both firms are present on the market, they each collect a duopoly payoff of π_d per period. When a single monopolistic firm operates on the market, it collects the monopoly profit π_m every period. We assume that $\pi_m > 2\pi_d$. We also define the discounted sum of profits under duopoly and monopoly as

$$\Pi_d = \frac{\pi_d}{1 - \delta}, \quad \Pi_m = \frac{\pi_d}{1 - \delta}.$$

Finally we denote by Δ the difference between the discounted sum of monopoly and

[3]This modeling assumption clarifies the analysis, without changing the results. In Section 6, we discuss an alternative model where firms have to wait an extra period before entering.

duopoly profits, $\Delta = \Pi_m - \Pi_d$. Notice that, by assumption, $\Delta > \Pi_d$.

We assume that, if firms faced high entry cost, they would never have an incentive to enter the market, even if they received monopoly profit. Instead, if firms had low costs, we assume that they would always make strictly positive profits by entering the market even if they received duopoly profit. Finally, when entry cost remains unknown, we suppose that a firm has an incentive to enter as a monopolist but not as a duopolist. Assumption 1 summarizes these properties.

Assumption 1

$$0 < \Pi_d < \frac{\theta}{2} < \Pi_m < \theta, \tag{1}$$

Assumption 1 allows us to concentrate on the interesting cases where (i) firms never enter when they learn that the cost is high and (ii) do not simultaneously enter at period 0, and wait to acquire information in equilibrium.

For most of the analysis, in order to simplify computations and obtain exact derivations, we consider the *continuous time limit of the discrete model*.[4] To this end, we introduce a constant time interval between periods, Λ, that will be taken to zero in the limit. We need to redefine the parameters of the model accordingly. First, we use the greek letter τ to index the *dates* at which decisions are made, with the correspondence $\tau = t\Lambda$. Similarly, we let the difference between two dates be denoted by $a = d\Lambda$. The per period discount factor satisfies $\delta = e^{-r\Lambda}$ where $r > 0$ is the pure rate of time preference. The rate at which the signal is received by the firms satisfies $\mu = \lambda\Lambda$, where λ is interpreted as the parameter of the Poisson process generating signals in the continuous time limit. Finally, the payoffs per period are given by $\pi_d = v_d\Lambda$ and $\pi_m = v_m\Lambda$ where v_d and v_m are interpreted as the flow duopoly and monopoly payoffs in the continuous time limit.

Given Assumption 1, it is a dominant strategy for a firm that learns the cost is high at period s to never enter and choose $e_i^t = 0$ for all $t \geq s$. If firm i has learned that the cost is low, has not yet entered at period t and observes entry of the firm j at sub-period $t.1$, we claim that it is a dominant strategy for firm i to enter at $t.2$.

[4]In Section 6, we show that the continuous time limit can alternatively be obtained as a game directly defined in continuous time.

By choosing $d_i^t = 1$, the firm obtains Π_d, which is assumed to be strictly positive by Assumption 1 and is higher than any other continuation value of the game, which would involve a delay in collecting Π_d.

Hence, we only need to determine the strategy of firm i at period t in the following three cases, (i) the entry decision e_i^t when firm i is uninformed, denoted p_i^t, (ii) the entry decision e_i^t when firm i has learned that the cost is low at some period $s \leq t$ denoted $q_i^{t,s}$, and (iii) the herding decision d_i^t when firm i is uninformed and has observed entry in t, denoted u_i^t.[5] These strategies depend on the publicly observable history of the game, h^t which records the past entry decisions of the firms, $h^t = ((e_i^s, e_j^s, d_i^s, d_j^s)_{s=1}^{t-1})$. A strategy for firm i is a mapping from any history h^t to probabilities $p_i^t(h^t), q_i^{t,s}(h^t)$ for any $s \leq t$ and $u_i^t(h^t)$ in $[0, 1]$. We also let $\chi_i^t(h^t)$ denote firm i's belief that the cost is high after history h^t.

The solution concept we consider is a *symmetric Bayesian perfect equilibrium*, where (i) every firm chooses an optimal strategy given her beliefs at any history, (ii) beliefs are updated according to Bayes' rule after every history, and (iii) the two firms select the same strategy if they have identical beliefs at any history.

Given that entry decisions are irreversible, firm i has no further decision to make after it has entered. Hence, the only relevant histories for firm i are the histories in which $e_i^s = d_i^s = d_i^t = 0$, and we can summarize the relevant histories for player i by whether firm j has entered or not and, in case firm j has entered, by the period s at which entry took place.

Consider a history h^t where firm j has entered at some period $s < t$. If firm i learns that the cost is low at $s' < t$, two situations can arise: if $s' \leq s$, then we claim as above that it is a dominant strategy for firm i to enter at period $s.2$. If $s' > s$, then we claim again that it is a dominant strategy for firm i to enter at period $s'.1$. By entering firm i obtains $\Pi_d > 0$ and any other strategy would involve a delay and result in a strictly lower payoff. This shows that the only history after which an informed firm has to make a nontrivial decision is the history at which firm j has

[5]Given that firm j has already entered, the herding decision at period s only involves an individual optimization for firm i. Hence, generically, firm i will not choose an interior probability $u_i^t \in (0, 1)$ but either $d_i^t = 0$ or $d_i^t = 1$. If firm i chooses an interior probability, the values of exit and entering are exactly identical, and the exact decision of firm i is irrelevant.

not yet entered.

Next, consider an uninformed firm at period t and suppose that firm j has entered at period s and $u_i^s \neq 1$. Notice that firm i has not received any additional information between period $s.2$ and t, so the beliefs of firm i are the same at period $s.2$ and at period t. Hence the optimal entry decision of firm i is the same at period $s.2$ and at period t. If firm i chooses not to enter with positive probability at $s.2$, it must be an optimal strategy not to enter at period t. This shows that the only history at which an uninformed firm has to make a nontrivial decision is the history at which firm j has not yet entered.

Hence a strategy can be summarized by the entry and herding decisions of an uninformed firm, and the entry decision of a firm that learns that the cost is low when the other firm has not yet entered at period t. A symmetric Bayesian perfect equilibrium is thus characterized by a collection $p^t, q^{t,s}, u^t$ for $t = 1, 2, \ldots, \infty$ and $s < t$, determining the probability of entry of the firms (i) when they are uninformed and no other firm has entered yet, (ii) when they learned that the cost is low at s and no firm has entered until t and (iii) when a firm is uninformed and the other firm has just entered at period $t.1$.

3 Preliminaries

3.1 Optimal Monopoly Strategy

As a benchmark, we first consider the optimal strategy of a monopoly operating both firms. The monopoly compares two strategies: (i) entering immediately before learning the cost or (ii) experimenting (using the facilities of both firms) and entering when it learns that the cost is low. The first strategy yields an expected discounted payoff

$$\Pi_1 = \Pi_d + \Delta - \frac{\theta}{2}.$$

The second strategy results in a discounted payoff

$$\Pi_2 = \sum_{t=0}^{\infty} \delta^t (1-\mu)^{2t} (1-(1-\mu)^2) \frac{\Pi_d + \Delta}{2} = \frac{\mu(2-\mu)(\Pi_d + \Delta)}{2(1-\delta(1-\mu)^2)},$$

and in the continuous time limit

$$\Pi_2 = \frac{(\Pi_d + \Delta)\lambda}{2\lambda + r}.$$

Hence a monopolist operating both firms has an incentive to wait if and only if $\Pi_1 > \Pi_2$, which in the continuous time limit is satisfied whenever the following condition on the parameters holds

$$2(\lambda + r)\Pi_d < \theta + 2\Delta\lambda.$$

3.2 Evolution of beliefs

Consider the common belief χ^t of an uninformed firm that the entry cost is high given that no firm has entered up to period t. At period 0, the belief is $\frac{1}{2}$. At period t, given that no firm has entered, three possibilities arise: (i) the other firm has learned that the cost is high between periods 0 and periods t, which has probability $\sum_{s=0}^{t-1} \mu(1-\mu)^s \frac{1}{2}$; (ii) the other firm has learned that the cost was low at some period $s < t$ but chose not to enter until t, which has probability $\sum_{s=0}^{t-1} \mu(1-\mu)^s \frac{1}{2} \prod_{s'=s}^{t-1}(1-q^{s',s})$; and (iii) the other firm has not learned the cost yet and has not entered, which has probability $(1-\mu)^t \prod_{s=0}^{t-1}(1-p^s)$.

The conditional belief that the cost of entry is high at period t is thus given by

$$\chi^t = \frac{\sum_{s=0}^{t-1} \mu(1-\mu)^s + (1-\mu)^t \prod_{s=0}^{t-1}(1-p^s)}{\sum_{s=0}^{t-1} \mu(1-\mu)^s (1 + \prod_{s'=s}^{t-1}(1-q^{s',s})) + 2(1-\mu)^t \prod_{s=0}^{t-1}(1-p^s)}.$$

Lemma 1 *The belief χ^t is weakly increasing over time, $\chi^{t+1} \geq \chi^t$.*

Hence, given that firms, which learn that the cost is high, never enter, "no news is bad news" and firms become more pessimistic over time, assigning a higher probability to the fact that the cost is high. In particular, given that $\chi^t \geq \chi^0 = \frac{1}{2}$ for all t, an uninformed firm always believes that the cost is more likely to be high than low.

3.3 Herding equilibrium

We first construct a simple symmetric Bayesian perfect equilibrium, which exists for all parameter values. In this equilibrium, uninformed firms never enter, firms that learn the cost is low enter immediately, and are immediately followed by the other firm. Formally,

Definition 1 (Herding equilibrium) *In a herding equilibrium,*

1. $p^t = 0$ *for all* t
2. $q^{t,s} = 1$ *at* $s = t$ *and* $q^{t,s} = 0$ *for all* $t > s$
3. $u^t = 1$ *for all* t

Proposition 1 (Existence of the herding equilibrium) *A herding equilibrium exists for all parameter values.*

Proposition 1 establishes the existence of a symmetric Bayesian perfect equilibrium in the entry game. As shown in the proof of Proposition 1, this is the *only equilibrium* of the game when uninformed firms do not enter. It results in an expected payoff for the two firms of

$$\Pi = \frac{\mu(2-\mu)\Pi_d}{2(1-\delta(1-\mu)^2)},$$

or, in the continuous time limit,

$$\Pi = \frac{\Pi_d \lambda}{2\lambda + r}.$$

4 Single-entry equilibrium

We continue the investigation of the Bayesian perfect equilibria of the entry game by considering equilibria where uninformed firms enter at a single period t (or at a single date τ in the continuous time limit). Fixing the period t, we first provide the conditions for the existence of the equilibrium, and derive comparative statics effects of changes in the parameters of the model on the single-entry equilibrium. We later compare equilibria for different values of the entry period t.

In a single entry equilibrium at period t, $p^t > 0$ and $p^s = 0$ for all $s \neq t$. We claim that this implies that $u^t = 0$ for almost all values of the parameters of the model. Suppose that $u^t > 0$, then except for a measure zero set of parameters of the model, firm i strictly prefers to enter at date t and $u^t = 1$. But then entry by firm i would be immediately followed and the expected profit would be equal to $\Pi_d - \chi^t\theta$ where the expected value of the cost satisfies $\chi^t\theta \geq \frac{\theta}{2}$. Because $\Pi_d - \frac{\theta}{2} < 0$ by Assumption 1, and because firm i can secure a payoff of 0 if it chooses never to enter, we conclude that an uninformed firm i has an incentive to deviate, proving that $u^t = 0$.

Next observe that firm i, when it learns that the cost is low at some date $s > t$, has no incentive to delay entry as entry will immediately be followed by the rival firm. So $q^{s,s} = 1$ and $u^s = 1$ for all $s > t$.

Consider now a period $s < t$. We first claim that if $q^{t,s} > 0$ then $q^{t,s'} > 0$ for all $s \leq s' \leq t$. Suppose by contradiction that $q^{t,s'} = 0$. Then, firm i when it learns that its cost is low at s' enters at some period $t' \neq t$ where only informed firms enter, and will be followed by their rival immediately. As firms have no incentive to delay entry when they are followed immediately, $q^{s',s'} = 1$ and $q^{t',s'} = 0$ for all $t' > s'$. But this implies that the firm prefers to enter immediately after learning that the cost is low, so $\Pi_d > \delta^{t-s'}E(\Pi_t)$ where $E(\Pi_t)$ is the expected profit of a firm, which has learned that its cost is low given that the other firm is uninformed at time t. As $s' \geq s$, we also have $\Pi_d > \delta^{t-s}E(\Pi_t)$, contradicting the fact that the firm waits with positive probability until period t before entering. We conclude that there exists a delay $d \leq t$ (possibly equal to 0) such that a firm that learns that the cost is low at a period between $t - d$ and t prefers to wait before entering, and all firms that learn that the cost is low before $t - d$ prefer to enter. As we have argued before, these firms must enter immediately, and $q^{s,s} = 1$ and $u^s = 1$ for all $s < t - d$. For any $t - d \leq s \leq t$, except for a zero measure set of parameters, firm i strictly prefers to wait and hide the cover of entry of uninformed firms at period t. Finally, we need to specify the herding behavior of firm i off the equilibrium path, when the other firm enters at some period $t - d \leq s < t$. We specify off-equilibrium path beliefs so that an uninformed firm prefers to follow immediately when it observes entry during the delay, $u^s = 1$ for $s = t - d, \ldots, t - 1$.

