Stable Networks with Bargaining and Heterogeneous Costs

Erik Darpö, Alvaro Domínguez & María Martín-Rodríguez

Nagoya University

May 22, 2020
Microeconomics Workshop, Keio University

Motivation I

People associate for many reasons

Motivation I

People associate for many reasons

- Trade

Motivation I

People associate for many reasons

- Trade
- Joint ventures

Motivation I

People associate for many reasons

- Trade
- Joint ventures
- Scientific research

  Economics: 76% of published papers are coauthored (1980 - 2016).

Motivation I

People associate for many reasons

- Trade
- Joint ventures
- Scientific research

  Economics: 76% of published papers are coauthored (1980 - 2016).

These collaborations generate networks

Motivation I

People associate for many reasons

- Trade
- Joint ventures
- Scientific research

  Economics: 76% of published papers are coauthored (1980 - 2016).

These collaborations generate networks

- Network: representation of a set of elements and their relationships.

Motivation II

But linking costs may differ across agents.

Motivation II

But linking costs may differ across agents.

For instance, this cost may depend on

Motivation II

But linking costs may differ across agents.

For instance, this cost may depend on

- the characteristics of the agents (types)

Motivation II

But linking costs may differ across agents.

For instance, this cost may depend on

- the characteristics of the agents (types)
- the characteristics of the joint activity (complementarities).

Motivation II

But linking costs may differ across agents.

For instance, this cost may depend on

- the characteristics of the agents (types)
- the characteristics of the joint activity (complementarities).

Motivation III

Assume that pairs of agents generate a joint surplus normalized to one.

Motivation III

Assume that pairs of agents generate a joint surplus normalized to one.

Strategic creation/breaking of links (trade-off):

Motivation III

Assume that pairs of agents generate a joint surplus normalized to one.

Strategic creation/breaking of links (trade-off):

- Share of surplus received depends on the relative position in the network

Motivation III

Assume that pairs of agents generate a joint surplus normalized to one.

Strategic creation/breaking of links (trade-off):

- Share of surplus received depends on the relative position in the network
- Creating links to collaborate is costly.

Motivation IV

Question: How do the stable networks look like?

Motivation IV

Question: How do the stable networks look like?

Answer: It depends on the cost structure!

Motivation IV

Question: How do the stable networks look like?

Answer: It depends on the cost structure!

For example, with two types of agents, these cost structures are natural:

Motivation IV

Question: How do the stable networks look like?

Answer: It depends on the cost structure!

For example, with two types of agents, these cost structures are natural:

- Cheap links across agents of the same type

Motivation IV

Question: How do the stable networks look like?

Answer: It depends on the cost structure!

For example, with two types of agents, these cost structures are natural:

- Cheap links across agents of the same type
- Cheap links across agents of different types.

Motivation IV

Question: How do the stable networks look like?

Answer: It depends on the cost structure!

For example, with two types of agents, these cost structures are natural:

- Cheap links across agents of the same type
- Cheap links across agents of different types.

Stable components:

Motivation IV

Question: How do the stable networks look like?

Answer: It depends on the cost structure!

For example, with two types of agents, these cost structures are natural:

- Cheap links across agents of the same type
- Cheap links across agents of different types.

Stable components:

- Equitable components: always pairs and odd cycles

Motivation IV

Question: How do the stable networks look like?

Answer: It depends on the cost structure!

For example, with two types of agents, these cost structures are natural:

- Cheap links across agents of the same type
- Cheap links across agents of different types.

Stable components:

- Equitable components: always pairs and odd cycles
- Inequitable components: certain bipartite graphs. More variety with the second cost structure than with the first.

Literature Review

R&D Networks: Creating a link leads to a cost reduction. Firms are ex-ante homogeneous but, ex-post, they can be homogeneous or heterogeneous.

- Goyal and Moraga-González (2001), Goyal and Joshi (2003).

Literature Review

R&D Networks: Creating a link leads to a cost reduction. Firms are ex-ante homogeneous but, ex-post, they can be homogeneous or heterogeneous.

