

# Bank Runs, Prudential Tools and Social Welfare in a Global Game General Equilibrium Model

Daisuke Ikeda

Bank of Japan

26 November 2019

Macroeconomics Workshop of Keio University

The views expressed in this presentation are those of the author and should not be interpreted as those of the Bank of Japan

# Need a simple model of prudential tools

- 10 years since the crisis
- Financial regulatory reforms such as Basel III
- No consensus about a model that can guide policymakers
- We are looking for a simple model
- Three essential ingredients:
  - 1 Systemic risk event
  - 2 Financial system resilience
  - 3 Sources of inefficiencies

# Bank runs as a systemic risk event

- Most of the crises feature bank runs (Gorton 2012)
- Financial panics as the culprit of the Great Recession (Bernanke 2018)

# What I did

- Developed a two-period general equilibrium model that features
  - ① Bank runs in a global game framework (systemic risk event)
  - ② Endogenous probability of bank runs (banking system resilience)
  - ③ Some sources of inefficiencies
- Conducted welfare analyses and studied prudential instruments:
  - Leverage restriction (capital requirement)
  - Liquidity requirement
  - Bank-specific/sectoral capital requirement
  - Restriction on concentration risk

# Main results

- ① Excessive bank leverage and insufficient liquidity  
⇒ Too high systemic risk
- ② Two sources of inefficiencies
  - ① Risk shifting (Jensen and Meckling 1976)
  - ② Pecuniary externalities (Christiano and Ikeda 2016)
- ③ Multiple tools needed; otherwise, risk migration
- ④ General equilibrium effect: which tool is more effective?
- ⑤ Applications
  - Bank-specific/sectoral capital requirements and risk weights
  - Concentration risk
  - Deposit insurance

# Today's presentation

- Two-period model with leverage only (Ikeda 2019)
- Infinite horizon dynamic model of leverage (work in progress)

## Related literature

- Global game bank run models
  - **Rochet and Vives (2004)**
  - Goldstein and Pauzner (2005)
- A two-period general equilibrium model with financial frictions
  - **Christiano and Ikeda (2013, 2016)**
- Closely related papers
  - **Gertler and Kiyotaki (2015)**
  - Kashyap et al. (2017); Vives (2014); Kara and Ozsoy (2016)
  - Allen and Gale (2017)  
'The literature on liquidity regulation is still at an early stage.'

# Outline

- 1 Two-period model with leverage only
- 2 Analytical results
- 3 Dynamic model
- 4 Numerical results
- 5 (Supplementary material)



# The two-period model: Overview

## Households



- Consume
- Save

## Banks



- Take in deposits
- Risky lending

## Fund managers



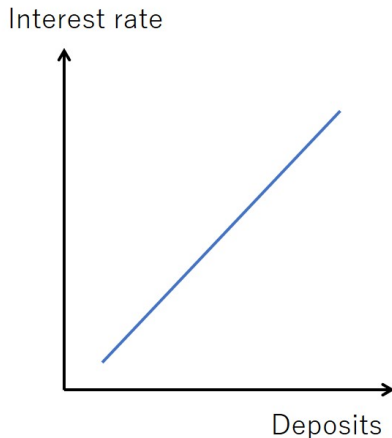
- Decide run or not
- Have private info

# Households



- Price taker
- Exogenous income
- Utility over consumption in periods 1 and 2
- Deposit contract
- Aware of bank default risk
- Owner of banks

## Supply curve of funds



## Households: analytical expression

$$\max_{\{c_1, c_2, d\}} u(c_1) + \mathbb{E}(c_2),$$

s.t.

$$c_1 + d \leq y, \quad c_2 \leq vRd + \pi,$$

where

$$v = \begin{cases} 1 & \text{with prob. } 1 - P \text{ (no bank default)} \\ < 1 & \text{with prob. } P \text{ (bank default)} \end{cases}$$

Solution: supply curve of funds:

$$R = \frac{u'(y - d)}{1 - P + \mathbb{E}(v|\text{default})P}$$

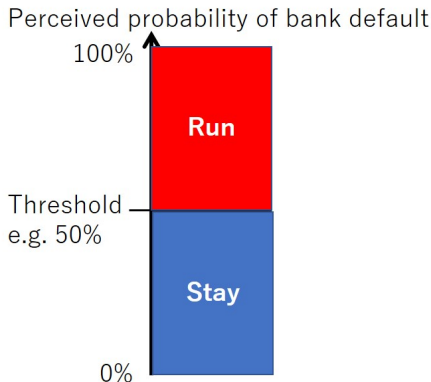
# Fund managers



- Risk neutral
- Private info about bank return (normally distributed)
- Decide run or not following a behavioral rule (Rochet and Vives 2004)
- Run iff perceived probability  $> \gamma$

