Expectation Effects of Switching Financial Frictions

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Main References

- A New Approach to Regime Switching
  - Chang, Choi and Park (2017) A New Approach to Model Regime Switching, *Journal of Econometrics*, 196, 127-143.
- Endogenous Policy Shifts in a Simple DSGE Model
  - Chang, Tan and Wei (2018) A Structural Investigation of Monetary Policy Shifts
  - Chang, Maih and Tan (2018) State Space Models with Endogenous Regime Switching
- DSGEs with Financial Friction
  - Christiano, Motto, and Rostagno (2014) Risk Shocks, AER
  - Linde, Smets, and Wouters (2016) Challenges for Macro Models Used at Central Banks, Macro Handbook

Research Question

How does switching financial condition affect macroeconomic variables?

- Emphasize expectation effect: outlook of financial market characterized by state transition probabilities.

- Rational expectation: use "optimistic" and "pessimistic" to highlight difference in transition matrices, not subjective beliefs.

- Focus on risk/uncertainty shock, and investment.

Motivation

- Connect the business cycle and the financial cycle (Claessens, Kose and Torrens [2012, JIE], among others).
- Christiano, Motto and Rostagno (2014, AER) shows risk-uncertainty shock in the financial market is the primary driver of the US business cycle.
    1. Entrepreneurs subject to idiosyncratic capital efficiency shocks. Will default if efficiency level is too low (a fixed threshold in equilibrium).
    2. Banks must pay monitoring cost to observe the efficiency level (the defaulting ones in equilibrium). Must charge a risk premium in addition to the risk-free rate.
    3. Higher risk (dispersion of the idiosyncratic efficiency shocks), higher default rate, higher premium, and vice versa.

Motivation: Regime Switching

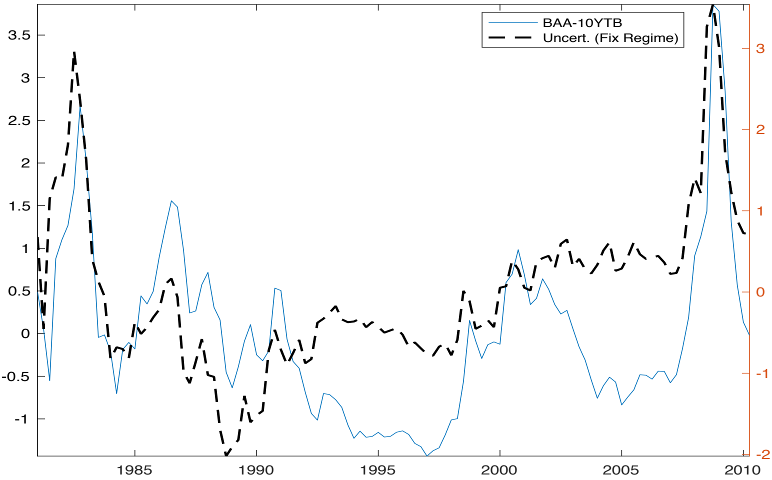
Anecdotal

- Spread data present significant and recurrent swings.
- Rises in recessions and declines in expansions.
- Credit conditions drastically loosened leading up to the recent financial crisis.

Quantitative

- Fixed-regime model unable to explain the dynamics of spread.
- Fixed-regime model reveals a disconnect between credit spreads and uncertainty in the mid-1990s and mid-2000s.
- Fixed-regime model generates risk spikes in both the 1982 and 2008 recessions with similar size, but the aftermaths are notably different.

Credit Spread and Idiosyncratic Uncertainty



Smoothed uncertainty under fixed regime.

Main Results

We find

- Expectation effect is quantitatively important through the lens of a conventional regime switching DSGE model with constant transition probabilities (RS-DSGE).
- Evidence of time-varying outlook of financial market condition
- Novel findings from a RS-DSGE with feedback to state transition probability matrix:
  - Historical shocks drive regime shift almost exclusively ($> 99\%$).
  - Zero to negative feedback from demand shocks, except inflation target shocks.
  - Positive feedback from supply shocks, except persistent TFP shocks.

Selected Literature

- Uncertainty shocks: Bloom (2009, Ecta), Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry (2018, Ecta).
- Uncertainty in medium-Scale DSGE: Christiano, Motto, Rostagno (2014, AER), Del Negro et al (2015, AEJ), Del Negro and Schorfheide (2016, Handbook), Lindé, Smets and Wouters (2016, Handbook).
- Expectation Effect: Leeper and Zha (2003, JME), Liu, Waggoner and Zha (2011, QE), Bianchi (2013, RES).

The Model

- Simplified CMR with synchronized
  - switching risk process
  - switching monitoring cost
- Real sector: Smets and Wouters (2007), standard neoclassical model plus
  - Price and wage rigidity
  - Consumption habit formation
  - Investment adjustment cost
  - Variable capital utilization and adjustment cost
- Financial sector: Bernanke, Gertler and Gilchrist (1999), financial accelerator in a business cycle with
  - Costly state verification
  - Idiosyncratic uncertainty in producing effective capital
  - One-period optimal contract between banks and entrepreneurs

The Model: Household

Representative household solves

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_{c,t} \left\{ \log(C_t - bC_{t-1}) - \psi_L \int_0^1 \frac{h_t(i)^{1+\sigma_L}}{1+\sigma_L} di \right\}$$

with β the discount factor, b the habit parameter, σ_L^{-1} the Frisch elasticity of labor, ψ_L the labor disutility parameter, $\zeta_{c,t}$ a preference shock, C_t the consumption, and $h_t(i)$ the differentiated labor, s.t.

$$P_t C_t + B_{t+1} + \left(\frac{P_t}{\Upsilon^t \mu_{\Upsilon,t}} \right) I_t + Q_{\bar{K},t}(1-\delta)\bar{K}_t$$

$$= \int_0^1 W_t(i) h_t(i) di + R_t B_t + Q_{\bar{K},t} \bar{K}_{t+1} + \Pi_t$$

where $\mu_{\Upsilon,t}$ is a investment technology shock, P_t is the nominal price for C_t, B_t the nominal bond with rate of return R_t, I_t the investment good, $\bar{K}_t$ the physical capital with price $Q_{\bar{K},t}$, $\bar{K}_{t+1}$ the end-of-period physical capital, $W_t(i)$ the wage for $h_t(i)$, and Π_t the lump-sum transfer of dividend payment after taxation.

The Model: Household as Capital Producer

The household produces $\bar{K}_{t+1}$ by translating one unit of C_t into $\Upsilon^t \mu_{\Upsilon,t}$ units of investment good I_t with a constant growth rate $\Upsilon > 1$ using technology

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \left(1 - S\left(\zeta_{I,t}\frac{I_t}{I_{t-1}}\right)\right) I_t, \quad \delta \in (0,1)$$

where $S(\cdot)$ is an adjustment cost of form

$$S(x_t) = \left[e^{\sqrt{S''}(x_t - x_{ss})} + e^{-\sqrt{S''}(x_t - x_{ss})} - 2\right] \Big/ 2, \quad x_t = \zeta_{I,t} I_t / I_{t-1}.$$

with x_{ss} the steady state value, and $\zeta_{I,t}$ the shock to the marginal efficiency of investment (MEI shock) and S'' is the cost of (dis)investing away from the steady state.

The Model: Final Good Packer

Competitive final good packer combines the intermediate goods $Y_t(j)$ for $j \in [0, 1]$ to produce homogeneous good Y_t with technology

$$Y_t = \left[\int_0^1 Y_t(j)^{1/\lambda_{f,t}} \right]^{\lambda_{f,t}}$$

where $\lambda_{f,t} \geq 1$ is the price markup shock. The j-th intermediate good is produced by a monopolist with production function

$$Y_t(j) = \max \left\{ 0, \epsilon_t K_t(j)^{\alpha} (z_t l_t(j))^{1-\alpha} - \Phi z_t^* \right\}$$

with a stationary shock ϵ_t and a shock of stationary growth z_t (permanent technological shock). $K_t(j)$ is the effective capital proportional to $\bar{K}_t(j)$. $l_t(j)$ is the labor employed by the producer j. The fixed cost Φz_t^* ensures zero long-run profit. To ensure balance growth, $z_t^* = z_t \Upsilon^{(\alpha/(1-\alpha))t}$.

