Bank Runs, Prudential Tools and Social Welfare in a Global Game General Equilibrium Model*

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Abstract

I develop a general equilibrium model that features endogenous bank runs in a global game framework. A bank-run-led crisis probability depends on banks' fundamentals such as leverage and liquid asset holdings. Bank risk shifting and pecuniary externalities induce excessive leverage and insufficient liquidity, resulting in an elevated crisis probability from a social welfare viewpoint. Addressing this problem requires policy tools on both leverage and liquidity. Imposing one tool only causes risk migration: banks respond by taking more risk in another area. Risk migration in other fields such as shadow banking and loan portfolios is also studied.

Keywords: Bank runs, global games, capital and liquidity requirements, risk migration.

JEL classification: E44, G01, G21, G28.

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1 Introduction

Ten years have passed since the global financial crisis. In 2019 the new regulatory framework, Basel III, to prevent the recurrence of such a crisis, will be fully implemented. The recovery phase of the banking system is over and we are moving toward the evaluation phase of the financial regulatory reform.

A key to the evaluation phase is to understand how and to what extent new regulations enhance financial system resilience, and to assess their social benefits and costs. Doing so is challenging, however, due to the still incomplete financial intermediation theory (Financial Stability Board, 2017). What we need is a model that helps us understand more about how the financial system responds to new regulations. In light of the objective of Basel III – building a more resilient financial system to financial crisis risk – and given its multiple-tool approach, I believe that three ingredients are essential for such a model: (i) a trigger event of a financial crisis, (ii) financial system resilience to such an event, and (iii) externalities that warrant the implementation of multiple tools.

In this paper, I develop a simple model that features such three ingredients. Specifically, I embed a bank run global game model studied by Rochet and Vives (2004) into a two-period general equilibrium model in the spirit of Christiano and Ikeda (2013, 2016). The model features bank runs as a crisis event, reflecting the historical fact that most of the financial crises have involved bank runs (Gorton, 2012 and Reinhart and Rogoff, 2009). The probability of no bank run – financial system resilience – is endogenously determined as a function of banks' fundamentals such as leverage and liquid asset holdings.

Using the model, I study its implications for social welfare and policy. The findings are twofold. First, in the laissez-faire economy, bank leverage is excessive and bank liquid asset holdings are insufficient, resulting in elevated financial crisis risk from a social welfare perspective. The culprit of such inefficiencies are bank risk shifting and pecuniary externalities. Second, addressing the inefficiencies requires prudential tools on both leverage and liquidity. With one instrument only, another risk area, either leverage or liquidity, is always at an inefficient level. I show these results analytically. In addition, I numerically illustrate risk migration: in response to a tightening in requirements in one area, risk can migrate from the targeted area to another, attenuating the intended effects of the requirements. Due to bank risk migration, social welfare may deteriorate if the coverage of prudential tools lacks comprehensiveness.

The model consists of three types of agents: households, banks, and fund managers.

Households and banks receive an endowment – household income and bank capital, respectively – in the beginning of the first period. Households allocate the income into consumption and deposits. Banks offer a deposit contract such that a promised interest rate is paid as long as they do not default; the contract allows early withdrawals of deposits. Banks invest the sum of deposits and bank capital in a portfolio of risky lending and safe liquidity. If the asset return is low enough, the banks, unable to pay the interest rate, default and the depositors incur losses. Households can avoid such losses by withdrawing deposits early. To do so effectively, the households delegate an early withdrawal decision to fund managers who have information advantages about the bank asset return. But early withdrawals are costly for banks. Banks have to sell some assets at a discounted price if their liquidity holdings are not enough to cover withdrawals. This costly liquidation causes an illiquidity-driven default if a large number of fund managers make a run on banks. This structure gives rise to a global game in which a bank run occurs if the bank asset return is lower than a certain threshold. Both households and banks take into account the possibility of bank runs and the default probability in choosing how much to lend and borrow, respectively. In the second period, bank runs occur, if any, and banks default, or banks survive and pay the deposit interest rate and distribute the profits to their owner households. The households consume everything at hand.

A unique feature of this model is that, unlike various financial frictions models such as Kiyotaki and Moore (1997), Bernanke et al. (1999), Jermann and Quadrini (2012), Gertler and Kiyotaki (2015) and Christiano and Ikeda (2016), bank leverage is pinned down without any binding constraints. Without bank runs, banks would increase leverage as long as the expected bank asset return is greater than the interest rate. With bank runs, however, higher leverage increases a bank-run-led default probability, which in turn dampens the expected profits. Thus, market discipline, if not perfect, restrains bank behavior. Balancing an asset return and a default probability, bank leverage has an interior solution.

To derive welfare implications, I set up a second best problem in which a benevolent regulator chooses leverage and liquidity subject to bank run risk. The first best outcome should involve no bank run because bank runs are caused by coordination failures. However, prohibiting early withdrawals is neither possible in the model nor practical in reality. Instead, by restricting leverage and liquidity, the regulator can affect banking system resilience to runs.

The analytical characterization of the competitive equilibrium and the regulator's problem reveals two sources of externalities that cause excessive leverage and insufficient liquidity: namely, risk shifting and pecuniary externalities as mentioned previously. Risk shifting arises because individual bank leverage and liquidity are assumed to be unobservable to bank depositors. Due to this inability to observe individual bank characteristics, individual bank default risk is not priced at the margin but on the basis of the expected losses on the banking industry as a whole as in Acharya (2009) and Mendicino et al. (forthcoming). As a result, banks end up with taking excessive risk in leverage and liquidity areas, a phenomenon known as risk shifting close in spirit to Jensen and Meckling (1976).

Second, the model has pecuniary externalities that work through an interest rate as in Christiano and Ikeda (2016). A bank run depends on the size of bank liabilities, specifically its important determinant, the *risky* deposit interest rate. This interest rate constitutes the expected rate of return on deposits, which is directly linked to the amount of deposits and leverage in a general equilibrium. The banks ignore this effect because they behave taking as given the expected rate of return on deposits in a competitive equilibrium.

The two sources of externalities affect leverage and liquidity differently. Bank risk shifting affects both leverage and liquidity, but pecuniary externalities affect leverage only. This is because a liquidity choice – how much liquidity banks hold relative to deposits – is about the composition of assets and does not affect the total amount of borrowing and leverage. Therefore, bank risk shifting is essential for obtaining the result of insufficient liquidity in this model.

The excessive leverage and insufficient liquidity warrant prudential instruments on leverage and liquidity to limit financial crisis risk. Doing so, however, involves a trade-off by restricting financial intermediation from households to the real sector. A leverage tool restricts the banks' capacity to borrow and a liquidity tool limits liquidity transformation – the amount of deposits that are transformed into lending. Prudential policy has to balance between stabilizing the financial system – decreasing a crisis probability – and promoting the real economy by maintaining the functioning of financial intermediation.

The model highlights a general equilibrium effect on jointly optimal leverage and liquidity requirements. A numerical example illustrates that a leverage restriction should be tightened more than a liquidity requirement, relative to the competitive equilibrium allocation, if the supply curve of funds, which is derived from households' optimization, is relatively steep. As the supply curve becomes steeper, restricting leverage lowers the interest rate more, yielding an additional benefit of reducing the crisis probability. This result suggests that jointly optimal requirements can differ significantly depending on the supply side of funds, e.g. a small open economy or a closed economy.

The model is so stylized that it has rich applications for banks' behavior and other prudential instruments including bank/sector specific capital requirements and caps on concentration risk. Yet, risk migration between two risk areas continues to be at the heart of the applications. In one applied model with heterogeneous banks, the two risk areas are leverage of regulated banks and leverage of unregulated 'shadow' banks. This model also allows us to study bank-specific capital requirements and sectoral capital requirements if both types of banks are regulated. In another applied model with a bank portfolio choice, the two risk areas are leverage and a portfolio choice. Because of risk shifting motives, banks prefer a riskier portfolio than perfectly diversified one. But, unlike Kareken and Wallace (1978), the banks do not necessarily choose the riskiest portfolio because doing so increases their default probability and could lower their expected profits. This model allows us to study concentration risk in lending portfolios.

As a further application, the paper considers a role of deposit insurance. Deposit insurance has been regarded as an institutional milestone for addressing bank runs by retail depositors. In the model, however, bank runs can occur as long as deposit insurance is imperfect, which is the case for large depositors and non-banks in practice. The paper shows that imperfect deposit insurance makes excessive leverage even more excessive and further increases the crisis probability.

Related literature

This paper contributes to the emerging literature on the interaction of multiple prudential tools. As I have emphasized, this paper features endogenous bank-run-led crises in a general equilibrium model. In this regard, the most closely related paper is Kashyap et al. (2017), who *numerically* study a general equilibrium version of Diamond and Dybvig (1983) in a global game framework and argue that no single regulation is sufficient to implement the social optimum. In contrast, this paper, building on a simpler global game bank run model à la Rochet and Vives (2004) and a simpler general equilibrium model à la Christiano and Ikeda (2013, 2016), derives *analytical* results on the sources of inefficiencies and the role of multiple prudential tools.[1]

In a global game framework, Vives (2014), using the partial equilibrium model of Rochet and Vives (2004), argues that regulations should focus on the balance sheet composition

[1] Because of its simplicity, the global game of Rochet and Vives (2004) is incorporated into the Bank of Canada's stress-test model for the banking sector (Fique 2017). Another important global game bank run model is Goldstein and Pauzner (2005). Yet, this paper adopts Rochet and Vives (2004) for analytical tractability in a general equilibrium framework.

of financial intermediaries. Ahnert (2016), extending the model of Morris and Shin (2000), studies intermediaries' choice of liquidity and capital separately and argues that prudential regulation should target liquidity rather than capital under fire sale externalities.[2] Unlike these papers, this paper endogenizes both the size and composition of a bank balance sheet using a general equilibrium framework and studies welfare implications of multiple prudential tools.

In a dynamic general equilibrium framework, De Nicolò et al. (2012), Covas and Driscoll (2014), Van den Heuvel (2016) and Boissay and Collard (2016) study capital and liquidity regulations. This paper's model is static but features endogenous financial crises, which allows us to study a link between a crisis probability and bank fundamentals, especially capital/leverage and liquidity. In a static setting but without endogenous bank runs, Goodhart et al. (2012a, 2012b) also study the role of multiple tools.

This paper shares similar policy implications with Kara and Ozsoy (2016). They study a model with fire sale externalities and analytically show that both capital and liquidity requirements are essential to achieve constrained efficiency. With only one tool imposed on one risk area, risk migrates to and increases in other areas. Focusing on similar externalities, Walther (2015) also studies the role of capital and liquidity requirements.

This paper can be placed in the huge literature on capital requirements and the emerging literature on liquidity requirements. Recent surveys on the literature on capital requirements include Rochet (2014), Martynova (2015), and Dagher et al. (2016). Regarding liquidity requirements, as put by Dewatripont and Tirole (2018), 'our theoretical understanding of how liquidity should be regulated is scant.' As sources that cause insufficient liquidity in an unregulated economy, this paper focuses on bank risk shifting, while others, for example, Allen and Gale (2004) and Dewatripong and Tirole (2018) consider market imperfection and a time inconsistency problem of a government's liquidity support, respectively. But, 'there is no wide agreement on the rationale for liquidity regulation' (Allen and Gale, 2017). For this early stage literature, see those cited in Diamond and Kshayp (2016) and Allen and Gale (2017).

Broadly this paper can be seen as an initial step toward the recent developments in macroeconomic models that incorporate bank runs: Ennis and Keister (2003) and Martin et al. (2014), both of which build on the idea of Diamond and Dybvig (1983); Gertler and

[2]Konig (2015) and Morris and Shin (2016) focus on the role of liquidity and liquidity tools only in a global game framework. Bebchuk and Goldstein (2011) study alternative government responses to an endogenous credit market freeze similar to a bank run considered in this paper.

Kiyotaki (2015) and Gertler et al. (2016, 2017), all of which study runs as self-fulfilling rollover crises, following Calvo (1988) and Cole and Kehoe (2011); Angeloni and Faia (2013) who extend Diamond and Rajan (2000, 2001) bank run models.

The rest of the paper is organized as follows. Section 2 presents a benchmark model in which banks choose leverage only. Section 3 conducts welfare analysis and clarifies the source of inefficiencies. Section 4 extends the model to incorporate bank liquidity and studies the role of and the interaction of leverage and liquidity requirements. Section 5 presents further extensions of the benchmark model to study bank/sector specific capital requirements, risk weights, concentration risk, deposit insurance, and shadow banks. Section 6 concludes by summarizing the paper's theoretical predictions for bank behavior.

2 Model with Leverage

In this section, I present a model in which banks choose leverage only. This is the simplest general equilibrium model that features bank runs in a global game framework in this paper. It serves as a benchmark model for extensions that incorporate liquidity and others, to be studied in Section 4 and 5, respectively. In the following I first describe the environment of the model and the behavior of agents. Then I define an equilibrium and conduct a comparative statics analysis. The derivation of non-trivial equations and the proof of all propositions are provided in the appendix.

2.1 Environment

The model has two periods, $t = 1, 2$. There is one type of goods, which can be used for consumption or investment. The economy is inhabited by three types of agents: households, banks, and fund managers. There are a continuum of households with measure unity, but for simplicity a representative household and a representative bank are considered. The bank is owned by the household.

In period $t = 1$, the household and the bank receive an endowment, y and n units of the goods, respectively.[3] The household consumes and saves in the bank for next period consumption. The bank offers a deposit contract to the household and then decides the

[3]The model can allow for bank heterogeneity with respect to endowment n, but such a heterogeneous bank model is essentially equivalent to the model with a representative bank because of linearity with respect to n as will be shown later.

amount of deposits taken in from the households. The bank invests the sum of bank capital n and deposits in a risky project with a stochastic gross return R^k, which follows a normal distribution:[4]

$$R^k \sim N\left(\mu, \sigma_k^2\right).$$

It is assumed that the mean return of the risky project is high enough to satisfy $\mu > R$, where R is the deposit interest rate, so that the bank always takes in deposits and invests in a risky project. Although there is no firm, this model is equivalent to that with firms with such a linear technology and with no frictions between the bank and firms. Hence, the bank's investment in a risk project should be interpreted as financial intermediation from the household to firms.

Fund managers, as delegates of the household, decide whether to withdraw funds early or not. The bank defaults if it cannot pay the deposit interest rate. In period $t = 2$, the bank pays the interest rate if it can, and transfers its profits to the household, who consumes everything at hand.

2.2 Household

There are a continuum of households with measure unity. A representative household has preferences, given by the following quasi-linear utility,

$$u(c_1) + \mathbb{E}(c_2),$$

where c_t is consumption in period t, $\mathbb{E}(\cdot)$ is an expectation operator, and $u(\cdot)$ is a strictly increasing, strictly concave and twice differentiable function and satisfies $\lim_{c_1 \to 0} u'(c_1) = \infty$. In period $t = 1$ the household consumes c_1 and makes a bank deposit of d, subject to the flow budget constraint, $c_1 + d \leq y$. The interest rate paid by the bank is given by vR, where R is the promised non-state-contingent interest rate – the deposit interest rate – and v is the recovery rate which takes 1 if the bank does not default and $v < 1$ if it defaults. The recovery rate v is given by the ratio of the bank's liquidation value to the debt obligation value of Rd. The household is assumed to delegate the management of deposits to fund managers who have information advantages. Fund managers can withdraw funds early at a

[4] This assumption is made following the literature. e.g. Rochet and Vives (2004) and Vives (2014). Under the assumption the gross return R^k can fall below zero in theory, but such a probability is essentially zero for practical parameter values. For example, the probability of R^k falling below zero is smaller than 1e-300 percent for parameter values set in Section 4.3 in this paper.

right timing as will be explained in Section 2.3. The household diversifies the management of their funds in the bank over a continuum of fund managers, so that the realization of v is independent of fund manager heterogeneity.[5] In period $t = 2$, the household consumes c_2, subject to $c_2 \leq vRd + \pi$, where π is bank profits. Both R and v are endogenously determined.

Given a bank default probability P – financial crisis risk – and a recovery rate v, solving the household's problem yields the upward-sloping supply curve of deposits:

$$R = \frac{u'(y-d)}{1 - P + \mathbb{E}(v|\text{default})P}, \tag{1}$$

where $\mathbb{E}(\cdot|\text{default})$ is an expectation operator conditional on the default of the bank. The household is willing to supply funds d for any combination of the interest rate R, the crisis probability P and the recovery rate v that satisfies equation (1).

2.3 Fund managers

There is a continuum of fund managers with measure unity. Fund managers have information advantages over the household about a stochastic bank asset return on a risky project, R^k. In the beginning of period $t = 2$, just after R^k is realized, but before it is known, fund manager $i \in (0, 1)$ receives a private noisy signal s_i about R^k, which follows a normal distribution:

$$s_i = R^k + \epsilon_i, \quad \text{with} \quad \epsilon_i \sim N\left(0, \sigma_\epsilon^2\right).$$

The standard deviation σ_ϵ captures the degree of the noise of the information and the distribution is public information.

A role of fund managers is to make a binary decision of withdrawing funds early or not. If a fund manager withdraws early and a bank is solvent at this stage, the fund manager secures R per unit of funds and the household receives R per unit of deposits. But if a fund manager does not withdraw and the bank defaults later, the household receives an interest rate strictly less than R. Only fund managers can provide this professional service of early withdrawals with a right timing.