- Goyal and Moraga-González (2001), Goyal and Joshi (2003).

Information networks: Creating a link leads to more information available. Cost heterogeneity directly derives from agent heterogeneity and results in core-periphery architectures.

- Galeotti et al. (2006), Galeotti and Goyal (2010).

Literature Review

R&D Networks: Creating a link leads to a cost reduction. Firms are ex-ante homogeneous but, ex-post, they can be homogeneous or heterogeneous.

- Goyal and Moraga-González (2001), Goyal and Joshi (2003).

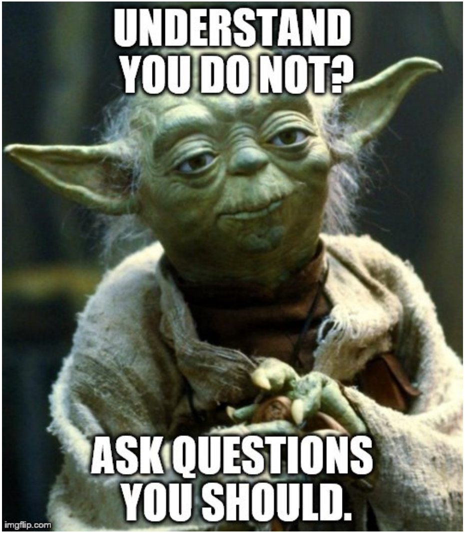
Information networks: Creating a link leads to more information available. Cost heterogeneity directly derives from agent heterogeneity and results in core-periphery architectures.

- Galeotti et al. (2006), Galeotti and Goyal (2010).

Bargaining networks: Creating a link may alter the bargaining power.

- Stationary networks: Manea (2011), Gauer and Hellmann (2017).
- Non-stationary networks: Kranton and Minehart (2001), Elliott and Nava (2019).

Questions



The Model

Two-stage game:

The Model

Two-stage game:

- $t = 0$: players form undirected, costly links.

The Model

Two-stage game:

- $t = 0$: players form undirected, costly links.
  - The linking cost differs.

The Model

Two-stage game:

- $t = 0$: players form undirected, costly links.
  - The linking cost differs.
- $t = 1, 2, \ldots$: given the network formed in the previous stage, infinite-horizon game in which pairs of players connected through a link are randomly matched to bargain *à la Manea*.

Networks I

Network, g: simple, undirected graph.

Networks I

Network, g: simple, undirected graph.

Set of players: $N = \{1, 2, ..., n\}$, $n \geq 3$, representing the nodes.

Networks I

Network, g: simple, undirected graph.

Set of players: $N = \{1, 2, ..., n\}$, $n \geq 3$, representing the nodes.

Link between players $i, j \in N$, $i \neq j$: $ij = ji$, representing the collaborations.

Networks I

Network, g: simple, undirected graph.

Set of players: $N = \{1, 2, ..., n\}$, $n \geq 3$, representing the nodes.

Link between players $i, j \in N$, $i \neq j$: $ij = ji$, representing the collaborations.

Neighbors of player i in network g: $N_i(g) = \{j \in N \mid ij \in g\}$

- *Degree* of player i: $\eta_i(g) = |N_i(g)|$
- A player is isolated if she has no neighbors.

Networks I

Network, g: simple, undirected graph.

Set of players: $N = \{1, 2, ..., n\}$, $n \geq 3$, representing the nodes.

Link between players $i, j \in N$, $i \neq j$: $ij = ji$, representing the collaborations.

Neighbors of player i in network g: $N_i(g) = \{j \in N \mid ij \in g\}$

- *Degree* of player i: $\eta_i(g) = |N_i(g)|$
- A player is isolated if she has no neighbors.

Path: sequence of links which joins a sequence of nodes which are all distinct.

Networks II

Component of network g: subnetwork in which any two nodes are connected to each other by paths, and which is connected to no additional nodes in the network.

Networks II

Component of network g: subnetwork in which any two nodes are connected to each other by paths, and which is connected to no additional nodes in the network.