The Model: Labor Packer

Competitive labor packer demands differentiated labor service $h_t(i)$ for $i \in [0, 1]$ and combines them into homogeneous labor with technology

$$l_t = \left[\int_0^1 h_t(i)^{1/\lambda_w} di \right]^{\lambda_w},$$

with wage markup parameter $\lambda_w \geq 1$. The labor packer then sells l_t to the intermediate good producers for nominal wage W_t.

The Model: Calvo Pricing in Goods and Labor Markets

- j-th intermediate good producer reoptimize $P_t(j)$ with probability $1 - \xi_p$.
  - The probability ξ_p characterizes the price rigidity of the intermediary good market.
  - The inflation rate of Y_t is $\pi_t = P_t / P_{t-1}$, and π_t^* denotes the inflationary target in the monetary policy rule.
  - With probability ξ_p, the producer set $P_t(j) = \tilde{\pi}_t P_{t-1}(j)$ where indexation factor $\tilde{\pi}_t = (\pi_t^*)^{\iota} (\pi_{t-1})^{1-\iota}$.
- i-th differentiated labor producer reoptimize $W_t(i)$ with probability $1 - \xi_w$.
  - The probability ξ_w characterizes the wage rigidity in the differentiated labor market.
  - With probability ξ_w, $W_t(i) = (\mu_{z^*,t})^{\iota_\mu} (\mu_{z^*})^{1-\iota_\mu} \tilde{\pi}_{w,t}$, where μ_{z^*} is the growth rate of z_t^* in the deterministic steady state, and $\tilde{\pi}_{w,t} = (\pi_t^*)^{\iota_w} (\pi_{t-1})^{1-\iota_w}$.

The Model: Entrepreneur and Financial Friction 1

After the production at t, an entrepreneur with net worth $N \geq 0$ borrows $B_{t+1}(N)$ from the banks to purchase $\bar{K}_{t+1}(N)$ following

$$Q_{\bar{K},t}\bar{K}_{t+1}(N) = N + B_{t+1}(N)$$

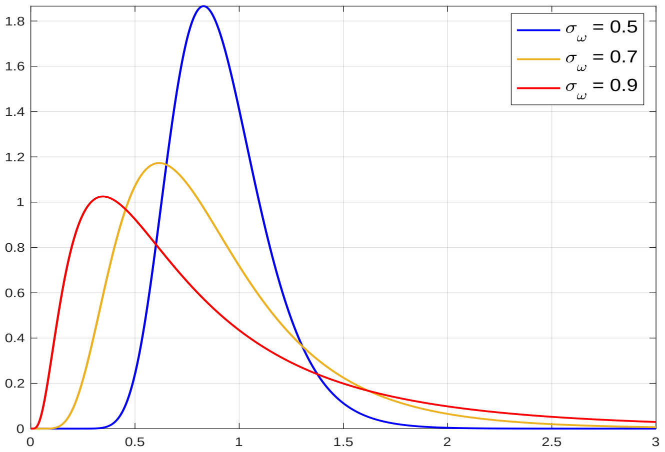
and turn it into effective capital $K_{t+1}(N) = \omega \bar{K}_{t+1}(N)$.

The efficiency level of capital is random and is distributed as

$$\omega_t \sim \text{log-normal}\left(-\frac{\sigma_{\omega,t}^2}{2}, \sigma_{\omega,t}^2\right)$$

to ensure a unit mean. $\sigma_{\omega,t}$ denotes the risk/uncertainty process.

Distribution of Efficiency (ω_t)



$\mathbb{E}(\omega_t) = 1$ and $\text{var}(\omega_t) = \exp(\sigma_\omega^2) - 1$. Larger σ_ω, higher risk.

The Model: Entrepreneur and Financial Friction 2

Given return rates, price and efficiency shock, entrepreneur chooses the utilization rate u_{t+1} of the effective capital to maximize the return of capital for a competitive rate r_{k+t}^{k}.

The *ex post* rate of return of the entrepreneur is

$$R_{t+1}^{k} = \frac{[u_{t+1} r_{t+1}^{k} - a(u_{t+1})] \Upsilon^{-(t+1)} P_{t+1} + (1-\delta) Q_{\bar{K},t+1}}{Q_{\bar{K},t}}$$

where $a(u_{t+1})$ is the adjustment cost

$$a(u) = r^{k} \left[\exp(\sigma_a (u-1)) - 1\right] / \, \sigma_a .$$

The curvature parameter $\sigma_a > 0$ characterizes the utilization cost and r^k is the steady state rental rate in the model.

The Model: Entrepreneur and Financial Friction 3

- Entrepreneurs will default if realized efficiency is too low.
- Banks must pay μ proportional to entrepreneur's realized return to reclaim the remaining value of defaulting entrepreneurs (ignore the repaying ones at equilibrium).
- Let $\bar{\omega}_{t+1}$ be the threshold that divides the repaying entrepreneurs and the defaulting ones. Must demand a rate Z_t s.t.

$$R^k_{t+1}\bar{\omega}_{t+1}Q_{\bar{K},t}\bar{K}_{t+1}(N) = B_{t+1}(N)Z_{t+1}.$$

- The law of motion of net worth after receiving transfer W^e follows

$$N_{t+1}(N) = \gamma_t \left[R^k_t Q_{\bar{K},t-1}\bar{K}_t(N) - Z_t(Q_{\bar{K},t-1}\bar{K}_t(N) - N)\right] + W^e$$

with γ_t is the shock to net worth of entrepreneurs (equity shock).

The Model: Entrepreneur and Financial Friction 4

The entrepreneur choose $\bar{\omega}_{t+1}, \bar{K}_{t+1}$ to optimize expected return

$$\max \mathbb{E}_t \left\{ \left[1 - \Gamma_t(\bar{\omega}_{t+1})\right] R^k_{t+1} Q_{\bar{K},t} \bar{K}_{t+1} \right\}$$

s.t. the bank's zero-profit condition

$$\left[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right] R^k_{t+1} Q_{\bar{K},t} \bar{K}_{t+1} = R_{t+1} B_{t+1}$$

with the expected monitoring cost for banks and the lenders' expected gross share of profit, respectively,

$$\begin{aligned} \mu G_t(\bar{\omega}_{t+1}) &= \mu \Phi(m_t - \sigma_{\omega,t}) \\ \Gamma_t(\bar{\omega}_{t+1}) &= G_t(\bar{\omega}_{t+1}) + \bar{\omega}_{t+1}(1 - \Phi(m_t)) \end{aligned}$$

where $\Phi(\cdot)$ is the standard normal CDF and

$$m_t = \left(\log \bar{\omega}_{t+1} + \frac{1}{2}\sigma^2_{\omega,t} \right) \Big/ \sigma_{\omega,t}.$$

The Model: Monetary, Fiscal Policy

- MP: (linearized)

$$
\begin{aligned}
R_t - R &= \rho_p(R_{t-1} - R) \\
&+(1-\rho_p)\left[\alpha_\pi(\pi_{t+1} - \pi_t^*) + \alpha_{\Delta y}\frac{1}{4}(\Delta y_t - \mu_{z^*})\right] \\
&+\frac{1}{400}\sigma_{e,p} e_{p,t}
\end{aligned}
$$

  with ρ_p the smoothing parameter, $e_{p,t}$ the MP shock, R the s.s. quarterly interest rate, Δy_t the quarterly GDP growth.

- FP:

$$
G_t = z_t^* g_t
$$

  with g_t the FP shock, and Y_t/z_t^* converges to a constant in the deterministic steady state.

