Bidding for Contracts under Uncertain Demand: Skewed Bidding and Risk Sharing

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Abstract

Procurement contracts often involve substantial uncertainty in project outcomes at the time of bidding. Whether the procurer of a contract bears such project risk depends on the specific contractual agreement. Using data from the Florida Department of Transportation, we document evidence that i) the procurer's choice over the type of contract depends on unobserved project heterogeneity, and ii) potential contractors behave opportunistically via skewed bidding for contracts wherein the contractor bears the project risk. We develop and estimate a model of bidding for contracts that captures the bidder's tradeoff between skewed bidding and risk exposure. Both efficient and inefficient bidders bid aggressively via skewed bidding. Counterfactual experiments suggest that the onus of bearing project risk should fall on the procurer (contractor) when project risk is large (small).

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1 Introduction

Infrastructure projects often involve significant risk, such as *ex-post* adjustments to inputs during project implementation. Which contracting party should be held responsible for such risk is a topic of heated debate.[1] On the one hand, a contractor who undertakes the project risk may demand a sizable risk premium in compensation. On the other hand, a procurer who undertakes the project risk may suffer from excessive cost overruns triggered by opportunistic contractors. Despite the empirical relevance of risk-allocation in contracts with uncertain demand, the empirical literature on this issue has been scarce.[2]

We study fixed-price (FP) contracts and unit-price (UP) contracts, collected by the Florida Department of Transportation (FDOT), to investigate the role of risk allocation via contractual arrangements on firm behavior and contracting outcomes.[3] Projects are procured through auctions. Costs associated with input adjustments are covered by the contractor under FP contracts and by the procurer under UP contracts. Contractual arrangements affect competition outcomes through shaping the effective project risk and incentives to win a contract, which in turn affect potential contractors' bidding and participation decisions in competition for a contract.[4]

For UP contracts, engineers at FDOT's procurement office first estimate the quantity of each input/item required to complete the project, and prospective contractors bid with a list of unit-prices that their firm intends to charge for each item.[5] Then, FDOT determines a score for each bidder by multiplying its quantity estimates by the bidder's unit prices, summing across all inputs. The bidder with the lowest score wins the contract and receives payment in the amount of its bidder score upon the completion of the project if the estimated inputs are actually used during project implementation. If there is any adjustment to any of the contracted items – i.e., the items specified by FDOT and bid on at the time of bidding –

[1]Resulting cost overruns could reach millions – if not billions – of dollars. For example, the Boeing Dreamliner program, announced in 2003, was supposed to cost \$6 billion, but the final bill was about \$32 billion. Another example is the construction of Berlin Brandenburg Airport, which was estimated to cost €1 billion initially, but ended up costing €6 billion.

[2]We use "project risk" and "uncertain demand" interchangeably in this paper.

[3]FP contracts are widely used in public procurements, including procurement of public transport, operation of water facilities, and electricity. UP contracts are more prevalent for construction procurements, including highway contracting, pipeline construction, defense procurement, and procurement projects supported by the World Bank. UP contracts are also used in timber auctions, as in Athey and Levin (2001).

[4]FDOT procures small infrastructure projects through either UP or FP contracts. Large projects are procured via so-called Design-Build Auctions.

[5]Construction items are measured in different units and are contracted in various forms. For example, labor is contracted on per-day basis and the quantity estimate is given in terms of the duration of the project.

the contractor is obliged to make input adjustments at the unit price.[6] Under FP contracts, prospective contractors submit a single-price bid to the FDOT procurement office for an entire project. The contracting firm with the lowest bid price wins the contract and receives its own price bid upon completion.[7] That is, inputs are provided at the cost of the contractor, regardless of the quantities required for project implementation.

We contribute to the empirical literature on contracts and auctions on various dimensions. First, we provide evidence suggesting that choice of the type of contract depends on unobserved project heterogeneity. The literature on procurement auctions has largely ignored unobserved differences between FP and UP contracts. Observations from auctions on the two contract types are often pooled, based on an OLS comparison that does not suggest any significant differences in bidding strategies. Second, we show evidence of opportunistic behavior by bidders. UP contracts mitigate costly *ex-post* renegotiation by fixing the price of items *ex-ante*, but prospective contractors strategically choose their unit-price bids. We document skewed bidding in our data – i.e., it is expected that contracting firms will bid high on items that are likely to overrun in order to obtain compensation in expectation, and conversely, bid low on items with no pay adjustments. Lastly, we construct, identify, and estimate a model of bidding for contracts, which nests the two contract types and allows for a variety of counterfactual experiments. To the best of our knowledge, our paper is among the first to identify and estimate a structural model of UP contracts in the presence of skewed bidding.[8]

We provide evidence that the contract choice of project managers at FDOT's procurement office depends on unobserved project heterogeneity in a way that is consistent with FDOT's belief: FP (UP) contracts should be employed for projects with small (large) project risk.[9] Although OLS results show no significant differences between the two contract types, the endogeneity of contract choice obscures their differences. That is, an FDOT project manager is more likely to employ a UP contract for projects entailing significant project risk, which likely increases contractor costs, and may not be fully captured

[6] In case of quantity changes on uncontracted items: the contracting parties renegotiate the prices of items not specified at the time of bidding and not bid on, for both FP and UP contracts. UP contracts differ from cost-plus contracts in that pay adjustment due to changes in the plan is fixed at the time of auction, and therefore leave no room for renegotiation on contracted items.

[7] Quantity estimates are also provided in FP contract.

[8] A recent work of Bolotnyy and Vasserman (2019) adopts a different model, which we compare with ours in the next subsection.

[9] FDOT's project guidelines explicitly list the tasks suited for FP and UP contracts. See Figure 1.

by the observables in the data. We resolve this endogeneity issue using variation in contract choice due to FDOT's caseload. As UP contracts require appreciably more FDOT personnel to keep track of inputs used during project implementation, adoption of the UP contract is less likely when FDOT has many projects to deal with. Our empirical results suggest that while the adoption of FP contracts is negatively correlated with bidder scores, the adoption of UP contracts is insignificantly correlated with these scores. The above findings are consistent with a hypothesis that contractors' unobserved costs are increasing in project risk under FP contracts, while UP contracts are robust to project risk. We further document that bidders submit substantially different compositions of unit-price bids under UP contracts, which suggests that bidders hedge against project risk through forming portfolios of unit-price bids.

We further show suggestive evidence that UP contracts induce bidders to behave opportunistically through skewed bidding. A bidder has an incentive to bid high on underestimated items and bid low on overestimated items, since positive *ex-post* adjustments increase bidder revenue, while negative *ex-post* adjustments reduce bidder revenue. Since bidders differ from each other in terms of their input estimates, the bidder with the highest estimate has the largest incentive to win the contract. In the unique setting of UP contracts, bidders behave strategically at the time of bidding rather than *ex-post*, in which the econometrician does not observe how non-winning bidders would have behaved. This allows us to directly control for unobserved project heterogeneity in showing evidence of skewed bidding, and we find that bidders who skew their unit-price bids are much more likely to win the contract than those who do not.

To quantify the effects of demand uncertainty, we construct a model of bidding for a contract, nesting both UP and FP contracts. The model captures the key tradeoff that the procurer faces in choosing the type of contract given the degree of project risk. On the one hand, UP contracts allow bidders to hedge against uncertain demand by forming a portfolio of unit-price bids while FP contracts do not. On the other hand, UP contracts induce skewed bidding which may result in higher procurement costs through the selection of inefficient contractors. Our model extends that of Ewerhart and Fieseler (2003) by introducing multidimensional bidder heterogeneity, risk aversion, and endogenous entry. From an empirical standpoint, it is difficult to rationalize the observed distribution of bids in the framework of unidimensional bidder heterogeneity, since multidimensional bidding strategies would be a function of unidimensional type.

Risk aversion explains why bidders do not completely skew their bids and rationalizes FDOT's beliefs.[10] Endogenizing the entry decision of bidders is important, as altering contractual arrangements affects not only bidding behavior but also the level of competition through entry. In particular, UP contracts bring in more competition than FP contracts, since the UP contractual arrangement incentivizes bidders to earn profit through the choice of portfolio of unit-price bids as a tool to hedge against project risk and to make profit through cost overruns, none of which exist under the FP contractual arrangement.

We demonstrate that the model is semiparametrically identified from UP contracts, accounting for unobserved project heterogeneity in cost and risk. The estimated model is consistent with empirical findings in that i) the UP contract is robust to project risk, ii) bidder scores are much more dispersed with FP versus UP contracts, iii) the composition of unit-price bids exhibits a substantial amount of within-auction heterogeneity, iv) bidders who skew their bids are more likely to win the contract. Structural estimates show a large amount of unobserved heterogeneity in both costs and project risk. Based on the estimated model, we numerically demonstrate that FP (UP) contracts perform well for projects with low (high) project risk and show that our model is consistent with empirical findings i)-iv) listed above. Counterfactual experiments suggest that UP contracts perform well, at least for those projects in the data that were procured through UP contract. Switching from UP to FP contracting would significantly increase the procurement cost.

Despite practical relevance, empirical work on evaluating the performance of contractual arrangements remain scarce. One of few related empirical works is Decarolis (2014), which compares contracts awarded via first-price auctions and average-price auctions. First-price auctions are found to have a perverse effect on *ex-post* contract performance relative to average-price auctions. Bajari et al. (2013), which also investigates UP contracts, shows that skewed bidding is not economically significant, which contradicts our finding. This empirical discrepancy is likely due to institutional differences across state departments of transportation. Bajari et. al. (2013) uses procurement data from the California Department of Transportation, where the contracting price can be renegotiated if quantity adjustments exceed 25% of the original estimated quantity, which contrasts sharply with FDOT's threshold of 125%.[11] In addition

[10] On theoretical grounds, bidder risk aversion is explained by imperfect capital markets so that procurement-specific risks matter to bidders (Samuelson, 1986).

[11] For details, see Section 4 at http://mcraemetcalf.com/wp-content/uploads/2016/07/FDOT-2016eBook-Standard-Specifications.pdf

to allowing for comparison of FP with UP contracts, our model differs from Bajari et. al. (2013) in that we allow for the expected quantity of work items to differ across bidders and bidders face uncertainty in the actual quantity of work items. Relaxing these assumptions explains not only why the composition of unit-price bids varies considerably in any given auction, but also why bidders do not completely skew their bids to hedge against project risk.

The rest of the paper is organized as follows. Section 2 presents the related literature. Section 3 describes the data and the procurement procedures under both FP and UP contracts. Section 4 provides evidence that the procurer's choice of contract depends on unobserved project risk, together with evidence of skewed bidding. Section 5 presents the model of bidding for a contract. Section 6 shows semiparametric identification of the model. Section 7 provides estimation steps together with the results. Section 8 provides counterfactual experiments and shows that UP works better than FP when project risk is large, while FP works better than UP when project risk is small. Section 9 concludes.

2 Related Literature

Skewed bidding has received considerable attention in the auction literature. Athey and Levin (2001) finds evidence of skewed bidding in the U.S. forest service timber auctions. They also find that bidders do not completely skew their bids, which is consistent with bidder risk-aversion. Our model differs from Athey and Levin (2001) in that we apply a private value framework while they assume a common value framework.

The most closely related work to ours is Ewerhart and Fieseler (2003), who study bidder behavior in a UP contract in independent private value framework. In their framework, uncertainty in *ex-post* item quantity does not matter to bidders as bidders are assumed to be risk-neutral. They show that bidders with a large estimate on the *ex-post* quantity has a larger incentive to win the contract and therefore, bid more aggressively to win the contract, which results in skewed bidding. We introduce bidder risk-aversion and endogenous entry decision into the framework of Ewerhart and Fieseler (2003) as bidders do not completely skew their bids empirically, and contract formats significantly affects bidders' expected payoffs, which in turn influences bidders' entry decisions.

This paper is more broadly related to the literature on contracting via auctions. The seminal work paper in the literature on procurement contracts through auction is McAfee and McMillan (1986), which compares the performance of fixed-price contracts and cost-plus contracts in an incomplete contract setting. The authorsy show that the optimal incentive contract is linear in bid and *ex-post* realized costs. Bajari, Houghton, and Tadelis (2014) structurally estimates a model of UP contracts in an incomplete contract setting, since the majority of *ex-post* adjustments originate from uncontracted items in their environment. Our framework differs from this strand of incomplete contracts literature in that we consider a complete contract setting, given that the majority of *ex-post* adjustments in Florida originate from adjustments on contracted items. Lewis and Bajari (2014) looks empirically at the tradeoff between effort and risk in the procurement setting. An and Tang (2017) considers the incomplete contracting setting, in which buyers endogenously specify the initial contract. Decarolis (2014) finds a perverse effect of first-price auctions on infrastructure procurement projects in Italy. A recent paper Bolotnyy and Vasserman (2019) also study unit price contracts with risk averse bidders. They adopt a framework similar to Bajari, Houghton, and Tadelis (2014): bidders observe a common signal about (ex-ante unknown) actual item quantities and individual scalar private information on costs. In contrast, bidders in our model have multidimensional private information on actual item quantities, which explains substantial heterogeneity in bid skewness among bidders.

Our work is also related to the vast literature on the identification and estimation of auction models with risk-averse bidders and endogenous entry.[12] Guerre, Perrigne, and Vuong (2009) shows that risk-averse bidder utility functions and private value distributions can be nonparametrically identified via an exclusion restriction and observed bids from first-price auctions. Campo, Guerre, Perrigne, Vuong (2011) shows that risk-averse bidders' utility function and private value distribution are semiparametrically identified under a conditional quantile restriction on the distribution of bidder private valuation and a parametrization of the bidder utility function. Luo, Perrigne, and Vuong (2018) develop a structural model with risk-averse bidders to analyze FP auctions subject to *ex-post* uncertainty. They derive the model restrictions and study identification under exogenous and endogenous participation. Li and Zheng (2009) estimates three competing endogenous entry models in procurement auctions and finds that the

[12]Bidding behavior consistent with risk-aversion is confirmed in both experimental and non-experimental studies (e.g., Cox, Smith, and Walker, 1988; Athey and Levin, 2001; Goeree, Holt, and Palfrey, 2002). Bajari and Hortacsu (2005) also show that a structural model with risk aversion provides the best fit to some experimental data among a set of competing models.

model with a common entry cost where bidders draw their private costs upon entry best fits the data.