Following Rochet and Vives (2004) and Vives (2014), I assume that fund managers and households adopt this behavioral rule: fund manager i withdraws early if and only if the

[5] This modeling is equivalent to that in which there are a continuum of households with measure unity; each household delegates the management of deposits to each fund manager; and households insure the risk of delegating to a heterogeneous fund manager by trading contingent claims.

perceived probability of bank default, P_i, exceeds some threshold $\gamma \in (0, 1)$:

$$P_i > \gamma. \tag{2}$$

This rule can be derived, for example, from the following payoff matrix for fund managers:

Action \ States	No bank default	Bank default
Not withdraw	R	$vR - \Gamma_1$
Withdraw	$R - \Gamma_0$	$vR - \Gamma_0$

In the case of no bank default, a fund manager secures R per unit of funds, but early withdrawal entails the cancellation cost of $\Gamma_0 > 0$. In the case of bank default, a fund manager eventually receives vR – the recovery rate times the promised interest rate, but no withdrawal entails the higher cost of $\Gamma_1 > \Gamma_0$. Even though all the creditors receive vR in the end, those who have not withdrawn early cannot secure vR right away in the event of bankruptcy, which is costly for households due to potential liquidity needs. The payoff matrix implies that fund manager i, who maximizes the expected payoff using the perceived bank default probability P_i, chooses to withdraw early if and only if $P_i > \Gamma_0/\Gamma_1$. In this general equilibrium model, the behavioral rule (2) can be considered as the limiting case, $\lim_{\Gamma_0,\Gamma_1 \to 0} \Gamma_0/\Gamma_1 = \gamma$, where the costs Γ_0 and Γ_1 are infinitesimally small.[6]

As shown by Rochet and Vives (2004), in this environment fund managers employ a threshold strategy such that they withdraw if and only if $s_i < \bar{s}$. The threshold $\bar{s}$ is determined jointly with the bank's problem described below.

The behavioral rule (2) is surely the source of inefficiencies that leads to a coordination failure, i.e. bank runs. But in this paper, I regard it as an inevitable nature of the financial system, and focus on the resilience of the banking system that is vulnerable to runs.

2.4 Bank

Deposit contract and risk shifting: A key friction is that bank choices such as leverage are unobservable to households. This friction restricts a contract between a representative

[6] Rochet and Vives (2004) note that the cancellation cost Γ_0 can be related to reputation for example because fund managers' reputation suffers if they have to recognize that they have made a bad investment. Vives (2014) notes that a larger γ is associated to a less conservative investor and thereby risk management may influence parameter γ.

bank and households to a simple deposit contract with interest rate R. In other words, the interest rate cannot be contingent on bank choices.[7]

A representative bank decides the amount of funds, k, invested in a risky project, or equivalently the leverage $L \equiv k/n$. In this simple model, the leverage decision is also equivalent to how much deposits the bank takes in from the household. But, because each household is small relative to the bank, households cannot observe bank-level deposits. In equilibrium, the bank asset k is equated to the bank liability $n + d$, i.e. $Ln = n + d$. The bank is protected by limited liability.

The setup of a simple deposit contract and the inability of households to observe bank choices is similar to that of Jensen and Meckling (1976) about the incentive effects associated with debt. Specifically, a borrower chooses an investment project between two, a good and a bad, in Jensen and Meckling, but in this model a borrower bank chooses leverage, which determines its riskiness – a bank-run-led default probability – as will be shown below. Similar in spirit to Jensen and Meckling, this setup will lead to bank risk shifting, which distorts the bank's choice of leverage and the crisis probability in equilibrium.

Costly liquidation and bank run: In the beginning of period $t = 2$, the gross return R^k is realized, but before it is observed, some fund managers may withdraw their funds from the bank. In response, the bank has to sell some assets. This early liquidation is costly: early liquidation of one unit of bank asset generates only a fraction $1/(1 + \lambda)$ of R^k, where $\lambda > 0$. The underlying assumption is that in response to early withdrawal requests the bank raises funds by selling illiquid assets, which have been invested in the risky project, to the household who have a linear but inferior technology than the bank. The technology transforms one unit of bank assets into $1/(1 + \lambda)R^k$ units of the goods. With perfect competition and no friction between the household and the bank, the fire sale price of the early liquidated asset is $1/(1 + \lambda)R^k$, where λ captures the degree of the discounting of the fire sale, or put simply the cost of early liquidation.

Let x denote the number of fund managers who withdraw funds. Then, to cover the early withdrawal of xRd, the bank has to liquidate $(1 + \lambda)xRd/R^k$ units of bank assets.[8] After the liquidation, the bank has $R^k(n + d) - (1 + \lambda)xRd$ in hand. If this amount is less than the promised payment for not-yet-withdrawn deposits, $(1 - x)Rd$, the bank goes

[7] Similar to this paper, Acharya (2009) and Mendicino et al. (forthcoming) consider a bank problem by imposing the assumption that individual bank default risk is not priced at the margin but on the basis of depositors' expectations about potential losses from bank default.

[8] To see this, let z denote the quantity of bank assets to be liquidated. Then, z should satisfy $1/(1 + \lambda)R^k z = xRd$, which leads to $z = (1 + \lambda)xRd/R^k$.

bankrupt. That is, the bank defaults if and only if

$$R^k < R\left(1-\frac{1}{L}\right)(1+\lambda x), \tag{3}$$

where $L = k/n$ is bank leverage.

Under the fund managers' withdrawal strategy that fund manager i withdraws if and only if $s_i < \bar{s}$, the number of fund managers who withdraw is given by $x(R^k, \bar{s}) = \Pr(s_i < \bar{s}) = \Pr(\epsilon_i < \bar{s} - R^k) = \Phi((\bar{s} - R^k)/\sigma_\epsilon)$, where $\Phi(\cdot)$ is a standard normal distribution function. Condition (3) implies that the probability of bank default perceived by fund manager i is given by

$$P_i = \Pr\left(R^k < R\left(1-\frac{1}{L}\right)\left[1+\lambda x(R^k, \bar{s})\right] |s_i\right). \tag{4}$$

Conditions (2), (3) and (4) imply that the equilibrium threshold $\bar{s}^*$ is a solution to the following set of equations:

$$\Pr\left(R^k < R^{k*} | \bar{s}^*\right) = \gamma, \tag{5}$$

$$R^{k*} = R\left(1-\frac{1}{L}\right)\left[1+\lambda x(R^{k*}, \bar{s}^*)\right]. \tag{6}$$

These two equations have a unique solution for $\bar{s}^*$ and R^{k*} if the standard deviation of the signal σ_ϵ is small relative to that of bank asset return σ_k as shown by Rochet and Vives (2004). From now on this condition is imposed on the model.

Both the thresholds $\bar{s}^*$ and R^{k*} depend on the interest rate R and leverage L. In particular, an increase in leverage raises $\bar{s}^*$ and R^{k*} so that more fund managers withdraw funds and the probability of bank default increases.

The bank's problem: Taking the interest rate R as given, the bank chooses leverage L to maximize the expected profits $\mathbb{E}(\pi)$:

$$\max_{\{L\}} \int_{R^{k*}(L)}^{\infty} \left\{R^k L - R\left[1+\lambda x\left(R^k, \bar{s}^*(L)\right)\right](L-1)\right\} n dF(R^k), \tag{7}$$

where $F(\cdot)$ is a normal distribution function with mean μ and standard deviation σ_k, and $\bar{s}^*(L)$ and $R^{k*}(L)$ are solutions for $\bar{s}^*$ and R^{k*} as a function of L, respectively. In this problem the bank is subject to a technical restriction such that leverage should not be too high: $L \leq L_{\max}$. This restriction differs from a prudential tool introduced later. With a high enough $L_{\max}$, the restriction is not binding in equilibrium, but it plays a role of excluding an uninteresting profitable deviation of $L = \infty$ as will be discussed shortly. One interpretation of this restriction is a physical limit $L_{\max} = (y-1)/n$ at which the household lends all funds to the bank.

Given that the technical restriction $L \leq L_{\max}$ is non-binding, the first-order condition of the bank's problem is:

$$0 = \int_{R^{k*}}^{\infty} (R^k - R) dF(R^k) - R\lambda (L-1) \int_{R^{k*}}^{\infty} \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} dF\left(R^k\right) - R\lambda \int_{R^{k*}}^{\infty} x dF\left(R^k\right). \quad (8)$$

The first term of the right-hand-side of equation (8) is the expected marginal return by increasing leverage and the remaining two terms comprise the expected marginal costs. The second term is the expected marginal liquidation cost. An increase in L raises threshold $\bar{s}^*$ and increases the number of fund managers who withdraw, which leads to an increase in the liquidation cost. The third term is the expected liquidation cost. Equation (8), with the assumption of $\mu > R$, implies that $\partial \mathbb{E}(\pi)/\partial L|_{L=1} > 0$, so that a unique solution to (8) satisfies the second-order condition as well.[9]

Is there no profitable deviation from the solution to (8)? This is where the technical restriction, $L \leq L_{\max}$, comes into play. For the sake of exposition and analytical tractability, consider a limit case in which the fund managers' noisy signal becomes perfectly accurate, i.e. $\sigma_\epsilon \to 0$. In this case, the thresholds are given by:

$$\bar{s}^* = R^{k*} = R\left(1 - \frac{1}{L}\right)\left[1 + \lambda(1-\gamma)\right], \quad (9)$$

and the optimality condition of the bank's problem (8) is reduced to:

$$0 = \int_{R^{k*}}^{\infty} (R^k - R) dF(R^k) - \lambda(1-\gamma) f\left(R^{k*}\right)\left[1 + \lambda(1-\gamma)\right] R^2 \frac{L-1}{L^2}. \quad (10)$$

Equation (9) implies that $\lim_{L\to\infty} R^{k*} = R\left[1 + \lambda(1-\gamma)\right]$, so that even with infinite leverage the default probability is strictly less than unity: $\lim_{L\to\infty} F\left(R^{k*}\right) < 1$. This observation and condition (10) suggest $\partial \mathbb{E}(\pi)/\partial L > 0$ for a large value of L. Were it not for $L \leq L_{\max}$, there would be a profitable deviation by choosing $L = \infty$. This issue has to do with the

[9] At $L = 1$, $R^{k*} = 0$ and there is essentially no bank run. Hence, given that the probability of the gross return R^k falling below zero is essentially zero, the final term in the right-hand-side of equation (8) is essentially zero.

fact that the domain of the distribution for R^k is unbounded above. Should it exist the upper bound, for example, as in a uniform distribution, there would be no need for such a technical restriction.[10]

Bank default probability: The bank leverage and the interest rate determine the bank default probability P and the recovery rate v. Specifically, the bank default probability is given by:

$$P = \Pr(R^k < R^{k*}) = F\left(R^{k*}\right), \tag{11}$$

where R^{k*} is a solution to equations (5) and (6). If the bank defaults, the bank is liquidated and its value is distributed among creditors. Consequently, the recovery rate v is given by:

$$v = \min\left\{1, \max\left\{\frac{R^k}{R}\frac{L}{L-1} - \lambda x(R^k, \bar{s}^*), \frac{1}{1+\lambda}\frac{R^k}{R}\frac{L}{L-1}\right\}\right\}. \tag{12}$$

If the bank has survived, the recovery rate is 1. If it has defaulted but has not sold all the assets, the recovery rate is given by the first term in the max operator in (12). The second term in the max operator corresponds to the recovery rate when the bank has sold all the assets and has defaulted. The bank sells all the assets if the return on bank assets is lower than $\underline{R}^k$, which is defined by:

$$\underline{R}^k = R\left(1 - \frac{1}{L}\right)(1+\lambda)x(\underline{R}^k, \bar{s}^*).$$

The threshold $\underline{R}^k$ is clearly lower than the default threshold R^{k*}.

2.5 Equilibrium

A competitive equilibrium for this economy consists of the interest rate R and leverage L that satisfy the supply curve for funds (1), the demand curve for funds (8) and the market clearing condition, $Ln = n + d$, where R^{k*}, $\bar{s}^*$, P and v in these curves are given by (5), (6), (11) and (12), respectively. With the solution of R and L at hand, household consumption series c_1 and c_2 are obtained from the household budget constraints.

A unique feature of the bank's problem that leads to the demand curve (8) is that bank leverage L is uniquely determined as an interior solution without any financial frictions

[10] A uniform distribution has also a lower bound, which implies that a crisis probability can fall to zero if leverage is sufficiently low. But, with a normal distribution the crisis probability is always positive, which is deemed to be a case in practice. This is a main reason why this paper considers a normal distribution.

that directly constrain leverage. Many papers have such frictions, which include banks' moral hazard of defaulting i.e. running away with borrowings (Gertler and Kiyotaki, 2015, Jermann and Quadrini, 2012), banks' hidden effort as moral hazard (Christiano and Ikeda, 2016), asymmetric information and costly state verification (Bernanke et al., 1999), and limited pledgeability (Kiyotaki and Moore, 1997). In this model, however, it is bank run risk and the resulting market discipline that help pin down bank leverage. This effect is captured by the second and third terms of the right-hand-side of equation (8) (equation (10) in the limit case). Too high leverage makes the bank's liability vulnerable to runs, increases the default probability, raises expected liquidation costs and lowers profits. Because of this effect the bank refrains from choosing too high leverage and as a result bank leverage has an interior solution.

2.6 Comparative Statics

The competitive equilibrium for this economy depends on parameters such as μ, γ, λ, y and n. The following proposition summarizes how the supply curve (1) and the demand curve (8) shift in response to changes in these parameter values.

Proposition 1 (Comparative statics)**.** *Consider the credit market described by the supply curve (1) and the demand curve (8). Consider a limit case where $\sigma_\epsilon \to 0$. Assume that bank default probability is not too high, $P \leq 0.5$, and the leverage is not too low, $L > 5/3 > \left(1 - \frac{0.4}{1+\lambda(1-\gamma)}\right)^{-1}$. Then, the following results hold.*

(i) An increase in the mean return of bank assets μ shifts the demand curve outward.

(ii) An increase in the liquidation cost λ (or a decrease in the threshold probability γ) shifts the demand curve inward.

(iii) An increase in the household endowment y shifts the supply curve outward.

(iv) An increase in the bank capital n shifts the supply curve inward.

The comparative statics analysis supports a view that credit booms are associated with vulnerability to financial crises (Schularick and Taylor, 2012). In the model a typical credit boom would feature increases in the mean return of bank assets μ, the household endowment y and the bank capital n. On the demand side, the demand curve for funds shifts outward as a result of an increase in the bank asset return. In addition, possible decreases in the liquidation costs λ and the threshold probability by fund managers γ would shift the demand curve outward further. This pushes up both leverage and the interest rate. On the supply side, if the effect of y dominates the effect of n, the supply curve shift outward as well. This puts upward pressure on leverage but also downward pressure on the interest rate. In total, these developments lead to an increase in leverage, and if the interest rate does not fall as the demand effect dominates the supply effect, the crisis probability increases surely. A credit boom builds up financial system vulnerability that triggers a bank-run-led crisis.

3 Welfare Analysis

In this section, I conduct a welfare analysis of the benchmark model presented in the previous section. The analysis shows that leverage is excessive in a competitive equilibrium relative to that chosen by a benevolent regulator, so that restraining leverage can improve welfare. The source of the inefficiencies is bank risk shifting and pecuniary externalities.

3.1 Regulator's Problem

What is an optimal allocation for this economy? The first best should involve no bank run. But, in this paper, a bank run is regarded as an inevitable feature of the financial system. Hence, I take a regulator perspective and set up a benevolent regulator's problem in which the regulator chooses leverage L to maximize social welfare subject to bank run risk and the supply curve for funds (1). In other words, in place of the bank the regulator chooses leverage, but unlike the bank the regulator maximizes social welfare and takes into account the general equilibrium effect of the choice of leverage on the interest rate. The social welfare, SW, is given by the expected households' utility, $SW = u(c_1) + \mathbb{E}(c_2)$, because the bank is owned by the household.

The regulator's problem is explicitly written as:

$$\begin{aligned} \max_{\{L\}} \;\; & u\left(y - (L-1)\,n\right) + \mathbb{E}(R^k)Ln \\ & -\lambda \left\{ \int_{\underline{R}^k}^{\infty} \left[x(R^k, \bar{s}^*(L, R(L)))\right] R(L)(L-1)ndF(R^k) + \int_{-\infty}^{\underline{R}^k} \frac{R^k L}{1+\lambda} dF(R^k) \right\} n, \end{aligned} \tag{13}$$

where $R(L)$ is implicitly defined by the supply curve (1) and threshold $\bar{s}^*$ is written as a function of R as well as L to take into account the effect of R on the threshold. The regulator balances the expected benefit of financial intermediation, which is given by the first row of the regulator's objective (13), and the expected loss due to the costly liquidation of bank assets, which is given by the second row of the regulator's objective (13). The loss is governed by the parameter, $\lambda > 0$, that captures the cost of early liquidation.

The first-order condition of the regulator's problem is given by:

$$
\begin{aligned}
0 = & -R\left[1 - P + \mathbb{E}(v|\text{default})P\right] + \mathbb{E}(R^k) - \lambda R \int_{\underline{R}^k}^{\infty} x dF - \lambda \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF \\
& -\lambda R(L-1) R \int_{\underline{R}^k}^{\infty} \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} dF - \lambda (L-1) \int_{\underline{R}^k}^{\infty} \left(\frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} R + x \right) \frac{\partial R}{\partial L} dF,
\end{aligned} \tag{14}
$$

where the supply curve (1) was used to substitute out for $u'(y-(L-1)n)$. Condition (14) distinguishes itself from the bank's optimality condition (8) in two respects. First, the regulator internalizes the impact of leverage L on the interest rate R, which is captured by the final term in (14), while the bank does not as it takes R as given. Second, while the regulator accounts for all possible states including bank run states, the bank focuses only on non-default states due to limited liability. However, as will be shown later, limited liability by itself is not the source of inefficiencies.