The Model: List of Fundamental Shocks

Shocks	Label
ϵ_t	Transitory Technological Shock
z_t	Persistent Technological Shock
g_t	Government Spending Shock
$e_{p,t}$	Monetary Policy Shock
π_t^*	Inflation Target Shock
$\mu_{\Upsilon,t}$	Investment-Specific Shock
γ_t	Equity Shock
$\lambda_{f,t}$	Price Markup Shock
$\zeta_{c,t}$	Preference Shock
$\zeta_{i,t}$	Marginal Efficiency of Investment Shock
$\sigma_{\omega,t}$	Uncertainty Shock

Switching Risk Process

- $\sigma_{\omega,t}$ governs the dispersion of idiosyncratic capital efficiency shock.

- Regime-switching process

$$\log \sigma_{\omega,t} - \log \sigma(s_t) = \rho_\sigma(\log \sigma_{\omega,t-1} - \log \sigma(s_{t-1})) + v_t$$

  with the shock v_t correlated with other exogenous shocks at $t-1$.

- Regime $s_t \in \{1,2\}$. The $\sigma(s_t)$ is regime-dependent mean. Set $\sigma(1) < \sigma(2)$ for identification.

- Following CMR, estimate steady state default probability $F(\bar{\omega})_{s_t}$ which maps 1-1 to the steady state levels of risk. $(F(\bar{\omega})_1 < F(\bar{\omega})_2)$

Switching Monitoring Cost

- $\mu_t \in [0, 1]$ is the cost to acquire the information of private capital efficiency level as the percentage of the realized return to capital.
- $\mu_t = \mu(s_t)$. Note s_t determines the regime of both μ and σ.
- Do not impose an order for $\mu(1), \mu(2)$ to include all combinations of μ and σ.

Regimes, Regime Factor and Feedback

Regime factor w_t determines regime s_t

$$\begin{aligned} s_t &= 1 + 1\{w_t \geq \tau\}, \\ w_t &= \alpha_w w_{t-1} + \nu_t, \quad |\alpha_w| < 1. \end{aligned}$$

Feedback takes form

$$(\varepsilon_{t-1}, \nu_t)' \overset{i.i.d.}{\sim} \mathbb{N}\left(0, \begin{pmatrix} I & \rho_{\varepsilon,\nu} \\ \rho'_{\varepsilon,\nu} & 1 \end{pmatrix}\right)$$

with ε_{t-1} the column vector of fundamental shocks at $t-1$ and $\rho_{\varepsilon,\nu}$ the column vector of correlation coefficients for each pair of ε_{t-1} and ν_t satisfying $\rho'_{\varepsilon,\nu}\rho_{\varepsilon,\nu} < 1$.

Feedback Channel

Given positive shocks, expect demand and supply shocks of distinct effects ("+" means "increase w_t")

Parameters	Label	Anticipated Effect
$\rho_{v,z}$	persist. technological shock	$-$
$\rho_{v,\epsilon}$	transitory technological shock	$+$
$\rho_{v,\gamma}$	equity shock	$+$
ρ_{v,μ_Υ}	investment technology shock	$+$
ρ_{v,ζ_i}	MEI shock	$+$
$\rho_{v,\sigma}$	risk shock	$+$
ρ_{v,λ_f}	price markup shock	$-$
$\rho_{v,g}$	government spending shock	$-$
$\rho_{v,p}$	MP shock	$-$
ρ_{v,π^*}	inflation target shock	$-$
ρ_{v,ζ_c}	preference shock	$-$

Agents' Information Set

- Agents know s_t and the transition probability matrix at t. But regime factor w_t is latent to agents.
- Regime factor w_t with endogenous feedback introduces a specification of time-varying transition.

Time-Varying Transition

The time-varying transition matrix is characterized by

$$P_{1|1,t} = \frac{\int_{-\infty}^{\tau\sqrt{1-\alpha_w^2}} \Phi_{\rho_{\varepsilon,\nu}}\left(\tau - \frac{\alpha_w w}{\sqrt{1-\alpha_w^2}} - \rho'_{\varepsilon,\nu}\varepsilon_t\right) d\Phi(w)}{\Phi(\tau\sqrt{1-\alpha_w^2})}$$

$$P_{1|2,t} = \frac{\int_{\tau\sqrt{1-\alpha_w^2}}^{\infty} \Phi_{\rho_{\varepsilon,\nu}}\left(\tau - \frac{\alpha_w w}{\sqrt{1-\alpha_w^2}} - \rho'_{\varepsilon,\nu}\varepsilon_t\right) d\Phi(w)}{1-\Phi(\tau\sqrt{1-\alpha_w^2})}$$

with $\Phi(\cdot)$ be CDF of standard normal and

$$\Phi_{\rho_{\varepsilon,\nu}}(w) = \Phi\left(w/\sqrt{1-\rho'_{\varepsilon,\nu}\rho_{\varepsilon,\nu}}\right).$$

To fix idea...Assume zero feedback ($\rho_{\varepsilon,\nu} = 0$)

- $P_{1|1}$ and $P_{1|2}$ are time-invariant.
- The map $(\alpha_w, \tau) \mapsto (P_{1|1}, P_{1|2})$ is 1-1. Chang, Choi and Park (2017, JOE)
- (s_t) is Markovian and the model is of rational expectation.

Decomposition of Regime Factor Innovation

By normality,

$$\nu_t = \underbrace{\rho'_{\varepsilon,\nu}\varepsilon_{t-1}}_{\text{feedback}} + \sqrt{1 - \rho'_{\varepsilon,\nu}\rho_{\varepsilon,\nu}}\eta_t, \quad \eta_t \sim \mathbb{N}(0,1).$$

Variance-decomposition of ν_t

- $\rho^2_{\varepsilon_i,\nu}$ the % contribution of *i*-th shock to the regime factor.
- $\rho'_{\varepsilon,\nu}\rho_{\varepsilon,\nu}$ the total % contribution of all fundamental shocks to regime factor.

Solution

With $s_t = i$ and $s_{t+1} = j$, we look for regime-dependent policy functions

$$X_t = T_i(X_{t-1}, \varepsilon_t)$$

to solve for the system of equations of FOCs and constraints

$$0 = E_t \left[\sum_{j=1}^{2} p_{i,j} f_i(\underbrace{T_j(T_i(x_{t-1}, \varepsilon_t), \varepsilon_{t+1})}_{x_{t+1}}, \underbrace{T_i(x_{t-1}, \varepsilon_t)}_{x_t}, x_{t-1}, \varepsilon_t) \right]$$

Solution Method 1

Perturbation method by Maih and Waggoner (2018, Mimeograph), to the 1st order.

Features:

1. State-dependent policy function perturbed around state-dependent steady states $\bar{x}_i$.

2. Perturbation parameter σ in the transition matrix $p_{i,j}$, and perturbed around identity matrix for consistent interpretation of the approximate solution.

3. Feedback effect disappears in the 1st order solution. Can generate time-varying generalized IRF by probability weighting.