3 Institutional Details and Data

This section describes the procurement procedure, overviews FDOT's project guidelines, and provides descriptive statistics of the data. Description of the auction procedure specifies who makes what decisions at what point in time. FDOT's project guidelines shed light on why one should be concerned about endogeneity of contract type. Lastly, we provide an OLS comparison of bidders' behavior and project outcomes across the two types of contractual arrangements.

3.1 Procurement Procedure

FDOT consists of seven district offices that procure infrastructure projects independently. Each district office announces a list of projects every month. The set of procured projects in any month is determined by FDOT's project managers and various department personnel. The procurement procedure can be decomposed into a design phase, followed by an auction phase, and finally, a construction phase.

In the design stage, FDOT's in-house engineers specify the plan of a project – namely, estimates of the quantity needed for each construction item and project cost. The project manager then decides whether to procure the project by FP or UP contract. A project guideline published by the FDOT explicitly states that FP contracts should be employed for "projects with low risk of unforeseen conditions."[13] Figure 1, extracted from the guideline, lists the project types for which FP contracts are and are not suited. Essentially, the guideline states that FP contracts should be used for simple projects, and UP contracts, otherwise. One to two months prior to project letting, the FDOT posts an advertisement online which lists information about project location, description of work, expected contract duration, and an engineer's estimate of the project cost.

Next, the project enters the auction phase. If a project is procured through a UP contract, every prospective contractor submits a list of unit prices on their bid form for each item given in the FDOT project plan quantity estimates. For example, if FDOT's project plan indicates that 10 units of electronic message signs need be implemented, each bidder must submit a dollar amount for how much their con-

[13] Lump-Sum Project Guideline is found at https://www.fdot.gov/roadway/PPMManual/2017PPM.shtm.

tracting firm intends to charge for one of 10 message signs. FDOT then determines a score for each bidder by multiplying its planned quantities with the bidder's unit-prices and summing across all construction items. Participating bidders are then ranked by their score, and the bidder with the lowest score wins the UP contract. The contracting firm is then obligated to provide the contracted items at the unit prices stated in their bid form. If the project were to be procured through an FP contract, on the other hand, prospective contractors would instead submit a single-price bid, which would also be their bidder score under FP, and the bidder with the lowest bid price would win the contract. The contracting firm would be obligated to implement the project at its bid price amount, unless significant changes are made to the contract during the construction phase.[14]

The auction phase is followed by the construction phase. Project implementation is closely monitored by an FDOT construction engineering inspector. If no changes are made to the construction plan, the contracting firm receives its own bid price upon delivery of the project under both UP and FP contracts. If the FDOT project manager finds a need to adjust the construction plan under a UP contract, contractor payment is adjusted based on FDOT's quantity adjustment(s) and the contractor bid form list of unit-prices. For example, if FDOT requires any additional days of construction work, and labour is contracted on a daily basis, then FDOT compensates the contractor by the number of additional days multiplied by the contractor's daily labour rate. More than 95% of these adjustments are initiated by FDOT, and not the contractors. Under an FP contract, no adjustments in payment would be made for changes to contracted items.

Adjustments could also occur on uncontracted items. For example, storms during construction may damage construction materials, and repairs may be needed. In this case, the FDOT project manager files a claim, describing the extra work needed, the reason for the change, the associated cost, and the time extension required to implement the change. These additional uncontracted tasks could involve negotiation, and the compensation for these uncontracted tasks is determined the same way for FP and UP contracts. Table 1 shows the distribution of the share of adjustments on contracted items out of total *ex-post* adjustments.

[14] Quantity estimates are also provided in FP contracts.

Table 1: Share of Contracted Item Adjustments in *Ex-post* Adjustments under UP contracts

Mean	5pctl	25pctl	50pctl	75pctl	95pctl
.801	.158	.608	1	1	1

We conduct a simple simulation exercise to demonstrate the extent of *ex-post* adjustments on the winner selection process. To implement the simulation, we assume that (i) bidders' behavior is fixed (i.e., unit-price bids are given by the data), (ii) exactly the same *ex-post* quantity adjustments are imposed on the project, regardless of which bidder wins the contract, and (iii) the auctioneer selects the winner based on the final payment, rather than bidder score – that is, the auctioneer is assumed to foresee the ultimate quantities required for project completion at the time of auction. We find that 10.3% of UP contracts in the data would have had a different winner if the auctioneer had been able to select the winner based on the final payment to the contractor.[15] We relax assumptions (i)-(iii) later in the structural modeling section, but this simple exercise demonstrates the extent of inefficiency introduced by *ex-post* adjustments.

A typical concern raised in the analysis of cost overrun is the possibility of default. Contractor default is particularly relevant in this context, since FP contracts may involve more frequent default than UP contracts if contractors are unable to supply extra work or items required to complete the project. During the sample period, 25 projects (1.3% of the sample size) that were procured through either FP or UP contracts defaulted.[16] The majority of these defaults were not due to adjustments in the project plan but due to contractors failing to perform work in accordance with the terms of the contract. Another possibility for default is binding FDOT district office project budget constraints. If a district office is unable to make additional payments for extra work or items under UP contracts, then project managers may decide not to complete the project due to insufficient funds. It turns out that the FDOT district offices pool their annual budget across projects to ensure that all procured projects are completed.[17]

[15]The probability of winner switch under FP contracts is not affected by this experiment, by construction.

[16]Our sample contains 22 of the 25 defaulted projects, of which 13 projects (9 projects) were procured through UP (FP) contract.

[17]FDOT requires every bidder to submit a surety bond, specifying a firm that would take over an incomplete project in case of contractor default. FDOT project managers state that every project is completed without exception. We also control for annual district budget amounts in the following regression analyses.

Examples of projects that may be good Lump Sum contracting candidates:

- Bridge painting
- Bridge projects
- Fencing
- Guardrail
- Intersection improvements (with known utilities)
- Landscaping
- Lighting
- Mill/Resurface (without complex overbuild requirements)
- Minor road widening
- Sidewalks
- Signing
- Signalization

Examples of projects that may not be good Lump Sum contracting candidates:

- Urban construction/reconstruction
- Rehabilitation of movable bridges
- Projects with subsoil earthwork
- Concrete pavement rehabilitation projects
- Major bridge rehabilitation/repair projects where there are many unknown quantities.

Figure 1: Excerpt from FDOT Project Guidelines

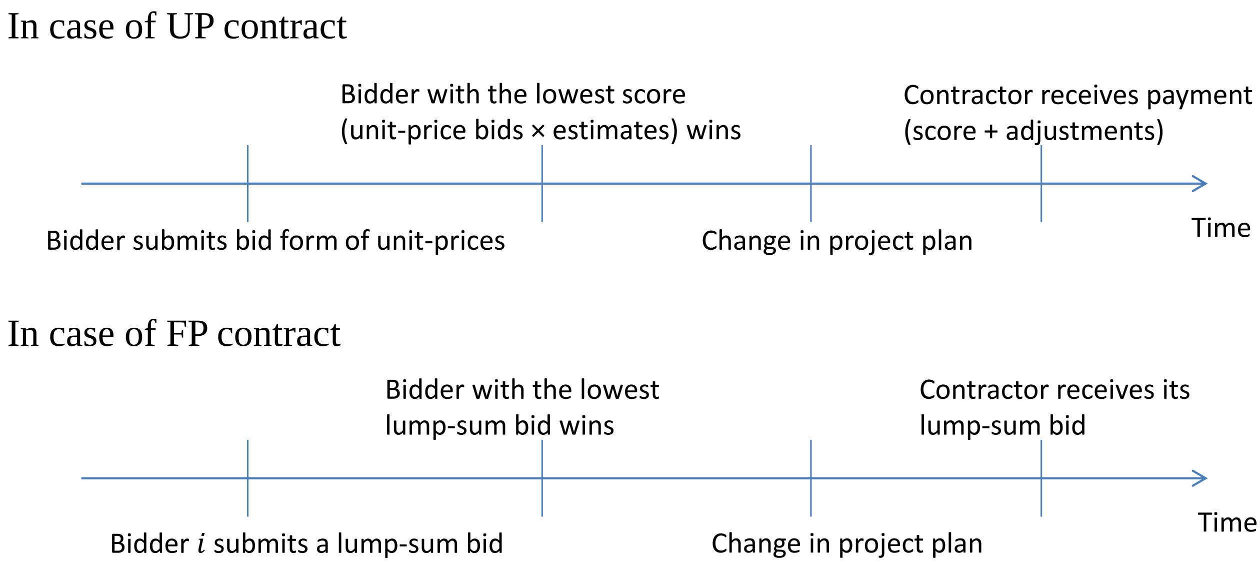

Figure 2: Timeline of Events

3.2 Data and Descriptive Analyses

We investigate a sample of infrastructure projects procured by FDOT under FP or UP contracts between the years 2004 and 2014.[18] The data contain rich information, including all participating bidders' bid prices (every unit-price bid for UP contracts), FDOT engineers' cost estimates, quantity estimates for UP contracts, final payment to contractors, project location, description of work, and identities of both participating and non-participating bidders. We define participating bidders as plan holders that submitted bids and non-participating bidders as those that did not.

FDOT's procurement office determines cost estimates based on historical unit-price bids.[19] There is no difference in the way the estimates are determined between FP and UP contracts. Work description provides information about the nature of the project (i.e., road repair, bridge construction, etc); as descriptions vary significantly across projects, we extract the tasks from these work descriptions and define project type as a linear combination of these tasks.[20]

[18]The sample consists of relatively small projects since FDOT uses another mechanism, the so-called Design-Build auction, for large projects. The average contracting price for Design-Build auctions during the sample period is about $14 million.

[19]Engineer's cost estimate and expected contract duration are explicitly given in the advertisement of a project, and therefore these project characteristics are known to bidders at the time of bidding.

[20]We create an indicator variable for each task. For example, we set an indicator variable equal to one if a work description contains the word "milling", another indicator variable equal to one if a work description contains the word "widening", and

Table 2: Summary Statistics of Fixed-Price and Unit-Price Contracts

	FP					UP				
Variable	**Mean**	**Std.**	**Min.**	**Max.**	**N**	**Mean**	**Std.**	**Min.**	**Max.**	**N**
Winning Score ($1,000)	1591	2065	14.0	14500	590	3806	6313	7.51	148000	1208
Engineer's Cost Estimates ($1,000)	1879	2338	23.3	16600	590	4638	7943	12.7	164000	1208
Expected Contract Duration (# of Days)	117	67.8	30	550	590	236	196	15	2034	1208
Final Payment to Contractor ($1,000)	1642	2138	14.3	14400	590	3977	6798	7.50	159000	1208
Ex-Post Pay Adjustment ($1,000)	51.0	177	-1640	1851	590	185	750	-2000	11200	1208
# of Participating Bidders / Auction	4.36	2.41	1	15	590	5.03	2.69	1	19	1208
# of Plan Holders / Auction	38.3	15.8	2	82	590	43.3	23.6	1	159	1208

Winning score is the winner's bid price for FP and the sum of unit-price multiplied by estimated quantity for UP, in thousands of dollars.

Table 2 presents summary statistics of key variables under UP and FP contracts. On average, fewer bidders participate in FP versus UP auctions. UP contracts are used for relatively large projects with longer expected time to completion than those of FP contracts. We also see that FP projects are less susceptible to cost overruns, and the average cost overrun of UP projects is six times greater than that of FP projects.

Table 3 presents OLS outcomes. We consider four dependent variables: entry, log(score), winner's log(score), and log(final payment). A potential bidder is considered to enter an auction if a plan holder submits a bid. For FP contracts, bidder score is equivalent to bid price for FP contracts; conversely, for UP contracts, the score is determined by bidder unit prices multiplied by FDOT quantity estimates and summed across all items. We find that bidders score 2% lower under FP versus UP contracting, despite no statistically significant differences in entry, winning score, and final payment to contractors across the two contract types.

Consistent with the procurement auction literature, we find that most of the variation in score and final payments are explained by the variation in FDOT engineer cost estimates. An additional participating rival bidder is associated with a 1% reduction in bidder score on average, suggesting that competition drives down price.[21]

so on. In total, we found 18 tasks with which to characterize project type.

[21] The strong negative correlation between the score and the number of participating bidders suggests that bidders are competing in a private value paradigm rather than a common value paradigm.

Table 3: OLS Comparison of Contract Formats, Bidder Behavior, and Auction Outcomes

Dependent Variable	*entry*	ln(*score*)		*winner's* ln(*score*)		ln(*final payment*)	
FP (=0 if UP, =1 if FP)	.00140	-.0176	-.0235	-.00699	-.0194	.00332	-.00708
	(.0035)	(.0072)	(.0072)	(.013)	(.013)	(.014)	(.014)
log(engineer's cost estimate)	-.00941	.974	.973	.983	.982	.998	.997
	(.0018)	(.0030)	(.0030)	(.0064)	(.0062)	(.0069)	(.0068)
# of pariticipating bidders			-.0102		-.0255		-.0246
			(.0011)		(.0025)		(.0028)
# of plan holders	-.000701	.000166	.000721	.000272	.00133	.000441	.00149
	(.00012)	(.00018)	(.00019)	(.00043)	(.00043)	(.00046)	(.00047)
log(district office budget)	-.00174	.00561	.0135	.0469	.0626	.0363	.0515
	(.0063)	(.015)	(.015)	(.030)	(.029)	(.033)	(.032)
District FE	yes	yes	yes	yes	yes	yes	yes
Year FE	yes	yes	yes	yes	yes	yes	yes
Month FE	yes	yes	yes	yes	yes	yes	yes
Bidder FE	yes	yes	yes	yes	yes	yes	yes
Project Type FE	yes	yes	yes	yes	yes	yes	yes
R^2	.533	.975	.975	.979	.980	.976	977
N	75714	8654	8654	1798	1798	1798	1798

Project type is defined as a linear combination of tasks, which are extracted from work descriptions of bid tabs. Bidders that win less than one percent of the total value of projects during the sample period are grouped together as fringe contractors and treated as the same bidder when controlling for bidder fixed effect. Standard errors are clustered at project/auction level and presented in parentheses.