Notation:

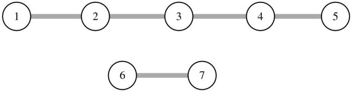
- $g + ij := g \cup \{ij\}$: network obtained by adding link ij to the existing network g
- $g - ij := g \setminus \{ij\}$: network obtained by deleting link ij from the existing network g.

Networks II

Component of network g: subnetwork in which any two nodes are connected to each other by paths, and which is connected to no additional nodes in the network.

Notation:

- $g + ij := g \cup \{ij\}$: network obtained by adding link ij to the existing network g
- $g - ij := g \setminus \{ij\}$: network obtained by deleting link ij from the existing network g.



Bargaining I

Given a network g, bargaining stage *à la Manea*

Bargaining I

Given a network g, bargaining stage *à la Manea*

- Each period $t = 1, 2, \ldots$ a link $ij \in g$ is selected with the same probability and then, with probability $1/2$, one of the two players is chosen to make an offer on how to split the unitary joint surplus

Bargaining I

Given a network g, bargaining stage *à la Manea*

- Each period $t = 1, 2, \ldots$ a link $ij \in g$ is selected with the same probability and then, with probability $1/2$, one of the two players is chosen to make an offer on how to split the unitary joint surplus
- If rejected, both players get zero; if accepted, both players exit the game with the shares agreed upon and they are replaced in the next period by identical players.

Bargaining I

Given a network g, bargaining stage *à la Manea*

- Each period $t = 1, 2, \ldots$ a link $ij \in g$ is selected with the same probability and then, with probability $1/2$, one of the two players is chosen to make an offer on how to split the unitary joint surplus
- If rejected, both players get zero; if accepted, both players exit the game with the shares agreed upon and they are replaced in the next period by identical players.

The stationary equilibrium payoffs are denoted by v.

Bargaining II

With sufficiently patient agents, the algorithm below calculates the equilibrium payoffs v.

Bargaining II

With sufficiently patient agents, the algorithm below calculates the equilibrium payoffs v.

For every network g and subset of nodes M, let $L^g(M) = \cup_{i \in M} N_i(g)$ be the set of neighbors of the nodes in M. M is *g-independent* if there is no link between two nodes in M.

Bargaining II

With sufficiently patient agents, the algorithm below calculates the equilibrium payoffs v.

For every network g and subset of nodes M, let $L^g(M) = \cup_{i \in M} N_i(g)$ be the set of neighbors of the nodes in M. M is *g-independent* if there is no link between two nodes in M.

Intuitively, at each step the algorithm identifies the players in strongest and weakest bargaining positions by minimizing $|L^{g_s}(M_s)|/|M_s|$.

Bargaining II

With sufficiently patient agents, the algorithm below calculates the equilibrium payoffs v.

For every network g and subset of nodes M, let $L^g(M) = \cup_{i \in M} N_i(g)$ be the set of neighbors of the nodes in M. M is *g-independent* if there is no link between two nodes in M.

Intuitively, at each step the algorithm identifies the players in strongest and weakest bargaining positions by minimizing $|L^{g_s}(M_s)|/|M_s|$.

At each step s, given the network g_s

Bargaining II

With sufficiently patient agents, the algorithm below calculates the equilibrium payoffs v.

For every network g and subset of nodes M, let $L^g(M) = \cup_{i \in M} N_i(g)$ be the set of neighbors of the nodes in M. M is g-*independent* if there is no link between two nodes in M.

Intuitively, at each step the algorithm identifies the players in strongest and weakest bargaining positions by minimizing $|L^{g_s}(M_s)|/|M_s|$.

At each step s, given the network g_s

- Identify a non-empty, g-independent set M_s such that $r_s = |L^{g_s}(M_s)|/|M_s|$ is minimized

Bargaining II

With sufficiently patient agents, the algorithm below calculates the equilibrium payoffs v.

For every network g and subset of nodes M, let $L^g(M) = \cup_{i \in M} N_i(g)$ be the set of neighbors of the nodes in M. M is g-*independent* if there is no link between two nodes in M.