The previous discussion shows that in a single-entry equilibrium at period t, generically, the following strategies are chosen:

1. $p^t > 0$, $p^s = 0$ for all $s \neq t$,

2. $q^{s,s} = 1$ for all $s < t - d$ and $s > t$ and $q^{t,s} = 1$ for all $t - d \leq s \leq t$, and

3. $u^s = 1$ for all $s \neq t$ and $u^t = 0$.

We now derive conditions on the parameters under which a single-entry equilibrium at period t exists.

Entry of an uninformed firm We first consider the behavior of an uninformed firm. Recall that $\Pi_d < \frac{\theta}{2}$ and that $\chi^t \geq \frac{1}{2}$ at any period t, so that we cannot support an equilibrium where $p^t = 1$. Hence $0 < p^t < 1$ and the uninformed firm must be indifferent between entering and waiting at period t. For simplicity, we drop the index t whenever it is not needed. We compute the expected profit of an uninformed firm entering and waiting, conditional on the fact that no firm has entered until period t as Π_E and Π_W. The expected profit from entry is given by

$$\begin{aligned}\Pi_E = &\frac{(1-\mu)^{t-d} - (1-\mu)^t}{2}\Pi_d + \frac{1-(1-\mu)^t}{2}(\Pi_d + \Delta - \theta) \\ &+ (1-\mu)^t \left(p(\Pi_d - \frac{\theta}{2}) + (1-p)(\Pi_d + \Delta - \frac{\delta\mu}{2(1-\delta(1-\mu))}\Delta - \frac{\theta}{2}) \right).\end{aligned}$$

We decompose the expected profit of entry as follows. With probability $\frac{(1-\mu)^{t-d}-(1-\mu)^t}{2}$, the rival firm has learned the entry cost is low between $t-d$ and t and also enters, giving a payoff of Π_d. With probability $\frac{1-(1-\mu)^t}{2}$, the rival firm has learned that the entry cost is high before τ and never enters, giving a payoff of $\Pi_m - \theta = \Pi_d + \Delta - \theta$. With probability $(1-\mu)^t$, the rival firm has not learned the cost until t and enters with probability p and waits with probability $1-p$. If the rival firm enters, the firm's payoff is $\Pi_d - \frac{\theta}{2}$. If the rival firm does not enter and continues to experiment, then, if the cost is low, the firm collects monopoly profits until the other firm learns the cost, and duopoly profits thereafter. If the cost is high, the firm collects monopoly profits with high entry cost. The expected payoff is given by $\Pi_d + \Delta - \frac{\delta\mu}{2(1-\delta(1-\mu))}\Delta - \frac{\theta}{2}$. We next compute the expected profit from waiting,

$$\begin{aligned}\Pi_W = &\frac{(1-\mu)^{t-d} - (1-\mu)^t}{2}\frac{\delta\mu}{1-\delta(1-\mu)}\Pi_d \\ &+ (1-\mu)^t \left(p\frac{\delta\mu}{2(1-\delta(1-\mu))}\Pi_d + (1-p)\frac{\delta(1-(1-\mu)^2)}{2(1-\delta(1-\mu)^2)}\Pi_d \right).\end{aligned}$$

With probability $\frac{(1-\mu)^{t-d}-(1-\mu)^t}{2}$, the rival firm has learned that the entry cost is low between $t-d$ and t and enters. In that case, the firm continues to experiment until it eventually learns the cost is low and also enters, giving a payoff of $\frac{\delta\mu}{1-\delta(1-\mu)}\Pi_d$. With probability $\frac{1-(1-\mu)^t}{2}$, the rival firm has learned that the entry cost is high before t and never enters. The firm continues to experiment until it eventually learns the cost is high and drops, giving a profit of 0. With probability $(1-\mu)^t$, the rival firm has not learned the cost until t and enters with probability p and waits with probability $1-p$. If the rival firm enters, the firm continues to experiment until it eventually learns the cost is low and also enters or until it eventually learns the entry cost is high and drops from the race, yielding an expected payoff of $\frac{\delta\mu}{2(1-\delta(1-\mu))}\Pi_d$. If the rival firm does not enter, both firms continue experimenting and enter in the period in which the first of them learns the entry cost is low or never enter, resulting in an expected payoff of $\frac{\delta(1-(1-\mu)^2)}{2(1-\delta(1-\mu)^2)}\Pi_d$. For an uninformed firm to be indifferent between entering or not, we must have $\Pi_E = \Pi_W$.

Taking the continuous time limit as the time interval approaches zero, and conditioning on the fact that the firm has not learned the cost until period τ, this equality results in

$$\begin{aligned}
e^{-\tau\lambda}\Pi_E &= \frac{\left(e^{a\lambda}-1\right)}{2}\Pi_d + \frac{e^{\tau\lambda}-1}{2}(\Pi_d+\Delta-\theta) + p(\Pi_d - \frac{\theta}{2}) + (1-p)(\Pi_d + \Delta - \frac{\lambda}{2(\lambda+r)}\Delta - \frac{\theta}{2}) \\
&= \frac{\left(e^{a\lambda}-1\right)}{2}\frac{\lambda}{\lambda+r}\Pi_d + p\frac{\lambda}{2(\lambda+r)}\Pi_d + (1-p)\frac{\lambda}{2\lambda+r}\Pi_d \\
&= e^{-\tau\lambda}\Pi_W.
\end{aligned}$$

This expression highlights the trade-off faced by an uninformed firm contemplating entry at date τ. By entering, the firm may discover that the cost is high, and be locked in a situation where it is a monopoly on the market, and obtains a negative discounted payoff of $\Pi_m < \theta$. This is the downside of entry when the firms is uninformed, as by waiting the firm only enters when it learns that the cost is low, and never faces the possibility of a negative discounted payoff. On the upside, if the cost is low and the firm enters when its rival does not, it will collect monopoly profits until the other firm learns that the cost is low. This benefit from entry will be higher the lower the probability that the other firm enters when it is uninformed.

Rearranging, we define the entry probability p as a function of the delay a,

$$p = f(a) = 1 + \frac{\frac{1}{2}\left(e^{a\lambda}-1\right)\frac{r}{\lambda+r}\Pi_d + \frac{1}{2}\left(e^{\lambda\tau}-1\right)(\Delta-\theta+\Pi_d) + \left(\Pi_d - \frac{\theta}{2}\right) - \frac{\lambda\Pi_d}{2(\lambda+r)}}{\frac{\lambda\Pi_d}{2(\lambda+r)} - \frac{\lambda\Pi_d}{2\lambda+r} + \frac{\Delta(\lambda+2r)}{2(\lambda+r)}}.$$

Entry of an informed firm We next turn to the strategy of a firm that learns that its cost is low at a period $s < t$. If the firm enters at any period different from t, entry will be followed immediately, and the firm's profit equal to $\Pi_E = \Pi_d$. If the firm waits $t - s$ periods and enters at period t, it will collect an expected profit of

$$\Pi_W = \delta^{t-s}(1-(1-\mu)^{t-s})\Pi_d + \delta^{t-s}(1-\mu)^{t-s}\left(p\Pi_d + (1-p)\left(\Pi_d + \Delta - \frac{\delta\mu}{1-\delta(1-\mu)}\Delta\right)\right)$$

With probability $(1-(1-\mu)^{t-s})$, the rival firm learns that the entry cost is low between s and t and also enters yielding a profit Π_d. With probability $(1-\mu)^{t-s}$, the rival firm does not learn the entry cost between s and t. In this case, the rival firm will enter with probability p and wait with probability $1-p$. If the rival firm enters, the firm's payoff is Π_d. If the rival firm does not enter and continues to experiment, the firm collects monopoly profits until the rival eventually learns the entry cost is low and enters, giving an expected payoff of $\Pi_d + \Delta - \frac{\delta\mu}{1-\delta(1-\mu)}\Delta$.

In equilibrium, we require that $\Pi_E \geq \Pi_W$ for $s \geq t-d$ and $\Pi_E < \Pi_W$ for $s < t-d$. In the continuous time limit, we define a to be the delay at which an informed firm is exactly indifferent between entering and waiting,

$$\begin{aligned}\Pi_E &= \Pi_d \\ &= e^{-ra}\Pi_d + (1-p)e^{-ra}e^{-a\lambda}\frac{r}{\lambda+r}\Delta \\ &= \Pi_W.\end{aligned}$$

The trade-off faced by an informed firm is simple: by waiting the firm benefits from being a monopolist on the market until the other firm learns that the cost is low. Waiting however entails a cost, as the firm will only be able to collect profits at a later date. The value of waiting is higher the lower the probability that an uninformed firm exits at date τ. Equating the payoff of entering and waiting, we obtain an expression for the entry probability,

$$p = k(a) = 1 - \frac{\left(1-e^{-ra}\right)\Pi_d}{e^{-(\lambda+r)a}\frac{r}{\lambda+r}\Delta}.$$

Herding decision by an uninformed firm We finally turn to the herding decision of uninformed firms. In equilibrium, uninformed firms prefer not to follow an entering firm at period t. By following suit, it obtains an expected payoff of

$$\Pi_E = \frac{(1-\mu)^{t-d} - (1-\mu)^t}{2}\Pi_d + p(1-\mu)^t(\Pi_d - \frac{\theta}{2}).$$

With probability $\frac{(1-\mu)^{t-d}-(1-\mu)^t}{2}$, the rival firm enters at t after having learned that the entry cost is low between $t-d$ and t, giving a payoff of Π_d. With probability $p(1-\mu)^t$, the rival firm enters at t without learning the cost, giving an expected payoff of $\Pi_d - \frac{\theta}{2}$.

If instead the firm does not follow suit and waits, it obtains an expected payoff of

$$\Pi_W = \frac{(1-\mu)^{t-d} - (1-\mu)^t}{2}\frac{\delta\mu}{1-\delta(1-\mu)}\Pi_d + p(1-\mu)^t\frac{\delta\mu}{2(1-\delta(1-\mu))}\Pi_d.$$

With probability $\frac{(1-\mu)^{t-d}-(1-\mu)^t}{2}$, the rival firm enters at t after having learned that the cost is low between $t-d$ and t. In that case, the firm continues to experiment until it eventually learns the cost is low and also enters, giving a payoff of $\frac{\delta\mu}{1-\delta(1-\mu)}\Pi_d$. With probability $p(1-\mu)^t$, the rival firm entered without being informed. In that case, the firm continues to experiment until it eventually learns that the cost is low and also enters or until it eventually learns that the entry cost is high and drops from the race, giving an expected payoff of $\frac{\delta\mu}{2(1-\delta(1-\mu))}\Pi_d$.

In the continuous time limit, conditioning on the fact that the firm is uninformed, we obtain

$$\begin{aligned} e^{-\tau\lambda}\Pi_E &= \frac{\left(e^{a\lambda}-1\right)}{2}\Pi_d + p(\Pi_d - \frac{\theta}{2}) \\ &\leq \frac{\left(e^{a\lambda}-1\right)}{2}\frac{\lambda}{\lambda+r}\Pi_d + p\frac{\lambda}{2(\lambda+r)}\Pi_d \\ &= e^{-\tau\lambda}\Pi_W \end{aligned}$$

If the firm follows a rival's entry without being informed, it risks competing with the rival when the entry cost is high. By waiting, the firm is guaranteed never to pay the high entry cost. In order to assess whether it should herd or not, the firm must take into account the composition of the pool of rivals who may have entered at date τ. If the probability of entry of uninformed firms, p, is high, the pool will contain a large number of uninformed firms, making herding less profitable. Hence for the equilibrium to exist, the entry probability must be sufficiently high. Formally, we write the no-herding constraint as

$$p \geq g(a) = \frac{\left(e^{a\lambda} - 1\right)\frac{r}{\lambda+r}\Pi_d}{\theta - \frac{\lambda+2r}{\lambda+r}\Pi_d}.$$

Equilibrium We now summarize the conditions under which an equilibrium exists in the continuous time limit. Given a fixed date τ, an equilibrium is characterized by a pair of delay and entry probability (a^*, p^*) such that

1. An uninformed firm is indifferent between entering and waiting at τ, $f(a^*) = p^*$
2. An informed firm prefers to enter before $\tau - a^*$ and to wait between $t - a^*$ and t, $k(a^*) = p^*$
3. An uninformed firm does not herd at τ: $p^* \geq g(a^*)$
4. The delay cannot be larger than the date: $a^* \leq \tau$.

The last condition is an obvious feasibility condition: firms cannot wait longer than the time which has elapsed before date τ. Taking into account this boundary condition on the delay a^*, and the fact that $f(\cdot)$ is increasing in a and $k(\cdot)$ is decreasing in a, two situations may arise:

1. Either the equilibrium is defined by a pair (a^*, p^*) which satisfies the three conditions $f(a^*) = p^*, k(a^*) = p^*$ and $p^* \geq g(a^*)$
2. Or the equilibrium is given by (τ, p^*) where $f(\tau) = p^*$, $k(\tau) > p^*$ and $p^* \geq g(\tau)$.