3.2 Roles of Leverage Restrictions

Now we are in a position to study whether leverage is excessive in the competitive equilibrium. If the slope of the social welfare evaluated at the competitive equilibrium allocation, $\partial SW/\partial L|_{\text{CE}}$, is negative, the leverage is excessive so that restricting it can improve welfare. Because the competitive equilibrium solves the bank's optimal condition, it has to be $\partial \mathbb{E}(\pi)/\partial L|_{\text{CE}} = 0$. Then, $\partial SW/\partial L|_{\text{CE}}$ is written and expanded as:

$$
\begin{aligned}
\left.\frac{\partial SW}{\partial L}\right|_{\text{CE}} = & \left.\frac{\partial SW}{\partial L}\right|_{\text{CE}} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{\text{CE}} \\
= & -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF\right] \\
& - \lambda(L-1)\left[\int_{\underline{R}^k}^{R^{k*}} R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\infty} \left(R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right) \frac{\partial R}{\partial L} dF \right],
\end{aligned} \tag{15}
$$

where $\partial x/\partial \bar{s}^* = \Phi'((\bar{s}^* - R^k)/\sigma_\epsilon)(1/\sigma_\epsilon) > 0$.[11] The first term on the right-hand-side of (15) is negative under the assumption that the probability of the gross return R^k falling below zero is essentially zero. As shown in the appendix, an increase in leverage L raises the threshold $\bar{s}^*$ and an increase in the interest rate R raises the threshold: $\partial \bar{s}^*/\partial L > 0$ and $\partial \bar{s}^*/\partial R > 0$. In addition, again as derived in the appendix, under a mild condition the supply curve (1) is upward-sloping, i.e. $\partial R/\partial L > 0$. In this case, equation (15) implies $\partial SW/\partial L|_{\mathrm{CE}} < 0$, which leads to the following proposition.

Proposition 2 (Excessive leverage)**.** *Assume that the supply curve (1) is upward sloping. Then, in a competitive equilibrium, bank leverage is excessive from a social welfare viewpoint. Restricting leverage can improve social welfare.*

A corollary of Proposition 2 is that the crisis probability is too high in a competitive equilibrium. The excessive leverage implies the high threshold R^{k*}, which in turn implies the elevated crisis probability.

The excessive leverage and the resulting elevated crisis probability in the competitive equilibrium provides a rational for policymakers to introduce a prudential tool to improve social welfare. The second best allocation, which solves the benevolent regulator's problem, can be achieved, for example, by imposing a leverage restriction $\bar{L}$ on the bank such that $L \leq \bar{L} = L^*$, where L^* is a solution to the regulator's problem, (14). Similarly, the second best can be achieved by imposing a restriction on a capital ratio, $n/(n+d)$, such that it is no less than $1/L^*$.

3.3 Source of Inefficiencies

To disentangle the source of inefficiencies, I consider the same problem but without bank risk shifting. In this economy, the household's optimality condition (1) stays the same, but what changes is the bank's behavior. Now the bank's choices are observable to households, so that the deposit interest rate can be contingent on bank leverage. Specifically, the bank now chooses a pair of leverage and the interest rate $\{L, R\}$ simultaneously to maximize the same expected profits (7) subject to the technical constraint $L \leq L_{\max}$ and the household's participation constraint

$$R[1 - P + \mathbb{E}(v|\text{default})P] \geq R^e, \tag{16}$$

[11] Bank capital n is abstracted away from the condition because it is proportional to n. The same applies from now on in calculating the slope of the social welfare.

where R^e is the expected rate of return on deposits.[12] The participation constraint (16) can be explicitly written as a function of L as:

$$R[1 - F(R^{k*})] + \int_{\underline{R}^k}^{R^{k*}} \left[R^k \frac{L}{L-1} - R\lambda x(R^k, \bar{s}^*) \right] dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} \frac{L}{L-1} dF \geq R^e, \quad (17)$$

where R^{k*}, $\bar{s}^*$ and $\underline{R}^k$ are all a function of L and R. The left-hand-side of (17) corresponds to the expected return received by households, $R[1-P+\mathbb{E}(v|\text{default})]$. As long as condition (17) holds, which promises the expected rate of return on deposits R^e, the household is willing to supply funds irrespective of a pair of L and R. In equilibrium, the constraint (17) is binding and $R^e = u'(y - (L-1)n)$.

The binding constraint (17) disciplines the bank's behavior. It implicitly defines R as a function of L, which is denoted as $R = R_B(L)$, where $\partial R_B / \partial L > 0$.[13] Hence, a higher leverage causes a higher interest rate and thus a greater cost of deposits and a higher crisis probability. Because of this negative effect of leverage, the bank chooses lower leverage than that of the benchmark model with bank risk shifting. The optimality condition of the bank's problem is relegated to the appendix.

Leverage is still excessive in a competitive equilibrium even in the economy without bank risk shifting, but the degree of excessiveness is mitigated. Let CE' denote a competitive equilibrium in such an economy. The slope of the social welfare evaluated at the competitive equilibrium is given by

$$\left. \frac{\partial SW}{\partial L} \right|_{\mathrm{CE}'} = \lambda(L-1) \left[\int_{\underline{R}^k}^{\infty} \left(\frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} R + x \right) dF \right] u''(c_1) \in \left(\left. \frac{\partial SW}{\partial L} \frac{1}{n} \right|_{\mathrm{CE}}, 0 \right). \quad (18)$$

This equation shows that the only source of inefficiencies is the pecuniary externalities that work through the expected rate of return on deposits R^e, which is captured by the second derivative of the utility function, $u''(c_1)$. This result is summarized in the following proposition.

[12] In the household problem in Section 2.2, there is no expected rate of return on deposits R^e. But, if there is a competitive mutual fund that collects deposits from the household and invests all the funds to the bank, the rate of return R^e can be regarded as the expected interest rate paid to the household from a competitive mutual fund. In addition, the expected interest rate can be regarded as the risk-free interest rate. In that case, the mutual fund trades a contingent security with the household to insure the bank default risk.

[13] A condition for $\partial R_B / \partial L > 0$ is the same as that for $\partial R / \partial L > 0$, which is assumed to hold. A relationship between $\partial R / \partial L$ and $\partial R_B / \partial L$ is such that $\partial R/\partial L - \partial R_B/\partial L \propto -u''(c_1) > 0$. Hence, the slope of $R(L)$ is sleeper than that of $R_B(L)$.

Proposition 3 (Excessive leverage in the model without bank risk shifting)**.** *Consider the benchmark model without bank risk shifting in which the supply curve (1) is upward sloping. In a competitive equilibrium, bank leverage is excessive because only of the pecuniary externalities that work through the expected rate of return on deposits.*

Propositions 2 and 3 reveal that the source of inefficiencies in the benchmark model is twofold: bank risk shifting and pecuniary externalities. Regarding bank risk shifting, in the benchmark model, the representative bank competes for attracting deposits by using the deposit interest rate only. Even if the bank attempts to become safer by promising to choose a lower leverage and offers a lower deposit interest rate reflecting its lower riskiness, depositors would not supply funds to such a bank. This is because they rationally expect that the bank will renege on the promise. Hence, such an attempt by the bank cannot be a profitable deviation from the equilibrium. In contrast, in the model without bank risk shifting, the bank can commit to being safe by choosing low leverage. In this case, the bank takes into account the effect of leverage on the deposit interest rate, and consequently the leverage becomes lower than in the benchmark model. Regarding pecuniary externalities, an increase in bank leverage raises the expected rate of return on deposits R^e through a general equilibrium effect and increases deposit interest rate R and the costs associated with bank asset fire sales, $\lambda Rx(L-1)$. This effect is ignored by the bank which takes the expected rate of return on deposits as given in a competitive equilibrium.

4 Leverage and Liquidity

In this section, I extend the benchmark model presented in Section 2 to incorporate liquid assets in a bank balance sheet. This section first presents the extended model and shows analytical results on a competitive equilibrium, social welfare, the source of inefficiencies and prudential tools on leverage and liquidity. It then proceeds to numerical analyses on the role of and the interaction between the two tools regarding social welfare and the crisis probability.

4.1 Model with Leverage and Liquidity

In this model, a bank balance sheet consists of safe liquidity as well as risky lending. Specifically, a representative bank has access to a safe technology with gross return unity.

Assets invested in a safe technology are called liquidity, which can be drawn at any time without any cost.

In period $t = 1$, after offering a deposit contract to households, the bank determines its leverage L and allocates the assets $k = Ln$ to liquidity M and lending $k - M$. In response to fund managers' early withdrawal claim of xRd, the bank uses liquidity first because doing so is not costly.[14] And then the bank sells the assets invested in a risky project to a household at a fire sale price if the amount of liquidity is not enough to cover the amount of the claim: $xRd > M$. In this case, the bank sells $(1 + \lambda)(xRd - M)/R^k$ units of the bank lending. If the bank's revenue, $R^k(n + d - M) - (1 + \lambda)(xRd - M)$, cannot cover the promised payment for deposits that have not been withdrawn, $(1 - x)Rd$, the bank defaults. Instead, if the bank can cover the early withdrawal request by using liquidity, i.e. $xRd < M$, it does not liquidate any risky assets and it is subject to only a fundamental default. Hence, the bank defaults if and only if

$$R^k < \frac{R - m}{\frac{L}{L-1} - m}\left(1 + \lambda \frac{\max\{xR - m, 0\}}{R - m}\right), \tag{19}$$

where $m \equiv M/(k - n)$ is a liquidity-deposit ratio (a liquidity ratio or liquidity for short from now on) and $L \equiv k/n$ is leverage. This condition is reduced to condition (3) in the case of bank leverage only, i.e. $m = 0$. Condition (19) implies that thresholds $\bar{s}^*$ and R^{k*} are determined by equation (5) and

$$R^{k*} = \frac{R - m}{\frac{L}{L-1} - m}\left(1 + \lambda \frac{x(R^{k*}, \bar{s}^*)R - m}{R - m}\right), \tag{20}$$

where $x(R^{k*}, \bar{s}^*) = \Phi((\bar{s}^* - R^{k*})/\sigma_\epsilon)$. At the thresholds of R^{k*} and $\bar{s}^*$, the amount of early withdrawals exceeds the bank liquidity, i.e. $x(R^{k*}, \bar{s}^*)R - m > 0$. Otherwise, the bank would not default for R^k close to but smaller than R^{k*}. Equation (20) is the extension of equation (6) to incorporate a bank liquidity choice.

Taking as given the interest rate R, the bank maximizes the expected profits $\mathbb{E}(\pi)$ by

[14] In this model, bank liquidity holdings, required by regulators or not, are usable liquidity and hence the model abstracts away from Goodhart (2008)'s concern on liquidity usability.

choosing leverage and liquidity,

$$\max_{\{L,m\}} \int_{R^{k*}(L,m)}^{\infty} \Big\{ R^k L - (R^k - 1)(L-1)m$$
$$- \left[R + \lambda \max\left\{x\left(R^k, \bar{s}^*(L,m)\right) R - m, 0\right\}\right](L-1) \Big\} n dF(R^k),$$

subject to $L \leq L_{\max}$ and $0 \leq m \leq L/(L-1)$, where the thresholds $\bar{s}^*(L,m)$ and $R^{k*}(L,m)$ are a solution to equations (5) and (20), written as a function of L and m. A marginal increase in the liquidity ratio m is associated with the opportunity cost of $(R^k - 1)(L-1)$, but it reduces the likelihood of fire sales and its cost $\lambda \max\{xR - m, 0\}(L-1)$. High enough liquidity, e.g. $m = L/(L-1)$, insulates the bank from bank runs and makes it perfectly bank-run-proof, but $R > 1$ is assumed so that such a choice cannot be a solution to the problem.[15]

To solve the bank's problem, let $\bar{R}^k$ define a threshold for R^k such that bank liquidity just covers the amount of early withdrawal, i.e. $x(\bar{R}^k, \bar{s}^*)R = m$. Solving for $\bar{R}^k$ yields:

$$\bar{R}^k = \bar{s}^* - \sigma_\epsilon \Phi^{-1}\left(\frac{m}{R}\right).$$

Now the first-order conditions of the bank's problem, which characterize an interior solution for leverage L and liquidity m, are given by:

$$\begin{aligned} 0 = & \int_{R^{k*}}^{\infty} [R^k - (R^k - 1)m - R] dF(R^k) \\ & - \int_{R^{k*}}^{\bar{R}^k} \left[\lambda(Rx - m) + (L-1)\lambda R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right] dF(R^k), \end{aligned} \tag{21}$$

$$0 = -\int_{R^{k*}}^{\infty} (R^k - 1) dF(R^k) + \lambda \int_{R^{k*}}^{\bar{R}^k} \left(1 - R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m}\right) dF(R^k). \tag{22}$$

Equation (21) corresponds to $0 = \partial \mathbb{E}(\pi)/\partial L$, which is reduced to the optimal condition in the benchmark model (10) when $m = 0$. The marginal cost of leverage – the second term of the right-hand-side of Equation (21) – emerges only when liquidity cannot cover the amount of early withdrawals, i.e. when $R^k < \bar{R}^k$. Similar to the benchmark model,

[15] If leverage is too low, the gross interest rate can fall below unity, violating the assumption of $R > 1$. One way to address this problem is to assume that the gross return of liquidity is lower than unity. Another way is to assume that the gross return of liquidity depends on R and is given by $R - \xi$, where $\xi > 0$ is a liquidity premium. In this case the presence of R would become another source of pecuniary externalities. To make the model as simple as possible, I restrict my attention to a case in which $R > 1$.

the marginal impact of raising the threshold $\bar{s}^*$ on x, the number of fund managers who withdraw funds early, is positive and the marginal impact of leverage L on the threshold $\bar{s}^*$ is positive: $\partial x/\partial \bar{s}^* > 0$ and $\partial \bar{s}^*/\partial L > 0$.

Equation (22) corresponds to $0 = \partial \mathbb{E}(\pi)/\partial m$. The first term in the right-hand-side of equation (22) is the opportunity cost of holing liquidity, i.e. the net expected return on the risky project the bank would have earned if it had not held liquidity but invested in the risky project only. The second term in the right-hand-side of equation (22) is the marginal benefit of holding liquidity by lowering the number of fund managers who withdraw early, x. As shown in the appendix, an increase in liquidity lowers the threshold $\bar{s}^*$, i.e. $\partial \bar{s}^*/\partial m < 0$ if the interest rate is not high enough to satisfy:

$$R < \frac{1+\lambda}{1+\lambda x}\frac{L}{L-1}. \tag{23}$$

Under condition (23) an increase in liquidity m reduces the thresholds $\bar{s}^*$ and R^{k*} and lowers the bank run probability $F(R^{k*})$. Put differently, an increase in liquidity increases bank resilience to a bank-run-led financial crisis. If condition (23) is violated, the interest cost of bank liabilities is so high that a decrease in the expected revenue due to an increase in liquidity holdings makes the bank to be more vulnerable to runs, raising the threshold R^{k*} and the default probability $F(R^{k*})$. From now on condition (23) is assumed.

Given a unique solution to the first-order condition with respect to liquidity, (22), the bank's liquidity holding is positive if and only if $\partial \mathbb{E}(\pi)/\partial m|_{m=0} > 0$ i.e.

$$\int_{R^{k*}}^{\infty} \left[-(R^k - 1) + \lambda \left(1 - R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m} \right) \right] dF(R^k) > 0.$$

Hence, given a unique solution to (22), the sufficient condition for $m > 0$ is:

$$\mathbb{E}(R^k | \text{no default}) < 1 + \lambda. \tag{24}$$

That is, the bank holds liquidity if the expected return of the risky loan conditional on no default is not so high, satisfying condition (24). In other words, the bank holds low return safe assets when the opportunity cost of doing so is not high.

It is worth noting that the sufficient condition for positive liquidity (24) does not apply to a limit case in which $\sigma_\epsilon \to 0$. In this case $\bar{R}^k \to R^{k*}$ for $m > 0$ and as a result the second term of the right-hand-side of condition (22) vanishes. Hence, in this case, the bank does not hold liquidity. This is intuitive. In the limit case, it is either all fund managers withdraw or no one withdraws. Because a marginal increase in liquidity is not enough to prevent the bank from defaulting due to runs by all fund managers, it generates no marginal benefit. However, if there is a region of fire sales with no default, i.e. $\bar{R}^k - R^{k*} > 0$, as in the case of $\sigma_\epsilon > 0$, building additional liquidity yields the benefits of reducing the costs of fire sales. Hence, the noisy information, $\sigma_\epsilon > 0$, is essential for analyzing positive bank liquidity holdings in this model.

The supply side of funds – the household problem – is the same as in the benchmark model except for the recovery rate v. Assuming that the bank can satisfy early withdrawal requests, a fraction x of fund managers who withdraw early receive R per unit of deposit. When the bank defaults, a remaining fraction, $1 - x$, of fund managers divide the bank's return $[R^k(n + d - M) - \lambda(xRd - M)]$ equally and receive $[R^k(n + d - M) - \lambda(xRd - M)]/[(1 - x)d]$ per unit of deposit. Because the household diversifies over fund managers, the household receives a weighted sum of the returns when the bank defaults: $R^k(L/(L - 1) - m) + m - \lambda(Rx - m)$. This recovery rate assumes that the bank has not sold all the risky assets. The recovery rate when the bank has done so is given by $(R^k/(1+\lambda))(L/(L - 1) - m) + m$. Consequently, the recovery rate is given by:

$$v = \min\left\{1, \max\left\{R^k\left(\frac{L}{L-1} - m\right) + m - \lambda(Rx - m), \frac{R^k}{1+\lambda}\left(\frac{L}{L-1} - m\right) + m\right\}\right\}. \tag{25}$$

This expression also applies to a case when the bank defaults because it cannot satisfy the request of early withdrawals. The recovery rate is increasing in liquidity m when the bank does not sell all the assets if $R^k < 1 + \lambda$, which holds under the assumption of (24). As in the benchmark model presented in Section 2, it is useful to define a threshold $\underline{R}^k$ under which the bank sells all the risky assets:

$$\underline{R}^k = (1+\lambda)\frac{Rx(\underline{R}^k, \bar{s}^*) - m}{\frac{L}{L-1} - m}.$$

From equation (20) it is clear that $\underline{R}^k < R^{k*}$.

A competitive equilibrium for this economy consists of the interest rate R, leverage L and liquidity m that satisfy the supply curve for funds (1), the demand curve for funds (21), the optimality condition for liquidity (22) and the market clearing condition, $Ln = n + d$, where R^{k*}, $\bar{s}^*$, P and v in these equations are given by (5), (20), (11) and (25), respectively.