Intuitively, at each step the algorithm identifies the players in strongest and weakest bargaining positions by minimizing $|L^{g_s}(M_s)|/|M_s|$.

At each step s, given the network g_s

- Identify a non-empty, g-independent set M_s such that $r_s = |L^{g_s}(M_s)|/|M_s|$ is minimized
- If $r_s \geq 1$, each player gets payoff $1/2$ and stop. Otherwise, players in M_s get payoff $x_s = r_s/(1 + r_s) = |L^{g_s}(M_s)|/(|L^{g_s}(M_s)| + |M_s|) < 1/2$ and players in $L^{g_s}(M_s)$ get $1 - x_s = |M_s|/(|L^{g_s}(M_s)| + |M_s|) > 1/2$

Bargaining II

With sufficiently patient agents, the algorithm below calculates the equilibrium payoffs v.

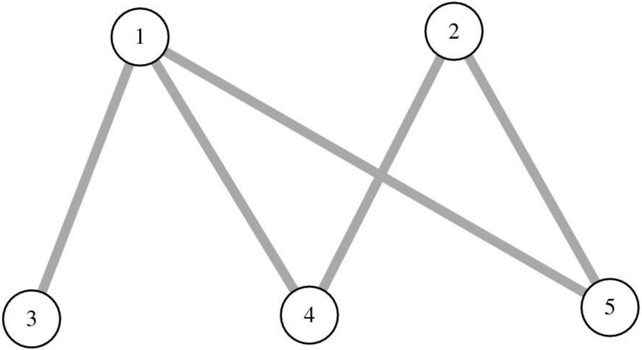
For every network g and subset of nodes M, let $L^g(M) = \cup_{i \in M} N_i(g)$ be the set of neighbors of the nodes in M. M is g-*independent* if there is no link between two nodes in M.

Intuitively, at each step the algorithm identifies the players in strongest and weakest bargaining positions by minimizing $|L^{g_s}(M_s)|/|M_s|$.

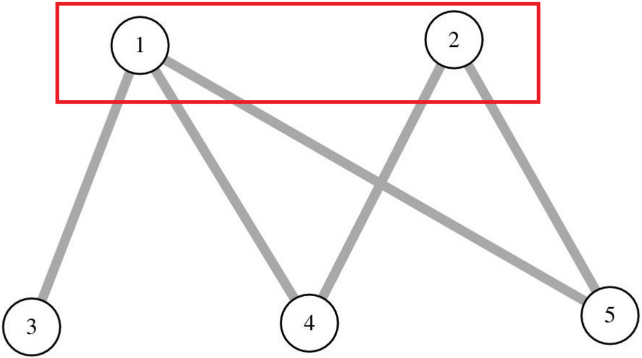
At each step s, given the network g_s

- Identify a non-empty, g-independent set M_s such that $r_s = |L^{g_s}(M_s)|/|M_s|$ is minimized
- If $r_s \geq 1$, each player gets payoff $1/2$ and stop. Otherwise, players in M_s get payoff $x_s = r_s/(1 + r_s) = |L^{g_s}(M_s)|/(|L^{g_s}(M_s)| + |M_s|) < 1/2$ and players in $L^{g_s}(M_s)$ get $1 - x_s = |M_s|/(|L^{g_s}(M_s)| + |M_s|) > 1/2$
- Set $g_{s+1} = g_s \setminus (M_s \cup L^{g_s}(M_s))$ and repeat.