The exact conditions on the parameters of the model guaranteeing existence of an equilibrium are not easy to derive. Our first result provides a necessary condition for existence of a single-entry equilibrium at any date τ.

Proposition 2 (Existence) *If a single-entry equilibrium exists, the following condition holds:*

$$\theta \leq \frac{\lambda + 2r}{\lambda + r}\Delta + \frac{2(\lambda + r)}{2\lambda + r}\Pi_d.$$

The condition of Proposition 2 is the necessary and sufficient condition for the existence of a single-entry equilibrium at date $\tau = 0$. In this *immediate entry equilibrium*, the two firms enter at date 0 with positive probability,

$$p = 1 + \frac{\Pi_d - \frac{\theta}{2} - \frac{\lambda}{2(\lambda+r)}\Pi_d}{\frac{\lambda+2r}{2(\lambda+r)}\Delta + \frac{\lambda}{2(\lambda+r)}\Pi_d - \frac{\lambda}{2\lambda+r}\Pi_d}.$$

Given that $\theta \leq \frac{\lambda+2r}{\lambda+r}\Delta + \frac{2(\lambda+r)}{2\lambda+r}\Pi_d$, the probability p is well-defined and belongs to the interval $(0,1)$. At date 0, $g(0) = 0$, as only uninformed firms leave and, hence, an uninformed firm has no incentive to herd at date 0. At any date $\tau > 0$, the only firms to enter are firms that learn that the cost is low. They have an incentive to enter immediately and are immediately followed by uninformed firms.

Proposition 2 thus shows that if a single-entry at date τ exists, there must also exist an immediate entry equilibrium. The intuition is easy to grasp: as time passes, firms become more pessimistic – no news is bad news – and the value of entry goes down. This in turn reduces the probability of entry that makes a firm indifferent between entering or not. It also encourages informed firms to wait longer, resulting over time in a lower probability of entry p^* and a longer delay a^*.[6] Recall that the last condition of the equilibrium – the condition on the herding decision of an uninformed firm – is independent of the date τ, but only requires the entry probability p^* to be sufficiently high. As time passes and p^* becomes smaller, this condition is less likely to be satisfied. Hence, if there exists a single-entry equilibrium at a date $\tau > 0$, there is also a single-entry equilibrium at date 0.

Our second result shows that, for any τ, the two equations defining the entry probability and delay have a unique solution. Hence, if a single-entry equilibrium exists at date τ, it must be unique.

Proposition 3 (Uniqueness) *Suppose that an equilibrium with single entry at date τ exists. Then the equilibrium (p^*, a^*) is unique.*

Our third result deals with the effect of changes in the parameters Π_d, Δ and θ on the equilibrium values (p^*, a^*).

[6] In this intuitive interpretation, we focus attention on the case where $a^* < \tau$. We discuss the case where $a^* \geq \tau$ in the proof of Proposition 2.

Proposition 4 (Comparative Statics) *Suppose that an equilibrium with single-entry at date τ exists. The following comparative statics hold for all values of the parameters: (i) $\partial a^*/\partial\theta \geq 0$, (ii) $\partial p^*/\partial\theta < 0$, (iii) $\partial a^*/\partial\Pi_d \leq 0$, (iv) $\partial p^*/\partial\Pi_d \gtreqless 0$, (v) $\partial a^*/\partial\Delta \gtreqless 0$, (vi) $\partial p^*/\partial\Delta > 0$.*

A higher entry cost lowers the probability that makes an uninformed firm indifferent between entering and waiting and has no effect on the decision of an informed firm. Hence it results in a lower equilibrium entry probability and a higher equilibrium delay. An increase in the duopoly profit makes immediate entry more profitable for an informed firm, and hence lowers the delay during which the firm is willing to wait. It has ambiguous effects on the entry probabilities, as the value of entry goes up both for the uninformed and informed firms, resulting in the first case in a higher probability that makes the uninformed firm indifferent between entering and waiting, and in the latter case in a lower probability that makes the informed firm indifferent between entering and waiting. On the other hand, an increase in the monopoly profit increases the value of waiting both for an uninformed firm and for an informed firm. For an informed firm, the value of immediate entry does not depend on the monopoly profit, so that an increase in the monopoly profit must be matched by an increase in the entry probability of uninformed firms, to keep the informed firm indifferent between entering and waiting. This increase will have ambiguous effects on the equilibrium delay, as an increase in the entry probability results in a higher delay to make an uninformed firm indifferent between entering and waiting, and a lower delay to make an informed firm indifferent between entering and waiting.

We next compare single-entry equilibria at different dates τ_1 and τ_2.

Proposition 5 (Comparison of equilibria across dates) *Let (p_1^*, a_1^*) and (p_2^*, a_2^*) be two single-entry equilibria at dates $\tau_1 < \tau_2$. Then $p_1^* > p_2^*$.*

Proposition 5 echoes Proposition 2 by showing that the entry probability p becomes smaller when time passes, as uninformed firms become more pessimistic. This, in turn, lowers the fraction of uninformed firms in the pool of firms, which enter at the specified entry date, making the existence of a single-entry equilibrium harder.

We now show that this implies that there is an upper bound on the date at which entry of an uninformed firm can occur.

We now treat the date τ as an endogenous variable and consider the system of three equations in three unknowns given by:

$$\begin{aligned} f(\overline{a},\overline{\tau}) &= \overline{p}, \\ g(\overline{a}) &= \overline{p}, \\ k(\overline{a}) &= \overline{p}. \end{aligned}$$

We first argue that this system always has a unique solution satisfying $0 < \overline{p} < 1$, $\overline{a} > 0$ and $\overline{\tau} > 0$ if $\theta \leq \frac{\lambda+2r}{\lambda+r}\Delta + \frac{2(\lambda+r)}{2\lambda+r}\Pi_d$. Notice first that $g(\cdot)$ is an increasing function of a satisfying $g(0) = 0$ and $\lim_{a\to\infty} = +\infty$ and $k(\cdot)$ is a decreasing function satisfying $k(0) = 1$ and $\lim_{a\to\infty} k(a) = -\infty$. This implies that $(g - k)(\cdot)$ is an increasing function on $(0, +\infty)$ with $g(0) - k(0) = -1 < 0$ and $\lim_{a\to\infty} g(a) - k(a) = +\infty$, so there is a unique pair $(\hat{p}, \hat{a})$ such that $\hat{p} = g(\hat{a}) = k(\hat{a})$ with $a > 0$ and $0 < p < 1$. Now, notice that $\lim_{\tau\to-\infty} f(\hat{a}, \tau) = -\infty$ and $\lim_{\tau\to\infty} f(\hat{a}, \tau) = +\infty$. Hence, there is a unique value of τ (possibly negative) such that $f(\hat{a}, \hat{\tau}) = \hat{p}$. To show that $\overline{\tau}$ is positive suppose by contradiction that $\overline{\tau} < 0$. Because $f(\cdot)$ is strictly decreasing in τ, this implies that $f(\overline{a}, 0) < 0$. Because f is strictly increasing in a and $\overline{a} > 0$, $f(0, 0) < f(\overline{a}, 0) < 0$. But if $\theta \leq \frac{\lambda+2r}{\lambda+r}\Delta + \frac{2(\lambda+r)}{2\lambda+r}\Pi_d$, we must have $f(0, 0) \geq 0$, in contradiction to the previous statement.

We next show that there is no single-entry equilibrium at a date later than $\overline{\tau}$.

Proposition 6 (Latest entry date) *There is no single-entry equilibrium at $\tau \geq \overline{\tau}$.*

The solution $(\overline{\tau}, \overline{a}, \overline{p})$ to the system of equations corresponds to the latest date, for which the single-entry equilibrium exists, $\overline{\tau}$, the corresponding equilibrium delay $\overline{a}$ (assuming that $\overline{a} < \overline{t}$ as otherwise the delay will be equal to $\overline{\tau}$) and the corresponding entry probability of uninformed firms, $\overline{p}$. For later entry dates, $\tau > \overline{\tau}$, a single-entry equilibrium does not exist. For $\tau > \overline{\tau}$, all (a, p) pairs that satisfy the no-herding constraint would violate the waiting constraint and (a, p) pairs that satisfy the waiting constraint would violate the no-herding constraint.

Example 1 *Let us parametrize our model as follows.* $\Pi_d = \alpha\Pi_m$ *with* $\alpha \in [0, \frac{1}{2}]$, $\Delta = \Pi_m - \alpha\Pi_m = (1-\alpha)\Pi_m$, *and* $\theta = \beta\Pi_m$ *with* $\beta \in [1+\alpha, 2]$. *Then*

$$f(a) = 1 + \frac{\frac{1}{2}\left(e^{a\lambda}-1\right)\frac{r}{\lambda+r}\alpha + \frac{1}{2}\left(e^{\lambda\tau}-1\right)(1-\beta) + \left(\alpha - \frac{\beta}{2}\right) - \frac{\lambda}{2(\lambda+r)}\alpha}{\frac{\lambda}{2(\lambda+r)}\alpha - \frac{\lambda}{2\lambda+r}\alpha + \frac{(\lambda+2r)}{2(\lambda+r)}(1-\alpha)}$$

$$g(a) = \frac{\left(e^{a\lambda}-1\right)\frac{r}{\lambda+r}\alpha}{\beta - \frac{\lambda+2r}{\lambda+r}\alpha}$$

$$k(a) = 1 - \frac{\left(1-e^{-ra}\right)\alpha}{e^{-(\lambda+r)a}\frac{r}{\lambda+r}(1-\alpha)}$$

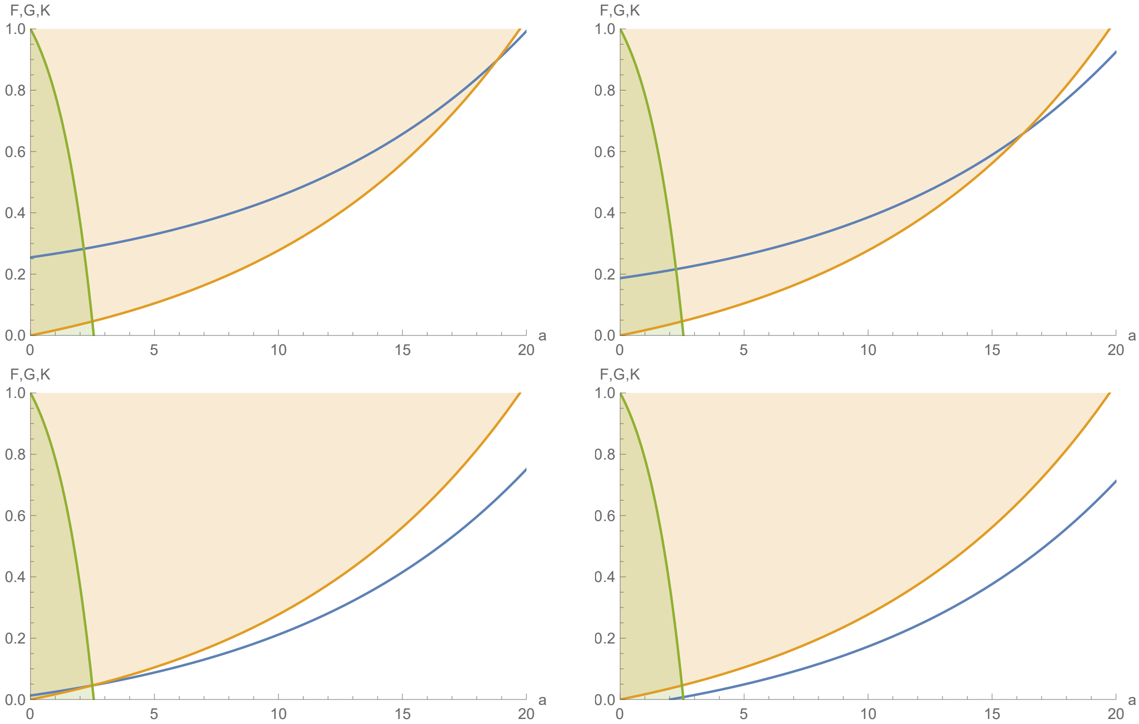

Figure 1: Blue: $f(a)$ entry probability; Green: $k(a)$ waiting constraint; Orange: $g(a)$ no-copy constraint. Parameters: $\alpha = 0.2$, $\beta = 1.4$, $r = 0.5$, $\lambda = 0.1$. Top left: $\tau = 1$, top right: $\tau = 3$, bottom left: $\tau = 6.81604$, bottom right: $\tau = 7.5$

For $\alpha = 0.2$, $\beta = 1.4$, $r = 0.5$, $\lambda = 0.1$, *we find that the latest equilibrium entry date for uninformed firms is* $\overline{\tau} = 6.81604$, *the corresponding equilibrium delay is given by* $\overline{a} = 2.49351$ *and the entry probability is given by* $\overline{p} = 0.0456762$. *Any earlier entry date also corresponds to a single-entry date equilibrium. For example,* $\tau = 3$ *corresponds to one, in which* $a^* = 2.2537$ *and* $p^* = 0.216038$. *Similarly,* $\tau = 1$ *corresponds to one, in which* $a = \tau = 1$ *and* $p^{**} = 0.266821$. *In this case, the intersection of the* $k(a)$ *and* $f(a)$ *would have led to an inadmissible delay of*

$a^* = 2.14959 > \tau = 1$. *For* $\tau = 7.5 > \overline{\tau}$, *the single-entry equilibrium does not exist. Figure 1 illustrates these four cases.*

5 Multiple-entry equilibrium

We now study equilibria, in which uninformed firms enter at a finite number of periods, $t_1, t_2, \ldots, t_M$. As in a single-entry equilibrium, for entry to be profitable it must be that the rival firm does not herd, that is, $u^t = 0$ for all $t \in \{t_1, \ldots, t_M\}$. An informed firm may delay entry for d_m periods before the fixed entry date t_m, $q^{s,s} = 1$ if $s \notin \{t_m - d_m, \ldots, t_m - 1\}$ for any m and, if $s \in \{t_m - d_m, \ldots, t_m - 1\}$, $q^{t_m,s} = 1$. Hence a multiple entry equilibrium at periods $t_1, \ldots, t_M$ is characterized by entry probabilities $p_1, \ldots, p_M$ and delays $d_1, \ldots, d_M$ for each of the entry dates.