4.2 Roles of Liquidity and Leverage Requirements

Is liquidity insufficient in a competitive equilibrium from a social welfare viewpoint? Does leverage continue to be excessive in the extended model? To address these questions, as in Section 3, I set up a benevolent regulator's problem, where the regulator chooses leverage L and liquidity m to maximize social welfare:

$$\begin{aligned}\max_{\{L,m\}} \quad u(y-(L-1)n)+&\Bigg\{\int_{\bar{R}^k}^{\infty}\left[R^k L-(R^k-1)(L-1)m\right]dF\\&+\int_{\underline{R}^k}^{\bar{R}^k}\left[R^k L-(R^k-1)(L-1)m-\lambda(xR-m)(L-1)\right]dF\\&+\int_{-\infty}^{\underline{R}^k}\left[\frac{R^k}{1+\lambda}L-\left(\frac{R^k}{1+\lambda}-1\right)(L-1)m\right]\Bigg\}n,\end{aligned}$$

where $R=R(L,m)$ is given by the supply curve (1) and $\bar{s}^*=\bar{s}^*(L,m,R)$ is given by a solution to equations (5) and (20). The interest rate depends on liquidity in addition to leverage because the interest rate depends on the recovery rate, which is affected by liquidity.

The first-order condition of the regulator's problem with respect to liquidity yields:

$$\begin{aligned}0=&-\int_{\underline{R}^k}^{\infty}(R^k-1)dF-\int_{-\infty}^{\underline{R}^k}\left(\frac{R^k}{1+\lambda}-1\right)dF\\&+\lambda\int_{\underline{R}^k}^{\bar{R}^k}\left[1-R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial m}-\left(R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial R}+x\right)\frac{\partial R}{\partial m}\right]dF.\end{aligned}\tag{26}$$

In contrast to equation (22) that characterizes the bank's privately optimal choice of liquidity, equation (26) takes into account the effect of liquidity on the interest rate. As shown in the appendix, if condition (24) holds and the supply curve (1) is upward sloping, an increase in liquidity lowers the interest rate, $\partial R/\partial m<0$, by decreasing the default probability and increasing the recovery rate.[16]

The slope of the social welfare with respect to liquidity, evaluated at the level of liquidity

[16] This negative relationship is consistent with the empirical finding by Miller and Sowerbutts (2018) for the major US banks.

$m = m^*$ implied by the privately optimal choice (22), is given by:

$$\begin{aligned}
\left.\frac{\partial SW}{\partial m}\right|_{m=m^*} &= \left.\frac{\partial SW}{\partial m}\right|_{m=m^*} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial m}\right|_{m=m^*} \\
&= \int_{\underline{R}^k}^{R^{k*}} (1 - R^k) dF + \int_{-\infty}^{\underline{R}^k} \left(1 - \frac{R^k}{1+\lambda}\right) dF \\
&+ \lambda \int_{-\infty}^{R^{k*}} \left(1 - R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m}\right) dF - \lambda \int_{-\infty}^{\bar{R}^k} \left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right) \frac{\partial R}{\partial m} dF.
\end{aligned}$$

The slope of the social welfare evaluated at m^* consists of four terms. The sign of the last two terms is positive if the supply curve (1) is upward sloping and conditions (23) and (24) hold. Hence, if the first two terms are positive as well, $\partial SW/\partial m|_{m=m^*} > 0$ follows. This leads to the following proposition.

Proposition 4 (Insufficient liquidity)**.** *Assume that the supply curve (1) is upward sloping and conditions (23) and (24) hold. Assume further that the threshold R^{k*} is low enough to satisfy $\int_{-\infty}^{R^{k*}} (1 - R^k) dF > 0$. Then, for given leverage, the bank's liquid asset holdings are insufficient from a social welfare viewpoint: raising liquidity can improve social welfare.*

Proposition 4 does not require that leverage is at the competitive equilibrium level. Indeed, Proposition 4 holds for an arbitrary level of leverage. Hence, Proposition 4 implies that bank liquidity is insufficient not only in a competitive equilibrium but also in an equilibrium with $m > 0$ where leverage is restrained e.g. by a prudential tool on leverage. This result suggests that there is room for imposing a liquidity tool to improve social welfare even if a leverage restriction is already put in place.

Turning to welfare implications for leverage, the first-order condition of the regulator's problem with respect to leverage is given by:

$$\begin{aligned}
0 = \frac{\partial SW}{\partial L} =& - R\left[1 - P + P\mathbb{E}(v|\text{default})\right] + \int_{\underline{R}^k}^{\infty} \left[R^k - (R^k - 1)m\right] dF \\
&- \lambda \int_{\underline{R}^k}^{\bar{R}^k} \left[(xR - m) + R(L-1)\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} + (L-1)\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L}\right] dF \\
&+ \int_{-\infty}^{\underline{R}^k} \left[\frac{R^k}{1+\lambda} - \left(\frac{R^k}{1+\lambda} - 1\right) m\right] dF.
\end{aligned}$$

The slope of the social welfare with respect to leverage, evaluated at the bank's privately optimal choice $L = L^*$ implied by condition (21), is given by:

$$\begin{aligned}\left.\frac{\partial SW}{\partial L}\right|_{L=L^*} =& \left.\frac{\partial SW}{\partial L}\right|_{L=L^*} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{L=L^*} \\ &= -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF\right] \\ &\quad -\lambda(L-1)\left[\int_{\underline{R}^k}^{R^{k*}} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\bar{R}^k}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF\right].\end{aligned}$$

The sign of $\partial SW/\partial L|_{L=L^*}$ is negative if the supply curve of funds (1) is upward sloping, $\partial R/\partial L > 0$. This leads to the following proposition.

Proposition 5 (Excessive leverage). *Assume that the supply curve (1) is upward sloping. Then, for given liquidity, bank leverage is excessive from a social welfare viewpoint: restricting leverage can improve social welfare.*

Similar to Proposition 4 that shows insufficient liquidity, Proposition 5 holds for any level of bank liquidity. Even if liquidity is at some regulated level, the bank would choose excessive leverage relative to the constrained optimal level. Hence, Propositions 4 and 5 warrant imposing both leverage and liquidity requirements.

The source of inefficiencies that give rise to excessive leverage and insufficient liquidity is the same as those in the benchmark model, i.e. bank risk shifting and pecuniary externalities. However, the choice of liquidity is free from the pecuniary externalities as the composition of bank assets does not affect the marginal utility of the household in period $t = 1$. Hence, without risk shifting the bank's liquidity choice would coincide with the solution to the regulator's problem. This result is formalized in the following proposition.

Proposition 6 (Optimal liquidity but excessive leverage without bank risk shifting). *Consider a version of the extended model in which the bank has no risk shifting motives. Suppose that the supply curve (1) is upward sloping and conditions (23) and (24) hold. In a competitive equilibrium, given leverage, liquidity is at the level that would be chosen by the benevolent regulator. But, given liquidity, leverage is excessive because of the pecuniary externalities that work through the expected rate of return on deposits.*

Proposition 6 highlights that bank risk shifting is essential for the model to generate insufficient liquidity in a competitive equilibrium.

4.3 Parameterization

The previous section analytically showed that leverage is excessive *given* liquidity and liquidity is insufficient *given* leverage in a competitive equilibrium. However, it is not clear whether the results hold *jointly*. In addition, how the bank responds to changes in leverage and liquidity requirements and changes in key parameter values are yet to be known. Addressing these questions requires numerical analyses. To this end, this section parameterizes the extended model presented in Section 4.1.

Parameter values are set so that the extended model generates key endogenous variables similar to those observed for major US banks. Yet, it should be noted that the model aims to capture a financial system as a whole which issues short-term liabilities vulnerable to runs. After all, the model is so stylized that numerical analyses are intended to show qualitative implications rather than quantitative ones.

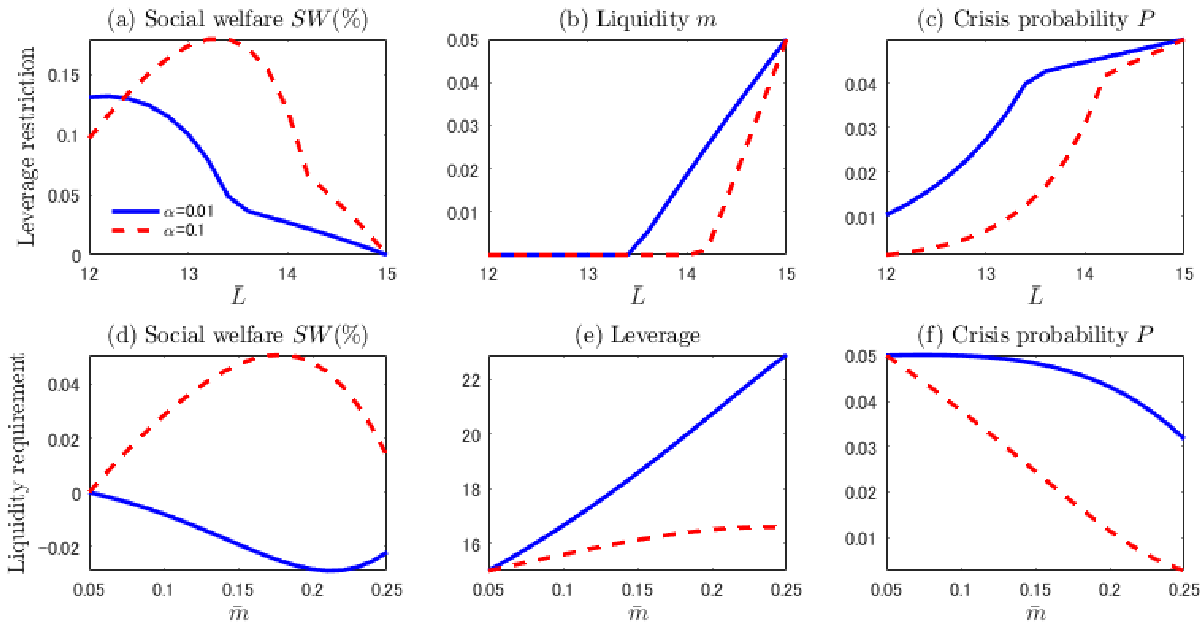
Parameters σ_ϵ, γ and λ and y are set so that the model hits the following target values in a competitive equilibrium: the leverage of $L = 15$, the liquidity ratio of $m = 0.05$, the crisis probability of $P = 0.05$ and the borrowing interest rate of $R = 1.02$. For the six largest US banks, over the period of 2008–2017 the leverage, measured by the ratio of total assets to Core Equity Tier 1 capital, is 13.4 on average and the liquidity ratio, measured by the ratio of liquid assets to total liabilities, is 0.037 on average (Miller and Sowerbutts, 2018).[17] The target values for L and m are not far from these observations. The target value of $P = 0.05$ is consistent with the historical fact that suggests that in any given country, banking crises occur on average once in every 20 to 25 years, i.e. the average annual crissi probability of 4–5 percent (Basel Committee on Banking Supervision 2010). The bank capital n is set so that the consumption in period 1 is close to the consumption in period 2. The resulting parameter values are $\sigma_\epsilon = 8.68/10000$, $\gamma = 0.66$, $\lambda = 0.17$, $y = 1.63$ and $n = 0.055$.

The mean return on bank lending is set to $\mu = 1.035$ so that the after-taxed return on equity at the mean return when there is no asset liquidation is about 15 percent, which is higher than those observed in the post-crisis period of 2008–17, but it is in line with the pre-crisis period of 2000–07.[18] The standard deviation of the return on bank lending is set

[17] The liquid assets are the sum of cash, withdrawable reserves and US treasury securities. The ratio of liquid assets to total assets, reported by Miller and Sowerbutts (2018), is transformed into the ratio of liquid assets to total liabilities using leverage. The data are available from 2008 only because two banks in the sample were purely investment banks until 2008 and their data source – the Federal Reserve's Financial Reports (form FRY-9C) – was not available before 2008 for these two banks.

[18] The after-taxed return on equity at the mean return when there is no fire sales is given by $(1-\tau)[\bar{R}^k L - (\mu-1)(L-1)m - R(L-1) - 1]$, where τ is the tax rate. In the calculation, the tax rate is assumed to be 30 percent.

Figure 1: Impacts of leverage (upper panels) and liquidity (lower panels) requirements



Note: In Panels (a) and (d), social welfare is measured as a percentage deviation from the level of social welfare at the competitive equilibrium without any restrictions.

to $\sigma_k = 0.025$ so that there exists an equilibrium that satisfy the target values discussed above.[19] Admittedly the return is highly volatile, but such volatility is required for the equilibrium to have the target level of a 5 percent crisis probability.

Finally, the functional form of the period-1 utility is assumed to be $u(c_1) = (c_1^{1-\alpha})/(1-\alpha)$ and two values $\alpha = 0.01$ and 0.1 are considered. A smaller value of α means that the utility function becomes close to be linear and the degree of the pecuniary externalities identified in the model becomes smaller. The two values of α are enough to show contrasting implications for prudential tools, highlighting a general equilibrium effect through the curvature of $u(\cdot)$.

4.4 Leverage and Liquidity Requirements

To understand the joint impact of leverage and liquidity requirements, I first consider cases of one tool only for leverage and liquidity, respectively, which is followed by an analysis on the joint effects of the two tools.

[19]If σ_k is set too low, there is no parameter value for e.g. $\gamma \in (0, 1)$ that supports the equilibrium with the target values.

4.4.1 Leverage restriction only

Consider the extended model presented in Section 4.1 in which only a restriction on leverage is put in place. This situation is reminiscent of the periods under the Basel I and II in which liquidity requirements were absent. Panels (a)-(c) of Figure 1 show the impacts of the leverage restriction, $L \leq \bar{L}$, on social welfare, liquidity, and the crisis probability, respectively, for the economies with $\alpha = 0.01$ (blue solid line) and 0.1 (red dashed line). Without any restriction the leverage is $L = 15$. As the leverage restriction is tightened from $L = 15$ to lower values, the social welfare is improved (Panel (a)) and the crisis probability is decreased (Panel (c)). However, the bank responds by reducing liquidity holdings (Panel (b)). Hence, imposing a leverage tool only induces the bank to migrate risk from leverage to liquidity.

The speed of a decrease in liquidity is faster for the economy with $\alpha = 0.1$ than that with $\alpha = 0.01$. This is because a tightening in leverage limits the amount of deposits and lowers the interest rate R, which further reduces the crisis probability. The decreased crisis probability allows the bank to take more risk in another area, i.e. liquidity, leading to a decrease in liquidity. The impact on liquidity is stronger for the economy with a greater general equilibrium effect of leverage on the interest rate, which is governed by parameter α, the curvature of the period-1 utility function.

Another consequence of the general equilibrium effect is the optimal level of leverage that maximises the social welfare. The economy with $\alpha = 0.01$ – a relatively small general equilibrium effect – calls for a tighter leverage restriction around $\bar{L} = 12$ than the economy with $\alpha = 0.1$ where such an optimal leverage restriction is above $\bar{L} = 13$. In the latter case, tightening leverage reduces the crisis probability more by lowering the interest rate, and as a result the optimal leverage restriction becomes milder.

4.4.2 Liquidity requirement only

Next, consider a situation in which only a liquidity requirement, $m \geq \bar{m}$, is put in place. Panels (d)-(f) of Figure 1 show the impacts of the liquidity tool on social welfare, leverage, and the crisis probability, respectively, for the economies with $\alpha = 0.01$ and $\alpha = 0.1$. As the liquidity requirement is tightened, the crisis probability is decreased for both economies (Panel (f)). However, while the social welfare is improved for the economy with $\alpha = 0.1$, it is deteriorated for the economy with $\alpha = 0.01$ (Panel (d)). This difference is driven by the divergent responses of leverage (Panel (e)). For the economy with the lower curvature of the utility function, the effect of increasing leverage on the interest rate is smaller, so that the bank responds by increasing leverage to a tightened liquidity requirement much more than in the economy with the higher curvature of the utility function. This negative effect dominates the benefit of increasing bank liquidity holdings, and as a result, imposing the liquidity requirement worsens welfare rather than improves it. This numerical example is still consistent with Proposition 4, which states that imposing a liquidity requirement can improve welfare *given* leverage. In this example, doing so worsens welfare, because leverage is not fixed; the bank responds by increasing leverage. This risk migration is a culprit of the welfare deterioration as a result of imposing the liquidity requirement only for the economy with $\alpha = 0.01$.

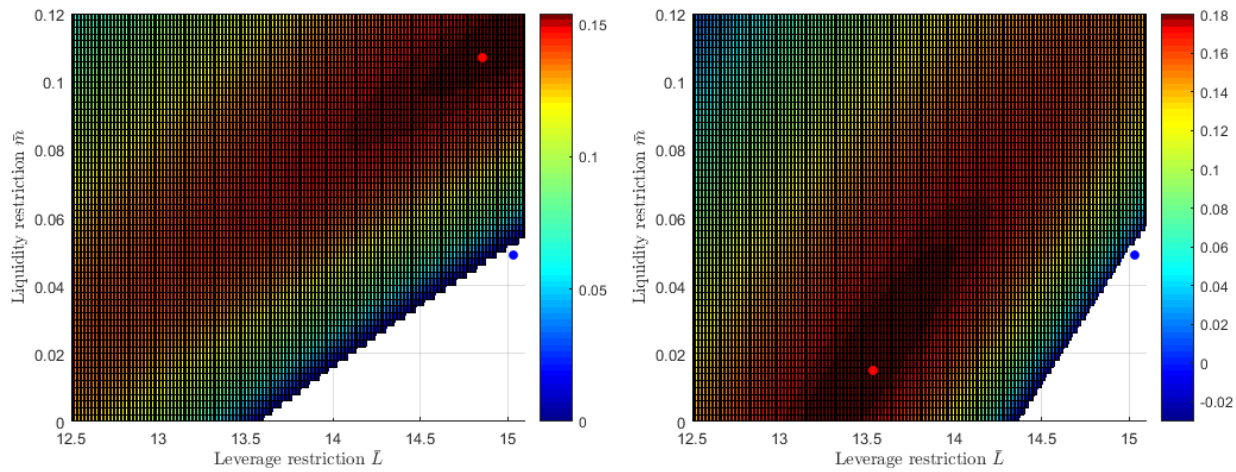
4.4.3 Coordination of leverage and liquidity tools

The previous analysis on one tool only highlights need for joint restrictions on leverage and liquidity to address risk migration from one area to another. Then, what is an optimal policy coordination between leverage and liquidity tools? How does the optimal coordination differ from the cases of one tool only?