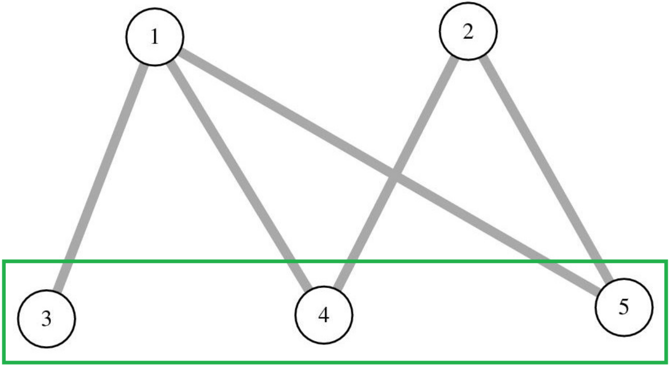
Bargaining III



Bargaining III

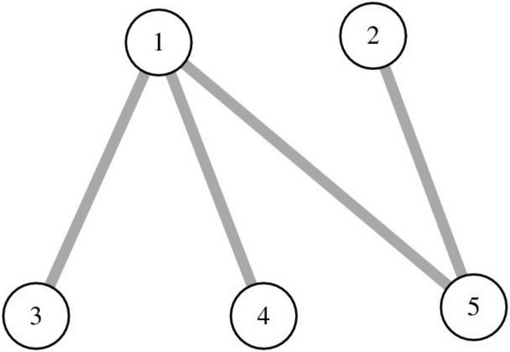


Bargaining III

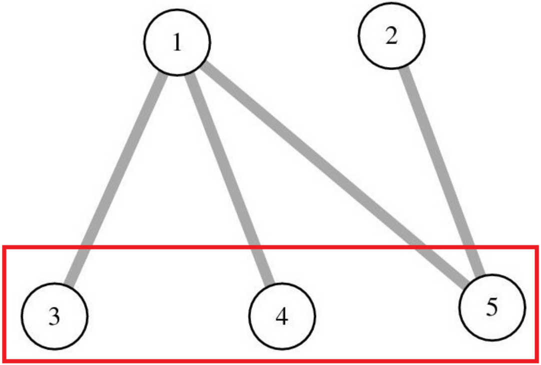


The algorithm finishes in one step:
$M_1 = \{3, 4, 5\}$, $L^{g_1}(M_1) = \{1, 2\}$; $r_1 = 2/3$, $x_1 = 2/5$, $1 - x_1 = 3/5$

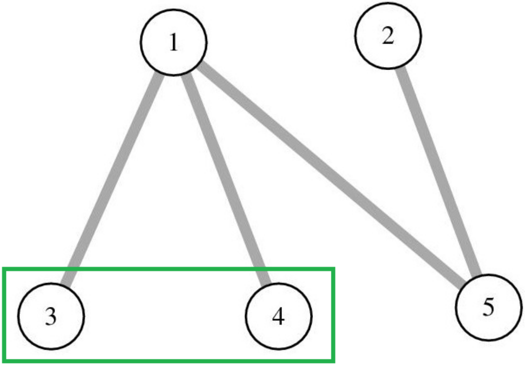
Bargaining IV



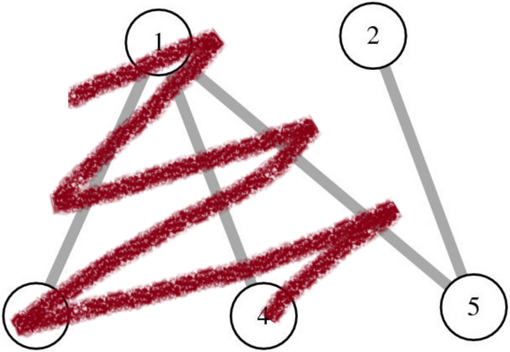
Bargaining IV



Bargaining IV



Bargaining IV



The algorithm finishes in two steps:

$$M_1 = \{3,4\},\ L^{g_1}(M_1) = \{1\};\ r_1 = 1/2,\ x_1 = 1/3,\ 1 - x_1 = 2/3$$
$$M_2 = \{5\},\ L^{g_2}(M_2) = \{2\};\ r_1 = 1,\ x_1 = 1 - x_1 = 1/2$$

Stability

When forming the network, the players anticipate the outcome of the bargaining stage v.

Stability

When forming the network, the players anticipate the outcome of the bargaining stage v.

The linking decisions reflect the following trade-off:

- creating a link may alter the bargaining power
- creating a link to collaborate is costly.