4 Unobserved Project Heterogeneity and Skewed Bidding

Institutional facts indicate that contract choice is unlikely to be random and could therefore confound the effects of contractual arrangements on project outcomes. If FDOT project managers follow FDOT project guidelines, then bids could be low in FP versus UP contracts, simply because simple projects are procured via FP contracting and more complex projects via UP contracting. Furthermore, if FP contracts indeed reduce procurement cost for simple projects, while UP contracts are well suited for projects with greater project risk, then the effects of FP relative to UP contracting depend on unobserved project heterogeneity: project heterogeneity observable to the bidders and FDOT project managers, but unobservable to the econometrician. We show that the FDOT project manager's contract choice indeed depends on unobserved project heterogeneity in a way that is consistent with FDOT's belief that UP contracts are well suited for projects involving much uncertainty.

UP contracts, however, may induce bidders to behave opportunistically through changing in the composition of unit-prices on their bid form. As UP contracts compensate for cost overruns, bidders have much incentive to bid aggressively to obtain compensation, in expectation, through cost overruns. We show that the composition of unit-price bids varies substantially within a given auction, and that the unit-price composition is strongly related to the bidder's likelihood of winning.

4.1 Endogeneity of Unobserved Project Heterogeneity

While the OLS comparison in Table 3 suggests there is little difference in project outcomes and bidder behavior across UP and FP contracts, contract choice is likely not exogenous. Here, we provide a simple framework based on the Heckman (1976) selection model to test whether the FP/UP contract choice depends on unobserved project heterogeneity via correlation of contract format choices and bidding strategies.

Let X be a vector of project and bidder characteristics, and let $Z \supset X$ be a vector of exogenous observables relevant to the FDOT's project manager's contract choice, denoted by V. Let $score_f$ and $score_u$ denote bidder score under FP and UP contracts, respectively. Then, we consider:

$$
\begin{aligned}
V &= Z\gamma + \varepsilon_p \\
\ln(score_f) &= X\beta_f + \varepsilon_f \\
\ln(score_u) &= X\beta_u + \varepsilon_u
\end{aligned}
$$

and the FDOT project manager's choice between FP and UP is governed by:

$$
FP = \begin{cases} 1 & \text{if} \quad V \geq 0 \\ 0 & \text{if} \quad V < 0 \end{cases}
$$

where γ, β_f, and β_u are vectors of parameters. We assume ε_p, ε_f, and ε_u are trivariate normal random unobservables with $\text{Var}(\varepsilon_f) \equiv \sigma_f^2$, $\text{Var}(\varepsilon_u) \equiv \sigma_u^2$, $corr(\varepsilon_p, \varepsilon_f) \equiv \rho_f$, and $corr(\varepsilon_p, \varepsilon_u) \equiv \rho_u$. We normalize $\text{Var}(\varepsilon_p)$ to 1. Unobservables are assumed to be independent of Z.

The expected bidder score, given contract format $j \in \{f, u\}$ and observables Z is given by:

$$E\left[\ln(score_j)|FP = 1, Z\right] = X\beta_j + \rho_j \sigma_j \frac{\phi(Z\gamma)}{\Phi(Z\gamma)} \quad (1)$$

$$E\left[\ln(score_j)|FP = 0, Z\right] = X\beta_j - \rho_j \sigma_j \frac{\phi(Z\gamma)}{1-\Phi(Z\gamma)} \quad \text{for } j \in \{f, u\} \quad (2)$$

where $\phi(.)$ and $\Phi(.)$ are the PDF and CDF of a standard normal random variable, respectively. We test $H_0 : \rho_j = 0$ against $H_A : \rho_j \neq 0$ for $j \in \{f, u\}$.

The intuition behind the test is as follows. If FDOT project managers are following FDOT project guidelines, recall that project managers should be less likely to employ FP contracts when project risk is high. Additionally, if project risk is not fully captured by the observables, then unobservable ε_p captures unobserved project risk. We also expect unobserved project risk to be captured by ε_f as bidders' unobserved costs are likely increasing in unobserved project risk. Thus, adoption of FP contracts and bidding strategy on FP projects are negatively correlated with project risk (i.e., $\rho_f < 0$) via project risk. Similarly, we would expect a weak correlation between the adoption of UP contracts and bidding strategy on UP projects if UP contracts are robust to project risk: bidders' unobserved costs are weakly related to project risk.

To provide identification of ρ_j without purely relying on functional form assumptions, we now introduce excluded variables in Z that affect FP/UP contract choice but do not enter X. We now turn to the description of our excluded variables.

4.2 Excluded Variables

Our excluded variables capture the extent of backlog experienced by the relevant parties of the auction process. First, for each FDOT district office, backlog is measured by the total dollar value of unfinished projects that the district office has at the time of procurement. Since UP contracts involve a large administrative cost – as FDOT would need considerably more personnel to keep track of the number of units of all materials used during the construction phase of a UP project – an FDOT project manager would therefore be more likely to employ FP contracts when the office is heavily backlogged. Second, since prospective contractors are also likely backlogged when district offices are backlogged, we construct bidder backlog in the same manner and directly control for it. That is, we argue that the level of backlog at an FDOT district office has nothing to do with prospective contractor bidding strategies (e.g., bidders' unobserved costs), conditional on bidders' backlogs. We construct backlog for each of FP and UP contracts, as the level of backlog depends on contract format.

4.3 Estimation Results

Table 4 indicates a strong negative correlation between ε_p and ε_f, consistent with anecdotal evidence.[22] When project risk is large, project managers are less likely to adopt the FP contract and prospective contractors' costs tend to be high, which is passed onto their scores. Note here that the bias-corrected expected value (1) indicates that, on average, FP contracts generate lower bidder scores than UP contracts, with $\rho_f < 0$.

The weak insignificant correlation between ε_p and ε_u is also consistent with anecdotal evidence that FDOT believes UP contracts are robust to project risk. If UP contracts are robust to project risk, then bidder costs ε_u would be uncorrelated with project risk, since project risk does not translate into bidder costs. Therefore, our estimation results are in line with FDOT's belief that UP contracts should be used for projects with greater project risk.[23]

One concern with our approach here is that the excluded variable may be correlated with unobserved project heterogeneity. The exclusion restriction would be violated if the FDOT project managers can anticipate the complexity of projects well before project letting, and accordingly, try to coordinate and decide when to procure which project based on the complexity of projects. Ideally, we would like to check if cost overruns are correlated with our excluded variables. However, we do not observe the actual cost overrun under FP contracts, by construction. Thus, we instead test if time overruns are correlated with our excluded variables. Statistically significant correlation between our excluded variables and *ex-post* auction outcomes would cast doubt on the validity of our excluded variables. To implement this idea, we regress the log-difference in completion time and expected contract duration on our excluded variables together with exogenous project characteristics. We then run an F-test to check whether the

[22] F-test for the first stage indicates that our excluded variables are relevant.

[23] The estimation results also suggest that $\sigma_f > \sigma_u$: scores are more dispersed under FP than UP contracts. Our model is also consistent with this observation.

coefficients on all excluded variables (district office backlog) are jointly zero, controlling for bidder and project characteristics, and various fixed effects. Table 5 displays the relevant p-values. Given that the p-values range from 20% to over 80% after controlling for time fixed effects, the correlation between FDOT district office backlogs and time overrun is not statistically significant, even at confidence levels well below 90%. Therefore, we conclude that there is no evidence that our excluded variables are correlated with project risk.

If UP contracts are robust to project risk and generate lower bidder scores than FP contracts, then why would FDOT use FP contracts? One reason may be to avoid the high administrative costs associated with UP contracts. In the following subsection, we illustrate another reason why the FDOT may wish to avoid using UP contracts.

Table 4: Endogenous Switching Model: Estimation Results

Dependent Variable	$\ln(score_j)$							
Specification	(1)		(2)		(3)		(4)	
Regime	FP	UP	FP	UP	FP	UP	FP	UP
ρ_f, ρ_u	-.754	.0790	-.688	.0897	-.792	.142	-.699	.131
	(.081)	(.074)	(.14)	(.054)	(.060)	(.051)	(.16)	(.049)
σ_f, σ_u	.370	.237	.316	.212	.314	.207	.294	.206
	(.025)	(.0067)	(.026)	(.0064)	(.022)	(.0063)	(.030)	(.0062)
District Office Backlog	yes	yes	yes	yes	yes	yes	yes	yes
Project Characteristics	yes	yes	yes	yes	yes	yes	yes	yes
District FE	yes	yes	yes	yes	yes	yes	yes	yes
Year FE	no	no	yes	yes	yes	yes	yes	yes
Month FE	no	no	yes	yes	yes	yes	yes	yes
Bidder FE	no	no	no	no	yes	yes	yes	yes
Project Type FE	no	no	no	no	no	no	yes	yes
N	8654	8654	8654	8654	8654	8654	8654	8654

Standard errors are clustered at the district-year-month level. Project characteristics include engineer's estimate of project cost, number of plan holders, project type fixed effects, month fixed effects, year fixed effects, and district fixed effects. Project types are defined as a linear combination of tasks, which are extracted from the work description of bid tabs. Bidder characteristics include bidder backlog from FP and UP contracts. Bidders that have won less than one percent of the total value of projects during the sample period are grouped together as fringe firms and treated as the same bidder when controlling for bidder fixed effects. District office backlog is calculated as the total dollar value of projects uncompleted at the time of project letting.

Table 5: Test of Endogeneity of Excluded Variable

Dependent Variable	Time Overrun				
F-Test ($p-$value)	.000	.278	0.208	0.869	.806
Bidder Characteristics	yes	yes	yes	yes	yes
Project Characteristics	yes	yes	yes	yes	yes
District FE	yes	yes	yes	yes	yes
Year FE	no	yes	yes	yes	yes
Month FE	no	no	yes	yes	yes
Bidder FE	no	no	no	yes	yes
Project Type FE	no	no	no	no	yes
N	1877	1877	1877	1877	1877

Time overrun is defined as log-difference in actual construction days and expected contract days. The null hypothesis for the F-test is the coefficients on all excluded variables (district office backlog) are jointly zero.

4.4 Skewed Bidding

Table 6presents the top 10 most frequently used items in UP contracts. It turns out that contractual arrangements differ across items. Indeed, some items are procured in a lump-sum manner; i.e., there would be no adjustment in payment associated with any adjustments in these items.[24] Figure 3 shows the distribution of the sum of unit prices across lump-sum items as a share of bidder score, which clearly exhibits considerable variation.

Table 7 shows the source of variation in non-lump-sum item values by decomposing the variance of the share of bids on non-lump-sum items. It reveals that 30% of the total variance is within-auction. The question here is why bidders differ so much in their composition of unit-price bids for any given auction.

Table 8 regresses bidder score and winning status on the share of non-lump-sum item bids. It shows that bidders who place a large share of bids on non-lump-sum items bid much more aggressively than those with a small non-lump-sum item bid share. We find that a bidder with one standard deviation higher share of non-lump-sum bids attains a 3.4% lower bidder score, and is 7.3% more likely to win a UP project, based on the specification with only auction fixed effect. The correlation is even stronger after controlling for bidder fixed effect. These results pose the question: why do bidders with a higher share of non-lump-sum bids have an incentive to bid more aggressively than others?

[24]Quantity estiamte for lump-sum item is always set equal to one.

Table 6: Contract Type of Top 10 Items in UP contracts

Item Category	**Contractual Arrangement**	**Frequency**
Mobilization	Lump-Sum	1241
Maintenance of Traffic	Lump-Sum	1239
Work Zone Sign	Per Day	1217
Temporary Barricade	Per Day	1168
Advance Warning / Arrow Board	Per Day	890
High Intensity Flashing Lights	Per Day	1200
Temporary Retro-reflective Pavement Marker	Each Unit	865
Portable Changeable Message Sign	Per Day	1004
Clearing & Grubbing	Lump-Sum	1067
Painted Pavement Markings	Lump-Sum	788

The means are calculated using the lowest bidder's unit-price bid from 1,341 unit-price auctions. Quantity is estimated by FDOT prior to auction.

One plausible explanation here is that a bidder who anticipates a large cost overrun for non-lump-sum item places a high unit-price bid on non-lump-sum item in expectation to get compensated through cost overrun. Since the bidder with a large estimate for non-lump-sum item still needs to compete against other bidders to win a contract, he/she places a low unit-price bid on lump-sum item, which is not affected by either cost overrun or underrun. Those bidders with skewed distribution of unit-price bids more aggressively to win a contract as they expect to get paid more from cost overruns.

One may suspect that more risk-averse bidders would want to place a larger share of bids on non-lump-sum items and bid more aggressively than less risk-averse bidders if bidders are characterized by decreasing absolute risk aversion.[25] If this were the case, however, the inclusion of bidder fixed effects should remove some of the confounding effects rather, than strengthen the result.

Figure 4 is a scatter plot showing a clearly positive relationship between cost overrun in UP contracts and winners' bids on non-lump-sum items in dollars. We find that cost overrun is increasing in bids on non-lump-sum items on average, suggesting that skewed bidding is associated with larger cost overruns.

[25] This point is made by Phil Haile in Athey and Levin (2001).