Figure 2 addresses these questions by plotting social welfare as a function of the two requirements for the model with $\alpha = 0.01$ (Panel (a)) and that with $\alpha = 0.1$ (Panel (b)). Let subscript $_{BR}$ and $_{CE}$ denote a solution to the benevolent regulator's problem and the competitive equilibrium, respectively. First, the optimal coordination $\{L_{BR}, m_{BR}\}$ depends crucially on the curvature of the period-1 utility function, i.e. the general equilibrium effect of leverage on the interest rate. Relative to the competitive equilibrium (a blue point), the solution to the regulator's problem (a red point) features tightened leverage and tightened liquidity, i.e. $L_{BR} < L_{CE}$ and $m_{BR} > m_{CE}$, in the case of $\alpha = 0.01$ (Panel (a)). But the solution features tightened leverage and loosened liquidity, i.e. $L_{BR} < L_{CE}$ and $m_{BR} < m_{CE}$, in the case of $\alpha = 0.1$ (Panel (b)). In this case, the general equilibrium effect of the leverage restriction on the crisis probability, through its effect on the interest rate, is so great that lowering leverage is more effective than increasing liquidity to address the inefficiencies. It is worth noting that even though the optimal level of liquidity is lower than that in the competitive equilibrium, the liquidity requirement is still binding. Without the requirement, the bank would choose a lower level of liquidity as shown in Figure 1(b). In the case of $\alpha = 0.01$, the general equilibrium effect is small and hence tightening both leverage and liquidity becomes optimal.

Figure 2: Impacts of leverage and liquidity requirements on social welfare

(a) $\alpha = 0.01$ (b) $\alpha = 0.1$



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium. A red circle corresponds to a solution to the constrained planner problem and a blue circle corresponds to the competitive equilibrium.

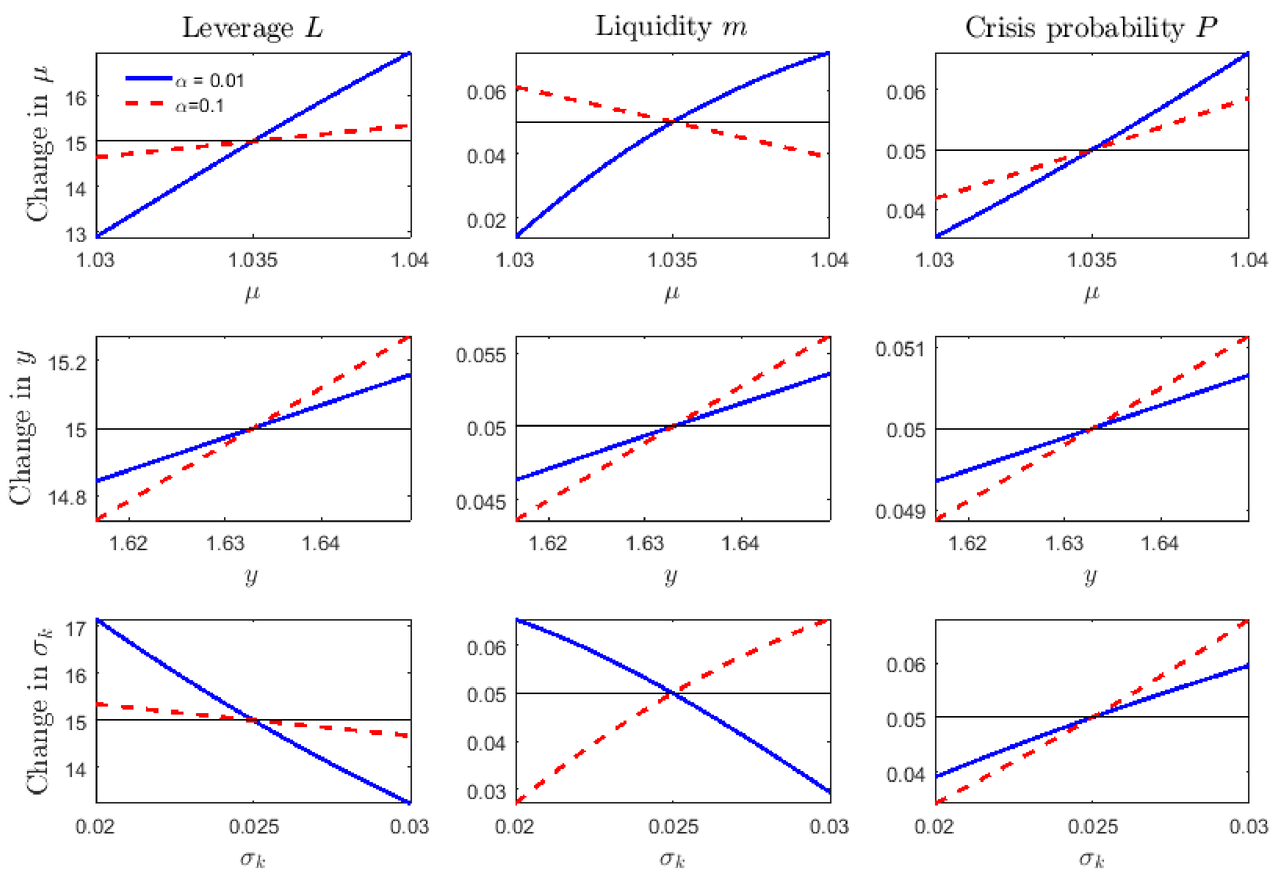
Next, relative to the cases of a leverage tool only, the optimal coordination between leverage and liquidity requirements calls for milder requirements on leverage. On the one hand, in the case of the leverage tool only, the optimal level of leverage that achieves the highest possible welfare is 12 and 13.2 for $\alpha = 0.01$ and 0.1, respectively. On the other hand, the optimal coordination requires the leverage of 14.9 and 13.5, respectively. Hence, with a liquidity requirement put in place, a less strict leverage restriction is called for to achieve the highest possible social welfare than in the case of a leverage tool only.

A similar implication holds for liquidity in the case of $\alpha = 0.1$: a liquidity tool only requires the liquidity ratio of around 0.18, while the optimal coordination calls for the liquidity ratio of only 0.016. However, this result does not hold for $\alpha = 0.01$ because tightening a liquidity requirement worsens welfare as discussed in Section 4.4.2.

4.5 Comparative Statics Analysis

Having studied the welfare implications of the model, in this section, I study how the economy without any restriction and the economy under jointly optimal leverage and liquidity requirements respond to changes in key parameter values.

Figure 3: Comparative statics of the competitive equilibrium



4.5.1 Comparative statics: competitive equilibrium

Figure 3 plots how leverage, liquidity, and the crisis probability react in response to changes in the mean return on bank assets μ, the household income y, and the standard deviation of the bank asset return σ_k for the cases of $\alpha = 0.01$ and 0.1, respectively

Figure 3 reveals three findings. First, similar to Proposition 1 for the baseline model with a bank leverage choice only, both leverage and the crisis probability increase as the mean return μ and the household income y increase. This result holds for both cases of $\alpha = 0.01$ and 0.1.

Second, in response to an increase in the standard deviation σ_k – uncertainty – of the bank asset return, the bank lowers leverage but the crisis probability increases for both cases of $\alpha = 0.01$ and 0.1. Although leverage is an important determinant of the crisis probability, the crisis probability increases when the bank is deleveraging.

Third, in response to changes in the mean bank asset return μ and the uncertainty of the bank asset return σ_k, in the case of $\alpha = 0.01$, leverage and liquidity move in the opposite direction in terms of contributions to a crisis probability. But in the case of $\alpha = 0.1$ leverage and liquidity move in the same direction, both of which contributes to increasing or decreasing the crisis probability. Specifically, when the mean return rises, the bank responds by increasing leverage and thereby contribute to raising a crisis probability in both cases of $\alpha = 0.01$ and 0.1, but it behaves differently in a liquidity choice: it increases liquidity, which restrains a crisis probability, in the case of $\alpha = 0.01$ while it decreases liquidity, which raises a crisis probability, in the case of $\alpha = 0.1$.

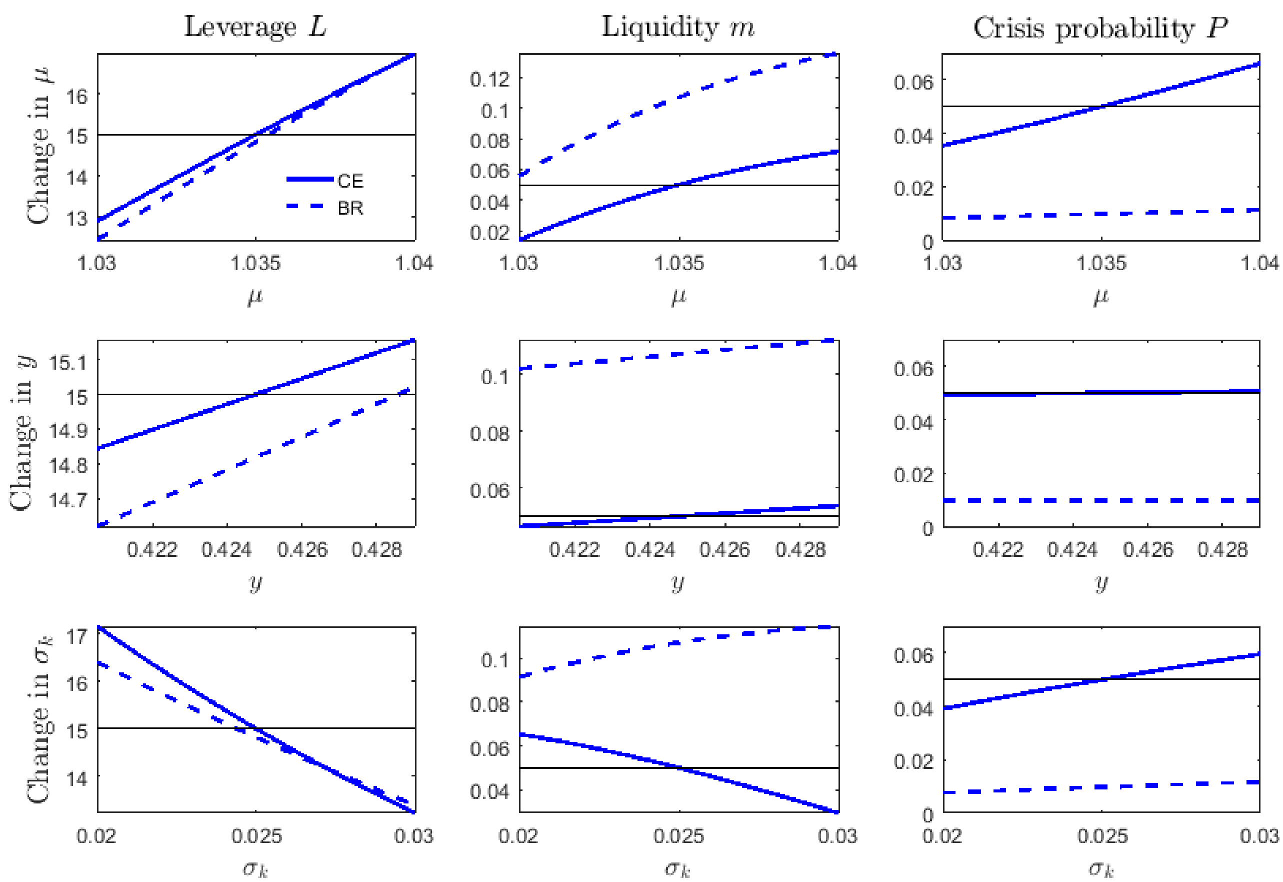
These different responses have to do with the general equilibrium effect of leverage on the interest rate and the crisis probability. When the curvature of the period-1 utility is relatively flat, $\alpha = 0.01$, a higher leverage is less associated with a rise in the interest rate than the case of $\alpha = 0.1$. Hence, the bank finds it profitable to increase leverage and limit the associated increase in the crisis probability by increasing liquidity holdings. If, instead, the general equilibrium effect is relatively strong, $\alpha = 0.1$, the bank uses leverage to restrain the crisis probability and uses liquidity to take more risk. In response to an increase in the average return μ the bank slightly increases leverage but at a slower pace than in the economy with $\alpha = 0.01$ and reduces liquidity to raise the asset return. A similar mechanism works in the case of a change in the uncertainty of the bank asset return.

4.5.2 Comparative statics: constrained optimal allocation

How does the constrained optimal allocation – a solution to the benevolent regulator's problem – react in response to changes in key parameter values? Figure 4 plots the constrained optimal allocation for leverage, liquidity, and the crisis probability in the case of $\alpha = 0.01$ in response to changes in the mean return on bank assets μ, the household income y, and the uncertainty of bank asset returns σ_k. The case of $\alpha = 0.1$ is omitted as its implications are similar.

Figure 4 reveals two findings. First, the constrained optimal levels of leverage and liquidity change in response to changes in the parameter values. In most cases the constrained optimal levels change in parallel with changes in the competitive equilibrium allocation. For example, both the constrained optimal level and the competitive equilibrium level of leverage increase as the mean return on bank assets rises. However, this is not always a case: the two levels can move in the opposite direction. For example, in response to an increase in the uncertainty of the bank asset return the constrained optimal level of liquidity increases while its counterpart in the competitive equilibrium decreases (bottom

Figure 4: Comparative statics of the constrained optimal allocation



Note: CE (blue solid line) and BR (blue dashed line) represent the competitive equilibrium and the solution to the benevolent regulator's problem, respectively.

medium panel of Figure 4). These observations suggest that the optimal prudential policy, which aims to achieve the constrained optimal allocation, differs in a non-trivial manner depending on parameter values that characterize the banking system and the economy.

Second, the constrained optimal level of the crisis probability is relatively stable around 1 percent, irrespective of changes in the parameter values. This makes a contrast with the volatile crisis probability in the competitive equilibrium. The stable crisis probability implies that the degree of the crisis probability curbed by the optimal prudential policy – a difference between P_{CE} and P_{BR} – becomes greater as P_{CE} increases. This is evident in response to increases in the mean return on bank assets (top right panel of Figure 4) and the uncertainty of bank asset returns (bottom right panel of Figure 4). The stable crisis probability in the constrained optimal allocation implies that if a default probability were observable for banks, instead of imposing multiple tools, setting a target level for the probability and letting banks to behave freely as long as the probability is no higher than the target level might be a robust way to improve welfare in various economies with a different banking system.

5 Extensions

The benchmark model presented in Section 2 is so stylised that it can be extended in various ways. In this section, I provide some extensions that can be used to discuss bank/sector specific capital requirements, risk weights and deposit insurance. In addition, the extensions bring some implications for shadow banking and concentration risk. Unless mentioned otherwise, the same parameter values set in Section 4.3 are used in this section. Main implications are unaffected by the discussed values of the curvature of the utility function, and hence $\alpha = 0.1$ is used in this section.

5.1 Model with Heterogeneous Banks

5.1.1 Overview of the model

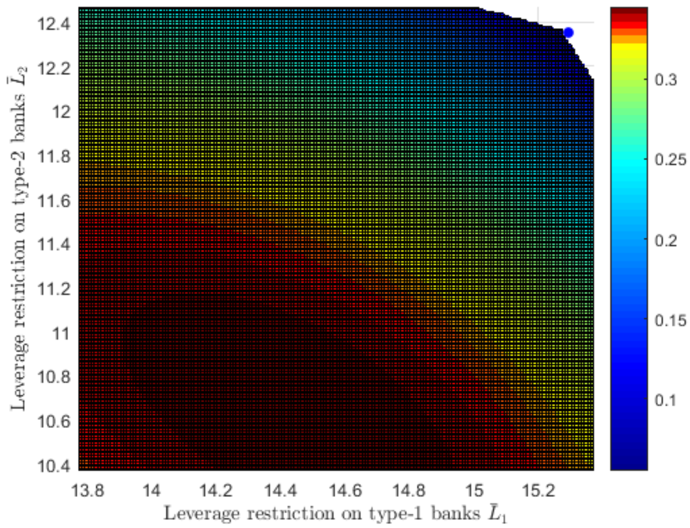
I extend the benchmark model to incorporate two types of banks, indexed by $j \in \{1, 2\}$. For simplicity, the two types of banks differ only in the riskiness of lending. The type-j bank specializes in lending to sector j and cannot lend to the other sector. Lending to sector j yields the same expected return μ, but the riskiness differs between the two sectors: $R_j^k \sim N(\mu, \sigma_j^2)$ with $\sigma_1 \neq \sigma_2$. The remaining part of the model is essentially the same as in the benchmark model.

The equilibrium for this economy is characterized by the following four equations with four unknowns $\{R_j, L_j\}_{j=1}^2$: for $j = 1, 2$,

$$R_j = \frac{u'(y - (L_1 - 1)n - (L_2 - 1)n)}{1 - P_j + \mathbb{E}(v_j|\text{default})P_j},$$
$$0 = \int_{R_j^{k*}}^{\infty} (R^k - R)dF_j - R_j\lambda\,(L_j - 1)\int_{R_j^{k*}}^{\infty} \frac{\partial x_j}{\partial \bar{s}_j^*}\frac{\partial \bar{s}_j^*}{\partial L_j} dF_j - R_j\lambda \int_{R_j^{k*}}^{\infty} x_j dF_j,$$

where $P_j = F_j(R_j^{k*})$ is the default probability for the type-j bank, $F_j(\cdot)$ is the cumulative normal distribution function with mean μ and standard deviation σ_j. The thresholds R_j^{k*} and $\bar{s}_j^*$ are given by equations (5) and (6) and the recovery rate v_j is given by (12) with a modification to add subscript j.