[Link](#)

## Fund managers' behaviour



## How do they form perceived probability of bank default?

- $x = \#$  of fund managers who run;  $R^k =$  bank return;  $L =$  leverage
- Early withdrawal of  $xRd$
- **Costly liquidation:** banks have to sell  $(1 + \lambda)xRd/R^k$  units of assets
- After the liquidation, the bank has  $R^k(n + d) - (1 + \lambda)xRd$  in hand
- The banks still has to pay  $(1 - x)Rd$  to remaining depositors
- The bank defaults iff  $R^k < R^{k*}$  where

$$R^{k*} = R \left( 1 - \frac{1}{L} \right) (1 + \lambda x)$$

## Fund managers: analytical expression

- Withdraw iff  $P_i > \gamma$
- $R^k$  = bank return;  $L$  = leverage;  $x$  = # of fund managers who run
- Private information:  $s_i = R^k + \epsilon_i$
- Threshold strategy: withdraw if private info  $s_i < \bar{s}$
- Equilibrium threshold  $\bar{s} = \bar{s}^*$ :

$$Pr(R^k < R^{k*} | \bar{s}^*) = \gamma,$$

$$R^{k*} = R \left( 1 - \frac{1}{L} \right) \left[ 1 + \lambda x(R^{k*}, \bar{s}^*) \right],$$

$$x(R^{k*}, \bar{s}^*) = Pr(R^{k*} + \epsilon_i < \bar{s}^*)$$

- Limit case in which private info becomes infinitely accurate:

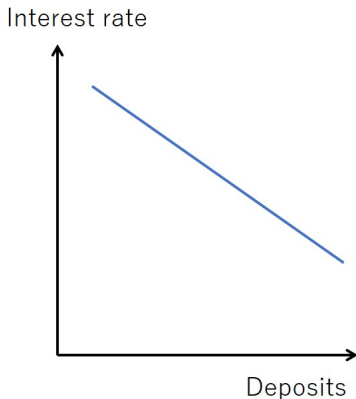
$$\bar{s}^* = R^{k*} = R \left( 1 - \frac{1}{L} \right) [1 + \lambda(1 - \gamma)]$$

# Banks



- Exogenous bank capital
- Simple debt contract; deposit interest rate independent of leverage or liquidity (Acharya 2009)
- Tradeoff: higher leverage
  - Higher return on equity
  - Higher default probability

## Demand curve for funds



## Banks: analytical expression

- Bank defaults iff  $R^k < R^{k^*}(L)$
- Deposits  $d = (L - 1)n$ , where  $n$  is bank capital
- Bank's problem:

$$\mathbb{E}(\pi) = \max_{\{L\}} \int_{R^{k^*}(L)}^{\infty} \left\{ R^k L - R \left[ 1 + \lambda x \left( R^k, \bar{s}^*(L) \right) \right] (L - 1) \right\} n dF(R^k).$$

- Optimality condition:

$$0 = \int_{R^{k^*}}^{\infty} (R^k - R) dF(R^k) - R\lambda \int_{R^{k^*}}^{\infty} x \left( R^k, \bar{s}^*(L) \right) dF(R^k),$$
$$- R\lambda (L - 1) \int_{R^{k^*}}^{\infty} \frac{\partial x \left( R^k, \bar{s}^* \right)}{\partial \bar{s}^*} \frac{\partial \bar{s}^*(L)}{\partial L} dF(R^k)$$



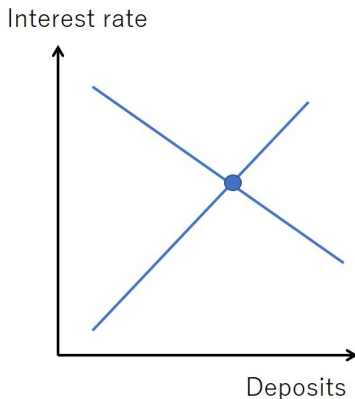
# Competitive equilibrium



## Endogenous variables

- Consumption  $c_1, c_2$
- Deposits  $d$
- Leverage  $L$
- Interest rate  $R$
- Recovery rate  $v$
- **Bank run probability  $P$**