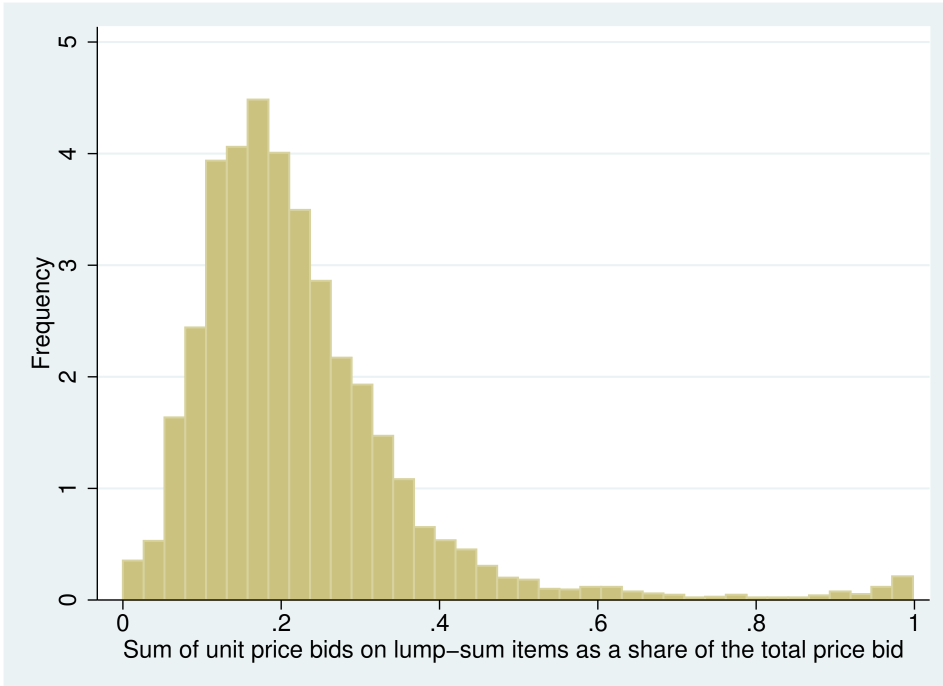

Figure 3: Distribution of the sum of unit prices across lump-sum items as a share of bidder score

Table 7: Variance Decomposition of Share of Non-Lump-Sum Bids

	Std. Dev.	Percentage
Between-Auction	.130	70%
	(.0027)	
Within-Auction Between-Bidder	.0560	30%
	(.0005)	

Standard errors in parentheses

Table 8: Share of Non-Lump-Sum Bid and Bidding Strategy

Dependent Variable	log(score)		win	
Share of Non-Lump-Sum Bid	-.636 (.030)	-.832 (.034)	1.35 (.10)	1.58 (.11)
Auction FE	Yes	Yes	Yes	Yes
Bidder FE	No	Yes	No	Yes
N	6373	6373	6373	6373

Standard errors in parentheses

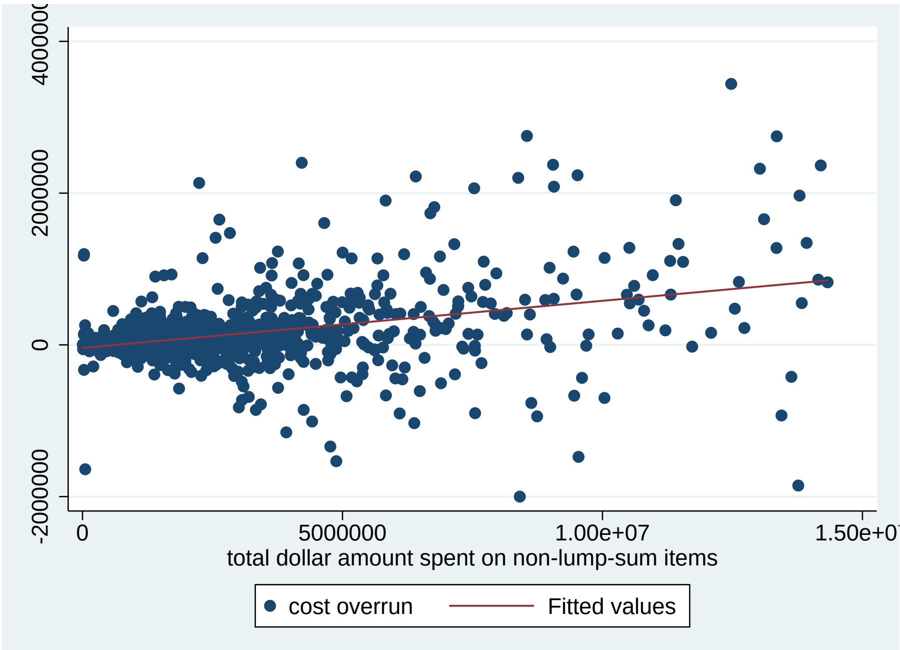

Figure 4: Cost Overrun and Bids on Non-Lump-Sum Items

5 Structural Model

The empirical evidence suggests that i) UP contracts are robust to project risk, ii) bidder scores are much more dispersed in FP versus UP auctions, iii) bidders are substantially heterogeneous with regards to a portfolio of unit-price bids, and iv) bidders who skew their bids towards non-lump-sum items are much more likely to win a project. Motivated by these empirical findings, we construct a structural model of bidding for contracts.

Our model nests both UP and FP contracts. UP contracts differ from FP contracts in that contractors are compensated through cost overruns on contracted items, and that bidders can hedge against project risk by forming a portfolio of unit-price bids. Since bidders may also differ in their quantity estimates, the portfolio of unit-price bids also differs: bidders may submit high unit-prices for underestimated items and low unit-prices for overestimated items. The bidder with the largest estimate has the greatest incentive to win the contract, and therefore would want to bid aggressively to get compensated through cost overruns. This incentive to skew unit-price bids dissipates with increasing project risk, since skewed bidding comes with an increase in payoff uncertainty.[26]

We extend the model of Ewerhart and Fieseler (2003) on various dimensions to capture empirically relevant features of the environment. First, we introduce multidimensional bidder heterogeneities to add flexibility in multidimensional bidding strategies. Without multidimensional bidder heterogeneities, multidimensional bidding strategies would be a function of a single type, which is restrictive and cannot rationalize the observed distribution of unit-price bids in the data. Second, we introduce risk aversion to account for the fact that complete skewing is not observed in the data. Lastly, we endogenize the entry decision of bidders, since changes in contract format affect bidders' incentive to participate in a given auction. In particular, we will see that UP contracts induce more competition than FP contracts since, all else equal, skewed bidding and risk hedging raise the expected return from entering an auction.

The timing of events is as follows.

1. **Entry Stage:** Consider N risk-averse potential bidders with constant absolute risk-aversion (CARA) utility $u(.)$, parametrized by $\alpha \geq 0$.[27] Each of N potential bidders independently draws entry cost

[26]We abstract from the moral hazard problem as i) the construction process is closely monitored by FDOT employees, and ii) most *ex-post* adjustments in the construction plan are initiated by FDOT project managers rather than contractors.

[27]The assumption of CARA may seem restrictive since projects are heterogeneous in project size and bidders may be

k_i, from a common distribution $F_{ec}(.)$. Bidders are privately informed about their own entry cost and make their entry decision simultaneously. All participating bidders learn the number of actual bidders n upon entry.[28]

2. **Bidding Stage:** A project involves two items: a lump-sum item and a non-lump-sum item.[29] Let θ_1 and θ_2 denote the cost of providing lump-sum and non-lump-sum items at the FDOT's quantity estimates. These costs are common to all participating bidders.[30] Upon entry, bidder i learns its own estimate of quantity, denoted by $e_{j,i}$ for item $j \in \{1, 2\}$.[31] Quantity estimate $e_{j,i}$ is the private information of bidder i, drawn independently across bidders from a common joint distribution H. The distribution H has a smooth density over a finite positive support. Assume that $\mathrm{E}[e_{j,i}] = 1$ and $e_{j,i}$ can be interpreted as the bidder's quantity estimate normalized against the FDOT's quantity estimate. Given the quantity estimates, all participating bidders simultaneously submit prices $b_{j,i}$ on item j. The bidder with the lowest score $s_i \equiv b_{1,i} + b_{2,i}$ wins the contract.

3. **Implementation Stage:** The FDOT project manager may make adjustments to the non-lump-sum item, denoted by ϵ which is independently drawn from a normal distribution (i.e., $\epsilon \sim N(0, \sigma)$).[32] The demand shock affects the quantity required to complete the project.[33] The contractor receives payment based on the contract format.

more risk averse for larger projects. In order to allow for heterogeneity in the level of risk aversion, we allow risk aversion to depend on project size (and project characteristics in general) later in the identification section.

[28] We assume that all the primitives that are common to all potential bidders are common knowledge at the time of entry.

[29] Extending the model to more than two items can be done without any difficulty in equilibrium characterization, but imposes difficulty in estimation. See Appendix 10.2 for an extension of the model with K non-lump-sum items.

[30] We assume that any given auction always has more than one participating bidder. The assumption that more than one bidder enters is to prevent the unintuitive bidding strategy in which a bidder submits an infinitely high score when it is revealed the bidder is the sole participant in the auction. There are only a few auctions with only one participating bidder in the data. This assumption is also adopted in Li and Zheng (2009).

[31] 32Our model differs from Bajari et. al. (2014) in that we allow for the expected quantity of work items to differ across bidders and for bidders to face uncertainty in actual item quantity. Relaxing these assumptions explains the considerable variation in composition of unit-price bids in any given auction and also explains why bidders do not completely skew their bids.

[32] It is possible to allow for correlation between bidders' private information $e_{2,i}$ and *ex-post* shock ϵ, but this makes the model significantly more notationally involved and turns out to be empirically not relevant. Thus, we present the model where *ex-post* shock is independently distributed of private $e_{2,i}$.

[33] Since the demand shock on lump-sum items does not affect the characterization of equilibrium bidding strategy and its dispersion is not identifiable, we set $\epsilon_1 = 0$. This abstraction of uncertainty in non-lump-sum item quantity is justified under the CARA assumption, since bidders would adjust their bids by exactly the risk premium. See Eso and Whilte (2004) for details.

Let us first consider UP contracts. The final payment to the winning bidder, denoted by $p_{u,i}$, is given by:

$$p_{u,i} = b_{1,i} + b_{2,i}(e_{2,i} + \epsilon), \tag{3}$$

where there is no uncertainty in payment for lump-sum items. The additive separability in private information $e_{j,i}$ and demand shock ϵ implies common uncertainty across bidders in the sense that the project risk does not depend on private information $e_{j,i}$.

Note that the final payment to a contractor could differ for two reasons. First, the final payment may differ from bidder score s_i due to $e_{2,i}$. For example, suppose that non-lump-sum item is contracted based on the number of work days. That is, $b_{2,i}$ specifies how much contracting firm i receives if it completes the project on the auctioneer's expected completion date. In practice, contractors differ in terms of speed in delivering the project. Some contractors are fast ($e_{2,i} < 1$) while others are slow ($e_{2,i} > 1$). Thus, the payment scheme implies that, all else equal, fast contractors receive a smaller payment than slow contractors. Second, the final payment may differ from score s_i due to demand shock ϵ. Bidder estimates are also imperfect and affected by unexpected changes in the project plan. Demand shock ϵ captures unexpected delay in project implementation, and $\mathrm{Var}(\epsilon) \equiv \sigma$ captures project risk.

The total cost of implementing the project, $tc_{u,i}$, is defined as:

$$tc_{u,i} = \theta_1 e_{1,i} + \theta_2(e_{2,i} + \epsilon). \tag{4}$$

Bidder i's interim expected payoff upon entry, $\pi_{u,i}$, is defined as:

$$\pi_{u,i} = \max_{b_{1,i}, b_{2,i}} \int \mathrm{E}\left[u\left(p_{u,i} - tc_{u,i}\right) | \boldsymbol{\mathcal{I}}\right] \boldsymbol{dF_{tc,-i}}, \tag{5}$$

where $\boldsymbol{\mathcal{I}} \equiv \{\theta_1, \theta_2, \alpha, \sigma, e_{1,i}, e_{2,i}, \boldsymbol{F_{tc}}, n\}$, $\mathrm{E}[.|\boldsymbol{\mathcal{I}}]$ is the expectation over the distribution of ϵ, and $\boldsymbol{F_{tc,-i}}$ is the distribution of rival bidder types.

The bidder's nested optimization problem consists of two parts: an inner loop to optimize portfolio $\{b_{1,i}, b_{2,i}\}$ given total bidder score s_i, and an outer loop to optimize the total bidder score s_i. Given the normal distribution assumption on ϵ and CARA utility, the inner problem of a bidder can be written as:

$$\max_{b_{j,i}\geq 0} \quad b_{1,i}+b_{2,i}e_{2,i}-(\theta_1 e_{1,i}+\theta_2 e_{2,i})-\frac{\alpha\sigma}{2}(b_{2,i}-\theta_2)^2$$
$$s.t. \quad s_i = b_{1,i}+b_{2,i},$$

where $\alpha \geq 0$ represents the CARA risk-aversion coefficient. Solving this simple constrained optimization problem gives:

$$b_{2,i}=\begin{cases} 0 & \text{if } \theta_2+\frac{e_{2,i}-1}{\alpha\sigma}<0 \\ \theta_2+\frac{e_{2,i}-1}{\alpha\sigma} & \text{if } \theta_2+\frac{e_{2,i}-1}{\alpha\sigma}\in[0,s_i] \\ s_i & \text{if } \theta_2+\frac{e_{2,i}-1}{\alpha\sigma}>s_i \end{cases} \tag{6}$$

See Appendix 10.2 for the derivation. Condition (6) shows an interesting relationship between project risk σ and bid skewness. For example, consider the case of no project risk, $\sigma = 0$. Bidders with a large cost of delivering the non-lump-sum item (i.e., $e_{2,i} > 1$) would skew their bid toward the non-lump-sum item completely, and submit $\{b^*_{1,i} = 0,\ b^*_{2,i} = s_i\}$. On the other hand, those bidders with a small cost of delivering the non-lump-sum item (i.e., $e_{2,i} < 1$) would skew their bid toward the lump-sum item completely, and submit $\{b^*_{1,i} = s_i,\ b^*_{2,i} = 0\}$. Now, consider the other extreme, in which project risk $\sigma = \infty$. The optimal bidding strategy of a bidder of any type $e_{2,i}$ would be $\{b^*_{1,i} = s_i - \theta_2,\ b^*_{2,i} = \theta_2\}$. Thus, the incentive to skew bids dissipates as project risk increases.

For the remainder of the paper, we consider the case of moderate project risk, in which the solution to the inner problem above is $b^*_{2,i} = \theta_2 + \frac{1}{\alpha\sigma}(e_{2,i} - 1)$, since we observe neither completely skewed bidding nor identical bid amounts on non-lump-sum items in the data.