Solution Method 2

We assume solution

$$X_t = T_i(X_{t-1}, \sigma, \varepsilon_t)$$

of perturbation parameter $\sigma \in [0,1]$ and

- $T_i\left(x_{t-1}, \varepsilon_t\right) = T_i\left(x_{t-1}, 1, \varepsilon_t\right)$
- $T_i\left(\overline{x}_i, 0, 0\right) = \bar{x}_i$ (easy to solve)

to the system of equations

$$\begin{aligned} 0 =& E_t\left[\sum_{j=1}^{2} p_{i,j}(\sigma) f_i\left(T_j\left(T_i\left(x_{t-1}, \sigma, \varepsilon_t\right)\right.\right.\right. \\ &\left.\left. + (1-\sigma)\left(\overline{x}_j - eT_i\left(\overline{z}_i\right)\right), \sigma, \sigma\varepsilon_{t+1}\right), T_i\left(x_{t-1}, \sigma, \varepsilon_t\right), x_{t-1}, \varepsilon_t\right] \end{aligned}$$

where

$$p_{i,j}(\sigma) = \begin{cases} \sigma p_{i,j} & \text{for } i \neq j \\ 1 - \sigma\left(1 - p_{i,i}\right) & \text{for } i = j \end{cases}$$

Expectation Effect

- Assume zero feedback, we consider generalized IRF under a state transition matrix P, regime s_t and a scaler structural shock e_t:
  - $GI_x^P(k, s_t, e_t)$
- Define expectation effect as the difference between GIRFs for different state transition matrices P and P^*.
  - Expectation effect: $GI_x^P(k, s_t, e_t) - GI_x^{P^*}(k, s_t, e_t)$.
- Parameters set at fixed-regime estimates, except for the switching parameters.
- Simulate GIRFs for high risk regime ($s_t = 2$).

Calibrated Parameters (Quarterly)

Parameter	Label	Value
β	discount rate	0.9987
σ_L	curvature, disutility of labor	1.0000
ψ_L	disutility weight on labor	0.7705
$\lambda_{w,ss}$	s.s. markup, labor	1.0500
μ_z	growth rate of economy	0.4100
Υ	trend of investment technology	0.4200
δ	capital depreciation rate	0.0250
α	capital share	0.4000
$\lambda_{f,ss}$	s.s. markup, intermediate good	1.2000
γ_{ss}	s.s. survival rate of entrepreneurs	0.9850
W_e	transfer to entrepreneurs	0.0050
η_g	s.s. spending-to-gdp ratio	0.2000
π^*	s.s. inflation target	2.4300

Posterior Modes (Fixed Regime)

Parameter	Label	Dist.	Prior Mean	SD	Pmode 1-Regime
b	consumption habit	B	0.5	0.1	0.7746
$F(\bar{\omega})$	probability of default	B	0.007	0.0037	0.0145
μ	monitoring cost	B	0.275	0.15	0.1838
σ_a	curvature, utilization cost	N	1	1	1.8454
S''	curvature, invest. adjust. cost	N	5	3	12.0885
α_π	MP weight on inflation	N	1.5	0.25	1.0818
$\alpha_{\Delta y}$	MP weight, output growth	N	0.25	0.1	0.3620
ρ_p	MP smoothing	B	0.75	0. 1	0.8481
ξ_p	price rigidity	B	0.5	0.1	0.7981
ι	price index	B	0.5	0.15	0.8710
ξ_w	wage rigidity	B	0.75	0.1	0.8243
ι_w	wage index, inflation target	B	0.5	0.15	0.4862
ι_μ	wage index, persist tech. growth	B	0.5	0.15	0.9333

Posterior Modes (Fixed Regime)

Parameter	Label	Dist.	Prior Mean	SD	Pmode 1-Regime
σ_{e,λ_f}	stddev price markup	invg2	0.002	0.0033	0.0116
σ_{e,μ_Υ}	stddev investment price	invg2	0.002	0.0033	0.0040
$\sigma_{e,g}$	stddev government spending	invg2	0.002	0.0033	0.0253
σ_{e,μ_z}	stddev persistent technological growth	invg2	0.002	0.0033	0.0073
$\sigma_{e,\gamma}$	stddev equity	invg2	0.002	0.0033	0.0039
$\sigma_{e,\epsilon}$	stddev transitory technology	invg2	0.002	0.0033	0.0047
$\sigma_{e,p}$	stddev MP	invg2	0.002	0.0033	0.5049
σ_{e,ζ_c}	stddev consumption preferece	invg2	0.002	0.0033	0.0259
σ_{e,ζ_i}	stddev MEI	invg2	0.002	0.0033	0.0209
$\sigma_{e,\sigma}$	stddev unanticipated uncertainty	invg2	0.002	0.0033	0.0369
ρ_{λ_f}	AR price markup	B	0.5	0.2	0.9959
ρ_{μ_Υ}	AR price of investment good	B	0.5	0.2	0.9928
ρ_g	AR government spending.	B	0.5	0.2	0.9111
ρ_{μ_z}	AR persistent technological growth	B	0.5	0.2	0.1035
ρ_ϵ	AR transitory technology	B	0.5	0.2	0.9928
ρ_σ	AR uncertainty	B	0.5	0.2	0.8977
ρ_{ζ_c}	AR preference	B	0.5	0.2	0.9830
ρ_{ζ_i}	AR MEI	B	0.5	0.2	0.4051

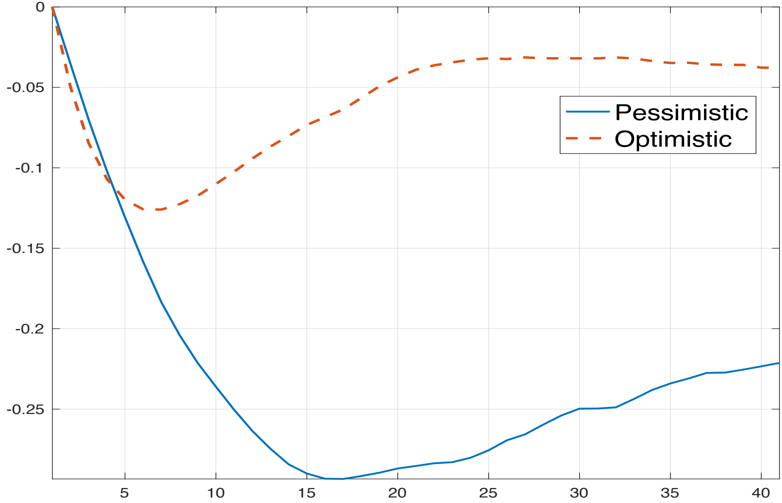
Expectation Effect - Numerical Experiment

Holding everything else fixed, unconditional low risk probability increases with τ. Consider the difference between $\tau = 0$ (Pessimistic) and $\tau = 1.2$ (Optimisitic)

Parameter	Label	Pessimistic	Optimistic
$F(\bar{\omega})_1$	low probability of default	0.01	-
$F(\bar{\omega})_2$	high probability of default	0.02	-
μ_1	low monitoring cost	0.20	-
μ_2	high monitoring cost	0.27	-
α	regime factor persistense	0.9	-
τ	threshold	0	1.2

Note: The "-" denotes the same value as in the "Pessimistic" case. The unconditional low risk probabiility are 0.5 and 0.7 in the pessimistic and optimistic cases, respectively.

Expectation Effect - Impulse Responses



Impulse responses of investment to a positive uncertainty shock under high-uncertainty steady state

Evidence of Time-varying Transition (Exogenous Switching)

- Measure outlook to financial market through the lens of RS-DSGE model with zero feedback.
- Quasi-Bayesian estimation on sub-samples: 1985-2005 (exclude financial crisis), 1990-2010 (include financial crisis) as first step of Bayesian estimation.
- Adjustment cost estimates are unstable across sub-samples. We assume these parameters are similar on the sub-samples. Calibrated at fixed-regime estimates.

Data Set

The dataset is a subset of CMR (1981:Q1 to 2010:Q2).

- Macro
  - GDP
  - consumption
  - investment
  - inflation
  - real wage
  - relative price of investment goods
  - labor hours
  - federal funds rate
- Financial
  - credit to non-financial firms
  - net worth of entrepreneurs (Dow Jones Wilshire 5000 index)
  - credit spread (BAA-10YTB)

Quasi-Bayesian Estimation

- $\hat{\theta} = \arg\max_{\theta \in \Theta} [\log p(y_{1:T}; \theta) + \log q(\theta)]$ with $q(\theta)$ the prior distribution, and $p(y_{1:T}; \theta)$ the likelihood of θ. $\hat{\theta}$ is the posterior mode.