Stability

When forming the network, the players anticipate the outcome of the bargaining stage v.

The linking decisions reflect the following trade-off:

- creating a link may alter the bargaining power
- creating a link to collaborate is costly.

At $t = 0$, each player i tries to maximize

$$u_i(g) := v_i(g) - \sum_{j \in N_i(g)} c^{ij}.$$

Stability

When forming the network, the players anticipate the outcome of the bargaining stage v.

The linking decisions reflect the following trade-off:

- creating a link may alter the bargaining power
- creating a link to collaborate is costly.

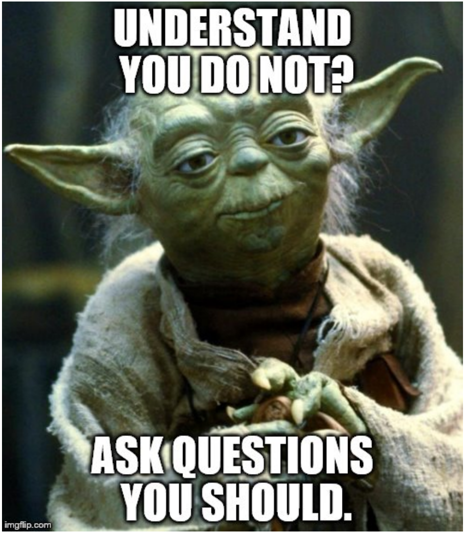
At $t = 0$, each player i tries to maximize

$$u_i(g) := v_i(g) - \sum_{j \in N_i(g)} c^{ij}.$$

A network g is *pairwise stable* if:

- for all $ij \in g : u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$, and
- for all $ij \notin g$: if $u_i(g + ij) > u_i(g)$, then $u_j(g + ij) < u_j(g)$

Questions



General Results: Architecture of Stable Components

These results about the architecture of the stable components do not depend on the cost structure, but the specific stability condition does.

General Results: Architecture of Stable Components

These results about the architecture of the stable components do not depend on the cost structure, but the specific stability condition does.

For all stable components, the algorithm finishes in one step.

General Results: Architecture of Stable Components

These results about the architecture of the stable components do not depend on the cost structure, but the specific stability condition does.

For all stable components, the algorithm finishes in one step.

Equitable components (exhaustive list)

- pairs
- odd cycles

General Results: Architecture of Stable Components

These results about the architecture of the stable components do not depend on the cost structure, but the specific stability condition does.

For all stable components, the algorithm finishes in one step.

Equitable components (exhaustive list)

- pairs
- odd cycles

Inequitable components: certain bipartite graphs such that all its leaves are elements of M.

Costs

Assume two types of players, E and T, such that $N = N_E \dot{\cup} N_T$.

Costs

Assume two types of players, E and T, such that $N = N_E \dot{\cup} N_T$.

The linking cost may include several components. For instance,

Costs

Assume two types of players, E and T, such that $N = N_E \dot{\cup} N_T$.

The linking cost may include several components. For instance,

- Communication costs

Costs

Assume two types of players, E and T, such that $N = N_E \dot{\cup} N_T$.

The linking cost may include several components. For instance,

- Communication costs
- Cost that depends on the complementarities required to complete the collaboration.

Costs

Assume two types of players, E and T, such that $N = N_E \dot{\cup} N_T$.

The linking cost may include several components. For instance,

- Communication costs
- Cost that depends on the complementarities required to complete the collaboration.

One can assume that it's easier to communicate with someone of the same type.

Costs

Assume two types of players, E and T, such that $N = N_E \dot{\cup} N_T$.

The linking cost may include several components. For instance,

- Communication costs
- Cost that depends on the complementarities required to complete the collaboration.

One can assume that it's easier to communicate with someone of the same type.

If strong complementarities among skills are necessary to complete the collaboration and generate the surplus, then the linking cost may be cheaper among agents of different types.

Costs

Assume two types of players, E and T, such that $N = N_E \dot{\cup} N_T$.