Note that $b^*_{2,i}$ is increasing in $e_{2,i}$, which implies that a high-cost bidder that uses many non-lump-sum items bids high on non-lump-sum items. For example, if a non-lump-sum item is contracted on a daily basis, then a slow bidder would bid high on those non-lump-sum items.

Given $b^*_{2,i}$, we have $b^*_{1,i} = s_i - \left(\theta_2 + \frac{1}{\alpha\sigma}(e_{2,i} - 1)\right)$ and so plugging $b^*_{1,i}$ and $b^*_{2,i}$ into the certainty equivalent payoff of bidder i gives:

$$u\left(b_{1,i}^{*}+b_{2,i}^{*}e_{2,i}-(\theta_{1}e_{1,i}+\theta_{2}e_{2,i})-\frac{\alpha\sigma}{2}(b_{2,i}^{*}-\theta_{2})^{2}\right) \quad = \quad u\left(s_{i}-c_{u,i}\right)$$

where $c_{u,i}$ is the pseudo-type of bidder i defined as:

$$c_{u,i} \quad \equiv \quad \theta_{1}e_{1,i}+\theta_{2}-\frac{1}{2\alpha\sigma}(e_{2,i}-1)^{2}. \tag{7}$$

Thus, bidder i's problem reduces to one-dimensional choice problem, such that:

$$\pi_{u,i} \quad = \quad \max_{s_{i}} \int u(s_{i}-c_{u,i})\boldsymbol{dF_{u,-i}}, \tag{8}$$

and $\boldsymbol{dF_{u,-i}}$ is the distribution of rival bidders' pseudo-types. Note that $c_{u,i}$ is inverse U-shaped in $e_{2,i}$ centered at $e_{2,i}=1$, which implies that bidder score s_i is non-monotone in $e_{2,i}$ in a monotone equilibrium where s_i is non-decreasing in $c_{u,i}$. This means that for those bidders whose estimate on the non-lump-sum items differs substantially from the auctioneer's estimate would bid more aggressively than those whose estimates are closer to the auctioneer's estimate. Using the example of workdays, the model captures that both efficient and inefficient bidders bid more aggressively than bidders who can deliver the project on time as expected by the FDOT district office. While it is standard for efficient bidders to submit lower bids in procurement auctions, UP contracts provide a unique incentive for inefficient bidders to do the same, as inefficient bidders know that they would receive more payment if they win the contract. Thus, less efficient contractors lower their bidder score by skewing their bids towards non-lump-sum items to get compensated in expectation through *ex-post* adjustments on non-lump-sum items.

Since the remaining equilibrium characterization relates to the auction literature, we summarize it in the following proposition.

Proposition 1. *The unique symmetric, monotone, and differentiable equilibrium bidding strategy is char-*

acterized by the following differential equation and the initial condition:

$$\begin{aligned}\frac{\partial s(c_u;n)}{\partial c_u} &= 1+\frac{(n-1)f_u(c_u)}{\alpha(1-F_u(c_u))}\left(\exp\left\{\alpha(s(c_u;n)-c_u)\right\}-1\right) \\ s(\bar{c}_u;n) &= \bar{c}_u,\end{aligned} \tag{9}$$

where F_u and f_u are, respectively, the CDF and PDF of c_u, which is continuous and bounded over $[\underline{c}_u, \bar{c}_u]$.

Given the unique bidding strategy above, a potential bidder enters an auction if the expected profit from entering outweighs the cost of entry. As shown in Krasnokutskaya and Seim (2011), the unique symmetric equilibrium entry strategy is given by the entry threshold utility $\bar{u}$, which is determined by:

$$\sum_{n}^{N}\binom{N-1}{n-1}\delta(\bar{u}(N))^{n-1}(1-\delta(\bar{u}(N)))^{N-n}\int u\left(s(c_u;n)-c_u\right)\boldsymbol{dF_{u,n}} = \bar{u}(N) \tag{10}$$

where the left-hand side of (10) is the equilibrium expected profit from entering an auction, and $\boldsymbol{F_{u,n}}$ is the distributions of c_u over all n entering bidders. The equilibrium entry probability is determined by $\delta(\bar{u}(N)) \equiv \Pr(u(-k) < \bar{u}(N))$. That is, a bidder participates in an auction if the bidder's entry cost k is below some threshold level which corresponds to the level of utility $\bar{u}(N)$.

Now, let us consider the case of FP contracts. FP payment $p_{f,i}$ is given by:

$$p_{f,i} = s_i \tag{11}$$

where s_i is bidder i's score in an FP auction. That is, the FP payment is the same as bidder i's score, which is equal to the bid submitted by bidder i. Therefore, the interim expected payoff of bidder i under FP with CARA utility is given by:

$$\pi_{f,i} \equiv \max_{s_i}\int u\left(s_i-c_{f,i}\right)\boldsymbol{dF_{f,-i}} \tag{12}$$

where the pseudo-type of bidder i, $c_{f,i} \in [\underline{c}_f, \bar{c}_f]$, is defined as:

$$c_{f,i} \equiv \theta_1 e_{1,i} + \theta_2 e_{2,i} + \frac{\alpha\sigma}{2}\theta_2^2. \tag{13}$$

The distributions of rival bidder pseudo-types is denoted by $\boldsymbol{F_{f,-i}}$. The remainder of the FP equilibrium characterization is similar to that of UP contracts.

5.1 Bidding Strategies and Contract Outcomes

We demonstrate the effect of contract type on the distribution of bidder pseudo-types and the expected final payment under varying levels of project risk. The pseudo-types from UP and FP contracts indicate that bidder cost structure changes endogenously with respect to the type of contractual arrangement. In particular, the cost of project risk increases more slowly under UP versus FP contracts. Thus, we expect UP contracts to be better suited than FP contracts for projects with large project risk. On the other hand, UP contracts may suffer from the selection of inefficient contractors who are able to submit competitive bids via skewed bidding and obtain compensation in expectation through cost overruns.

To see the effect of project risk on bidder pseudo-types under FP/UP contracts, we simulate the distribution of pseudo-types under the two contract types and vary the level of project risk. Figure 6 shows that i) pseudo-cost increases faster in project risk under FP versus UP contracts, and ii) the distribution of bidder pseudo-types under FP is more dispersed than under UP contracts and shows relatively lower costs when project risk is large. These differences in pseudo-costs are directly translated into bidder scores, and hence inefficient bidders are able to outbid moderately efficient bidders. Figure 6 plots expected final payment against project risk. The expected final payment is lower under FP (UP) contracts when project risk is low (high).

The intuition behind the results is best explained with an example. Suppose that the pay scheme for the non-lump-sum item under UP is based on days required to complete the project. All else equal, contractors that deliver a project quickly would be paid less than contractors who work more slowly. Therefore, less efficient firms would have a larger incentive to win the contract and get compensated in expectation through larger cost overruns than efficient firms. This leads to a selection towards less efficient contractors with large cost overruns, because the more aggressive bidders are the ones that expect larger cost overruns. Since there is a tradeoff between risk hedging and skewed bidding, the incentive to skew bids decreases in project risk. Therefore, FP projects result in lower final payments than UP projects when project risk is low, since FP contracts always select the most efficient contractor among the set of bidders. UP outperforms FP contracts when project risk is high, since UP contracts allow bidders to hedge against project risk and the incentive to skew bidding decreases with project risk.

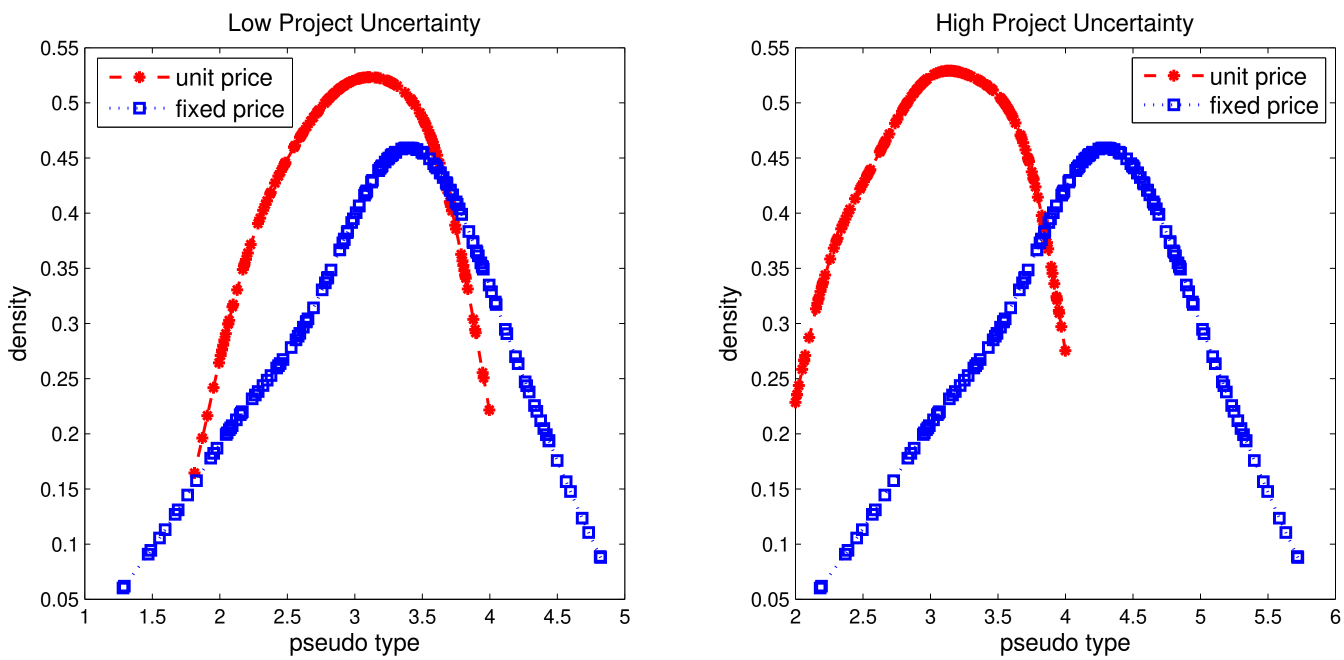

Figure 5: Distributions of Pseudo-cost Types under UP and FP Contracts

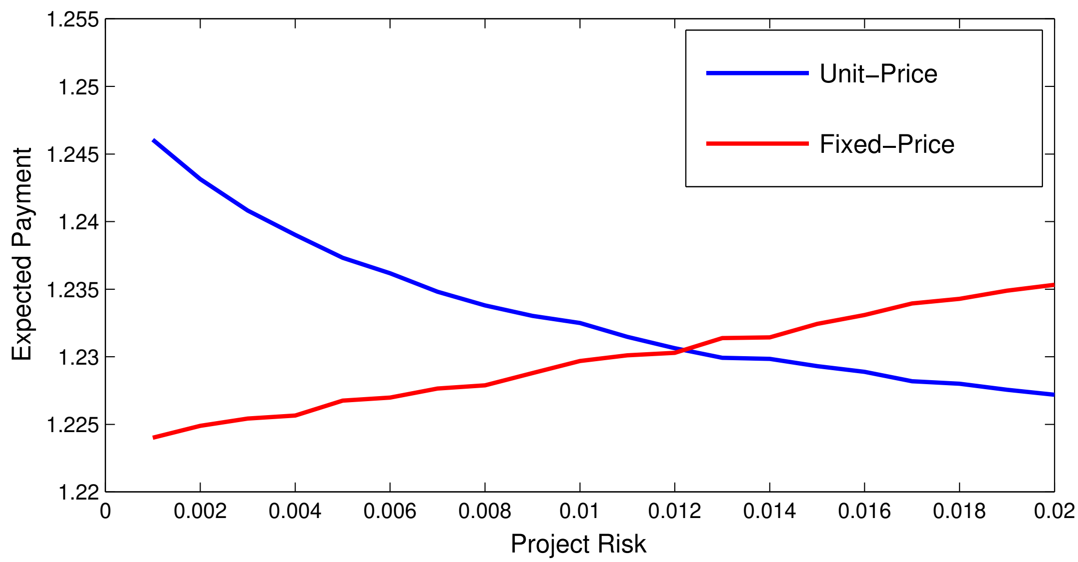

Figure 6: Expected Final Payment and Project Risk in FP and UP Contracts

6 Identification

First, this section specifies the information structure, the set of model primitives to be identified, and the set of observables used to identify the model primitives. Then, we show that the model is semiparametrically identified.

6.1 Observables, Primitives, and Information Structure

Let X denote a vector of exogenous project characteristics, and let $W \subset X$ denote exogenous variables that affect bidding strategy but not entry decision. The econometrician observes the number of potential bidders N, the number of actual participating bidders n, bids on lump-sum and non-lump-sum items for all participating bidders, $\{b_{1,i}, b_{2,i}\}_{i=1}^{n}$, and cost overruns on contracted items Δ in UP contracts. Without loss of generality, we rank bidders based on their score $s_{u,i} = b_{1,i} + b_{2,i}$, and the winner of an auction is assigned $i = 1$.

The primitives to be identified are the joint distribution of bidder types H, common cost components $\theta_j(X)$ for $j \in \{1, 2\}$, risk-aversion parameter $\alpha(X)$, project risk $\sigma(X)$, and distribution of entry cost F_{ec}. Let $\mathcal{I}_i$ denote bidder i's state at the time of bidding. Identifying Assumption 1 summarizes what bidders know at the time of bidding.

Identifying Assumption 1. *At the time of bidding, state $\mathcal{I}_i$ of bidder i consists of auction heterogeneities, bidder i's private information, the joint distribution of rival bidders' private information, the number of participating bidders, estimated costs, project risk, and the number of actual bidders: $\mathcal{I}_i \equiv \{\theta_1(X), \theta_2(X), \alpha(X), \sigma(X), e_{1,i}, e_{2,i}, H, n\}$.*

Identifying Assumption 1 is standard in the empirical auction literature. The assumption that the number of actual bidders is common knowledge can be tested. We find that bidder scores, and thereby bidding strategy, are strongly negatively correlated with the number of actual bidders, suggesting that auction entrants know how many rivals they face at the time of bidding and bid more aggressively as the number of participating bidders increases.