- Take the following steps to evaluate $p(y_{1:T}; \theta)$
  1. For each θ, solve $X_t = T_i(X_{t-1}, \epsilon_t; \theta)$.
  2. Stack observation equations, regime transitions and solutions in (1) to form SSR.
  3. Apply Chang, Maih and Tan (2018) filter to obtain approximated $p(y_{1:T}; \theta)$.

- Optimization methods
  - Local (Derivative-based, Derivative-free)
  - Global (Derivative-based, Derivative-free)
  - Mixture of global and local methods

Chang, Maih and Tan filter, Setup

- Exact filter requires complete history of $\{s_t\}_{t=1}^{T} \in \{1,2\}^T$. Costly to compute.
- Approximate using "marginalization-collapsing" procedure.
- State Space Model

$$
\begin{aligned}
y_t &= D_{s_t} + Z_{s_t} x_t + F_{s_t} z_t + Q_{s_t} u_t \\
x_t &= C_{s_t} + G_{s_t} x_{t-1} + E_{s_t} z_t + R_{s_t} \epsilon_t
\end{aligned}
$$

with s_t specified by

$$
\begin{aligned}
w_t &= \alpha w_{t-1} + \nu_t \\
s_t &= 1 + 1\{w_t \geq \tau\}
\end{aligned}
$$

allowing correlation between ν_t and ϵ_{t-1} with vector of correlation coefficients ρ.

Chang, Maih and Tan filter, Notation

- Let $d_t = \epsilon_t$. An equivalent SSM

$$y_t = \underbrace{D_{s_t} + F_{s_t} z_t}_{\tilde{D}_{s_t}} + \underbrace{\begin{pmatrix} Z_{s_t} & 0 \end{pmatrix}}_{\tilde{Z}_{s_t}} \underbrace{\begin{pmatrix} x_t \\ d_t \end{pmatrix}}_{\zeta_t} + Q_{s_t} u_t$$

$$\underbrace{\begin{pmatrix} x_t \\ d_t \end{pmatrix}}_{\zeta_t} = \underbrace{\begin{pmatrix} C_{s_t} + E_{s_t} z_t \\ 0 \end{pmatrix}}_{\tilde{C}_{s_t}} + \underbrace{\begin{pmatrix} G_{s_t} & 0 \\ 0 & 0 \end{pmatrix}}_{\tilde{G}_{s_t}} \underbrace{\begin{pmatrix} x_{t-1} \\ d_{t-1} \end{pmatrix}}_{\zeta_{t-1}} + \underbrace{\begin{pmatrix} R_{s_t} \\ I \end{pmatrix}}_{\tilde{R}_{s_t}} \epsilon_t$$

- Let

$$\begin{aligned}
p^{i,j}_{t|t-1} &= \mathbb{P}(s_t = j, s_{t-1} = i | Y_{1:t-1}) \\
p^{i,j}_{t|t} &= \mathbb{P}(s_t = j, s_{t-1} = i | Y_{1:t}) \\
p^{j}_{t|t} &= \mathbb{P}(s_t = j | Y_{1:t}) \\
X^{j}_{t|t} &= \mathbb{E}(X_t | s_t = j, Y_{1:t}) \\
P^{j}_{x,t|t} &= \text{var}(X_t | s_t = j, Y_{1:t})
\end{aligned}$$

Chang, Maih and Tan filter, Recursion 1

Step 0. Initialize $(\zeta^i_{0|0}, P^i_{0|0})$ using invariant distribution under regime i. Set $p^1_{0|0} = \Phi(\tau\sqrt{1-\alpha^2})$ and $p^1_{0|0} = 1 - p^0_{0|0}$. (Note $w_0 \sim N(0, 1/(1-\alpha^2))$)

Step 1. Given inputs $(\zeta^i_{t-1|t-1}, P^i_{t-1|t-1}, p^i_{t-1|t-1})_{i=1,2}$,

a. Forecast

$$
\begin{aligned}
\zeta^{(i,j)}_{t|t-1} &= \tilde{C}_j + \tilde{G}_j \zeta^i_{t-1|t-1} \\
P^{(i,j)}_{\zeta,t|t-1} &= \tilde{G}_j P^i_{\zeta,t|t} \tilde{G}'_j + \tilde{R}_j \tilde{R}'_j \\
p^{(i,j)}_{t|t-1} &= \underbrace{\int_{-\infty}^{\infty} \mathbb{P}(s_t = j, s_{t-1} = i | \rho'\epsilon_{t-1}, Y_{1:t-1}) p(\rho'\epsilon_{t-1} | Y_{1:t-1}) d\rho'\epsilon_{t-1}}_{\text{with a trivariate normal CDF representation}}
\end{aligned}
$$

Note by construction

$$
\mathbb{P}(s_t = 0, s_{t-1} = 0 | \rho'\epsilon_{t-1}, Y_{1:t-1}) = \underbrace{\mathbb{P}(s_t = 0 | s_{t-1} = 0, \rho'\epsilon_{t-1})}_{\text{with exact representation}} p^0_{t-1|t-1}
$$

Approximate

$$
p(\rho'\epsilon_{t-1} | Y_{1:t-1}) \approx \mathbb{N}(\rho'\epsilon_{t-1}; \underbrace{\rho'\zeta^0_{d,t-1|t-1}, \rho' P^0_{d,t-1|t-1}\rho}_{d \text{ section of the inputs}})
$$

Chang, Maih and Tan filter, Recursion 2

Step 1. Given outputs of 1a,

b. Forecast

$$\begin{aligned} y^{(i,j)}_{t|t-1} &= \tilde{D}_j + \tilde{Z}_j \zeta^{(i,j)}_{t|t-1} \\ P^{(i,j)}_{y,t|t-1} &= \tilde{Z}_j P^{(i,j)}_{\zeta,t|t-1} \tilde{Z}_j' + Q_j Q_j' \end{aligned}$$

Evaluate conditional density

$$p(y_t|Y_{1:t-1}) = \sum_{i,j} \underbrace{p(y_t|y^{(i,j)}_{t|t-1}, P^{(i,j)}_{y,t|t-1})}_{\text{normal dist.}} p^{(i,j)}_{t|t-1}$$

c. Update

$$\begin{aligned} p^{(i,j)}_{t|t} &= \frac{p(y_t|y^{(i,j)}_{t|t-1}, P^{(i,j)}_{y,t|t-1}) p^{(i,j)}_{t|t-1}}{p(y_t|Y_{1:t-1})}, \quad p^j_{t|t} = \sum_i p^{(i,j)}_{t|t} \\ \zeta^{(i,j)}_{t|t} &= \zeta^{(i,j)}_{t|t-1} + P^{(i,j)}_{\zeta,t|t-1} \tilde{Z}_j' (P^{(i,j)}_{y,t|t-1})^{-1} (y_t - y^{(i,j)}_{t|t-1}) \\ P^{(i,j)}_{\zeta,t|t} &= P^{(i,j)}_{\zeta,t|t-1} - P^{(i,j)}_{\zeta,t|t-1} \tilde{Z}_j' (P^{(i,j)}_{y,t|t-1})^{-1} \tilde{Z}_j P^{(i,j)}_{\zeta,t|t-1} \end{aligned}$$

Collapse

$$\zeta^j_{t|t} = \sum_i \frac{p^{(i,j)}_{t|t} \zeta^{(i,j)}_{t|t}}{p^j_{t|t}}, \quad P^j_{\zeta,t|t} = \sum_i \frac{p^{(i,j)}_{t|t} [P^{(i,j)}_{t|t} + (\zeta^{(j)}_{t|t} - \zeta^{(i,j)}_{t|t})(\zeta^{(j)}_{t|t} - \zeta^{(i,j)}_{t|t})']}{p^j_{t|t}}$$

Posterior Modes (No Feedback)