## Demand and supply curve for funds



Market clearing:

$$d = (L - 1)n$$

# Competitive equilibrium: analytical expression

- Household optimality condition:

$$R = \frac{u'(y - (L - 1)n)}{1 - P + \mathbb{E}(v|\text{default})P}$$

- Bank optimality condition:

$$0 = \int_{R^{k*}}^{\infty} (R^k - R) dF(R^k) - R\lambda \int_{R^{k*}}^{\infty} x(R^k, \bar{s}^*(L)) dF(R^k),$$
$$-R\lambda(L - 1) \int_{R^{k*}}^{\infty} \frac{\partial x(R^k, \bar{s}^*)}{\partial \bar{s}^*} \frac{\partial \bar{s}^*(L)}{\partial L} dF(R^k)$$

- Recovery rate

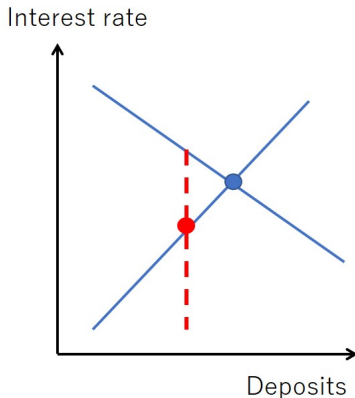
$$v = \min \left\{ 1, \max \left\{ \frac{R^k}{R} \frac{L}{L - 1} - \lambda x(R^k, \bar{s}^*), \frac{1}{1 + \lambda} \frac{R^k}{R} \frac{L}{L - 1} \right\} \right\}$$

# Regulator's problem



- Leverage too high?
- Liquidity too low?
- Bank run probability too high?
- Improve social welfare?
- Sources of inefficiencies?

## Leverage restriction



## Regulator's problem: analytical expression

- Regulator sets leverage and liquidity (liquidity-deposit ratio)
- Otherwise, everything is the same as competitive equilibrium
- Regulator does so to maximize social welfare:

$$\max_{\{L,m\}} SW = u(c_1) + \mathbb{E}(c_2),$$

subject to

Household optimality conditions

Bank run risk (fund managers' behaviour)

# Outline

- 1 Two-period model with leverage only
- 2 Analytical results
- 3 Dynamic model
- 4 Numerical results
- 5 (Supplementary material)

# Analytical result 1: Elevated bank run probability

## Proposition

*In a competitive equilibrium:*

- *Leverage is excessive, given any choice of liquidity*
- *(Liquidity is insufficient, given any choice of leverage)*
- *Consequently, bank run probability is too high*

## Policy implications

- Need leverage restriction
- (Need liquidity requirement)
- **(Need both)**

## Analytical results 2: Sources of inefficiencies

### Proposition

*There are two sources of inefficiencies:*

- 1 *Bank risk shifting: affects both leverage and liquidity*
- 2 *Pecuniary externalities: affect only leverage*

### Intuition

- 1 Risk shifting: banks do not internalize the effects of their choice of riskiness (leverage and liquidity) on the (risky) interest rate
- 2 Pecuniary externalities: costs associated with bank runs depend on the (risk-neutral) interest rate (households' willingness to supply funds)

# Outline

- 1 Two-period model with leverage only
- 2 Analytical results
- 3 **Dynamic model**
- 4 Numerical results
- 5 (Supplementary material)



# Overview: dynamic model

- Real business cycle model with banks
- Banks intermediate funds from households to firms

$$k_{t+1} = n_t + d_t \quad \text{(Demand for capital)}$$

- Bank runs as in the two-period model
- Capital depreciation (quality) shock

$$k_{t+1} = (1 - \delta)\xi_t k_t + i_t \quad \text{(Supply of capital)}$$

# Households

- GHH preferences

$$E_0 \sum_{t=0}^{\infty} \beta \log \left( c_t - \psi \frac{h_t^{1+1/\nu}}{1+1/\nu} \right), \quad \psi, \nu > 0$$

- Budget constraint

$$c_t + d_t \leq R_t d_{t-1} + w_t h_t + \Theta_t,$$

- Deposit interest rate

$$R_t = \begin{cases} \bar{R}_{t-1} & \text{if banks do not default} \\ v_t \bar{R}_{t-1} & \text{if banks default} \end{cases}$$

where  $0 \leq v_t < 1$  is the recovery rate.