We now argue that the entry strategy an informed firm and the herding strategy of an uninformed firm are identical in a multiple-entry equilibrium and in a single-entry equilibrium. Fix a period t_m. By entering immediately after learning that the cost is low at s between t_{m-1} and t_m, the firm obtains Π_d. If the firm waits until t_m to enter, its entry will not be followed: it will either obtain Π_d with probability p_m or, with probability $1 - p_m$, the rival firm will wait until it learns that the cost is low before entering. The condition stating that the informed firm prefers to wait or enter at date t_m only depends on the entry probability p_m and the delay between s and t_m, as in the case of a single-entry equilibrium. In other words, the function defining the delay, $k(a)$, remains the same as in the single-entry equilibrium.

Consider the herding strategy of an uninformed firm. By herding at period $t_m.2$, it obtains Π_d if the other firm has learned that the cost is low during the delay d_m and $\Pi_d - \frac{\tau}{2}$ is the rival firm has entered at t_m uninformed. By waiting, it will obtain a duopoly profit when and if it learns that its cost is low, a payoff which only depends on p_m. Hence the herding strategy of the uninformed firm only depends on the entry probability p_m and the delay d_m, as in the single-entry equilibrium. The function defining the entry parameter for which herding occurs, $g(a)$, remains the same as in the single-entry equilibrium.

By contrast, the entry decision of the uninformed firm at t_m, p_m, is not the same in a single-entry equilibrium and in a multiple-entry equilibrium. In a multiple-entry equilibrium, the entry of uninformed firms before t_m affects the conditional probability that the rival firm has learned the cost is high. The entry of uninformed firms after t_m affects the expected payoff of an uninformed firm when it decides to wait as the other firm is also uninformed and does not enter. Hence, the function defining the entry probability at period t_m, $f^m(a)$, depends on the entry probabilities $p_1, \dots, p_{m-1}$ and $p_{m+1}, \dots, p_M$.

Multiple-entry vs. single-entry equilibrium We start by analyzing the entry probability at the last entry period t_M. We introduce some notation on the belief of firm i on the signal received by firm j. Let $\xi(0)$ be the probability the rival firm has learned the cost is low between $t_M - d_M$ and t_M, $\xi(\theta)$ be the probability the rival firm has learned the cost is high and $\xi(\hat{\theta})$ the probability the rival firm remains uninformed at t_M. By entering at period t_M, the firm obtains an expected profit

$$\begin{aligned}\Pi_E = {} & \xi(0)\Pi_d + \xi(\theta)(\Pi_d + \Delta - \theta) \\ & + \xi(\hat{\theta})\left(p\left(\Pi_d - \frac{\theta}{2}\right) + (1-p)\left(\Pi_d + \Delta - \frac{\delta\mu}{2(1-\delta(1-\mu))}\Delta - \frac{\theta}{2}\right)\right).\end{aligned}$$

After t_M, if both firms are uninformed and wait, they only enter when one of them discovers that the cost is low as in the single-entry equilibrium. Hence the expected profit when waiting as in the single-entry case is given by

$$\Pi_W = \xi(0)\frac{\delta\mu}{1-\delta(1-\mu)}\Pi_d + \xi(\hat{\theta})\left(p\frac{\delta\mu}{2(1-\delta(1-\mu))}\Pi_d + (1-p)\frac{\delta(1-(1-\mu)^2)}{2(1-\delta(1-\mu)^2)}\Pi_d\right).$$

For an uninformed firm i to be indifferent between entering and waiting, we need $\Pi_E = \Pi_W$. Taking the continuous time limit and conditioning on the fact that the rival firm has not entered before $\tau_M - a_M$, this equality results in

$$\begin{aligned}& \xi(0)\Pi_d + \xi(\theta)(\Pi_d + \Delta - \theta) + \xi(\hat{\theta})\left(p\left(\Pi_d - \frac{\theta}{2}\right) + (1-p)\left(\Pi_d + \Delta - \frac{\lambda}{2(\lambda+r)}\Delta - \frac{\theta}{2}\right)\right) \\ & \quad = \xi(0)\frac{\lambda}{\lambda+r}\Pi_d + \xi(\hat{\theta})\left(p\frac{\lambda}{2(\lambda+r)}\Pi_d + (1-p)\frac{\lambda}{2\lambda+r}\Pi_d\right),\end{aligned}$$

yielding an entry probability

$$p_M = f_M(a) = 1 + \frac{-\frac{\xi(0)}{\xi(\hat{\theta})}\frac{r}{\lambda+r}\Pi_d + \frac{\xi(\theta)}{\xi(\hat{\theta})}(\Delta - \theta + \Pi_d) + \left(\Pi_d - \frac{\theta}{2}\right) - \frac{\lambda\Pi_d}{2(\lambda+r)}}{\frac{\lambda\Pi_d}{2(\lambda+r)} - \frac{\lambda\Pi_d}{2\lambda+r} + \frac{\Delta(\lambda+2r)}{2(\lambda+r)}}.$$

Now, we compute

$$\xi(0) = \frac{1}{2}\prod_{m=1}^{M-1}(1-p_m)(e^{-(\tau_m-a_m)\lambda} - e^{-\tau_m\lambda}),$$

$$\xi(\theta) = \frac{1}{2}(1-e^{-\tau_m\lambda}\prod_{m=1}^{M-1}(1-p_m))$$

$$\xi(\hat{\theta}) = \prod_{m=1}^{M-1}(1-p_m)e^{-\tau_m\lambda}.$$

Notice that

$$\frac{\xi(0)}{\xi(\hat{\theta})} = \frac{1-e^{-a_m\lambda}}{2e^{-a_m\lambda}},$$

is independent of the entry probabilities $p_1, \ldots, p_{M-1}$ and of the date τ_m, while

$$\frac{\xi(\theta)}{\xi(\hat{\theta})} = \frac{1-e^{-\tau_m\lambda}\prod_{m=1}^{M-1}(1-p_m)}{2\prod_{m=1}^{M-1}(1-p_m)e^{-\tau_m\lambda}}$$

is increasing in each of the values $p_1, \ldots, p_{M-1}$ and in the date τ_m. As more uninformed agents enter before date τ_M, the conditional probability a firm has learned the cost is high, given it has not yet entered goes up. Hence firms are more pessimistic in a multi-entry equilibrium with last date τ_M than in a single-entry equilibrium at any date $\tau < \tau_M$. We use this insight to provide a necessary condition for the existence of a multi-entry equilibrium.

Proposition 7 (Existence) *If a multiple-entry equilibrium with last entry date τ_M exists, then there exists a single-entry equilibrium at date $\tau_M - \tau_{M-1}$.*

Proposition 7 shows that the range of parameters for which multiple-entry equilibria exist is a subset of the set of parameters for which a single-entry equilibrium exists. Hence it is harder to support multiple-entry equilibria than single-entry equilibria, and more stringent conditions are needed to guarantee the existence of these equilibria. This result also shows that there exists a last-entry date in a multiple-entry equilibrium, which is at most equal to M times the last-entry date of the single-entry equilibrium.

Interestingly, the existence of a multiple-entry equilibria with last entry date τ_M does not guarantee the existence of a single-entry equilibrium at that date, but at the earlier date $\tau_M - \tau_{M-1}$. Given that, as stated in Proposition 5, entry probabilities in a single-entry equilibrium are decreasing over time, the existence of a single-entry equilibrium at date $\tau_M - \tau_{M-1}$ does not imply existence of a single-entry equilibrium at date τ_M. The reason why existence of a multiple-entry equilibrium with last entry date τ_M only guarantees existence of a single entry equilibrium at date $\tau_M - \tau_{M-1}$ is as follows. Suppose that the delay in the multiple-entry equilibrium is constrained by the difference between two consecutive entry dates and is exactly equal to $\tau_M - \tau_{M-1}$. Then the number of informed firms entering between $\tau_M - \tau_{M-1}$ is bounded, relaxing the constraint that an uninformed firm does not want to follow entry at date τ_M. This implies that the no-herding constraint may be easier to satisfy in a multiple-entry equilibrium at date τ_M than in a single-entry at date τ_M because multiple entries put a constraint on the delay $\tau_M - \tau_{M-1}$. If one considers instead a single-entry equilibrium at date $\tau_M - \tau_{M-1}$ rather than date τ_M, the constraint on the delay induced by the entry date is identical in the multiple-entry and single-entry equilibrium, making this effect disappear.

Uniqueness of multiple-entry equilibria Proposition 3 shows that, for a fixed entry date τ, the single-entry equilibrium, if it exists, is unique. We now extend this result to multiple-entry equilibria, fixing the entry dates $t_1, \ldots, t_M$. Uniqueness of equilibrium is harder to prove because the entry probability at period t_m given by $p_m = f_m(a_m)$ is a function of the entry probabilities at all other entry dates, creating an inter-dependence between the different entry probabilities which could result in multiplicity of equilibria. To analyze the problem further, we compute the entry probability p_m at an arbitrary period $t_m < t_M$. We observe that, because uninformed firms will be indifferent between and waiting at further periods $t_{m+1}, \ldots, t_M$ when no firm has entered, we can compute the continuation value of a firm waiting at period t_m assuming that it will exit with probability 1 at period t_{m+1}. Using this observation and the algebraic derivations contained in an Online Appendix, we compute the entry probability at period t_m as:

$$p_m = f_m(a) = 1 - \frac{\Pi_d - \frac{\theta}{2} - \frac{\Pi_d \lambda}{2(\lambda+r)} - \frac{\xi(0)}{\xi(\hat{\theta})}\Pi_d \frac{r}{\lambda+r} - \frac{\xi(\theta)}{\xi(\hat{\theta})}(\Pi_d + \Delta - \theta)(1 - e^{-(\lambda+r)(\tau_{m+1}-\tau_m)})}{\frac{\Pi_d \lambda}{2(\lambda+r)} + \frac{\Delta(2r+\lambda)}{2(\lambda+r)} - A(p_{m+1})}.$$

where

$$A(p_{m+1}) \equiv \frac{1-e^{-(2\lambda+r)(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}\frac{2\lambda}{2\lambda+r}\Pi_d$$
$$+\frac{e^{-2\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}(1-e^{-a_{m+1}\lambda})e^{-r(\tau_{m+1}-\tau_m)}\left(\Pi_d+\frac{\Delta r}{\lambda+r}(1-p_{m+1})e^{-a_{m+1}\lambda}\right)$$
$$+e^{-(r+\lambda)(\tau_{m+1}-\tau_m)}\left[\frac{e^{-\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}-e^{-\lambda(\tau_{m+1}-\tau_m)}}{2}\Pi_d+\frac{(1-e^{-\lambda(\tau_{m+1}-\tau_m)})}{2}(\Pi_d+\Delta-\theta)\right.$$
$$\left.+e^{-\lambda(\tau_{m+1}-\tau_m)}\left(\Pi_d+(1-p_{m+1})\frac{(2r+\lambda)\Delta}{2(\lambda+r)}-\frac{\theta}{2}\right)\right].$$

We need an additional Assumption to verify that the probability $f_m(a)$ is well-defined. As we prove in Proposition 8, if $\frac{2r+\lambda}{\lambda+r}\Delta<\theta$, the equilibrium entry probability belongs to the interval $(0,1)$ and $f_m(\cdot)$ is decreasing in $p_1,\ldots,p_{m-1}$ and increasing in p_{m+1}. We use this fact to prove uniqueness of the equilibrium for a sequence of entry dates $t_1,\ldots,t_M$.