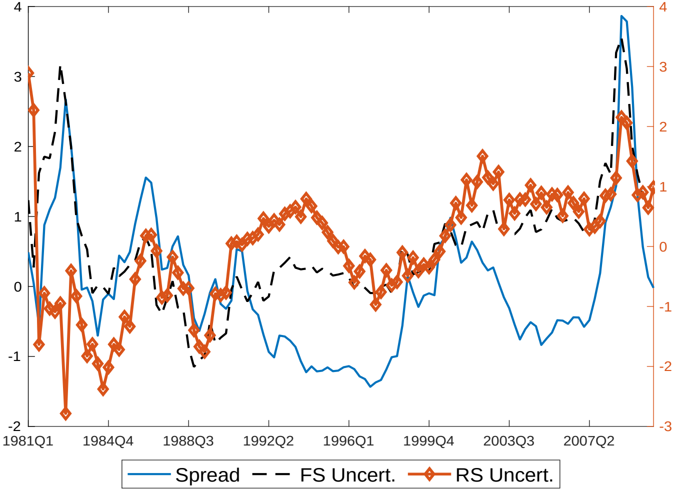
Parameters	Prior	Mean	SD	1985-2005	1990-2010
ξ_w	B	0.7	0.9	0.9099	0.5435
σ_{e,λ_f}	IG	0.0005	0.0015	0.0101	0.0195
$\sigma_{e,\Upsilon}$	IG	0.002	0.006	0.0047	0.0070
$\sigma_{e,g}$	IG	0.001	0.0033	0.0322	0.0601
σ_{e,μ^*}	IG	0.003	0.01	0.0188	0.0734
$\sigma_{e,\gamma}$	IG	0.003	0.01	0.0441	0.0318
$\sigma_{e,\varepsilon}$	IG	0.003	0.01	0.0792	0.0813
$\sigma_{e,p}$	IG	0.01	1	0.7066	0.4616
σ_{e,ζ_c}	IG	0.003	0.01	0.1900	0.1532
σ_{e,ζ_i}	IG	0.003	0.01	0.1331	0.0336
$P_{2\|1}$	B	0.001	0.1	0.0281	0.3017
$P_{1\|2}$	B	0.001	0.5	0.2271	0.0029
$F(\bar{\omega})_1$	B	0.003	0.01	0.0047	0.0030
$F(\bar{\omega})_2$	B	0.01	0.02	0.0067	0.0032
μ_1	B	0.2	0.36	0.0695	0.0996
μ_2	B	0.2	0.36	0.1260	0.1187

Note: Prior means of $P_{2|1}$ and $P_{1|2}$ maps to $\alpha = 0.999$ and $\tau = 0$.

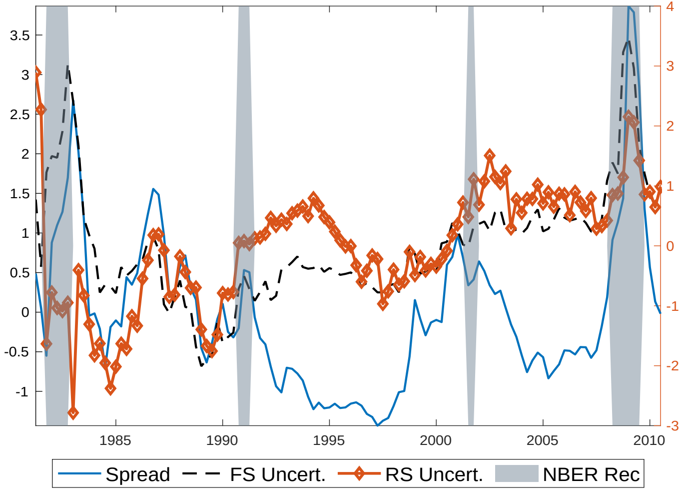
Posterior Modes (No Feedback)

Parameters	Prior	Mean	SD	1985-2005	1990-2010
b	B	0.7	0.1	0.1440	0.9584
ξ_p	B	0.8	0.1	0.8021	0.8630
α_p	N	2.5	0.25	3.8766	3.0903
ρ_p	B	0.75	0.1	0.9536	0.9410
ι	B	0.5	0.15	0.9875	0.9960
ι_w	B	0.5	0.15	0.0969	0.9154
ι_μ	B	0.5	0.15	0.7616	0.1244
$\alpha_{\Delta y}$	N	0.25	0.1	0.0458	0.4850
ρ_{λ_f}	B	0.9	0.2	0.9990	0.9977
ρ_Υ	B	0.9	0.2	0.9173	0.7585
ρ_g	B	0.9	0.2	0.9445	1.0000
ρ_{μ^*}	B	0.1	0.2	0.2810	0.0002
ρ_ε	B	0.9	0.2	0.8345	0.6844
ρ_σ	B	0.9	0.2	0.6986	0.5736
ρ_{ζ_c}	B	0.9	0.2	0.1127	0.8004
ρ_{ζ_i}	B	0.9	0.2	0.9608	0.1289
$\sigma_{e,\sigma}$	IG	0.05	0.04	2.3927	0.8332

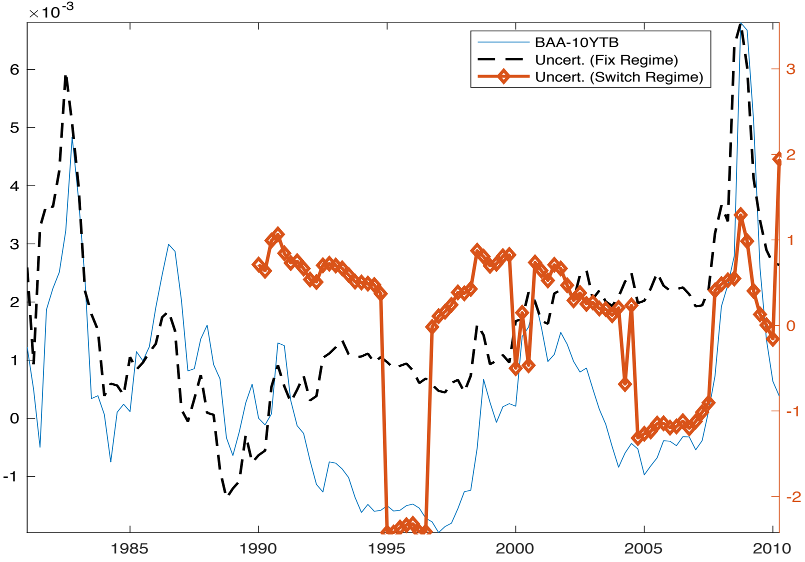
Estimated Uncertainty Process (No Feedback)



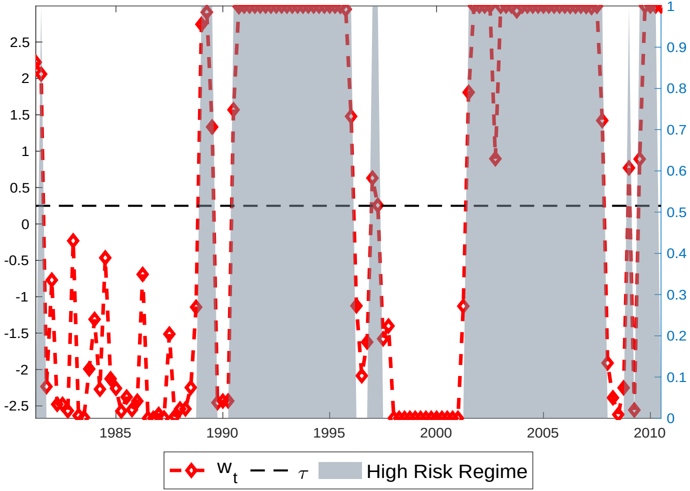
Estimated Uncertainty Process (No Feedback)