# Firms

- Cobb-Douglas production function

$$y_t = k_t^\alpha h_t^{1-\alpha}, \quad 0 < \alpha < 1$$

- Factor prices

$$r_t^k = \alpha y_t / k_t,$$
$$w_t = (1 - \alpha) y_t / h_t$$

- Law of motion for capital

$$k_{t+1} = (1 - \delta) \xi_t k_t + i_t, \quad 0 < \delta < 1$$

- Capital depreciation (quality) shock

$$\xi_t - \xi = \rho_\xi (\xi_{t-1} - \xi) + \epsilon_{\xi,t}, \quad 0 < \rho_\xi < 1$$

- Good market clearing

$$y_t = c_t + i_t$$

# Fund managers

- Risk neutral fund managers

- Private information  $s_{i,t+1}$  about bank asset return  $R_{t+1}^k$

$$s_{i,t+1} = R_{t+1}^k + \epsilon_{i,t+1}, \quad \epsilon_{i,t+1} \sim N(0, \sigma_\epsilon^2)$$

- $P(s_{i,t+1})$ : probability of bank default perceived by fund manager  $i$
- Fund managers withdraw funds iff

$$P(s_{i,t+1}) > \gamma, \quad 0 < \gamma < 1$$

# Banks

- Bank balance sheet

$$k_{t+1} = n_t + d_t$$

- Bank asset return

$$R_{t+1}^k = \underbrace{r_{t+1}^k}_{\text{pre-determined}} + (1 - \delta)\xi_{t+1}$$

- Expected bank asset return:  $R_{t+1|t}^k = r_{t+1}^k + (1 - \delta)E_t\xi_{t+1}$
- $R_{t+1}^k \sim N(R_{t+1|t}^k, \sigma_{R^k}^2)$  where  $\sigma_{R^k} = (1 - \delta)\sigma_\xi$
- Early liquidation: banks sell assets to households at discounted prices
- Bank problem is the same as in the two-period model

## Banks (cont'd)

- Bank net profits

$$\pi_t^b = R_t^k L_{t-1} n_{t-1} - \bar{R}_{t-1} (L_{t-1} - 1) n_{t-1} - n_{t-1}$$

- Banks remit a fraction  $1 - \chi_0$  of net profits to households
- After the remittance, a fraction  $1 - \chi_1$  of bankers become workers
- The same number of workers becomes bankers with net worth  $n_0$
- If banks have defaulted, the government injects bank capital  $\bar{n}$ , financed by lump-sum taxes on households
- Law of motion for bank net worth

$$n_t = \begin{cases} \chi_1 \left\{ \chi_0 \pi_t^b + (1 - \chi_0) \pi_t^b \mathbf{1}_{\{\pi_t^b < 0\}} + n_{t-1} \right\} + n_0 & \text{if no default} \\ \bar{n} & \text{if default} \end{cases}$$

# Limit equilibrium

- Limit equilibrium  $\sigma_\epsilon \rightarrow 0$
- Bank run (default) iff  $R_{t+1}^k < R_{t+1}^{k*}$

$$R_{t+1}^{k*} = \bar{R}_t \left( 1 - \frac{1}{L_t} \right) [1 + \lambda(1 - \gamma)]$$

- Probability of bank default

$$P_t = \Phi \left( \frac{R_{t+1}^{k*} - R_{t+1|t}^k}{\sigma_{R^k}} \right)$$

# Outline

- 1 Two-period model with leverage only
- 2 Analytical results
- 3 Dynamic model
- 4 Numerical results
- 5 (Supplementary material)



# Numerical solution

- Target values in stochastic SS

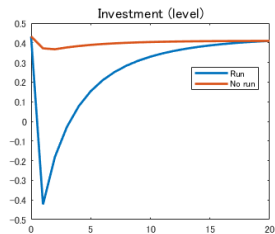
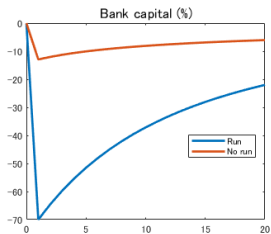
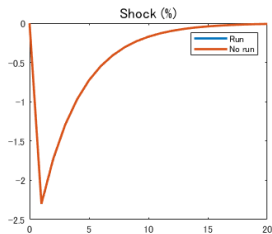
$$L = 15, \sigma_{R^k} = 0.01, P = 0.05/4, h = 1.$$