The linking cost may include several components. For instance,

- Communication costs
- Cost that depends on the complementarities required to complete the collaboration.

One can assume that it's easier to communicate with someone of the same type.

If strong complementarities among skills are necessary to complete the collaboration and generate the surplus, then the linking cost may be cheaper among agents of different types.

Anyhow, each link has a cost either $\underline{c}$ or $\overline{c}$ for both players, with $\underline{c} < \overline{c}$. This cost depends only on the types of the adjacent players.

Cheap Connections with Same Type: Components I

Equitable components: essentially the same as in Gauer and Hellmann.

$\underline{c} \leq 1/2$

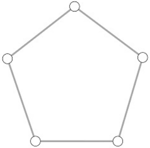
$\overline{c} \leq 1/2$

Cheap Connections with Same Type: Components I

Equitable components: essentially the same as in Gauer and Hellmann.

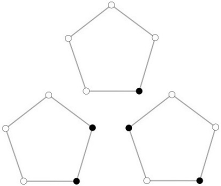
$\underline{c} \leq 1/2$

$\overline{c} \leq 1/2$



$\underline{c} \leq \frac{1}{6}$,

with at most $\frac{1}{2\underline{c}}$ players

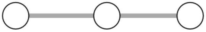


$\overline{c} \leq \frac{1}{6}$,

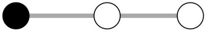
with at most $\frac{1}{2\overline{c}}$ players

Cheap Connections with Same Type: Components II

Inequitable components: essentially the same as in Gauer and Hellmann.



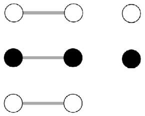
$\underline{c} = 1/6$



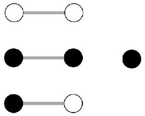
$\overline{c} = 1/6$

Cheap Connections with Same Type: Networks I

Very minor differences with respect to Gauer and Hellmann.



(a)

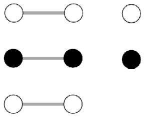


(b)

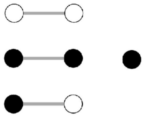
There can be networks formed by pairs of the same types and two isolated nodes of different types (the former depend on $\underline{c}$ and the latter on $\overline{c}$).

Cheap Connections with Same Type: Networks I

Very minor differences with respect to Gauer and Hellmann.



(a)



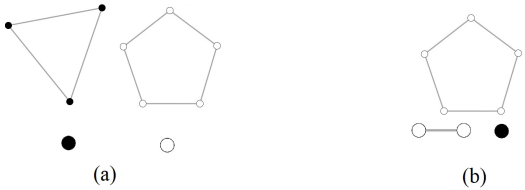
(b)

There can be networks formed by pairs of the same types and two isolated nodes of different types (the former depend on $\underline{c}$ and the latter on $\overline{c}$).

However, if there are pairs of players of different types, there can only be one isolated node.

Cheap Connections with Same Type: Networks II

Very minor differences with respect to Gauer and Hellmann.

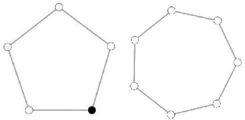


There can be networks formed by odd cycles of the same types and two isolated nodes of different types (the former depend on $\underline{c}$ and the latter on $\overline{c}$).

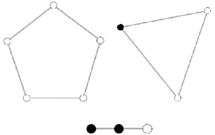
If in addition to the cycles of same types there are pairs, all of the same type, there can only be one isolated node of type different from the pairs.

Cheap Connections with Same Type: Networks II

Very minor differences with respect to Gauer and Hellmann.



(c)

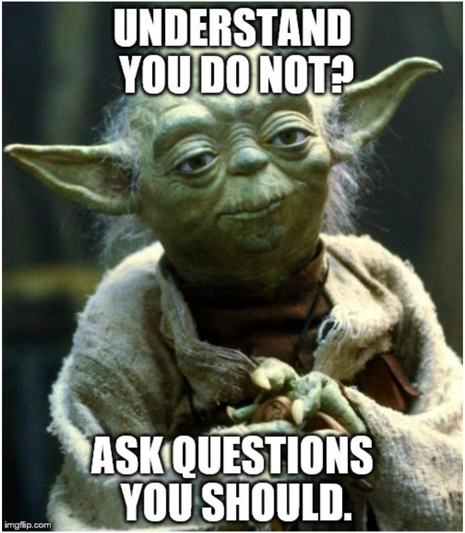


(d)

Given the maximum size of the cycles that connect players of the same type, the cycles that connect players of different types cannot be larger.