Identifying Assumption 2. *Bidders' private information is i.i.d. across bidders and also independently distributed from entry cost, conditional on project characteristics. That is, the bid preparation cost is* *irrelevant of its productivity, conditional on project characteristics.*

Identifying Assumption 2 2 is required for identifying the distribution of bidders' private types H. Intuitively, the econometrician has no way of detecting which of bidders' private information, $e_{1,i}$ or $e_{2,i}$, is correlated with its entry cost from the data, precluding the possibility of allowing for selective entry.

Identifying Assumption 3. *Ex-post adjustments on non-lump-sum bids ϵ are independently distributed from non-lump-sum bid $b_{2,1}$.*

Identifying Assumption 3 abstracts from the possibility that FDOT project manager's demand for *ex-post* adjustments are endogenous – i.e., FDOT project managers reduce (increase) demand for *ex-post* adjustments when the contractor's non-lump-sum bid is high (low). We argue that *ex-post* adjustments are exogenous in this context based on two grounds. First, if FDOT does not commit, the point of using UP contracts is jeopardized and bidders would adjust their beliefs about the distribution of *ex-post* adjustments accordingly.[34] Second, construction items and tasks are typically non-storable, so FDOT has little incentive to purchase non-lump-sum items to store for later use, even if they are priced low.

Identifying Assumption 4. *There is at least one variable $W \subset X$ that affects project implementation cost without affecting entry cost.*

Identifying Assumption 4 required for identifying the entry cost distribution. Without variable W, all we can identify is the probability of entry, and any distribution of entry cost can be rationalized by the data.[35] To this end, we assume that bid preparation costs are independent of project size, conditional on project type.

6.2 Semiparametric identification

We show that the model primitives are identified from the data on UP contracts and do not rely on variation in the use of contract formats.

[34] Based on private conversations with FDOT project managers, we confirm that this is indeed a concern of FDOT.
[35] See the online appendix of Krasnokutskaya and Seim (2011) for details.

Proposition 2. *Under Identifying Assumption 1-4, all the model primitives are identified.*

First, consider the non-lump-sum bidding strategy given in (6). It is straightforward to see that $\theta_2(X)$ is directly identified from equation (6), such that:

$$\mathrm{E}\left[b_{2,i}|X\right] \quad = \quad \theta_2(X), \tag{14}$$

since $\mathrm{E}[e_{j,i}] = 1$. Given knowledge about $\theta_2(X)$, we identify $\alpha(X)$ and $\sigma(X)$ from the mean and variance of cost overruns. Note here that cost overrun is defined as $\Delta \equiv b_{2,1}(e_{2,1} - 1 + \epsilon)$ where $i = 1$ denotes the winning bidder. Substituting the non-lump-sum bidding strategy given in (6) into cost overrun gives:

$$\frac{\Delta}{b_{2,1}} \quad = \quad e_{2,1} - 1 + \epsilon \tag{15}$$

$$\quad = \quad \alpha(X)\sigma(X)(b_{2,1} - \theta_2(X)) + \epsilon \tag{16}$$

where the second equality follows from the first-order inversion with respect to $b_{2,1}$. Therefore, the extent of bidder risk-aversion $\alpha(X)$ and project risk $\sigma(X)$ is identified from the mean and variance of cost overruns, conditional on $b_{2,1}$, such that:

$$\mathrm{E}\left[\frac{\Delta}{b_{2,1}}|b_{2,1}, X\right] \quad = \quad \mathrm{E}\left[\alpha(X)\sigma(X)(b_{2,1} - \theta_2(X)) + \epsilon|b_{2,1}, X\right] \tag{17}$$

$$\quad = \quad \alpha(X)\sigma(X)(b_{2,1} - \theta_2(X)) \tag{18}$$

$$\mathrm{Var}\left[\frac{\Delta}{b_{2,1}}|b_{2,1}, X\right] \quad = \quad \mathrm{Var}\left[\alpha(X)\sigma(X)(b_{2,1} - \theta_2(X)) + \epsilon|b_{2,1}, X\right] \tag{19}$$

$$\quad = \quad \sigma(X). \tag{20}$$

Given knowledge about $\alpha(X)$, $\sigma(X)$, and $\theta_2(X)$, the distribution of $e_{2,i}$ can now be nonparametrically identified from the solution to the bidders' inner problem,

$$\alpha(X)\sigma(X)(b_{2,i} - \theta_2(X)) + 1 \quad = \quad e_{2,i} \tag{21}$$

Now, let $G_n(.|X)$ and $g_n(.|X)$ denote, respectively, the CDF and PDF of score distributions with n participating bidders conditional on observables X. Expressing the first-order optimality condition (9) in terms of bid distributions gives:

$$\mathrm{E}\left[s_{u,i}-\theta_2(X)-\frac{1}{\alpha(X)}ln\left(1+\alpha(X)\frac{1-G_n(s_{u,i}|X)}{(n-1)g_n(s_{u,i}|X)}\right)+\frac{1}{2}\alpha(X)\sigma(X)(b_{2,i}-\theta_2(X))^2|b_{2,i},X\right]=\theta_1(X).\text{(22)}$$

See Appendix 10.3 for the derivation. Thus, we identify $\theta_1(X)$. Given $\theta_1(X)$, we can now identify the distribution of $e_{1,i}$ nonparametrically:

$$\left[s_{u,i}-\theta_2(X)-\frac{1}{\alpha(X)}ln\left(1+\alpha(X)\frac{(1-G_n(s_{u,i}|X))}{(n-1)g_n(s_{u,i}|X)}\right)+\frac{1}{2}\alpha(X)\sigma(X)(b_{2,i}-\theta_2(X))^2\right]/\theta_1(X)=e_{1,i},\ (23)$$

which corresponds to the first-order inversion of Guerre, Perrigne, and Vuong (2000). Equation (23), together with (21), identify the joint distribution of bidder private information H.

Lastly, the entry cost k is identified from the equilibrium entry condition, given by equation (10). Knowing $\theta_j(X)$ and H_j for $j\in\{1,2\}$ gives us the pseudo-type distribution F_u, as well as interim expected payoff $\int u\left(s_u-c_u\right)\boldsymbol{dF_{u,n}}$ for each number of participating bidders n. In order to identify the distribution of entry costs, we need an additional identifying assumption. In order to identify the distribution of entry costs, we need an additional identifying assumption – specifically, we need a variable that affects the expected payoff but not the entry cost of bidders.[36]

7 Structural Estimation

Nonparametric estimation of the model places a burden on the small sample size and suffers from the curse of dimensionality. Further, the model is highly nonlinear due to the CARA utility assumption. Applied works in the procurement auction literature often assume constant *relative* risk aversion (CRRA), rather than CARA, because of its simplicity and goodness of fit to the data. However, we assume CARA utility, because using CRRA would require the approximation of certainty equivalent payoffs via Taylor expansion, which is valid only for small *ex-post* adjustments. Since the data contain a large degree of *ex-post* uncertainty, we assume CARA utility together with normally distributed *ex-post* adjustments.

On top of the nonlinearity caused by the CARA assumption, our reduced-form evidence suggests exis-

[36]Without this exclusion restriction, we would only be able to identify entry probabilities which can be rationalized by any continuous distribution of entry costs. See the online appendix in Krasnokutskaya and Seim (2011) for details.

tence of unobserved project heterogeneity. Therefore, the econometrician needs to address the possibility that the extent of project risk may differ across projects in a way that is unobserved to the econometrician.

We estimate a finite mixture model that allows for a finite number of discrete unobserved states in project risk σ and mean cost estimates θ_j. Our econometric specification suggests an easy-to-estimate multi-step estimation procedure, which allows for the estimation of model primitives, together with the distribution of unobserved project heterogeneity, via indirect inference. For clarity, we introduce auction index a from here onwards.

7.1 Econometric Specification

We specify the model in a single-index framework. More specifically, we rescale all the model primitives by observables X_a for a given auction a. Define:

$$\begin{aligned} \theta_j^t(X_a) &= \theta_j^t \exp\{X_a\beta\} \\ \sigma^t(X_a) &= \sigma^t \\ \alpha(X_a) &= \alpha/\exp\{X_a\beta\} \end{aligned} \tag{24}$$

where superscript t denotes the state of the world (unobserved to the econometrician) and captures unobserved project heterogeneity in mean and variance parameters. That is, the mean cost estimate and project risk are both allowed to differ across projects in a way that is unobserved to the econometrician. The multiplicatively separable cost specification is commonly employed in the auction literature to account for project heterogeneity.[37] Rescaling wealth in CARA utility requires normalization of the CARA coefficient by observed project characteristics.[38]

One implication of the above econometric specification on the equilibrium bidding strategy is that the scoring strategy, non-lump-sum bidding strategy, and cost overrun are all multiplicatively separable in observables. To see this, let us make explicit the dependency of outcome variables on the primitives. Let $b_{2,ia} \equiv b_2\left(\theta_2(X_a), \sigma(X_a), \alpha(X_a), e_{2,ia}\right)$, $s_{u,ia} \equiv s_u\left(\theta_1(X_a), \theta_2(X_a), \sigma(X_a), \alpha(X_a), e_{1,ia}, e_{2,ia}, n\right)$, and $\Delta_a \equiv \Delta\left(\theta_2(X_a), \sigma(X_a), \alpha(X_a), e_{2,1a}, \epsilon_a\right)$. Define $b^0_{2,ia} \equiv b_2\left(\theta_2(0), \sigma(0), \alpha(0), e_{2,ia}\right)$,

[37]See Haile, Hong, and Shum (2003).
[38]See Theorem 1 in Raskin and Cochran (1986).

$s^0_{u,ia} \equiv s_u(\theta_1(0), \theta_2(0), \sigma(0), \alpha(0), e_{1,ia}, e_{2,ia}, n)$, and $\Delta^0_a \equiv \Delta(\theta_2(0), \sigma(0), \alpha(0), e_{2,1a}, \epsilon_a)$ as "normalized" non-lump-sum score, normalized score, and normalized cost overrun, respectively. This multiplicative separability of project characteristics allows for the bid-homogenization approach in a setting with CARA bidders and reduces computational burden by reducing the number of auctions the econometrician has to solve.

Proposition 3. *Given the econometric specification above, the unique equilibrium non-lump-sum bidding strategy, scoring strategy, and cost overrun are all multiplicatively separable in project characteristics, such that:*

$$\begin{aligned} b_{2,ia} &= b^0_{2,ia} \exp\{X_a\beta\} \\ s_{u,ia} &= s^0_{u,ia} \exp\{X_a\beta\} \\ \Delta_a &= \Delta^0_a \exp\{X_a\beta\}. \end{aligned}$$

See Appendix 10.4 for the proof. Our econometric specification of the model can be interpreted as bidders exhibiting "decreasing" absolute risk aversion in project size. That is, our econometric specification captures the intuitive property that bidders care less about project risk as project size becomes large. Suppose, for example, that there are two projects/auctions of different sizes with the same level of project risk. If the econometrician assumes that bidders participating in these two auctions have the same CARA coefficient (i.e., constant α), then bidders must also care about the project risk in exactly the same way across the two projects. However, one can imagine that bidders participating in the auction for a large project may care less about project risk, since by virtue of selection, these bidders tend to be larger contracting firms with more diversified operations, i.e., care less about uncertainty. Normalizing the CARA coefficient by project characteristics allows the above specification to incorporate the intuition that bidders care less about project risk as the project size becomes larger.

7.2 Estimation Steps

Step 1: Partial out the impact of project characteristics on the bidder scoring strategy by running a log-regression of score $b_{2,ia}$ on the project characteristics:

$$\ln b_{2,ia} = X_a\beta + \ln b^0_{2,ia}$$

Step 2: Estimate CARA parameter α, the distribution of unobserved mean cost estimate for non-lump-sum item θ^t_2, the distribution of unobserved project risk σ^t, and the marginal distribution of $e_{2,ia}$, by maximum likelihood using the following equations on the non-lump-sum bidding strategy and cost overrun:

$$\begin{aligned} b^0_{2,ia} &= \theta^{t'}_2 + \frac{e_{2,ia}-1}{\alpha\sigma^t} \\ \frac{\Delta^0_a}{b^0_{2,1a}} &= \alpha\sigma^t(b^0_{2,1a} - \theta^{t'}_2) + \epsilon_a \end{aligned}$$

where $t, t' \in \{L, H\}$ and log-likelihood function $l(\Theta)$ to be maximized is:

$$l(\Theta) = \sum_a \ln\left(\sum_{t,t'\in\{L,H\}} P_{tt'} \frac{\phi\left(\tilde{\Delta}^{tt'}_a\right)}{(\sigma^t)^{1/2}} \prod_{i\in\mathcal{N}_a} \frac{\alpha\sigma^t\phi\left(\tilde{b}^{tt'}_{2,ia}\right)}{(\sigma_{e2})^{1/2}}\right),$$

where $\mathcal{N}_a$ is the set of participating bidders in auction a, $\phi(.)$ is standard normal PDF, $\tilde{b}_{2,ia} \equiv \alpha\sigma^t\left(b^0_{2,ia} - \theta^{t'}_2\right) / (\sigma_{e2})^{1/2}$, and $\tilde{\Delta}^{tt'}_a \equiv \left(\left(\Delta^0_a/b^0_{2,1a}\right) - \alpha\sigma^t\left(b^0_{2,1a} - \theta^{t'}_2\right)\right) / (\sigma^t)^{1/2}$.

Step 3: Given the estimates obtained from Steps 1 and 2, estimate the distribution of the lump-sum item's cost by indirect inference following Li (2010). To this end, we match the moments of the bidder score distribution in the data with the moments of the scores generated via simulation. More specifically, we:

1. Estimate the conditional moments of homogenized scores $s^0_{u,ia}$ by regressing the second, third, and fourth moments of $s^0_{u,ia}$ on the project characteristics. Denote the resulting vector of estimated coefficients by ψ_{data}.

2. 2. Guess the parameters of the lump-sum item's cost distribution. Simulate the equilibrium scoring

strategy, denoted by $s_{u,ia}^{sim}$, for each bidder in the data using the parameter estimates obtained from the previous steps.