- Other parameters

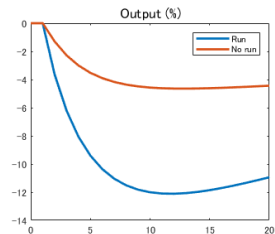
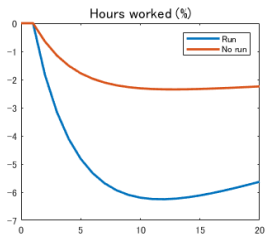
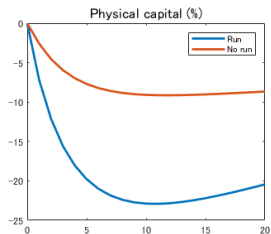
$$\beta = 0.995, \nu = 1, \alpha = 0.33, \delta = 0.025, \\ \lambda = 0.5, \chi_0 = 0.05, \chi_1 = 0.95, \rho_\xi = 0.75$$

- Four state variables:  $k_t, \bar{R}_{t-1}, n_{t-1}, \xi_t$
- Solution method: parameterized expectations
- Simulate  $T = 5000$  periods;  $\mathcal{R}^2 = 0.9995$ .

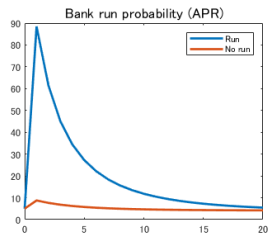
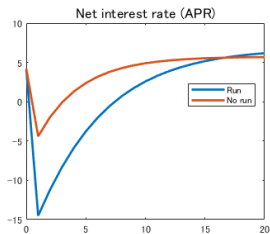
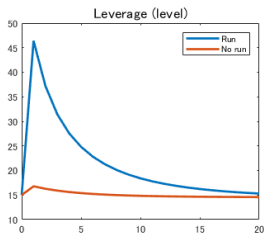
# Impulse responses: shock, bank capital, and investment



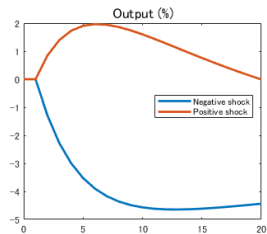
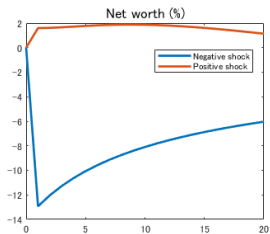
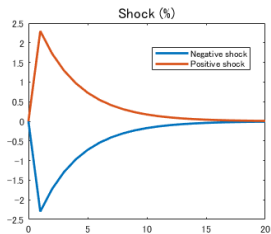
# Impulse responses: capital, hours, and output



# Impulse responses: leverage, interest rate, and crisis Pr



# Asymmetric responses: no bank run



## Conclusion: future work

- More micro-foundations for the bank's problem (Mendicino et al. forthcoming)
- Introduce adverse selection and an idiosyncratic shock to endogenize  $\lambda$  (fire sale parameter) and the number of defaulted banks
- More numerical results on the dynamic model
- Macroprudential policy: capital requirements; CCyB

**Thank you**

# Outline

- 1 Two-period model with leverage only
- 2 Analytical results
- 3 Dynamic model
- 4 Numerical results
- 5 (Supplementary material)



# Interpretation of $\gamma$

- Fund manager's action and payoff

Action \ States	No bank default	Bank default
Not withdraw	$R$	$vR - \Gamma_1$
Withdraw	$R - \Gamma_0$	$vR - \Gamma_0$