Figure 5: Impacts of bank-specific leverage restrictions on social welfare



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium. A blue circle at the upper right corner indicates the competitive equilibrium.

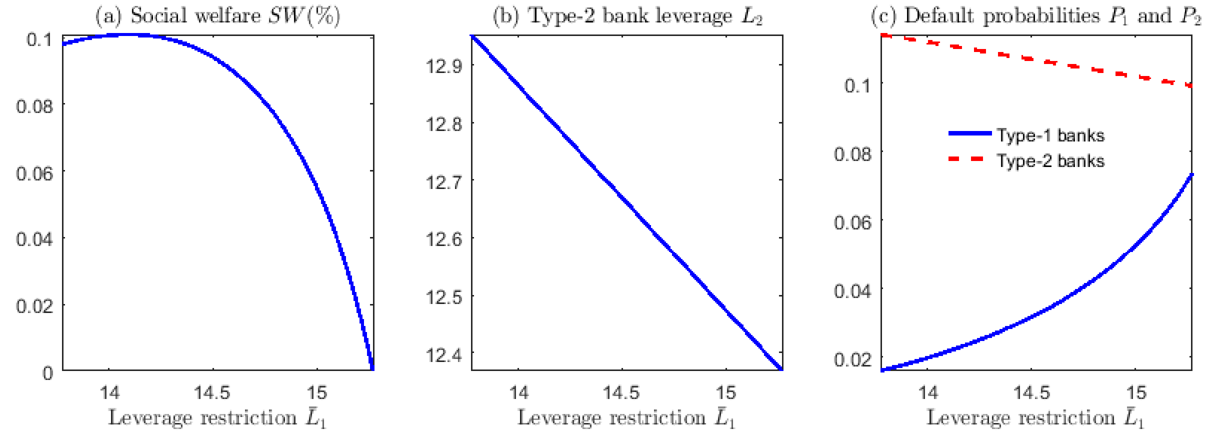
For a numerical illustration, the type-2 bank is assumed to be riskier than the type-1 bank. Specifically, the standard deviation of the type-2 bank asset return is 1.5 times as high as that of the type-1 bank. The bank net worth is set to a half of the level set in Section 4.3 for each type of banks so that the aggregate bank net worth remains the same.

In a competitive equilibrium, the type-2 bank has a lower leverage but a higher default probability than does the type-1 bank, reflecting the higher riskiness of the bank asset return. The leverage and the default probability are $L_1 = 15.3$ and $P_1 = 0.074$ for the type-1 bank and $L_2 = 12.4$ and $P_2 = 0.099$ for the type-2 bank. Hence, in this model, low leverage reflects the riskiness of the bank and does not necessarily signals the safety of the bank.

5.1.2 Heterogeneous capital requirements and risk weights

A heterogeneity in bank riskiness calls for bank-specific leverage/capital requirement. Figure 5 shows the joint effects of bank-specific leverage restrictions on social welfare. Limiting leverage for both types of banks improves social welfare and the optimum is attained around $\bar{L}_1 = \bar{L}_1^* \equiv 14.5$ and $\bar{L}_2 = \bar{L}_2^* \equiv 10.6$. Reflecting the heterogeneous riskiness of bank assets, the leverage restriction imposed on banks differs between the two types of banks.

Figure 6: Impacts of a leverage restriction on the type-1 banks only



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium.

A single capital/leverage restriction can achieve the same outcome if it is complemented by risk weights. This is so-called risk-weighted-based capital requirements. For example, consider risk weights normalized at 100 percent for the type-1 bank loans and 100ω percent for the type-2 bank loans. In addition, suppose that a risk-weighted-based capital requirement is $1/\bar{L}_1^*$. By construction, the capital ratio (or leverage) is restrained at the optimal level for the type-1 bank. To achieve the optimal level for the type-2 bank i.e. $n/(n+d_2) = 1/\bar{L}_2^*$, the risk weight ω has to be such that $n/(n+\omega d_2) = 1/\bar{L}_1^*$. Solving the equations for ω yields

$$\omega = \omega^* \equiv \frac{\bar{L}_1^* - 1}{\bar{L}_2^* - 1} > 1.$$

The optimal risk weight for the type-2 bank loans is more than 100 percent, reflecting the bank's high riskiness.

5.1.3 Shadow banks

Shadow banks, by definition, lie outside the reach of banking regulations. In the model with heterogeneous banks, the type-2 bank, which specializes in riskier loans, can be interpreted as a shadow bank if it is free from regulations, while the type-1 bank, which specializes in less risky loans, can be seen as a traditional bank if it is regulated.

With restrictions imposed only on the type-1 bank – a traditional bank, the bank becomes safer, but the type-2 bank – a shadow bank – becomes riskier. Figure 6 plots the impacts of a leverage restriction on the type-1 bank only on social welfare, the type-2 bank leverage, and the default probabilities. As the leverage restriction is tightened, the type-2 bank reacts by increasing leverage and as a result its default probability rises. The social welfare is improved for somewhat, but its highest achievable level of around 0.1 percent is far below the optimum of above 0.3 percent when both types of banks are regulated.

5.2 Model with a Portfolio Choice

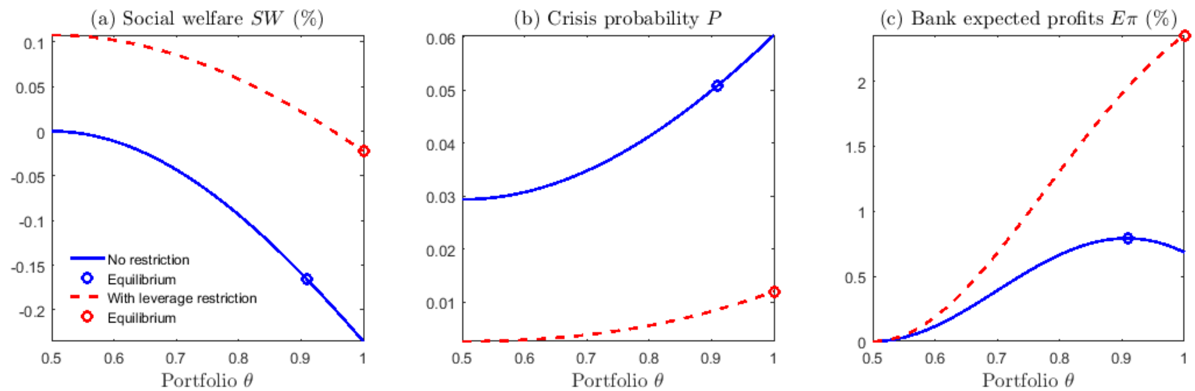
Banks may choose a less-diversified and riskier portfolio than socially desirable one when they have risk shifting motives. To formalize this idea, I extend the baseline model presented in Section 2 to incorporate a portfolio of loans. Specifically, a representative bank makes loans to two sectors, indexed by $j \in \{1, 2\}$. The returns of the two sectors follow a joint normal distribution, $\boldsymbol{R}^k \sim N(\boldsymbol{\mu}, \Sigma)$, where $\boldsymbol{R}^k \equiv [R_1^k, R_2^k]'$ is a vector of returns on lending to the two sectors. In addition to leverage the bank chooses a portfolio of loans, $\boldsymbol{\theta} \equiv [\theta, 1-\theta]'$, where $\theta \in [0, 1]$ is a fraction of total loans invested in sector $j = 1$. The return of the bank asset portfolio is then given by $R^k(\theta) \equiv \boldsymbol{\theta}'\boldsymbol{R}^k$, which follows $N(\mu(\theta), \sigma_k(\theta)^2)$, where $\mu(\theta) \equiv \boldsymbol{\theta}'\boldsymbol{\mu}$ is the mean return and $\sigma_k(\theta) \equiv (\boldsymbol{\theta}'\Sigma\boldsymbol{\theta})^{\frac{1}{2}}$ is the standard deviation of the portfolio. Each fund manager i observes a bank portfolio as well as leverage and receives independent signals $s_{ij} = R_j^k + \epsilon_{ij}$ with $\epsilon_{ij} \sim N(0, \sigma_{\epsilon_j}^2)$ for $j = 1, 2$. Given a bank portfolio, this extended model works essentially the same way as in the benchmark model. Fund manager i withdraws deposits early if and only if $\boldsymbol{\theta}'\boldsymbol{s}_i$ is less than the threshold $\bar{s}^*(L, \theta)$, where $\boldsymbol{s}_i \equiv [s_{i1}, s_{i2}]'$ is a vector of noisy signals. A difference is that now the threshold depends on bank asset portfolio θ as well as leverage L.

To illustrate concentration risk,[20] I assume that the two sectors are identical. The only difference from the benchmark model is that the bank can reduce their loan risk by diversifying over loans to the two sectors. Specifically, the bank is able to minimize the risk of its loan portfolio by setting $\theta = 0.5$. Not surprisingly, the smallest portfolio risk achieves the highest social welfare, as shown by the solid line in Figure 7(a). However, the bank does not choose such a portfolio but instead picks the riskier and more concentrated portfolio of around $\theta = 0.9$ to maximize the profits (Figure 7(c)). As a result, the crisis probability rises to 5 percent from 3 percent, a level which would be realized if the bank chose the perfectly diversified portfolio (Figure 7(b)).

The numerical illustration of the model highlights need for addressing concentration risk with a unique prudential instrument. Imposing a leverage restriction can improve welfare,

[20] Basel Committee on Banking Supervision (2014) points out the concentration risk as potential risk to the financial system.

Figure 7: Bank portfolio choice and concentration risk



Note: Social welfare is measured as a percentage deviation from that in the economy with no restriction and with $\theta = 0.5$. Bank expected profits are measured as a percentage deviation from those with $\theta = 0.5$ for each curve.

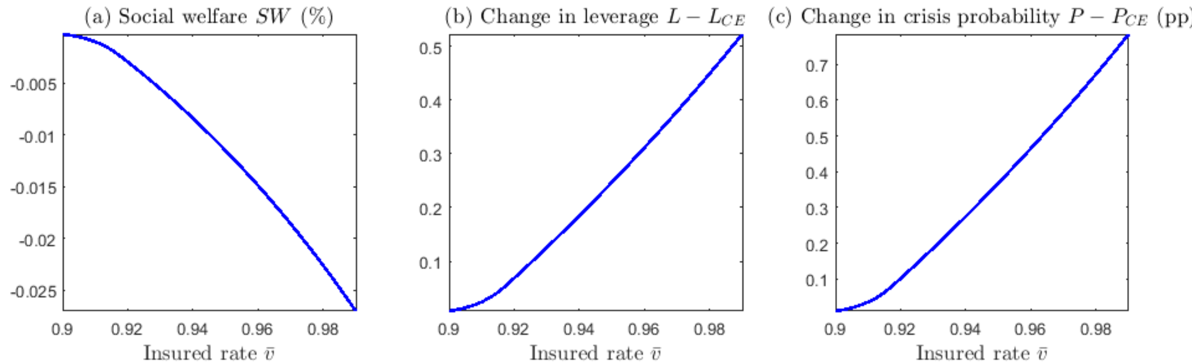
but as in the model with liquidity and leverage and the model with heterogeneous banks, doing so causes risk to migrate to a non-regulated area, which is a portfolio choice in this model. For example, if a regulator imposes the leverage restriction, $\bar{L} = L_{CE} - 1$, which is tighter by 1 than what the bank would choose without any restriction, the bank responds by concentrating completely in sector-1 lending, i.e., by setting $\theta = 1$, as shown in the red dashed line in Figure 7(c). As a result, the crisis probability becomes materially higher and the social welfare gets significantly lower than what would be achievable if the bank chose the perfectly diversified portfolio of $\theta = 0.5$. Hence, a prudential tool that limits exposure to single type of lending – a cap on concentration risk – is required to address the risk migration from the leverage area to the portfolio area.

5.3 Model with Deposit Insurance

Perfect deposit insurance, which ensures the recovery rate of unity, $v = 1$, will eliminate bank runs in theory, but such an insurance is hardly put in place in practice. Typically, the coverage of bank deposit insurance is limited and there is no insurance for money-like short-term debt. In short, deposit insurance is imperfect in practice.

Imperfect deposit insurance falls short of eliminating bank runs. As long as a representative household has a chance of losing some deposits and fund managers follow the behavioral rule (2), bank runs can still occur. A key modeling assumption behind this argument is that the fund managers' incentive to run, summarized by parameter γ in (2), is unaffected by the presence of deposit insurance.

Figure 8: Impacts of imperfect deposit insurance



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium with no deposit insurance. L_{CE} and P_{CE} denote the leverage and the crisis probability in the competitive equilibrium with no deposit insurance.

To explore the impact of deposit insurance on financial stability and social welfare, the benchmark model presented in Section 2 is extended to incorporate imperfect deposit insurance that protect the household from incurring losses more than $100(1-\bar{v})$ percent of the promised interest rate R. Hence, $\bar{v}$ forms the floor of the recovery rate. The government finances $(\bar{v}-v)R$ per unit of funds by imposing lump-sum taxes on the household in period $t=2$. Then, the supply curve for funds (1) is modified to:

$$R = \frac{u'(y-(L-1)n)}{1-P+\mathbb{E}(\max\{v,\bar{v}\}|\text{default})P}. \tag{27}$$

Equation (27) implies that an increase in the insurance rate $\bar{v}$ shifts the supply curve outward and makes excessive leverage even more excessive and increases the crisis probability.

Figure 8 confirms this prediction. As the coverage rate of the deposit insurance rises, the leverage becomes more excessive (Panel (b)), the crisis probability increases further (Panel (c)), and as a result, the social welfare deteriorates (Panel (a)).

6 Conclusion

Bank runs are an essential feature of sudden banking crises. This paper has developed a model of endogenous bank runs in a global game general equilibrium framework. The benchmark model presented in Section 2 has highlighted risk shifting and pecuniary externalities as the source of inefficiencies that give rise to an elevated financial crisis probability. The paper has extended the benchmark model and studied the role of multiple prudential tools for addressing the inefficiencies: leverage and liquidity tools in Section 4; bank/sector specific capital requirements in Section 5.1; a leverage restriction and a cap on concentration risk in Section 5.2. These tools are closely related to and motivated by the actual regulations and reforms implemented after the global financial crisis (Basel Committee on Banking Supervision 2011, 2013, 2014). The benchmark model, upon which the extended models are built and used to study these tools, hence provides a basic framework for studying banking crises, banks' behavior and prudential policy tools.

The models studied in the paper offer several empirical predictions. Their common theme is that risk can migrate from one area to others. And this is a main reason why multiple restrictions are required to address the issue. In the case of capital/leverage and liquidity discussed in Section 4, a tightening in a leverage restriction causes a bank to reduce the holdings of liquid assets. In the case of traditional and shadow banks discussed in Section 5.1, tightening a leverage restriction for a traditional bank induces a shadow bank to grow and makes the shadow bank riskier. In the case of leverage and a portfolio choice discussed in Section 5.2, restricting leverage induces a bank to choose a riskier asset portfolio by increasing exposure to one sector.

This paper has highlighted risk migration between two different risk spaces, e.g. capital/leverage and liquidity, for simplicity and clarity. In practice there would be risk migration among more than two areas, e.g. capital/leverage, liquidity, and portfolios, under the name of 'balance sheet optimization.' The paper abstracts away from a heterogeneity in bank liabilities, but this can be another area of risk migration. Analyzing risk migration in all possible areas would be extremely difficult, if not impossible. Yet, the models presented in this paper allow us to disentangle the impacts of one or two prudential tools on two risk spaces, a crisis probability, and social welfare. In the case of leverage and liquidity tools, the model has also shed light on the general equilibrium effect through the interest rate on the constrained optimal allocation.

The models presented in this paper have considered various prudential tools on risk spaces, but they still lack an important dimension: time. Adding a time dimension is essential for considering time-varying tools, e.g. countercyclical capital buffers, and also for highlighting other potential sources of externalities. Having kept this potential extension in mind, I have constructed the benchmark model so that it would be easily incorporated into a dynamic general equilibrium model. I plan to tackle on this problem in a future work.

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Appendix

Derivation of equation (9). As shown in Section 2 the threshold R^{k*} is a solution to equations (5) and (6). These equations are written explicitly as:

$$\Phi\left(\sqrt{\frac{1}{\sigma_k^2}+\frac{1}{\sigma_\epsilon^2}}R^{k*}-\frac{\frac{1}{\sigma_k^2}\mu+\frac{1}{\sigma_\epsilon^2}\bar{s}^*}{\sqrt{\frac{1}{\sigma_k^2}+\frac{1}{\sigma_\epsilon^2}}}\right)=\gamma, \tag{28}$$

$$R^{k*}=R\left(1-\frac{1}{L}\right)\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right], \tag{29}$$

where $\Phi(\cdot)$ is the standard normal distribution function. Equation (28) implies that $\lim_{\sigma_\epsilon\to 0}\Phi((R^{k*}-\bar{s}^*)/\sigma_\epsilon)=\gamma$. Therefore, $\lim_{\sigma_\epsilon\to 0}\Phi((\bar{s}^*-R^{k*})/\sigma_\epsilon)=1-\gamma$. Substituting this result into equation (29) yields (9).