3. Following the first step, estimate the conditional moments of $s_{u,ia}^{sim}$ by regressing the second, third, and fourth moments of $s_{u,ia}^{sim}$ on the project characteristics. Repeat this simulation many times and take average of the estimates, denoted by ψ_{sim}.

4. Search for the structural parameter values of the primitives such that $\psi_{sim} = \psi_{data}$.

Step 4: Given the parameter estimates obtained from Steps 1-3, and following the procedure analogous to Step 3, estimate the distribution of entry costs by matching the moments of the entry decisions in the data and the moments of entry decisions generated via simulation.

7.3 Estimation Results

A finite mixture model requires *a priori* knowledge about the number of unobserved states. To this end, we conduct an elbow test based on the mean and variance of (normalized) non-lump-sum bids, $b_{2,i}^0$. More specifically, we apply K-means clustering on the mean and variance of $b_{2,i}^0$ for each number of potential clusters, and determine the number of clusters where the sum of squared errors stops dropping radically. The results of the elbow test are presented in Figure 7 and 8. The data seem to contain two unobserved states in both mean and variance of $b_{2,i}^0$. We interpret this as a suggestive evidence that the data contain two unobserved states in the mean cost of lump-sum θ_1^t, non-lump-sum items θ_2^t, and the extent of project risk σ^t.

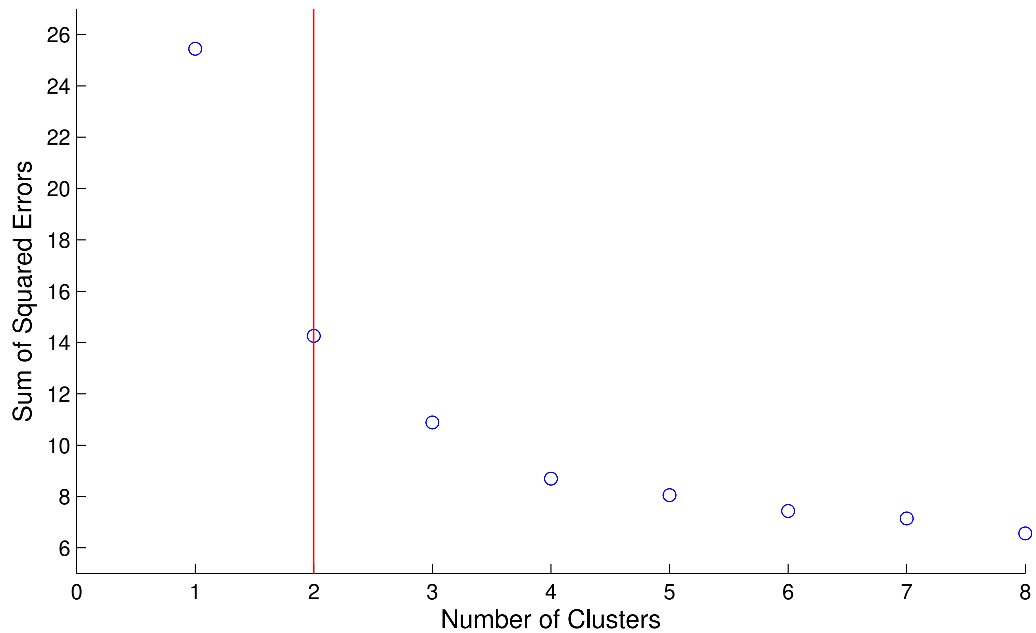

Figure 7: Elbow Test on The Mean of Nonlumpsum Bids

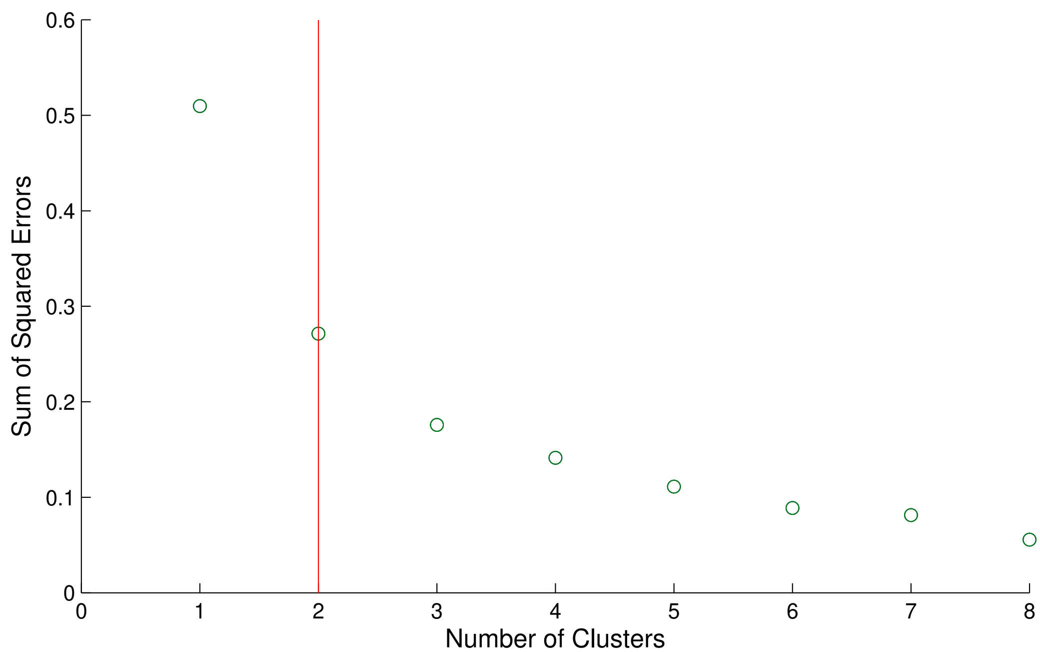

Figure 8: Elbow Test on The Variance of Nonlumpsum Bids

We parameterize the distribution of bidder types by bivariate normal: $[e_{1,i}, e_{2,i}] \sim \boldsymbol{N(1, \Sigma)}$ with correlation $\rho \equiv corr(e_{1,i}, e_{2,i})$ to allow for within-bidder correlation in types.[39] There are four unobserved states (i.e., all possible combinations of θ_2^t and σ^t for $t \in \{L, H\}$) and the probability distribution of each state occuring is denoted by $\Pr_j$ for $j \in \{LL, LH, HL, HH\}$ where $\Pr_{HH} = 1 - \Pr_{LL} - \Pr_{LH} - \Pr_{HL}$.

As shown in Krasnokutskaya and Seim (2011), exclusion restrictions are needed for the identification of the entry cost distribution.[40] To this end, we assume that entry costs (or equivalently, bid preparation costs) are independent of FDOT engineer estimates of project cost, which serve as a proxy for project size, conditional on project types. To determine which project types are relevant to entry cost, we regress the number of participating bidders on a linear combination of tasks (extracted from project work descriptions), together with FDOT engineer cost estimates. We find that projects that involve at least one of "milling" or "guardrail" are associated with lower bidder entry rates. We define a project that involves at least one of these tasks as a "minor" project and estimate the distribution of entry costs for minor and major (non-minor) projects.

Table 9 displays the estimation results. The CARA parameter is precisely estimated and largely in line with estimates found in the literature. There is considerable unobserved heterogeneity in both mean cost estimates θ_j^t and project risk σ^t. An important observation here is that bidder types are highly positively correlated (i.e., $\rho = 0.5$). This finding has an important implication for the effect of employing UP contracts over FP. If bidder types ($e_{1,i}$and $e_{2,i}$) are highly correlated, winning bidders tend to be bidders with low e_{2i}, since efficient bidders (i.e., those with estimate $e_{2,i}$ lower than θ_2) also bid aggressively. Therefore, we expect increases in ρ to be associated with increases in allocative efficiency. That is, the adverse effect of skewed bidding for UP versus FP contracts diminishes as ρ increases, and therefore, the use of UP contracts can be justified, even when project risk is small.

[39] We truncate the top and bottom 1% of the distribution.

[40] Without such exclusion restriction, the econometrician can only recover the probability of entry, which can be rationalized by any entry cost distribution.

Table 9: Structural Estimation Results

Parameter	α	σ^L	θ_1^L	θ_2^L	σ^H	θ_1^H	θ_2^H	σ_{e1}	σ_{e2}	ρ
Estimate	0.716	0.0531	0.232	1.18	0.109	0.421	1.53	1.56	0.0134	0.510
Standard Error	(0.13)	(0.0031)	(0.022)	(0.055)	(0.0032)	(0.049)	(0.054)	(0.43)	(0.0011)	(0.098)

Parameter	μ_{ec}^1	σ_{ec}^1	μ_{ec}^2	σ_{ec}^2	$\Pr_{LL}$	$\Pr_{LH}$	$\Pr_{HL}$
Estimate	0.101	0.012	0.145	0.013	0.341	0.211	0.0564
Standard Error	(0.034)	(0.0042)	(0.040)	(0.0033)	(0.017)	(0.019)	(0.010)

Block-bootstrapped standard errors are presented in parentheses. Auction level characteristics include engineer estimates of project cost and project type. The engineer's cost estimate is an estimate of the winning bid price, as predicted by an FDOT engineer prior to auction. Project type is assigned to each project based on the project description on bid tabs.

8 Does UP contract do well?

A natural question here is whether the UP contract is a mechanism that minimizes procurement costs. In this section, we consider two hypothetical scenarios: i) switching from a UP contract to FP, and ii) imposing a cap on non-lump-sum bids. FP contracts are an obvious alternative to UP contracts, especially when project risk is relatively small.

We consider imposing a cap r on non-lump-sum bids (or equivalently, reserve price) at the estimated mean cost of non-lump-sum item θ_2^t. This experiment allows us to see how the performance of UP contracts can be improved in a simple and costless manner.[41] The intuition is simple but differs from its role of reserve price for a typical first-price auction. A cap on the non-lump-sum bid at θ_2^t would preclude only inefficient types ($e_{2,i} \geq 1$) from skewing their bids and continue to allow efficient types ($e_{2,i} < 1$) to skew their bids. Since cost overruns occur due to skewing of inefficient bidders, setting a cap $r = \theta_2^t$ reduces the extent of cost overruns for inefficient types, which in turn limits their incentive to bid aggressively, and results in efficient selection of contractors via UP contract. More specifically, a bidders' non-lump-sum bidding strategy under reserve price r is given by:

$$b_{2,i} = \begin{cases} \theta_2^t + \frac{e_{2,i}-1}{\alpha\sigma} & \text{if } e_{2,i} < 1 \\ \theta_2^t & \text{if } e_{2,i} \geq 1 \end{cases}$$

[41] Item-wise reserve price is a very common practice in timber auctions, which generally employ UP contracts to select a contractor. See Athey and Levin (2001), for example.

and pseudo-cost of a bidder, c_r, is given by:

$$c_{r,i} = \begin{cases} \theta_1^t e_{1,i} + \theta_2^t - \frac{(e_{2,i}-1)^2}{2\alpha\sigma} & \text{if } e_{2,i} < 1 \\ \theta_1^t e_{1,i} + \theta_2^t & \text{if } e_{2,i} \geq 1 \end{cases}$$

and the scoring strategy is a function of $c_{r,i}$ where $c_{u,i}$ in equation (9) is replaced by c_r.

Table 10 presents the percentage change in the expected final payment by switching contract formats. We find that switching from a UP contract to FP would increase the expected procurement cost in all cases, rationalizing the use of UP contracts by FDOT. We also find that the cost saving effect of UP contracts is larger when project risk is large, and that the cost of project risk is large when the mean estimated cost of non-lump-sum item θ_2^t is large.

Table 11 shows the effect of the non-lump-sum reserve price on the expected final payment. We find that the effect of the reserve price is surprisingly small. There are two explanations for this phenomenon. First, since within-bidder type is highly positively correlated, winning contractors tend to be efficient (i.e., low $e_{1,i}$ and low $e_{2,i}$) and thus, placing a cap on non-lump-sum bids does not do much in affecting the final payment. Second, the scope of skewing is limited by large project risk. Risky projects shift the attention of bidders from skewing to risk hedging, and therefore, leave little difference between efficient and inefficient bidders in terms of non-lump-sum bids.

Table 10: Effect of switching from UP to FP on final payment

	State LL $(\theta_1^L, \theta_2^L, \sigma^L)$	State LH $(\theta_1^L, \theta_2^L, \sigma^H)$	State HL $(\theta_1^H, \theta_2^H, \sigma^L)$	State HH $(\theta_1^H, \theta_2^H, \sigma^H)$
Change in final payment	2.78%	5.17%	3.27%	6.89%

$\theta_1^L = .232$, $\theta_2^L = 1.18$, $\sigma^L = .0531$, $\theta_1^H = .421$, $\theta_2^H = 1.53$, $\sigma^H = .109$.

Table 11: Effect of reserve price on final payment in UP

	State LL $(\theta_1^L, \theta_2^L, \sigma^L)$	State LH $(\theta_1^L, \theta_2^L, \sigma^H)$	State HL $(\theta_1^H, \theta_2^H, \sigma^L)$	State HH $(\theta_1^H, \theta_2^H, \sigma^H)$
Change in final payment	-0.17%	-0.06%	-0.13%	-0.06%

$\theta_1^L = .232$, $\theta_2^L = 1.18$, $\sigma^L = .0531$, $\theta_1^H = .421$, $\theta_2^H = 1.53$, $\sigma^H = .109$. Reserve price on non-lumpsum bid is set at θ_2^t.

9 Conclusion

This paper analyzes the performance of UP contracts relative to FP contracts and finds that procurer choice of contract type depends on unobserved project heterogeneity, consistent with the Florida Department of Transportation (FDOT)'s belief that UP contracts should be used for projects with larger project risk. Skewed bidding for UP contracts is economically and statistically significant, suggesting that UP projects may select inefficient contractors that expect extra payment for cost overruns. We build a simple and estimable model of bidding for contracts, which is consistent with the empirical findings. Our empirical specification of the model allows for unobserved project heterogeneity in both expected cost and project risk. We find that UP (FP) contracts are ideal for projects with large (small) project risk, and the estimated model rationalizes FDOT's practice.