Proposition 8 (Uniqueness) *Suppose that $\frac{2r+\lambda}{\lambda+r}\Delta<\theta$ and that an equilibrium with multiple entry at dates $\tau_1,\ldots,\tau_M$ exists. Then the equilibrium entry probabilities and delays $(p_1^*,a_1^*,\ldots,p_M^*,a_M^*)$ are unique.*

Comparison of entry probabilities We complete the investigation of multiple-entry equilibria by comparing the entry probabilities $p_1,\ldots,p_M$ at the different entry dates $\tau_1,\ldots,\tau_M$ when the difference between two successive dates, $D\tau$, is constant. Consider again the expected profit of an uninformed firm when it waits at date τ_m with $m<M$. Because the firm is indifferent between entering and waiting at any later date, we can compute the continuation payoff assuming that the firm waits at every date $\tau_{m+1},\ldots,\tau_M$, leading to an alternative formulation of the entry probability

$$p_m=f_m(a)=1-\frac{\Pi_d-\frac{\theta}{2}-\frac{\Pi_d\lambda}{2(\lambda+r)}-\frac{\xi(0)}{\xi(\hat\theta)}\Pi_d\frac{r}{\lambda+r}-\frac{\xi(\theta)}{\xi(\hat\theta)}(\Pi_d+\Delta-\theta)}{\frac{\Pi_d\lambda}{2(\lambda+r)}+\frac{\Delta(2r+\lambda)}{2(\lambda+r)}-W_{m+1}}.$$

where the continuation value W_m is given by the recursive formula

$$
\begin{aligned}
W_m &= \frac{1-e^{-(2\lambda+r)(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}\frac{2\lambda}{2\lambda+r}\Pi_d \\
&+ \frac{e^{-2\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}(1-e^{-a_{m+1}\lambda})e^{-r(\tau_{m+1}-\tau_m)}\left(\Pi_d+\frac{\Delta r}{\lambda+r}(1-p_{m+1})e^{-a_{m+1}\lambda}\right) \\
&\quad + \frac{e^{-2\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}(1-e^{-a_{m+1}\lambda})e^{-r(\tau_{m+1}-\tau_m)}\Pi_d\frac{\lambda}{\lambda+r}e^{-a_{m+1}\lambda} \\
&\qquad + e^{-\lambda(\tau_{m+1}-\tau_m)}\left(p_{m+1}\frac{\pi_d\lambda}{2(\lambda+r)}+(1-p_{m+1})W_{m+1}\right)
\end{aligned}
$$

with terminal condition at the last entry date:

$$
W_{M+1} = \frac{2\lambda}{2\lambda+r}\Pi_d.
$$

As we have already argued, $\frac{\xi(\theta)}{\xi(\hat{\theta})}$ increases over time, and for a fixed equilibrium, is higher at date τ_{m+1} than at date τ_m. As p_m is decreasing in $\frac{\xi(\theta)}{\xi(\hat{\theta})}$, this effect suggests, as in Proposition 7, that entry probabilities should be lower at later dates. However, when one considers entry at dates τ_m which are *not* the last entry dates, a second effect arises, as the continuation value W_{m+1} also depends on the date τ_m. It is easy to check that the entry probability p_m is decreasing in W_{m+1}. If the continuation value goes up, the expected profit of waiting also goes up, and the probability that makes the uninformed firm indifferent between entering and waiting must go down. Hence, a sufficient condition for the equilibrium entry probabilities p_m to decrease over time is that the continuation value W_m be increasing over time. We now show that if the difference between two successive entry dates is sufficiently small, the continuation value increases over time, and the entry probability thus decreases over time.

Proposition 9 *There exists an upper bound $\overline{D\tau}$ such that, if $D\tau < \overline{D\tau}$, the equilibrium continuation values are increasing over time, $W_{m+1} > W_m$ and the equilibrium entry probabilities are decreasing over time, $p_m > p_{m+1}$.*

Proposition 9 shows that, when the difference between entry dates is sufficiently small, the continuation value is increasing over time, implying that equilibrium entry probabilities are smaller for more distant entry dates. The proof relies on the fact that the interval between two consecutive dates is small, so that, at any date τ_m, the expected profit earned during the next time interval $D\tau$ is small with respect to the continuation value at τ_{m+1}. The continuation value at τ_{m+1} is computed taking account of two possibilities. If the rival firm enters as an uninformed firm (with probability p_{m+1}), the firm will only obtain the expected profit of a firm drawing the signal alone $\frac{\lambda}{2(\lambda+r)}$, which is the lowest possible continuation payoff. If instead the rival firm does not enter (with probability $1-p_{m+1}$), the firm will collect the expected continuation payoff W_{m+1}. At the terminal date, $W_{M+1} = \frac{\lambda}{2\lambda+r}$, a continuation value which is higher than $\frac{\lambda}{2(\lambda+r)}$, so that the continuation value must be lower at M than at $M+1$. This in turn implies that the entry probability is higher at date τ_{M-1} than at date τ_M. An inductive argument then shows that the continuation value must be lower at m than at $m+1$, and the entry probabilities higher at m than at $m+1$.

6 Interpretation and robustness

Comparison of equilibria The preceding analysis shows that multiple equilibria may exist in the game. How are equilibria ranked from the point of view of the two symmetric firms ex ante, before any information is revealed?

In the no-entry equilibrium, the expected payoff in period 0 is given by

$$\Pi = \frac{\lambda \Pi_d}{2\lambda + r}.$$

Consider a single-entry equilibrium at date τ. The expected payoff in period 0 is given by

$$\begin{aligned}\Pi = (1 - e^{-(2\lambda+r)(\tau-a)})\frac{\lambda}{2\lambda+r}\Pi_d \\ + \frac{e^{-2\lambda(\tau-a)}}{2}(1-e^{-a_\lambda})e^{-r\tau}\left(\Pi_d + \frac{\Delta r}{\lambda+r}(1-pe^{-a\lambda})\right) \\ + e^{-(r+\lambda)\tau}\frac{e^{-\lambda(\tau-a)} - e^{-\lambda\tau}}{2}\Pi_d\frac{\lambda}{\lambda+r} + e^{-(r+2\lambda)\tau}\left(p_{m+1}\frac{\pi_d\lambda}{2(\lambda+r)} + (1-p_{m+1})\frac{\lambda}{2\lambda+r}\Pi_d\right)\end{aligned}$$

We compare the two expected profits. In the single-entry equilibrium the firm benefits from a period in which it collects monopoly profits (when it learns that the cost is low, enters at τ and is not followed by the rival firm). It also suffers from the fact that it will be the only firm drawing signals (thereby delaying the date at which it starts collecting duopoly profits) when the firm remains uninformed while the rival firm enters at date τ, and the entry cost is low. The trade-off between these two effects depends on the parameters. If the monopoly surplus Δ is high (for example if firms compete in prices à la Bertrand with identical marginal costs), the single-entry equilibrium dominates the no-entry equilibrium. If the entry date τ is small, as in Proposition 9, the expected profit is higher in the no-entry equilibrium than in the single-entry equilibrium because the firm is unlikely to learn the value of the cost before the entry date. However, in general, the comparison between the two profits is ambiguous and the two equilibria cannot be ranked.

Common values and private values We have assumed that the two firms face the same entry cost. In a previous study, Bloch et al. (2015), we analyzed the case of private values where the entry costs of the two firms are independently drawn from the same distribution. The differences between the two models are striking.

First, the evolution of beliefs leads to opposite conclusions in the two models. As time passes, as shown in Lemma 1, the firm believes with higher probability that the rival firm has learned that the cost is high. In the common values model, this makes the firm more pessimistic and less likely to enter. In the private values model, if the rival firm learns that its cost is high, the firm is more optimistic as it becomes more likely that it will be a monopolist. Hence, firms become more pessimistic over time, and are less likely to enter when uninformed in the common values model, but become more optimistic over time and are more likely to enter when uninformed in the private values model. In fact, in the common values model, as shown in Proposition 2, if an equilibrium with single entry at date τ exists, there must also be an equilibrium with immediate entry. In the private values model, as shown in Bloch et al. (2015), preemption arises at a unique date τ at which one of the two uninformed firms enters with probability one.

Second, entry by an informed firm in the common values model is immediately followed by the other firm, whereas entry in the private values model, as shown in Bloch et al. (2015), leads the other firm to wait until it learns its cost before entering. Hence for a firm that learns that the cost is low, entry has a negative impact on the behavior of the rival firm in the common values model but a positive impact in the private values model. In the private values model, the informed firm has no incentive to delay entry whereas in the common values model, the informed firm may want to delay entry in order to hide under the cover of the entry of uninformed firms at a specified date τ.

Game in continuous time We model the interaction between the two firms in discrete time, but then consider the continuous time limit because closed form expressions are easier to handle. We find the discrete time formulation more sensible in the entry game because uninformed firms coordinate on a specific entry period, and it is easier to coordinate on a fixed, discrete period than in a date in continuous time. Nonetheless, we could have alternatively modeled the entry game in continuous time and obtained the same formulae for equilibrium entry probabilities and delays.

Consider a closed interval $[0, T]$ and suppose that time is continuous, $\tau \in [0, T]$. In order to specify outcomes of the game in continuous time, assume, as in Perry and Reny (1993, 1994), there is an exogenous delay between any two actions of the players. More specifically, suppose that if $p^t > 0$ then $p^\tau = 0$ for all $\tau \in (t, t + \nu)$; if $q^{t,s} > 0$ then $q^{\tau,s} = 0$ for all $\tau \in (t, t + \nu)$ and if $r^t > 0$ then $r^\tau = 0$ for all $\tau \in (t, t + \nu)$. Under this assumption, and the fact that there exists a final date T, the game is well specified. Every firm will only enter at a *finite* number of dates, and we can compute equilibrium entry probabilities and delays directly in the continuous time formulation, instead of analyzing the continuous time limit of a discrete time game.

Alternative timing In the model we consider, a rival has the opportunity to follow a firm's entry in the same period. Alternatively, we could consider a model where the rival has to wait until the next period before entering. Instead of computing the probability of rival entry at period t, we would need to compute the probability of entry of an uninformed firm at period $t + 1$ after a history where the rival firm has entered at period t. We claim that the model is equivalent. Assuming that the delay between periods is small, as we do in the continuous time limit, the incentive of an uninformed firm to follow at period t or at period $t + 1$ will be identical, resulting in the same equilibrium characterization.

Imperfect signals We suppose that the signals received by the firms are perfectly informative. If the signals were noisy, the beliefs would evolve in a continuous way, and we would need to study a firm's strategy, not as a function of three possible states (cost is low, high or uninformed), but as a function of a continuous belief on the value of the cost. This would clearly complicate the analysis of the game, but we conjecture that it would be possible to define threshold beliefs (where the thresholds vary over time) such that firms choose to enter whenever their belief that the cost is high falls below the threshold. We expect the analysis of single-entry and multiple-entry equilibria could be extended, assuming that firms, whose beliefs belong to an intermediate range, coordinate on entry at fixed dates. The full study of the model with imperfect signals remains to be done.

7 Conclusion

We model the interplay of experimentation and entry decisions into a new market with uncertain common entry costs. Two firms gradually learn about a binary entry cost and decide whether to enter. We show that an equilibrium where firms enter immediately when they learn that the cost is low and are immediately followed always exist. Under certain restrictions on the parameters, we also show existence of equilibria where uninformed firms coordinate to enter at specific entry dates with positive probability. They are not followed by the rival firm. Firms that learn that the cost is low delay to hide under the cover of the entry of uninformed firms. We show that these equilibria are more likely to exist when the entry date is closer to zero, that they are unique given the fixed entry date and that the probability of entry is decreasing over time. We also investigate equilibria where uninformed firms enter a finite number of times, show that a multiple-entry equilibrium exists if a single-entry equilibrium exists, that given a distribution of fixed entry dates, equilibrium is unique when it exists, and that equilibrium probabilities are decreasing over time when the interval between two successive entries is sufficiently small.

The entry game we consider gives rise to a rich set of equilibria. The equilibrium analysis relies on some simplifying assumptions: the entry cost can only take two values, firms do not control the level of experimentation, and all signals are perfect. While these simplifying assumptions proved useful to get our first insights into the entry game, we believe they could be relaxed in future work to improve our understanding of dynamic duopoly competition with gradual learning.

Appendix

A Proofs

Proof of Proposition 1. INFORMED FIRMS: Given that entry will be followed immediately in each period, delaying is not profitable for informed firms ($\Pi_d > \delta\Pi_d$). Hence $q^{t,s} = 1$ if and only if $s = t$.