A single line of length three that connects players of different types can coexist with cycles of size three that connect players of different types and with cycles that connect players of the same type. If $\underline{c}$ is large enough, there can also be pairs.

Questions



Cheap Connections with Different Type: Components I

Pairs and odd cycles are still the only equitable components.

But now there are inequitable components besides the line of length three!

Cheap Connections with Different Type: Components I

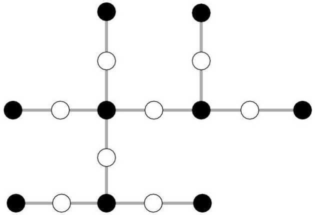
Pairs and odd cycles are still the only equitable components.

But now there are inequitable components besides the line of length three!

Odd lines of length m

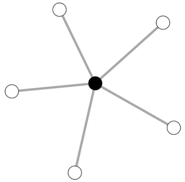
$$M_1 = \{\text{blacks}\}$$

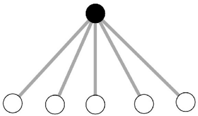
Cheap Connections with Different Type: Components II



$M_1 = \{\text{blacks}\}$

Cheap Connections with Different Type: Components III

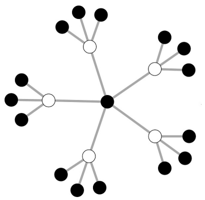


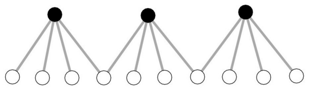


Stars with n leaves, all of the same type and different from the root

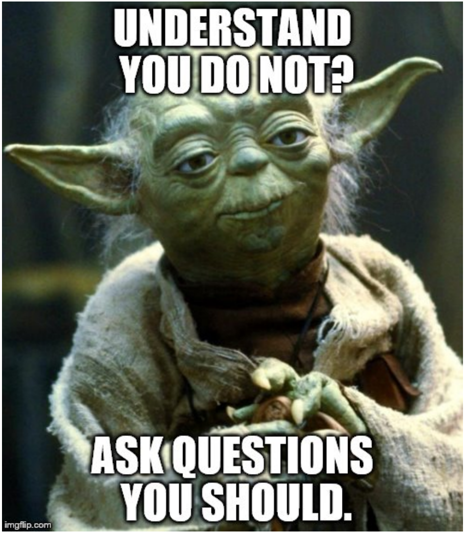
$$M_1 = \{\text{leaves}\}$$

Cheap Connections with Different Type: Components IV





Questions



Conclusions

- Model to study the stability of the network resulting from agents decisions to collaborate.

Conclusions

- Model to study the stability of the network resulting from agents decisions to collaborate.

- Factors like the agents' heterogeneity and the severity of the complementarities determine the overall cost structure.

Conclusions

- Model to study the stability of the network resulting from agents decisions to collaborate.
- Factors like the agents' heterogeneity and the severity of the complementarities determine the overall cost structure.
- When it is cheaper in the overall to collaborate between types that are alike, there are more architectures of equitable components.

Conclusions

- Model to study the stability of the network resulting from agents decisions to collaborate.
- Factors like the agents' heterogeneity and the severity of the complementarities determine the overall cost structure.
- When it is cheaper in the overall to collaborate between types that are alike, there are more architectures of equitable components.
- When it is cheaper in the overall to collaborate between types that are different, there are more architectures of inequitable components.

Future Work

- Characterize the inequitable components as much as possible.

Future Work

- Characterize the inequitable components as much as possible.

- Empirical check.


THANK
YOU!