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10 Appendix

Table 12: Top 10 Contractors in FP and UP

Top Contractors in FP	# of FP contracts	Top Contractors in UP	# of UP contracts
APAC-Southeast	73	Anderson Columbia Co	103
Anderson Columbia Co	70	Community Asphalt	101
AJAX Paving	47	APAC-Southeast	73
Lane Construction	33	Ranger Construction	72
Better Roads	31	Weekley Asphalt Paving	71
L-J Construction Co	23	Hubbard Construction	51
C.W. Roberts Contracting	21	C.W. Roberts Contracting	47
Ranger Construction	19	General Asphalt Co	38
Hubbard Construction	16	AJAX Paving	34
D.A.B. Constructors	14	P&S Paving	32

10.1 State Dependence in Contract Formats

There is also a large amount of heterogeneity in the use of these two contractual arrangements across the district offices of the FDOT. Figure 9 plots the varying level of intensity in the use of FP relative to UP for the seven district offices across time. As a district office procures multiple projects at a time, the intensity of FP use is measured by the share of all FP projects over the sum of FP and UP projects procured during a year.

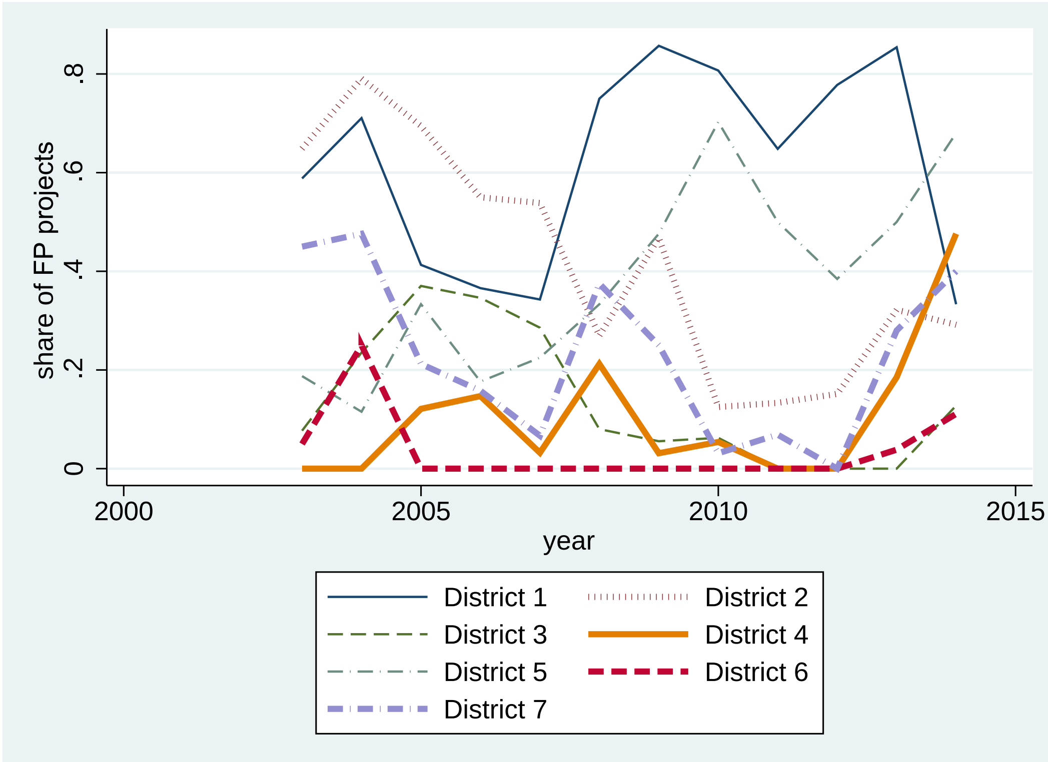

Figure 9: Use of FP over UP at each FDOT's district office

Two observations can be made from Figure 9. First, there is state dependency in the use of FP over UP while exhibiting much variation across time, which could be a product of turnover in project managers. Second, there is a common sharp increase in the use of FP over UP for the year following the financial crisis in 2008. In February 2009, the American Recovery and Reinvestment Act was signed into law. This stimulus package had an emphasis on infrastructure investment, which raised the number of procurements significantly. If the FDOT is capacity constrained, then the FDOT may choose to procure those additional projects via FP. UP could involve a higher transaction costs in order to estimate quantity of each construction item, and keep track of materials used. Indeed, the FDOT engineer mentions that the bulk of the administrative costs associated with UP comes from keeping track of materials used.

10.2 Derivation of (6)

We consider a bidder's inner problem where the bidder maximizes its profit with respect to non-lump-sum bid $b_{2,i}$ conditional on score s_i and non-negativity constraint on $b_{j,i}$ for $j \in \{1,2\}$. That is, we consider:

$$\begin{aligned} \max_{b_{2,i}} \quad & b_{1,i} + b_{2,i}e_{2,i} - (\theta_1 e_{1,i} + \theta_2 e_{2,i}) - \frac{\alpha\sigma}{2}(b_{2,i} - \theta_2)^2 \\ s.t. \quad & b_{1,i} \geq 0,\ b_{2,i} \geq 0 \end{aligned}$$

which can be rewritten as:

$$\max_{b_{2,i}} \quad s_i - b_{2,i} + b_{2,i}e_{2,i} - (\theta_1 e_{1,i} + \theta_2 e_{2,i}) - \frac{\alpha\sigma}{2}(b_{2,i} - \theta_2)^2 + \lambda_1(s_i - b_{2,i}) + \lambda_2 b_{2,i}$$

where λ_j for $j \in \{1,2\}$ is a lagrangian multiplier. First-order condtion with respect to $b_{2,i}$ gives:

$$b_{2,i} = \begin{cases} 0 & \text{if } \theta_2 + \frac{e_{2,i}-1}{\alpha\sigma} < 0 \\ \theta_2 + \frac{e_{2,i}-1}{\alpha\sigma} & \text{if } \theta_2 + \frac{e_{2,i}-1}{\alpha\sigma} \in [0, s_i] \\ s_i & \text{if } \theta_2 + \frac{e_{2,i}-1}{\alpha\sigma} > s_i \end{cases}$$

We now consider an extension of the model in which there is an arbitrary number of M independent non-lump-sum items. Allowing for correlation among M non-lump-sum items is trivial but we abstract from it here. A bidder's inner problem is:

$$\begin{aligned} \max_{\{b_{m,i}\}_{m=2}^{M+1}} \quad & b_{1,i} + \sum_{m=2}^{M+1} b_{m,i}e_{m,i} - \left(\theta_1 e_{1,i} + \sum_{m=2}^{M+1} \theta_m e_{m,i}\right) - \frac{\alpha}{2}\sum_{m=2}^{M+1} \sigma_m (b_{m,i} - \theta_m)^2 \\ s.t. \quad & b_{1,i} \geq 0,\ b_{m,i} \geq 0\ \forall\, m = 2, 3, ..., M+1 \end{aligned}$$

which can be again rewritten as:

$$\begin{aligned} \max_{\{b_{m,i}\}_{m=2}^{M+1}} \quad & s_i + \sum_{m=2}^{M+1} b_{m,i}(e_{m,i} - 1) - \left(\theta_1 e_{1,i} + \sum_{m=2}^{M+1} \theta_m e_{m,i}\right) \\ & - \frac{\alpha}{2}\sum_{m=2}^{M+1} \sigma_m (b_{m,i} - \theta_m)^2 + \lambda_1\left(s_i - \sum_{m=2}^{M+1} b_{m,i}\right) + \sum_{m=2}^{M+1} \lambda_m b_{m,i} \end{aligned}$$

where λ_j for $j = 1, 2, 3, ..., M+1$ is a lagrangian multiplier. First-order condtion with respect to $b_{m,i}$ gives:

$$b_{m,i} = \theta_m + \frac{e_{m,i}-1}{\alpha\sigma_m} \quad \forall m = 2,3,4,...,M+1 \text{ if } \sum_{m=2}^{M=1} \theta_m + \frac{e_{m,i}-1}{\alpha\sigma_m} < s_i \text{ and } \theta_m + \frac{e_{m,i}-1}{\alpha\sigma_m} > 0$$

in case of the inner solution. The conditions for corner solutions are non-trivial but can be obtained.

10.3 Derivation of (22)

A bidder's utility maximization problem in UP contract, who has a pseudo-cost c_u, is given by:

$$\max_{s_u} \quad [1 - G_n(s_{u,i}|X)]^{n-1} u\left(s_{u,i} - c_{u,i}|X\right),$$

where $u(.)$ is CARA utility function.

The first order optimality condition gives:

$$\frac{u(s_{u,i} - c_{u,i}|X)}{u'(s_{u,i} - c_{u,i}|X)} = \frac{1 - G_n(s_{u,i}|X)}{(n-1)g_n(s_{u,i}|X)}.$$

Rewriting the left hand side of the above equation explicitly, we have:

$$\frac{u(s_{u,i} - c_{u,i}|X)}{u'(s_{u,i} - c_{u,i}|X)} = \frac{1}{\alpha(X)}\left(\exp\left\{\alpha(s_{u,i} - c_{u,i})\right\} - 1\right).$$

Rearranging the above first order condition, we have:

$$s_{u,i} - \frac{1}{\alpha(X)} ln\left(1 + \alpha(X)\frac{1 - G_n(s_{u,i}|X)}{(n-1)g_n(s_{u,i}|X)}\right) = c_{u,i}.$$

Since we know that $b_{2,i} = \theta_2(X) + \frac{1}{\alpha(X)\sigma(X)}(e_{2,i} - 1)$ and $c_{u,i} = \theta_1(X)e_{1,i} + \theta_2(X) - \frac{1}{2\alpha(X)\sigma(X)}(e_{2,i} - 1)^2$, we have:

$$s_{u,i} - \theta_2(X) - \frac{1}{\alpha(X)} ln\left(1 + \alpha(X)\frac{1 - G_n(s_{u,i}|X)}{(n-1)g_n(s_{u,i}|X)}\right) + \frac{\alpha(X)\sigma(X)}{2}(b_{2,i} - \theta_2(X))^2 = \theta_1(X)e_{1,i}.$$

Therefore, we have:

$$E\left[s_{u,i} - \theta_2(X) - \frac{1}{\alpha(X)} ln\left(1 + \alpha(X)\frac{1 - G_n(s_{u,i}|X)}{(n-1)g_n(s_{u,i}|X)}\right) + \frac{\alpha(X)\sigma(X)}{2}(b_{2,i} - \theta_2(X))^2 | b_{2,i}, X\right] \quad = \quad \theta_1(X).$$

10.4 Proof of (3)

We show that the unique equilibrium bidding strategies and cost overruns are multiplicatively separable in project characteristics X given the econometric specification in (24). First, consider non-lump-sum bidding strategy $b_{2,i} \equiv b_{2,i}(\theta_2(X), \sigma(X), \alpha(X), e_{2,i})$. We know that:

$$\begin{aligned} b_{2,i}(\theta_2(X), \sigma(X), \alpha(X), e_{2,i}) &= \theta_2(X) + \frac{e_{2,i} - 1}{\alpha(X)\sigma(X)} \\ &= \left(\theta_2 + \frac{e_{2,i} - 1}{\alpha\sigma}\right)\exp\{X\beta\} \\ &= b^0_{2,i}\exp\{X\beta\}, \end{aligned}$$

where the second line follows directly from the normalization assumption (24). Thus, non-lump-sum bidding strategy is multiplicatively separable in X.

Second, we show that scoring strategy is multiplicatively separable in X. To see this, let us first consider the pseudo-cost $c_{u,i} \equiv \theta_1(X)e_{1,i} + \theta_2(X) - \frac{1}{2\alpha(X)\sigma(X)}(e_{2,i} - 1)^2$ and $c^0_{u,i} \equiv c_{u,i}(0)$. We have:

$$\begin{aligned} c_{u,i} &= \left(\theta_1 e_{1,i} + \theta_2 - \frac{(e_{2,i} - 1)^2}{2\alpha\sigma}\right)\exp\{X\beta\} \\ &= c^0_{u,i}\exp\{X\beta\}, \end{aligned}$$

and thus, pseudo-cost is multiplicatively separable in X. Now, conjecture that $s_{u,i} \equiv s_{u,i}(\theta_1(X), \theta_2(X), \sigma(X), \alpha(X), e_{1,i}$ $s^0_{u,i}\exp\{X\beta\}$ constitutes an equilibrium scoring strategy. Consider first order condition with respect to score given by:

$$\begin{aligned}
s_{u,i} - \frac{1}{\alpha(X)} ln \left(1 + \alpha(X) \frac{1 - G_n(s_{u,i}|X)}{(n-1)g_n(s_{u,i}|X)}\right) &= c_{u,i} \\
s^0_{u,i} - \frac{1}{\alpha} ln \left(1 + \alpha \frac{1 - G_n(s^0_{u,i}|X=0)}{(n-1)g_n(s^0_{u,i}|X=0)}\right) \exp\{X\beta\} &= c^0_{u,i} \exp\{X\beta\} \\
s^0_{u,i} - \frac{1}{\alpha} ln \left(1 + \alpha \frac{1 - G_n(s^0_{u,i}|X=0)}{(n-1)g_n(s^0_{u,i}|X=0)}\right) &= c^0_{u,i}
\end{aligned}$$

where the second line follows because G_n is homogeneous of degree 0 while g_n is homogeneous of degree -1. Therefore, $s_{u,i} = s^0_{u,i} \exp\{X\beta\}$ constitutes an equilibrium scoring strategy if $s^0_{u,i}$ is the equilibrium scoring strategy corresponding to pseudo-cost $c^0_{u,i}$. Since we know that the equilibrium is unique, $s_{u,i} = s^0_{u,i} \exp\{X\beta\}$ is the unique equilibrium scoring strategy with $X \neq 0$.

Lastly, it is straightforward to see that $\Delta = \Delta^0 \exp\{X\beta\}$ from the cost overrun equation.

$$\begin{aligned}
\Delta &= b_{2,1}(e_{2,1} - 1 + \epsilon) \\
&= b^0_{2,1}(e_{2,1} - 1 + \epsilon) \exp\{X\beta\} \\
&= \Delta^0 \exp\{X\beta\}
\end{aligned}$$

This completes the proof.