UNINFORMED FIRMS: Given that only informed firms enter, delaying to follow entry by another firm is not profitable ($\Pi_d > \delta\Pi_d$) so $u^t = 1$. Given that entry is followed immediately ($u^t = 1$), entry by uninformed firms is not profitable. Entering gives an expected profit of $\Pi = \Pi_d - \frac{\theta}{2} < 0$, whereas waiting gives a positive value, equal to $\frac{\Pi_d}{2}(\mu + \frac{(1-\mu)\mu(2-\mu)\delta}{2(1-\delta(1-\mu)^2)}) > 0$. ■

Proof of Lemma 1. Note that we can write the difference in beliefs that the cost is high and low as

$$\chi^t - (1-\chi^t) = \frac{\sum_{s=0}^{t-1} \mu(1-\mu)^s (1 - \prod_{s'=s}^{t-1}(1-q^{s',s}))}{1 - \sum_{s=0}^{t-1} \mu(1-\mu)^s \sum_{s'=s}^{t-1} q^{s',s} \prod_{s''=s}^{s'-1}(1-q^{s'',s})}.$$

The denominator is the probability that no firm has entered until t, which is decreasing in t. The numerator is the difference between the probability that the firm has learned that the cost was high and the probability that the firm has learned that the cost is low, conditional on the fact that the firm has not entered until t. This difference is increasing in t. Hence $2\chi^t - 1$ is increasing in t, implying that the common belief χ^t is also increasing in t. ■

Proof of Proposition 2. Suppose that the condition fails,

$$\theta > \frac{\lambda + 2r}{\lambda + r}\Delta + \frac{2(\lambda + r)}{2\lambda + r}\Pi_d.$$

We first observe that

$$\begin{aligned}\frac{\lambda+2r}{\lambda+r}\Delta+\frac{2(\lambda+r)}{2\lambda+r}\Pi_d &= \Delta+\frac{r}{\lambda+r}\Delta+\Pi_d+\frac{r}{2\lambda+r}\Pi_d \\ &> \Delta+\Pi_d+\Pi_d\frac{r(3\lambda+r)}{(\lambda+r)(2\lambda+r)}, \\ &> \Delta+\Pi_d+\Pi_d\frac{r}{\lambda+r}.\end{aligned}$$

Now, consider the entry probability when $a=\tau$: informed firms have been waiting to enter since the beginning of the game. This entry probability satisfies

$$p=\phi(\tau)=1+\frac{\frac{1}{2}\left(e^{\tau\lambda}-1\right)\frac{r}{\lambda+r}\Pi_d+\frac{1}{2}\left(e^{\lambda\tau}-1\right)(\Delta-\theta+\Pi_d)+\left(\Pi_d-\frac{\theta}{2}\right)-\frac{\lambda\Pi_d}{2(\lambda+r)}}{\frac{\lambda\Pi_d}{2(\lambda+r)}-\frac{\lambda\Pi_d}{2\lambda+r}+\frac{\Delta(\lambda+2r)}{2(\lambda+r)}}.$$

It is easy to check that the sign of $\phi'(a)$ is the same as the sign of

$$\Delta-\theta+\Pi_d+\frac{r}{\lambda+r}\Pi_d,$$

which as we shown is negative. Hence $\phi(0)>\phi(\tau)$ for all $\tau>0$. Now, suppose by contradiction that there exists an equilibrium at date τ with delay $a\leq\tau$. Because $f(\cdot)$ is increasing in a, $f(a)\leq f(\tau)=\phi(\tau)<\phi(0)=f(0)$. Because the condition for rival herding does not depend on the date τ and is increasing in a, $g(a)>g(0)$. Hence we obtain

$$f(0)>f(a)\geq g(a)>g(0)=0,$$

or

$$f(0)=1+\frac{\Pi_d-\frac{\theta}{2}-\frac{\lambda}{2(\lambda+r)}\Pi_d}{\frac{\lambda+2r}{2(\lambda+r)}\Delta+\frac{\lambda}{2(\lambda+r)}\Pi_d-\frac{\lambda}{2\lambda+r}\Pi_d}>0,$$

in contradiction to the fact that

$$\theta>\frac{\lambda+2r}{\lambda+r}\Delta+\frac{2(\lambda+r)}{2\lambda+r}\Pi_d.$$

This completes the proof of the Proposition. ■

Proof of Proposition 3. Define $\widehat{a}$ to solve $f(\widehat{a})=k(\widehat{a})$ and assume first $f(\widehat{a})=k(\widehat{a})\geq g(\widehat{a})$. Because $f(a)$ is a strictly increasing continuous function and $k(a)$ is a strictly decreasing continuous function, then if it exists, $\widehat{a}$ is unique. Then if $\widehat{a}<\tau$, there is a unique equilibrium delay and entry probability, (a^*,p^*), such that $a^*=\widehat{a}$ and $p^*=f(\widehat{a})$. If $\widehat{a}\geq\tau$, the boundary condition constrains the solution. The equilibrium delay and entry probability pair is given by $(a^*, p^*) = (\tau, f(\tau))$ which is of course unique. ■

Proof of Proposition 4. We consider the set of parameters for which $\widehat{a} \neq \tau$. Suppose first that $\widehat{a} < \tau$ so that the equilibrium satisfies $a^* < \tau$ and consider variations of the parameters which are sufficiently small so that a^* remains bounded away from τ. Three equilibrium value of the delay a^* is given by the solution to

$$f(a) - k(a) = 0.$$

To compute the comparative statics effects of a change in any parameter y of the model, we compute

$$\frac{\partial a^*}{\partial y} = -\frac{\frac{\partial f}{\partial y} - \frac{\partial k}{\partial y}}{\frac{\partial f}{\partial a} - \frac{\partial k}{\partial a}}.$$

As $\frac{\partial f}{\partial a} - \frac{\partial k}{\partial a} > 0$, the sign of $\frac{\partial a^*}{\partial y}$ is the negative of the sign of $\frac{\partial f}{\partial y} - \frac{\partial k}{\partial y}$.

We now use implicit differentiation of the functions $f(a)$ and $k(a)$ to obtain the following comparative statics results:

Claim 1 *Suppose that* $0 < f(a; \Pi_d, r, \lambda, \Delta, \theta, \tau) < 1$. *Then* $\partial f/\partial a > 0$, $\partial f/\partial \Pi_d > 0$, $\partial f/\partial r > 0$, $\partial f/\partial \Delta > 0$, $\partial f/\partial \theta < 0$, $\partial f/\partial \tau < 0$ *and* $\partial f/\partial \lambda \gtreqless 0$.

Claim 2 *Suppose that* $0 < k(a; \Pi_d, r, \lambda, \Delta, \theta, \tau) < 1$. *Then* $\partial k/\partial a < 0$, $\partial k/\partial \Pi_d < 0$, $\partial k/\partial r \gtreqless 0$, $\partial k/\partial \lambda < 0$, $\partial k/\partial \Delta > 0$, $\partial k/\partial \theta = 0$, $\partial k/\partial \tau = 0$.

Using Claims 1 and 2, $\frac{\partial f}{\partial \theta} < 0$ and $\frac{\partial k}{\partial \theta} = 0$. Hence $\frac{\partial a^*}{\partial \theta} > 0$. Because the $k(\cdot)$ function does not depend on θ, and is decreasing we conclude that $p^* = k(a^*)$ is decreasing in θ. We also have $\frac{\partial f}{\partial \Pi_d} > 0$ and $\frac{\partial k}{\partial \Pi_d} < 0$, so that $\frac{\partial a^*}{\partial \Pi_d} < 0$. The effect of a change in Π_d on p^* is indeterminate. Next, using the fact that $f(\cdot)$ and $k(\cdot)$ are strictly monotonic, let $a = f^{-1}(p) = k^{-1}(p)$. Now if $p - f(a, y) = 0$, we have $\frac{\partial a}{\partial y} = -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial a}}$. Hence, as f is strictly increasing $\frac{\partial f^{-1}}{\partial y}$ has the opposite sign of $\frac{\partial f}{\partial y}$, whereas because k is strictly decreasing, the sign of $\frac{\partial k^{-1}}{\partial y}$ is the same as the sign of $\frac{\partial f}{\partial y}$. We deduce that $\frac{\partial f^{-1}}{\partial \Delta} < 0$ and $\frac{\partial k^{-1}}{\partial \Delta} > 0$, establishing that $\frac{\partial p^*}{\partial \Delta} > 0$.

Next suppose that $\widehat{a} > \tau$ so that the equilibrium is given by $a^* = \tau, p^* = f(\tau)$. Then, a^* is independent of the parameters and an increase in θ results in a decrease in p^* whereas an increase in Δ results in an increase in p^*. ■

Proof of Proposition 5. Suppose first that $\widehat{a_1} > \tau_1$ so that $a_1^* = \tau_1$. Then, if $a_2^* = \tau_2$, we have $p_1^* = \phi(\tau_1) > p_2^* = \phi(\tau_2)$ as ϕ, as defined in the proof of Proposition 2, is a decreasing function when the equilibrium exists. Finally, let $a_1^* = \tau_1$ and $a_2^* < \tau_2$. Then $p_2^* < \phi(\tau_2) < \phi(\tau_1) = p_1^*$.

Suppose next that $\widehat{a_1} < \tau_1$ so that $a_1^* < \tau_1$ and $k(a_1^*) = p_1^*$. Suppose by contradiction that $p_2^* \geq p_1^*$. Then as (p_2^*, a_2^*) is an equilibrium, $k(a_2^*) \geq p_2^*$. Hence $k(a_2^*) \geq p_2^* \geq p_1^* = k(a_1^*)$. Furthermore, as $k(\cdot)$ is strictly decreasing in a, this implies that $a_1^* \geq a_2^*$. Now recall that $f(\cdot)$ is strictly increasing in a and decreasing in τ. Hence, $p_1^* = f(a_1^*, \tau_1) > f(a_2^*, \tau_2) = p_2^*$, contradicting the original assumption $p_2^* \geq p_1^*$. ■

Proof of Proposition 6. We first prove the following Claim:

Claim 3 *Suppose that* $\theta \leq \frac{\lambda+2r}{\lambda+r}\Delta + \frac{2(\lambda+r)}{2\lambda+r}\Pi_d$. *Then* $f(a) - g(a)$ *is decreasing in* a.

Proof of the Claim: The sign of $f(a) - g(a)$ is the same as the sign of

$$\frac{1}{\frac{\lambda\Pi_d}{2(\lambda+r)} - \frac{\lambda\Pi_d}{2\lambda+r} + \frac{\Delta(\lambda+2r)}{2(\lambda+r)}} - \frac{a}{\theta - \frac{\lambda+2r}{\lambda+r}\Pi_d}.$$

A straightforward computation shows that this sign is the same as the sign of

$$\theta - \frac{\lambda+2r}{\lambda+r}\Delta - \frac{2(\lambda+r)}{2\lambda+r}\Pi_d.$$

This completes the proof of the claim.

Suppose that $\overline{\tau} > 0$. Then if $\overline{a} \leq \overline{\tau}$, there is an equilibrium with $p^* = \overline{p}, a^* = \overline{a}$. If $\overline{a} > \overline{\tau}$, then by the preceding claim, $f(\overline{\tau}) - g(\overline{\tau}) > f(\overline{a}) - g(\overline{a}) = 0$. So $p^* = f(\overline{\tau}) > g(\overline{t})$ and $p^* = f(\overline{\tau}), a^* = \overline{\tau}$ form an equilibrium.

Now, suppose by contradiction that there exists an equilibrium at $\tau > \overline{\tau}$, denoted (p^{**}, a^{**}). By Proposition 5, $p^{**} < p^*$. We now claim that $a^{**} > \overline{a}$. Suppose by contradiction that $a^{**} \leq \overline{a}$. Then, necessarily $a^{**} < \tau$ so $p^{**} = k(a^{**})$. But we then have $k(\overline{a}) \geq p^* > p^{**} = k(a^{**})$, where the first inequality is due to the fact that $k(\overline{a}) = p^*$ if $\overline{a} < \overline{\tau}$ and $p^* = k(\overline{\tau}) > k(\overline{a})$ if $\overline{a} > \overline{\tau}$, the second inequality from the statement that $p^* > p^{**}$ and the last equality due to the fact that $a^{**} < \tau$. Because $k(\cdot)$ is strictly decreasing, we obtain $a^{**} > \overline{a}$, contradicting the initial assumption.

Now if $a^{**} > \overline{a}$, we have $g(a^{**}) > g(\overline{a}) = k(\overline{a}) \geq p^* > p^{**}$, where the first inequality is due to the fact that $g(\cdot)$ is increasing, the equality by the definition of $\overline{a}$, the next inequality because $k(\overline{a}) \geq p^*$ at the equilibrium (p^*, a^*) and the last inequality by Proposition 5. But notice that $g(a^{**}) > p^{**}$ contradicts the fact that (p^{**}, a^{**}) is an equilibrium of the game at τ. ■

Proof of Proposition 7. We first show that $f_M(a) > f(a)$ for all a. To this end, it suffices to observe that the fraction$\frac{\xi(\theta)}{\xi(\hat{\theta})}$ is always higher in the multiple-entry equilibrium than in the single-entry equilibrium.

Let (a_M^*, p_M^*) be the equilibrium delay and entry probability in the multi-entry equilibrium. If $a_M^* < \tau_M - \tau_{M-1}$, then $g(a_M^*) \leq p_M^* = f_M(a_M^*) = k(a_M^*)$. Now given that the single-entry probability satisfies $f(a) \geq f_M(a)$ for all a, $f(a) - k(a) \geq f_M(a) - k(a)$ for all a. Since $f(a)$ and $f_M(a)$ are increasing in a and $k(a)$ is decreasing in a, there is a unique point a^{**} such that $f(a^{**}) - k(a^{**}) = 0$, which must satisfy $a^{**} < a_M^* < \tau_M - \tau_{M-1}$. Now notice that $p^{**} = k(a^{**}) > k(a_M^*) = p_M^* \geq g(a_M^*) > g(a^{**})$, so that (a^{**}, p^{**}) is a single-entry equilibrium at date $\tau_M - \tau_{M-1}$.