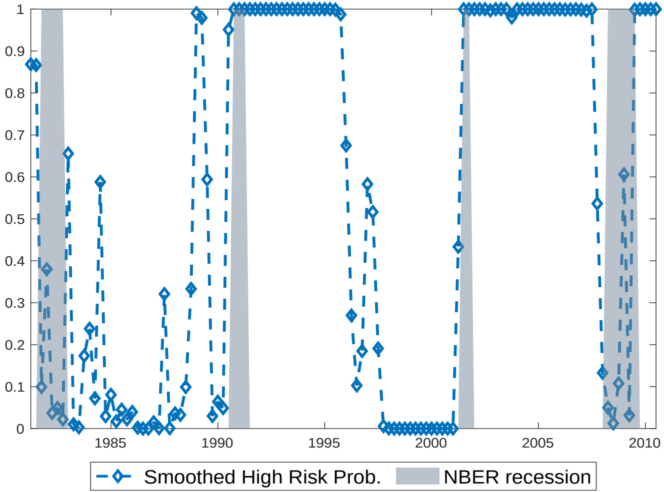
Estimated Uncertainty Process (No Feedback)



Latent Factor and Implied High Risk Regime (No Feedback)



High Risk Regime Probability and NBER Recessions (No Feedback)



Feedback and Time-varying Transition

- Conventional RS-DSGE model appears inadequate.
- Allow feedback. Quasi-Bayesian estimation on full sample.
- Priors for structural parameters identical to previous estimations, similar to CMR.
- Uniform$[-1, 1]$ priors for feedback parameters ρ.

Priors of (α, τ)

- Sub-sample estimates implies unconditional probability of low-risk state are 0.9 and 0.01 on 85-05 and 90-10 samples, respectively.
- Beta prior for α with $Q_{0.05} = 0.5$ and $Q_{0.95} = 0.95$.
- Normal prior for τ with $Q_{0.05} = 0$ and $Q_{0.95} = 1$.
- Unconditional low risk probability $(\Phi(\tau(1-\alpha^2)))$ decreases in α and increases in τ.
  - $\Phi(0) = 0.5$
  - $\Phi(1 - 0.5^2) = 0.8$

Posterior Modes, Regime Switching and Feedback Channel

Parameters Switching	Label	Prior	Endo	Pmode Exo	Const. μ
α	persistense of regime factor	B	0.8709	0.9531	0.8131
τ	threshold of regime factor	N	0.7994	0.2495	0.0141
$\rho_{v,z}$	persistent technological shock	U	-0.2422	-	0.0028
$\rho_{v,\epsilon}$	transitory technological shock	U	0.5469	-	-0.5767
$\rho_{v,\gamma}$	equity shock	U	0.1105	-	0.0215
ρ_{v,μ_Υ}	investment specific tech. shock	U	0.4219	-	0.5767
ρ_{v,ζ_i}	MEI shock	U	0.3142	-	0.0211
$\rho_{v,\sigma}$	risk shock	U	0.0979	-	0.0080
ρ_{v,λ_f}	price markup shock	U	-0.2121	-	-0.5767
$\rho_{v,g}$	government spending shock	U	0.0511	-	-0.0075
$\rho_{v,p}$	MP shock	U	-0.2500	-	0.0049
ρ_{v,π^*}	inflation target shock	U	0.2188	-	0.0002
ρ_{v,ζ_c}	preference shock	U	-0.4312	-	-0.0218
$F(\bar{\omega})_1$	default probability (regime 1)	B	0.0100	0.0100	0.0100
$F(\bar{\omega})_2$	default probability (regime 2)	B	0.0197	0.0200	0.0200
μ_1	monitoring cost (regime 1)	B	0.1212	0.0884	0.1258
μ_2	monitoring cost (regime 2)	B	0.1116	0.0999	0.1258
log-MDD	Laplace approximation		4021.8751	3995.405	3958.9613

Estimated Feedback Channel

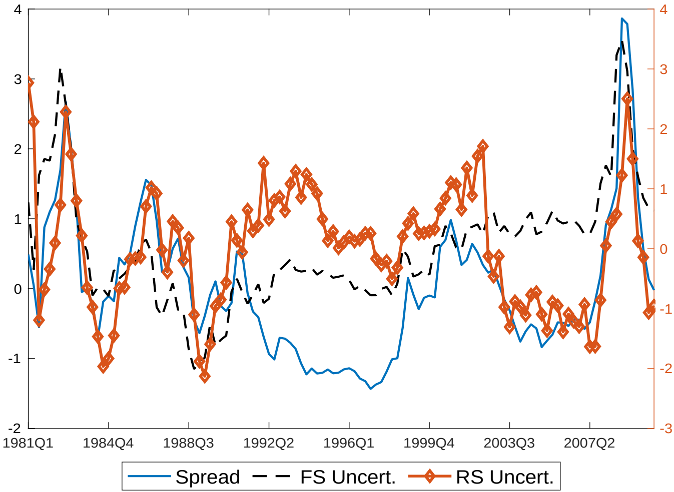
Given positive shocks

- Transitory supply shocks increase regime factor.
- Persistent supply shock decreases regime factor.
- Demand shocks likely decrease regime factor.
- FP and Inflation target shocks increase regime factor.

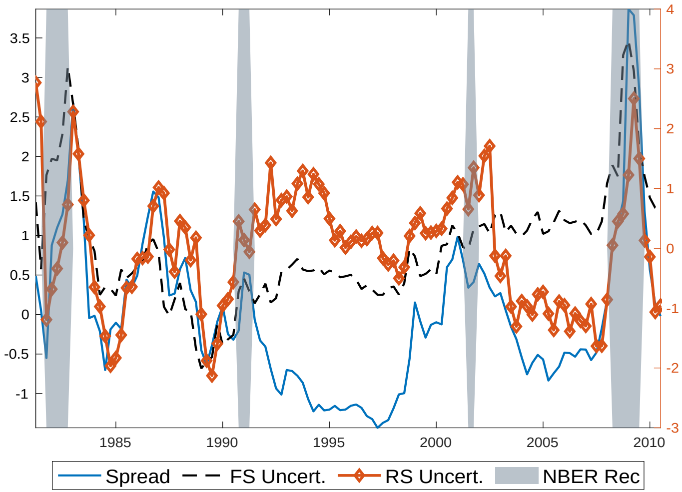
Posterior Modes (With Feedback)

Parameters	Label	Prior	Endo	Exo	Const. μ
ξ_w	wage rigidity	B	0.9107	0.8549	0.8265
ξ_p	price rigidity	B	0.7103	0.7450	0.7769
b	consumption habit	B	0.9013	0.8730	0.8534
α_π	MP weight on inflation	N	1.0340	1.0004	1.0841
$\alpha_{\delta y}$	MP weight on output growth	N	0.3018	0.2995	0.2873
ρ_p	MP smoothing	B	0.9148	0.8587	0.8462
ι	price indexation	B	0.2350	0.4491	0.6055
ι_w	wage indexation on inflation target	B	0.2361	0.3684	0.6282
ι_μ	wage indexation on presist tech. growth	B	0.7959	0.7802	0.7973
$\rho_{\lambda,f}$	AR price markup	B	0.7080	0.8759	0.8517
ρ_{μ_Υ}	AR investment specific technology	B	0.9870	0.9704	0.9928
ρ_g	AR government spending	B	0.9207	0.9245	0.9021
ρ_{μ_z}	AR persistent technological growth	B	0.0648	0.0689	0.0809
ρ_ε	AR transitory technology	B	0.9928	0.9844	0.8713
ρ_σ	AR risk	B	0.9770	0.9827	0.9737
ρ_{ζ_c}	AR preference	B	0.9774	0.8391	0.7834
ρ_{ζ_i}	AR marginal efficiency of investment	B	0.6716	0.7754	0.7001
$\sigma_{e,\lambda,f}$	std. dev. Price markup	IG	0.0166	0.0108	0.0116
σ_{e,μ_Υ}	std. dev. Investment specpfic technology	IG	0.0039	0.0039	0.0040
$\sigma_{e,g}$	std. dev. Government spending	IG	0.0227	0.0221	0.0229
$\sigma_{e,p}$	std. dev. MP	IG	0.5656	0.6346	0.5815
σ_{e,μ_z}	std. dev. Persistent technological growth	IG	0.0078	0.0076	0.0073
σ_ε	std. dev. Transitory technology	IG	0.0051	0.0047	0.0047
$\sigma_{e,\gamma}$	std. dev. Equity	IG	0.0074	0.0145	0.0050
$\sigma_{e,\sigma}$	std. dev. Risk	IG	0.0432	0.0826	0.1151
σ_{e,ζ_c}	std. dev. Preference	IG	0.0486	0.0310	0.0259
σ_{e,ζ_i}	std. dev. MEI	IG	0.0259	0.0209	0.0299