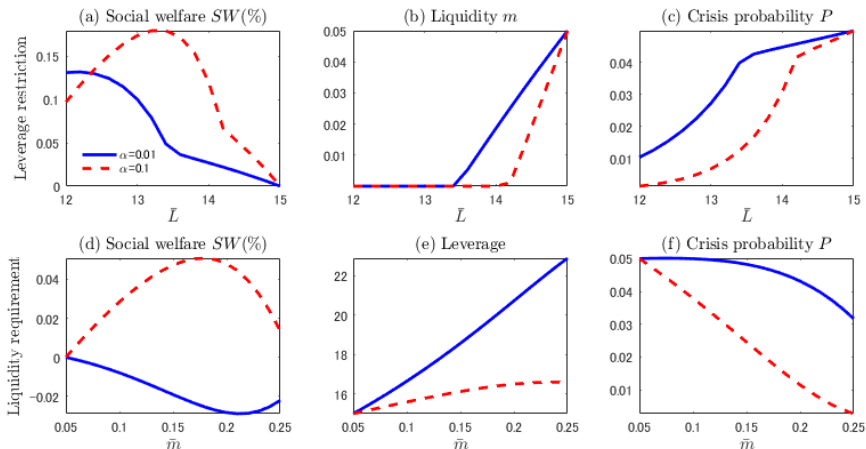
- Not withdraw:  $(1 - P_i)R + P_i(vR - \Gamma_1)$
- Withdraw:  $(1 - P_i)(R - \Gamma_0) + P_i(vR - \Gamma_0)$
- Withdraw iff  $P_i > \Gamma_0/\Gamma_1 \equiv \gamma$
- Costs  $\Gamma_0, \Gamma_1 \rightarrow 0$

Return

# Parameterization for numerical analyses

- US banks 2008-2017 (Miller and Sowerbutts 2018)
- Target values
  - Leverage = 15
  - Liquidity ratio relative to deposits = 5%
  - Crisis probability = 5% (BCBS 2010)
  - Deposit interest rate = 2%
- Average bank asset return = 3.5% (after-taxed RoE = 15%)
- Standard deviation of bank asset return = 2.5%
- Supply curve of funds: relatively flat or steep

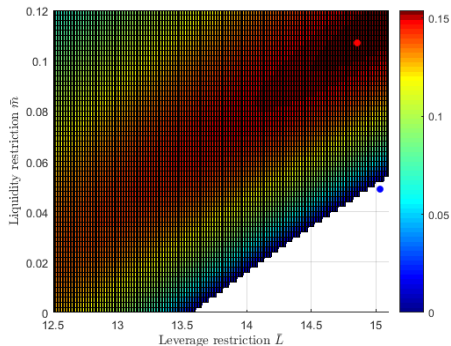
# Risk migration: leverage or liquidity requirements only



- Risk migrates from one area to another
- Tightening liquidity requirement worsens welfare when the supply curve is relatively flat ( $\alpha = 0.01$ ).

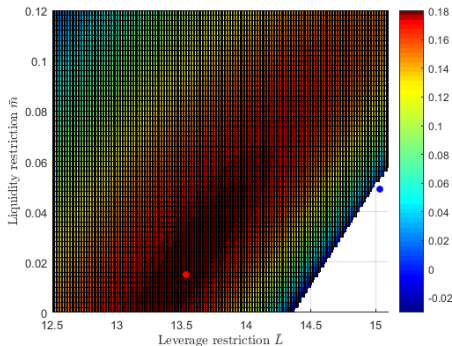
# Joint effects of leverage and liquidity requirements on social welfare

$\alpha = 0.01$  (flat supply curve)



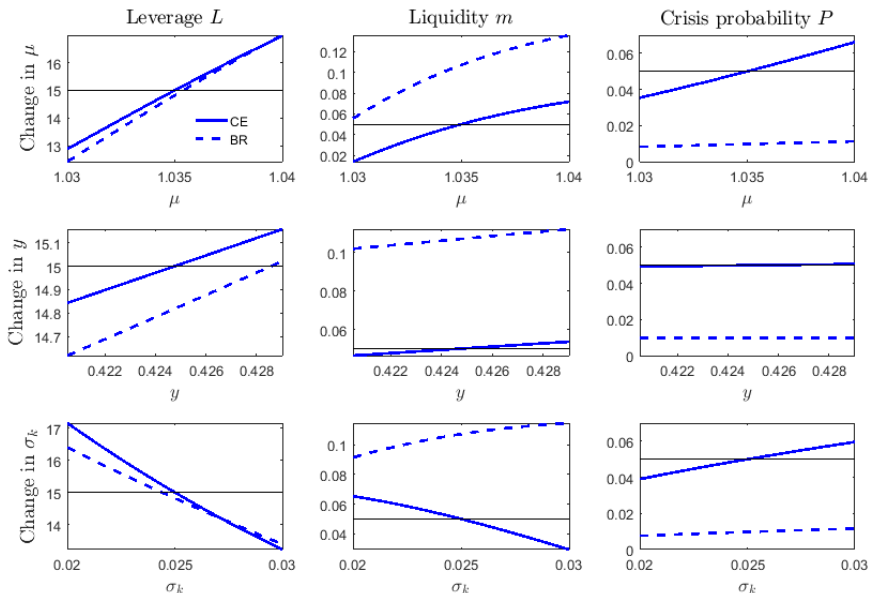
- Liquidity requirement is more tightened.