Next suppose that $a_M^* = \tau_M - \tau_{M-1}$, so that $g(\tau_M - \tau_{M-1}) \leq p_M^* = f_M(\tau_M - \tau_{M-1}) \leq k(\tau_M - \tau_{M-1})$. Let a^{**} be the unique delay such that $f(a^{**}) - k(a^{**}) = 0$. If$a^{**} < \tau_M - \tau_{M-1} = a_M^*$, by the Claim of Proposition 6, $f(a^{**}) - g(a^{**}) > f(a_M^*) - g(a_M^*)$. And hence $f(a^{**}) - g(a^{**}) > f(\tau_M - \tau_{M-1}) - g(\tau_M - \tau_{M-1}) > f_M(\tau_M - \tau_{M-1}) - g(\tau_M - \tau_{M-1}) \geq 0$, where the second inequality is due to the fact that $f(a) > f_M(a)$ for all a, and the last inequality to the assumption that the multiple-entry equilibrium exists. Hence we conclude that (a^{**}, p^{**}) is a single-entry equilibrium at date $\tau_M - \tau_{M-1}$. Finally, if $a^{**} \geq \tau_M - \tau_{M-1}$ then let $a^{**} = \tau_M - \tau_{M-1}$ and $p^{**} = f(\tau_M - \tau_{M-1}) > f_M(\tau_M - \tau_{M-1}) \geq g(\tau_M - \tau_{M-1})$. Again, we obtain that (a^{**}, p^{**}) is a single-entry equilibrium at date $\tau_M - \tau_{M-1}$. ■

Proof of Proposition 8.

We first observe that $A(p_{m+1})$ is a decreasing function of p_{m+1} and compute

$$
\begin{aligned}
A(0) = & \frac{1 - e^{-(2\lambda + r)(\tau_{m+1} - a_{m+1} - \tau_m)}}{2} \frac{2\lambda}{2\lambda + r} \Pi_d \\
& + \frac{e^{-2\lambda(\tau_{m+1} - a_{m+1} - \tau_m)}}{2} (1 - e^{-a_{m+1}\lambda}) e^{-r(\tau_{m+1} - \tau_m)} (\Pi_d + \frac{\Delta r}{\lambda + r} e^{-a_{m+1}\lambda}) \\
+ e^{-(r+\lambda)(\tau_{m+1} - \tau_m)} & \left[\frac{e^{-\lambda(\tau_{m+1} - a_{m+1} - \tau_m)} - e^{-\lambda(\tau_{m+1} - \tau_m)}}{2} \Pi_d + \frac{(1 - e^{-\lambda(\tau_{m+1} - \tau_m)})}{2} (\Pi_d + \Delta - \theta) \right. \\
& \left. + e^{-\lambda(\tau_{m+1} - \tau_m)} \left(\Pi_d + \frac{(2r + \lambda)\Delta}{2(\lambda + r)} - \frac{\theta}{2} \right) \right].
\end{aligned}
$$

Now, as

$$e^{-\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}(1-e^{-a_{m+1}\lambda}) < 1-e^{-\lambda(\tau_{m+1}-\tau_m)},$$

$$\begin{aligned} A(0) < \Pi_d \Bigg(& \frac{1-e^{-(2\lambda+r)(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}\frac{2\lambda}{2\lambda+r} \\ & + e^{-(r+\lambda)(\tau_{m+1}-\tau_m)}\frac{e^{-\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}-e^{-\lambda(\tau_{m+1}-\tau_m)}}{2}\Bigg) \\ + \left(\Pi_d + \frac{(2r+\lambda)\Delta}{2(\lambda+r)} - \frac{\theta}{2}\right) & \left(e^{-(r+2\lambda)(\tau_{m+1}-\tau_m)} + e^{-(r+\lambda)(\tau_{m+1}-\tau_m)}(1-e^{-\lambda(\tau_{m+1}-\tau_m)})\right). \end{aligned}$$

It is easy to see that $\frac{2r+\lambda}{\lambda+r}\Delta < \theta$ is a sufficient condition for $A(0) < \Pi_d$, so that

$$\frac{\Pi_d \lambda}{2(\lambda+r)} + \frac{\Delta(2r+\lambda)}{2(\lambda+r)} - A(p_{m+1}) > \frac{(\Delta-\Pi_d)(2r+\lambda)}{2(\lambda+r)} > 0.$$

guaranteeing that the entry probability belongs to the interval $(0,1)$.

We next prove the following Claim.

Claim 4 *Suppose that there are two equilibria at date* τ_m, $(p_m^*, a_m^*, p_m^{**}, a_m^{**})$. *Suppose that* $p_m^* > p_m^{**}$, *then* $a_m^* < a_m^{**}$.

Proof of the Claim: Suppose by contradiction that $a_m^{**} < a_m^*$ Then necessarily $a_m^{**} < \tau_m - \tau_{m-1}$ and hence $p_m^{**} = k(a_m^{**})$. Now, $k(a_m^*) \geq p_m^* > p_m^{**} = k(a_m^{**})$, which implies, because $k(\cdot)$ is decreasing that $a_m^{**} > a_m^*$, contradicting the original hypothesis. This completes the proof of the Claim.

Now suppose by contradiction that there exist two equilibria with $p_1^* \neq p_1^{**}$. Without loss of generality, suppose that $p_m^* > p_1^{**}$. We claim that we must have $p_2^* > p_2^{**}$. Suppose by contradiction that $p_2^* \leq p_2^{**}$. Then because $f_1(a, p_2^*) \leq f_1(a, p_2^{**})$ for all a, $a_1^{**} > a_1^*$ and f_1 is increasing in a and p_2, so that $p_1^* = f_1(a_1^*, p_2^*) < f_1(a_1^{**}, p_2^*) \leq f_1(a_1^{**}, p_2^{**}) = p_1^{**}$, contradicting the original assumption $p_1^* > p_1^{**}$. Hence we have $p_1^* > p_1^{**}$ and $p_2^* > p_2^{**}$.

We now claim that this implies that $p_3^* > p_3^{**}$. Suppose by contradiction that $p_3^* \leq p_3^{**}$. Because $f_2(a, p_1, p_3)$ is increasing in a and p_3 and decreasing in p_1 and $a_2^* < a_2^{**}$ by the preceding Claim, we conclude $p_2^* = f_2(a_2^*, p_1^*, p_3^*) < f_2(a_2^{**}, p_1^*, p_3^*) < f_2(a_2^{**}, p_1^{**}, p_3^*) \leq f_2(a_2^{**}, p_1^{**}, p_3^{**}) = p_2^{**}$, contradicting the original assumption $p_2^* > p_2^{**}$. Hence $p_3^* > p_3^{**}$.

The same argument can be repeated to show that if $p_1^* > p_1^{**}$, then $p_m^* > p_m^{**}$ for all $m = 2, \ldots, M$. But then $p_M^* > p_M^{**}$ and $p_m^* > p_m^{**}$ for all $m < M$. Hence, $p_M^* = f_M(a_M^*, p_1^*, \ldots, p_{M-1}^*) < f_M(a_M^{**}, p_1^*, \ldots, pM-1^*) < f_M(a_M^{**}, p_1^{**}, \ldots, pM-1^{**}) = p_M^{**}$, contradicting the fact that $p_M^* > p_M^{**}$. This last contradiction shows that we cannot have $p_1^* \neq p_1^{**}$. Now notice that the same arguments hold if there exist some $\tilde{m}$ such that $p_m^* = p_m^{**}$ for all $m < \tilde{m}$ and $p_{\tilde{m}}^* \neq p_{\tilde{m}}^{**}$. Hence equilibrium must be unique, concluding the proof of the Proposition. ■

Proof of Proposition 9. The proof is by induction, starting at the terminal date. To compute W_{M+1} we decompose the equilibrium continuation value considering different possible trajectories of the learning process: (1) Either one of the two firms learns that the cost is low before $\tau_M - a_M$ and both firms receive Π_d immediately, or (2) one of the two firms learns that the cost is low between $\tau_M - a_M$ and τ_M and the two firms receive an expected payoff equal to $\Pi_d + e^{-a\Delta\tau}\frac{\Pi_d + \frac{\lambda\Pi_d + r\Delta}{2(\lambda+r)}}{2}$ at period τ_M, or (3) none of the firms learns that the cost is low before τ_M, the rival enters with probability p_M and the firm waits until it learns that the cost is low and then collects Π_d, or (4) none of the firms learns that the cost is low before τ_M, the rival waits with probability $1 - p_M$, and both firms wait until they learnt hat the cost is low to collected Π_d.

If the difference between two entry dates is sufficiently small, e^{-a_M} and $e^{-\Delta\tau}$ are bounded above and there exists $\epsilon > 0$ such that

$$W_M < \frac{\epsilon}{2}\frac{2\lambda}{2\lambda + r}\Pi_d + \frac{\epsilon}{2}\left(\Pi_d + \epsilon\frac{\lambda\pi_d}{2(\lambda+r)} + \frac{r\Delta}{2(\lambda+r)}\right) + \left(p_M\frac{\pi_d\lambda}{2(\lambda+r)} + (1-p_M)\frac{\lambda\Pi_d}{2\lambda+r}\right)$$

Now, choosing ϵ sufficiently small,

$$W_M < \left(p_M\frac{\pi_d\lambda}{2(\lambda+r)} + (1-p_M)\frac{\lambda\Pi_d}{2\lambda+r}) < \frac{\lambda\Pi_d}{2\lambda+r}\right) = W_{M+1}.$$

We now claim that this implies that $p_M < p_{M-1}$. Because $W_{M+1} > W_M$, $f_M(a) < f_{M-1}(a)$ for all a. Suppose by contradiction that $p_{M-1} \leq p_M$. Then we must have $a_{M-1} < a_M$ since otherwise, if $a_{M-1} \geq a_M$, we would have $p_{M-1} = f_{M-1}(a_{M-1}) \geq f_{M-1}(a_M) > f_M(a_M) = p_M$, a contradiction. But now, if $a_{M-1} < a_M$, we have $p_{M-1} = k(a_{M-1}) > k(a_M) \geq p_M$, contradicting our original assumption.

Consider then the inductive step. Suppose that for all $W_{m+1} < W_{m+2}$, and

$p_{m+1} > p_{m+2}$. By the same computation as the terminal step, we have

$$|W_m - \left(p_{m+1} \frac{\pi_d \lambda}{2(\lambda + r)} + (1 - p_{m+1}) W_{m+1} \right) | < \epsilon \text{ as } \Delta\tau \to_0$$

Now

$$\begin{aligned} W_m &= \left(p_{m+1} \frac{\pi_d \lambda}{2(\lambda + r)} + (1 - p_{m+1}) W_{m+1} \right) \\ &< \left(p_{m+2} \frac{\pi_d \lambda}{2(\lambda + r)} + (1 - p_{m+2}) W_{m+1} \right) \\ &< \left(p_{m+2} \frac{\pi_d \lambda}{2(\lambda + r)} + (1 - p_{m+2}) W_{m+2} \right) \\ &= W_{m+1} \end{aligned}$$

where the first inequality is due to the fact that $p_{m+2} < p_{m+1}$ and $W_{m+1} > \frac{\Pi_d \lambda}{2(\lambda+r)}$, as the latter is the expected profit if the firm were the only firm to draw a signal, a lower bound on the expected continuation value, and the second inequality to the fact that $W_{m+2} > W_{m+1}$.

By the same argument as for the terminal step, if $W_m < W_{m+1}$ then we must have $p_m > p_{m-1}$, completing the proof of the inductive step. ■

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B Online Appendix (Not for print publication)

This online Appendix contains the derivations for the entry probabilities in a multi-entry equilibrium. The payoff of an uninformed firm when it enters only depends on the entry probabilities at dates $t_1, \ldots, t_{m-1}$ and is given as before by

$$\Pi_E = \xi(0)\Pi_d + \xi(\theta)(\Pi_d + \Delta - \theta) + \xi(\hat{\theta}) \left(p_m \left(\Pi_d - \frac{\theta}{2} \right) + (1 - p_m) \left(\Pi_d + \Delta - \frac{\delta\mu}{2(1 - \delta(1 - \mu))}\Delta - \frac{\theta}{2} \right) \right).$$

In the continuous time limit,

$$\Pi_E = \xi(0)\Pi_d + \xi(\theta)(\Pi_d + \Delta - \theta) + \xi(\hat{\theta}) \left(p_m \left(\Pi_d - \frac{\theta}{2} \right) + (1 - p_m) \left(\Pi_d + \Delta - \Delta\frac{\lambda}{2(\lambda + r)} - \frac{\theta}{2} \right) \right).$$