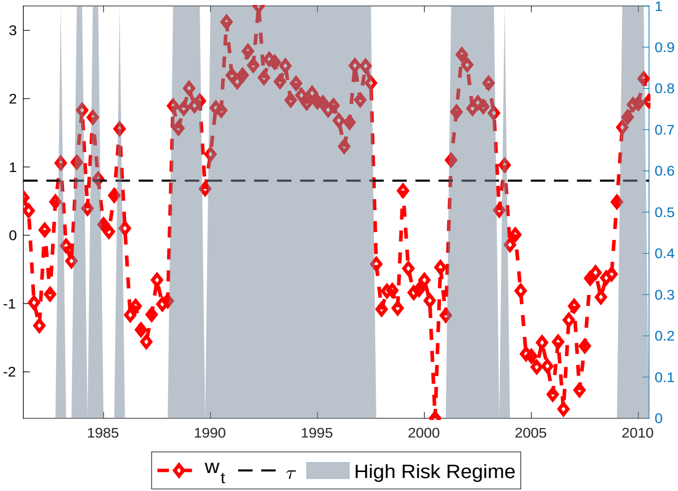
Estimated Uncertainty Process (With Feedback)



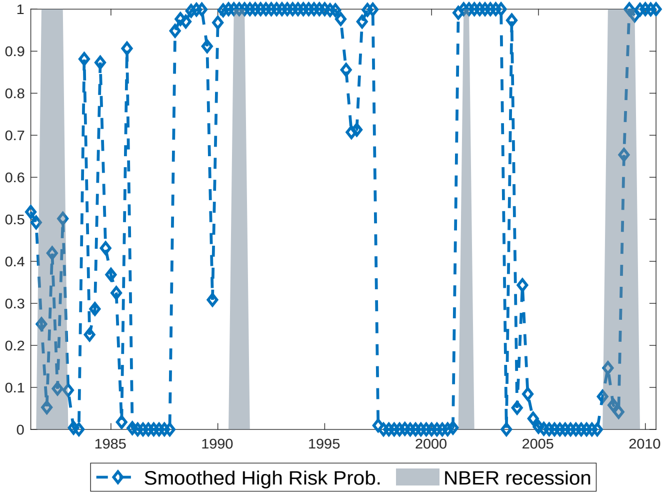
Estimated Uncertainty Process (With Feedback)



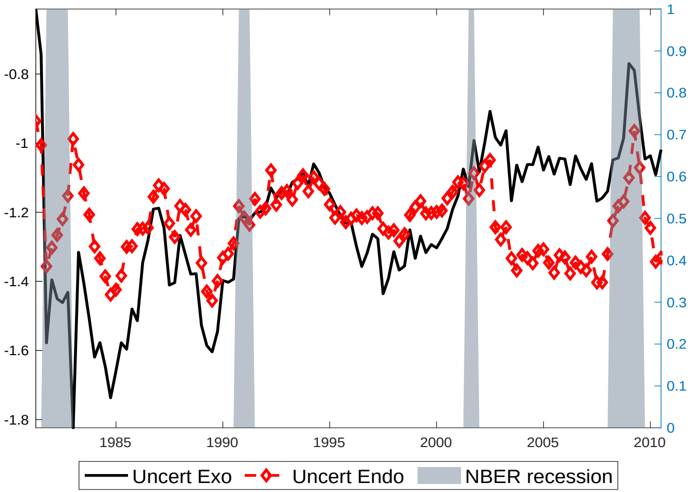
Latent Factor and Implied High Risk Regime (With Feedback)



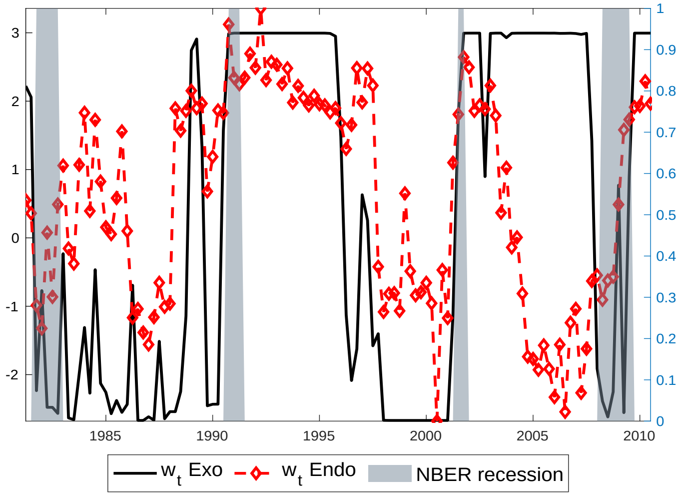
High Risk Regime Probability and NBER Recessions (With Feedback)



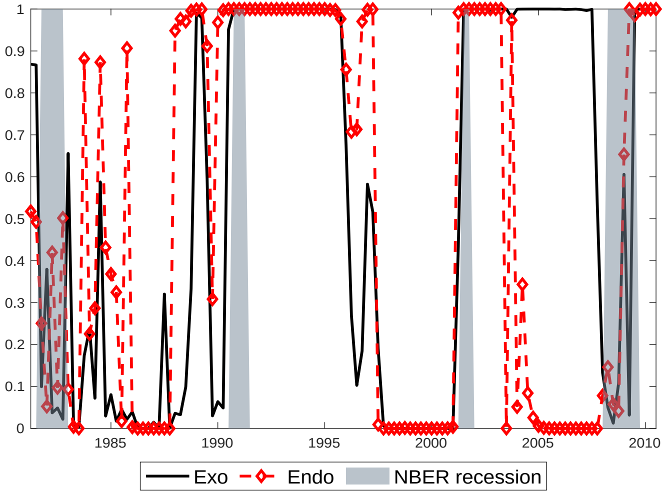
Uncert. No Feedback vs. Feedback



Regime Factor. No Feedback vs. Feedback



High Risk Regime Probability. No Feedback vs. Feedback



Inference (In Progress)

- Use standard Random Walk MH.
- Draw a chain of $\{\theta^i\}$ taking the following steps:

  0. Use $\hat{\theta}$ as θ^1.
  1. Given $\theta^{i-1}, p(Y|\theta^{i-1})$ and $q(\theta^{i-1})$, draw $\vartheta = \theta^{i-1} + \eta$ with $\eta \sim \mathbb{N}(0, c^2\Sigma)$.
  2. Let $\theta^i = \vartheta$ with probability $\alpha = \min\left\{\frac{p(\vartheta|Y)}{p(\theta^{i-1}|Y)}, 1\right\}$, and $\theta^i = \theta^{i-1}$ otherwise.

- Burn-in, Thinning, Fine-tuning c and Σ. Convergence tests.

Concluding Remarks

1. Introduce time-varying transition to RS-DSGE and study the expectation effect induced by RS.
2. Expectation effect appears quantitatively important.
3. Evidence of time-varying transition probability of financial market from conventional RS-DSGE.
4. Novel findings from RS-DSGE with feedback.
    - Strong feedback: historical shocks drive regime shift almost exclusively ($> 99\%$).
    - Zero to Negative feedback from demand shocks, except inflation target shocks.
    - Positive feedback from supply shocks, except persistent TFP shocks.