$\alpha = 0.1$  (steep supply curve)



- Leverage restriction is more tightened.

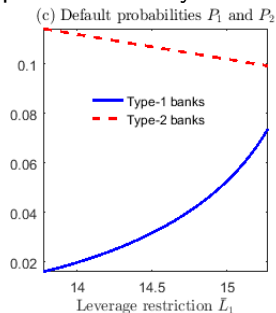
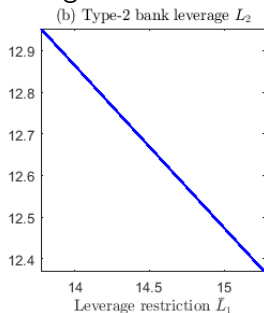
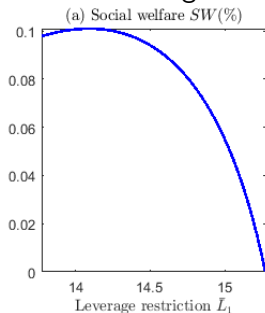
# Comparative statics: constrained optimal allocation



# Application 1: Regulated banks and shadow banks

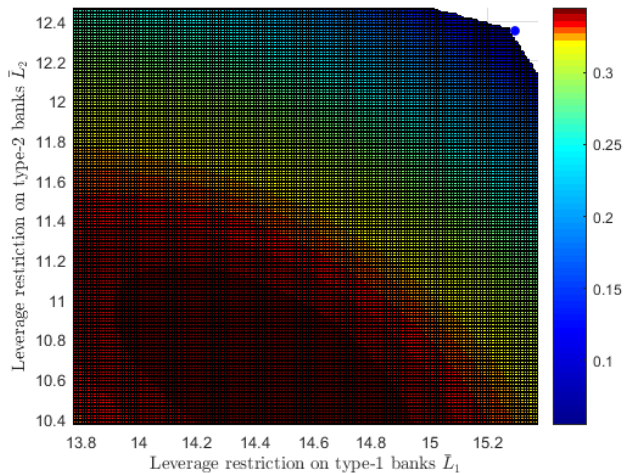
- Two types of banks; leverage choice only
- Type- $j$  bank specializes in lending to sector  $j \in \{1, 2\}$
- Sector 2 is riskier than sector 1

## Risk migration: leverage restriction on type-1 banks only



# Application 1 (cont'd): Bank-specific/sectoral capital requirements

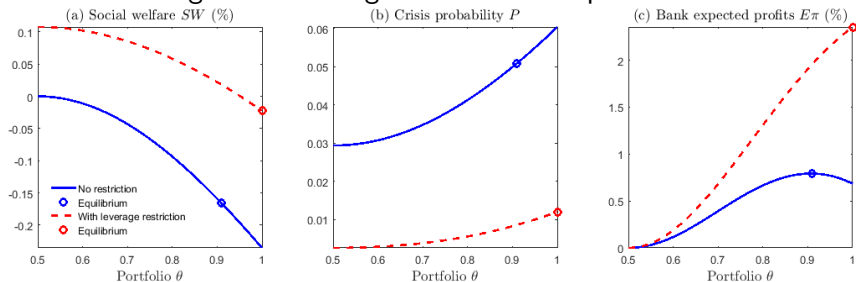
Joint effects of type-specific leverage restrictions



## Application 2: Concentration risk

- One type of banks; leverage and portfolio choices
- Identical and independent two types of lending
- Portfolio  $[0.5, 0.5]$  minimizes the riskiness of bank assets

### Risk migration: leverage restriction and portfolio choice





# Recap: macroprudential perspective

- Leverage and liquidity
- Regulated banks and shadow banks
- Leverage and portfolio choice