The payoff of an uninformed firm when it decides to wait becomes more complex when there are further entry dates $t_{m+1}, \ldots, t_M$. With probability $\xi(0)$ the rival firm has learned that the cost is low and enters at τ_m. The firm will then wait until it learns the value of the cost and enter immediately thereafter. With probability $\xi(\theta)$, the rival firm has learned that the cost is high. The firm will then either learn the value of the cost and not enter or enter as an informed firm at a future date $t_{m+1}, \ldots, t_M$. With probability $\xi(\hat{\theta})$, the rival firm is uninformed. If the rival firm enters with probability p_m, the firm will wait to learn the value of the cost and enter immediately after it learns that the cost is low. With probability $1 - p_m$ both firms remain in the game. Different possibilities can arise. Either one of the two firms learns that the cost is low between t_m and $t_{m+1} - d_{m+1}$, enters immediately and is followed by the rival so that both firms receive the duopoly profit. If the firm learns that the cost is low during the delay, it will delay entry, enter at t_{m+1}, and either be followed immediately (if the other firm has also learned that the cost is low between $t_{m+1} - d_{m+1}$ and t_{m+1} or enters as uninformed at period t_{m+1} with probability p_{m+1}) or receive a monopoly profit and be followed only after the other firm learns that the cost is low (if the rival firm remains uninformed at t_{m+1} and does not enter with probability $1 - p_{m+1}$). If the firm learns that the cost is high before t_{m+1} it does not to enter. The final situation is the one where the firm remains uninformed at period t_{m+1}. In that case, it will consider three different possibilities. First, the rival firm may have learned the cost is low during the delay between $t_{m+1} - d_{m+1}$ and t_{m+1}. Second, the rival firm may have learned the cost is high between t_m and t_{m+1}. Third, the rival firm may not have learned the cost between t_m and t_{m+1}. In all three cases, the firm will enter with probability p_{m+1} and wait with probability $1-p_{m+1}$. If any of the two firm enters, the expected payoff can be computed easily. If both firms remain in the game at period $m+1$, we let W_{m+1} denote the common continuation value of the game. Formally,

$$
\begin{aligned}
\Pi_W = \xi(0)&\frac{\mu\delta}{1-\delta(1-\mu)}\Pi_d \\
&+ \xi(\theta)\sum_{s=1}^{M-m}\left(\delta(1-\mu)^{t_{m+s}-t_m}p_{m+s}\prod_{s'=1}^{s-1}(1-p_{m+s'})\right)(\Pi_d+\Delta-\theta) \\
&+ \xi(\hat{\theta})\left\{p_m\frac{1}{2}\frac{\mu\delta}{1-\delta(1-\mu)}\Pi_d + (1-p_m)\left[\sum_{s=0}^{t_{m+1}-d_{m+1}-t_m}\mu(1-\mu)^{2s}\delta^s\Pi_d\right.\right. \\
+\sum_{s=0}^{d_{m+1}}&\frac{1}{2}\mu(1-\mu)^{s+t_{m+1}-d_{m+1}-t_m}\delta^{t_{m+1}-t_m}\left(\Pi_d+(1-p_{m+1})(1-\mu)^{t_{m+1}-t_m}\frac{\Delta}{1-\delta(1-\mu)}\right) \\
&+ (1-\mu)^{t_{m+1}-t_m}p_{m+1}\delta^{t_{m+1}-t_m}\left[\frac{(1-\mu)^{t_{m+1}-d_{m+1}}-(1-\mu)^{t_{m+1}-t_m}}{2}\Pi_d\right. \\
&\qquad + \frac{1-(1-\mu)^{t_{m+1}-t_m}}{2}(\Pi_d+\Delta-\theta) \\
&\left. +(1-\mu)^{t_{m+1}-t_m}p_{m+1}(\Pi_d-\frac{\theta}{2})+(1-p_{m+1})(\Pi_d-\frac{\theta}{2}+\frac{\Delta}{2}+\frac{\Delta}{2(1-\delta(1-\mu))}\right] \\
+(1-\mu)^{t_{m+1}-t_m}&(1-p_{m+1})\delta^{t_{m+1}-t_m}\left[\frac{(1-\mu)^{t_{m+1}-d_{m+1}}-(1-\mu)^{t_{m+1}-t_m}}{2}\frac{\Pi_d\mu\delta}{1-\delta(1-\mu)}\right. \\
&\left.\left.\left.+(1-\mu)^{t_{m+1}-t_m}\left(p_{m+1}\frac{\delta\mu\Pi_d}{2(1-\delta(1-\mu))}+(1-p_{m+1})W_{m+1}\right)\right]\right]\right\}.
\end{aligned}
$$

In the continuous time limit,

$$\Pi_W = \xi(0)\Pi_d \frac{\lambda}{\lambda + r}$$
$$+ \xi(\theta)(\Pi_d + \Delta - \theta) \sum_{s=1}^{M-m} \left(p_{m+s} \prod_{s'=1}^{s-1} (1 - p_{m+s'}) e^{-(r+\lambda)(\tau_{m+s} - \tau_m)} \right)$$
$$+ \xi(\hat{\theta}) \left\{ p_m \frac{\Pi_d \lambda}{2(\lambda + r)} + (1 - p_m) \left[\frac{1 - e^{-(2\lambda + r)(\tau_{m+1} - a_{m+1} - \tau_m)}}{2} \frac{2\lambda}{2\lambda + r} \Pi_d \right. \right.$$
$$+ \frac{e^{-2\lambda(\tau_{m+1} - a_{m+1} - \tau_m)}}{2} (1 - e^{-a_{m+1}\lambda}) e^{-r(\tau_{m+1} - \tau_m)} \left(\Pi_d + \frac{\Delta r}{\lambda + r} (1 - p_{m+1}) e^{-a_{m+1}\lambda} \right)$$
$$+ e^{-(r+\lambda)(\tau_{m+1} - \tau_m)} p_{m+1} \left[\frac{e^{-\lambda(\tau_{m+1} - a_{m+1} - \tau_m)} - e^{-\lambda(\tau_{m+1} - \tau_m)}}{2} \Pi_d + \frac{(1 - e^{-\lambda(\tau_{m+1} - \tau_m)})}{2} (\Pi_d + \Delta - \theta) \right]$$
$$+ e^{-\lambda(\tau_{m+1} - \tau_m)} \left(\Pi_d + (1 - p_{m+1}) (\frac{2r + \lambda\Delta}{2(\lambda + r)} - \frac{\theta}{2}) \right)$$
$$+ e^{-(r+\lambda)(\tau_{m+1} - \tau_m)} (1 - p_{m+1}) \left[\frac{e^{-\lambda(\tau_{m+1} - a_{m+1} - \tau_m)} - e^{-\lambda(\tau_{m+1} - \tau_m)}}{2} \Pi_d \frac{\lambda}{\lambda + r} \right.$$
$$\left. \left. \left. + e^{-\lambda(\tau_{m+1} - \tau_m)} \left(p_{m+1} \frac{\Pi_d \lambda}{2(\lambda + r)} + (1 - p_{m+1}) W_{m+1} \right) \right] \right] \right\}.$$

We next use the fact that in equilibrium, if no firm has entered at t_{m+1}, an uninformed firm is indifferent between entering and waiting at that period. This implies that the expected profit if a firm waits at period t_m is the same as the expected profit it would obtain if it were to enter with probability 1 as an uninformed firm at period t_{m+1}. We can then rewrite the expected profit of waiting as

$$\Pi_W = \xi(0)\Pi_d \frac{\lambda}{\lambda + r} + \xi(\theta)(\Pi_d + \Delta - \theta) p_{m+1} e^{-(r+\lambda)(\tau_{m+1} - \tau_m)}$$
$$+ \xi(\hat{\theta}) \left\{ p_m \frac{\Pi_d \lambda}{2(\lambda + r)} + (1 - p_m) \left[\frac{1 - e^{-(2\lambda + r)(\tau_{m+1} - a_{m+1} - \tau_m)}}{2} \frac{2\lambda}{2\lambda + r} \Pi_d \right. \right.$$
$$+ \frac{e^{-2\lambda(\tau_{m+1} - a_{m+1} - \tau_m)}}{2} (1 - e^{-a_{m+1}\lambda}) e^{-r(\tau_{m+1} - \tau_m)} \left(\Pi_d + \frac{\Delta r}{\lambda + r} (1 - p_{m+1} e^{-a_{m+1}\lambda}) \right)$$
$$+ e^{-(r+\lambda)(\tau_{m+1} - \tau_m)} \left[\frac{e^{-\lambda(\tau_{m+1} - a_{m+1} - \tau_m)} - e^{-\lambda(\tau_{m+1} - \tau_m)}}{2} \Pi_d + \frac{(1 - e^{-\lambda(\tau_{m+1} - \tau_m)})}{2} (\Pi_d + \Delta - \theta) \right.$$
$$\left. \left. \left. + e^{-\lambda(\tau_{m+1} - \tau_m)} \left(\Pi_d + (1 - p_{m+1}) \frac{(2r + \lambda)\Delta}{2(\lambda + r)} - \frac{\theta}{2} \right) \right] \right] \right\}.$$

Notice that, after this rearrangement, the expected profit Π_W only depends on

$p_1, \dots, p_m, p_{m+1}$ and not on the entry probabilities for periods after p_{m+1}. Let

$$
\begin{aligned}
A(p_{m+1}) \equiv & \frac{1-e^{-(2\lambda+r)(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}\frac{2\lambda}{2\lambda+r}\Pi_d \\
+ & \frac{e^{-2\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}(1-e^{-a_{m+1}\lambda})e^{-r(\tau_{m+1}-\tau_m)}\left(\Pi_d+\frac{\Delta r}{\lambda+r}(1-p_{m+1})e^{-a_{m+1}\lambda}\right) \\
+ & e^{-(r+\lambda)(\tau_{m+1}-\tau_m)}\left[\frac{e^{-\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}-e^{-\lambda(\tau_{m+1}-\tau_m)}}{2}\Pi_d+\frac{(1-e^{-\lambda(\tau_{m+1}-\tau_m)})}{2}(\Pi_d+\Delta-\theta)\right. \\
& \left.+e^{-\lambda(\tau_{m+1}-\tau_m)}\left(\Pi_d+(1-p_{m+1})\frac{(2r+\lambda)\Delta}{2(\lambda+r)}-\frac{\theta}{2}\right)\right].
\end{aligned}
$$

Equating Π_E and Π_W, we find the entry probability which makes an uninformed firm indifferent between entering or waiting at period t_m in the multiple-entry equilibrium:

$$
p_m = f_m(a) = 1 - \frac{\Pi_d - \frac{\theta}{2} - \frac{\Pi_d\lambda}{2(\lambda+r)} - \frac{\xi(0)}{\xi(\hat{\theta})}\Pi_d\frac{r}{\lambda+r} - \frac{\xi(\theta)}{\xi(\hat{\theta})}(\Pi_d+\Delta-\theta)(1-e^{-(\lambda+r)(\tau_{m+1}-\tau_m)})}{\frac{\Pi_d\lambda}{2(\lambda+r)}+\frac{\Delta(2r+\lambda)}{2(\lambda+r)}-A(p_{m+1})}.
$$

Alternatively, we compute the expected profit if a firm waits at period t_m assuming that it will wait at any further periods as

$$
\begin{aligned}
\Pi_W = & \xi(0)\Pi_d\frac{\lambda}{\lambda+r}+\xi(\hat{\theta})\left\{p_m\frac{\Pi_d\lambda}{2(\lambda+r)}\right. \\
& + (1-p_m)\left[\frac{1-e^{-(2\lambda+r)(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}\frac{2\lambda}{2\lambda+r}\Pi_d\right. \\
+ & \frac{e^{-2\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}(1-e^{-a_{m+1}\lambda})e^{-r(\tau_{m+1}-\tau_m)}\left(\Pi_d+\frac{\Delta r}{\lambda+r}(1-p_{m+1})e^{-a_{m+1}\lambda}\right) \\
& + e^{-(r+\lambda)(\tau_{m+1}-\tau_m)}\left[\frac{e^{-\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}-e^{-\lambda(\tau_{m+1}-\tau_m)}}{2}\Pi_d\frac{\lambda}{\lambda+r}\right. \\
& \left.\left.\left.+e^{-\lambda(\tau_{m+1}-\tau_m)}\left(p_{m+1}\frac{\pi_d\lambda}{2(\lambda+r)}+(1-p_{m+1})W_{m+1}\right)\right]\right]\right\},
\end{aligned}
$$

where the continuation value at date τ_m given that none of the firms has received the signal at τ_m, W_m, is given by the recursive formula

$$
\begin{aligned}
W_m = & \frac{1-e^{-(2\lambda+r)(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}\frac{2\lambda}{2\lambda+r}\Pi_d \\
+ & \frac{e^{-2\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}(1-e^{-a_{m+1}\lambda})e^{-r(\tau_{m+1}-\tau_m)}\left(\Pi_d+\frac{\Delta r}{\lambda+r}(1-p_{m+1})e^{-a_{m+1}\lambda}\right) \\
& + \frac{e^{-2\lambda(\tau_{m+1}-a_{m+1}-\tau_m)}}{2}(1-e^{-a_{m+1}\lambda})e^{-r(\tau_{m+1}-\tau_m)}\Pi_d\frac{\lambda}{\lambda+r}e^{-a_{m+1}\lambda} \\
& + e^{-\lambda(\tau_{m+1}-\tau_m)}\left(p_{m+1}\frac{\pi_d\lambda}{2(\lambda+r)}+(1-p_{m+1})W_{m+1}\right)
\end{aligned}
$$

with terminal condition at the last entry date:

$$W_{M+1} = \frac{2\lambda}{2\lambda + r}\Pi_d.$$

Equating the expected profit of entering and waiting at date τ_m results in an entry probability p_m given by

$$p_m = f_m(a) = 1 - \frac{\Pi_d - \frac{\theta}{2} - \frac{\Pi_d \lambda}{2(\lambda+r)} - \frac{\xi(0)}{\xi(\hat{\theta})}\Pi_d \frac{r}{\lambda+r} - \frac{\xi(\theta)}{\xi(\hat{\theta})}(\Pi_d + \Delta - \theta)}{\frac{\Pi_d \lambda}{2(\lambda+r)} + \frac{\Delta(2r+\lambda)}{2(\lambda+r)} - W_{m+1}}.$$