Derivation of equation (10). Equation (10) is the limiting case of equation (8) where $\sigma_\epsilon\to 0$. First, consider the derivation of $\partial\bar{s}^*(L)/\partial L$ in equation (8). Totally differentiating equations (28) and (29) yields:

$$dR^{k*}=\frac{1}{\frac{\sigma_\epsilon^2}{\sigma_k^2}+1}d\bar{s}^*,$$

$$dR^{k*}=\frac{R}{L^2}\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right]dL+R\left(1-\frac{1}{L}\right)\lambda\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}(d\bar{s}^*-dR^{k*})$$

Combining these equations yields:

$$\frac{d\bar{s}^*}{dL}=\frac{\left(\sigma_k^2+\sigma_\epsilon^2\right)\frac{R}{L^2}\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right]}{\sigma_k^2-\left(1-\frac{1}{L}\right)\lambda\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\sigma_\epsilon},$$

where $\phi(\cdot)$ is the standard normal pdf. Note that $\lim_{\sigma_\epsilon\to 0}\phi((\bar{s}^*-R^{k*})/\sigma_\epsilon)=\phi(\lim_{\sigma_\epsilon\to 0}(\bar{s}^*-R^{k*})/\sigma_\epsilon)=\phi(\Phi^{-1}(1-\gamma))$. Then, in the limit, $d\bar{s}^*/dL$ is given by:

$$\lim_{\sigma_\epsilon\to 0}\frac{d\bar{s}^*}{dL}=\frac{R}{L^2}\left[1+\lambda(1-\gamma)\right].$$

Next, consider $\int_{R^{k*}}^{\infty}[\partial x(R^k,\bar{s}^*)/\partial\bar{s}^*]dF(R^k)$ in equation (8), where $F(\cdot)$ is the normal distribution function with mean μ and variance σ_k^2. It is explicitly written as:

$$\begin{aligned}\int_{R^{k*}}^{\infty}\frac{\partial x(R^k,\bar{s}^*)}{\partial\bar{s}^*}dF(R^k)&=\int_{R^{k*}}^{\infty}\phi\left(\frac{\bar{s}^*-R^k}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}dF(R^k)\\&=\int_{R^{k*}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\bar{s}^*-R^k}{\sigma_\epsilon}\right)^2}\frac{1}{\sigma_\epsilon}\frac{1}{\sqrt{2\pi}\sigma_k}e^{-\frac{1}{2}\left(\frac{R^k-\mu}{\sigma_k}\right)^2}dR^k.\end{aligned}$$

The terms in the power of e are arranged as:

$$
\begin{aligned}
& -\frac{1}{2}\left(\frac{\bar{s}^{*}-R^{k}}{\sigma_{\epsilon}}\right)^{2}-\frac{1}{2}\left(\frac{R^{k}-\mu}{\sigma_{k}}\right)^{2} \\
= & -\frac{1}{2}\left[\frac{\bar{s}^{*2}-2\bar{s}^{*}R^{k}+R^{k2}}{\sigma_{\epsilon}^{2}}+\frac{R^{k2}-2R^{k}\mu+\mu^{2}}{\sigma_{k}^{2}}\right] \\
= & -\frac{1}{2}\left[\left(\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}\right)R^{k2}-2\left(\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu}{\sigma_{k}^{2}}\right)R^{k}+\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu^{2}}{\sigma_{k}^{2}}\right] \\
= & -\frac{1}{2}\left(\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}\right)\left[R^{k2}-2\frac{\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu}{\sigma_{k}^{2}}}{\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}}R^{k}+\frac{\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu^{2}}{\sigma_{k}^{2}}}{\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}}\right] \\
= & -\frac{1}{2}\left(\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}\right)\left[\left(R^{k}-\frac{\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu}{\sigma_{k}^{2}}}{\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}}\right)^{2}-\left(\frac{\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu}{\sigma_{k}^{2}}}{\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}}\right)^{2}+\frac{\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu^{2}}{\sigma_{k}^{2}}}{\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}}\right] \\
= & -\frac{1}{2}\left(\frac{R^{k}-\frac{\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu}{\sigma_{k}^{2}}}{\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}}}{\sqrt{\frac{\sigma_{\epsilon}^{2}\sigma_{k}^{2}}{\sigma_{\epsilon}^{2}+\sigma_{k}^{2}}}}\right)^{2}+\frac{1}{2}\left[\frac{\left(\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu}{\sigma_{k}^{2}}\right)^{2}}{\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}}-\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}-\frac{\mu^{2}}{\sigma_{k}^{2}}\right].
\end{aligned}
$$

Then, $\int_{R^{k*}}^{\infty}[\partial x(R^{k},\bar{s}^{*})/\partial\bar{s}^{*}]dF(R^{k})$ is written as:

$$
\int_{R^{k*}}^{\infty}\frac{\partial x(R^{k},\bar{s}^{*})}{\partial\bar{s}^{*}}dF(R^{k})=\left(\int_{z^{*}}^{\infty}\phi(z)dz\right)\frac{1}{\sqrt{2\pi}}\sqrt{\frac{1}{\sigma_{\epsilon}^{2}+\sigma_{k}^{2}}}\exp\left\{\frac{1}{2}\left[\frac{\left(\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu}{\sigma_{k}^{2}}\right)^{2}}{\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}}-\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}-\frac{\mu^{2}}{\sigma_{k}^{2}}\right]\right\},
$$

where

$$
z^{*}=\frac{R^{k*}-\frac{\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu}{\sigma_{k}^{2}}}{\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}}}{\sqrt{\frac{\sigma_{\epsilon}^{2}\sigma_{k}^{2}}{\sigma_{\epsilon}^{2}+\sigma_{k}^{2}}}}
$$

Note that $\lim_{\sigma_{\epsilon}\to 0}=\Phi^{-1}(\gamma)$ and

$$
\lim_{\sigma_{\epsilon}\to 0}\frac{1}{2}\left[\frac{\left(\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}+\frac{\mu}{\sigma_{k}^{2}}\right)^{2}}{\frac{1}{\sigma_{\epsilon}^{2}}+\frac{1}{\sigma_{k}^{2}}}-\frac{\bar{s}^{*}}{\sigma_{\epsilon}^{2}}-\frac{\mu^{2}}{\sigma_{k}^{2}}\right]=-\frac{1}{2}\left(\frac{\bar{s}^{*}-\mu}{\sigma_{k}}\right)^{2}.
$$

Therefore, the limit of $\int_{R^{k*}}^{\infty}[\partial x(R^{k},\bar{s}^{*})/\partial\bar{s}^{*}]dF(R^{k})$ is given by:

$$
\lim_{\sigma_{\epsilon}\to 0}\int_{R^{k*}}^{\infty}\frac{\partial x(R^{k},\bar{s}^{*})}{\partial\bar{s}^{*}}dF(R^{k})=(1-\gamma)f(\bar{s}^{*}),
$$

where $f(\cdot)$ is the pdf of the normal distribution with mean μ and variance σ_{k}^{2}.

Finally, the term, $\int_{R^{k*}}^{\infty}x(R^{k},\bar{s}^{*}(L))dF(R^{k})$, in equation (8) goes to zero as $\sigma_{\epsilon}\to 0$. Therefore, in the limit of $\sigma_{\epsilon}\to 0$, equation (8) is reduced to equation (10).

Proof of Proposition 1.

(i) The first-order condition of the banks' problem in the limit equilibrium (10) is written as $0 = \partial \mathbb{E}(\pi)/\partial L$, where

$$\frac{\partial \mathbb{E}(\pi)}{\partial L} = \int_{\frac{R^{k*}-\mu}{\sigma_k}}^{\infty} (\mu + \sigma_k z)\, d\Phi(z)$$
$$- \left\{ \left[1 - \Phi\left(\frac{R^{k*}-\mu}{\sigma_k} \right) \right] R + \lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right] \phi\left(\frac{R^{k*}-\mu}{\sigma_k} \right) R^2 \frac{L-1}{L^2} \right\}.$$

A marginal change in this derivative with respect to a marginal increase in μ is given by:

$$\begin{aligned} \frac{\partial^2 \mathbb{E}(\pi)}{\partial L \partial \mu} &= 1 - \Phi\left(\frac{R^{k*}-\mu}{\sigma_k} \right) + \left[R^{k*} - R\phi\left(\frac{R^{k*}-\mu}{\sigma_k} \right) \right] \frac{1}{\sigma_k} \\ &+ \frac{\lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right]}{\sigma_k} \phi'\left(\frac{R^{k*}-\mu}{\sigma_k} \right) R^2 \frac{L-1}{L^2}. \end{aligned}$$

Because $\max_z \phi(z) < 0.4$, the assumptions of this proposition imply $R^{k*} > R\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right)$ and $\phi'(\cdot) > 0$, and thereby the sign of the above derivative is positive: $\partial^2 \mathbb{E}(\pi)/(\partial L \partial \mu) > 0$. Given that the solution L is an optimal solution, the $\partial \mathbb{E}(\pi)/\partial L$ curve is downward sloping. Then, $\partial^2 \mathbb{E}(\pi)/(\partial L \partial \mu) > 0$ implies that the $\partial \mathbb{E}(\pi)/\partial L$ curve shifts upward, implying that the optimal L increases. Hence, the demand curve shifts outward.

(ii) A marginal change in $\partial \mathbb{E}(\pi)/\partial L$ with respect to a marginal increase in λ is given by:

$$\begin{aligned} \frac{\partial^2 \mathbb{E}(\pi)}{\partial L \partial \lambda} &= - \left[R^{k*} - R\phi\left(\frac{R^{k*}-\mu}{\sigma_k} \right) \right] \frac{1}{\sigma_k} \frac{\partial R^{k*}}{\partial \lambda} \\ &- \frac{\lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right]}{\sigma_k} \phi'\left(\frac{R^{k*}-\bar{R}^k}{\sigma_k} \right) R^2 \frac{L-1}{L^2} \frac{\partial R^{k*}}{\partial \lambda} \\ &-(1-\gamma)[1+2\lambda(1-\gamma)]\phi\left(\frac{R^{k*}-\mu}{\sigma_k} \right) R^2 \frac{L-1}{L^2}, \end{aligned}$$

where $\partial R^{k*}/\partial \lambda = R(1-1/L)(1-\gamma) > 0$. Hence, $\partial^2 \mathbb{E}(\pi)/(\partial L \partial \lambda) < 0$, which implies that an increase in λ shifts the demand curve inward. Similarly, a marginal change in $\partial \mathbb{E}(\pi)/\partial L$ with respect to a marginal increase in γ is given by:

$$\begin{aligned} \frac{\partial^2 \mathbb{E}(\pi)}{\partial L \partial \gamma} &= - \left[R^{k*} - R\phi\left(\frac{R^{k*}-\mu}{\sigma_k} \right) \right] \frac{1}{\sigma_k} \frac{\partial R^{k*}}{\partial \gamma} \\ &- \frac{\lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right]}{\sigma_k} \phi'\left(\frac{R^{k*}-\mu}{\sigma_k} \right) R^2 \frac{L-1}{L^2} \frac{\partial R^{k*}}{\partial \gamma} \\ &+\lambda[1+2\lambda(1-\gamma)]\phi\left(\frac{R^{k*}-\mu}{\sigma_k} \right) R^2 \frac{L-1}{L^2} \end{aligned}$$

where $\partial R^{k*}/\partial \gamma = -R(1-1/L)\lambda < 0$. Hence, $\partial^2 \mathbb{E}(\pi)/(\partial L \partial \gamma) > 0$, which implies that a decrease in γ shifts the demand curve inward.

(iii) The supply curve (1) is written as:

$$R = \frac{u'\left(y - (L-1)\,n\right)}{1 - P + \mathbb{E}(v|\text{default})P}.$$

From this it is clear that an increase in y shifts the supply curve outward.

(iv) Similarly, the supply curve implies that an increase n shifts the curve inward.

Derivation of equation (15). The first-order condition of the regulator's problem is $\partial \mathrm{SW}/\partial L = 0$, where

$$\begin{aligned}\frac{\partial \mathrm{SW}}{\partial L} =& - R\left[1 - P + \mathbb{E}(v|\text{default})P\right] + \mathbb{E}(R^k) - \lambda R \int_{\underline{R}^k}^{\infty} x dF - \lambda \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF \\ & -\lambda(L-1)\int_{\underline{R}^k}^{\infty} R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF - \lambda(L-1)\int_{\underline{R}^k}^{\infty} R\left(\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF,\end{aligned}$$

The first-order condition of the bank's problem is $\partial \mathbb{E}(\pi)/\partial L = 0$, where

$$\frac{\partial \mathbb{E}(\pi)}{\partial L} = \int_{R^{k*}}^{\infty} (R^k - R) dF - \lambda\,(L-1)\int_{R^{k*}}^{\infty} R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF - R\lambda \int_{R^{k*}}^{\infty} x dF.$$

Then, $\partial \mathrm{SW}/\partial L$ evaluated at the competitive equilibrium is given by:

$$\begin{aligned}\left.\frac{\partial \mathrm{SW}}{\partial L}\right|_{\mathrm{CE}} =& \left.\frac{\partial \mathrm{SW}}{\partial L}\right|_{\mathrm{CE}} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{\mathrm{CE}} \\ =& \int_{\underline{R}^k}^{R^{k*}} R^k dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} R^k dF - R\mathbb{E}(v|\text{default})P - \lambda R \int_{\underline{R}^k}^{R^{k*}} x dF \\ & -\lambda(L-1)\left[\int_{\underline{R}^k}^{R^*} R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\infty}\left(R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF\right]\end{aligned}$$

Because the recovery rate v is given by equation (12), $R\mathbb{E}(v|\text{default})P$ is given by:

$$R\mathbb{E}(v|\text{default})P = \int_{\underline{R}^k}^{R^{k*}} \left(R^k \frac{L}{L-1} - \lambda R x\right) dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} R^k \frac{L}{L-1} dF.$$

Then, the first-order condition of the regulator's problem, evaluated at the competitive equilibrium, is written as:

$$\begin{aligned}\left.\frac{\partial \mathrm{SW}}{\partial L}\right|_{\mathrm{CE}} =& -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} R^k dF\right] \\ & -\lambda(L-1)\left[\int_{\underline{R}^k}^{R^*} R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\infty}\left(R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF\right].\end{aligned}$$

This completes the derivation of (15).

Derivation of $\partial \bar{s}^*/\partial L$ and $\partial \bar{s}^*/\partial R$ in the benchmark model. Totally differentiating equations (28) and (29) with respect to R, $\bar{s}^*$ and R^{k*} yields:

$$dR^{k*} = \frac{\sigma_k^2}{\sigma_\epsilon^2 + \sigma_k^2} d\bar{s}^*,$$
$$dR^{k*} = \left(1 - \frac{1}{L}\right)(1 + \lambda x)dR + R\left(1 - \frac{1}{L}\right)\phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}(d\bar{s}^* - dR^{k*}).$$

Also, totally differentiating equation (29) with respect to L, $\bar{s}^*$ and R^{k*} yields:

$$dR^{k*} = \frac{R}{L^2}(1 + \lambda x)dL + R\left(1 - \frac{1}{L}\right)\phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}(d\bar{s}^* - dR^{k*}).$$

Rearranging these equations leads to:

$$\frac{\partial \bar{s}^*}{\partial R} = \frac{\left(1 + \frac{\sigma_\epsilon^2}{\sigma_k^2}\right)\left(1 - \frac{1}{L}\right)(1 + \lambda x)}{1 - \frac{\sigma_\epsilon}{\sigma_k^2} R\left(1 - \frac{1}{L}\right)\phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right)} > 0,$$
$$\frac{\partial \bar{s}^*}{\partial L} = \frac{\left(1 + \frac{\sigma_\epsilon^2}{\sigma_k^2}\right)\frac{R}{L^2}(1 + \lambda x)}{1 - \frac{\sigma_\epsilon}{\sigma_k^2} R\left(1 - \frac{1}{L}\right)\phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right)} > 0.$$

The sign of these derivatives is positive because the denominator, which is identical between the two, is positive for the threshold $\bar{s}^*$ to uniquely exist, which is assumed to hold.

The slope of the supply curve (1). The supply curve (1) is written in terms of leverage as:

$$R(1 - P) + \int_{\underline{R}^k}^{R^{k*}} \left(R^k \frac{L}{L-1} - R\lambda x\right) dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} R^k \frac{L}{L-1} dF = u'(y - (L-1)n),$$

Totally differentiating the equation with respect to R and L yields:

$$\left\{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left[R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right] dF\right\} dR$$
$$= \left\{\int_{\underline{R}^k}^{R^{k*}} \left[\frac{R^k}{(L-1)^2} + R\lambda \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right] dF + \frac{1}{1+\lambda}\int_{-\infty}^{R^{k*}} \frac{R^k}{(L-1)^2} dF - u''(c_1)n\right\} dL.$$

Then, the slope of the supply curve is given by:

$$\frac{dR}{dL} = \frac{\int_{\underline{R}^k}^{R^{k*}} \left[\frac{R^k}{(L-1)^2} + R\lambda \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right] dF + \frac{1}{1+\lambda}\int_{-\infty}^{R^{k*}} \frac{R^k}{(L-1)^2} dF - u''(c_1)n}{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left[R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right] dF}. \tag{30}$$

The numerator of (30) is positive. Hence, the slope of the supply curve is positive if and only if the denominator is positive.

Banks' problem without bank risk shifting motives. Banks choose leverage L to maximize

$$\int_{R^{k*}}^{\infty} \left\{R^k L - R\left[1 + \lambda x(R^k, \bar{s}^*(L, R))\right](L-1)\right\} n dF(R^k),$$

subject to the technical constraint $L \leq L_{\max}$ and the households' participation constraint (17), which is rewritten here for convenience:

$$R[1-F(R^{k*})]+\int_{\underline{R}^k}^{R^{k*}}\left[R^k\frac{L}{L-1}-R\lambda x(R^k,\bar{s}^*)\right]dF+\int_{-\infty}^{\underline{R}^k}\frac{R^k}{1+\lambda}\frac{L}{L-1}dF \geq R^e.$$

The technical constraint is non-binding as in the benchmark model. It is obvious that the households' participation constraint is binding. The binding constraint implicitly defines the interest rate as a function of leverage: $R = R_B(L)$. The slope of this curve is derived in a similar manner as in the supply curve and is given by:

$$\frac{dR_B}{dL}=\frac{\int_{\underline{R}^k}^{R^{k*}}\left[\frac{R^k}{(L-1)^2}+R\lambda\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right]dF+\frac{1}{1+\lambda}\int_{-\infty}^{R^{k*}}\frac{R^k}{(L-1)^2}dF}{1-P-\lambda\int_{\underline{R}^k}^{R^{k*}}\left[R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}+x\right]dF} \tag{31}$$

Compared to the slope of the supply curve, (30), the only difference in the slope of R_B is the absence of $-u''(c_1)n$ in the numerator.

Substituting $R = R_B(L)$ into the banks' objective function, the first-order condition with respect to L is written as:

$$\begin{aligned}0=\frac{\partial \mathbb{E}(\pi)}{\partial L}&=\int_{R^{k*}}^{\infty}R^k dF-(1-P)R-\lambda R\,(L-1)\int_{R^{k*}}^{\infty}\frac{\partial x}{\partial \bar{s}^*}\left(\frac{\partial \bar{s}^*}{\partial L}+\frac{\partial \bar{s}^*}{\partial R}\frac{\partial R_B}{\partial L}\right)dF\\&-\lambda R\int_{R^{k*}}^{\infty}x dF-\frac{\partial R_B}{\partial L}(L-1)\int_{R^{k*}}^{\infty}(1+\lambda x)dF.\end{aligned}$$

Derivation of equation (18). The slope of the social welfare function is given by equation (14), which is rewritten here for convenience:

$$\begin{aligned}\frac{\partial SW}{\partial L}&=-R\left[1-P+\mathbb{E}(v|\text{default})P\right]+\mathbb{E}(R^k)-\lambda R\int_{\underline{R}^k}^{\infty}x dF-\lambda\int_{-\infty}^{\underline{R}^k}\frac{R^k}{1+\lambda}dF\\&-\lambda R(L-1)R\int_{\underline{R}^k}^{\infty}\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}dF-\lambda(L-1)\int_{\underline{R}^k}^{\infty}\left(\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}R+x\right)\frac{\partial R}{\partial L}dF,\end{aligned}$$

where constant proportional term n is omitted for simplifying notations. Let CE' denote the competitive equilibrium without bank risk shifting motives. Then, the slope of the social welfare evaluated at this competitive equilibrium is given by:

$$\begin{aligned}
\left.\frac{\partial SW}{\partial L}\right|_{\mathrm{CE}'} =& \left.\frac{\partial SW}{\partial L}\right|_{\mathrm{CE}'} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{\mathrm{CE}'} \\
=& -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} R^k dF\right] \\
& -\lambda(L-1)\left[\int_{\underline{R}^k}^{R^*} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\infty}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF\right] \\
& + \frac{\partial R_B}{\partial L}\left[\int_{R^{k*}}^{\infty}\left(\lambda\frac{\partial \bar{s}^*}{\partial R} + 1 + \lambda x + \lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}\right) dF\right](L-1) \\
=& -\int_{\underline{R}^k}^{R^{k*}}\left[\frac{R^k}{L-1} + R\lambda(L-1)\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right] dF + \frac{1}{1+\lambda}\int_{-\infty}^{R^{k*}}\frac{R^k}{L-1} dF \\
& + \frac{\partial R_B}{\partial L}\left[1 - P - \lambda\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right) dF\right](L-1) \\
& - \lambda(L-1)\Delta R\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right) dF,
\end{aligned}$$

where $\Delta R \equiv \partial R/\partial L - \partial R_B/\partial L \propto -u'' > 0$. Substituting $\partial R_B/\partial L$ out using (31) yields:

$$\left.\frac{\partial SW}{\partial L}\right|_{\mathrm{CE}'} = \lambda(L-1)\left[\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right) dF\right] u''(c_1) < 0.$$

Derivation of $\partial \bar{s}^*/\partial L$, $\partial \bar{s}^*/\partial m$, $\partial \bar{s}^*/\partial R$ and condition (23) in Section 4. In the model with leverage and liquidity, the thresholds $\bar{s}^*$ and R^{k*} are characterized by equations (5) and (20). Equation (20) is written as:

$$R^{k*} = \frac{R-m}{\frac{L}{L-1} - m}\left[1 + \lambda\frac{\Phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right) R - m}{R-m}\right]. \tag{32}$$

Totally differentiating equations (5) with respect to R^{k*} and $\bar{s}^*$ yields:

$$dR^{k*} = \frac{1}{\frac{\sigma_\epsilon^2}{\sigma_k^2} + 1} d\bar{s}^*.$$

Totally differentiating equation (32) with respect to R^{k*}, $\bar{s}^*$ and L yields:

$$dR^{k*} = \frac{1}{[L - m(L-1)]^2}\left(1 + \lambda\frac{xR - m}{R-m}\right) dL + \frac{\lambda R x'}{\frac{L}{L-1} - m}\frac{d\bar{s}^* - dR^{k*}}{\sigma_\epsilon},$$

where $x' \equiv \phi((\bar{s}^* - R^{k*})/\sigma_\epsilon)$. Then, $d\bar{s}^*/dL$ is given by:

$$\frac{d\bar{s}^*}{dL} = \frac{\frac{\sigma_\epsilon^2/\sigma_k^2 + 1}{[L - m(L-1)]^2}\left(1 + \lambda\frac{xR - m}{R-m}\right)}{1 - \frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R x'}{\frac{L}{L-1} - m}} > 0.$$

Note that the denominator is positive for the model to have a unique solution for $\bar{s}^*$ and R^{k*}. Next, totally differentiating equation (32) with respect to R^{k*}, $\bar{s}^*$ and m yields:

$$dR^{k*} = \frac{-(1+\lambda)\frac{L}{L-1} + (1+\lambda x)R}{[L/(L-1)-m]^2}dm + \frac{\lambda R x'}{\frac{L}{L-1}-m}\frac{d\bar{s}^* - dR^{k*}}{\sigma_\epsilon},$$

Then, $d\bar{s}^*/dm$ is given by:

$$\frac{d\bar{s}^*}{dm} = \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{[L/(L-1)-m]^2}\left[-(1+\lambda)\frac{L}{L-1} + (1+\lambda x)R\right]}{1 - \frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R x'}{\frac{L}{L-1}-m}}.$$

Hence, $d\bar{s}^*/dm < 0$ if the interest rate is low enough to satisfy condition (23):

$$R < \frac{1+\lambda}{1+\lambda x}\frac{L}{L-1}.$$

Finally, totally differentiating equation (32) with respect to R^{k*}, $\bar{s}^*$ and R yields:

$$dR^{k*} = \frac{1}{\frac{L}{L-1}-m}(1+\lambda x)dR + \frac{\lambda R x'}{\frac{L}{L-1}-m}\left(d\bar{s}^* - dR^{k*}\right).$$

Then, $d\bar{s}^*/dR$ is given by:

$$\frac{d\bar{s}^*}{dR} = \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{L/(L-1)-m}(1+\lambda x)}{1 - \frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R x'}{\frac{L}{L-1}-m}} > 0.$$

Derivation of $\partial R/\partial L$ and $\partial R/\partial m$ in Section 4. Using the recovery rate in the model with liquidity, the supply curve of funds (1) is written as:

$$\begin{aligned} u'(y-(L-1)n) = R(1-P) &+ \int_{\underline{R}^k}^{R^{k*}} \left[R^k\left(\frac{L}{L-1}-m\right) + m - \lambda(Rx-m)\right]dF \\ &+ \int_{-\infty}^{\underline{R}^k}\left[\frac{R^k}{1+\lambda}\left(\frac{L}{L-1}-m\right)+m\right]dF. \end{aligned}$$

Totally differentiating this equation with respect to L and R yields:

$$\begin{aligned} -u''(c_1)ndL = &\left[1 - P - \lambda\left(\int_{\underline{R}^k}^{R^{k*}} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)dF\right]dR \\ &- \left[\int_{\underline{R}^k}^{R^{k*}}\left(\frac{R^k}{(L-1)^2} + \lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right)dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k}\frac{R^k}{(L-1)^2}dF\right]dL. \end{aligned}$$

Similarly, totally differentiating it with respect to R and m yields:

$$0 = \left[1 - P - \lambda\left(\int_{\underline{R}^k}^{R^{k*}} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right) dF\right] dR$$
$$+\left[\int_{\underline{R}^k}^{R^{k*}} \left(-R^k + 1 + \lambda - \lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m}\right) dF + \int_{-\infty}^{\underline{R}^k} \left(-\frac{R^k}{1+\lambda} + 1\right) dF\right] dm.$$

Hence, $\partial R/\partial L$ and $\partial R/dm$ are given by:

$$\frac{\partial R}{\partial L} = \frac{\int_{\underline{R}^k}^{R^{k*}} \left(\frac{R^k}{(L-1)^2} + \lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right) dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} \frac{R^k}{(L-1)^2} dF - u''(c_1)n}{1 - P - \lambda\int_{\underline{R}^k}^{R^{k*}} \left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right) dF},$$
$$\frac{\partial R}{\partial m} = \frac{\int_{\underline{R}^k}^{R^{k*}} \left[\lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m} - (1 + \lambda - R^k)\right] dF - \int_{-\infty}^{\underline{R}^k} \left(1 - \frac{R^k}{1+\lambda}\right) dF}{1 - P - \lambda\int_{\underline{R}^k}^{R^{k*}} \left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right) dF}$$

The numerator of the equation for $\partial R/\partial L$ is positive. Hence, the slope of the supply curve is positive, i.e. $\partial R/\partial L > 0$, if and only if

$$1 - P - \lambda\int_{-\infty}^{R^{k*}} \left[x + R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}\right] dF(R^k) > 0.$$

The numerator of the equation for $\partial R/\partial m$ is negative under the assumptions of (23) and (24). If the slope of the supply curve is positive, the slope of the interest rate curve with respect to liquidity is negative, i.e. $\partial R/\partial m < 0$.

Proof of Proposition 5. As provided in Section 4, the first-order condition of the regulator's problem with respect to leverage is given by:

$$0 = \frac{\partial SW}{\partial L} = -R\left[1 - P + P\mathbb{E}(v|\text{default})\right] + \int_{\underline{R}^k}^{\infty} \left[R^k - (R^k - 1)m\right] dF$$
$$-\lambda\int_{\underline{R}^k}^{\bar{R}^k} \left[(xR - m) + R(L-1)\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} + (L-1)\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L}\right] dF$$
$$+\int_{-\infty}^{\underline{R}^k} \left[\frac{R^k}{1+\lambda} - \left(\frac{R^k}{1+\lambda} - 1\right) m\right] dF.$$

where $RP\mathbb{E}(v|\text{default})$ is given by:

$$RP\mathbb{E}(v|\text{default}) = \int_{\underline{R}^k}^{R^{k*}} \left[R^k\left(\frac{L}{L-1} - m\right) + m - \lambda(Rx - m)\right] dF$$
$$+\int_{-\infty}^{\underline{R}^k} \left[\frac{1}{1+\lambda}R^k\left(\frac{L}{L-1} - m\right) + m\right] dF.$$

On the other hand, as provided in the main text, the first-order condition of the banks' problem with respect to leverage is given by:

$$0 = \frac{\partial \mathbb{E}(\pi)}{\partial L} = \int_{R^{k*}}^{\infty} [R^k - (R^k - 1)m - R] dF - \lambda \int_{R^{k*}}^{\bar{R}^k} \left[(xR - m) + R(L-1) \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} \right] dF.$$

Hence, the slope of the social welfare, evaluated at the banks' privately optimal choice of leverage $L = L^*$, is given by:

$$\begin{aligned} \frac{\partial SW}{\partial L}\bigg|_{L=L^*} =& - \frac{1}{L-1} \left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF \right] \\ & - \lambda(L-1) \left[\int_{\underline{R}^k}^{R^{k*}} R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\bar{R}^k} \left(R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right) \frac{\partial R}{\partial L} dF \right]. \end{aligned}$$

Hence, under the assumption of the upward-sloping supply curve, $\partial R/\partial L > 0$, the sign of $\partial \mathrm{SW}/\partial L|_{L=L^*}$ is negative. This completes the proof of Proposition 5.

Proof of Proposition 6. When banks do not have risk shifting motives, they maximize the profits subject to the households' participation constraint, $R[1 - P + \mathbb{E}(v|\text{default})] \geq R^e$ for some R^e, and the technical constraint $L \leq L_{\max}$. Given R^e, the households' participation constraint implicitly defines the interest rate as a function of leverage and liquidity, $R = R_B(L, m)$. In particular, the derivatives with respect to L and m respectively are given by:

$$\begin{aligned} \frac{\partial R_B}{\partial L} =& \frac{\int_{\underline{R}^k}^{R^{k*}} \left(\frac{R^k}{(L-1)^2} + \lambda R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} \right) dF + \frac{1}{1+\lambda} \int_{-\infty}^{\underline{R}^k} \frac{R^k}{(L-1)^2} dF}{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left(R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right) dF}, \\ \frac{\partial R_B}{\partial m} =& \frac{\partial R}{\partial m} = \frac{\int_{\underline{R}^k}^{R^{k*}} \left[\lambda R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial m} - (1 + \lambda - R^k) \right] dF - \int_{-\infty}^{\underline{R}^k} \left(1 - \frac{R^k}{1+\lambda} \right) dF}{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left(R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right) dF} \end{aligned}$$

Taking into account $R = R_B(L, m)$, the first-order condition of the banks' problem with respect to m is given by:

$$\begin{aligned} 0 =& - \int_{R^{k*}}^{\infty} (R^k - 1) dF(R^k) + \lambda \int_{R^{k*}}^{\bar{R}^k} \left(1 - R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial m} \right) dF(R^k) \\ & - (1-P) \frac{\partial R_B}{\partial m} - \lambda \frac{\partial R_B}{\partial m} \int_{R^{k*}}^{\bar{R}^k} \left(x + R \frac{\partial x}{\partial \bar{s}^*} \frac{\bar{s}^*}{\partial R} \right) dF(R^k). \end{aligned}$$

The last two terms in the right-hand-side of the equation correspond to those related to the effect of liquidity on the interest rate. Because the sign of these terms are positive, banks which have no risk shifting motives have higher liquidity holdings than otherwise would be the case. Evaluating the first-order condition of the regulator's problem with respect to liquidity at the competitive equilibrium level of liquidity $m = m^*$ yields:

$$\begin{aligned}
\left.\frac{\partial SW}{\partial m}\right|_{m=m^*} &= \frac{\partial R_B}{\partial m}\left[1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m} + x\right) dF\right] \\
&- \left\{\int_{\underline{R}^k}^{R^{k*}} \left[\lambda R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m} - (1 + \lambda - R^k)\right] dF - \int_{-\infty}^{\underline{R}^k} \left(1 - \frac{R^k}{1+\lambda}\right) dF\right\} \\
&= 0.
\end{aligned}$$

The final equality was derived by using the expression for $\partial R_B/\partial m$. The first-order condition of the regulator's problem, evaluated at the competitive equilibrium level of leverage, can be derived similarly to the benchmark model. This completes the proof of Proposition 6.

Calibration: the extended model with bank leverage and liquidity. The unit of time is annual. The calibration strategy is to set target values for endogenous variables L, m, R and P and pin down parameter values for σ_ϵ, γ, λ and y jointly. The four parameters, σ_ϵ, γ, λ and y, are set as follows. The probability of bank default is given by $P = \Phi((R^{k*} - \mu)/\sigma_k)$, so that the threshold R^{k*} is given by $R^{k*} = \mu + \sigma_k \Phi^{-1}(P)$. Condition (5) is arranged as:

$$\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon} = \frac{\sigma_\epsilon}{\sigma_k^2}(R^{k*} - \mu) - \sqrt{1 + \frac{\sigma_\epsilon^2}{\sigma_k^2}}\Phi^{-1}(\gamma).$$

Also, condition (20) is arranged as:

$$\lambda = \left[R^{k*}\frac{\frac{L}{L-1} - m}{R - m} - 1\right] \frac{R - m}{\Phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right) R - m} \equiv \lambda(\sigma_\epsilon, \gamma),$$

where the equation for $(\bar{s}^* - R^{k*})/\sigma_\epsilon$ was used in deriving the final equivalence. The first-order conditions (21) and (22) are written as:

$$\begin{aligned}
0 &= \int_{R^{k*}}^{\infty} [R^k - (R^k - 1)m - R] dF(R^k) - \int_{R^{k*}}^{\bar{R}^k} \left[\lambda(Rx - m) + (L-1)\lambda R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right] dF(R^k), \\
0 &= -\int_{R^{k*}}^{\infty} (R^k - 1) dF(R^k) + \lambda \int_{R^{k*}}^{\bar{R}^k} \left(1 - R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m}\right) dF(R^k),
\end{aligned}$$

where

$$\begin{aligned}
\mu =& \bar{s}^* - \sigma_\epsilon \Phi^{-1}\left(\frac{m}{R}\right), \\
\frac{\partial x}{\partial \bar{s}^*} =& \phi\left(\frac{\bar{s}^* - R^k}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}, \\
\frac{\partial \bar{s}^*}{\partial L} =& \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{[L-m(L-1)]^2}\left(1+\lambda\frac{xR-m}{R-m}\right)}{1-\frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R}{\frac{L}{L-1}-m}\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)}, \\
\frac{\partial \bar{s}^*}{\partial m} =& \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{[L/(L-1)-m]^2}\left[-(1+\lambda)\frac{L}{L-1}+(1+\lambda x)R\right]}{1-\frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R}{\frac{L}{L-1}-m}\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)}.
\end{aligned}$$

These two equations are solved for σ_ϵ and γ. In solving the simultaneous equations, σ_ϵ and γ have to satisfy conditions (23) and (24). Also, these parameters have to be such that the denominator of $\partial\bar{s}^*/\partial L$ is positive. With σ_ϵ and γ at hand, parameter λ is determined. Finally, y is set to satisfy equation (1), i.e.

$$y = (L-1)\,n + \frac{1}{[R\,(1-P+\mathbb{E}(v|\text{default})P)]^\alpha},$$

where $\mathbb{E}(v|\text{default})P$ is given by:

$$R\mathbb{E}(v|\text{default})P = \int_{\underline{R}^k}^{R^{k*}}\left[R^k\left(\frac{L}{L-1}-m\right)+m-\lambda(Rx-m)\right]dF + \int_{-\infty}^{\underline{R}^k}\left[\frac{1}{1+\lambda}R^k\left(\frac{L}{L-1}-m\right)+m\right]dF.$$